An Anti-jamming Strategy When it Is Unknown Which Receivers Will Face with Smart Interference

Andrey Garnaev[1,3](✉), Wade Trappe[1], and Athina Petropulu[2]

[1] WINLAB, Rutgers University, North Brunswick, USA
garnaev@yahoo.com, trappe@winlab.rutgers.edu

[2] Department of Electrical and Computer Engineering, Rutgers University, Piscataway, USA
athinap@rutgers.edu

[3] Saint Petersburg State University, Saint Petersburg, Russia

Abstract. The paper considers a communication system consisting of a communication node utilizing multiple antennas in order to communicate with a group of receivers, while potentially facing interference from one or more jammers. The jammers impact the scenario by possibly interfering some of the receivers. The objective of the jammers is to reduce the throughput of nearby receivers, while taking into account the cost/risk of jamming. The fact that jammers face a cost implies that they might not choose to interfere, and thus the communication node faces uncertainty about which of its receivers will be jammed. This uncertainty is modeled by the communicator having only a priori probabilities about whether each receiver will face hostile interference or not, and if he does face such jamming, whether the jamming attack is smart or not. The goal of the communication node is to distribute total power resources to maximize the total throughput associated with communicating with all of the receivers. The problem is formulated as a Bayesian game between the communication system and the jammers. A waterfilling equation to find the equilibrium is derived, and its uniqueness is proven. The threshold value on the power budget is established for the receivers to be non-altruistic.

Keywords: Multicast communication · Jamming · Bayesian game

1 Introduction

Due to the openness of the wireless channel, wireless networks are vulnerable to a variety of physical layer security threats, including interference in the form of jamming attacks. For this reason, wireless security has continued to receive considerable attention by the research community and one can find a comprehensive survey of security threats in cognitive radio networks [1,2], particular, jamming

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threats, in [3]. One of the challenges in dealing with interference attacks is to develop a strategy to cope with such interference, i.e. an anti-jamming strategy. A popular tool to design anti-jamming strategies is game theory [4] since it gives a framework involving competitive agents from which solutions can be formulated. In game theory, the agents (say, a user and a jammer) are considered as active, i.e., they respond to a variety of environmental parameters as well as to the rival's action. The main aim of such active jammers is to magnify their jamming effect in the network they intend to jam, while also balancing the costs associated with their efforts (e.g. they might aim to conserve their energy).

The term *smart jammer* is used to label jammers that aim to balance their efforts against costs/risks, while *naive jammer* is used to for jammers that do not consider their costs/risks. We now provide a quick, and wide sampling of the research literature involving smart jammers. For example, in [5], applying a Stackelberg game approach, the problem of maximizing the secure transmission rate between sensors and a controller while in the presence of a malicious eavesdropper and smart jammer was investigated. In [6], interactions between the user and a smart jammer with SNR as the user's utility was modeled as an anti-jamming Bayesian Stackelberg game involving uncertainties for the channel gain and transmission cost, and in which the user acted as the game's leader. In [7], a modified Q-learning algorithm against a user's fixed strategy and a smart jammer for multi-channel transmission was developed and compared with the corresponding Nash equilibrium. In [8], a jamming defense problem, in which the jammer can quickly learn the transmission power for the user and adaptively adjust its transmission power to maximize the damaging effect, was investigated. In [9], an adaptive rapid channel-hopping scheme was introduced using the notion of a dwell window and a deception mechanism to mitigate smart jammer attacks. In [10], a smart jammer suppression algorithm for a GPS receiver was designed combining spatial amplitude and phase estimation method and high resolution coherent subspace estimation method. In [11], the interactions between a user and a smart jammer regarding their respective transmit power choices was studied using prospect theory. In [12], the optimal user's strategy is designed when unknown whether the user faces with jamming or eavesdropping attack. In [13], a jamming attack against an LTE network was modeled by a repeated game where a smart adversary can be a cheater or a saboteur. The cheater intends to gain more resources for itself, and thus the adversary's intent is not to damage the channel resources, but rather jams the network as a side effect of its malicious activity by reducing competition among the other users, while the saboteur intends to cause the most possible damage to the network resources. In [14], the optimal bandwidth scanning strategy facing interference attacks aimed at reducing spectrum opportunities is designed. In [15], the interactions between a jammer and a communication node that exploits a timing channel to improve resilience to jamming attacks is studied in a game-theoretic framework involving a smart jammer who starts transmitting its interference signal only after detecting activity by the node. In [16], an evolutionary algorithm is proposed to find the equilibrium strategy of a collection of IoT devices seeking to thwart a jamming attack by distributing power among communication subcarries in a smart way so as to decrease the aggregate bit error rate caused by the jammer. In [17], the problem of secure multicast communications was formulated as Stackelberg game where smart private jammers were allocated nearby to eavesdroppers to increase the secrecy capacity of multicast communication.

In all of these papers, the players are assumed to be smart, i.e., rational, which is reflected by employing the best response strategy in the actions taken by the rivals. Recently, behavioral economics has challenged such rational and selfish individual behavior [18]. It has suggested that human decisions vary across time and space, and is subject to cognitive biases, emotions, and thus there may actually be uncertainty regarding the motivations of the rivals. Thus, some players, for whatever reason might not apply the best response strategy in response to their rival's strategies, and thus can be considered as non-rational strategies.

In this paper, we explore *a complementary aspect* of designing anti-jamming strategies for physical layer, multicast communication systems by recognizing a new challenge for such a system that is motivated by the observation, inherited from behavioral economics, that there may be non-rational behavior by the agents. Namely, we investigate the impact that incomplete information regarding a communication node (CN), whether its receivers face jamming attacks, and whether these attacks are smart or not can have on the anti-jamming strategy.

In order to explore this impact, we examine the specific case of a multicast system consisting of a CN employing multiple antennas to communicate with a group of receivers. The communication with some of the receivers might be targeted by jammers, and the CN knows only a priori probabilities about whether each receiver will face hostile interference or not, and if he does face such jamming, whether the jamming attack is smart or not. We formulate this problem as a Bayesian game between the CN and several jammers that might be allocated near the receivers. Existence and uniqueness of the equilibrium strategies are proven, and a waterfilling equation to find the equilibrium strategies is formulated.

The organization of this paper is as follows: in Sect. 2, the model is described. In Sect. 3, auxiliary notation and results are given. In Sect. 4, uniqueness of the equilibrium is proven, and then the waterfilling equation to find the equilibrium is derived. Finally, in Sect. 5, conclusions are given.

2 Communication Model

We begin our formulation by considering an operational scenario involving a *communication node* (CN) that is capable of supporting multiple antenna communication [19]. The multiple antenna interfaces allow this node to operate in a P2P fashion with many receivers simultaneously. We suppose the CN is equipped with n antennas allowing it to communicate with n receivers. We assume that the signals employed are orthogonal, and, thus, they do not interfere each other [20, 21]. There is the possibility that a jammer might be present near each receiver, with the intent to jam the communication. Thus, the number of jammers is at most n. In our scenario, we assume each of the jammers is equipped with a single antenna, and he can jam only its neighboring receiver. We portray this scenario in Fig. 1. In order to maintain the reliable communication with the receivers, the CN has to allocate its total power $\overline{P}$ across the orthogonal signals so as to maximize the total throughput. Thus, a strategy of the CN is a vector $\boldsymbol{P} = (P_1, \dots, P_n)$ with P_i being the power assigned to communicate with receiver i. Let Π be the set of feasible strategies for the CN.

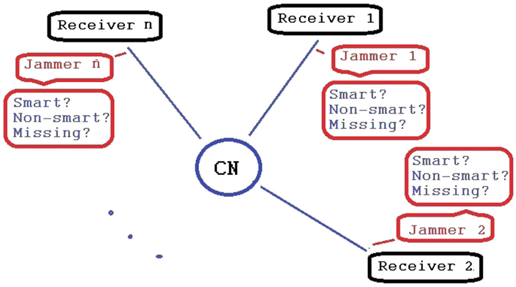

Fig. 1. The CN, receivers and jammers.

The strategy for jammer i is a power J employed to jam communication with receiver i. Let q_i be a priori probability that the jammer i is *smart*. Under the case of a smart jammer, we assume that he employs the best response strategy, i.e. he is focused on getting the best result. Thus, for a smart jammer the set of feasible strategies is $\mathbb{R}_+$. In contrast, a naive jammer, which employs a constant jamming strategy, i.e., *a constant* jammer, will be considered as our *non-smart* jammer since he does not adapt, and in particular does not use the best response strategy. Thus, for the non-smart jammer the set of feasible strategies is reduced to the single point $\{J\}$. Let $\overline{q}_i = 1 - q_i$ be the probability that the jammer i is non-smart. Let $\mathcal{G}_i(J)$ be the probability distribution function associated with applying jamming power J by the non-smart jammer i. We include here also an important particular case $J = 0$ which reflects the possibility that the jammer might be *missing* (or not active), and thus communication with the receiver might not suffer from hostile interference. Let $\boldsymbol{J} = (J_1, \dots, J_n)$. Then, the payoff to the CN is the expected total throughput, i.e.,

$$v_{CN}(\boldsymbol{P}, \boldsymbol{J}) = \sum_{i=1}^{n} \left(q_i \ln \left(1 + \frac{h_i P_i}{\sigma_i^2 + g_i J_i} \right) + \overline{q}_i \int_{\mathbb{R}_+} \ln \left(1 + \frac{h_i P_i}{\sigma_i^2 + g_i J} \right) d\mathcal{G}_i(J) \right), \tag{1}$$

where h_i and g_i are fading channel gains, while σ_i^2 is the background noise.

For a boundary scenario, where the feasible strategy for each non-smart jammer is $J = 0$, the a priori probability $\overline{q}_i$ is defined as the probability that the jammer for receiver i is missing. Then, the payoff (1) can be simplified as follows:

$$v_{CN}(\boldsymbol{P}, \boldsymbol{J}) = \sum_{i=1}^{n} \Big(q_i \ln \left(1 + h_i P_i / (\sigma_i^2 + g_i J_i) \right) + \overline{q}_i \ln \left(1 + h_i P_i / \sigma_i^2 \right) \Big). \tag{2}$$

Since the smart jammer i intends to harm communication with receiver i, this throughput can be considered as the smart jammer's cost utility. Hence, as payoff to the smart jammer i, we consider the difference between his payoff utility (which is his cost utility multiplied by minus one) and the cost of the employed jamming efforts, i.e.,

$$v_i(P_i, J_i) = -\ln\big(1 + h_i P_i/(\sigma_i^2 + g_i J_i)\big) - C_i J_i, \tag{3}$$

where C_i is the jamming cost per power unit. Note that (3) is a classical payoff for the jammer in CDMA-style transmission problems with resource power costs, which has been studied in literature for different scenarios [8,22].

Each of the rivals wants to maximize its payoff, and thus we look for the Nash equilibrium [4]. Recall that $(\boldsymbol{P}_*, \boldsymbol{J}_*)$ is an equilibrium if and only if, for any $(\boldsymbol{P}, \boldsymbol{J})$, the following inequalities hold:

$$v_{CN}(\boldsymbol{P}, \boldsymbol{J}_*) \le v_{CN}(\boldsymbol{P}_*, \boldsymbol{J}_*), \tag{4}$$

$$v_i(P_{*i}, J_i) \le v_i(P_{*i}, J_{*i}) \text{ for } i = 1, \dots, n. \tag{5}$$

We may consider the smart jammer and non-smart jammer as a single jammer, but able to have different types. From this perspective, the CN knows only a priori probability about type of the jammer it faces with, and this allows us to interpret the game as a Bayesian game between the CN and several jammers [4].

Theorem 1. *The considered game has at least one equilibrium.*

Proof. It is clear that $v_{CN}(\boldsymbol{P}, \boldsymbol{J})$ is concave in $\boldsymbol{P}$, and $v_i(P_i, J_i)$ is concave in J_i. Also, the set Π of feasible strategies for the CN is compact. To be in the framework of Nash's theorem for the existence of equilibrium [4], the set of feasible strategies for each jammer also has to be compact, while it is $\mathbb{R}_+$. Note that

$$\frac{\partial v_i(P_i, J_i)}{\partial J_i} = \frac{h_i P_i}{\sigma_i^2 + g_i J_i + h_i P_i} \frac{g_i}{\sigma_i^2 + g_i J_i} - C_i \le \frac{g_i}{\sigma_i^2 + g_i J_i} - C_i.$$

Thus, $\partial v_i(P_i, J_i)/\partial J_i < 0$ for $J_i > 1/C_i - \sigma_i^2/g_i$. This means that none of the strategies $J_i > 1/C_i - \sigma_i^2/g_i$ can be an equilibrium strategy. Thus, the set of feasible strategies for the jammer i can be reduced to the compact set $[0, \max\{0, 1/C_i - \sigma_i^2/g_i\}]$, and the result follows. ■

3 Auxiliary Notations and Results

Let us introduce auxiliary notations:

$$L_i(P_i, J_i) := \frac{q_i h_i}{\sigma_i^2 + g_i J_i + h_i P_i} + \int\limits_{\mathbb{R}_+} \frac{\overline{q}_i h_i}{\sigma_i^2 + h_i P_i + g_i J} \, d\mathcal{G}_i(J), \tag{6}$$

$$M_i(P_i, J_i) := h_i g_i P_i/((\sigma_i^2 + g_i J_i + h_i P_i)(\sigma_i^2 + g_i J_i)). \tag{7}$$

In the next two propositions, important properties of these functions L_i and M_i, which will be employed further to design the equilibrium strategy, are gathered.

Proposition 1. *(a) $L_i(P_i, J_i)$ is decreasing in P_i and J_i.*
(b) $L_i(P_i, J_i)$ tends to zero, while P_i tends to infinity.
(c) Let $0 < \omega \le L_i(0,0)$. Then

$$L_i(P_i, \mathcal{J}_i(P_i)) = \omega \text{ for } P_i \in [\underline{P}_i, \overline{P}_i], \tag{8}$$

where

$$\mathcal{J}_i(x) := \frac{h_i}{g_i} \frac{q_i}{\omega - \int_{\mathbb{R}_+} \overline{q}_i h_i/(\sigma_i^2 + h_i x + g_i J)\, d\mathcal{G}(J)} - \frac{\sigma_i^2 + h_i x}{g_i}, \tag{9}$$

and $\underline{P}_i = \underline{P}_i(\omega)$ and $\overline{P}_i = \overline{P}_i(\omega)$ are unique positive roots of the equations:

$$\begin{aligned} &L_i(\overline{P}_i(\omega), 0) = \omega, \\ &L_i(\underline{P}_i(\omega), \infty) := \int_{\mathbb{R}_+} \overline{q}_i h_i/(\sigma_i^2 + h_i \underline{P}_i(\omega) + g_i J)\, d\mathcal{G}_i(J) = \omega. \end{aligned} \tag{10}$$

(d) $\mathcal{J}_i(P_i)$ is strictly decreasing from infinity for $P_i \downarrow \underline{P}_i$ to zero for $P_i = \overline{P}_i$.
(e) $\overline{P}_i(\omega)$ is continuous and decreasing from infinity for $\omega \downarrow 0$ to zero for $\omega = L_i(0,0)$.

Proof. (a) and (b) follow directly from (6). (a) and (b) jointly with (8) imply (c). (a), (8) and (10) yield (d). (a), (6) and (10) imply (e), and the result follows. ∎

Proposition 2. *(a) $M_i(P_i, J_i)$ is increasing in P_i and decreasing in J_i.*
(b) $M_i(\overline{P}_i(\omega), 0)$ is decreasing from g_i/σ_i^2 for $\omega \downarrow 0$ to zero for $\omega = L_i(0,0)$.
(c) If $g_i/\sigma_i^2 \le C_i$ then $M_i(\overline{P}_i(\omega), 0) < C_i$ for $\omega < Ł_i(0,0)$, while if $g_i/\sigma_i^2 > C_i$ then is a unique $\omega_i \in (0, L_i(0,0))$ such that

$$M_i(\overline{P}_i(\omega), 0) \begin{cases} > C_i, & 0 < \omega < \omega_i, \\ = C_i, & \omega = \omega_i, \\ < C_i, & \omega_i < \omega < Ł_i(0,0). \end{cases} \tag{11}$$

Let $\Omega_i = 0$ for $g_i/\sigma_i^2 \le C_i$ and $\Omega_i = \omega_i$ for $g_i/\sigma_i^2 \le C_i$.

Proof. (a) follows from (7). Proposition 1, (a) and (7) imply (b). (b) yields (c), and the result follows. ∎

4 Equilibrium Strategies

In this section, to find the equilibrium and prove its uniqueness, we employ a *constructive* approach. We first use a parameter (Lagrange multiplier) to describe the form each equilibrium must have. This allows us to obtain a continuum of candidates for the equilibrium. Then, we prove that only one of them is the equilibrium.

Proposition 3. *Each equilibrium has to have the following form* $(\boldsymbol{P}(\omega), \boldsymbol{J}(\omega))$ *where* $\omega > 0$ *is a parameter:*

(a) If

$$i \in I_a(\omega) := \{i : L_i(0,0) \le \omega\} \tag{12}$$

then

$$P_i(\omega) := 0 \textit{ and } J_i(\omega) := 0, \tag{13}$$

(b) If

$$i \in I_b(\omega) := \{i : \Omega_i \le \omega < L_i(0,0)\} \tag{14}$$

then

$$P_i(\omega) := \overline{P}_i(\omega) \textit{ and } J_i(\omega) := 0, \tag{15}$$

(c) If

$$i \in I_c(\omega) := \{i : \omega < \Omega_i\} \tag{16}$$

then $P_i(\omega)$ *is the unique root in* $[\underline{P}_i(\omega), \overline{P}_i(\omega)]$ *of the equation*

$$\mathcal{F}_i(P_i(\omega)) = C_i \tag{17}$$

while

$$J_i(\omega) := \mathcal{J}_i(P_i(\omega)), \tag{18}$$

with

$$\mathcal{F}_i(x) := M_i(x, \mathcal{J}_i(x)). \tag{19}$$

Moreover, $J_i(\omega)$ *and* $P_i(\omega)$ *have the following monotonic properties on* ω*:*

Property $(\Pi - J)$: $J_i(\omega)$ *is continuous and decreasing to zero* $\omega = \Omega_i$.
Property $(\Pi - P)$: $P_i(\omega)$ *is continuous and decreasing on* ω *from infinity for* $\omega \downarrow 0$ *to* $\overline{P}_i(\omega)$ *for* $\omega = \Omega_i$.

Proof. By (4) and (5), $(\boldsymbol{P}, \boldsymbol{J})$ is an equilibrium if and only if these strategies are the best response to each other, i.e., they are solutions of the following best response equations:

$$\boldsymbol{P} = \mathrm{BR}_{CN}(\boldsymbol{J}) := \operatorname{argmax}\{v_{CN}(\boldsymbol{P}, \boldsymbol{J}) : \boldsymbol{P} \in \Pi\}, \tag{20}$$

$$J_i = \mathrm{BR}_{J,i}(\boldsymbol{P}) := \operatorname{argmax}\{v_i(P_i, J_i) : J_i \in \mathbb{R}_+\} \text{ for } i = 1, \dots, n. \tag{21}$$

Since (20) is a concave NLP problem, to solve it we have to introduce a Lagrangian depending on a Lagrange multiplier ω as follows: $\mathcal{L}_\omega(\boldsymbol{P}) := v_{CN}(\boldsymbol{P}, \boldsymbol{J}) + \omega(\overline{P} - \sum_{i=1}^n P_i)$. Then, taking into account notation (8), KKT Theorem implies that $\boldsymbol{P} \in \Pi$ is the best response strategy if and only if the following condition holds:

$$\frac{\partial \mathcal{L}_\omega(\boldsymbol{P})}{\partial P_i} = L_i(P_i, J_i) - \omega \begin{cases} = 0, & P_i > 0, \\ \le 0, & P_i = 0. \end{cases} \tag{22}$$

Since (21) is an optimization problem involving one real variable J_i and v_i is concave in J_i, in notation (7), J_i is the best response strategy if and only if the following relations hold:

$$\frac{\partial v_i(P_i, J_i)}{\partial J_i} = M_i(P_i, J_i) - C_i \begin{cases} = 0, & J_i > 0, \\ \le 0, & J_i = 0. \end{cases} \tag{23}$$

By (23), if $P_i = 0$ then $J_i = 0$. Substituting both of them into (22) implies that $i \in I_a(\omega)$ given by (12), and (a) follows. Thus, we have only to consider separately two cases: (A) $P_i > 0$, $J_i = 0$ and (B) $P_i > 0$, $J_i > 0$.

(A) Let $P_i > 0$ and $J_i = 0$. Then, by (22) and (23),

$$L_i(P_i, 0) = \omega, \tag{24}$$

$$M_i(P_i, 0) \le C_i. \tag{25}$$

By Proposition 1(d), the Eq. (24) has a positive root if and only if

$$\omega < L_i(0, 0), \tag{26}$$

and, moreover, it is equal to $\overline{P}_i(\omega)$. This implies (15). Substituting this $P_i = \overline{P}_i(\omega)$ into (27) implies that the following inequality must hold:

$$M_i(\overline{P}_i(\omega), 0) \le C_i. \tag{27}$$

By Proposition 2(b) and (c), (27) holds if and only if $\Omega_i \le \omega \le L_i(0, 0)$. This and (26) yield that $i \in I_b(\omega)$ given by (14), and thus (b) follows.

(B) Let $P_i > 0$ and $J_i > 0$. Then, by (22) and (23),

$$L_i(P_i, J_i) = \omega, \tag{28}$$

$$M_i(P_i, J_i) = C_i. \tag{29}$$

Solving (28) on J_i implies that $J_i = \mathcal{J}_i(P_i)$ with $\mathcal{J}_i$ given by (9). By (28), P_i and J_i are functions of ω, i.e., $P_i = P_i(\omega)$ and $J_i = J_i(\omega)$. By (8) and (28),

$$\lim_{\omega \downarrow 0} P_i(\omega) = \infty. \tag{30}$$

By Proposition 2(a), the left side of (29) is increasing in P_i and decreasing in J_i. Thus, to be a solution for (29), these $P_i(\omega)$ and $J_i(\omega)$ have to be either decreasing or increasing simultaneously on ω. This and (30) imply that $P_i(\omega)$ and $J_i(\omega)$ are decreasing on ω. By (8), (11), (28) and (29), $J_i(\Omega_i) = 0$ and $P_i(\Omega_i) = \overline{P}_i(\Omega_i)$. This, jointly with (30) yield properties $(\Pi - J)$ and $(\Pi - P)$.

For a fixed ω, by (28), J_i is a function on P_i, namely, $J_i = \mathcal{J}_i(P_i)$. Substituting this J_i into (28) implies that P_i has to be the root of equation (17). By Proposition 1(d), $\mathcal{J}_i(P_i)$ is decreasing from infinity for $P_i \downarrow \underline{P}_i$ to zero for $P_i = \overline{P}_i$ with $\underline{P}_i$ and small $\overline{P}_i$ given by (10). Thus, by Proposition 2(a), we have that $\mathcal{F}_i(P_i)$ is increasing on P_i. Moreover, since $\lim_{J_i \uparrow \infty} M_i(P_i, J_i) = 0$ for each fixed P_i then

$$\mathcal{F}_i(\underline{P}_i) = M_i(\underline{P}_i, \infty) = 0, \tag{31}$$

$$\mathcal{F}_i(\overline{P}_i) = M_i(\overline{P}_i, 0). \tag{32}$$

Thus, since $\mathcal{F}_i$ is increasing, (17) has root if and only if $M_i(\overline{P}_i, 0) > C_i$. By (11), this is equivalent to $\omega < \Omega_i$, i.e., $i \in I_c(\omega)$ given by (16), and (c) follows. ■

Proposition 4. *(a) The function* $H(\omega) := \sum_{i=1}^{n} P_i(\omega)$ *is continuous and decreasing from infinity for* $\omega \downarrow 0$ *to zero for* $\overline{L} := \max_i L_i(0, 0)$.
(b) The following water-filling equation has the unique root in $(0, \overline{L})$*:*

$$H(\omega) = \overline{P}. \tag{33}$$

Proof. The result follows directly from Proposition 3. ■

Theorem 2. *The game has a unique equilibrium, namely* $(\boldsymbol{P}, \boldsymbol{J}) = (\boldsymbol{P}(\omega), \boldsymbol{J}(\omega))$*, with* $\boldsymbol{P}(\omega)$ *and* $\boldsymbol{J}(\omega)$ *given by Proposition 3 and* ω *given by (33).*

Proof. Proposition 3 describes such a parameterized set of functions $\boldsymbol{P}$: $\mathbb{R}_+ \to \mathbb{R}_+^n$ that each CN equilibrium strategy belongs to. To ascertain whether each of them is an equilibrium, we have to verify whether it utilizes all of the power resource, i.e., whether (33) holds. By Proposition 4, (33) has the unique root, and the result follows. ■

If the total power budget is enough large then the CN can maintain communication with all the receivers, namely, the following result holds.

Corollary 1. $P_i > 0$ *for all* i *if and only if* $\overline{P} \geq H(\underline{L})$ *where* $\underline{L} := \min_i L_i(0, 0)$.

Proof. By Proposition 3, $P_i(\omega) > 0$ for any i if and only if $\omega < \underline{L}$. Then, Theorem 2 straightforward implies the result. ■

Finally, the two boundary cases of Theorem 2: (a) none of the jammers are smart, and (b) each of the jammers is smart, can be simplified as follows:

Corollary 2. *(a) Let* $q_i = 0, \forall i$*, i.e., none of the jammers is smart. Then, the CN optimal strategy* $\boldsymbol{P} = \boldsymbol{P}(\omega)$ *is*

$$P_i(\omega) \text{ is } \begin{cases} \int_{\mathbb{R}_+} h_i/(\sigma_i^2 + h_i P_i(\omega) + g_i J)\, d\mathcal{G}_i(J) = \omega, & \int_{\mathbb{R}_+} h_i/(\sigma_i^2 + g_i J) d\mathcal{G}_i(J) > \omega, \\ 0, & \text{otherwise.} \end{cases}$$

(b) Let $q_i = 1, \forall i$*, i.e., each jammer is smart. Then, the equilibrium strategies* $\boldsymbol{P} = \boldsymbol{P}(\omega)$ *and* $\boldsymbol{J} = \boldsymbol{J}(\omega)$ *are:*

$$P_i(\omega) = \begin{cases} C_i h_i / ((C_i h_i + \omega g_i)\omega), & \omega < h_i/\sigma_i^2 - h_i C_i/g_i, \\ 1/\omega - \sigma_i^2/h_i, & h_i/\sigma_i^2 - h_i C_i/g_i \leq \omega < h_i/\sigma_i^2, \\ 0, & h_i/\sigma_i^2 \leq \omega, \end{cases}$$

$$J_i(\omega) = \begin{cases} (h_i/g_i)\left(1/\omega - (\sigma_i^2 + h_i P_i(\omega))/h_i\right), & \omega < h_i/\sigma_i^2 - h_i C_i/g_i, \\ 0, & \omega \geq h_i/\sigma_i^2 - h_i C_i/g_i. \end{cases}$$

In both cases, (a) and (b), ω *is given as the unique root of water-filling equation* $\sum_{i=1}^{n} P_i(\omega) = \overline{P}$.

5 Discussion of Results and Conclusions

In this paper, we formulated and solved a Bayesian game between a communicating node and several jammers that might be allocated near several receivers. Our model also allowed for jammers to be smart (i.e. follow equilibrium strategy), or non-smart. The considered game combines two types of strategies: (a) *power allocation* akin to OFDM style for the CN, and (b) *power assigning* akin to CDMA style for each jammer. It is interesting that the obtained equilibrium strategies have a hierarchical structure although we look for the Nash equilibrium. Namely, in the first step, the CN equilibrium strategy is found by solving the water-filling Eq. (33). In the second step, the jamming strategies $\boldsymbol{J}$ are designed as the derivative of $\boldsymbol{P}$. In particular, it allows to establish the threshold value on the power budget to maintain non-altruistic communication with all the receivers. This makes the problem completely different from the OFDM jamming problem in the general SNR regime, where a system of two water-filling equations must be solved simultaneously [23], i.e., user and jammer strategies have to be designed simultaneously. The equilibrium strategy of the considered game coincides with the equilibrium in OFDM jamming problem only for a boundary case where all the smart jammers are non-active due to high jamming costs, i.e., when the CN strategy is classical OFDM transmission strategy [24]. It is interesting that although in OFDM jamming problem for the low SNR regime the equilibrium jammer's strategy also can be found in one step, multiple user equilibrium strategies might arise [25], while in the considered game the equilibrium is always unique. Figure 2(a) and (b) illustrate the dependence of the equilibrium strategies on a priori probabilities for the case when non-smart jammers employ jamming power according to uniform distribution in $[0, b]$ with $b = 0.1$, and $n = 5$, $\overline{P} = 1$, $\sigma^2 = (1, 1, 1, 1, 1)$, $h = (0.5, 0.9, 0.6, 0.9, 0.7)$, $g = (1, 2, 1.3, 1.8, 0.9)$, $C = (0.1, 0.04, 0.03, 0.05, 0.02)$, while a priori probabilities are given as follows: $q = (Q, Q, Q, Q, Q)$ with $Q \in \{0.2, 0.4, 0.6, 0.8\}$. Fig. 2(c) illustrates zone in plane (b, Q) where the total power budget is enough for the CN to maintain communication with all the receivers, i.e., when the CN strategy does not assume altruistic behaviour for none of the receivers.

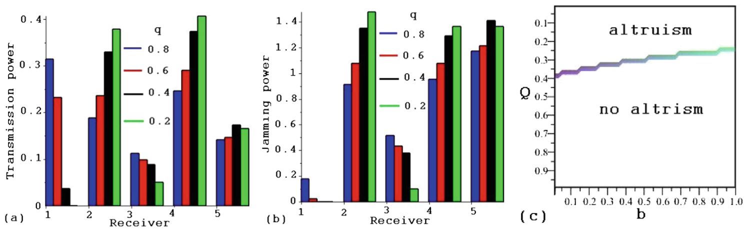

Fig. 2. (a) Equilibrium strategy of the CN, (b) equilibrium strategies of the jammers and (c) zone where none of the receivers is altruistic.

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