Linear and Nonlinear Double Diffusive Convection in a Couple Stress Fluid Saturated Anisotropic Porous Layer with Soret Effect and Internal Heat Source

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1 Introduction

Studies of double diffusive convection in porous media play a significant role in many areas, such as the petroleum industry, solidification of binary mixtures, and migration of solutes in water-saturated soils. Other examples include geophysics systems, crystal growth, electrochemistry, the migration of moisture through air contained in fibrous insulation, the Earth's oceans, and magma chambers. The problem of double diffusive convection in a porous media has been presented by Ingham and Pop [1], Nield and Bejan [2], Vafai [3, 4], and Vadasz [5]. The study was continued by Poulikakos [6], Trevison and Bejan [7], and Momou [8] among others. The first study of double diffusive convection in porous media was mainly concerned with linear stability analysis and was performed by Nield [9].

The growing importance of non-Newtonian fluids with suspended particles in modern technology and industries makes the investigation of such fluids desirable. These fluids are applied in the extrusion of polymer fluids in industry, exotic suspensions, fluid film lubrication, solidification of liquid crystals, cooling of metallic plates in baths, and colloidal and suspension solutions. Non-Newtonian stress fluids have specific features, such as the polar effect. The theory of polar fluids and related theories are models for fluids whose microstructure is mechanically significant. The theory for couple stress fluid was proposed by Stokes [10]; it is a simpler polar fluid theory, that shows all the important features and effects of such fluids that occur inside a deforming continuum. The stabilizing effect of the couple stress parameter is reported in the works of Sharma and Thakur [11], who

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investigated thermal instability in an electrically conducting couple stress fluid with a magnetic field. Sunil et al. [12] studied the effect of suspended particles on double diffusive convection in a couple stress fluid-saturated porous medium, Sharma and Sharma [13] investigated the effect of suspended particles on couple stress fluid, heated from below, in the presence of rotation and a magnetic field. Malashetty et al. [14] performed an analytical study of linear and nonlinear double diffusive convection with the Soret effect in couple stress liquids. Gaikwad and Kamble [15] analyzed the linear stability of double diffusive convection in a horizontal, sparsely packed, rotating, anisotropic porous layer in the presence of the Soret effect. Malashetty and Kollur [16] investigated the onset of double diffusive convection in an anisotropic porous layer saturated with couple stress fluid. Shivakumara et al. [17] analyzed the linear and nonlinear stability of double diffusive convection in a couple stress fluid-saturated porous layer.

In the study of double diffusive convection in the Soret effect, in some of the important areas of application in engineering, including geophysics, oil reservoirs, and groundwater, researchers have developed a great interest in these type of flows. In the presence of cross diffusion two transport properties are produced: the Soret effect and the Dufour effect. The Soret effect describes the tendency of a solute to diffuse under the influence of a temperature gradient. There are only a few studies available on double diffusive convection in the presence of the Soret effect. The diffusion material is heated unevenly. A mixture of gases or a solution is caused by the presence of temperature gradient in the system. The effect was described by Swiss scientist J. Soret, who was the first to study thermodiffusion (1879). Hurle and Jakeman argue that the liquid mixture, the Dufour term, is indeed small, and thus the Dufour effect will be negligible when compared with the Soret effect. They conducted an experimental and theoretical study of Soret-driven thermosolutal convection in a binary fluid mixture [18]. Malashetty et al. [19] performed an analytical study of linear and nonlinear double diffusive convection with the Soret effect in couple stress liquids. Rudraiah and Malashetty [20] discussed double diffusive convection in a porous medium in the presence of the Soret and Dufour effects. Bahloul et al. [21] studied double diffusive convection and Soret-induced convection in a shallow horizontal porous layer analytically and numerically. Malashetty and Biradar [22] carried out an analytical study of linear and nonlinear double diffusive convection in a fluid-saturated porous layer with Soret and Dufour effects. Also in another study, Bhadauria and Hashim et al. [23] performed linear and nonlinear double diffusive convection in a saturated anisotropic porous layer with couple stress fluid. Hill [25] showed linear and nonlinear double diffusive convection in a saturated anisotropic porous layer with a Soret effect and an internal heat source. Bhadauria et al. [26] studied effect of internal heating on double diffusive convection in a couple stress fluid saturated anisotropic porous medium. A study concerning an internal heat source in porous media was provided by Tveitereid [24], who performed thermal convection in a horizontal porous layer with internal heat sources. Srivastava et al. [27] performed linear and nonlinear analyses of double diffusive convection in a porous layer with a concentration-based internal heat source. Bhadauria [28], Horton and Rogers [29], and Lapwood [30] studied the effect of internal heating on double diffusive convection in a couple stress fluid-saturated anisotropic porous medium. Govender [31] showed that the Coriolis effect on the stability of centrifugally driven convection in a rotating anisotropic porous layer is subject to gravity. Kapil [32] performed at the onset of convection in a dusty couple stress fluid with variable gravity through a porous medium in hydromagnetics.

The aim of this chapter was to study the Soret effect and an internal heat source with a couple stress fluid. However, in the present study, stability analysis of the Soret and internal heating effect on double diffusive convection in an anisotropic porous layer with a couple stress fluid was performed.

1.1 Nomenclature

Table 1

Latin symbols	
a	wave number
C	Couple stress parameter $C = \frac{\mu_c}{\mu d^2}$
Ł$_e$	Lewis number $Le = \frac{\kappa_T}{\kappa_s}$
d	height of porous layer
$\vec{g}$	acceleration due to gravity
D	Cross diffusion due to T component
Da	Darcy number $Da = \frac{\kappa_z}{d^2}$
Ra_T	thermal Rayleigh number $Ra_T = \frac{\beta_T g \Delta T K_z d}{\nu \kappa_{Tz}}$
Ra_S	solutal Rayleigh number $Ra_S = \frac{\beta_S g \Delta S K_z d}{\nu \kappa_{Tz}}$
K	permeability of porous medium $K_x(ii + jj) + K_z(kk)$
K_x	permeability in x-direction
K_z	permeability in z-direction
T	temperature
ΔT	temperature difference across the porous layer
t	time
p	reduced pressure
q	fluid velocity(u,v,w)
P_r	Prandtl number $P_{rd} = \frac{\varepsilon \gamma \nu d^2}{\kappa_T K}$
R_i	Internal heat source parameter $R_i = \frac{Qd^2}{\kappa_T}$
V_a	Vadasz number $V_a = (\frac{P_r}{D_a})$
S	solute concentration
N_u	Nusselt number
S_h	Sherwood number
ΔS	solute difference across the porous layer
(x,y,z)	space co-ordinates

(continued)

Table 1 (continued)

Greek symbols	
β_T	coefficient of thermal expansion
β_S	coefficient of solute expansion
ξ	mechanical anisotropic parameter
η	thermal anisotropic parameter
κ_s	effective concentration diffusivity
κ_{Tz}	effective thermal diffusivity
σ	dimensionless oscillatory frequency
μ	dynamic viscosity of the fluid
μ_c	couple stress viscosity of the fluid
k	porosity
γ	heat capacity ratio $\frac{(\rho c_p)_m}{(\rho c_p)_f}$
ν	kinematic viscosity $(\frac{\mu}{\rho_0})$
ρ	fluid density
Other symbols	
∇_1^2	$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$
∇^2	$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$
Subscripts	
b	basic state
c	critical
0	reference value
Superscripts	
’	perturbed quantity
*	dimensionless quantity
osc	oscillatory
st	stationary

2 Mathematical Formulation

We consider an infinitely extended horizontal plane at z=0 and z=d a fluid-saturated porous medium, which is heated from below and cooled from above. The Darcy model has been employed in the momentum equation. Further, an internal heat source term has been included in the energy equation. A cartesian frame of reference is chosen in such a way that the origin lies on the lower plane and the z-axis is

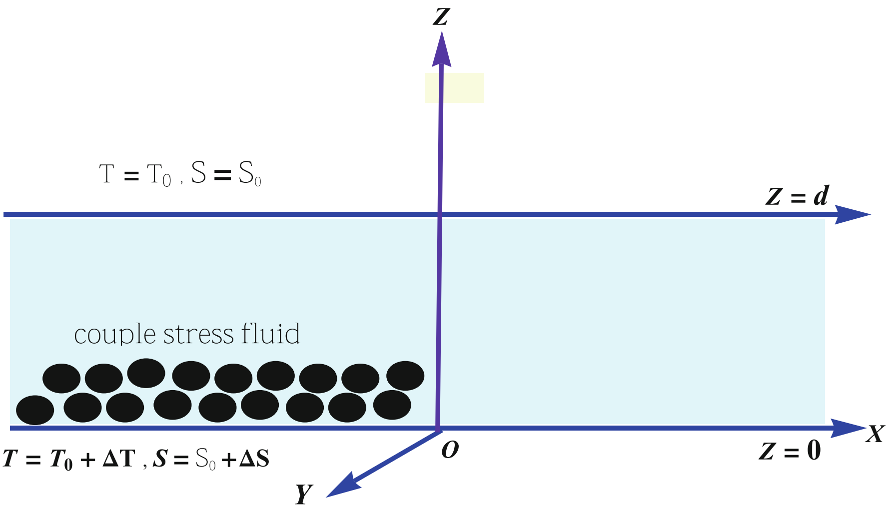

Fig. 1 Physical configuration of the problem

vertical upward. An adverse temperature gradient is applied across the porous layer and the lower and upper planes are kept at temperatures $T_0 + \Delta T$ and T_0, with a concentration $S_0 + \Delta S$ and S_0 respectively. The physical configuration of the model is reported in the Figure 1. The governing equations are given below

$$\begin{cases} \nabla.\vec{q} = 0, \\ \frac{\rho_0}{\varepsilon}(\frac{\partial q}{\partial t}) = -\nabla p + \rho g - \frac{1}{K}(\mu - \mu_c \nabla^2)\vec{q}, \\ \gamma \frac{\partial T}{\partial t} + (\vec{q}.\nabla)T = \nabla(\kappa_{Tz}.\nabla T) + Q(T - T_0), \\ \varepsilon \frac{\partial S}{\partial t} + (\vec{q}.\nabla)S = \kappa_s \nabla^2 S + D\nabla^2 T, \\ \rho = \rho_0[1 - \beta_T(T - T_0) + \beta_S(S - S_0)] \end{cases} \tag{1}$$

where the physical variables have their usual meanings as given in the nomenclature. The externally imposed thermal and solutal boundary conditions are given by

$$\begin{cases} T = T_0 + \Delta T, \quad at \quad z = 0 \quad and \quad T = T_0, \quad at \quad z = d, \\ S = S_0 + \Delta S, \quad at \quad z = 0 \quad and \quad S = S_0, \quad at \quad z = d, \end{cases} \tag{2}$$

3 Basic State

In this state, the velocity, pressure, temperature, and density profiles are given by

$$\vec{q_b} = 0,\ p = p_b(z),\ T = T_b(z),\ S = S_b(z),\ \rho = \rho_b(z). \tag{3}$$

Substituting Equation (3) in Equation (1), we obtain the following relations:

$$\frac{dp_b}{dz} = -\rho_b g, \tag{4}$$

$$\kappa_T \frac{d^2(T_b - T_0)}{dz^2} + Q(T_b - T_0) = 0, \tag{5}$$

$$K_s \frac{d^2 S_b}{dz^2} + D \frac{d^2 T_b}{dz^2} = 0, \tag{6}$$

$$\rho_b = \rho_0[1 - \beta_T(T_b - T_0) + \beta_S(S_b - T_0)]. \tag{7}$$

The solution of equation (5), subject to the boundary conditions (2), is given by

$$T_b = T_0 + \Delta T \frac{\sin((\sqrt{\frac{Qd^2}{\kappa_T}})(1 - \frac{z}{d}))}{\sin\left(\sqrt{\frac{Qd^2}{\kappa_T}}\right)}. \tag{8}$$

The solution of equation (6), subject to the boundary conditions (2),

$$S_b = S_0 + (\Delta S + \frac{D\Delta T}{K_s})(1 - \frac{z}{d}) - \frac{D\Delta T}{K_s} \frac{\sin((\sqrt{\frac{Qd^2}{\kappa_T}})(1 - \frac{z}{d}))}{\sin\left(\sqrt{\frac{Qd^2}{\kappa_T}}\right)} \tag{9}$$

Now, we superimpose finite amplitude perturbations on the basic state in the form:

$$\vec{q} = q_b + q',\ T = T_b + T',\ p = p_b + p',\ S = S_b + S',\ \rho = \rho_b + \rho', \tag{10}$$

Infinitesimal perturbation was applied to the basic state of the system and then the pressure term was eliminated by taking the curl twice of Equation (1). The resulting equations were nondimensional using the following transformations:

$$(x, y, z) = (x^*, y^*, z^*)d, \quad t = t^*(\frac{\gamma d^2}{\kappa_{T_z}}), \tag{11}$$

$$(u, v, w) = (u^*, v^*, w^*)(\frac{\kappa_{Tz}}{d}), \quad T = (\Delta T)T^*, S = (\Delta S)S^*$$

T_b, S_b in dimensionless forms are given

$$T_b = \frac{\sin\sqrt{R_i}(1-z)}{\sin\sqrt{R_i}}, \tag{12}$$

$$S_b = \frac{S_r L_e Ra_T \sin(\sqrt{R_i}(1-z))}{Ra_S \sin\sqrt{R_i}} - (\frac{S_r L_e Ra_T}{Ra_S} + 1)(1-z)$$

to obtain nondimensional equation (on dropping the asterisks for simplicity), and use the stream function $u = \frac{\partial\psi}{\partial z}$, $w = -\frac{\partial\psi}{\partial x}$

$$\frac{1}{V_a}\frac{\partial}{\partial t}\nabla_1^2\psi + (\frac{\partial^2}{\partial x^2} + \frac{1}{\xi}\frac{\partial^2}{\partial z^2})(1 - C\nabla_1^2)\psi = Ra_T\frac{\partial T}{\partial x} - Ra_S\frac{\partial S}{\partial x} = 0 \tag{13}$$

$$\Big[\frac{\partial}{\partial t} - \frac{\partial^2}{\partial z^2} - \eta\frac{\partial^2}{\partial x^2} - R_i\Big]T - f(z)\frac{\partial\psi}{\partial x} - \frac{\partial(\psi, T)}{\partial(x, z)} = 0 \tag{14}$$

$$\Big[\frac{\partial}{\partial t} - \frac{1}{L_e}(\frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial x^2})\Big]S - S_r\frac{Ra_T}{Ra_S}\nabla^2 T - b(z)\frac{\partial\psi}{\partial x} - \frac{\partial(\psi, S)}{\partial(x, z)} = 0 \tag{15}$$

where $V_a = \frac{\varepsilon P_r}{D_a}$ is Vadasz number, $Ra_T = \frac{\beta_T g \Delta T K_z d}{\nu\kappa_{Tz}}$ is the thermal Rayleigh number, $Ra_S = \frac{\beta_S g \Delta S K_z d}{\nu\kappa_{Tz}}$ is the solute Rayleigh number, $R_i = \frac{Qd^2}{\kappa_{Tz}}$ is the internal heat source parameter, $C = \frac{\mu_C}{\mu d^2}$ is the couple stress fluid, $L_e = \frac{\kappa_{Tz}}{\kappa_S}$ is the Lewis number, and $\chi = \frac{\varepsilon}{\gamma}$ is normalized porosity. The above system will be solved by considering stress-free and isothermal boundary conditions as given below:

$$w = \frac{\partial^2 w}{\partial z^2} = T = S = 0 \quad on \quad z = 0, z = 1. \tag{16}$$

4 Linear Stability Analysis

To study linear stability analysis according to solving the eigenvalue problem defined by Equations (13)–(15) subject to the boundary condition by Equations (5), (6), using time-dependent periodic disturbance in the horizontal plane:

$$(w, T, S) = (W, \Theta, \phi)exp[i(lx + my) + \sigma t] \tag{17}$$

where l, m are horizontal wave number and $\sigma = \sigma_r + i\sigma_j$ the growth rate. Substituting Equation (17) into the linearized equations (13)–(15), we obtain

$$\left[\frac{\sigma}{V_a}\delta^2 + \delta_1^2(1 - C\delta^2)\right]W + aRa_T\Theta - aRa_S\phi = 0 \tag{18}$$

$$[\sigma + \eta_1 - R_i]\Theta - 2aFW = 0 \tag{19}$$

$$[\sigma + \frac{\delta^2}{L_e}]\phi - 2aBW + S_r\delta^2\frac{Ra_T}{Ra_S}\Theta = 0. \tag{20}$$

Where $D = d/dz \ \ and \ \ a^2 = l^2 + m^2$. The boundary conditions are (17). Now read
W= $D^2W = \Theta = \phi = 0 \ \ at \ \ z = 0, 1$:
We assume that the solutions of equations (13)–(15) satisfying the boundary conditions (17),
$(W(z), \Theta(z), \phi(z)) = (W_0, \Theta_0, \phi_0)sinn\pi z \qquad (n = 1, 2, 3, \ldots)$
in the form of the thermal Rayleigh number can be obtained as

$$Ra_T = \frac{Ri - (\sigma + \eta_1)}{2a^2F}\left[\frac{(\delta^2 + L_e\sigma)(\frac{\sigma}{V_a}\delta^2 + \delta_1^2(1 - C\delta^2)) - 2a^2BL_eRa_S}{\sigma + \delta^2 + \delta^2S_rL_e}\right] \tag{21}$$

where $a^2 = l^2 + m^2$, $\delta^2 = \pi^2 + a^2$, $\delta_1^2 = \frac{\pi^2}{\xi} + a^2$, $\eta_1 = \pi^2 + \eta a^2$, F=$\int_0^1 \frac{dT_b}{dz}sin^2(\pi z)dz$, B= $\int_0^1 \frac{dS_b}{dz}sin^2(\pi z)dz$, η is a representative viscosity of the fluid. The growth rateσ is in general a complex quantity such that $\sigma = \sigma + i\sigma_i$. The system with σ_r <0 is always stable, whereas for σ_r >0 it will become unstable. For the neutral stability state $\sigma_r = 0$.

4.1 Stationary State

The values of the thermal Rayleigh number and the corresponding wave number of the system for a stationary mode of convection are given below:

$$Ra_T^{st} = \frac{Ri - \eta_1}{2a^2F}\left[\frac{\delta^2\delta_1^2(1 - C\delta^2) - 2a^2BRa_SL_e}{\delta^2(1 + L_eS_r)}\right], \tag{22}$$

It is important to note the critical wave number $a = a_c^{St}$, which is the result given by Malashetty et al. [19]. For single component fluid, $Ra_S = 0$, i.e., in the absence of a solute Rayleigh number, Equation (22) gives

$$Ra_T^{st} = \frac{(Ri - \eta_1)\delta_1^2(1 - C\delta^2)}{2a^2F(1 + L_eS_r)}. \tag{23}$$

For the system without internal heating, i.e., $R_i = 0$,F=-1/2, we get

$$Ra_T^{st} = \frac{(\eta_1)\delta_1^2(1 - C\delta^2)}{a^2(1 + L_e S_r)} \tag{24}$$

which is the one obtained by Shivakumara et al. [17]. When C = 0 (i.e., Newtonian fluid case), Eq. (3.11) reduces to

$$Ra_T^{st} = \frac{(\pi^2 + \eta^2 a^2)(a^2 + \frac{\pi^2}{\xi})}{a^2(1 + L_e S_r)} \tag{25}$$

In the case of no Soret effect

$$Ra_T^{st} = \frac{(\pi^2 + \eta^2 a^2)(a^2 + \frac{\pi^2}{\xi})}{a^2} \tag{26}$$

Lastly, in the case of isotropic porous medium, put $\eta = \xi = 1$

$$Ra_T^{st} = \left(\frac{\pi^2 + a^2}{a}\right)^2 \tag{27}$$

which has the critical value $Ra_c^{St} = 4\pi^2$ for $a_c^S t = \pi^2$ and which are the classical results obtained by Horton and Rogers [29] and Lapwood [30].

4.2 Oscillatory State

For the corresponding wave number of the system for the oscillatory mode of convection, we now set $\sigma = i\sigma_i$ in Equation (21) and clear the complex quantities from the denominator, to obtain

$$Ra_T^{osc} = \Delta_1 + i\sigma_i\Delta_2.$$

$$\Delta_1 = \frac{1}{2a^2F}\frac{A_1B_1 + \sigma^2 A_2B_2}{B_1^2 + \sigma^2 B_2^2} \tag{28}$$

$$\Delta_2 = \frac{1}{2a^2F}\frac{A_2B_1 - A_1B_2}{B_1^2 + \sigma^2 B_2^2}, \tag{29}$$

where, $A_1 = (R_i - \eta_1)(\delta^2\delta_1^2(1 - C\delta^2) - \frac{\sigma^2}{V_a}L_e\delta^2) + \sigma^2(L_e\delta_1^2(1 - C\delta^2) + \frac{\delta^4}{V_a}) - (R_i - \eta_1)Ra_S 2a^2 BL_e$,
$A_2 = (R_i - \eta_1)(L_e\delta_1^2(1 - C\delta^2) + \frac{\delta^4}{V_a}) - \delta^2\delta_1^2(1 - C\delta^2) + \frac{\sigma^2}{V_a}L_e\delta^2 + Ra_S 2a^2 BL_e$
$B_1 = \delta^2(1 + S_r L_e)$
$B_2 = 1$

For oscillatory onset $\Delta_2 = 0$ and ($\sigma_i \neq 0$), where σ is the oscillatory frequency, which is not given for brevity.

We have the necessary expression for the oscillatory Rayleigh number as:

$$Ra_T^{osc} = \Delta_1. \tag{30}$$

5 Nonlinear Stability Analysis

In this section, we study the nonlinear stability analysis using a minimal truncated Fourier series. For simplicity, we consider only two-dimensional rolls, so that all the physical quantities are independent of y. Consider the stream function ψ such that $u = \frac{\partial\psi}{\partial z}$, $w = \frac{-\partial\psi}{\partial x}$, then taking curl to eliminate the pressure term from Equation (1) and then the resulting nondimensional equations by using transformation given by Equation (11) and the following equation

$$\Big(\frac{1}{V_a}\frac{\partial}{\partial t}\nabla 1^2\psi + (\frac{\partial^2}{\partial x^2} + \frac{1}{\chi}\frac{\partial^2}{\partial z^2})(1 - C\nabla 1^2)\psi\Big) + Ra_T\frac{\partial T}{\partial x} - Ra_S\frac{\partial S}{\partial x} = 0, \tag{31}$$

$$\Big(\frac{\partial}{\partial t} - \frac{\partial^2}{\partial z^2} - \eta\frac{\partial^2}{\partial x^2} - R_i\Big)T - f(z)\frac{\partial\psi}{\partial x} - \frac{\partial(\psi, T)}{\partial(x, z)} = 0, \tag{32}$$

$$\Big[\frac{\partial}{\partial t} - \frac{1}{L_e}(\frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial x^2})\Big]S - \frac{\partial\psi}{\partial x}b(z) - \frac{\partial(\psi, S)}{\partial(x, z)} - S_r\frac{Ra_T}{Ra_S}\nabla^2 T = 0 \tag{33}$$

It should be noted that the effect of nonlinearity is to distort the temperature and concentration fields through the interaction of ψ and T, ψ, and S. As a result, a component of the form $sin(2\pi z)$ will be generated, where V is zonal velocity induced by rotation. A minimal Fourier series that describes the finite amplitude convection is given by

$$\psi = A_1(t)sin(ax)sin(\pi z), \tag{34}$$

$$T = B_1(t)cos(ax)sin(\pi z) + B_2(t)sin(2\pi z), \tag{35}$$

$$S = C_1(t)cos(ax)sin(\pi z) + C_2(t)sin(2\pi z), \tag{36}$$

where the amplitudes $A_1(t)$, $B_1(t)$, $B_2(t)$, $C_1(t)$, $C_2(t)$ are functions of time and are to be determined. Substituting the above expressions in Equations (31)–(33) and equating the like terms, the following set of nonlinear autonomous differential equations were obtained

$$\frac{dA_1(t)}{dt} = \frac{-V_a}{\delta^2}(\delta 1^2(1 + C\delta^2)A_1 + aRa_T B_1 - aRa_S C_1) \tag{37}$$

$$\frac{dB_1(t)}{dt} = 2aFA_1 - \pi a A_1 B_2 + (R_i - \eta_1)B_1 \tag{38}$$

$$\frac{dB_2(t)}{dt} = \frac{\pi a}{2} A_1 B_1 + (R_i - 4\pi^2)B_2 \tag{39}$$

$$\frac{dC_1(t)}{dt} = 2aBA_1 - \delta^2 S_r \frac{Ra_T}{Ra_S} B_1 - \delta^2 \frac{1}{L_e} C_1 - \pi a A_1 C_2 \tag{40}$$

$$\frac{dC_2(t)}{dt} = \pi \frac{a}{2} A_1 C_1 - 4\pi^2 S_r \frac{Ra_T}{Ra_S} B_2 - \frac{4\pi^2}{L_e} C_2 \tag{41}$$

where $A = 1 + 4c\pi^2$. The numerical method was used to solve the above nonlinear differential equation to find the amplitudes.

5.1 Steady Finite Amplitude Convection

For steady-state finite amplitude convection we have to set the left-hand side of the Equations (37)–(41) to zero.

$$(\delta 1^2(1 + C\delta^2)A_1 + aRa_T B_1 - aRa_S C_1) = 0 \tag{42}$$

$$2aFA_1 - \pi a A_1 B_2 + (R_i - \eta_1)B_1 = 0 \tag{43}$$

$$\frac{\pi a}{2} A_1 B_1 + (R_i - 4\pi^2)B_2 = 0 \tag{44}$$

$$2aBA_1 - \delta^2 S_r \frac{Ra_T}{Ra_S} B_1 - \delta^2 \frac{1}{L_e} C_1 - \pi a A_1 C_2 = 0 \tag{45}$$

$$\pi \frac{a}{2} A_1 C_1 - 4\pi^2 S_r \frac{Ra_T}{Ra_S} B_2 - \frac{4\pi^2}{L_e} C_2 = 0 \tag{46}$$

on solving for the amplitudes in terms of A_1, we obtain

$$B_1 = \frac{4aF(z)(4\pi^2 - R_i)A_1}{a^2 A_1^2 \pi^2 - 8\pi^2 R_i + 2R_i^2 + 8\pi^2 \eta_1 - 2R_i \eta},$$

$$B_2 = \frac{2a^2F(z)\pi A_1^2}{a^2A_1^2\pi^2 - 8\pi^2R_i + 2R_i^2 + 8\pi^2\eta_1 - 2R_i\eta},$$

$$C_1 = \frac{\begin{array}{l}16\left(8A_1BL_e\pi^2Ra_SR_ia + 2A_1BL_eRa_SRi^2a + A_1^3BL_e\pi^2Ra_Sa^3\right.\\ +A_1^3FL_e^2\pi^2Ra_Tsa^3 - 8A_1FL_e\pi^2Ra_TS_ra\delta^2 + 2A_1FL_eRa_TR_iS_ra\delta^2\\ \left.+8A_1BL_e\pi^2Ra_Sa\eta - 2A_1BL_eRa_SR_ia\eta_1\right)\end{array}}{Ra_S(A_1^2L_e^2a^2 + 8\delta^2)(-8\pi^2R_i + 2R_i^2 + A_1^2\pi^2a^2 + 8\pi^2\eta_1 - 2R_i\eta_1)},$$

$$C_2 = \frac{\begin{array}{l}2\left(8A_1^2BL_e^2\pi^2Ra_SR_ia^2 + 2A_1^2BL_e^2Ra_SR_i^2a^2 + A_1^4BL_e^2\pi^2Ra_Sa^4\right.\\ -8A_1^2FL_e\pi^2Ra_TSa^2\delta^2 - 8A_1^2FL_e^2\pi^2Ra_TSa^2\delta^2 + 2A_1^2FL_e^2Ra_TR_iSa^2\delta^2\\ \left.+8A_1^2BL_e^2\pi^2Ra_Sa^2\eta_1 - 2A_1^2BL_e^2Ra_SR_ia^2\eta_1\right)\end{array}}{Ra_S\pi(A_1^2L_e^2a^2 + 8\delta^2)(-8\pi^2R_i + 2R_i^2 + A_1^2\pi^2a^2 + 8\pi^2\eta_1 - 2R_i\eta_1)}.$$

To solve the above equation, a quadratic equation in $\frac{A_1^2}{8}$ is given by
$a_0x^2 + a_1x + a_2 = 0$
where x=$\frac{A_1^2}{8}$,
$a_0 = L_e^2a^4\pi^2\delta_1^2Ra_S(1 + C\delta^2)$
$a_1 = \frac{1}{4}\delta_1^2Ra_S(1+C\delta^2)(R_i-\eta_1)L_e^2a^2(R_i-4\pi^2) - \frac{1}{2}(R_i-4\pi^2)FRa_TRa_SL_e^2a^4 - 2L_ea^4\pi^2Ra_S(B + L_eFS_r) + a^2\pi^2\delta^2\delta_1^2Ra_S(1 + C\delta^2)$
$a_2 = \frac{(R_i-4\pi^2)}{4}(\delta^2\delta_1^2Ra_S(1 + C\delta^2)(R_i - \eta_1) - 2L_ea^2BRa_S(R_i - \eta_1) - 2a^2\delta^2FRa_S(L_eS_r + Ra_T))$
The required root of the above equation is
$x = \frac{-a_1+\sqrt{a_1^2-4a_oa_2}}{2a_0}$

5.2 *Steady Heat and Mass Transport*

In the study of this type of problem, quantification of heat and mass transport is very important in porous media. Let Nu and Sh be noted as the rate of heat and mass transport per unit for the fluid phase.
The Nusselt number and Sherwood number are defined by

$$Nu = 1 + \left[\frac{\int_0^{2\pi/a}\frac{\partial T}{\partial z}dx,}{\int_0^{2\pi/a}\frac{\partial T_b}{\partial z}dx,}\right]_{z=0} \tag{47}$$

$$Sh = 1 + \left[\frac{\int_0^{2\pi/a}\frac{\partial S}{\partial z}dx,}{\int_0^{2\pi/a}\frac{\partial S_b}{\partial z}dx,}\right]_{z=0} \tag{48}$$

substituting the value of T, T_b,S, and S_b in Equations (47)–(48),

$$Nu = 1 - \frac{2\pi B_2}{\sqrt{R_i}\cot\sqrt{R_i}}, \tag{49}$$

$$Sh = 1 - \frac{2\pi C_2 Ra_S \sin\sqrt{R_i}}{-S_r Ra_T \cos\sqrt{R_i}\sqrt{R_i} + \sin\sqrt{R_i} Ra_S + \sin\sqrt{R_i} S_r Ra_T}$$

substituting B_2, C_2 of Equations (5.1) into (49) gives

$$Nu, \quad Sh \tag{50}$$

6 Results and Discussion

This chapter investigates the combined effect of internal heating and the Soret effect on stationary and oscillatory convection in a anisotropic porous medium with couple stress fluid. In this section, we discuss the effects of the parameters in the governing equations on the onset of double diffusive convection numerically and graphically. The stationary and oscillatory expressions for different values of the parameters such as the Vadasz number, the couple stress parameter, the solute Rayleigh number, the mechanical anisotropic parameter, and the thermal anisotropic parameter are computed, and the results are depicted in the figures. The neutral stability curves in the (Ra_T,a) plane for various parameter values are shown in Figure 2a–e. We fixed the values for the parameters as Va = 5, C = 2, Ra_S = 100, L_e = 20, ξ = .5, η = .5, S_r = .05, and R_i = 2, except for the varying parameter. The effect of the Vadasz number Ta on the neutral curves is shown in Figure 2. We find that for fixed values of all other parameters, the minimum value of the Rayleigh number for the oscillatory mode increases as a function of increasing Va, indicating that the effect of the Vadasz number is to stabilize the system. In addition, the critical wave number increases with increasing Va. We observed that by increasing the value of internal heat source R_i, the mechanical anisotropic parameter ξ decreased the stationary and oscillatory Rayleigh number, which means that the internal heat source R_i, mechanical anisotropic parameter ξ destabilized. Figure 2 depicts the effect of the couple stress parameter C on the neutral stability curves. We find that with an increase in the value of the couple stress parameter, the value of the Rayleigh number for both stationary and oscillatory mode is enhanced, indicating that it stabilizes the onset of double diffusive convection and depicts the effect of the solute Rayleigh number Ra_S on the stability curve for stationary and oscillatory convection. We show that the effect of increasing Ra_S is to decrease the value of the Rayleigh number for stationary and oscillatory convection and the corresponding wave number. Thus, the solute Rayleigh number becomes unstable. We also show that the effect of an increasing Lewis number L_e and the thermal anisotropic

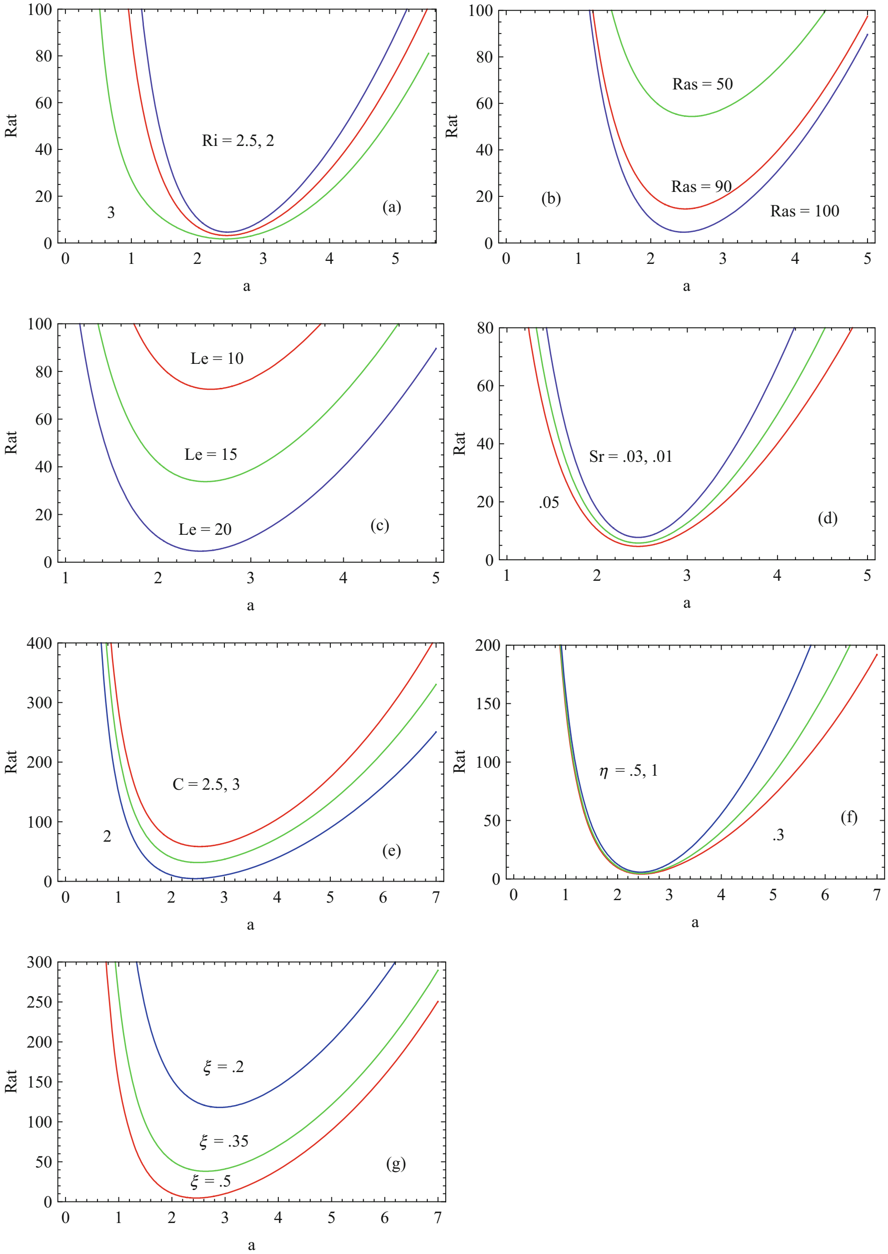

Fig. 2 Stationary neutral stability curves for the different values of (a), (b), (c), (d), (e), (f), (g)

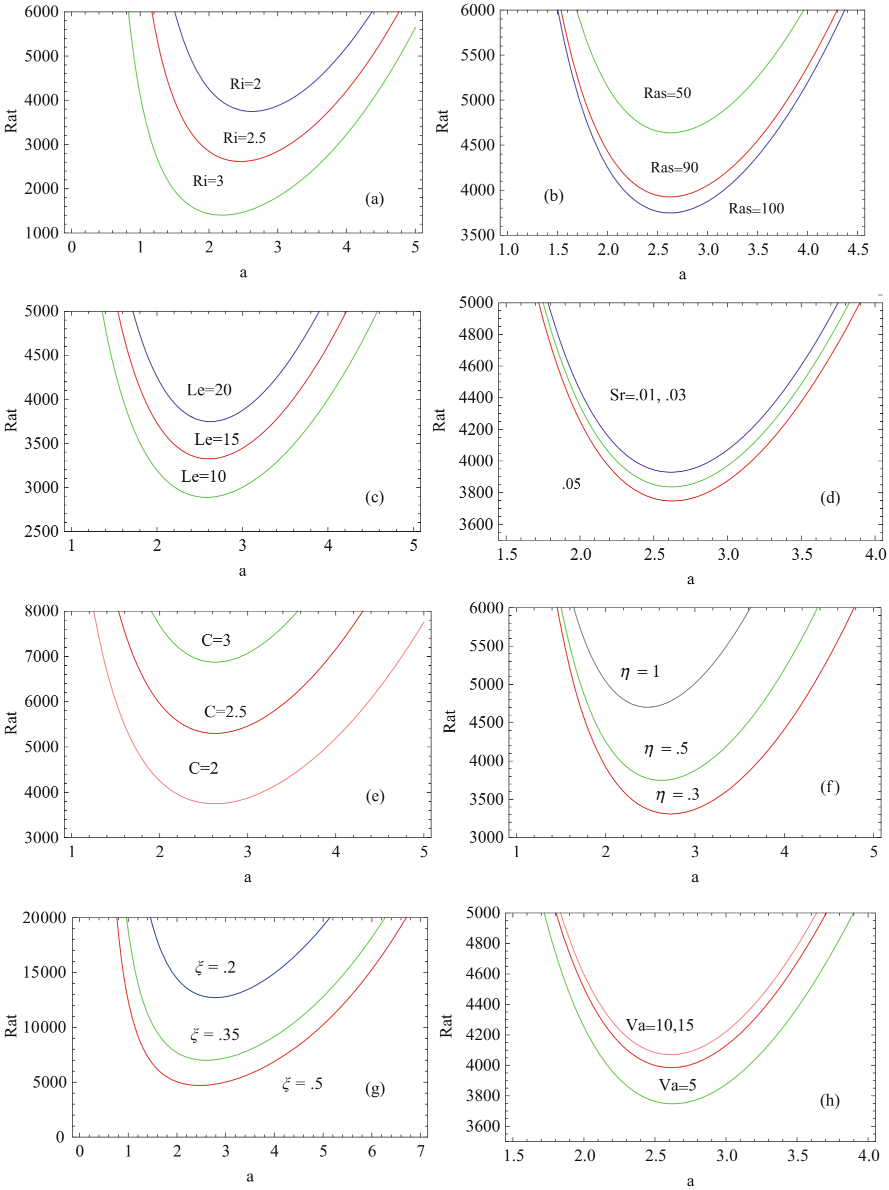

Fig. 3 Oscillatory neutral stability curves for the different values of (a), (b), (c), (d), (e), (f), (g), (h)

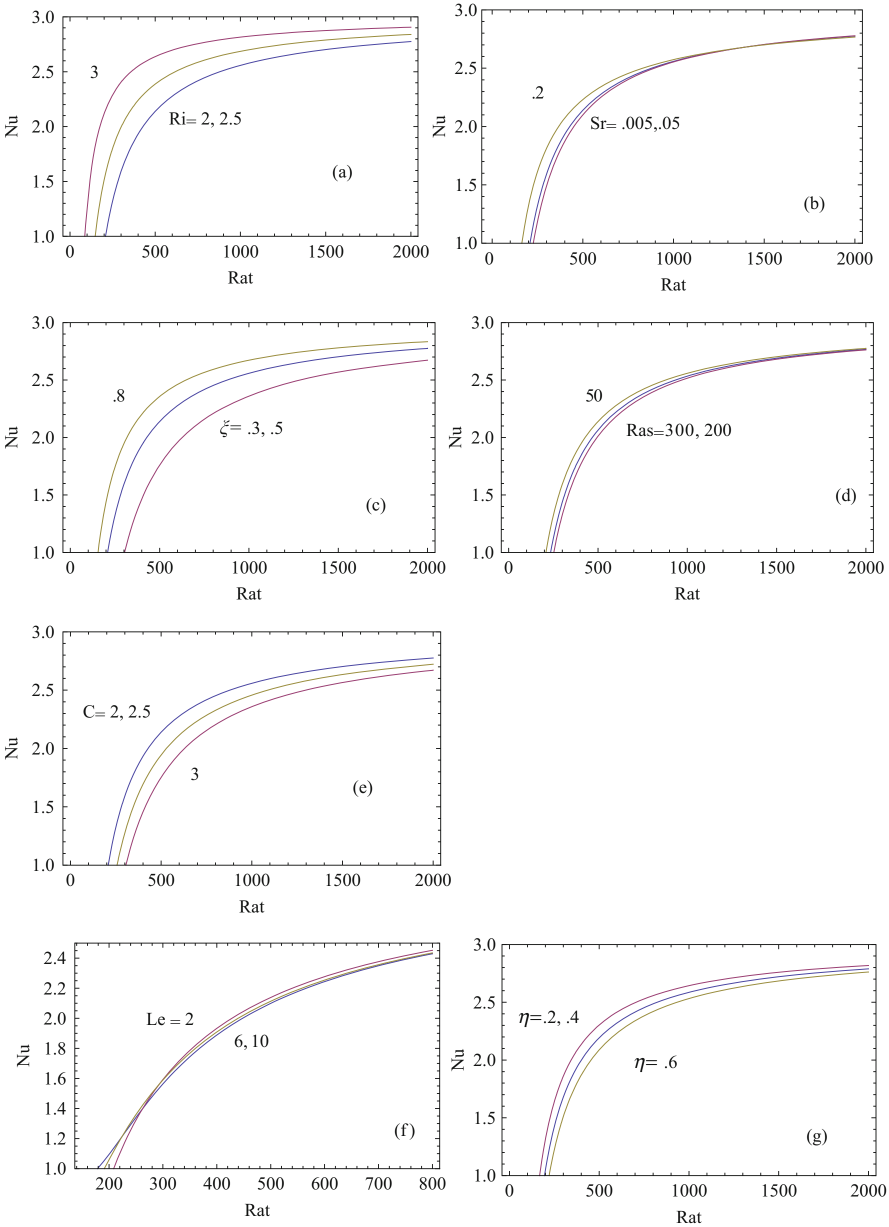

Fig. 4 Variation of Nu with Rat for different values of parameters

parameter η is to increase the value of the Rayleigh number for stationary convection and decrease the value of oscillatory convection. With regard to the corresponding wave number, we found it unstable for the stationary and stable for the oscillatory

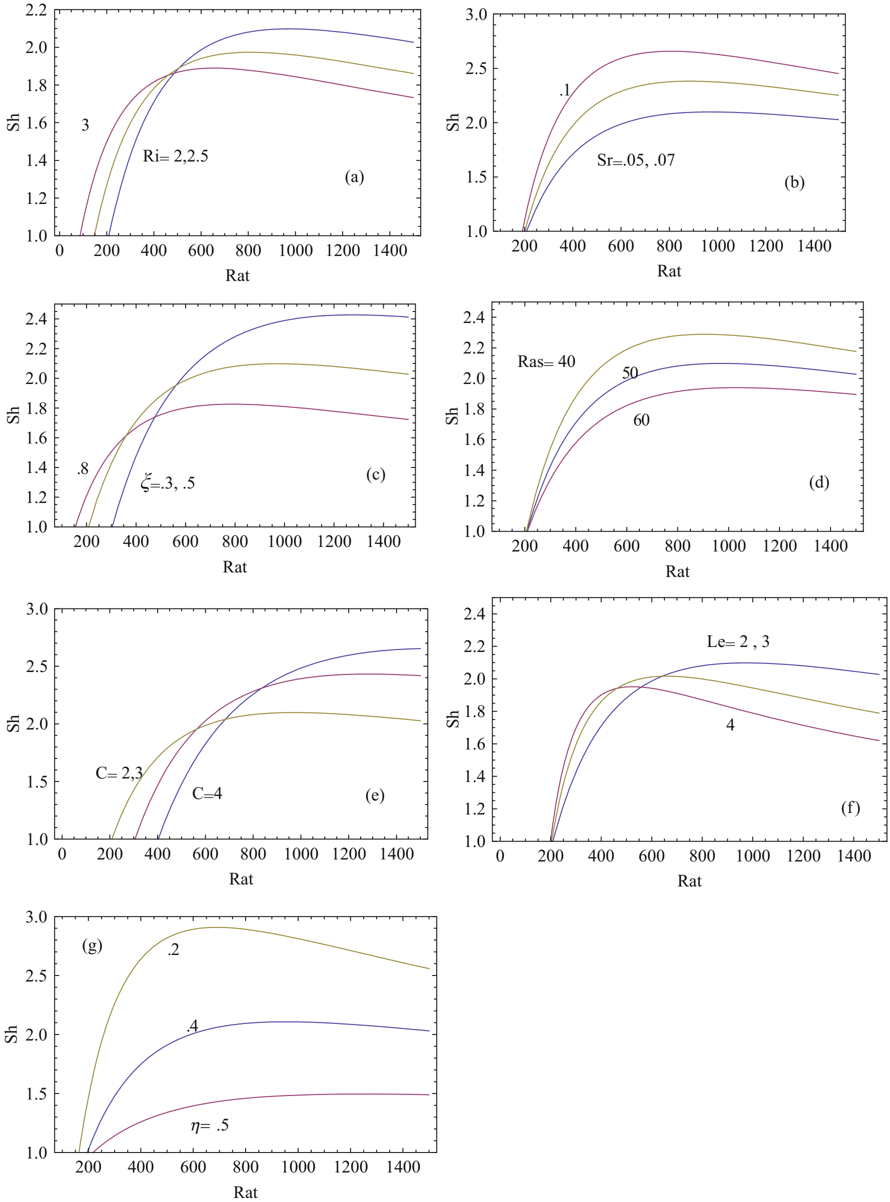

Fig. 5 Variation of Sh with Rat for different values of parameters

modes. We find that Figures 2 and 3 show that an increase in the value of the Soret parameter S_r decreases the Rayleigh number, indicating that the Soret parameter destabilizes the onset of stationary and oscillatory convection.

We use the parameter in a graph of the Nusselt and Sherwood number C = 2, $Ra_S = 20$, $L_e = 2$, $\xi = .5$, $\eta = .5$, $S_r = .05$, and $R_i = 2$, and Figures 4a and 5c show that an increase in the value of the internal Rayleigh number R_i decreases the rate of heat and increases mass transfer. We note that the effect of increasing the solute Rayleigh number Ra_S and the thermal anisotropic parameter η is to increase the value of the Nusselt number N_u and the Sherwood number S_h, thus reducing the heat and mass transfer. In Figures 4b and 5a, it can be found that with an increase in the value of the Soret parameter S_r, the mechanical anisotropic parameter ξ and then the value of the Nusselt number N_u and the Sherwood number S_h decrease; thus, the heat and mass transfer across the porous layer also decrease.

7 Conclusions

The Soret effect and the internal heating effect on double diffusive convection in a anisotropic porous medium saturated with a couple stress fluid that is heated and salted from below was investigated using linear and nonlinear stability analysis. The linear analysis is carried out using the normal mode technique. The following conclusions were drawn:

1) The Vadasz number Va has a stabilizing effect on oscillatory convection.
2) The internal heat parameter R_i, the solute Rayleigh number Ra_s, the Soret parameter S_r, and the mechanical anisotropic parameter ξ destabilize the system in the stationary and oscillatory modes.
3) The couple stress fluid C has a stabilizing effect on both the stationary and the oscillatory convection.
4) The normalized porosity parameter η and the Lewis number L_e have a destabilizing effect in the case of stationary and opposite oscillatory convection.
5) With the increasing value of the mechanical anisotropic parameter ξ, the Soret parameter S_r then increases the value of the Nusselt number N_u, i.e., increasing heat transfer, but increasing the value of the internal Rayleigh number R_i, and the normalized porosity parameter η and the solutal Rayleigh number Ra_S decrease the value of the Nusselt number N_u.
6) Mass transfer S_h increases with the increasing value of the internal Rayleigh number R_i, the mechanical anisotropic parameter ξ, the Soret parameter S_r, and decreases with the normalized porosity parameter η and the solutal Rayleigh number Ra_S.

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