Chapter 14
Starting with Physics: A Problem-Solving Activity for High-School Students Connecting Physics and Mathematics

E. Bagno, H. Berger, E. Magen, C. Polingher, Y. Lehavi, and B. Eylon

14.1 Introduction

The interrelations between physics and mathematics in the learning of high-school physics are manifested in several aspects of physics teaching (Sherin 2001; Bing and Redish 2009; Uhden et al. 2012; Karam 2014; Redish and Kuo 2015). These interrelations, used by teachers, have been conceptualized into four "phys-math patterns," each of which addresses different teaching goals (Pospiech and Oese 2014; Lehavi et al. 2015, 2017; Pospiech and Geyer 2016). The phys-math patterns reflect how teachers "travel" in their teaching between the two domains and within each of them, always starting from the physics domain. One of these patterns, the "application pattern," describes how teachers employ the phys-math interrelations in problem-solving – an endeavor that occupies much of high-school physics teachers' time and attention. Here we focus mainly on this pattern.

E. Bagno (✉) · H. Berger · B. Eylon
The Weizmann Institute of Science, Rehovot, Israel
e-mail: esther.bagno@weizmann.ac.il; Bat-sheva.Eylon@weizmann.ac.il

E. Magen
The Weizmann Institute of Science, Rehovot, Israel

Ostrovsky High School, Raanana, Israel

C. Polingher
The Weizmann Institute of Science, Rehovot, Israel

Hemda Schwartz-Reisman Science Education Center, Tel Aviv, Israel

Y. Lehavi
The Weizmann Institute of Science, Rehovot, Israel

The David Yellin Academic College of Education, Jerusalem, Israel

© Springer Nature Switzerland AG 2019

G. Pospiech et al. (eds.), *Mathematics in Physics Education*,
https://doi.org/10.1007/978-3-030-04627-9_14

Research on the problem-solving habits of high-school physics students shows that students often start solving problems by using mathematical manipulations or by looking for seemingly relevant formulae (Mason and Singh 2010; Kim and Pak 2002; Van Heuvelen 1991; Byun and Lee 2014; Heller and Heller 2010). Research also indicates that a technical use of formulae may decrement the development of students' understanding of physics (Bagno et al. 2008; Karam 2014).

Chi and her collaborators (Chi et al. 1981) report on a fundamental difference between how experts and novices address problem-solving. For example, they found that novices tend to sort problems according to their surface features (e.g., blocks on inclined planes), whereas experts sort them according to the underlying physics principles. Chi et al. claim that "experts in physics problem solving, engage in qualitative analysis of the problem prior to working with the appropriate equations . . . and . . . this method of solution for the experts occurs because the early phase of problem solving (the qualitative analysis) involves the activation and confirmation of an appropriate principle-oriented knowledge structure, a "schema." Apparently, qualitative analysis of a problem by using physics terms and principles is an essential skill that can assist high-school physics students in narrowing the gap between how they approach a problem in physics and how experts do it.

Here we describe the "Starting with Physics" activity, which attempts to activate the "principle-oriented knowledge structures" mentioned above. Students are asked to carry out an activity by focusing on the use of appropriate physics concepts and principles together with their mathematical manifestations (e.g., graphs and their descriptions) and to delay the use of formulas and other technical mathematical manipulations. Our goal is to stress, in the context of problem-solving, the power of a concise set of physics principles for explaining a phenomenon described in a problem, before using mathematical manipulations and techniques.

Another important goal that guided us in the design of this activity is an attempt to build a "learner-centered activity" supporting students' learning. In this regard we used the knowledge integration (KI) perspective on learning, (Linn and Eylon 2006), according to which learners build their knowledge when teachers stimulate four learning processes:

1. Eliciting prior knowledge: learners become aware of their preexisting knowledge
2. Adding new ideas: learners are introduced to ideas that are new to them. These ideas may originate from various sources such as a teacher, a textbook, a peer, or the Internet.
3. Developing criteria to evaluate ideas: questions and tests that the learners use to determine whether they consider the ideas as acceptable. Examples of such criteria are whether the origin of the new ideas is reliable (i.e., based on scientific principles) and whether contradictions exist within the ideas acquired or between them and the ideas that are already known to the learner.
4. Sorting out and reflecting: this is a metacognitive learning process in which learners reflect on and differentiate between their preexisting ideas and the newly acquired ones based on specific criteria.

The four processes do not necessarily appear one after another and not always in the same order. These learning processes formed the basis for designing the procedure through which our activity was carried out. However, many other teaching methods can promote these learning processes (e.g., peer instruction, context-rich problems).

The research literature reports on a large number of empirical studies, investigating the relationships between designs of such teaching methods that attempt to promote KI and learning outcomes (Linn and Eylon 2011).

We carried out a study in the context of implementing the activity in high-school physics classes. The research in this study was aimed at examining students' reasoning throughout the activity and their reflections regarding how the activity contributed to their learning. In addition, we investigated teachers' views regarding the activity and its contribution to physics learning.

The following sections describe the activity, the study, and teachers' views.

14.2 The "Starting with Physics" Activity

One of the main goals of physics instruction is to promote students' "physics understanding," as manifested in their ability to describe a phenomenon qualitatively and explain it by using physics concepts and principles. However, the usual structure of a standard physics problem allows students to have an "escape route" from this important goal. A problem in physics often consists of a paragraph describing a phenomenon, followed by a set of questions. Both experienced physics teachers and physics education researchers agree that students tend not to thoroughly read the introductory paragraph nor try to understand the problem. Instead, they turn to formulas and mathematical manipulations that seem relevant to them, without examining whether they are valid in explaining the phenomenon under consideration.

The "Starting with Physics" activity was designed as follows:

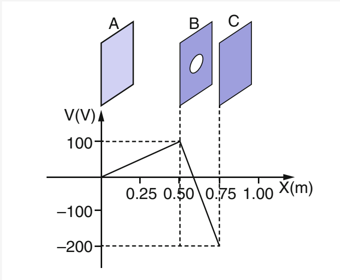
(a) Students receive only the first part of the problem consisting of a textual description of the phenomenon and the relevant mathematical information, without any subsequent questions. Thus, they are prompted to address the problem conceptually first with nothing to calculate.
(b) At the beginning of the activity, students are asked to divide the phenomenon into events and to describe and explain each event by using physical concepts and principles without using equations.
(c) Then, students are asked to list the physical concepts and principles on which they based each event's description and explanation.

Figure 14.1 shows an example of the "Starting with Physics" activity in the context of electrostatics. Based on the KI perspective, we implemented the activity in a four-phase learning cycle. The cycle consists of "individual work" in which each student fills in the table in Fig. 14.1. In order to save class time, this phase may be carried out as homework. This is followed by a "group work" phase that usually takes place in class. The students work in small groups on the same activity, evaluate their individual work, add new ideas, and reach a consensus (or

The motion of a charged particle between charged plates

1. Individual work

Many electric systems (for example, a particle acceleration system) contain charged plates similar to the system shown below.



The system contains three charged plates A, B and C parallel to each other. The distance between plates A and B is different from the distance between plates B and C. There is a small hole in the center of plate B (see the illustration, but assume that the plates are much larger than the distances between them).

The attached graph describes the electric potential between the plates.

Consider the following phenomenon:
A negatively charged particle is released from rest at the center of plate A and it starts moving.
Fill in the table below according to the following:

a. If possible, divide the phenomenon into events that differ from each other regarding the nature of the moving particles, the acting forces, and more. For each event indicate its starting and ending points. Use as much as possible diagrams, graphs, or illustrations. If needed, add rows to the table.
b. The "physics" of each event must include a description of the event and its explanation using physical concepts and principles (do not use equations).
c. List, in a separate column, the physical concepts and principles on which you based the event's description and explanation.

Events			"Starting with Physics"		
1	**2**	**3**	**4**	**5**	**6**
	Start	End	Describe and explain the event by using physical concepts and principles	List the physical concepts and principles	Diagrams
Event I					
Event II					

2. Group work
Discuss your individual work with your friends. If necessary, modify your table.
3. Whole-class discussion
Group work is discussed under the teacher's guidance.
4. Individual reflection
If you were helped by the activity, describe how.

Fig. 14.1 The "Starting with Physics" activity have a disagreement). The next phase is a "whole-class discussion" in which a representative of each group presents to the plenum the group's consensus as well as any disagreements; all the issues raised in the group work are discussed, under the teacher's guidance, and the class formulates a summary. The activity culminates in "individual reflection" in which each student individually accounts for what he or she has learned during the activity.

As can be seen, the design of this activity balances "problematizing" and "structuring" two complementary mechanisms of scaffolding problem-solving:

(1) *Structuring* a task refers to reducing its complexity and limiting the choices of the problem-solver. (2) *Problematizing* directs one's attention to aspects that one might otherwise overlook. Instruction should be balanced between structuring and problematizing so that tasks will be manageable to learners yet challenging and engaging (Reiser 2004; Yerushalmi and Eylon 2016). In our study, this activity (see Fig. 14.1) was carried out by two experienced 12th grade teachers with 31 students.

14.3 Research on Students' Use of Physical Concepts and Principles in Performing the Activity

14.3.1 Research Questions

We studied students' answers in the table, focusing on the following questions:

1. How did students in this activity use the *physical concepts and principles* in describing and explaining the events in a phenomenon? (From column 4 in the table)
2. How did students list the *physical concepts and principles* on which they based each event's description and explanation? (From column 5 in the table)

14.3.2 Methodology

The phenomenon in the activity exemplifies two apparently different events that share the same underlying physical principles. The two events are not identical, since in the first event the electric charge moves from a low potential to a high potential, whereas in the second event it moves from a high potential to a low potential. This information is conveyed by a graph (see Fig. 14.1) and leads to differences between the description and explanation of the two events in the direction of the electric field, the electric force, the acceleration, and the velocity of the charge (column 4 in Fig. 14.1).

However, the list of the physical concepts and principles should be the same (column 5 in Fig. 14.1).

Our considerations in the content analysis of the students' answers in the table were based on the answers written by a top-level student (see Fig. 14.2). This student focused mainly on the following two aspects:

1. The connections between physical concepts and principles
2. The connections between mathematics and physics

We will indicate below how the above two aspects were manifested in the paragraph written by this top-level student.

(a) *Graph V(x) has a constant slope, and therefore, the electric field is uniform.*
(b) *The force exerted on the particle is constant because the electric field is constant.*
(c) *The electric field is directed to the left due to the higher electric potential at plate B.*
(d) *Since the particle is negatively charged, the electric force acting on it is directed toward plate B.*
(e) *Due to this force, the particle moves with constant acceleration, and its speed increases.*

Manifestations of the Two Aspects in the Paragraph Statement *(a)* in this paragraph – *Graph V(x) has a constant slope* – is a mathematical statement leading to a physical conclusion – *…the electric field is uniform*. This conclusion is followed in statement *(b)* by a sequence of physical concepts, starting with the relationship between the field and the force and then the electric charge – *the force exerted on the particle is constant because the electric field is constant.*

In statement *(c)* the direction of the electric field is determined by referring back to the graph (a mathematical representation) – *the electric field is directed to the left due to the higher electric potential at plate B*. Next, in statement *(d)* an important relationship exists between three central physical concepts (field, force, and charge) – *since the particle is negatively charged, the electric force acting on it is directed toward plate B*. Finally, in statement *(e)*, the student relates to dynamics and kinematic concepts and concepts within kinematics – *due to this force, the particle moves with constant acceleration, and its speed increases*.

14.3.3 Findings on Research Question 1

How did students in this activity use *physical concepts and principles* in describing and explaining events in a phenomenon?

The findings are based on all students' answers in column 4 of the table in Fig. 14.1.

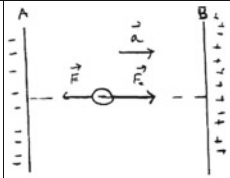
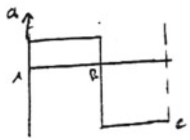
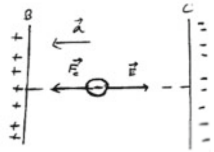
Events			"Starting with Physics"		
1	**2**	**3**	**4**	**5**	**6**
	Start	End	Describe and explain the event by using physical concepts and principles	List the physical concepts and principles	Diagrams
Event I	Particle at plate A	Particle at plate B	*a) Graph V(x) has a constant slope and therefore the electric field is uniform.* *b) The force exerted on the particle is constant because the electric field is constant.* *c) The electric field is directed to the left due to the higher electric potential at plate B.* *d) Since the particle is negatively charged, the electric force acting on it is directed towards plate B.* *e) Due to this force, the particle moves with constant acceleration and its speed increases.*	*a) Existence of the field, the gradient of the potential* *b) The magnitude of the force is proportional to the strength of the field.* *c) The force on a negative charge points in an opposite direction to that of the field.* *d) Newton's 2nd law* *e) The acceleration is in the direction of the velocity.*	
Event II	Particle at plate B	Particle at plate C (or before C)	*a) All the events have the same arguments; however, the directions are opposite.* *b) The slope of V(x) is negative, but the field is positive.* *c) The force is to the left, but the field is to the right.*	*The slope (the gradient)* ↓ *Field* ↓ *Force* ↓ *Acceleration* ↓  • *Newton's 2nd law* • *The acceleration is in the opposite direction of the velocity.*	

Fig. 14.2 The filled-in table of a top-level student

1. **Most of the students' statements dealt with the two aspects mentioned above: the connections between physical concepts and the connections between mathematics and physics.**

Students took advantage of the mathematics in the connections between physics and mathematics to enhance their understanding of physics; they formed connections between concepts or ideas within physics (either **within a domain** such as electrostatics or **between domains** such as kinematics and dynamics). In this respect, they employed what we termed "a phys-math exploration pattern," characterized by beginning with a certain physical phenomenon or system; then a mathematical representation is studied, and finally, the ramifications of the mathematical analysis for the case in hand are discussed with new physical insights (Lehavi et al. 2015).

2. **Students realized physics-related similarities between seemingly different events**.

This was reflected by the fact that most students used the same concepts and principles in describing the two events. Moreover, the findings regarding their individual reflections, described below, indicate that they were cognizant of this.

3. **There was a progression from the description of the first event to that of the second one.**

About 70% of the students described and explained the second event, in a more general manner than the first event. This finding was more frequent among top-level students: *The events are similar; however, the directions of the forces are opposite.*

14.3.4 Findings on Research Question 2

How did students list *physical concepts and principles* on which they based each event's description and explanation?

The findings are based on all students' answers in column 5 of the table in Fig. 14.1

1. **Most of the students used the same principles for the two apparently different events.**

In most of the students' tables, the list of the physical concepts and principles was the same for the two events. Some of the students did not even bother to write the same concepts and principles again for the second event. Some left the relevant box in the table empty and noted that it should be the same. Further support for this finding comes from the "whole-class discussion" in one of the classes. When the classroom summary was formulated under the teacher's guidance, the students suggested leaving the box of the concepts and principles for the second event empty, since it is identical to that of the first event.

2. **Some students summarized, in the second event, the whole sequence of reasoning by a concept map representing the connections between underlying physical concepts and principles** (typical of top-level students).

This is exemplified in column 5 of Fig. 14.2. In addition, this particular student also described the acceleration by representing it in a graph.

14.4 Research on Students' Views on the "Starting with Physics" Activity

14.4.1 Research Questions

1. How did students, in their individual reflections, refer to the *goals of the activity and its contribution* to their learning?
2. What congruence can be found between the students' individual reflections and their use of concepts and principles in performing the activity?

14.4.2 Methodology

The data for this analysis originates from students' individual reflections on the activity.

Whereas the first part of the activity involved team learning and **whole-class discussions, the individual reflection required students to report on what** they had learned from the activity. In order to enable the students to come up with a variety of ideas, the individual reflection was phrased in an open manner: "If you were helped by the activity, describe how."

We started the analysis by dividing students' reflections into statements. All together, we identified 50 statements. In the analysis, we looked for congruency between students' reflective statements and their answers in the tables. Accordingly, the analysis was guided in a top-down manner and referred to the following:

1. Reflections about connections and their congruence with the ones students wrote in column 4 of the tables
2. Reflections about physical relationships and their congruence with the ones students wrote in column 5 of the tables
3. Other ideas that students brought up

Note that some of the statements provided information on more than one of the three foci of reflection.

14.4.3 Findings on Research Questions 1 and 2

1. **Reflections about connections**: Of the 50 reflective statements, about 50% dealt with connections. The different types of connections that were found in column 4 of the tables were also found in students' reflections.

   The following are some examples:

- **Connections between physics and mathematics**: About 25% of the statements dealt with the ways by which the students understood the physical meaning of the mathematical representations.
  - *I understood that the slope of the graph can also indicate whether the field is constant.*
  - *I understood that the gradient is the derivative of the potential.*
  - *The activity helped me understand the meaning of the formula:* $E = -\Delta V/\Delta x$.
  - *The activity helped me mainly in better understanding graphs and in relating to and connecting between a graph and an event.*
- **Connections between concepts or ideas within a physics domain and/or between physics domains**: About 20% of the statements dealt with different aspects of physical connections.
  - *It helped me in better understanding the relationship between distance, potential, and the field.*
  - *I understood that the field is the slope of the potential.*
  - *It helped me understand how the potential affects the forces acting on a charged particle.*
  - *It clarified for me that a relationship exists between the potential, the field, the force, the acceleration, and the velocity.*

2. **Reflections about physical principles**: About 20% of the statements dealt with physical principles resembling those we found in column 5 of the table.
   - *It helped me to better understand the motion of a charged particle in an electric field.*
   - *It clarified for me that a relationship exists between the potential, the field, the force, the acceleration, and the velocity.*
3. **Reflections dealing with metacognitive issues:** About 50% of the statements dealt with different types of metacognitive issues:
   - Understanding the goals of the activity and how they are promoted by its structure:
     - *I was helped by the activity. I now better understood the material that we learned and how one can describe and analyze better an exercise before starting to solve it.*

- *Yes, I better understood the theory as well as interpreting and understanding a graph.*
- *Yes, the activity enhanced my understanding of how many events and parts are in a problem and what happens to the particle in each part.*

- Promotion of various learning capabilities:
  - *It underscored the rule that one should always check the given information in order to verify what really occurred.*
  - *How to analyze a situation according to a graph of V* vs. *x and what consequences can be derived from this graph.*
- Realizing the relationships between the studied topics:
  - *Yes, the activity summarized for me the materials studied and connected all the relevant topics.*

14.5 Summary of Research on Students

In studying students' use of concepts and principles in the activity, we found that the activity achieved its goal: students indeed engaged in physics during the activity rather than "jumping" to formulae and technical mathematical manipulations. Most of their statements actually dealt with various types of connections: the connections between physical concepts and the connections between mathematics and physics. We also found that students managed to describe the two apparently different events similarly and some of them even provided a more comprehensive and general description in the transition from the first event to the second one.

In studying students' views concerning the activity, we found that students, in their individual reflections, mentioned explicitly the formation of connections between physics and mathematics, between concepts or ideas within a physics domain, and/or between physics domains. They also referred to the important role of physics principles in describing events. Interestingly, we also found in students' reflections different types of metacognitive issues such as how the activity contributed to their learning capabilities.

14.6 Teachers' Reports on Using the Activity in Their Practice

Two important questions are to what extent and how is this activity useful for teachers in their practice and what did teachers think about its contribution to learning physics. We had an opportunity to examine these questions in the context of professional development programs for teachers in which they were introduced to several “learner-centered” activities. They implemented the activities in their classes and brought materials from their classes (such as students’ answers in responding to questions on the activities) for collaborative reflections with their peers. This “evidence-based” approach is a powerful method for teachers’ learning and impacts teachers’ practice (Berger et al. 2008; Harrison et al. 2008; Eylon et al. 2008).

We audiotaped the discussions and also interviewed several teachers. Most teachers rated the “Starting with Physics” activity as the highest one. The teachers referred to both the physics learning aspect and to various phases of the activity.

A common finding is that the teachers found that the activity contributes to their practice and to students’ learning of physics. They also reported the importance of carrying out the different phases. In addition, their reports indicate that this activity can be used in various formats and in different physics domains (e.g., mechanics, electrostatics), and therefore, it provides ample opportunities for teachers to use it in their practice on a regular basis. The following are some examples from reports of three teachers who participated in these professional development programs: Ella, Ziva, and Tibi (all pseudonyms).

Ella became convinced that the activity has a real impact on her students’ ability to relate physical principles to the events in a problem. She also pointed out that each of the phases has its own importance. In her words: “In the individual work, each student is forced to expose his or her own knowledge, whereas in the group work, they learn from each other; in the class discussion, the teacher helps them to correct mistakes that are found during the activity.” Ella also reported that, “Decomposing a complex situation into several events and dealing with each of them separately simplifies the activity for most of the students.”

Ziva was very enthusiastic about this activity as well. She uses it in her classes on a regular basis. In order to save class time, she usually asks her students to perform the individual phase at home. In an interview held with Ziva, she said: “This activity, which I am so attached to, no doubt caused a new language to develop in my classes. This language includes, for example, the term ‘event’. This word is now familiar to my students in the context of problem solving. I find myself solving with my students complex problems by decomposing them into their events. I even started to include tasks such as ‘decompose the problem into its events and give the event an appropriate title’ in my exams. Ninety percent of the exams are better organized now. I think that this organization has to do with my explicit request to relate to each event separately.” Ziva claimed that the activity enables her to emphasize the common underlying physical principles of apparently different problems: “Usually I spend a whole lesson solving each of the very similar problems I gave for homework. My students insist on it. With this activity, they leave me alone, since they realize that you can solve many problems by using the same ideas; and it serves as a supporting framework for problem solving in physics.”

Another teacher Tibi reported that in analyzing his students’ worksheets he found that the group discussions had greatly contributed to students’ understanding. He also said that in the “whole-class discussion” phase, his students easily realized the similarity between this electrostatics problem and other problems from mechanics, having the same underlying principles. He suggested that it is necessary to carry it out with students several times in order to bring about its habitual use.

Indeed, in an interview held with Tibi, several years after the professional development program, he said that he uses the activity regularly in his classes in the following format: he invites students to write on the blackboard their descriptions of the events, and he encourages others to justify the descriptions. Tibi also said that he encourages his students to reflect on the activity and to express explicitly what ideas they have learned during the activity and what still remains unclear. In his words: "Describing the underlying physics of the problem, before they start with the formulas and going back to the physics after they have finished with the formulas, is so important. This resembles debugging."

We also found other teachers' views that were similar to those illustrated here. Teachers used the activity in a wide range of formats that they found were feasible and useful for their students.

14.7 Discussion and Implications

In this paper we described an activity that aims to promote students' ability to describe and explain a phenomenon qualitatively by using physical concepts and principles rather than engaging in technical mathematical manipulations. The "Starting with Physics" activity was very effective in activating "principle-oriented knowledge structures" (Chi et al. 1981). Instead of technically misusing the phys-math relationships, students focused on physics concepts and principles and their relations to mathematical aspects.

Research on implementing the activity in physics high-school classes indicated that in carrying out the activity, most of the students managed to describe the two apparently different events similarly. They referred to various types of connections between physics concepts and principles and connections between physics and mathematics. Furthermore, some of the students' responses may indicate that they use mathematical ideas (e.g., the slope of the electric potential as an indicator of the electric field) rather than technics when analyzing a physical event. Such a perspective (an exploration phys-math pattern rather than an application one) was found to characterize more expert teachers (Lehavi et al. 2015). This positive finding may encourage further research on examining in detail the above described activity and models for its implementation in frameworks such as professional learning communities of teachers.

In their reflections the students explicitly mentioned different types of connections as well as the role of physical principles in describing events. They also referred to metacognitive issues. In particular, students mentioned the rationale underlying the activity's design and its important contribution to their learning. In a more detailed analysis of students' actual work in the table and their reflections (not reported here), we found congruency between their answers and their views. Some students even suggested that activities of this kind should be encouraged by giving them extra credit.

This activity was highly appreciated by physics teachers. They claimed that it emphasizes the common underlying physical principles of apparently different problems and supports problem-solving in physics. However, it is necessary to carry it out with students several times in order to bring about its habitual use. Since this activity is generic, it is suitable for many standard A level physics problems. We already have a large pool of problems in the format of this activity filled out by teachers and tried out by many students.

Several directions can be explored in future research: What can be learned from the data that students bring from the individual work to the peer discussion and from the discourse that follows? How do students evolve in their ability to fill in the tables in the activity correctly and exhaustively (i.e., use properly all the relevant concepts and their interrelations)? What impact may such an activity have on low grades students?

Such studies can enable one to better understand the underlying mechanisms leading to student and teacher learning in the context of this activity.

References

Bagno, E., Berger, H., & Eylon, B. S. (2008). Meeting the challenge of students' understanding of formulae in high-school physics: A learning tool. *Physics Education, 43*(1), 75–82.

Berger, H., Eylon, B., & Bagno, E. (2008). Professional development of physics teachers in an evidence-based blended learning program. *Journal of Science Education and Technology, 17*(4), 399–409.

Bing, T. J., & Redish, E. F. (2009). Analyzing problem solving using math in physics: Epistemological framing via warrants. *Physical Review Special Topics – Physics Education Research, 5*, 020108.

Byun, T., & Lee, G. (2014). Why students still can't solve physics problems after solving over 2000 problems. *American Journal of Physics, 82*, 906.

Chi, M. T. H., Feltovich, P. J., & Glaser, R. (1981). Categorization and representation of physics problems by experts and novices. *Cognitive Science, 5*(2), 121–152.

Eylon, B., Berger, H., & Bagno, E. (2008). An evidence-based continuous professional development program on knowledge integration in physics: A study of teachers' collective discourse. *International Journal of Science Education, 30*, 619–641.

Harrison, C., Hofstein, A., Eylon, B., & Simon, S. (2008). Evidence-based professional development of teachers in two countries. *International Journal of Research in Science Education, 30*, 577–591.

Heller, K., & Heller, P. (2010). *Cooperative problem solving in physics – A User's manual: Why? What? How?* Alexandria: The National Science Foundation.

Karam, R. (2014). Framing the structural role of mathematics in physics lectures: A case study on electromagnetism. *Physical Review Special Topics – Physics Education Research, 10*, 010119.

Kim, E., & Pak, S.-J. (2002). Students do not overcome conceptual difficulties after solving 1000 traditional problems. *American Journal of Physics, 70*, 759.

Lehavi, Y., Bagno, E., Eylon, B. S., Mualem, R., Pospiech, G., & Böhm, U. (2015). Towards a PCK of physics and mathematics interplay. In The *GIREP MPTL 2014 Conference Proceedings*.

Lehavi, Y., Bagno, E., Eylon, B. S., Mualem, R., Pospiech, G., Böhm, U., Krey, O., & Karam, R. (2017). Classroom evidence of teachers' PCK of the interplay of physics and mathematics. In T. Greczyło & E. Dębowska (Eds.), *Key competences in physics teaching and learning* (Springer proceedings in physics) (Vol. 190, pp. 95–104). Cham: Springer.

Linn, M. C., & Eylon, B. S. (2006). Science education: Integrating views of learning and instruction. In P. A. Alexander & P. H. Winne (Eds.), *Handbook of Educational Psychology* (2nd ed., pp. 511–544). Mahwah: Lawrence Erlbaum Associates.

Linn, M. C., & Eylon, B. S. (2011). *Science learning and instruction: Taking advantage of technology to promote knowledge integration*. New York: Routledge.

Mason, A., & Singh, C. (2010). Helping students learn effective problem solving strategies by reflecting with peers. *American Journal of Physics, 78*, 748.

Pospiech, G., & Geyer, M.-A. (2016). Physical – Mathematical modelling in physics teaching. In *Electronic proceedings – Key competences in physics teaching and learning* (pp. 38–44). Wroclaw: Institute of Experimental Physics, University of Wrocław. Abgerufen von. http://girep2015.ifd.uni.wroc.pl/.

Pospiech, G., & Oese, E. (2014). Use of mathematical elements in physics – Grade 8. In *Active learning – In a changing world of new technologies* (pp. S. 199–S. 206). Prag: Charles University in Prague, MATFYZPRESS Publisher.

Redish, E. F., & Kuo, E. (2015). Language of physics, language of math: Disciplinary culture and dynamic epistemology. *Science & Education, 24*, 561. https://doi.org/10.1007/s11191-015-9749-7.

Reiser, B. J. (2004). Scaffolding complex learning: The mechanisms of structuring and problematizing student work. *Journal of the Learning Science, 13*, 273–304. https://doi.org/10.1207/s15327809jls1303_2.

Sherin, B. (2001). How students understand physics equations. *Cognition & Instruction, 19*, 479.

Uhden, O., Karam, R., Pospiech, G., & Pietrocola, M. (2012). Modelling mathematical reasoning in physics education. *Science & Education, 20*(4), 485. https://doi.org/10.1007/s11191-011-9396-6.

Van Heuvelen, A. (1991). Learning to think like a physicist: A review of research-based instructional strategies. *American Journal of Physics, 59*(10), 891–897.

Yerushalmi, E., & Eylon, B. (2016). Problem solving in science learning. In R. Gunstone (Ed.), *Encyclopedia of science education*. Heidelberg: Springer.