

Theorizing Professional Learning through Lesson Study Using the Interconnected Model of Professional Growth



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Abstract Lesson study is highly regarded as a model for teacher professional learning. Yet there are few studies that attempt to theorize the learning process for participating teachers. In an earlier paper, we used Clarke and Hollingsworth's (Teach Teach Educ 18:947–967, 2002) *Interconnected Model of Professional Growth* (IMPG) to map the professional growth of a research lesson planning team participating in a lesson study project over two school terms. This chapter uses the IMPG to examine the professional learning experiences of individual participants from the other planning team in the same project. The analysis is based on interviews carried out at the beginning of the lesson study project and after the first and second lesson study research cycles.

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1 Introduction

Lesson study is widely regarded as a model for teacher-led professional learning that centers on teachers working collaboratively with their colleagues to plan, observe, and reflect on their practice (Groves et al. 2016; Lewis et al. 2009; Takahashi and McDougal 2016). Features of lesson study that are considered as indicators of high-quality professional development include a focus on student learning, time for reflection and inquiry into practice, a focus on the development of teachers' content knowledge and pedagogical skills grounded in practice, support from school leadership, and the involvement of outside experts (Darling-Hammond and Richardson 2009; Guskey and Yoon 2009).

Despite its long-standing history in Japan and its widespread adaptation in other countries, there have been few studies that investigate the mechanisms through which teacher professional growth occurs through lesson study (Elliott 2012; White et al. 2011; Widjaja et al. 2017). Adaptations of lesson study in other countries have resulted in different “versions” of lesson study, some of which bear little resemblance to Japanese Lesson Study (JLS) and ignore some of its salient features (Fujii 2016). However, Stigler and Hiebert (2016) claim that explicit theories underpinning lesson study would make it possible to “adapt what is essentially a cultural practice or routine to a new setting” (p. 582).

This chapter investigates the professional growth of *individual teachers* from the second planning team participating in the *Implementing structured problem-solving mathematics lessons through lesson study* project. The Clarke and Hollingsworth's (2002) *Interconnected Model of Professional Growth* (IMPG) is used to identify “change sequences and growth networks” (p. 957) in order to explain the mechanisms through which professional growth occurred for this group of teachers.

2 Literature and Theoretical Framework

This section discusses lesson study and Clarke and Hollingsworth's (2002) *Interconnected Model of Professional Growth* (IMPG).

2.1 *Lesson Study*

Japanese Lesson Study is a professional learning activity, the origins of which can be traced back for over a century. While there are variations in the lesson study process (see, e.g., Fujii 2016; Takahashi and McDougall 2016), in general, the school-based version of lesson study consists of four components: (i) formulation of overarching school goals related to students' learning and long-term development; (ii) group planning of a *research lesson* addressing these goals; (iii) one team member teaching the research lesson, while the planning group, and others, observe in order to gather evidence of student learning; and (iv) the post-lesson discussion where the planning group and other observers (including an "outside expert") discuss and reflect on the evidence gathered during the lesson, using it to improve the lesson, the unit, and teaching more generally (Perry and Lewis 2008 p. 366).

Teachers play a central role in researching classroom practice and exploring ways to improve students' learning in lesson study. The post-lesson discussion are informed by students' work samples and observers' notes collected during the research lessons (Lewis and Tsuchida 1998; Takahashi and Yoshida 2004). The planning process involves setting the goals for the lesson, studying curriculum documents, identifying appropriate teaching resources, and, in mathematics, finding and solving a suitable mathematical problem and anticipating students' solutions. Observers collect evidence of students' learning and document salient moments in the teaching and learning process during the research lesson. Teachers, researchers, and observers then discuss their "evidence" of students' learning and share ideas to improve the teaching and learning process during the post-lesson discussion (Takahashi and Yoshida 2004; Watanabe 2002). It should be noted that the focus of the post-lesson discussion is on the teaching and on students' learning and *not* on the teacher.

2.2 *Structured Problem-Solving Mathematics Lessons*

The impetus for the widespread interest in Lesson Study originated from the video and print descriptions of the "typical" Japanese Year 8 mathematics lessons captured as part of Stigler and Hiebert's (1997) TIMSS video study. These lessons followed a pattern which Stigler and Hiebert referred to as structured problem-solving. Typically, the research lesson in JLS in mathematics takes the form of a structured problem-solving lesson.

These lessons have a single focus and address a single problem designed to "achieve a single objective in a topic" (Takahashi 2006, p. 4). According to Shimizu

(1999), major characteristics of these structured problem-solving lessons are (i) the *hatsumon*, the thought-provoking question or problem students engage with, which is the key to students’ mathematical development; (ii) *kikan-shido*, the *purposeful scanning* that takes place while students are working individually or in groups, which allows teachers not only to monitor students’ strategies but also to orchestrate their reports on their solutions in the *neriage* phase of the lesson; (iii) *neriage*, the *kneading* stage of a lesson that allows students to compare, polish, and refine solutions through the teacher’s orchestration and probing of student solutions; and (iv) *matome*, the summing up and careful review of students’ discussion in order to guide them to higher levels of mathematical sophistication. Critical in the process of planning, a mathematics research lesson is anticipating student responses, which help guide the teacher in selecting students to share their solutions in the *neriage* stage of the lesson.

2.3 The Interconnected Model of Professional Growth

Clarke and Hollingsworth’s (2002) *Interconnected Model of Professional Growth* (IMPG) (Fig. 1) has been used in many recent studies of teacher professional growth, including a number on lesson study (e.g., Goldsmith et al. 2014; Schipper et al. 2017; White et al. 2011; Widjaja et al. 2017). The model posits that teacher change happens through the process of enactment and reflection in four domains: the personal domain (teacher knowledge, beliefs, attitudes), the domain of practice

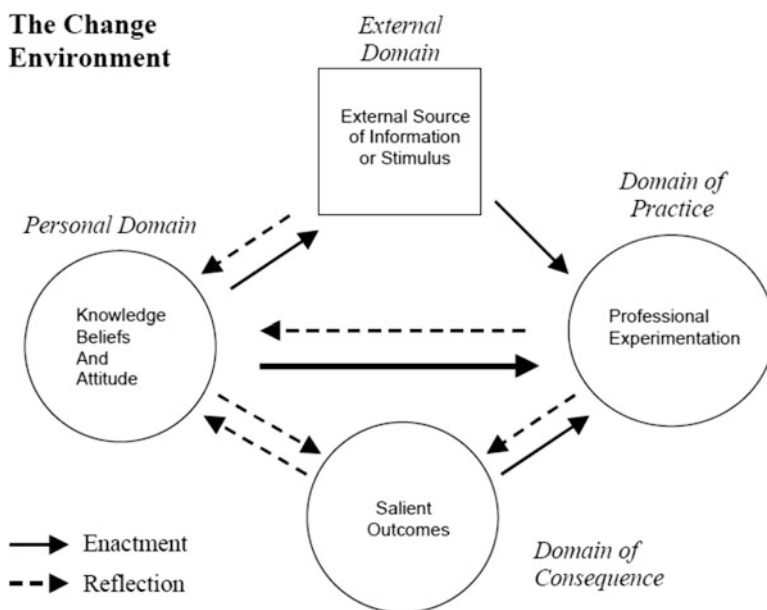


Fig. 1 Interconnected Model of Professional Growth (Clarke and Hollingsworth 2002, p. 951)

(classroom experimentation), the external domain (professional development, external support, and stimuli), and the domain of consequence (salient outcomes).

Drawing on Guskey's (1986) linear model of teacher change, the IMPG model depicts a nonlinear, iterative model that takes into account the "various dynamics at work in social behavior and [how] these interact and combine in different ways" (Clarke and Hollingsworth 2002, p. 378). Clarke and Hollingsworth (2002) argue that multiple "growth pathways" between the domains are possible (p. 950). Opfer and Pedder (2011) argue that the IMPG model is useful in capturing the complexity of teacher learning, particularly in explaining the "cyclic nature of the learning and change process" (p. 385).

3 Method

This section provides an overview of the project and the methods of data collection and analysis.

3.1 Overview of the Project

Three schools from an Australian local school network participated in the *Implementing structured problem-solving mathematics lessons through lesson study* project. A cross-school implementation of JLS was chosen to explore ways in which key elements of JLS could be embedded into mathematics teaching and professional learning for three schools through an interschool collaborative approach. Six teachers, two from each school, the numeracy¹ coach² or curriculum coordinator from each school, and the local network numeracy coach participated in the project. Following a professional learning day to introduce participants to JLS processes and JLS-structured problem-solving lessons, participants were divided into two cross-school planning teams, self-named the Matomes and the Bobbies. Each team consisted of three Year 3 or 4 teachers, one from each school, and two coaches. Widjaja et al. (2017) reported on the professional growth of the other team, the Bobbies. In this chapter we explore the professional growth of members of the Matomes team.

¹Numeracy is the term used to describe the mathematics subject in primary schools in Victoria at the time of the study.

²Numeracy coaches were teachers who were nominated by the school principal to support continuing professional learning in mathematics within schools. They were also involved in professional conversations with other coaches in the school networks.

Table 1 Members of the Matomes Planning Team

Name (pseudonym)	School	Role	Teaching experiences (Years)
Trevor	C	Year 3 teacher Teacher of research lesson 1	2
Camilla	B	Year 3–4 teacher Teacher of research lesson 2	5
Sandra	A	Year 4 teacher Numeracy coordinator Curriculum coordinator	12
George	A	Numeracy coach	7
Narah	B	Curriculum coordinator	6

3.2 *The Matomes Planning Team*

The school, role, and number of years teaching experience of each participant of the Matomes planning team are shown in Table 1. Trevor was the least experienced teacher, with the others having taught in primary schools for at least 5 years. Prior to her 5 years as a primary teacher, Camilla had spent 2 years working as a casual relief teacher. Narah, the curriculum coordinator at School B, had spent 2 years teaching in England. She started as a numeracy coach at School B before taking the role of the curriculum coordinator. George was an experienced teacher who was a numeracy coach with a leadership role in his school. He was studying for a Master's degree in Educational Leadership. Sandra, the most experienced participant, had two leadership roles at her school, coordinator of the school's curriculum program and the school's mathematics program, in addition to teaching a Year 4 class. The second and third authors were also participant-observers in the Matomes planning team.

3.3 *The Lesson Study Process*

The Matomes and the Bobbies each completed one lesson study cycle during each of two 10-week school terms, planning their own research lesson, based on the same problem provided by the research team. Participants planned each research lesson during four 2-hour sessions. One member of each team taught the research lesson using the problems presented by the researchers (see Appendixes B and C). In the case of the Matomes, Trevor taught the research lesson in Cycle 1 and Camilla in Cycle 2. All members of both planning teams, key staff at each school, together with all interested teachers who could be released from their classes, observed the research lessons and took part in the post-lesson discussions. In all between 20 and 30 people, including members of the leadership teams from other schools, staff from the regional office, mathematics educators, and an outside expert observed each research lesson and took part in the post-lesson discussions. However, due to a change in the date for the research lessons and the subsequent post-lesson discussions, Sandra was unable to attend the research lesson in Cycle 2, although she played a full part in the planning.

3.4 Data Collection and Analysis

Throughout the study, the researchers kept field notes of planning meetings, research lessons, and post-lesson discussions. In addition, all sessions, including the professional learning day, were video-recorded. Planning meeting agenda, together with lesson plans and notes prepared by members of the planning teams were collected, as was students' written work from all research lessons. These data were complemented by individual, audio-recorded, 30-min interviews with participants on three occasions: at the beginning of the project and following the first and second Lesson Study Cycles. Interviews and post-lesson discussions were transcribed.

In Widjaja et al. (2017), we reported on the professional growth of the Bobbies team, using the data collected during planning meetings, interviews, and post-lesson discussions. Due to technical issues with the video-recordings of the Matomes' planning sessions, systematic analysis of data from these meetings was not possible, so this chapter is based on transcripts of the three interviews.

As was the case in our earlier paper, Clarke and Hollingsworth's (2002) IMPG provided the framework for the data analysis. Ethnographic methods involving open coding using a constant comparative method (Corbin and Strauss 2008) were used to analyze the interview data. All four authors jointly coded substantial parts of the interview data and adapted and agreed on a final list of much more detailed codes than those used in the previous analysis for the Bobbies team (see Appendix A for a full list of codes). As the IMPG is a dynamic model, the arrows from one domain to another indicating Enactment and Reflection are a critical part of the model. Therefore, where applicable, interview segments were coded with the Arrows, the Domains, and the detailed codes. As would be expected, there were very few arrows identified in the initial interview transcripts as no Professional Experimentation had yet taken place. However, arrows were prominent in the coding in the later interviews.

This coding was used to construct summaries and Change Environment figures showing the professional growth for each member of the Matomes team.

A brief extract showing part of the summary for Camilla is shown in Fig. 2. This extract shows that Camilla was responding to a question about what were the main things she had learned through her professional experimentations with the lesson study process (JL) in the Domain of Practice (DP). Her answer indicates that her reflection (R) on this aspect resulted in a realization of "just how important it is for the teacher to have a really good understanding of the content" (CK) – a change in her beliefs in the Personal Domain (PD).

In the Change Environment figure for Camilla (Fig. 4), this resulted in a Reflection (R) arrow from the Domain of Practice (DP) to the Personal Domain (PD).

R: DP-JL → PD-CK – main things that you think you've learnt: Just how important it is for the teacher to have a really good understanding of the content

Fig. 2 An extract from the summary for Camilla illustrating the coding process for arrows

A further layer of coding was constructed for the Change Environment figures, designed to show whether the changes observed were an extension (or enhancement) of previous beliefs or practices (coded with an X) or changes that were new understandings (coded with a C). These letters were then numbered to show in which cycles the extensions and changes were noted as occurring – for example, the code C1 is used to indicate *Changing in Cycle 1*.

4 Findings

This section presents the cases for the five members of the Matomes team that were constructed using the process described above.

4.1 Trevor

Trevor enjoyed teaching mathematics and appreciated opportunities to observe other teachers teach and to receive feedback from the literacy and numeracy coaches to enhance his knowledge and practice of teaching. He was aware that not all people are comfortable with receiving feedback about how they could improve their practice:

I really value observing others but probably a big one for me is getting feedback and giving feedback and taking it on as being constructive, not taking it as a personal attack on what you've taught, but just being able to make you better, make others better in their teaching. (Interview 1, Ln 110–114)

Trevor was therefore looking forward to the project and the opportunity to observe or teach and participate in post-lesson discussions. Planning at Trevor's school involved the Year level team planning the content and sequence and number of lessons for a term. Pairs of teachers, one experienced the other inexperienced, then planned detailed lessons that all teachers would follow. His school (School C) used the Education Department Region's lesson model that involved a warm-up, introduction, student activity and share time. The share time was supposed to be 10–15 min but Trevor found that he often did not have enough time left to conduct this part of the lesson. He recognised that he needed to put more emphasis on “different ways we can solve problems in terms of mathematical thinking” and using the whole class time at the end of the lesson to do this (Interview 1, Ln 719–724). Trevor believed that mathematics teaching needed to be inclusive, that is, to cater for the needs of the individual kids, the ones that need that support and extra guidance, and the case where those ones that need that extension to get to the next level of their learning. (Interview 1, Ln 259–262).

Cycle 1: During the third planning meeting, the Matomes team randomly selected Trevor to teach the first research lesson using the matchstick problem (see Appendix B). The changes implemented and their consequences and impact on Trevor's personal domain and professional practice are displayed in Fig. 3. Trevor trialed

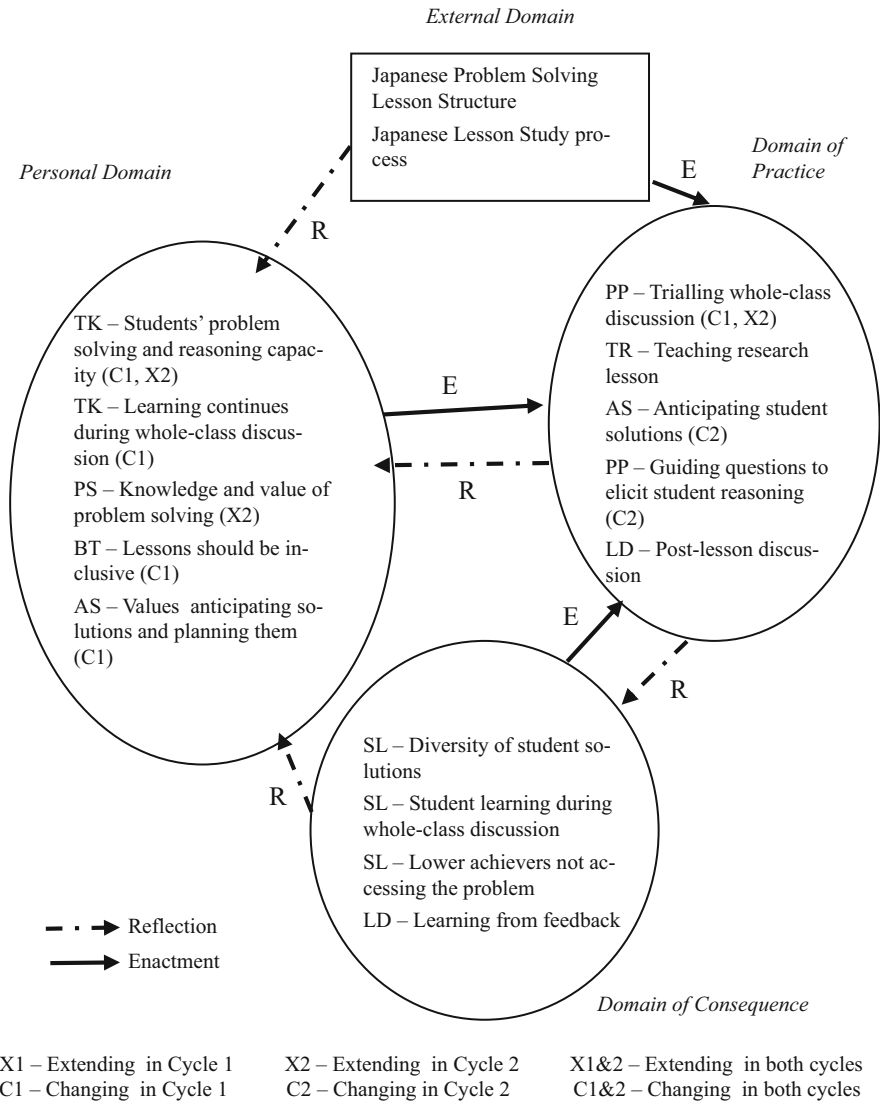


Fig. 3 Trevor’s change environment during Cycles 1 and 2

the Matomes’ draft lesson plan with another Year 3 grade at the school prior to teaching the research lesson in his class. The numeracy coach at his school (who was a member of the Bobbies LS team rather than the Matomes) worked with him to trial the lesson. Together they reflected on the lesson afterward and confirmed that a larger amount of time devoted to the whole-class discussion would work – that is, students could stay engaged longer than expected. After teaching the research lesson, Trevor reiterated the value of the whole-class discussion for students’ learning:

Actually over the past couple of weeks we've been doing that more and spending more time in the reflection part and it seems to work really, really well, I've seen that. (Interview 2, Ln 39–51) . . . And I think the more times we practice [whole-class discussions] with the kids they'll get better and better at it. (Interview 2, 115–20)

He was surprised by the range of solutions generated by students in his class, and this revealed to him of the value of planning for the possible solutions of students:

I know looking at this problem we've done today, I would never have that many possible solutions, but it just shows you how kids think differently. (Interview 2, Ln 153–5)

Trevor used the questions and prompts included in the lesson plan to elicit students' reasoning during the whole-class discussion. He concluded that along with anticipating student solutions, planning questions was also important:

more the questioning and allowing students to answer, giving open questions and allowing them to explain, not just sort of guide, like ask them guiding questions where you know what the answers going to be. Asking open ones where they're explaining and thinking about stuff. (Interview 2, Ln 218–225)

However, Trevor was not convinced that the lesson had worked well for all students in the class especially the lower achievers who used materials and counted by ones and did not see a pattern. He wondered about what enabling prompts he could have used to engage these students:

But I know there's a couple of kids who got no benefit out of it because I wasn't able to give them any prompting or any guidance to get them going. (Interview 2, Ln 430–434)

The post-lesson discussion was driven by the evidence of students' work and included discussion of teacher actions that could have been used to engage these particular students. The post-lesson discussion confirmed Trevor's beliefs about the value of feedback for his learning.

I think I got it from the feedback we got from the post lesson discussion, and that I need feedback, especially in teaching, if you don't get any feedback you're not going to improve. (Interview 2, Ln 359–361)

Cycle 2: In the second LS cycle, Trevor contributed to planning the research lesson, observed Lyn from School B teach it, and participated in the post-lesson discussion. As a consequence of planning and observing a second structured problem-solving lesson, Trevor realized that problems did not have to be open-ended to engage all students: "they're open. . . [but] they're still directed in a certain area as well" (Interview 3, Ln 311–312). They could be designed to achieve a specific learning goal but still allow students to solve them or express their solution in various ways. He also realized that observing a lesson did not just allow him to learn from other teachers but that it provided an opportunity to learn more about students' thinking:

I just watched like four students. . . . I really got to understand how a student thinks. You can make generalisations about how they think but I could never be able to sit there and . . . I really valued that. (Interview 3, Ln 69–74)

Apart from changing his classroom practice to use problems with specific learning goals that provide access for students and various methods of solution increasing the time given to whole-class discussion at the end of the lesson, and using open

questions to elicit students' explanations, Trevor stated that he would also press for changes to the way his school planned lessons. He valued the opportunities to collaborate and learn from others:

I've really enjoyed [the lesson study project], it's given me a lot of opportunities to do things I wouldn't have done before as a – like as a graduate. It's given me the chance to improve my teaching and I've thoroughly enjoyed it. I got to work with a number of different people and listen to the opinions and thoughts and suggestions of you know experts in numeracy and maths. And it's continued my interest in numeracy and makes me want to continue it in the future. (Interview 3, Ln 401–407)

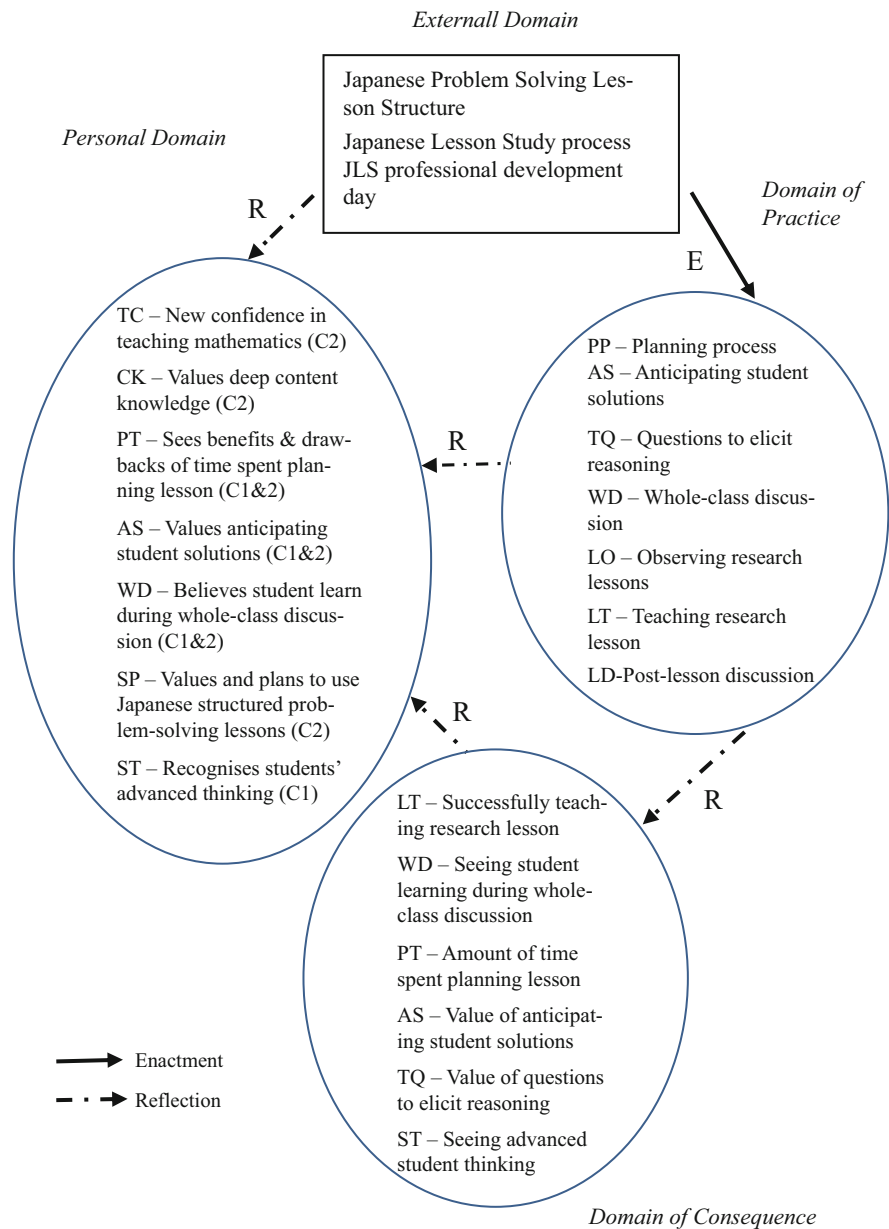
Trevor's reflection on the practices enacted during the lesson study project and in his classroom during the project illuminated salient consequences regarding students' learning, lesson planning, and teacher actions. He acknowledged new understandings of his students and their potential. He continued to develop his practice in conducting whole-class discussions and using questions to elicit children's solutions and reasoning.

4.2 *Camilla*

At the time of this research, Camilla was teaching Year 3 and 4 composite classes at School B, the same school where Narah was the curriculum coordinator. Camilla was in her seventh year of teaching, five of which had been at these year levels, with the other two being as a casual replacement teacher. According to Camilla, teachers at School B were encouraged to work together, with students in mathematics lessons grouped in hubs based on their needs. Camilla stated that she put a lot of effort into her mathematics teaching “because maths isn't my strong point, I always struggled with maths” (Interview 1, Ln 9–10). She had a bit of intermittent mathematics coaching 2 years earlier and more intensive mathematics coaching in her second year of teaching.

Camilla particularly valued professional learning experiences that were “hands-on. the ones that you get to experience the things yourself” (Interview 1, Ln 645–646). She regarded having experts on hand and understanding and support from leadership, and being trusted to do your job as important elements in supporting her professional learning.

Camilla regarded problem-solving as an important aspect of mathematics “because life is about problem solving . . . if you don't teach the kids how to solve the problems they can't do it” (Interview 1, Ln 575; 579–580). She described a problem-solving session as students working together, coming up with a solution on their own, and then sharing it with the others, and talking about the problem-solving strategies they used – which she saw as a major purpose for discussion. However, Camilla had never specifically focused on anticipating student responses to use when sharing student solutions. She described as one of the challenges she faced as the need to “differentiate very much in my class because I have such a wide scope . . . [with] a lot of children that can't work in small group independently” (Interview 1, Ln 543–546).



X1 – Extending in Cycle 1 X2 – Extending in Cycle 2 X1&2 – Extending in both cycles
 C1 – Changing in Cycle 1 C2 – Changing in Cycle 2 C1&2 – Changing in both cycles

Fig. 4 Camilla’s change environment during Cycles 1 and 2

Regarding her assessment practices, Camilla said that she used “lots of anecdotal notes, lots of observations, but pre-tests and post-tests and stuff as well” (Interview 1, Ln 591–592). She reflected on her teaching by thinking about whether the students learned what she intended to teach them and discussing her reflections with other people, rather than writing them down.

Cycle 1: Camilla contributed to planning the Matomes’ research lesson and observed both the Matomes’ research lesson and the Bobbies’ research lesson that used the same task.

While Camilla appreciated the benefits of having such a lot of planning time, she also felt that there were drawbacks, stating that “the amount of time that I’ve had to spend on this has been a challenge” (Interview 2, Ln 49). However, she saw this investment of time as having paid off in terms of her discovering “how important it is to know where you want to take the lesson . . . finding out what the kids are thinking or knowing what the kids are going to say” (Interview 2, Ln 178–179). Further, Camilla believed that in her own planning, she would be working “backwards a bit more, thinking more ‘this is what I want to get out of them through the reflection’ and being prepared for any eventuality and any student response” (Interview 2, Ln 214–216).

As an observer of the two research lessons, Camilla found it interesting to see “how other people would have approached the problem and the different ways teachers react to their students and interact with their students” and “watch the students, [when] you knew where they were going” (Interview 2, Ln 225–229). Overall, she believed the most beneficial aspect of observing a research lesson was being able to discuss it in the post-lesson discussion with other people who might notice something that you hadn’t noticed. Camilla appreciated the fact that the focus of post-lesson discussion was on the teaching and the learning of the students, and not the teacher, and believed that as a result of participating in these discussions, she would try to “remember to be a little bit more reflective in my lessons” (Interview 2, Ln 406). She also believed that she would be spending more time on thinking about her questioning strategies.

As a result of observing the two research lessons, as well as the trial of earlier problem-solving lessons, Camilla was surprised by how advanced some of her students’ thinking was and the way in which some students were able to think algebraically without her realizing it. She saw this as influencing her future teaching with a “lot more of the teaching [being] done during the sharing time . . . [and] helping me think about how I would present future problem solving” (Interview 2, Ln 71–72; 99–100).

Cycle 2: In Cycle 2, as well as participating in the lesson planning and taking part in the post-lesson discussion, Camilla taught the research lesson to her own class. Camilla claimed that, while a lot of what she learned through teaching the research lesson came from:

the planning and preparation. From the actual specific teaching part of the lesson, at the moment all I can say is I learned that I can do it . . . I wanted to prove to myself and to the other people that I could do it. (Interview 3, Ln 330–332; 311–312)

As further evidence of the increase in her confidence, Camilla stated that she was “hoping to have my own lesson study group or lead a lesson study group next year in the school” (Interview 3, Ln 214–215).

Regarding changes she had made or intended to make to her classroom practice as a result of being part of the project, Camilla said:

I have already changed my classroom practice in that I am spending a lot more time on the reflection part of it and thinking – trying to get the teaching done. I am making sure that I have a really good understanding . . . of the content of what I’m trying to teach. (Interview 3, Ln 190–194)

Camilla’s focus on content was also evident in her response to the question of what changes she intended to make to her planning, where she said she intended to start “spending a little bit more time really getting to know the content really deeply” (Interview 3, Ln 299–300) and later that one of the main things she had learned was “Just how important it is for the teacher to have a really good understanding of the content” (Interview 3, Ln 95–96).

According to Camilla, a main benefit of implementing lesson study was:

Changing the teachers’ thinking . . . Getting the teachers to really understand what it is they’re teaching and where they want to go – the point of the lessons, the point of the units. And getting them to do that research before teaching the lesson to sort of think about – why do I want to teach them how to do that. (Interview 3, Ln 582–586)

Regarding the use of the Japanese problem-solving structure, Camilla stated that she intended to use it much more frequently, adding that she had learned that you could

spend most of your time doing the teaching during the reflection stage . . . And that *neriage* – the way of presenting the answers in the order that leads towards an end point – and summarising it all to the final point . . . [has] been fantastic. (Interview 3, Ln 99–103)

Camilla’s views about discussion had also changed; “I always knew that they were really important. But now I can really see how important it is. Just how rich it can be and how good it can be” (Interview 3, Ln 512–514).

However, a major challenge for Camilla was that

You need to really, really know your kids. You need to really know where you want to go. And you need to . . . have those anticipated responses so that you know which examples to have. So you need to be more prepared prior to the lesson. (Interview 3, Ln 402–405)

This challenge was partly overcome by “having the other teachers have their little practice sessions with the same lesson so that you can gather up the anticipated responses . . . it felt like I’d never been more prepared for a lesson” (Interview 3, Ln 133–137). Camilla believed that in her everyday practice she would “do a lesson with the kids, get their responses and then keep that bank of responses for your anticipated responses for the next time that lesson comes around” (Interview 3, Ln 175–177).

Overall, while Camilla described her experiences in the project and the JLS model as “really worthwhile and really beneficial.” One of the problems she had “with it was the amount of time that I was out of the classroom” and “how much extra work was involved” (Interview 3, Ln 572–574; 89–90).

4.3 Sandra

At the beginning of the project, Sandra stated that teaching mathematics “is a passion of mine” (Interview 1, Ln 85–86). She believed that mathematics was all about problem-solving:

I think [problem solving] underpins what we do in mathematics, and how we use mathematics, we use mathematics to solve everyday problems and people use different strategies to come to an end result and everybody uses a different strategy to get there and in the end it doesn't really matter how you get there, as long as you understand how you got there and you're happy with the result. (Interview 1, Ln 332–337)

Sandra believed teaching should engage students in rich tasks with multiple solutions, and that students should be encouraged to explain and justify their solutions. She also focused on developing students' vocabulary so that they could explain their thinking. She believed that using effective questioning strategies during share time and whole-class discussion was important. Teachers at her school (School A) planned in teams. They were encouraged to engage in professional reading as part of the planning process. They also used formative assessment to plan and sequence lessons. This included pretests, student observations-checklists and anecdotal records, and student work samples. They moderated assessment of student work samples. The school followed the Education Department Region's lesson model, though Sandra would sometimes conduct the “share time” in the middle of the lesson.

Cycle 1: Sandra contributed to planning the Matomes' research lesson and observed both the Matomes' and the Bobbies' first research lessons that used the same task. Sandra recognized the value of anticipating student solutions when planning the lessons.

I liked the anticipating the student responses, something that you don't really do in your everyday teaching . . . Certainly thinking about them in the depth that we have . . . has really helped, particularly with that solution selection at the end . . . understanding the range of strategies and where they fit in the scheme of, you know, developmentally. (Interview 1, Ln 29–35)

While observing the research lessons, Sandra focused her attention on a group of students in each lesson and noticed their level of engagement – that is, whether they could access the problem or not (Matomes' Research Lesson) and whether they were challenged to extend their thinking and to generalize (Bobbies' Research Lesson). She noticed that the students tended to rely on less efficient processes or strategies such as counting all by ones, to solve the problem rather than those anticipated and documented in the lesson plan. She noticed that they also struggled to use appropriate vocabulary.

The little people at my table were just not trusting what they were doing, and changing their thinking, but not really having the vocabulary or the means to sort of explain their thinking with their drawings or their words, they changed their labels . . . you'd think they would, you know, trust the count. (Interview 1, Ln 279–282, 289)

She noted the difference in the introductions to the problem in the two research lessons and the difference in types and number of responses from students in the two grades: “[I’m] wondering . . . how the Grade 4s would have attempted our lesson, compared with the [Grade] 3s” (Interview 2, Ln 270–272).

She valued the “getting other people’s ideas, questioning people’s thinking, questioning your own thinking” (Interview 2, Ln. 159–160) that occurred through the planning, observation, and post-lesson discussion.

Cycle 2: As was the case in Cycle 1, Sandra contributed to planning the Matomes’ research lesson, although she was not able to be present on the day the two research lessons were taught. Following this cycle Sandra noted how students’ engagement and learning, problem selection and representation, and anticipating students’ thinking and solutions were impacting on her planning, teaching, and assessment practice.

It has definitely been the most valuable professional learning I’ve done this year. And it has changed my focus with my own numeracy lessons so the way I look at the way students work through a problem had definitely become more focussed, more process oriented and more questions to find out well why are they thinking the way that they’re thinking. (Interview 3, Ln 32–37)

At the same time, the lesson study project had also reinforced or refined her beliefs about teaching and learning of mathematics. The impact of the lesson study project on her professional growth is depicted in Fig. 5.

Sandra came to realize the challenge and importance of selecting the right problem and matching it to the learning goal and students’ readiness:

Probably looking at the problem itself and then trying to match it to specific goals and content area that’s probably been the most challenging. . . . more especially with the last feedback we got. Well why that number? It made perfect sense well yeah why that number when quite easily we could have shown the same thing with simpler number the same sorts of goals that we had so it really identifies the need for picking the right problem for the goal. (Interview 3, Ln 60–68)

She changed the way she conducted whole-class discussion to follow the process used for structured problem-solving lessons in JLS.

I find I’m . . . spending more time on the share time and the share time for me has always been how have you solved the problem but it’s now more looking at the different types of strategies that students use to solve a problem. (Interview 3, Ln 301–305)

She remained committed to using the “why?” questioning when interacting with students but also came to realize the importance of providing visual representations of students’ work and strategies during whole-class discussions.

Sandra also reported changes to her assessment practices, in particular, in her “in the moment” observations of students. Sandra really valued her experience of observing different lessons that used the same task and focusing her observation on a small group of students.

I only focussed on a small group or a particular table and you do realise how much you don’t notice about what students are doing when you’re an observer and you just wish you had the eyes all over the place all the time. (Interview 3, Ln 168–171)

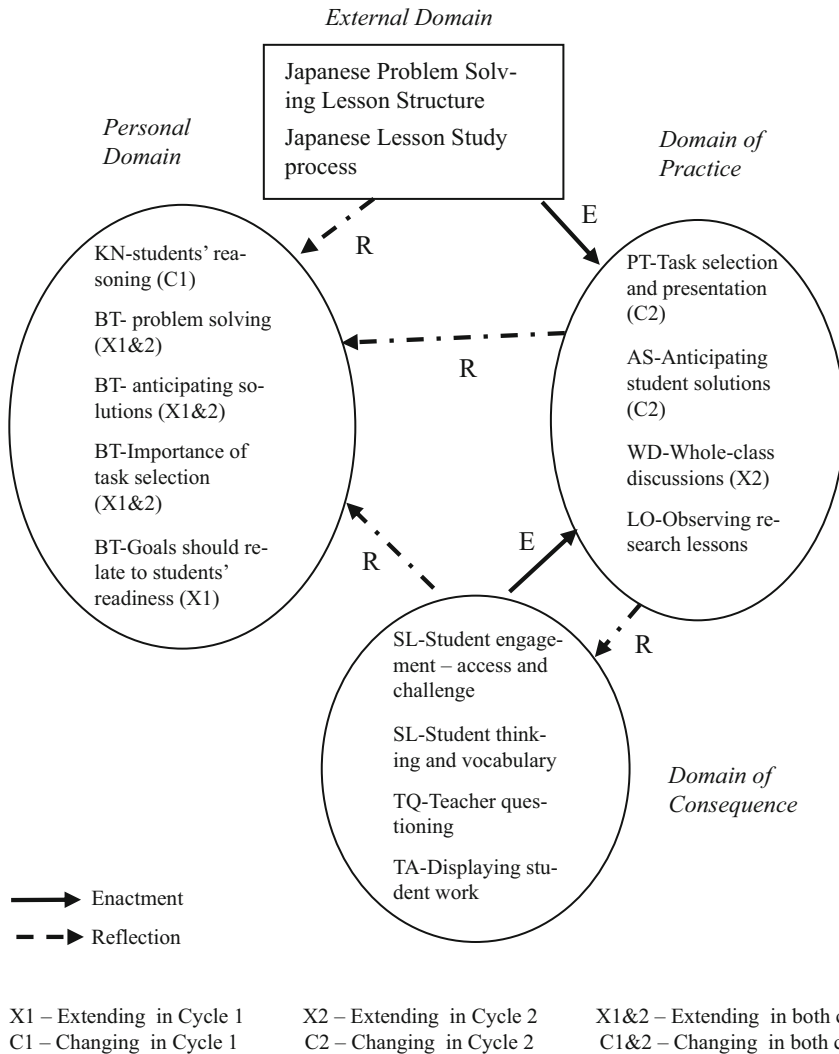


Fig. 5 Sandra's change environment during Cycles 1 and 2

She wants her school to introduce observations of students in other teachers' lessons for teacher professional learning. Sandra reflected on the planning and teaching of the research lessons to identify salient outcomes regarding students' engagement, representation, and launch of the task and the practice of observing students. The experience and consequence consolidated Sandra's beliefs about the value of problem-solving and the importance of teacher questioning during whole-class discussion to support and challenge students when communicating their solutions and reasoning.

4.4 George

George was a numeracy coach and lead teacher at his school, with responsibilities for providing professional development within the school. His focus at the time of the interviews was on the Year 3 and Year 4 teachers, a level at which he had taught for some years. He was “currently undertaking [a] Master[s] [degree] . . . in Numeracy Leadership” (Interview 1, Ln 61–62).

George gave a lot of information about the organization of the classes and the school in general. He was a firm believer in professional learning communities, which he organized for the Year 3 and 4 teachers, and had an emphasis on “student work and assessment” (Interview 1, Ln 84). Under his tutelage the teams planned lessons using the “whole-part-whole” model (Interview 1, Ln 177), with the warm-up “roughly would go for around 5 min . . . then the student activity normally runs for approximately 30 to 40 min . . . then the reflection . . . normally 5 to 10 min” (Interview 1, Ln 188–189). With respect to discussion, George said that he liked “the students to work together for at least the first 10 min of the activity. And then we actually stop as a class and have a class discussion about some strategies . . . then share some different strategies which hopefully allows students . . . to learn from each other” (Interview 1, Ln 221–226). Typical goals for these lessons were for the “students to articulate their thinking . . . show their reasoning and how they actually solve a problem” (Interview 1, Ln 332–334).

Teacher observation of other teachers in their classroom was routinely conducted by George in his role as the numeracy coach as well as by other teachers. He said that “We have collegiate classroom visits . . . that involves a meeting before the lesson to talk about it, the observations and then the meeting after” (Interview 1, Ln 410–411).

George had read about lesson study, as adapted in the United States, and had “taken away and tried to use in [his] team, . . . the idea of research and finding the best way . . . to teach concepts” (Interview 1, Ln 525–528).

Cycle 1: In the second interview, George said that the important things that he had learned during the lesson study cycle were “thinking through the planning and thinking about what student responses might come up” (Interview 2, Ln 22–23). He expressed surprise at how the long planning time was not enough. A further change had occurred in his thinking about the lesson structure: “a really significant shift is how the lesson is structured [with] the share time at the end . . . [and] it’s a great increase in the amount of time . . . we are normally used to here” (Interview 2, Ln 26–9). He further explained that

I’ve been able to go in and really model the planning process, to really think through what the task’s going to evoke from the students and then how I use the students to, sort of, do the teaching through the share time . . . I’ve really tried to model that since starting this [lesson study] process. (Interview 2, Ln 77–81).

While appreciating the advantages of the JLS model, and the structured problem-solving lesson in particular, he didn’t “think that we can jump all the way there, at least to start with” (Interview 2, Ln 145–146). George suggested that teachers would learn a lot about their students’ thinking by using the structured problem-solving

lesson approach, and that this would benefit the students as later lessons could build on the teacher's knowledge of the students.

When asked what changes he had made in his teaching as a result of his involvement in the lesson study cycle, he said

the timing of the lesson and how the lesson is structured that minimising the instruction time at the start, letting the students work on the problem, and then having that larger share time at the end where the students are doing the thinking, and learning, and the talking. (Interview 2, Ln 190–194)

In response to a question about changes that are critical to make in lesson planning, he replied that “building the content knowledge [of teachers] through the process, [so that] teachers [were] not just grabbing activities [but were] looking at . . . the continuum of skills for [the topic]” (Interview 2, Ln 284–287). As a numeracy coach, George observed many lessons, but thought that the lesson study observation was “great” (Interview 2, Ln 329), because, if “you're the teacher a lot of stuff gets forgotten and missed” (Interview 2, Ln 370). He felt that the benefits for the students of the structured problem-solving lesson was that the students were “spending the time to really . . . sinking their teeth into a problem and then also listening to each other, shar[ing] their responses” (Interview 2, Ln 377–379).

The post-lesson discussion aspect of the lesson study cycle had the potential to be difficult, as it might “become an attack on the person who delivered the lesson . . . [but] I think, in general, we kept our focus on the team of teachers” (Interview 2, Ln 452–454) and “how overwhelmingly positive it was” (Interview 2, Ln 489). However, summing up his challenges with regard to lesson study, George reiterated a theme running throughout his interview, in that, “we are time poor” (Interview 2, Ln 502–503).

Cycle 2: The main issue for George was the time needed to plan, observe, and discuss, and yet, despite this constraint, he thought that the strengths of lesson study lie in the fact that “it's a very rigorous process . . . [and] that it's a collaborative process” (Interview 3, Ln 47–49), all of which takes time. In his role as a numeracy coach, George was able to visit classrooms and see the effect the lesson study experience was having on the teachers.

In his school, some of the staff had already started to implement “orchestrating and ordering, and organizing, the student responses during share time” (Interview 3, Ln 71–72), which he described as a big change in teachers' practice. This was underscored in his comment that “an amazing part of the second [lesson study] cycle was when we started talking about the order that we'd wanted to select students in to share” (Interview 3, Ln 189–190). Clearly, this was an important, and powerful, revelation to both George and his colleagues. A further feature of the whole-class discussion, which had an impact on George, was that he now realized that “teaching can still happen there . . . [and] that's been a very powerful lesson from this project” (Interview 3, Ln 85–86) (Fig. 6).

When asked about feedback from the teachers about the lesson study experience, he said that “the idea of the structure of the lesson is something that has [had] a very

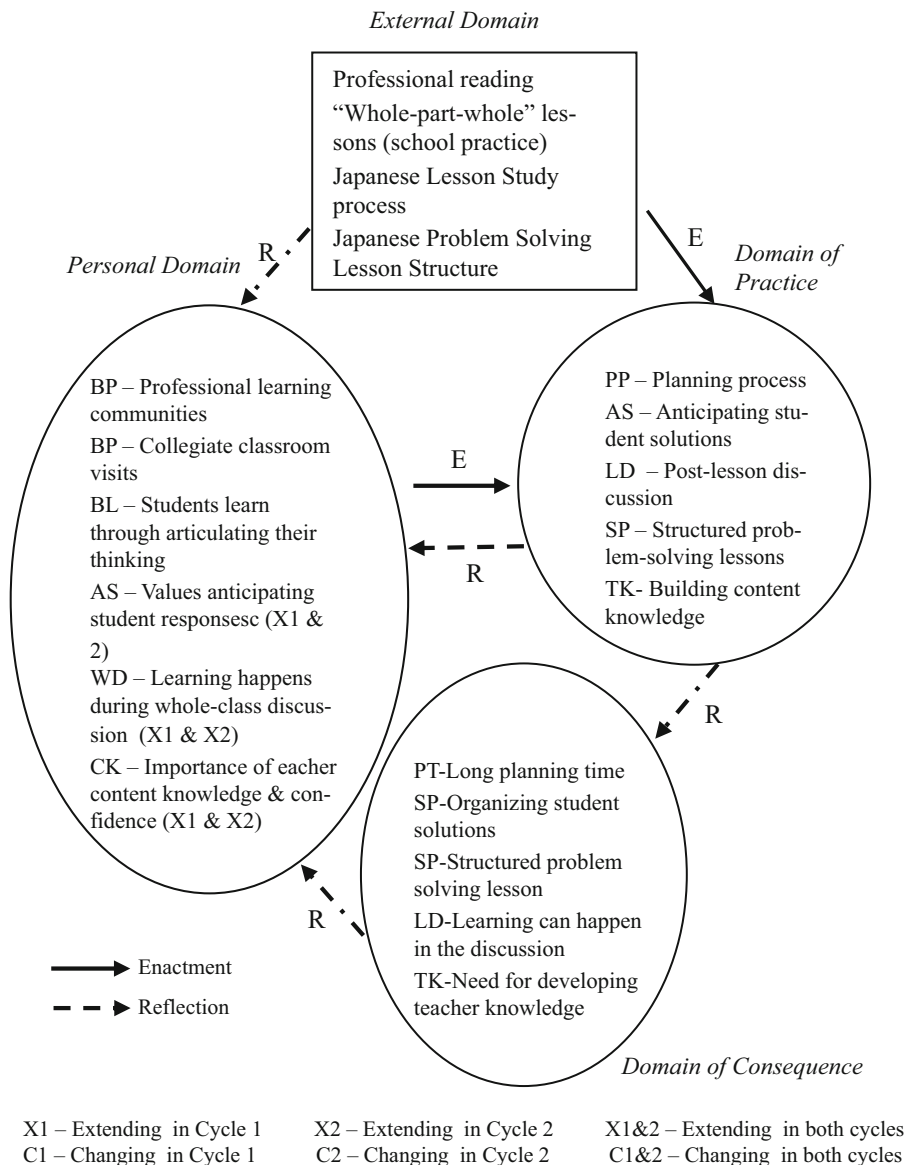


Fig. 6 George’s change environment during Cycles 1 and 2

positive comment and it’s something we’ve actually written into our annual implementation plan for next year” (Interview 3, Ln 302–305).

At the end of the research lessons, the observers provided feedback to the planning team, and then a *knowledgeable other*, or outside expert, made concluding remarks aimed at providing an extension to the mathematical topic of the lesson or

remarking on a pedagogical issue. What struck George were the comments made by the knowledgeable other on the specific numbers used for the problem set for the lesson. Reflecting on the comments, George admitted that normally:

You just pick them randomly, whereas that very deliberate thinking through why you want just those numbers, not even a whole problem or question . . . I think that was just really, really, good. (Interview 3, Ln 359–362)

In summary, George was very enthusiastic about the lesson study process but worried about the time needed to do it properly. The aspects that he thought were very positive were teacher collaboration, planning, and discussion by both students and teachers. An issue for George was that he thought that teacher knowledge was in need of improvement with regard to student misconceptions. While the structure of the lessons, very different from current practice, was seen in a very positive light.

4.5 *Narah*

Narah is the curriculum coordinator with 8 years of teaching experience. She moved to School B 18 months ago, and in previous years, she was the mathematics coach at the school. In her recent role as a curriculum coordinator, Narah participated in regular professional development by the regional coach for the network of schools every third week as well as those delivered by the school coach. Narah believed that teaching mathematics should be inquiry-based and underscored the importance of using assessment data to inform planning in order to ensure students' learning progressions and needs being catered for.

We need to look at what big areas we need to address and then we use a continuum of learning for the children, so we – we need to know where our children are at on that continuum, so based on where they're at that's how we decide our learning intentions. (Interview 1, Ln 355–359)

Her understanding of problem-solving was informed by the regional model problem-solving strategies or heuristics:

We use the WMR problem solving strategies, so those strategies that are linked to different learning styles, so for example, make a model, make a table, look for a pattern and so on, so those are explicitly taught and used across the school as well, to varying degree of success at this point. (Interview 1, Ln 157–161)

She saw the purpose of discussion in the lesson to “bringing all learning together is happening amongst the children in the discussion at the end.” Narah argued that the whole-class discussion should provide students with teaching and learning opportunities beyond just sharing time.

It's the climax of the lesson, it's the drama, it's that point where it's like wow, but yes that's probably a big weakness in our classes that people see it as a sharing time, not as a learning time. I see that discussion at the end -I almost hesitate to call it reflection as well because it's not, it's – that's when the real learning's happening. (Interview 1, Ln 587–592)

Cycle 1: Narah contributed to planning the Matomes' research lesson and observed both the Matomes' research lesson and the Bobbies' research lesson that used the same task. While Narah thought four planning sessions was "a bit much" at the start, she valued the reflective and collaborative discussion and noticed "a real development in our thinking about, . . . the maths that's involved in the lesson and the questioning and really thinking down to every detail." She identified the need for schools to provide time for professional learning and support for teachers "to research and source their own problems" and "resources to support teacher knowledge." She recognized teachers' lack of content knowledge and confidence in teaching mathematics as one of the real challenges and valued the lesson study process in the development of teacher knowledge and teacher questioning skills. She identified the whole-class discussion at the end of the lesson as the most challenging aspect to implementing it in schools.

The sharing part at the end, sharing strategies, is the crux of the lesson, it's so crucial. So therefore that's the hardest part to implement because if you don't get it right, the lesson's essentially a waste of time. . . and that all comes back again to teacher knowledge and unpacking the thinking and the progression of thinking and learning within a task and knowing the maths, in the task. (Interview 2, Ln 395–399)

Narah was initially worried that the post-lesson discussion might be a bit shallow. She underscored the importance of establishing the trust between the teams at the start of the project and the thorough planning process to allow for productive post-lesson discussion.

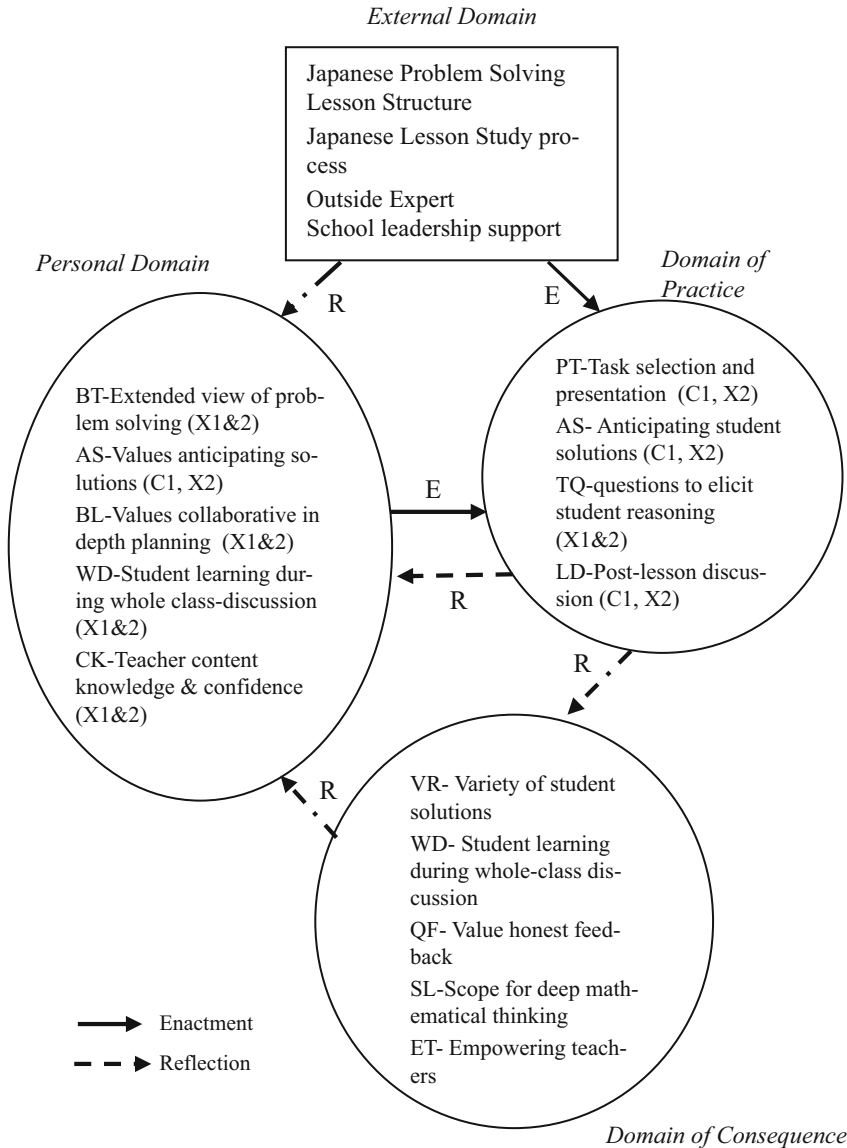
Cycle 2: Narah reiterated the important role of teacher content knowledge and teacher ability to question children and extend their learning as an important driver in teacher professional learning and growth in her Cycle 2 interview. She highlighted the vital role of school leadership to support teachers in terms of structuring time for teachers to collaboratively plan together and embedding this in their professional learning meetings. While acknowledging the importance of collaboration in planning, her view of collaborative planning shifted more toward a community of inquiry perspective.

I think just the importance of collaboration and a big thing that's dawning on our teachers here, is that collaboration doesn't mean, someone else does the work for you and its less work. It's about really challenging each other's thinking and questioning each other, and that's been a big feature of this. (Interview 3, Ln 140–144)

In her role as the curriculum coordinator, she valued the detailed planning in JLS. In particular, she identified the anticipated student responses as one of her biggest learning in the project that she had implemented in her practice in other subject areas.

But the big one is, the big one that can be applied in lots of different contexts, be it maths or your unit of enquiry or literacy, is the anticipated responses. That's a huge one and I've used that now, with a lot of teachers, even those who haven't been involved in this project. (Interview 3, Ln 132–135)

Narah acknowledged the role of the knowledgeable other and appreciated his honest and constructive feedback during the post-lesson discussion. She also realized different cultural contexts and reflected on her own experience as the mathematics



X1 – Extending in Cycle 1 X2 – Extending in Cycle 2 X1&2 – Extending in both cycles
 C1 – Changing in Cycle 1 C2 – Changing in Cycle 2 C1&2 – Changing in both cycles

Fig. 7 Narah’s change environment during Cycles 1 and 2

coach and the curriculum coordinator in providing feedback and that teachers sometimes take it personally when given honest feedback. She commented that “obviously you’ve still got to balance the scales, there has to be support, there’s also constructive criticism” [Interview 3, Ln101–102] (Fig. 7).

I just love his bluntness and it's just such a breath of fresh air to have someone come in and say "Hey you didn't do this" or "Why didn't you do this?" or "You got this wrong" or something, we pussyfoot around so much in Australia I think, with worrying about hurting teachers feeling and things. But as we've always said... it's not about the teacher, it's about the thinking and the learning. (Interview 3, Ln 84–89)

Narah's views of problem-solving shifted after her participation in the lesson study project. Initially during the first interview, her knowledge of problem-solving seemed to be closely linked to Polya's problem-solving heuristics such as "make a model, act it out, draw a list, work backwards." After participating in lesson study, she noticed that teaching these heuristics led to "structuring [students'] thinking perhaps a little too much." She contrasted this with the structured problem-solving approach in JLS, which encouraged students to use a variety of strategies.

Narah highlighted the importance of teacher content knowledge in providing rich learning opportunities for students during whole-class discussion. She has developed an appreciation of sequencing students' strategies according to increasing levels of sophistication – one salient feature of Japanese problem-solving lesson structure.

5 Discussion

Our previous analysis of the Bobbies planning team (Widjaja et al. 2017) involved the use of the IMPG to analyze the interaction between the experiments occurring in the Domain of Practice and the other domains as it occurred – that is, during the planning meetings, the research lessons, and the post-lesson discussions. The analysis documented the change environment that lesson study provides for teacher professional growth. In this chapter we have relied on participants' reflections on the enactment of each of the JLS processes to identify the nature of the change environment, map the interconnections in this environment, and report on participants' professional growth. Using the three individual interviews with each participant, we have been able to identify the connections between the various domains in the change environment and the mediating processes of enactment and reflection across the two cycles as discussed below.

Figure 8 maps the interconnections between each of the domains over the two study lesson cycles, which we have called *The Interconnected Trajectory of Professional Growth*. PD1 is the personal domain prior to enacting the JLS process, while the personal domain, PD3, represents participants' knowledge, beliefs, and attitudes following the second lesson study cycle.

Different experiences among the members of the Matomes planning team were reflected in the varying pace and degree of growth observed within one cycle and between the two cycles. This is consistent with Clarke and Hollingsworth's (2002) argument that teacher professional learning is nonlinear and happens through an iterative process. At the beginning of the study (PD1), the three teachers in the study who were full-time classroom teachers – Trevor, Camilla, and Sandra – valued "hands-on" professional learning tasks for their professional learning and either

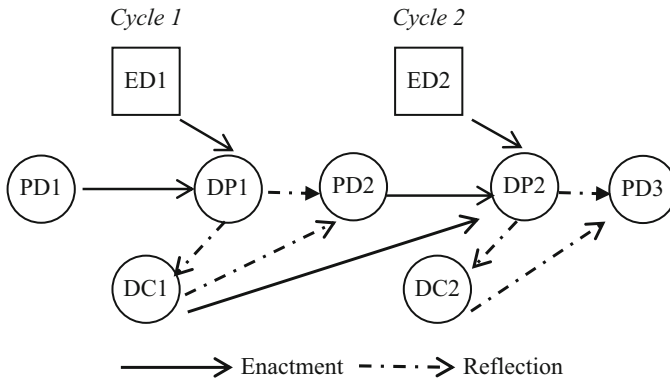


Fig. 8 The *Interconnected Trajectory of Professional Growth* for the Matomes across the two lesson study cycles

feedback from experienced teachers or coaches or the opportunity to share alternate perspectives. All three teachers believed that problem-solving was an important element of the mathematics curriculum and teaching. They believed that small group work was appropriate for problem-solving tasks and that children should share their thinking. Trevor and Sandra had very positive attitudes toward teaching mathematics, whereas Camilla was less confident. In their roles as numeracy coach and curriculum coordinator, George and Narah conducted their coaching and delivered professional development for teachers in their schools. They highlighted the importance of school factors to support teacher professional learning.

Reflecting on the planning and teaching of the first structured problem-solving research lesson (DP1) and the consequences for student engagement, student learning, and teacher actions (DC1), the teachers either extended their knowledge and beliefs about teaching and learning or changed their beliefs (PD2) (See Fig. 8). For example, Trevor and Camilla, the least experienced teachers, developed new understandings of the diversity of students’ thinking and solutions and came to believe that learning continues during whole-class discussion when students are encouraged to explain their solutions to others. All three teachers changed their perception of the role of the whole-class discussion and agreed that they needed to develop their students’ capacity to discuss their solutions and thinking. Sandra’s learning was deeper. She extended her beliefs regarding whole-class discussion and argued that she had learned that students needed to be able to explain and justify their thinking, not just share their solutions. Trevor and Sandra began to change their practice from brief “share time” at end of the mathematics lesson to implement longer whole-class discussions and anticipate student solutions when planning lessons (DP2). All three thought that planning questions to elicit teaching was a practice they should adopt. Sandra also began to think about the way in which the task is represented influences students’ thinking. She realized that she needed to pay more attention to the type of problem she selected to achieve the specific learning goal and to ways of representing and posing the problem.

Initially, both George and Narah raised the same concern about the amount of planning time required to plan one research lesson. Following the first JLS cycle both appreciated the value of structured problem-solving lessons in allowing students to take ownership of their learning and realized the need to develop teachers' questioning skills. Narah saw the benefits of anticipating student solutions and embedded this into her practice. While both of them acknowledged the challenges for teachers in conducting productive whole-class discussions, they saw these as a critical part of the lesson. They also valued the opportunity to focus on a few students during research lessons to learn the progression of students' mathematical thinking.

In the Cycle 2, the teachers used their new or extended understandings of the diversity of their students' thinking and the structured problem-solving approach in general (PD2) to plan and teach or observe the second research lesson (DP2). It was during this cycle that Camilla's professional growth was strongest. Camilla taught the second research lesson and successfully teaching this research lesson greatly enhanced her confidence. She deepened her knowledge of her students and appreciated the need to deepen her content knowledge for structured problem-solving mathematics lessons (PD3). During the Cycle 2, Trevor extended his knowledge of students to develop understanding of the nature and range of thinking and strategies students used to solve the problem (PD3). He also began to change his perception of problem-solving tasks to realize that they could be used to achieve specific learning goals. Sandra was the only one of the three teachers to connect her new understandings of students' thinking and the value of observing students closely while they worked on a problem with her assessment practice (PD3). She had begun to record the strategies and reasoning that students were using in her anecdotal notes during her mathematics lessons (DP2). Teaching the research lesson and the post-lesson discussion that followed impacted on their knowledge of students and planning for teacher actions, such as whole-class discussions. Sandra, the most experienced of these teachers, deepened her knowledge and practice of informal formative assessment and task selection through student observation.

At the end of the project, all three teachers aspired to make changes to the planning and evaluation processes at their school (PD3). Each of them recognized the value of research and the use of larger planning teams to bring together a range of experience and knowledge. Sandra was particularly keen to enabling teachers to observe each other's lessons with a particular focus on students' thinking. Camilla wanted to enact lesson study with her colleagues. This largely arose as a consequence of the different teachers' observations and reflections that occurred during the post-lesson discussions (DC1 and DC2), where the focus was on the students' learning and the planned lesson, rather than on the teacher.

Both George and Narah valued collaborative planning in lesson study. Narah perceived the order of comments during the post-lesson discussions – which started with the teacher who taught the lesson, followed by the planning team – as empowering the participating teachers to own the professional learning. In the Cycle 2, their appreciation of the whole-class discussion and the process of anticipating student solutions were strengthened. Both George and Narah also reiterated the value of the outside expert in providing constructive feedback and extending

their professional learning. They also emphasized the importance of support from school leadership to provide time for teachers to engage in professional learning through lesson study.

The analysis of the participants' professional growth during the lesson study project reveals the iterative and interconnected nature of their learning. It was not based on a discrete set of events or tasks; rather it involved participants in an ongoing process of research, experimentation, and reflection in their own classrooms, at the same time as they were participating in the formal activities of the lesson study process – the planning meetings, research lessons, and post-lesson discussions that occurred over two school terms.

6 Conclusion

There were some challenges in applying Clarke and Hollingsworth's (2002) *Interconnected Model of Professional Growth* (IMPG) to the data from the interviews of teachers and numeracy coaches. The four domains in the model were to be fleshed out with data drawn from the interview transcripts, but as the transcripts were analyzed, it was discovered that pieces of evidence were being lost in the overarching domain descriptions. Thus, it seemed necessary to define codes that kept the individual pieces of evidence visible, rather than obscured. The number of codes grew as more interviews were analyzed, with the final list being shown in Appendix A.

It was imperative in this research to trace the changes in individual participants' beliefs, understandings, and attitudes as a result of participating in lesson study. Thus, the issue was how to register changes and differences in a static representation, for example, on paper.

Our findings imply that investing in in-depth, quality planning, with a focus on advancing students' thinking, leads to teachers' professional growth (Hargreaves and Fink 2009). It also highlights the importance of establishing support within the school and across the school network, such as providing time for teachers to plan together, observe each other, and learn from each other's teaching. Ongoing collaborative enactment and reflection was vital throughout the two lesson study cycles to facilitate teachers' professional growth through lesson study.

The IMPG depicts the change environment and shows the interconnections between domains. The IMPG model enables us to identify elements in each domain and trace individual teacher' change as a result of their professional experimentation mediated by the processes of enactment and reflection. However, the IMPG model does not allow us to track when the change occurs and whether the change is sustained over a period of time. We posit that the use of the *Interconnected Trajectory of Professional Growth* (Fig. 8) adds the dimension of time so that we can track and map professional growth over time, while acknowledging the interactive elements of the change environment.

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Appendices

Appendix A: List of Codes

Code	Meaning	Code	Meaning
AP	Assessment practice	PL	Professional learning
AS	Anticipated student solutions	PP	Planning process
BD	Belief about discipline	PR	Professional reading
BL	Belief about learning	PS	Problem-solving
BP	Belief about professional learning	PT	Planning time
BS	Belief about support	QF	Constructive quality feedback
BT	Belief about teaching	RC	Resources
CC	Collaborative coaching	RF	Regional factors
CH	Challenges	RP	Reflective practice
CK	Content knowledge (teachers')	SC	Student confidence
DL	Deep learning	SD	Shallow discussion
DP	Domain of practice	SF	School factors
ED	External domain	SG	Small group discussion
EF	Expert feedback	SL	Student learning
ET	Empowering teachers	SM	Student misconceptions
GM	Good models (Japanese teachers)	SO	Salient outcomes
JD	Japanese professional learning day	SP	Structured problem-solving pattern
JL	Japanese lesson study process and model	ST	Articulate student thinking
JP	Japanese lesson study project	TA	Teacher actions
LD	Post-lesson discussion	TC	Teacher confidence teaching mathematics
LG	Learning goal	TI	Teaching/teachers' ideas
LO	Research lesson observation	TK	Teacher knowledge
LT	Research lesson teaching	TP	Teaching practice
LV	Research lesson variations (between two teams)	TQ	Teacher questioning
OE	Outside expert	TR	Trial lessons (and observation of these)
PD	Personal domain	TT	Teachers' thinking time
PE	Professional experience	VS	Variety of student solutions
PI	Practical implementation	WD	Whole-class discussion

Appendix B: The Matchstick Problem (Cycle 1 Research Lessons)

I used some matchsticks to make squares connected side by side as shown below.
How many matchsticks would I need if I was to make eight squares?
What if there were 100?



Appendix C

The 23×3 multiplication problem as presented by the researchers for *Cycle 2 Research Lessons* and used by the Bobbies and the Matomes

Here is a diagram of some dots.
Can you work out how many dots are in the diagram without counting them one by one?
Please make sure that you're showing your thinking in the space provided.



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