

# Chapter 18

## Critical Mathematics Education and Statistics Education: Possibilities for Transforming the School Mathematics Curriculum



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**Abstract** This chapter discusses how ideas from critical mathematics education and statistics education intersect and could be used to transform the types of experiences that students have with both mathematics and statistics in the school mathematics curriculum. Key ideas from the critical mathematics literature are described to provide a background from which to discuss what a critical statistics education could be. The chapter ends with a discussion of some of the major barriers that need to be considered to make such a vision a reality and possible future directions for moving towards making a critical statistics education a reality.

**Keywords** Critical literacy · Critical mathematical education · Statistics education  
Statistical literacy

### 18.1 Importance of Statistics in School

Data are everywhere in society today, aimed at influencing our decisions about what toothpaste to buy, what politician we should vote for, or what medicine is the best treatment for what ails us (Steen 2001). Today huge breakthroughs in science, medicine, economics, and public policy are being made using advanced data modeling techniques (Davidian and Louis 2012). Statistics, which is often described as the science of data, is becoming an increasingly important topic of study because of our society's reliance on data (Ben-Zvi and Garfield 2008; Gattuso and Ottaviani 2011). Experts and policy makers in many fields are increasingly basing their decisions on statistical results, using data to draw out new insights about the world (Pfannkuch 2008). In data driven societies, it is crucial that individuals are able to interpret and critically analyze quantitative data and statistics (Ben-Zvi and Garfield 2004) to be critical citizens (Skovsmose and Valero 2008). As the need has increased for governments to become more transparent in their operation and decision making,

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it has become crucial for citizens to have a strong statistical literacy to make sense of technical reports (Ullmann 2016). Furthermore, recent votes in both England (Brexit) and the United States (2016 Presidential Election) were fraught with misinformation campaigns many of which included the misuse of data based arguments (Belham 2016), which points to the importance of statistical literacy for being an engaged critical citizen. One of the commonly held goals of public education in the United States is to prepare students to become citizens of society (Labaree 1997). In light of the data centric focus of modern societies, a goal of public K-12 education should include teaching students to be statistically literate active citizens in their data-driven societies.

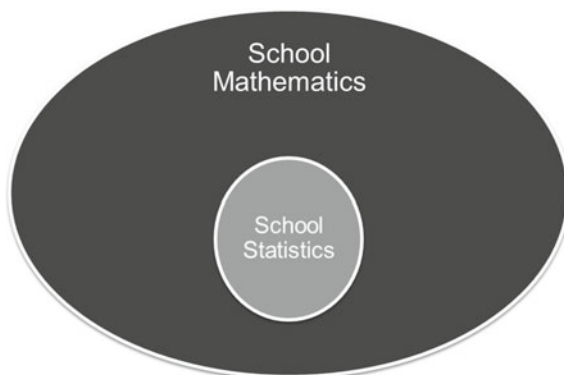
As the prevalence of discourse using data increases it is important that students are able to critically interpret such discourses in their context in society. The term critical is used here not in the critical thinking sense that is generally used in education drawing upon the traditional ways of knowing of different disciplines. Instead it is used in the way critical or emancipatory pedagogical writings describe to interrogate, problematize, and reconstitute discourses that are dehumanizing, unjust, and position groups as others (Darder 2014; Freire 1970; Giroux 2011; Gutstein 2003). In the case of statistics this involves using statistics to investigate the underlying structures and hidden assumptions present in society and also to critique and understand the hidden assumptions in the use of statistics. Many assumptions come with the use of quantitative data that are not always apparent in the contexts students are given to investigate in their school classrooms.

This chapter presents an argument, rooted in past scholarship, for a critical statistics education in conjunction with critical mathematics education (Skovsmose 1994a) in K-12 education with the intention of giving students experiences with reading and writing the world with mathematics (Gutstein 2003, 2006, 2013) and statistics. This is not an entirely new idea as Lesser (2007) has introduced the notion of teaching statistics with social justice, and a number of scholars have begun to delve into the complexities of raising sociopolitical issues in statistics classes (e.g. Bergen 2016; Engel 2016; Gray 2016; Poling and Naresh 2014). Also Gal (2002) began to draw Freire's (1970) ideas into his work on statistical literacy more than a decade ago in terms of critical dispositions. However, the majority of this work has been done or written for a post-secondary statistics education audience, and only a small subset of most societies' citizens goes on to post-secondary education. The use of data based arguments and statistics today is exploding in almost every setting making it crucial for all individuals to have experiences to learn and use statistical concepts and practices to make sense of the types of sociopolitical issues they will need to navigate and make sense of in their daily lives as citizen's in today's modern societies.

## 18.2 The Complex Nature of Statistics in the School Setting

K-12 educational settings are very different and far more complex than most post-secondary settings in several regards. To begin with students in K-12 settings are not yet considered adults and are typically under the direct care of their parents

**Fig. 18.1** The situatedness of statistics in school



and families in many nations, which also means that families also commonly have a voice in their children's education. Past research has shown parents can be quite influential in shaping what students are taught in mathematics classrooms (Boaler 2002; Herbel-Eisenmann et al. 2006). Unlike most post-secondary settings where students predominantly self-select the institution and curriculum in which they wish to participate, students in K-12 settings have little agency over the institutions they attend or the curriculum they experience, which is predominantly determined by geography and governmental policy. Furthermore, what students are taught in schools must be negotiated by a significant number of stakeholders including: parents, policy makers, teachers, researchers, disciplinary experts, politicians, etc. (Apple 1992). These stakeholders in turn bring all their own beliefs, values, and perspectives to bear on shaping the school mathematics curriculum. Unfortunately, the result has often been a very neutral mathematics curriculum, which does not address the truly political nature of mathematics education or the formatting power it has in shaping the world around us (Gutiérrez 2013a; Skovsmose 1994b).

The teaching and learning of statistics in the school setting is further complicated because it is situated in the mathematics curriculum, as modeled in Fig. 18.1, where it has only begun to gain a foothold in terms of statistical thinking and reasoning (Scheaffer and Jacobbe 2014). Furthermore, statistics is generally taught by mathematics teachers who may have had little to no past experience with statistics (Shaughnessy 2007), not by statisticians who generally teach much of post-secondary statistics. This means that many of the teachers directly shaping the curriculum that students experience in the mathematics classroom are likely more enculturated in the practices of the discipline of mathematics than they are of the discipline of statistics. Unfortunately, as Eichler and Zapata-Cardona (2016) point out, empirical research on mathematics teachers' teaching of statistics in the K-12 setting has been limited up to this point. Pointing out some of the complexity of considering statistics education in the K-12 school setting illustrates how different it is in many regards compared to post-secondary settings where much of the statistics education work around sociopolitical issues has been done.

### 18.3 Critical Mathematics

The world today is faced with a multitude of challenges such as economic collapse, poverty, resource depletion, climate change, polarization in wealth, extreme nationalism, and immigration/migration. As a result more and more scholars are advocating for bringing these issues into school classrooms (Apple and Beane 2007; Giroux 2011; Ladson-Billings 1995). Scholars in mathematics education have been advocating for similar efforts in the teaching of mathematics as well. In consideration of the social, political, and ethical dimensions of mathematics education, scholars in mathematics education over the past two decades have begun calling for the use of critical (Frankenstein 1994; Skovsmose 1994a; Wager and Stinson 2012) and culturally relevant pedagogies (Gutstein et al. 1997; Ladson-Billings 1995). These scholars seek to create mathematics classrooms where students learn how to understand their social, cultural, and political context in society as well as how to change that context. There has also been a growing literature base in the field of mathematics education based around incorporating social and political critique into the mathematics curriculum (Gutiérrez 2009, 2013a; Skovsmose 1994a; Skovsmose and Valero 2008). Many of these scholars argue for centering pedagogy around problem posing and connecting content areas to fundamental questions of society rather than focusing on neutral or trivial problems or contexts (Frankenstein 2009; Freire 1970; Gutstein 2006; Gutstein and Peterson 2013). This is a serious problem because as Skovsmose (1994b) points out,

It is important to relate the idea of the invisibility of mathematics to the assumption about the formatting power of mathematics, because if both assumptions are correct, we witness a challenging and critical situation for mathematics education. This conflict has been formulated as the paradox of relevance: on the one hand, mathematics has a pervasive social influence and, on the other hand, students and children are unable to recognize this relevance. (p. 82)

Critical mathematics scholars argue that students need opportunities to see and experience the pervasive influences mathematics has on the social world. To clarify how the word critical is used in this scholarship, Gutstein et al. (1997) bring up an interesting point about critical mathematical thinking and how critical has two meanings in this instance. One meaning for critical is in the mathematical sense as in making sense of problems, creating arguments, making conjectures, critiquing the reasoning of others, ideas that generally fall under the term critical thinking. We see these in NCTM's (2000) *Principles and Standards*, which have been taken up in a wide variety of educational settings worldwide. For example, the reasoning and proof standard includes "make and investigate mathematical conjectures" and "develop and evaluate mathematical arguments and proofs" (NCTM 2000, p. 56). There is also the meaning of critical in the broad sense, using multiple perspectives to look at an issue, and questioning the context in which one is situated and in education questioning standards, curriculum and practices (Gutstein et al. 1997). It is this second meaning of critical that critical mathematics education scholarship contributes to mathematics

education. As Skovsmose (1994b) describes in his book *Towards a Philosophy of Critical Mathematics Education*,

*If educational practice and research are to be critical, they must address conflicts and crises in society. Critical education must disclose inequalities and suppression of whatever kind. A critical education must not simply contribute to the prolonging of existing social relationships. It cannot be the means for continuing existing inequalities in society. To be critical, education must react to the critical nature of society. (p. 22)*

In Gutstein's (2003, 2006) writings he discusses his work teaching mathematics for social justice in urban public schools incorporating issues brought by his students such as urban planning, stop and frisk, gentrification, and AIDs/HIV. He discusses how he created challenging mathematics curriculum with students that pushed students to be academically successful in mathematics but also provided students with experiences using mathematical concepts to investigate and critique their own context in society. Gutstein (2006) draws heavily from Paulo Freire's literacy work, which was done predominantly in Brazil to help the marginalized of that nation in the latter half of the twentieth century to become literate, to make sense of and in turn influence and improve their reality and position in their world. Freire and Macedo (1987) discussed literacy in terms of reading the word and the world, learning to make sense of symbol systems by using them in conjunction with making sense of the world around oneself. They also discussed writing the word and the world, emphasizing how literacy can empower people to make sense of the world around them but to also influence and shape the world around them. Gutstein (2006) draws heavily from these notions to describe how to envision reading and writing the world with mathematics. He describes reading the world with mathematics as meaning:

to use mathematics to understand relations of power, resource inequities, and disparate opportunities between different social groups and to understand explicit discrimination based on race, class, gender, language, and other differences. Further, it means to dissect and deconstruct media and other forms of representation. It means to use mathematics to examine these various phenomena both in one's immediate life and in the broader social world and to identify relationships and make connections between them. (Gutstein 2003, p. 45)

This definition emphasizes how mathematical literacy can be used to read the word, which increasingly includes mathematical and quantitative language (Steen 2001) and also to read the world, which has been structured based on quantitative and technological discourses rooted in the abstract language of mathematics (Skovsmose 1994b). Reading the world with mathematics can in turn lead to writing the world with mathematics, which Gutstein defines as:

using mathematics to change the world... I view writing the world with mathematics as a developmental process, of beginning to see oneself capable of making change, and I refer to writing the world for youth as developing a *sense of social agency*. A "sense" of social agency captures the gradual nature of students' growth-it is not an all-or-nothing proposition. (Gutstein 2006, p. 27)

Writing the world with mathematics in this sense also implies being able to use mathematics in a meaningful way for positively changing the world, which would

seem to be very much in line with the goals of mathematics curriculum policy documents such as the Common Core Standards for Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers 2010) in the United States. However, in that document's description of its goal for mathematics education, making every student college and career ready, there is no specific connection made to changing the world for the better. It is neutral with respect to ethics to guide what constitutes a positive change or to even consider positive change in the context of mathematics, which is something that critical literacy emphasizes (Giroux 1993) and that can be seen in Gutstein's (2003, 2006) definitions of reading and writing the world with mathematics.

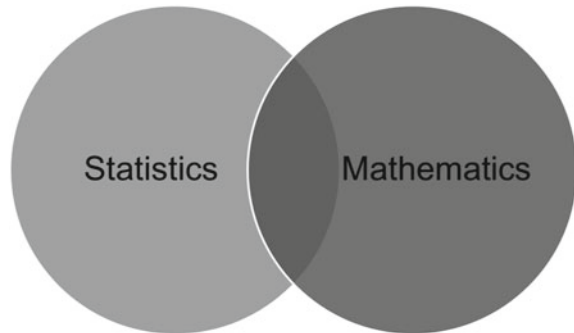
Mathematics can be used to change the world in very unjust ways. For example, consider the recent economic recession after the housing bubble burst. A number of bankers were blamed for using their abilities with mathematics, in unscrupulous ways, to cheat individuals out of large sums of their money, which sent the economy into a downward spiral (Cohan 2015). Just because an individual has the mathematics skills and abilities to be ready for a career does not mean they have any ethical principles behind how to use those abilities. If the goal of education truly is to prepare critical citizens for participation in democracy (Apple and Beane 2007; Giroux 1989) there needs to be some sense of developing ethics to guide such participation (Giroux 1993). Ethics is often linked to notions of fairness and social justice in mathematics education (Boylan 2016) but is a very murky term used and taken up in a wide variety of ways. I use the term ethics here broadly to mean a set of philosophical or moral values that people use to make decisions in relation to others, their communities, and the world at large.

The notions of critical mathematics and the idea of reading and writing the word and the world with mathematics can be drawn on in considering how to in turn foster students in K-12 school settings to read and write both the word and the world with statistics. Critical statistics education is needed in spite of a growing literature base around critical mathematics education because, as described in the next section, the disciplines of mathematics and statistics are distinct.

## 18.4 The Disciplines of Mathematics and Statistics

Mathematics is a socially created dynamic body of knowledge with a social history, and with areas expanding and contracting over time situated in context (Bishop 1988; Davis and Hersh 1981). A common definition of mathematics drawing from the Oxford English Dictionary is "the abstract science of number, quantity, and space" (Mathematics n.d.). However, what is considered to be the specific scope and knowledge of mathematics is socially agreed upon by members of the discipline and changes with time (Davis and Hersh 1981), which makes it inherently political. As a point of clarification, I am using the term political in the way it is commonly

**Fig. 18.2** The relationship between the disciplines of mathematics and statistics



used in sociopolitical perspectives, which is to describe any situation that involves making a choice or decision as there are always multiple options or perspectives in such situations, where one is chosen and advantaged over others and such decisions are always situated in power relations. From the sociopolitical perspective defining mathematics is political in the sense that its boundaries, practices, and what is considered knowledge are situated in, and shaped by, power relations between individuals and institutions, where certain views and perspectives may advantage some, while disadvantaging or silencing others (Gutiérrez 2013b). Statistics formally came into fruition in the 18th century (Katz 2009; Stigler 1986) and has similar characteristics. However, instead of being looked at as the science of quantity and space it is often viewed as the science of data or measurement (Davidian and Louis 2012; Stigler 1986). Both mathematics and statistics are part of the mathematical sciences (Steen 2001). As Moore and Cobb (2000) describe, “mathematicians and statisticians share a commitment to a process of pattern searching, generalization, and verification that operates at a deep level, despite surface differences” (p. 22). Furthermore, mathematics and statistics are linked through probability, which is a part of mathematics that is crucial to statistical inference (Fienberg 1992). However, statistics is its own distinct discipline not a sub-discipline or branch of mathematics (Cobb and Moore 1997; Franklin et al. 2007; Gattuso and Ottaviani 2011; Groth 2013), but it is inextricably linked to mathematics as modeled in Fig. 18.2. As Steen (2001) points out, “Although each of these subjects shares with mathematics many foundational tools, each has its own distinctive character, methodologies, standards, and accomplishments” (p. 4).

Some of the main differences between statistics and mathematics are discussed in the literature and are described in the next section. This is done not to create divisions or to argue for taking statistics out of the mathematics curriculum but instead to embrace the diversity of these two disciplines and how they provide different ways of looking at the world. Furthermore, such differences can position statistics as a powerful entry point for interrogation of sociopolitical issues in the school mathematics curriculum, which is discussed later.

## 18.5 Differences Between Statistics and Mathematics

There is a strong literature base that discusses the differences between the discipline of mathematics and that of statistics (Cobb and Moore 1997; Franklin et al. 2007; Gattuso 2006; Gattuso and Ottaviani 2011; Groth 2007, 2013; Pfannkuch 2008). The major differences discussed in these works are context, variability, uncertainty, and inductive versus deductive reasoning.

In statistics, “data are not just numbers they are numbers with a context” (Cobb and Moore 1997, p. 801). This is a departure from mathematics, particularly what is taught in schools, where numbers are frequently presented and used in their abstract form without any connection to context (Gattuso and Ottaviani 2011). As Cobb and Moore (1997) discuss, in mathematics, context is generally stripped away from a problem to try and uncover, or abstract, the underlying mathematical structure of the context. However, in statistics, the analysis of data cannot be considered without thinking about the context of the data (Cobb and Moore 1997; Franklin et al. 2007; Wild and Pfannkuch 1999). There is a constant interplay between considering a statistical problem and the context of the problem (Groth 2007; Wild and Pfannkuch 1999).

The need for the discipline of statistics comes from the omnipresence of variability (Cobb and Moore 1997) in the world. Simply stated in most cases individuals or objects that we study are not all the same for every attribute. Therefore statistics focuses on how attributes can vary from individual to individual or object to object. In statistics there are four main kinds of variation: measured, natural, induced, and sampling (Franklin et al. 2007).

It is important to point out that variation is not absent from mathematics, but that it is considered in a very deterministic way. Consider a linear function, the values of dependent and independent variables covary with one another but in a specific unchanging way (e.g. “as  $x$  increases by 2  $y$  increases by 3”). The discipline of statistics uses the concept of linear functions. However, instead of determining what the value of the dependent ( $y$ ) variable will be, given the value of the independent ( $x$ ) variable, the function is used to “fit” the data. In other words, a linear function can be used to summarize the relationship between an independent or explanatory variable and a dependent or response variable, where the accuracy of a prediction depends on the amount of variation between the observed and predicted values of the response variable explained by the explanatory variable in the model. This procedure is used in an effort to create a linear function that best “fits” the data, but will only provide predictions, which may not be very accurate depending on the amount of variation the model can account for in the actual data values.

As a result of the omnipresence of variation in statistical investigations, there is no certainty in the solutions. The end product of a statistical investigation is better thought of as a well principled argument (Abelson 1995). Mathematics on the other hand is generally treated in a very deterministic way, logically deducing a single solution to a problem using theorems, axioms, and definitions from the community of mathematics (Gattuso and Ottaviani 2011).



Another main difference between statistics and mathematics is the type of reasoning generally used. Mathematics primarily relies on deductive reasoning using definitions, axioms, and theorems, in a logical chain of reasoning, to come to a conclusion. For example, a student could use Euclid's definition of a circle and his first and third postulates to construct an equilateral triangle. At the same time Euclidean geometry is based on certain unprovable assumptions such as the parallel postulate, which if changed creates an entirely new type of geometry and way of viewing the world (Katz 2009). The practice of statistics is often driven by a question for which data are collected analyzed and interpreted to answer the question (Franklin et al. 2007; Wild and Pfannkuch 1999). It is from the data that information is empirically derived, which is the hallmark of inductive reasoning. Similar to uncertainty this can lead to issues in teaching statistics as teachers who have had few experiences with statistics may attempt to deduce solutions from rules and assumptions to find a single certain answer rather than inducing them from the data to find a range of possibilities.

These differences can also lead to common statistics teaching practices that are different from those of mathematics. In this regard, teaching concepts from statistics is not the same as teaching concepts from mathematics, though clearly there are parallels between the two (Franklin et al. 2007; Gattuso 2006; Gattuso and Ottaviani 2011). Since statistics is often situated within the mathematics curriculum at the K-12 level, this position can give the impression that statistics is just a branch of mathematics (Groth 2007). This is not to say that statistics should be taught outside of mathematics at the K-12 level (Franklin et al. 2007; Usiskin 2014). However, it is important to understand that statistics is a distinct discipline and as such there are different strategies, habits of mind, and practices involved in teaching concepts from statistics (Cobb and Moore 1997; Groth 2007). One approach is that of Gattuso and Ottaviani (2011) who aim "to emphasize the necessity of complementing statistical thinking and mathematical thinking in school and generating didactic strategies allowing statistics and mathematics to evolve together, in a harmonious way" (p. 122). They also state that statistics concepts and problems can be used to compliment mathematical thinking and bring more context and students' interests into mathematics classrooms. For these topics to evolve together in K-12 curriculum teachers should know both their similarities and their differences. How critical statistics could be envisioned differently than critical mathematics based on some of the disciplinary differences described in this section is elaborated on in the next section.

## 18.6 Differences Between Critical Statistics and Critical Mathematics

Critical mathematics has had several decades to build a base of literature and create examples for using mathematics for social justice. A number of the examples of critical mathematics activities in the classroom involve concepts from statistics. This

section presents two examples of such activities and points out how these activities could be expanded to provide students with robust experiences involving statistics.

Skovsmose and Valero (2008) in their paper on *Democratic access to mathematically powerful ideas* use an example that involves drawing samples of eggs from a population of eggs and seeing how many are infected based on a rate of infection reported by the Dutch government. This is inherently a statistical task creating a model based on chance to model getting an egg infected with salmonella. Some of the questions the authors had the students consider as part of this task were:

The basic question to be addressed by this experiment has to do with the *reliability* of information provided by samples. How can it be that a sample does not always tell the “truth” about the whole population? And how should we operate in a situation where we do not know anything about the whole population, except from what a sample might tell? How can we, in this case, evaluate the reliability of numerical information? (p. 9)

These questions are very rooted in statistical practices. This activity discussed by Skovsmose and Valero is meant to begin to get students to discuss the differences between ideal mathematical calculations and figures from empirical data collection, which is an important difference between the ways of knowing in mathematics and those of statistics. However though sampling was discussed, there was no emphasis on a discussion of variability, which would become a focal point if there had been an explicit focus on teaching statistics in this activity. For example, students could compare their samples across groups to facilitate a discussion of sampling variability, which could be expanded upon to begin to introduce the idea of sampling distributions as well as develop the ideas of standard error and margins of error to emphasize the variability present in empirical work.

Again the point is not to separate mathematics and statistics as they are deeply connected and statistics relies heavily on mathematics (Groth 2007, 2013). Instead, the aim is to point out the differences so that they are emphasized and not lost in classroom mathematics instruction. For example, in Skovsmose and Valero’s (2008) discussion of the egg task, they move into the mathematical realm of calculating theoretical probabilities instead of highlighting the idea of variation and how it can be measured and interpreted. Now this move might be because Skovsmose and Valero are trying to communicate to the mathematics education community in this particular example. However, in many cases this is the same community that is tasked with teaching statistics in the school setting and could potentially benefit from seeing specific examples of statistical practices in conjunction with the teaching of mathematics.

Another example comes from a project that Gutstein (2003) used in one of his mathematics classes described here:

For example, I developed a project in which students analyzed racially disaggregated data on traffic stops. The mathematical concepts of proportionality and expected value are central to understanding racial profiling. Without grasping those concepts, it is hard to realize that more African American and Latino drivers are stopped than one would expect, and this disproportionality should lead one to examine the root causes of the anomaly (p. 49).

Similar to the egg problem, the mathematical concepts become the focus of the discussion where there are also opportunities to discuss and investigate important

statistical concepts. In this example Gutstein (2003) focuses on the idea of proportionality to discuss how the African American and Latino/a drivers were pulled over disproportionately due to probabilistically determined expected values. However, these disaggregated data can be considered an example of a sample, which provides an opportunity to talk about variation in samples and to look at this proportion in terms of a sampling proportion. A sample proportion has a margin of error that needs to be considered when making inferences to a population. This context also provides the opportunity to introduce the idea of sampling distributions, which could be developed using a simulation to bootstrap a sampling distribution from the sample drawn. In this way, students could also begin to investigate what it means for a sample proportion to be unlikely to be drawn due to chance and to begin to make inferences about the population proportion.

The discussion above illustrates how rich mathematical lessons could be used to teach statistical concepts in mathematics classrooms to possibly help students to read and write both the word and the world with both mathematics and statistics. It is important that statistics educators begin to contribute more to these conversations based on their own expertise, which could be used to begin to emphasize statistical and mathematical concepts and practices in school curriculum.

## 18.7 Critical Statistics Education

In this section I outline a possible vision for critical statistics education beginning by briefly drawing a broad connection between Freire's (Darder 2014; Freire 1998; Freire and Macedo 1987) notions of literacy and Gal's (2002) description of statistical literacy in conjunction with statistical enquiry (Franklin et al. 2007; Wild and Pfannkuch 1999) to situate a view of critical statistics education in an overarching literacy perspective. The remainder of the discussion focuses on elaborating several points of intersection that connect the ideas discussed in earlier sections relative to critical mathematics education to statistics education, namely: considering context, variation, subjectivity, transnumeration, and problem posing.

### 18.7.1 Literacy Perspective

The term statistical literacy has been used by many scholars with many different meanings attributed to it (Ben-Zvi and Garfield 2004). In this chapter I have chosen to draw from Gal's definition of statistical literacy because of its seminal importance and because it is still one of the most commonly used definitions. Gal's (2002) states:

The term *statistical literacy* refers broadly to two interrelated components, primarily (a) people's ability to *interpret and critically evaluate* statistical information, data-related arguments, or stochastic phenomena, which they may encounter in diverse contexts, and when relevant (b) their ability to *discuss or communicate* their reactions to such statistical infor-

mation, such as their understanding of the meaning of the information, their opinions about the implications of this information, or their concerns regarding the acceptability of given conclusions (p. 49).

In his discussion, Gal (2002) describes two different types of contexts. The first, which is the focus of his definition of statistical literacy, he refers to as the reading context, which he describes as “people’s ability to act as effective “data consumers” in diverse life contexts” (p. 50). The other is the enquiry context, which is described as, “in enquiry contexts individuals serve as ‘data producers’ or ‘data analyzers’ and usually have to interpret their own data and results and report their findings and conclusions” (p. 50). A connection can be made here to the idea of reading and writing the world with mathematics (Gutstein 2006), discussed earlier. I propose that reading the world with statistics is what Gal (2002) describes as the reading context in statistical literacy and writing the world with statistics is partially what Gal (2002) refers to as enquiry contexts. I say partially because writing the world refers to changing one’s context, which goes beyond just producing and analyzing data to using it to take action to change the context it describes. Gal’s definition of statistical literacy focuses on the reading context, though he does mention the enquiry context. To further elaborate on the enquiry context to describe writing the world with statistics from the critical literacy perspective, I am choosing to draw on the statistical investigative cycle from the GAISE framework (Franklin et al. 2007) and from Wild and Pfannkuch’s (1999) work on statistical enquiry.

### ***18.7.2 Context***

One aspect of statistics that makes it particularly powerful for dealing with issues of race, gender, sexuality, immigration, sustainability and other sociopolitical issues is that it is the science of data, and data are inherently situated in context. Statistics can in fact be a gateway to introducing contextual discussions, situated in the daily realities of students, into the mathematics classroom, and in this way it can serve to act as a lens for reading the world. Statistics helps provide tools, practices, and habits of mind to measure and make sense of patterns in the world around us. It is this very focus that makes it so powerful for reading the world.

In relation to statistics education this perspective requires that teaching and learning statistics is situated in actual meaningful contexts for students. Frankenstein (2009) in her work discusses the importance of real real-world problems in mathematics education, where the context does not just serve as “window dressing” for a mathematics problem, but the actual focus and purpose of the problem is to explore and learn more about a context. From a disciplinary standpoint statistics is aptly suited to take on this task. However, to do so the issue must be taken head on in the school setting where mathematics and statistics are often portrayed as neutral, situated in fictitious contextualized situations (Frankenstein 2009; Skovsmose 1994a). This means contexts such as people’s favorite color, age, or height should not be used

as window dressing for tasks focused on calculation or following routine algorithms; instead instruction should focus on contexts that are from students daily lives and the contexts in which they are situated and using those as spring boards for students to explore and learn statistics in practice as part of making sense of a meaningful context.

For example, when looking at a data set with categories of male and female for gender a discussion around what is gender could begin. A discussion could revolve around whether these two categories are adequate for capturing the gender diversity of a population or as some argue whether they really exist at all except as a social construct (Butler 1990). This discussion could also move from reading context to enquiry context (Gal 2002) discussing issues of how to collect data on a person's gender to actively investigate issues around gender identity, which could then be interpreted and reported to write the word. Could data be collected by allowing an individual to self-identify using a fill in the blank item versus a dichotomous choose-one item? What implications does such a choice have on how the data can be analyzed and interpreted? Such investigations might also be focused on identifying and uncovering issues of genderism related to access to education, wages, or representation in government and society in an effort to transform conditions to write the world. Discussions of gender and other social constructs also relate to the notion of operational definitions of social constructs, which is an issue specific to statistics, which should be part of critical statistics education and has been discussed by others (e.g. Lesser 2007).

Drawing in such context also inherently brings in other issues particularly at the K-12 level where mathematics has traditionally been taught in a very neutral form (Gutiérrez 2013a; Skovsmose 1994b). Bringing sociopolitical contexts common in today's modern societies into the classroom also means opening up the classroom to the divisive and at times very insensitive and confrontational discourse that is also prevalent in societies around such issues. This raises a number of issues for the implementation of such curriculum that will be discussed and elaborated on later in the chapter.

### 18.7.3 *Variability*

One of the main purposes for the discipline of statistics is in attending to the omnipresence of variability inherent in our world (Cobb and Moore 1997). It is this very focus that makes statistics so powerful for reading the world. Variability comes in many forms, from how we measure things, to how we sample things, to how we try to show "cause and effect" by creating conditions to induce variability, to the fact that populations of things vary in measure of their attributes from one thing to the next. The word things is appropriate because statistics really is that broad in its application that it is applied to living things (people, animals, plants, etc.) and inanimate objects both created by people (machines, products, emissions, etc.) and created naturally (rocks, planets, stars, geological formations, etc.). In statistics there is an explicit emphasis

on trying to make sense of variability, measure it, visualize it, and at times control it. In the educational setting, explicit acknowledgement and treatment of variability is crucial (Cobb and Moore 1997; Franklin et al. 2007; Shaughnessy 2007), not only because of its central role in the discipline, but also because it helps to provide students with the perspective that everything does not always fit into a single numeric answer.

#### ***18.7.4 Subjectivity***

Another issue to consider is how people's subjectivity plays a role both in the production of data based arguments and in the interpretation of such arguments. This relates to reading and writing both the word and the world as our subjectivities influence everything we see, say, do, and make sense of. Our subjectivities act to filter how we can experience and act upon the world (Foucault 1972; Gutiérrez 2013b; Harding 1991). In statistics education, subjectivity is often treated in relation to biases in data production, such as how survey questions are worded, samples chosen, and participants or things are assigned to groups (Franklin et al. 2007; Utts 2003). However, what is not always considered is how the subjectivity of the reader of data based arguments plays a role in the reader's interpretation of the arguments. For example, consider the current anti-vaccination movements that are growing in spite of overwhelming amounts of scientific data that support the benefits of vaccination and the lack of scientific data to support claims of the purported negative effects of vaccinations. Some of my own subjectivities are likely evident in this statement and also throughout this chapter. People's subjectivities are always present in making sense of statistical arguments or carrying out statistical investigations whether they are lurking below the surface or transparently made explicit. Therefore it is crucial that authors make their subjectivities explicit in their data based arguments and that they reflect on how such subjectivity might influence their arguments.

There is a reason why "alternative facts" can spread like wild-fire in spite of a lack of supporting evidence, and it relates to the subjectivities through which people filter the world, making some statements more plausible, while others less. Open discussions of such issues in the classroom can prepare students to make sense of such alternative facts and should be a part of a critical statistics education.

#### ***18.7.5 Transnumeration***

The construct of transnumeration comes from past research in statistics education on the types of statistical thinking involved in statistical enquiry and is defined as "numeracy transformations made to facilitate understanding" (Wild and Pfannkuch 1999, p. 227). A number of different transformations of representations are involved in modeling data including the initial measurement of some real-world phenomenon,

applying aggregate measures to represent data, constructing data visualizations to represent data, creating statistical arguments that are convincing and understandable to communicate to an intended audience related to the problem situation being investigated (Pfannkuch et al. 2002). Transnumeration is related to context, variability, and subjectivity that were discussed previously. It is related to context as the initial operationalization and measurement of things involves transnumeration by quantifying and classifying the things, changing their representation to better understand them. This is also related to subjectivity because it is the person organizing and carrying out a statistical investigations that determines how to represent reality through how they decide to classify and quantify the aspects of reality they are measuring.

The focus on variability also connects to the notion of transnumeration where data visualization and statistical measures are used in service of making sense of a context to learn more about the context itself. In the context of variation different measures such as the range, interquartile range, and standard deviation can be used to represent variation as well as graphical displays such as boxplots, histograms, and bar graphs, which allow the investigator to “see” variation in the data. This relates back to statements earlier that the focus needs to be on statistics in service of reading and writing the world. Through visualizing variation in a data set one can begin to look for patterns and structures in the data that relay information about the context being measured. With the current explosion of technology in the world today, exploring data through visualizations and basic statistical measures has become increasingly easy to do even from the palm of your hand. This trend makes the notion of transnumeration increasingly important in statistics education and a crucial aspect of any vision of a critical statistics education as it is deeply intertwined in the process of statistical enquiry and relates to writing the world.

### ***18.7.6 Problem Posing***

A final aspect that is important for a critical statistics education is pedagogical and that is the practice of teaching through problem posing (Freire 1970). Problem posing does not mean simply giving students problems to solve. Instead, Freire (1970) describes it as a pedagogy where the teacher/student dichotomy is broken down, and both teachers and students collaborate in dialogue with one another. This pedagogy pairs well with teaching and learning statistics as teachers and students can explore issues together bringing their own prior experiences to bear to make sense of data based arguments. Furthermore, statistical investigations are based around problem posing as they begin with asking questions (Franklin et al. 2007). It is crucial that students be given opportunities to pose their own problems, which are meaningful and relevant to their lives that they can then investigate. Teachers in turn can bring their strong background in mathematics and statistics into the conversation to show different lens for making sense of issues and reading the world. More specifically in the case of fostering statistical literacy, students need experience posing such problems and then going through the stages of a statistical investigative cycle, ending with interpretations

of their results that include critiques of their process. Such experiences should include considering the implications the interpretations made may have in terms of the context and what actions might be suggested as a result.

## 18.8 Issues of Implementation

There are a number of significant issues related to implementing a critical statistics education perspective in school classrooms. One is the consideration of context. It is central to statistics and to critical mathematics education, yet the discussion of sociopolitical issues such as race, sexuality, or even political campaigns is generally considered very controversial in school classrooms and in some cases is forbidden by administrators or policy. Yet those are the kind of issues students face in their lives and that are visible in the world around them. Ignoring such issues within the walls of the mathematics classroom only perpetuates the paradox of relevance that Skovsmose (1994b) discussed. An important consideration here is what does this mean for the classroom teacher who is tasked to plan and carry out mathematics and statistics curriculum in the classroom?

To facilitate meaningful discussions around mathematical and statistical concepts and practices as well as their application to contextual issues is a significant task and requires some knowledge of the context being explored. This does not mean that teachers need to be experts in a multitude of content areas related to the contexts they explore to be able to carry on meaningful conversations and investigations with their students. However, they do need to have some familiarity with the major discussion points around such issues and the different viewpoints that are relevant. Furthermore, they should have taken some time to consider their own subjectivity towards such issues and how they might influence their instruction. Without prior consideration and reflection, classroom discussions could fall into ideological arguments with little basis in supporting evidence or more importantly in using mathematics and statistics to explore such issues. Considering contextual issues also means teachers need to be comfortable with not knowing everything that is being discussed in the classroom and to be comfortable in the role as learners along with their students and that students may at times challenge the teacher's own positioning in discussions. This relates directly to taking up a problem posing pedagogy (Freire 1970), discussed previously.

Another issue for implantation is related to teacher education and considering how to shape experiences for teachers to be prepared to take on the tasks necessary for implementing a vision of critical statistics education in their classrooms. Teachers are already expected to enter the classroom with content knowledge, and increasingly pedagogical content knowledge is being included in teacher training (Ball et al. 2008; Shulman 1987), but what is not common is discussion of what knowledge of contexts that can be investigated using mathematics and statistics might be useful or necessary.



It is not realistic to require teachers to have an extensive knowledge of contexts for the use of mathematics and statistics, particularly given the already overloaded curriculum of many teacher education programs. However, where advances towards such teaching could be made is in terms of pedagogy in methods courses, focusing on how to make sense of contexts using mathematics and statistics in the classroom and how to facilitate meaningful discussions around such issues. Furthermore, there is still a need for teachers to have more preparation in statistical concepts and practices. The *Statistical Education of Teachers* (Franklin et al. 2015) is taking strides in this direction. However, it will take many years for such policy to effect widespread changes in classrooms, and such policy documents do not explicitly advance a critical perspective of statistics education, which has been the focus of this chapter. What this means is that for teachers to be prepared and have the proper resources to facilitate experiences for students consistent with a critical statistical education, significant strides will need to be made at the teachers education level. A possible way to do this would be to work from the guidelines of the *Statistical Education of Teachers* (Franklin et al. 2015) providing pre-service teachers with statistics and statistics methods courses and focusing on the types of pedagogy discussed in this chapter. Furthermore, in statistics content courses an emphasize could be made on using statistics to investigate real real-world problems, which there has already been a call to do at the undergraduate level (Frankenstein 1994, 2009; Lesser 2007).

Taking up the type of pedagogy described in this chapter of course is easier said than done, and the reform movement in mathematics education has met serious challenges in a similar endeavor (Schoenfeld 2004). One such relevant issue is that of families, parents and their beliefs and values around what and how their children should be taught in schools. A number of mathematics reform movements in the U.S. have been crippled and collapsed by parents (Orrill 2016). This in part could be managed by not only working to create learning environments based on open dialogue between teachers and students but also by opening up such spaces through dialoguing with parents and the community as well. In a very real sense it is such open dialogue that is a hallmark of democracy, and to prepare students to be citizens in their democratic societies we can begin by creating learning environments that function as democracies where all vested parties have a say. Now this is not to say that such an opening up of learning spaces does not bring its own issues, but it does allow for dialogue and negotiation between parties to create a balance of perspectives and goals in shaping the learning environments for students to experience. That being said, taking this approach has not always worked in the past with reform efforts in mathematics, and there is a need for more research around how to initiate and sustain productive dialogue between all the stakeholders in education. Though I have only presented a few of the challenges I would argue they are some of the largest that need to initially be tackled to begin to shift towards both a critical mathematics and critical statistics education in schools.

## 18.9 Conclusion

This chapter presented the foundations and a possible vision for what a critical statistical education could be in K-12 mathematics classrooms. In moving forward however, there needs to be more work investigating what a critical statistics education could look like in the classroom. This type of work requires partnerships between statistics and mathematics educational researchers and mathematics teachers to develop a better understanding of how critical statistics education can be implemented in the classroom as well as what are some of the affordances and constraints of such implementation. Implications that this type of education has for mathematics teacher education also need to be considered and studied. For example, what type of experiences do mathematics teachers need to have to develop the flexible understanding of statistics and mathematics necessary for this type of teaching? Another question of concern is how to get important stakeholders of the mathematics community on board and involved with such changes.

Statistics' value in K-12 education with the goal of preparing students to become citizens in today's information based societies comes from the core practices that make up the statistical process: to pose questions, collect relevant data, analyze the data in the context of a problem (Franklin et al. 2007), and then verbalize the story that the data tell about an issue to others in a precise well principled argument (Abelson 1995). These practices situated in mathematics classrooms can begin to provide students with experiences in critically investigating and critiquing their own context in society, while developing the statistical concepts and practices that will enable them to make sense of their context. The task is dynamic and complex. The goal is not for students to completely understand or solve issues of society but to instead grapple with ideas using statistics to read and write the word and the world.

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