Adaptive Finite-Time Synchronization of Inertial Neural Networks with Time-Varying Delays via Intermittent Control

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Abstract. In this paper, the adaptive finite-time synchronization is investigated for inertial neural networks with time-varying delays. The second-order inertial systems can be transformed into two first-order differential systems by selecting the appropriate variable substitution. Using the adaptive periodically intermittent controllers, the slave system can realize synchronization with the master system in finite time. By the several differential inequalities and finite-time stability theory, some simple finite-time synchronization criteria for an array of inertial neural networks are derived. A numerical example is finally provided to illustrate the effectiveness of the obtained theoretical results.

Keywords: Finite-time synchronization · Inertial neural networks
Time-varying delays · Adaptive intermittent control

1 Introduction

Over the last few decades, periodic oscillation, chaotic behaviors, and stability analysis for neural neural network has aroused the discussion and research of many scholars. Meanwhile, neural networks play a significant role in different areas, since neural networks can be applied to image processing, combinatorial optimization, secure communication, and pattern recognition [1–6]. Synchronization means agreement or correlation of different processes in time. Among many dynamical behaviors of neural networks, synchronization is one of the most significance ones that has aroused widespread attentions of many researchers. Research on synchronization phenomena has been an active subject, such as the analysis of synchronization of chaotic system. So far, there are many different types of synchronization, for example, projective synchronization [7], lag synchronization [8], cluster synchronization [9], complete synchronization [10], phase synchronization [11] etc.

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Most previous literature has mainly been devoted to the stability analysis and periodic oscillations of different kinds of neural networks. The problem about delayed Hopfield neural networks with global exponential stability has been discussed in [12]. The authors in [13] investigated exponential stability for stochastic BAM networks with discrete and distributed delays. Exponential stability of complex-valued memristor-based neural networks with time-varying delays have been studied in [14]. Note that many of the studies focused on neural networks, and only the first derivative of states is important for introducing inertial terms into neural networks. The inertial terms are considered as key tools for generating complex bifurcations and chaos. Up to now, the inertial neural networks have attracted the attention of many researchers. The authors in [15] discussed the robust stability of inertial BAM neural networks with time delays and uncertainties via impulsive effect. In [16], the inertial Cohen-Grossberg-type neural networks with time delays was proposed and its stability analysis were discussed.

Lately, a volume of the existing research on inertial neural networks were mainly focused on exponential synchronization or asymptotical synchronization of networks [17,18]. That it to say, as the time goes to infinity, the dynamical systems only can achieve stability. However, in many actual situations, the dynamical system might be hoped to be stabilised as speedy as possible in a finite time. Since then, problems related to finite time synchronization for networks becomes a hot topic [19,20]. The work in [21] only investigated finite-time stability for inertial neural networks. As it is well known, delays are ubiquitous in the real world, and the introduction of delays may make neural networks their dynamical behaviors much more complicated, even in causing instability [22,23]. However, the authors have ignored the time-varying delays in [21]. The problem on finite-time and fixed-time synchronization analysis for inertial memristive neural networks via state feedback control has been investigated in [24]. The advantages of discontinuous control with different continuous control strategies are non control sections. As far as we know, there are few results on finite-time synchronization of inertial neural networks with time-varying delays.

Motivated by the aforementioned discussion, this paper addresses the problem of adaptive finite-time synchronization of inertial neural networks with time-varying delay via periodically intermittent control. Rather, by the finite-time stability analysis techniques and the linear matrix inequalities, some effective criteria are derived, which can guarantee the master system synchronizes to the slave system in finite time. Meanwhile, the general continuous feedback control is discussed with inertial neural networks. In the end, an example is given to demonstrate the effectiveness of the proposed synchronization criteria.

The rest of this paper is organized as follows. In Sect. 2, the model description and some preliminaries are proposed. In Sect. 3, the main results and remark for finite time synchronization of inertial neural networks with time-varying delay. Moreover, in Sect. 4, an example is given to show the effectiveness of our results. Finally, in Sect. 5, conclusions are given.

2 Model Description and Preliminaries

Considering the inertial neural network with time-varying delays. The model is described by the following equations:

$$\frac{d^2x_i(t)}{dt^2} = -c_i\frac{dx_i(t)}{dt} - d_ix_i(t) + \sum_{j=1}^{n} a_{ij}f_j(x_i(t)) + \sum_{j=1}^{n} b_{ij}f_j(x_i(t-\tau(t))) + I_i, \quad (1)$$

where $x_i(t)(i = 1, 2, ..., n)$ is the state vector of the ith neuron at time t, the second derivative of $x_i(t)$ is called an inertial neural term of system (1), c_i and d_i are positive constants. The nonlinear function f_j denotes activation function of the jth neuron at time t, a_{ij} and b_{ij} are constants and denotes the connection strengths, I_i is an external inputs for the ith neuron, $\tau(t)$ is the time-varying delay of inertial neural network that satisfies $0 \leq \tau(t) \leq \tau_1, \dot{\tau}(t) \leq \mu_m < 1$, τ_1 and μ_m are constants.

Remark 1. The chaotic neural network is a highly nonlinear dynamic system. The research on chaotic neural network mainly lies in recognizing the chaotic characteristics of individual neurons and the behavior analysis of simple chaotic neural networks. The second-order inertial neural networks (1) has chaos, which is different from the first order neural network chaos, such as chen's system and chua's system.

Next, let the following variable transformation be: $y_i(t) = \frac{dx_i(t)}{dt} + \theta_ix_i(t), \ i = 1, 2, ..., n$. Denote $x(t) = (x_1(t), x_2(t), ..., x_n(t))^T$, $y(t) = (y_1(t), y_2(t), ..., y_n(t))^T$, then inertial neural network (1) can be written as:

$$\begin{cases} \frac{dx(t)}{dt} = -\Theta x(t) + y(t), \\ \frac{dy(t)}{dt} = -Cy(t) - Dx(t) + Af(x(t)) + Bf(x(t-\tau(t))) + I, \end{cases} \quad (2)$$

where $\overline{c}_i = c_i - \theta_i$, $\overline{d}_i = \theta_i(\theta_i - c_i) + d_i$, $C = diag(\overline{c}_1, \overline{c}_2, ..., \overline{c}_n)$, $D = diag(\overline{d}_1, \overline{d}_2, ..., \overline{d}_n)$, $A = (a_{ij})_{n\times n}$, $B = (b_{ij})_{n\times n}$, $\Theta = diag(\theta_1, \theta_1, ..., \theta_n)$, $f(x(t)) = (f_1(x_1(t)), f_2(x_2(t))...f_n(x_n(t)))^T$, $f(x(t-\tau(t))) = (f_1(x_1(t-\tau(t))), f_2(x_2(t-\tau(t))), ..., f_n(x_n(t-\tau(t))))^T$, $I = (I_1, I_2, ..., I_n)^T$. For simplicity, we choose (3) as the master system, the corresponding slave system is formulated as follows:

$$\begin{cases} \frac{dv(t)}{dt} = -\Theta v(t) + w(t) + u_1(t), \\ \frac{dw(t)}{dt} = -Cw(t) - Dv(t) + Af(v(t)) + Bf(v(t-\tau(t))) + I + u_2(t), \end{cases} \quad (3)$$

where $v(t) = (v_1(t), v_2(t), ..., v_n(t))^T$, $w(t) = (w_1(t), w_2(t), ..., w_n(t))^T$, are the state variables of the slave system, $u_1(t), u_2(t)$ are the appropriate control inputs to be designed later.

Denote the synchronization error $e(t) = v(t) - x(t)$, $\overline{e}(t) = w(t) - y(t)$, we can get the following error system

$$\begin{cases} \frac{de(t)}{dt} = -\Theta e(t) + \overline{e}(t) + u_1(t), \\ \frac{d\overline{e}(t)}{dt} = -C\overline{e}(t) - De(t) + A(f(v(t)) - f(x(t))) + B(f(v(t-\tau(t))) \\ \qquad - f(x(t-\tau(t)))) + u_2(t). \end{cases} \quad (4)$$

In order to realize adaptive finite-time synchronization of inertial neural networks between the master system (3) and slave system (4), the intermittent control $u_i(t)$ is defined by:

$$\begin{cases} u_1(t) = k \odot e(t) - \lambda sign(e(t)) \\ \qquad\qquad - \lambda(\int_{t-\tau(t)}^{t} e^T(s)e(s)ds)^{\frac{1}{2}} \frac{e(t)}{\|e(t)\|^2}, & lT \leq t \leq lT + \delta \\ u_2(t) = \varepsilon \odot \overline{e}(t) - \lambda sign(\overline{e}(t)), & lT \leq t \leq lT + \delta, \\ u_1(t) = u_2(t) = 0, & lT + \delta \leq (l+1)T, \end{cases} \tag{5}$$

where $k = (k_1, k_2, ..., k_n)^T$, $\varepsilon = (\varepsilon_1, \varepsilon_2, ..., \varepsilon_n)^T$ are the adaptive laws, and the $\odot$ is defined as $k \odot e(t) = [k_1 \cdot e_1(t), k_2 \cdot e_2(t), ..., k_n \cdot e_n(t)]^T$, $\lambda > 0$ is real constant. $\pounds = \{1, 2, ..., l\}$ is a finite natural number set. $T > 0$ is called the control period. $\theta = \frac{\delta}{T}$ denote the control rate.

At the same time, the adaptive rule defined as follows:

$$\dot{k}_i = \begin{cases} -\alpha_i\Big(e^T(t)e(t) + \frac{\lambda}{\sqrt{\alpha_i}} sign(k_i) + \frac{\eta_i e(t)^T e(t)}{k_i}\Big), & lT \leq t \leq lT + \delta, \\ 0, & lT + \delta \leq (l+1)T, \end{cases} \tag{6}$$

$$\dot{\varepsilon}_i = \begin{cases} -\alpha_i\Big(\overline{e}^T(t)\overline{e}(t) + \frac{\lambda}{\sqrt{\alpha_i}} sign(\varepsilon_i) + \frac{\epsilon_i \overline{e}(t)^T \overline{e}(t)}{\varepsilon_i}\Big), & lT \leq t \leq lT + \delta, \\ 0, & lT + \delta \leq (l+1)T, \end{cases} \tag{7}$$

where $\alpha_i > 0$ is a positive constant. $\eta_i > 0, \epsilon_i > 0$ are nonnegative constants denotes the control gain.

Assumption 1. For all x, $y \in \mathbb{R}^n$, suppose that the activation function $f(\cdot)$ satisfies the following condition,

$$\| f(x) - f(y) \| \leq \| J(x-y) \| .$$

where $J \in \mathbb{R}^{n \times n}$ is a known constant matrix.

Definition 1 ([25]). *The slave system* (4) *is said to reach finite-time synchronization with the master system* (3)*, if there exists a constant* $t_1 \geq 0$ *such that*

$$\lim_{t \to t_1} \| e(t) \| = 0,$$

and $\| e(t) \| = 0$ *for* $t \geq t_1$ *, where* t_1 *denotes the settling time.*

Lemma 1 ([26]). *If* $b_1, b_2, ..., b_n \geq 0$ *,* $0 < k \leq 1$*, after that*

$$(\sum_{i=1}^{n} b_i)^k \leq \sum_{i=1}^{n} b_i^k.$$

Lemma 2 ([27]). *If* $X, Y,$ *and* Q *are real matrices with appropriate dimensions, there exists a constant* $\sigma > 0$ *and* $Q = Q^T > 0$ *such that*

$$2X^T Y \leq \sigma X^T Q X + \sigma^{-1} Y^T Q^{-1} Y.$$

Lemma 3 ([28]). *If there exist a continuous, positive definite $V(t)$ satisfies the following inequality:*

$$\dot{V}(t) \leq -\alpha V^{\eta}(t) + hV(t), \forall t \geq t_0, V^{1-\eta}(t_0) \leq \frac{\alpha}{h},$$

where $\alpha > 0, 0 < \eta < 1, h > 0$ are three constants, then the settling time t_1 is given by

$$t_1 \leq \frac{\ln(1 - \frac{h}{\alpha}V^{1-\eta}(0))}{h(\eta - 1)}.$$

3 Main Results

Now, we are in a position to present our results. We will introduce the synchronization criteria between the master system and the salve system in finite time with time-varying delay via adaptive intermittent controllers.

Theorem 1. *Suppose that Assumption 1 hold. For given positive constants ρ, σ, δ, if there exist two diagonal positive definite matrices $\Xi = diag(\eta_1, \eta_2, ..., \eta_n)$ and $\Lambda = diag(\epsilon_1, \epsilon_2, ..., \epsilon_n)$ such that the following conditions hold.*

$$\Phi = \begin{pmatrix} \Phi_{11} & \frac{1}{2}(I_n - D) & 0 \\ * & \Phi_{22} & 0 \\ * & * & \rho J^T J - \frac{1}{2}(1 - \mu_m)I_n \end{pmatrix} < 0, \tag{8}$$

$$(\frac{1}{2} + \delta - \beta)I_n + \sigma J^T J - \Theta \leq 0, \tag{9}$$

$$\sigma^{-1}AA^T + \rho^{-1}BB^T + \delta^{-1}(I_n - D)(I_n - D)^T - \beta I_n - C - \Lambda \leq 0, \tag{10}$$

$$\rho J^T J - \frac{1}{2}(1 - \mu_m)I_n \leq 0. \tag{11}$$

Then the master system (3) and slave system (4) can be finite-time synchronized under the adaptive periodically intermittent control:

$$t_1 = \frac{V^{1-\eta}(0)e^{(1-\eta)(1-\theta)\gamma t}}{\alpha\theta(1 - \eta)}, \tag{12}$$

where $\Phi_{11} = \sigma J^T J + \frac{1}{2}I_n - \Theta - \Xi$, $\Phi_{22} = \sigma^{-1}AA^T + \rho^{-1}BB^T - C - \Lambda$, let $e(t) = (e_1^T(t), e_2^T(t), ..., e_n^T(t))^T \in \mathbb{R}^n$.

Proof. Constructing the following Lyapunov-Krasovskii function: $V(t) = V_1(t) + V_2(t) + V_3(t)$,

$V_1(t) = \frac{1}{2}e^T(t)e(t) + \frac{1}{2}\overline{e}^T(t)\overline{e}(t), V_2(t) = \frac{1}{2}\int_{t-\tau(t)}^{t} e^T(s)e(s)ds$,

$V_3(t) = \frac{1}{2}\sum_{i=1}^{n} \frac{1}{\alpha_i}k_i^2 + \frac{1}{2}\sum_{i=1}^{n} \frac{1}{\alpha_i}\varepsilon_i^2$.

When $t \in [lT, lT + \theta T]$, the time derivative of $V(t)$ along the trajectories of the error system (5) and using Assumption 1, one have

$$\begin{aligned}
\dot{V}_1(t) &= e^T(t)\Big(-\Theta e(t) + \overline{e}(t) + k \odot e(t) - \lambda sign(e(t)) \\
&\quad - \lambda (\int_{t-\tau(t)}^{t} e^T(s)e(s)ds)^{\frac{1}{2}} \frac{e(t)}{\| e(t) \|^2}\Big) + \overline{e}^T(t)\Big(-C\overline{e}(t) - De(t) \\
&\quad + \overline{A}(f(v(t)) - f(x(t))) + B(f(v(t-\tau(t))) - f(x(t-\tau(t)))) \\
&\quad + \varepsilon \odot \overline{e}(t) - \lambda sign(\overline{e}(t))\Big) \\
&\leq -e^T(t)\Theta e(t) + e^T(t)\overline{e}(t) + e^T(t)k \odot e(t) - \lambda \mid e^T(t) \mid \\
&\quad - \lambda (\int_{t-\tau(t)}^{t} e^T(s)e(s)ds)^{\frac{1}{2}} - \overline{e}^T(t)C\overline{e}(t) - \overline{e}^T(t)De(t) \\
&\quad + \sigma^{-1}\overline{e}^T(t)AA^T\overline{e}(t) + \sigma e^T(t)J^TJe(t) + \rho^{-1}\overline{e}^T(t)BB^T\overline{e}(t) \\
&\quad + \rho e^T(t-\tau(t))J^TJe(t-\tau(t)) + \overline{e}^T(t)\varepsilon \odot \overline{e}(t) - \lambda \mid \overline{e}^T(t) \mid . \\
\dot{V}_2(t) + \dot{V}_3(t) &= \frac{1}{2}\Big(e^T(t)e(t) - (1-\dot{\tau}(t))e^T(t-\tau(t))e(t-\tau(t))\Big) \\
&\quad + \sum_{i=1}^{n} \frac{1}{\alpha_i}k_i\Big[-\alpha_i\Big(e^T(t)e(t) + \frac{\lambda}{\sqrt{\alpha_i}}sign(k_i) + \frac{\eta_i e(t)^T e(t)}{k_i}\Big)\Big] \\
&\quad + \sum_{i=1}^{n} \frac{1}{\alpha_i}\varepsilon_i\Big[-\alpha_i\Big(\overline{e}^T(t)\overline{e}(t) + \frac{\lambda}{\sqrt{\alpha_i}}sign(\varepsilon_i) + \frac{\epsilon_i \overline{e}(t)^T \overline{e}(t)}{\varepsilon_i}\Big)\Big] \\
&\leq \frac{1}{2}\Big(e^T(t)e(t) - (1-\mu_m)e^T(t-\tau(t))e(t-\tau(t))\Big) \\
&\quad - \sum_{i=1}^{n} k_i e^T(t)e(t) - \sum_{i=1}^{n} \frac{\lambda}{\sqrt{\alpha_i}} \mid k_i \mid - \sum_{i=1}^{n} \eta_i e^T(t)e(t) \\
&\quad - \sum_{i=1}^{n} \varepsilon_i \overline{e}^T(t)\overline{e}(t) - \sum_{i=1}^{n} \frac{\lambda}{\sqrt{\alpha_i}} \mid \varepsilon_i \mid - \sum_{i=1}^{n} \epsilon_i \overline{e}^T(t)\overline{e}(t).
\end{aligned} \tag{13}$$

By lemma 1 and combine (10) and (11), we get

$$\begin{aligned}
\dot{V}(t) \leq\ & e^T(t)\Big(\sigma J^T J + \frac{1}{2}I_n - \Theta - \Xi\Big)e(t) \\
& + \overline{e}^T(t)\Big(\sigma^{-1}AA^T + \rho^{-1}BB^T - C - \Lambda\Big)\overline{e}(t) \\
& + e^T(t-\tau(t))\Big(\rho J^T J - \frac{1}{2}(1-\mu_m)I_n\Big)e(t-\tau(t)) \\
& + e^T(t)(I_n - D)\overline{e}(t) - \lambda \mid e^T(t) \mid - \lambda \mid \overline{e}^T(t) \mid \\
& - \lambda\Big(\int_{t-\tau(t)}^{t} e^T(s)e(s)ds\Big)^{\frac{1}{2}} - \sum_{i=1}^{n} \frac{\lambda}{\sqrt{\alpha_i}} \mid k_i \mid - \sum_{i=1}^{n} \frac{\lambda}{\sqrt{\alpha_i}} \mid \varepsilon_i \mid \\
\leq\ & \xi^T(t)\Phi\xi(t) - \sqrt{2}\lambda\Big(\frac{1}{2}e^T(t)e(t) + \frac{1}{2}\overline{e}^T(t)\overline{e}(t) \\
& + \frac{1}{2}\int_{t-\tau(t)}^{t} e^T(s)e(s)ds + \frac{1}{2}\sum_{i=1}^{n}\frac{1}{\alpha_i}k_i^2 + \frac{1}{2}\sum_{i=1}^{n}\frac{1}{\alpha_i}\varepsilon_i^2\Big)^{\frac{1}{2}}
\end{aligned} \tag{14}$$

where $\xi(t) = (e^T(t), \overline{e}^T(t), e^T(t-\tau(t)))^T$, based on the condition (9) that $\Phi \leq 0$, we can obtain as $V(t) \leq -\sqrt{2}\lambda V^{\frac{1}{2}}(t)$.

When $lT + \theta T \leq (l+1)T$, based on the conditions (10)–(12), then the time derivative of V for $t > 0$ is given by

$$\begin{aligned}
\dot{V}(t) =\ & e^T(t)\Big(-\Theta e(t) + \overline{e}(t)\Big) + \overline{e}^T(t)\Big(-C\overline{e}(t) - De(t) + A(f(v(t)) - f(x(t))) \\
& + B[f(v(t-\tau(t))) - f(x(t-\tau(t)))]\Big) + \frac{1}{2}e^T(t)e(t) \\
& - \frac{1}{2}(1-\dot{\tau}(t))e^T(t-\tau(t))e(t-\tau(t)) \\
\leq\ & -e^T(t)\Theta e(t) + e^T(t)\overline{e}(t) - \overline{e}^T(t)C\overline{e}(t) - \overline{e}^T(t)De(t) \\
& + \sigma^{-1}\overline{e}(t)AA^T\overline{e}(t) \\
& + \sigma e^T(t)J^T Je(t) + \rho^{-1}\overline{e}^T(t)BB^T\overline{e}(t) + \rho e^T(t-\tau(t))J^T Je(t-\tau(t)) \\
& + \frac{1}{2}e^T(t)e(t) - \frac{1}{2}(1-\mu_m)e^T(t-\tau(t))e(t-\tau(t)) \\
\leq\ & e^T(t)\Big((\frac{1}{2} + \delta - \beta)I_n + \sigma J^T J - \Theta\Big)e(t) \\
& + e^T(t-\tau(t))\Big(\rho J^T J - \frac{1}{2}(1-\mu_m)I_n\Big)e(t-\tau(t)) \\
& + \overline{e}^T(t)\Big(\sigma^{-1}AA^T + \rho^{-1}BB^T + \delta^{-1}(I_n - D)(I_n - D)^T - \beta I_n\Big)\overline{e}(t) \\
& + \beta(e^T(t)e(t) + \overline{e}^T(t)\overline{e}(t)) \leq \beta V_1(t).
\end{aligned} \tag{15}$$

Then $\dot{V}(t) \leq \beta V(t)$, by Theorem 1, it follows from (18) and (20), let $\alpha = \sqrt{2}\lambda, \eta = \frac{1}{2}$, we get

$$\begin{cases} \dot{V}(t) \leq -\sqrt{2}\lambda V^{\frac{1}{2}}(t), & lT \leq t \leq lT + \delta T. \\ \dot{V}(t) \leq \beta V(t), & lT + \delta T \leq (l+1)T. \end{cases} \tag{16}$$

By Lemma 3, we obtain $t \leq \frac{V^{\frac{1}{2}}(0)e^{\frac{1}{2}(1-\theta)\beta t}}{\sqrt{2}\lambda\theta} = t_1$. therefore, the synchronization of inertial neural networks for the master system (3) and the slave system (4) is achieved in a finite time. So the Theorem 1 is proved.

The adaptive periodically intermittent controller is degenerated to continuous feedback control strategy when $\theta = 1$. A new controller (6)–(8) are proposed as follows:

$$\begin{cases} u_1(t) = k \odot e(t) - \lambda sign(e(t)) - \lambda(\int_{t-\tau(t)}^{t} e^T(s)e(s)ds)^{\frac{1}{2}} \frac{e(t)}{\|e(t)\|^2}. \\ u_2(t) = \varepsilon \odot \overline{e}(t) - \lambda sign(\overline{e}(t)). \end{cases} \tag{17}$$

$$\begin{cases} \dot{k}_i = -\alpha_i\Big(e^T(t)e(t) + \frac{\lambda}{\sqrt{\alpha_i}} sign(k_i) + \frac{\eta_i e(t)^T e(t)}{k_i}\Big). \\ \dot{\varepsilon}_i = -\alpha_i\Big(\overline{e}^T(t)\overline{e}(t) + \frac{\lambda}{\sqrt{\alpha_i}} sign(\varepsilon_i) + \frac{\epsilon_i \overline{e}(t)^T \overline{e}(t)}{\varepsilon_i}\Big), \end{cases} \tag{18}$$

then the following corollary can be obtained.

Corollary 1. *Under Assumptions 1, for given positive constants ρ, σ, δ, if there exist two diagonal positive definite matrices $\varXi = diag(\varepsilon_1, \varepsilon_2, ..., \varepsilon_n)$ and $\varLambda = diag(\epsilon_1, \epsilon_2, ..., \epsilon_n)$ such that the following conditions hold:*

$$\varPsi = \begin{pmatrix} \varPsi_{11} & 0 & 0 \\ * & \varPsi_{22} & 0 \\ * & * & \rho J^T J - \frac{1}{2}(1-\mu_m)I_n \end{pmatrix} < 0, \tag{19}$$

afterwards the master system (3) and the slave system (4) can achieve finite time synchronization with the continuous controller in the setting time: $t_2 = \frac{2V^{\frac{1}{2}}(0)}{\sqrt{2}\lambda}$. where $\varPsi_{11} = \sigma J^T J + (\frac{1}{2} + \delta)I_n - \varTheta - \varXi$, $\varPsi_{22} = \sigma^{-1}AA^T + \rho^{-1}BB^T + \delta^{-1}(I_n - D)(I_n - D)^T - C - \varLambda$.

4 Numerical Example

In this section, an example is given to verify the effectiveness of the synchronization for inertial neural networks scheme obtained in the previous section. Considering the following inertial neural networks:

$$\frac{d^2x_i(t)}{dt^2} = -c_i\frac{dx_i(t)}{dt} - d_i x_i(t) + \sum_{j=1}^{2} a_{ij} f_j(x_i(t)) + \sum_{j=1}^{2} b_{ij} f_j(x_i(t-\tau(t))) + I_i, \quad i = 1, 2. \tag{20}$$

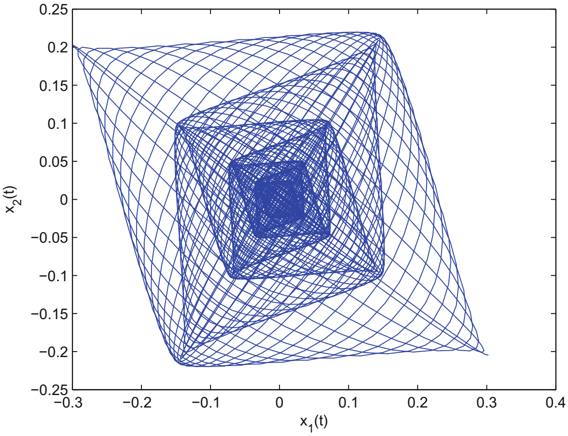

Fig. 1. Phase portrait of the system (20).

then, the corresponding slave system is formulated as follows:

$$\begin{cases} \frac{dv(t)}{dt} = -\Theta v(t) + w(t) + u_1(t), \\ \frac{dw(t)}{dt} = -Cw(t) - Dv(t) + Af(v(t)) + Bf(v(t-\tau(t))) + I + u_2(t), \end{cases} \quad (21)$$

we can get the corresponding matrix

$$C = \begin{pmatrix} 0.1 & 0 \\ 0.1 & 0 \end{pmatrix}, D = \begin{pmatrix} 0.11 & 0 \\ 0 & 0.11 \end{pmatrix}, A = \begin{pmatrix} -0.95 & 0.01 \\ 0.01 & -1 \end{pmatrix}, B = \begin{pmatrix} 0.6 & -0.5 \\ 1.8 & 0.5 \end{pmatrix},$$

where $\Theta = diag(0.5, 0.5)$, $f(x) = tanh(x)$, let $\tau(t) = 0.5\sin(t) + 0.3$, Obviously, $\dot{\tau}(t) \leq \mu_m = 0.5$. Moreover, the initial values are given as: $x(0) =$

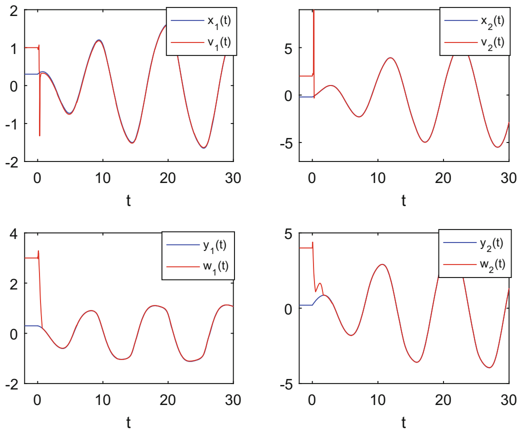

Fig. 2. Trajectory of the synchronization $x(t), v(t), y(t), w(t)$ with the intermittent controller.

$(0.3, -0.2)^T$, $y(0) = (0.3, 0.2)^T$, $v(0) = (1, 2)^T$, $w(0) = (3, 4)^T$. Given $\theta = 0.6$, $T = 3$, $\lambda = 3$, $k = 2.1$. By the Matlab LMI Control Toolbox to solve the LMI in the Theorem 1. We have a set of feasible solutions: $\Xi = diag(13.3641, 13.3641)$, $\Lambda = diag(24.7574, 24.7574)$. Then, the Fig. 1 show the phase portrait of the system (33). Then, trajectory of the synchronization $x(t), v(t)$ and $y(t), w(t)$ with the adaptive periodically intermittent strategy in Fig. 2. Finally, the synchronization errors are shown for the systems by using the adaptive intermittent controller in Fig. 3.

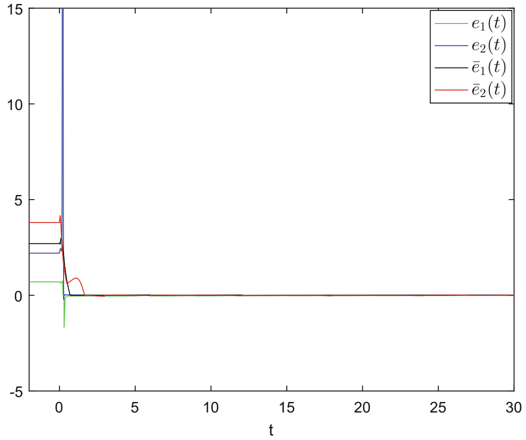

Fig. 3. Trajectory of the synchronization errors $e(t), \overline{e}(t)$ with the intermittent controller.

5 Conclusion

In this paper, the finite-time synchronization for a class of inertial neural networks with time-varying delay was studied. By selecting suitable variable substitution, the original system can changed to two first-order differential equations. The discontinuous intermittent controller was proposed to adjust the system to realize synchronization with finite time. By the some adequate conditions and finite-time stability theory, we have proposed the finite-time synchronization of master-slave systems. In the end, the numerical simulation given to demonstrate the effectiveness the proposed method. In the future, a mixed intermittent controller with different control rates may be researched. Hence, it is worth learning the finite-time synchronization for coupled inertial neural networks under aperiodically intermittent control, impulsive control, sampled-data control and so on.

References

1. Wang, D.L.: Pattern recognition: neural networks in perspective. IEEE Expert. **8**, 52–60 (1993)
2. Cochocki, A., Unbehauen, R.: Neural Networks for Optimization and Signal Processing. Wiley, Chichester (1993)
3. Kwok, T., Smith, K.A.: Experimental analysis of chaotic neural network models for combinatorial optimization under a unifying framework. Neural Netw. **13**, 731–744 (2000)
4. Kwok, T., Smith, K.A.: A unified framework for chaotic neural-network approaches to combinatorial optimization. IEEE Trans. Neural Netw. **10**, 978–981 (1999)
5. Pecora, L.M., Carroll, T.L.: Synchronization in chaotic systems. Phys. Rev. Lett. **64**, 821 (1990)
6. Xu, Y., Wang, H., Li, Y.G., Pei, B.: Image encryption based on synchronization of fractional chaotic systems. Commun. Nonlinear Sci. Numer. Simul. **19**, 3735–3744 (2014)
7. Mahmoud, G.M., Mahmoud, E.E., Arafa, A.A.: Projective synchronization for coupled partially linear complex-variable systems with known parameters. Math. Methods Appl. Sci. **40**, 1214–1222 (2017)
8. Guo, W.: Lag synchronization of complex networks via pinning control. Nonlinear Anal. Real World Appl. **12**, 2579–2585 (2011)
9. Wang, Y.L., Cao, J.D.: Cluster synchronization in nonlinearly coupled delayed networks of non-identical dynamic systems. Nonlinear Anal. Ser. B Real World Appl. **14**, 842–851 (2013)
10. Mahmoud, G.M., Mahmoud, E.E.: Complete synchronization of chaotic complex nonlinear systems with uncertain parameters. Nonlinear Dyn. **62**, 875–882 (2010)
11. Mahmoud, G.M., Mahmoud, E.E.: Phase and antiphase synchronization of two identical hyperchaotic complex nonlinear systems. Nonlinear Dyn. **61**, 141–152 (2010)
12. Chen, T.P.: Global exponential stability of delayed Hopfield neural networks. Neural Netw. **14**, 977–980 (2001)
13. Bao, H.B., Cao, J.D.: Exponential stability for stochastic BAM networks with discrete and distributed delays. Appl. Math. Comput. **218**, 6188–6199 (2012)
14. Shi, Y.C., Cao, J.D., Chen, G.R.: Exponential stability of complex-valued memristor-based neural networks with time-varying delays. Appl. Math. Comput. **313**, 222–234 (2017)
15. Zhang, W., Huang, T.W. Li, C.D., Yang, J.: Robust stability of inertial BAM neural networks with time delays and uncertainties via impulsive effect. Neural Process. Lett. **48**, 1–12 (2017)
16. Ke, Y.Q., Miao, C.F.: Stability analysis of inertial Cohen-Grossberg-type neural networks with time delays. Neurocomputing **117**, 196–205 (2013)
17. Zhang, W., Li, C.D., Huang, T.W., Tan, J.: Exponential stability of inertial BAM neural networks with time-varying delay via periodically intermittent control. Neural Comput. Appl. **26**, 1781–1787 (2015)
18. Cao, J.D., Wan, Y.: Matrix measure strategies for stability and synchronization of inertial BAM neural network with time delays. Neural Netw. **53**, 165–172 (2014)
19. Sowmiya, C., Raja, R., Cao, J.D., Rajchakit, G.: Enhanced robust finite-time passivity for Markovian jumping discrete-time BAM neural networks with leakage delay. Adv. Differ. Equ. **2017** Article no. 318 (2017)

20. Bao, H.B., Cao, J.D.: Finite-time generalized synchronization of nonidentical delayed chaotic systems. Nonlinear Anal. Model. Control **21**, 306–324 (2016)
21. Cui, N., Jiang, H.J., Hu, C., Abdujelil, A.: Finite-time synchronization of inertial neural networks. J. Assoc. Arab. Univ. Basic Appl. Sci. **24**, 300–309 (2017)
22. Li, Y.N., Sun, Y.G., Meng, F.W.: New criteria for exponential stability of switched time varying systems with delays and nonlinear disturbances. Nonlinear Anal. Hybrid Syst. **26**, 284–291 (2017)
23. Guo, Y.X.: Globally robust stability analysis for stochastic Cohen-Grossberg neural networks with impulse control and time-varying delays. Ukr. Math. J. **69**, 1220C–1233 (2017)
24. Wei, R.Y., Cao, J.D., Ahmed, A.: Finite-time and fixed-time synchronization analysis of inertial memristive neural networks with time-varying delays. Cogn. Neurodyn. **12**, 1–14 (2017)
25. Shen, J., Cao, J.D.: Finite-time synchronization of coupled neural networks via discontinuous controllers. Cogn. Neurodyn. **5**, 373–385 (2011)
26. Wang, L., Xiao, F.: Finite-time consensus problems for networks of dynamic agents. IEEE Trans. Autom. Control **55**, 950–955 (2010)
27. Yang, Z.C., Xu, D.Y.: Stability analysis and design of impulsive control systems with time delay. IEEE Trans. Autom. Control **52**, 1448–1454 (2007)
28. Shen, Y.J., Xia, X.H.: Semi-global finite-time observers for nonlinear systems. Automatica **44**, 3152–3156 (2008)