Chapter 10
Comparing Student Understanding of Graphs in Physics and Mathematics

Maja Planinic, Ana Susac, Lana Ivanjek, and Željka Milin Šipuš

10.1 Introduction and Background

Research suggests that many students at high school or introductory university level lack the ability to understand and interpret graphs. This has been documented in several physics education studies (e.g., McDermott et al. 1987; Brasell and Rowe 1993; Beichner 1994; Forster 2004; Araujo et al. 2008; Nguyen and Rebello 2011; Christensen and Thompson 2012), as well as mathematics education studies (Dreyfus and Eisenberg 1990; Leinhardt et al. 1990; Swatton and Taylor 1994; Graham and Sharp 1999; Kerslake 1981; Hadjidemetriou and Williams 2002; Habre and Abboud 2006). Student difficulties with calculating and interpreting slope of a graph and area under a graph were common.

The concept of slope (gradient) is very important for physics since many physical quantities are defined as gradients (e.g., velocity, acceleration) and represented with line graphs. The concept of slope is also important for mathematics as a necessary

M. Planinic (✉)
Department of Physics, Faculty of Science, University of Zagreb, Zagreb, Croatia
e-mail: maja@phy.hr

A. Susac
Department of Physics, Faculty of Science, University of Zagreb, Zagreb, Croatia

Department of Applied Physics, Faculty of Electrical Engineering and Computing, University of Zagreb
e-mail: ana.susac@fer.hr

L. Ivanjek
Austrian Competence Center Physics, University of Vienna, Vienna, Austria
e-mail: lana.ivanjek@univie.ac.at

Ž. Milin Šipuš
Department of Mathematics, Faculty of Science, University of Zagreb, Zagreb, Croatia
e-mail: milin@math.hr

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prerequisite for the development of the concept of derivation. Students study line graph slope in both mathematics and physics, but because of the differences in contexts, they may not necessarily realize that they are studying the same concept.

Student difficulties concerning the interpretation of the area under a graph may be even stronger than those concerning graph slope, since interpretation of slope is usually more emphasized by school mathematics and physics teaching than the interpretation of area under a graph. Yet, the area interpretation, with the idea of accumulation of infinitesimal quantities, underlies the concept of definite integral, important for both mathematics and physics teaching.

Student difficulties with graphs were identified through both physics and mathematics education research. Most of the research on student understanding of graphs in physics was done in the context of kinematics, because of the very broad use of graphical representations in kinematics. Students were found to have difficulties with linking the graph and the verbal descriptions of a given event and with understanding graphs as symbolic representations of relationships among variables (Brasell and Rowe 1993; Beichner 1994). They often have trouble discriminating the slope and height of a graph and interpret changes in height as changes in slope (McDermott et al. 1987; Beichner 1994). Many students are unable to choose which feature of the graph represents the information that is needed to answer the question (e.g., they calculate slope when they should have been calculating the area) (McDermott et al. 1987; Beichner 1994). Very few students seem to be able to interpret the area under an a vs. t graph as a change in velocity, whereas they have far less problems interpreting the area under a v vs. t graph as a distance travelled (Beichner 1994). Some of the research in mathematics education was also based on kinematics motion graphs and had similar general findings (Graham and Sharp 1999; Kerslake 1981), while studies in purely mathematical context have in addition shown that student understanding of mathematical concepts (such as functions) tends to be typically algebraic and not visual. Visual information, including graphs, seems to be more difficult for students to learn and is considered by them to be less mathematical (Dreyfus and Eisenberg 1990; Habre and Abboud 2006).

Student difficulties with graphs are sometimes classified as interval-point confusions, slope-height confusions, and iconic confusions (Leinhardt et al. 1990). The iconic confusion is usually characteristic of younger students, although traces of it can also be found in older populations, sometimes even university students (McDermott et al. 1987; Beichner 1994). It consists in students' incorrect interpretation of the graph as an actual picture of the motion. Students who show this difficulty will tend to interpret, for example, a curved v vs. t graph as representing the motion along a curved trajectory. Such students do not yet see the graph as a symbolic representation of an abstract relationship between the variables on its axis but as a concrete picture of body's motion. It is therefore difficult for them to see why the graph should change if the variables on the axes change, and they will generally expect the graph to remain the same.

The slope-height confusion happens when students mistake the height of the graph for its slope (McDermott et al. 1987; Beichner 1994; Leinhardt et al. 1990). For example, when asked to reason about the slope of a graph, students sometimes just read off the y coordinate (the height of the graph at the point of interest). If they observe, for example, the constant diminishing of the y coordinate of the graph, they usually conclude that the slope of the graph shows the same behavior (e.g., the slope of the straight line constantly diminishes, because the y coordinate constantly diminishes).

The interval-point confusion refers to the cases where students focus on a single point of the graph when they should be using an interval. This difficulty will be displayed, for example, when students attempt to determine the slope of a graph from one point only, instead of choosing two points and calculating $\Delta y/\Delta x$. Slope-height and interval-point confusions are quite common among students at high school and university level (McDermott et al. 1987; Beichner 1994; Forster 2004; Leinhardt et al. 1990; Hadjidemetriou and Williams 2002; Wemyss and van Kampen 2013). Overall, the findings of both physics and mathematics education research are rather similar and point to the presence of similar student difficulties in both domains.

The important issue of transfer of knowledge between mathematics and physics (usually expected to occur from mathematics to physics) was also tackled in several studies on graphs (Christensen and Thompson 2012; Wemyss and van Kampen 2013; Woolnough 2000), with mostly negative results. It was suggested in one of the studies that most secondary students, even those who do well in mathematics and physics, do not make substantial links between the two domains and that some students may even think that it is not appropriate to transfer concepts from mathematics to physics (Woolnough 2000). For transfer to occur, it is necessary that students possess the required mathematical knowledge, but this is not always the case, especially when advanced concepts such as derivative or integral are concerned (Nguyen and Rebello 2011; Christensen and Thompson 2012). The problem of transfer of knowledge between mathematics and physics was addressed in cognitive psychology, unrelated to graphs. One study that investigated interdomain transfer between isomorphic topics in algebra and physics (kinematics) found very high transfer from algebra to physics, but almost no transfer from physics to algebra, and suggested that "transfer from physics to other domains is blocked by the embedding of physics equations within a specific content domain" (Bassok and Holyoak 1989). The problem of domain specificity of knowledge is not limited to physics; it is also present in mathematics. Michelsen (2005) suggests that it is not just the mathematical formalism that presents a barrier in learning physics but that the problem lies in the missing link between mathematics and physics. He suggests that the mathematical domain should be expanded by using examples from physics and from everyday life contexts in mathematics teaching, in order to solve the problem of domain specificity. In such an expanded domain, modeling of real-life situations could be a way of bridging the gap between mathematics and physics. We will take a closer look at the key issue of transfer of learning from theoretical viewpoint in the following chapter.

10.2 Theoretical Framework

Transfer of learning is usually defined as the ability to extend what has been learned in one context to new contexts (Bransford et al. 1999) and is sometimes regarded as one of the ultimate goals of education. Hammer et al. (2005) suggest that it would generally be more appropriate to speak of activation of cognitive resources than of transfer, since knowledge and reasoning abilities are comprised of many resources that may, or may not, be activated in a particular context. They oppose the view of knowledge and abilities as objects which are acquired, manipulated, and transferred as intact units, with the exception of locally coherent sets of resources which activate together and possess internal structural stability. Such cognitive units, whose mechanism of stability is structural rather than contextual, can be viewed as transferable (Hammer et al. 2005). In our opinion, students' concepts of the graph slope and of the area under a graph can be examples of such transferable units in cases when they are well formed and stable.

Whether or not transfer will happen depends not only on the presence or absence of relevant resources but also on students' framing of the situation (Hammer et al. 2005). Framing means that students have to interpret what is going on in a certain situation or in a certain problem and decide accordingly which resources to use or which epistemic game to play (Tuminaro and Redish 2007). In physics education we usually expect students to transfer their mathematical knowledge from mathematics to physics. There are several reasons why the expected transfer could fail: either the required resource does not exist, or the resource exists, but is not activated due to the wrong framing of the problem, or the resource is activated, but its mapping to the problem is not appropriate (Tuminaro 2004). Research suggests that transfer is more likely to happen when students have seen the given idea in at least two separate contexts or when they receive metacognitive scaffolding (Bransford et al. 1999).

Many studies that have looked for transfer of knowledge have usually come up with mostly negative results, which may be due, among other things, also to the design of those studies (Bransford and Schwartz 1999). Bransford and Schwartz (1999) have suggested to shift the view on transfer from the direct application perspective (successful application of knowledge acquired in one context to similar problems in different contexts) to a more dynamical view of preparation for future learning (PFL). The PFL perspective can be demonstrated through questions about and approaches to the new problem, which were shaped and influenced by the previous learning, even if students are not able to completely solve the new problem. The PFL perspective is very important for learning, because it reveals more about students' useful learning trajectories than the direct application perspective. The focus is not only on what students can or cannot directly transfer and solve but whether students are able to learn while they transfer. In this way transfer can be considered a dynamical way of reconstructing knowledge (Cui 2006) rather than just an application of previously acquired knowledge in a different situation. This dynamical view of transfer is in agreement with knowledge-as-elements perspective, because it assumes activation of different knowledge elements in a new context and dynamical creation of the response on the spot.

Theories of transfer of knowledge are based upon the idea that knowledge can be transferred from one situation to another and linked with a new situation (Potgieter et al. 2008). Some researchers disagree and argue that learners' mental processes are structured by the context and the implemented activities and tools (Lave 1988). Teachers often expect students to rise above the context, but that is not easy for students. Recognizing mathematics in a different context requires good understanding of the context (which is often missing), along with mathematical knowledge (Potgieter et al. 2008). To investigate transfer of knowledge in more detail, some comparative studies in mathematics and physics were conducted and produced interesting results.

10.3 Results of Comparative Studies on Graphs in Mathematics and Physics

Few studies attempted to compare student reasoning difficulties about graphs in different contexts and domains (Wemyss and van Kampen 2013; Woolnough 2000). Such comparison, on the other hand, can provide interesting and important insights in student knowledge and learning and the issue of possible transfer between domains. An example is the study of Wemyss and van Kampen (2013), in which first-year university students solved three different context problems including line graphs, found that the number of students' correct answers to a problem involving water level vs. time graph, which students had not encountered in the formal educational setting before, was much higher than the number of correct answers to the supposedly more familiar problem of determining the speed of object from a distance-time graph. The reason for students' poorer performance on physics problems was attributed to students' reliance on learned procedures in physics (e.g., use of formulas). This study also found evidence that students' mathematical knowledge of slope does not guarantee their success on problems involving slope in kinematics.

In our first study on graphs (Planinic et al. 2012), we compared second-year high school students' ($N = 114$) understanding of the line graph slope in the domains of physics and mathematics. Student answers to two pairs of parallel (isomorphic) questions regarding line graph slope from mathematics and physics (kinematics) were analyzed and compared. Also, a sample ($N = 90$) of Croatian physics teachers were asked to rank the isomorphic questions according to their expected difficulty for students. Physics teachers largely thought that the physics questions would be easier for students because they were regarded as less abstract than the mathematics questions. Many also expressed the belief that the lack of mathematical knowledge would present the main problem for students when solving physics questions. It was found however that, contrary to the prevalent belief of physics teachers, students did better on mathematics than on physics questions. The main source of student difficulties with the concept of line graph slope in physics seemed not to be their lack of mathematical knowledge but rather their lack of ability to interpret the meaning of the line graph slope in physics context. Many students successfully solved the mathematical questions but were unable to solve parallel physics questions or used different strategies for solving analogous mathematics and physics problems. It was observed that the transfer of knowledge from mathematics to physics did not always occur, even though many students possessed the needed mathematical knowledge. (Interestingly, beside the expected transfer from mathematics to physics, which was relatively weak, some occasional cases of transfer from physics to mathematics were also observed.) Also, the same student difficulty known as slope-height confusion was detected in both domains, but it occurred far more frequently in physics than in mathematics (about twice as often).

After this study it was natural to pose the question about the reason for the observed higher difficulty of physics questions relative to parallel mathematics questions: Is the higher difficulty of physics questions the consequence of students' lack of relevant physics conceptual knowledge, or would the same effect be observed to the same extent also in parallel questions situated in different contexts, which did not require additional content knowledge? We attempted to investigate this issue by using sets of three parallel (isomorphic) questions and to analyze and compare item difficulties as well as student strategies in different domains. The three domains were mathematics without context, physics (kinematics), and mathematics in contexts other than physics, which did not require additional conceptual knowledge. Eight such sets of parallel (isomorphic) mathematics, physics, and other context questions about graphs were developed by the authors and administered to 385 first-year students at Faculty of Science, University of Zagreb in Zagreb, Croatia, and later also to 417 first-year students at University of Vienna. Students were either prospective physics or mathematics teachers or prospective physicists or mathematicians. Students were tested at the beginning of the first semester, before any formal instruction on graphs, so their knowledge on graphs came only from high school mathematics and physics instruction. Five sets of questions referred to the concept of graph slope and three to the concept of area under a graph. Four sets were in a multiple choice format, and four sets were open-ended (the whole test can be accessed through the link in reference (http://journals.aps.org/prstper/supplemental/10.1103/PhysRevSTPER.9.020103/Planinic_TEST_PRST_PER.pdf)). In addition to choosing the correct answer in multiple choice questions, or providing the answer in open-ended questions, students were asked to provide explanations for their answers and/or necessary calculations where appropriate, so that insight into the underlying student reasoning could be obtained. Rasch analysis (Linacre 2006, n.d.; Bond and Fox 2001) was performed to evaluate the functioning of the test and obtain linear measures of item difficulties. Both sets of data (Croatian and Austrian students) seemed to fit the Rasch model. The functioning of the test as a whole for Croatian students was found to be satisfactory with very high item reliability (0.99) and somewhat lower, but satisfactory, person reliability (0.85) and Cronbach alpha (0.88) (Planinic et al. 2013). For Austrian students the test functioned similarly: item reliability was found to be 0.99, person reliability 0.86 and Cronbach alpha 0.90 (Ivanjek et al. 2015). The analysis of item fit showed that no test items in either data set were degrading for measurement (all had infit and outfit MNSQ values within the range of 0.5–1.5). The point-biserial correlations of items were all positive and greater than 0.3 (Planinic et al. 2013; Ivanjek et al. 2015). It can be concluded that all items worked together in defining the underlying variable (student understanding of graphs) and that a reliable scale of item difficulties was obtained for the items in the test, which allowed further analysis of difficulties of different groups of items. Interestingly, parallel questions of the same set usually differed quite significantly in difficulty.

In order to compare the difficulties of items in each investigated context, the average values of item difficulties over three different domains (mathematics without context, physics, mathematics in context) and two investigated concepts (slope, area) were calculated. The comparison of average difficulties of slope and area items for the two samples is presented in Fig. 10.1. Since in Rasch analysis the average difficulty of items in the test is usually assigned the value of zero logits, positive values in the graph indicate higher than average difficulty (harder items) and negative values lower than average difficulty (easier items).

The comparison of the results of the two samples indicated the stability of the construct of the test. Although some differences in the performance of students in the two groups were noticed, the general trends were the same. For both groups of students, it was noticed that mathematics without context was the easiest domain. Adding context to questions generally had the effect of increasing the difficulty. Kinematics was found to be a difficult context for the students in both samples, in

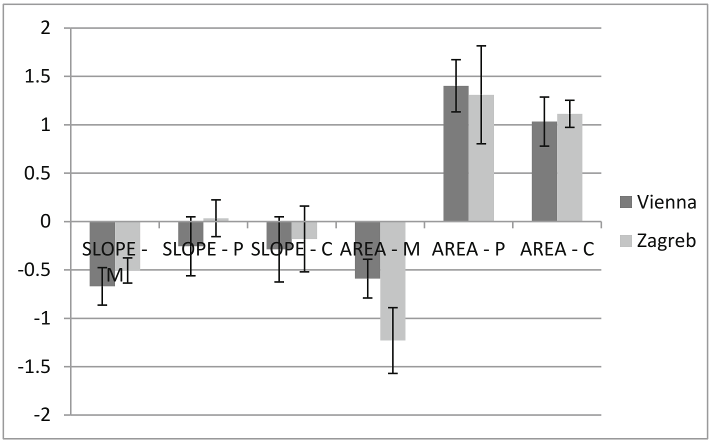

Fig. 10.1 Average difficulties of slope and area items in three different domains for the two groups of students – students from University of Zagreb and from University of Vienna (*M* stands for mathematics without context, *P* for Physics, and *C* for other contexts) (Ivanjek et al. 2015). Error bars indicate the combined uncertainties of each average value

spite of the presumed students' familiarity with the type of questions (kinematics questions used in the test were of the type that is often used in physics teaching, whereas other context questions were typically new to students). When comparing student understanding of slope and area, it was found that, on the average, slope seemed to be better understood. It was also found to be more homogenous – the differences between domains were less pronounced than in the case of area under a graph. Interpretation of area in kinematics and other context was the most difficult aspect of graph interpretation for the students in the both samples (Fig. 10.1). The difficulty of the concept of area under a graph differs dramatically between mathematics on one side and physics and other contexts on the other. This is consistent with the findings of Nguyen and Rebello (2011) that very few students are able to apply this concept in physics problems.

Another aspect of the study conducted on Croatian sample was the analysis of students' strategies and expressed difficulties, obtained through the analysis of their explanations and procedures provided with the answers to questions (Ivanjek et al. 2016). The main findings can be summarized as follows:

1. *Strategies used on parallel questions are often context-dependent and domain-specific. The preferred strategy on physics questions seems to be the use of physics formulas.*

Only a small fraction of students typically used the same strategy on all three questions of the same set of questions, although some have used the same strategy on two of the three questions. It seems that in many cases, students perceived the questions from the same set as different and approached them in different ways. The strategy that was used usually depended on the domain and the context of the problem.

It seems that if students acquire domain-specific procedures for solving a certain class of problems (such as determining the slope of the straight line in mathematics with the use of mathematical formulas or calculating acceleration in physics with the use of physics formulas), they will tend to stick to those procedures and will generally not seem to recognize the mathematical similarity of the problems in different domains. This may be an indication of the absence of transfer of knowledge between the domains, but it could also be a consequence of students' different learning experiences in different school subjects, where they had implicitly learned that each discipline has its own language and conventions and that they have to answer questions in the way that the particular discipline requires. How students framed the problem (Hammer et al. 2005) may have determined their choice of strategy for its solving.

Even though students demonstrated that they were capable of using different strategies for reasoning about graphs, the preferred strategy in physics domain tended to be the use of kinematics formulas. On all area problems and some slope problems, students chose the use of formulas as the main strategy for solving physics problems. The application of the incorrect or inappropriate formulas led them to many incorrect conclusions on physics questions, even on the questions where calculations were not necessary. At the same time, it was not uncommon for students to give correct answers to parallel questions in mathematics and other contexts domains, demonstrating that they were able to reason correctly about the same problem in a different context. The very extensive use of the formula $a = v/t$ on the test indicates, for example, that many students may not have understood the very meaning of the concept of acceleration (as the *rate* of change of velocity) and therefore cannot be expected to understand its representation as the slope of the v vs. t graph. All these findings suggest that students not only have problems with graph interpretation but also with the understanding of the meaning and applicability of physics formulas.

2. *Students use a wider spectrum of strategies on other context problems than on physics problems. Other context problems could be potentially useful in physics and mathematics teaching.*

Other context problems seemed to activate more of students' cognitive resources, and students displayed a wider variety of strategies on those problems than on physics problems. Some students came to the idea that multiplication is needed and others to calculate the area under a graph on other context questions by using some form of dimensional analysis. Dimensional analysis is an approach primarily developed in physics, but surprisingly students did not use the same approach on physics questions.

Many of the student approaches to other context problems could have helped them to solve physics problems as well, but the reliance on formulas as the primary strategy in physics prevented students from using other approaches of which they were capable. Some instances of transfer of knowledge in the sense of preparation for future learning were evidenced in students' use of knowledge and techniques (e.g., dimensional analysis, modeling), acquired in one domain (usually physics), in some other domain (usually other context questions). Some students seemed to think more creatively and used more of the available resources on other context questions than on physics questions, where they seemed to be bound too much by how they perceived the conventions of the discipline. Other context problems could therefore be a potentially useful tool in teaching of both mathematics and physics.

3. *Students show similar difficulties with graph interpretation in all domains, but there are differences between their understanding of graph slope and area under a graph.*

The same patterns of naïve reasoning (slope-height confusion and interval-point confusion) were present in all three domains, but not equally often in each one of them (more frequently in physics than mathematics domain). This is something that we had already noticed in a previous study on high school students' understanding of line graph slope (Planinic et al. 2012).

However, differences were found in students' understanding of the concepts of slope and area and their interpretation. Student explanations on mathematics slope items revealed that for many students, slope may not be more than the vague notion of how steep a straight line is, sometimes identified with the angle that the straight line forms with one of the coordinate axis. In problems which demand only qualitative comparison of slopes, this may often be enough to produce the correct answer. However, when it comes to calculating slope, this vague idea no longer helps. Even though students did not do too well on determining slope in mathematics domain, they did even worse in other domains. The percentage of Croatian students in the study who knew how to determine slope mathematically (54%) was roughly the same as was found in two other studies on first-year university students (Beichner 1994; Wemyss and van Kampen 2013), whereas the respective percentage for Austrian students was somewhat higher (66%). Calculation of slope, as some other studies also suggest (Hadjidemetriou and Williams 2002), may be the most difficult aspect of the concept of slope.

An important aspect of the understanding of the concept of slope is the understanding of the meaning of negative slope. Negative slope is obviously more difficult to understand than positive slope. It seems that students who used vague explanations of negative slope on the basis of graph appearance (e.g., "*straight line is going down*") do not fully understand the concept but have some visual rule for recognizing it.

When it comes to area under a graph, most students know how to determine it, but the interpretation of the meaning of that area seems to be a much bigger problem. Few students seem to be able to interpret areas under graphs in new situations. Unlike slope, whose meaning is more often discussed during teaching, and which is encountered in a greater variety of situations than the area under a graph, interpretation of area seems to be limited to a few isolated examples in physics and learned without sufficient understanding and without necessary reasoning required to transfer that knowledge to other situations. It is interesting that students are more likely to come to the correct interpretation of area in other context questions than in physics, because in physics they often seem to be blocked in their thinking by their overreliance on physics formulas.

10.4 Conclusions and Implications for Teaching

We have attempted through several studies to compare student performance on mathematically similar problems in different domains. The results suggest that students interpret graphs best in mathematics without context. Even though mathematics questions appear more abstract, they are more direct and require less processing of information and less conceptual understanding than parallel physics (kinematics) questions. Kinematics was found to be a difficult context for students, even though it was rather extensively covered in high school. It can be concluded that context generally seems to increase the difficulty of items. Context added to the mathematical slope or area problem will usually increase the cognitive demand on the students, acting as an additional barrier in the problem, and will therefore also increase the difficulty of the item. The only exception may be very familiar contexts for students. Teachers should realize that it is very important to work on students' conceptual understanding and interpretation of physical and mathematical quantities as well as on building stronger links between the two subjects.

Many physics teachers attribute student difficulties with graphs in physics to their presumed lack of mathematical knowledge. But even if students have the needed mathematical knowledge, which was generally the case in our studies (although some problems were noticed in that area too), the transfer to a different domain is not guaranteed. The interpretation of the mathematical quantities in physics or in other contexts is a crucial step which most students in our sample were not able to perform. Some cases of transfer of the problem-solving strategies from physics to other contexts were found on the area items (e.g., dimensional analysis). During teaching of kinematics, the interpretation of slope is usually much more emphasized than the interpretation of area under a graph. An important implication for physics teaching is that we should work more on building student reasoning which leads toward the interpretation of area (which is essentially the idea of integral) and not only provides ready-made interpretation for specific cases in physics. That could also help later to strengthen student understanding of the concept of a definite and indefinite integral in mathematics.

Student reasoning about problems is often very much bound by the contexts and conventions of the disciplines in which their knowledge was acquired. The observed dependence of student strategies on the domain and context of the questions seems to support the knowledge-in-pieces framework, which explains this dependence through context-dependent activation of cognitive resources and the importance of framing. Students seemed to think more freely and creatively, and to transfer more of their knowledge, in problems which in their perception probably did not fall in the category of either physics or mathematics (other context problems). Other context problems may have a potential to expose and develop student reasoning more than the standard domain-specific mathematics and physics questions. They should be used more, in both mathematics and physics teaching. Both disciplines should work more on establishing links between common concepts and procedures in mathematics and physics and promote their integration in students' minds to a much larger extent than is the case now. Students' almost exclusive reliance on formulas in physics presents, in our opinion, an important obstacle for the development of students' deeper reasoning in physics and sometimes even an obstacle for the application of their already existing knowledge and reasoning developed in other domains.

The comparison of the results of Croatian and Austrian students has confirmed the stability of the test and its relevance beyond just Croatian educational system. Currently we continue the research on graphs on other groups of students, besides physics and mathematics students, using also other techniques, such as eye tracking. Some preliminary results suggest better success of nonspecialist groups of university students (e.g., psychology students) on qualitative than quantitative slope and area questions and higher transfer of strategies from physics to finance problems for physics students.

On the basis of our findings, we can summarize some teaching recommendations that might help in the effort of building stronger and more unified student knowledge about graphs:

- Use of other context problems in both mathematics and physics teaching
- Use of multiple strategies on graph problems, which can help remove emphasis from the use of physics formulas as the primary strategy
- Promoting conceptual understanding of graph slope and area in both mathematics and physics teaching
- Building better understanding of the meaning and applicability of physics formulas (and their graphical interpretations where possible)
- Encouraging transfer between mathematics and physics by using and linking different contexts when teaching graphs (e.g., using kinematics examples in mathematics teaching and relating kinematics graphs and formulas to their mathematical origin and meaning in physics teaching)
- Strengthening and operationalizing student understanding of the concept of slope and its calculation (the practice of drawing the rise and run triangle on a line graph – *Steigungsdreieck* in German – seems to help for calculation of slope)
- Promoting interpretation of area under a graph in physics teaching wherever possible by leading students to the idea of accumulation

The presented findings confirm once again that human knowledge is very complex and multifaceted. Students' answers to questions and problems are influenced by the context and formulation of the question, students' framing of the question, the procedures and conventions of the domain in which a certain piece of knowledge was first acquired, the existing or missing links between the domains, as well as many other factors. Using many contexts during teaching and constantly building and strengthening links between different domains could be a good way to building stronger student knowledge. This could help education efforts in both mathematics and physics.

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