Chapter 10
R&D Activities in a Differentiated Goods Duopoly with Quadratic Cost Function

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Abstract The purpose of this paper is to assess the relationship between the firms' behavior in the differentiated product market and the decisions regarding the R&D investments. The comparison is made between the case of the Stackelberg-type duopolistic competition in the final product market and the situation of a cartelized industry under the assumption of quadratic cost functions. On the one hand, different levels of product differentiation are considered. And, on the other hand, the impact of the extent of research spillovers is analyzed. The numerical analysis leads to the conclusion that, under the assumption of quadratic cost functions, it is always beneficial for both firms to form an industry cartel. This result is similar to the case of the linear cost functions with one exception: the threat of cartelizing the industry was not present when the final products were homogenous.

Keywords R&D activities · Industry cartelization · Stackelberg competition Heterogeneous products · Quadratic cost functions

10.1 Introduction

The purpose of this paper is to assess the relationship between the firms' behavior in the differentiated product market and the decisions regarding the R&D investments when the production is characterized by the quadratic cost function. Recently, Prokop and Karbowski (2018) analyzed the case of differentiated products in the industries characterized by the linear production costs. They concluded that as long as the final products are even slightly differentiated, it is always beneficial to firms competing in the Stackelberg fashion to fully cartelize the industry. It was shown that the threat of industry cartelization is not present only in the case of competition in homogenous goods.

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The basic framework of the analysis has been set by d'Aspremont and Jacquemin (1988). Following these authors, two-stage games have been used in the literature to analyze the relationship between the research activities and the behavior of firms in the final product market. In the first stage firms simultaneously choose the size of R&D investments and in the second stage they decide about their conduct in the final product market.

An important element of the analysis are technological spillovers, i.e., R&D investments made by one company generate positive externalities for the remaining firms in the industry. The extent of the knowledge spillovers could vary due to the type of industry and due to the behavior of firms (see, e.g., Geroski 1995). The maximum level of research externalities is known to be achieved when companies form a joint venture.

In addition to the varying size of research externalities, also product differentiation is a significant factor affecting the firm conduct in the final goods market (see, e.g., Symeonidis 2003). It is usually observed that greater homogeneity of products leads to more fierce competition. However that does not exclude possibility for the firms to create a cartel.

The decision regarding research investments could be made independently, or it may be a result of cooperation by companies. Also, the production decisions could be a result of competition, or they may be set by firms in a coordinated way. These possibilities have been pointed out by Kamien et al. (1992).

In this paper, the focus is on the impact of R&D spillovers and product differentiation on the firm research activities and performance. The comparison is made between the case of Stackelberg-type competition and the situation of an industry cartel when the marginal costs of production are increasing.

Due to the difficulties in obtaining algebraic form of equilibrium solutions, the numerical analysis is applied.

The rest of the paper is organized as follows. In the following section, the case of a Stackelberg leadership duopoly in the final product market is analyzed. Section 10.3 is focused on the conduct and performance of companies in the cartelized industry. Based on the comparison of the above two cases, the evaluation of firm behavior and performance is given in Sect. 10.4. Concluding remarks close the paper.

10.2 Quantity Leadership in a Differentiated Product Market

We consider an industry composed of two firms, denoted 1 and 2. Firms produce q_1 and q_2 units of a heterogeneous product, respectively. The market demand for the product is given as a linear price function:

$$p_i = a - q_i - sq_j, \tag{10.1}$$

where p_i is the market price, q_i denotes the production supplied by firm i, while a is a given demand parameter and s captures the extent of substitutability between different goods. Clearly, both goods are perfect substitutes when $s = 1$, and each firm becomes a monopolist when $s = 0$.

Each of the firms produces at the total costs given by the following quadratic function:

$$C_i\left(q_i, x_i, x_j\right) = \frac{q_i^2}{c + x_i + \beta x_j} \tag{10.2}$$

where c is an initial level of efficiency of firm i, x_i denotes the amount of research investments made by firm i, and x_j denotes the level of research investments made by company j. Following the previous literature, parameter $\beta\,(0 \leq \beta \leq 1)$ measures the extent of R&D spillovers, i.e. the benefits for a given firm resulting from the research investments undertaken by the competitor. Greater size of parameter β means that the research undertaken by one firm reduces the manufacturing costs of the other firm by a bigger amount.

It is assumed that the entry barriers to the industry are high, so there is no issue of new competitors in this market.

Each company i incurs the costs of research investments according to the following quadratic function:

$$\gamma \frac{x_i^2}{2}, \tag{10.3}$$

where $\gamma\,(\gamma > 0)$ is a given parameter.

First, we consider the case when the competition of firms in the final product market is characterized by quantity leadership, i.e., company 1 assumes the role of the Stackelberg leader, and company 2, is the follower. Thus, firm 1 is the first to decide about the quantity of its production level, q_1, and firm 2, knowing the output level chosen by the leader, chooses its own amount of supply, q_2.

There are two stages of decision making by firms. At the first stage, both of them simultaneously and independently choose their levels of research investments, x_i. These decisions affect the manufacturing costs of both companies. At the second stage, the firms compete in the final product market according to the Stackelberg leadership model.

We use backward induction to find the equilibrium of the presented game with the two companies as players. Consider the profit of the follower firm at the second stage of the game for a given amount of R&D investments, x_1 and x_2:

$$\pi_2 = (a - q_2 - sq_1)q_2 - \frac{q_2^2}{c + x_2 + \beta x_1} - \gamma \frac{x_2^2}{2}. \tag{10.4}$$

Given the output level of the leader q_1, the follower maximizes its own profit by setting the production level at:

$$q_2 = \frac{(a - sq_1)(c + \beta x_1 + x_2)}{2(1 + c + \beta x_1 + x_2)}, \tag{10.5}$$

which is calculated by solving the first order optimality condition $\frac{\partial \pi_2}{\partial q_2} = 0$ with respect to q_2.

Taking into account the reaction function of the follower given by (10.5), the leader maximizes its own profit for given levels of research investments x_1 and x_2:

$$\pi_1 = (a - q_1 - sq_2)q_1 - \frac{q_1^2}{c + x_1 + \beta x_2} - \gamma \frac{x_1^2}{2}. \tag{10.6}$$

From the first order condition for profit maximization, $\frac{d\pi_1}{dq_1} = 0$, the optimal output level for the leader is given as:

$$q_1 = \frac{a(2 + (2 - \mathrm{s})(c + \beta x_1 + x_2))(c + x_1 + \beta x_2)}{2(2(1 + c + \beta x_1 + x_2)(1 + c + x_1 + \beta x_2) - \mathrm{s}^2(c + x_1 + \beta x_2)(c + \beta x_1 + x_2))}. \tag{10.7}$$

By substituting (10.7) into (10.5), we obtain the optimal output level of the follower as a function of R&D investments, x_1 and x_2:

$$q_2(x_1, x_2). \tag{10.8}$$

The production levels q_1 and q_2 given by (10.7) and (10.8) constitute the Nash-Stackelberg equilibrium.

After substituting (10.7) and (10.8) into (10.4) and (10.6), we obtain the equilibrium profits of both firms as functions of R&D investments, x_1 and x_2:

$$\pi_1(x_1, x_2), \tag{10.9a}$$

$$\pi_2(x_1, x_2). \tag{10.9b}$$

The Nash equilibrium strategies at the first stage of the game are found as a solution to the following system of two equations with two unknowns x_1 and x_2:

$$\frac{\partial \pi_1}{\partial x_1} = 0, \tag{10.10a}$$

$$\frac{\partial \pi_2}{\partial x_2} = 0. \tag{10.10b}$$

Under certain restrictions on the values of parameters a, c, β, γ, and s, the above system has exactly one solution; denote it by x_1^* and x_2^*.

Substituting x_1^* and x_2^* into (10.9a) and (10.9b), we obtain the equilibrium profits of the leader and the follower; denote them by π_1^*, and π_2^*.

Since the algebraic solution of our model is practically hard to present due to the quadratic cost function, we will use a numerical analysis in order to show possibilities of certain outcomes. For the purpose of this paper, we will restrict our considerations to the case when three parameters of the model are: $a = 100$, $c = 10$, and $\gamma = 20$. The results of the calculations for $s = 0.5$ and various levels of parameter β are given in Table 10.1.

Based on Table 10.1, let us consider the impact of parameter β, i.e. the size of research externalities, on the equilibrium conduct of firms. The size of R&D investments of both companies is a declining function of the extent of research externalities measured by the parameter β. It can also be observed that the quantity leader invests in R&D more than the follower.

The supply of the final product offered by the firms behaves nonmonotonically with respect to the level of research spillovers. The largest output offered by the leader takes place for the parameter $\beta = 0.5$, but the highest production of the follower is observed for $\beta = 0.6$. The lowest prices are offered by both suppliers for parameter $\beta = 0.6$. Thus, the medium extent of research spillovers generates the highest gains for the consumers in this industry.

The profits of each firms are an increasing function of the size of technological externalities. Thus, both competing firms are interested in the largest extent of technological spillovers.

Table 10.1 Quantity leadership for $a = 100$, $c = 10$, $\gamma = 20$, $s = 0.5$ and $\beta \in [0, 1]$

β	x_1^*	x_2^*	q_1^*	q_2^*	p_1^*	p_2^*	π_1^*	π_2^*
0.0	0.68000	0.64665	39.3860	36.7059	42.2611	43.6012	1514.62	1469.69
0.1	0.64764	0.61116	39.3944	36.7126	42.2493	43.5902	1515.27	1470.33
0.2	0.61622	0.57654	39.4010	36.7181	42.2399	43.5814	1515.84	1470.90
0.3	0.58564	0.54267	39.4059	36.7224	42.2329	43.5746	1516.33	1471.41
0.4	0.55581	0.50943	39.4090	36.7256	42.2282	43.5699	1516.74	1471.85
0.5	0.52664	0.47672	39.4104	36.7277	42.2257	43.5671	1517.08	1472.25
0.6	0.49806	0.44445	39.4102	36.7287	42.2255	43.5662	1517.35	1472.59
0.7	0.47000	0.41252	39.4082	36.7286	42.2275	43.5673	1517.55	1472.88
0.8	0.44240	0.38083	39.4045	36.7275	42.2317	43.5702	1517.69	1473.12
0.9	0.41519	0.34930	39.3991	36.7253	42.2383	43.5752	1517.75	1473.31
1.0	0.38833	0.31784	39.3918	36.7221	42.2472	43.5820	1517.75	1473.46

Table 10.2 Quantity leadership for $a = 100$, $c = 10$, $\gamma = 20$, $\beta = 0.2$ and $s \in [0, 1]$

s	x_1^*	x_2^*	q_1^*	q_2^*	p_1^*	p_2^*	π_1^*	π_2^*
0.0	0.86294	0.86294	45.8456	45.8456	54.1544	54.1544	2284.83	2284.83
0.1	0.78938	0.78904	43.9002	43.8035	51.7194	51.8064	2088.22	2087.80
0.2	0.72926	0.72665	42.3057	41.9144	49.3115	49.6245	1916.25	1913.12
0.3	0.68094	0.67237	41.0314	40.1323	46.9289	47.5583	1765.26	1755.09
0.4	0.64339	0.62325	40.0627	38.4146	44.5714	45.5603	1632.46	1609.05
0.5	0.61622	0.57654	39.4010	36.7181	42.2399	43.5814	1515.84	1470.90
0.6	0.59972	0.52922	39.0668	34.9939	39.9369	41.5660	1414.05	1336.76
0.7	0.59512	0.47752	39.1059	33.1816	37.6670	39.4443	1326.41	1202.64
0.8	0.60513	0.41607	39.6027	31.1981	35.4388	37.1197	1253.07	1063.96
0.9	0.63505	0.33639	40.7075	28.9179	33.2664	34.4453	1195.32	915.04
1.0	0.69532	0.22415	42.6929	26.1319	31.1752	31.1752	1156.42	748.27

Now, we look at the effect of changes in the substitutability (parameter s) on the behavior of both companies in the leader-follower setting. Table 10.2 reports the Stackelberg equilibrium for various levels of s, and the R&D spillover parameter $\beta = 0.2$.

It follows from Table 10.2 that the biggest size of research investments by both firms is observed when there is maximum product differentiation. However, the growing level of substitutability (increasing parameter s) induces the follower to reduce the R&D spendings by more than the leader. The decline is monotonic for the follower, and nonmonotonic for the leader. When the product substitutability becomes relatively high ($s \geq 0.7$), the research investments of the leader start growing with an increasing s.

A decline in product differentiation reduces the profits of both companies. It is not surprising that both firms enjoy the highest profits when product differentiation is maximal, i.e., $s = 0$; the lowest profits are observed when products are homogenous, i.e., $s = 1$. An increase in product substitutability reduces the follower's profit faster than the leader's.

The consumers enjoy the lowest prices when the product differentiation is minimized, i.e., s = 1.

We move on to analyze the cooperation of firms in the industry cartel.

10.3 Industry Cartel

Let us consider a model in which the firms form a cartel both at the R&D stage, and at the final product market. We assume that the demand function as well as the cost functions of the firms stay the same as in the case of Stackelberg competition.

At the second stage of the game, the companies decide about their production levels q_1 and q_2 to maximize the joint profit, given the size of research investments, x_1 and x_2:

$$\pi = (a - q_1 - sq_2)q_1 - \frac{q_1^2}{c + x_1 + \beta x_2} - \frac{\gamma x_1^2}{2} + (a - q_2 - sq_1)q_2 - \frac{q_2^2}{c + x_2 + \beta x_1} - \frac{\gamma x_2^2}{2}. \tag{10.11}$$

In the case of the symmetric equilibrium, i.e., $x_1 = x_2 = x$, the optimal production level of each cartel member is:

$$q_1 = q_2 = q = \frac{a(c + (1+\beta)x)}{2(1 + c + cs + (1+s)(1+\beta)x)}. \tag{10.12}$$

After substituting (10.12) into the market demand function described by (10.1), we arrive at the symmetric equilibrium price of the final product:

$$p_1 = p_2 = p = \frac{a(2 + c + cs + (1+s)(1+\beta)x)}{2(1 + c + cs + (1+s)(1+\beta)x)}. \tag{10.13}$$

When companies simultaneously choose the levels of R&D investments x_1 and x_2 at the first stage of the game, their joint profit becomes:

$$\pi = \frac{1}{2}\left(\frac{a^2(c + (1+\beta)x)}{1 + c + cs + (1+s)(1+\beta)x} - 2\gamma x^2\right). \tag{10.14}$$

When the companies form a cartel at the research stage and in the final product market, the symmetric equilibrium takes place when the R&D investments of each of the firms (x) satisfy the following first order condition for profit maximization:

$$\frac{\partial \pi}{\partial x} = 0. \tag{10.15}$$

Under certain restrictions on the values of parameters a, c, β, γ, and s, the above equation has exactly one solution; denote it by $\tilde{x}$. After substituting $\tilde{x}$ for x into (10.12), we obtain the production level of each of the firms; denoted by $\tilde{q} = \tilde{q}_1 = \tilde{q}_2$.

The equilibrium price of the final product offered by each company is obtained by substituting $\tilde{x}$ for x into (10.13); denote it by $\tilde{p}$.

Next, by substituting $\tilde{x}$ for x into (10.14), we obtain the equilibrium joint profit of the companies; denote it by $\tilde{\pi}$. Thus every company earns:

$$\tilde{\pi}_1 = \tilde{\pi}_2 = \frac{1}{2}\tilde{\pi}. \tag{10.16}$$

Table 10.3 Full industry cartel for $a = 100$, $c = 10$, $\gamma = 20$, $s = 0.5$ and $\beta \in [0, 1]$

β	$\tilde{x}$	$\tilde{q}_i$	$\tilde{p}$	$\tilde{\pi}_i$
0.0	0.44958	31.3343	52.9986	1564.69
0.1	0.48697	31.3496	52.9756	1565.11
0.2	0.52267	31.3657	52.9515	1565.55
0.3	0.55667	31.3824	52.9265	1566.02
0.4	0.58901	31.3995	52.9008	1566.51
0.5	0.61971	31.4170	52.8745	1567.01
0.6	0.64882	31.4348	52.8478	1567.53
0.7	0.67639	31.4527	52.8209	1568.06
0.8	0.70248	31.4708	52.7938	1568.60
0.9	0.72716	31.4889	52.7666	1569.16
1.0	0.75049	31.5070	52.7395	1569.72

For the sake of a comparison with the equilibria obtained in the previous section, we will restrict our numerical calculations to the case when the four parameters are $a = 100$, $c = 10$, $\gamma = 20$, and $s = 0.5$. The equilibrium results for various levels of parameter β have been presented in Table 10.3.

Based on Table 10.3, let us consider the impact of technological spillovers in research and development on the equilibrium conduct and performance of firms in the cartelized industry. In this case, we find positive correlation between the size of R&D externalities and research investments aimed at the cost reduction. It should be observed that it is exactly opposite relationship than the one observed in the case of Stackelberg competition in the final product market reported in Table 10.1, where that relationship was negative. The lowest level of cost-reducing investments is observed when there are no research externalities $(\beta = 0)$.

This result is different from the case of a cartelized industry with homogenous product $(s = 1)$ discussed by Prokop (2016). When the final products are homogenous, the R&D spending of every cartel member is declining together with the larger extent of technological spillovers. Thus the highest investments in research in a fully cartelized industry are expected when there are no technological externalities.

Together with the growing research investments, each company supplies a higher amount of final output as a result of greater size of externalities. That leads to the reduction of market price. However, it should be noticed that the prices are still significantly higher than those in the noncollusive case. Despite the declining price, the profits of each cartel member are increasing with the higher level of research spillovers. These results are similar to the case of a cartelized industry with linear production costs analysed by Prokop and Karbowski (2018).

Additional regularities can be observed by changing the degree of product differentiation measured by parameter s. Table 10.4 reports the calculation of cartel equilibrium for various size of s, and for $\beta = 0.2$.

Table 10.4 shows that the R&D investments by a cartel member $(\tilde{x})$ are a declining function of the extent of product differentiation (parameter s). Similar

Table 10.4 Full industry cartel for $a = 100$, $c = 10$, $\gamma = 20$, $\beta = 0.2$ and $s \in [0, 1]$

s	$\tilde{x}$	$\tilde{q}_i$	p	$\tilde{\pi}_i$
0.0	1.00651	45.9043	54.0957	2285.08
0.1	0.86800	41.9968	53.8035	2092.31
0.2	0.75575	38.7091	53.5491	1929.74
0.3	0.66355	35.9034	53.3255	1790.77
0.4	0.58696	33.4802	53.1277	1670.56
0.5	0.52267	31.3657	52.9515	1565.55
0.6	0.46821	29.5041	52.7935	1473.01
0.7	0.42169	27.8523	52.6511	1390.84
0.8	0.38167	26.3766	52.5221	1317.37
0.9	0.34700	25.0501	52.4049	1251.30
1.0	0.31678	23.8511	52.2978	1191.55

relationship was observed about the behavior of Stackelberg follower reported in Table 10.2. However, the Stackelberg leader's investment in R&D was nonmonotonic.

Using Table 10.4, it can be concluded that the production levels, prices, and profits of each cartel member are declining with an increasing homogeneity of the final product. These results do not differ from the case of linear cost function discussed by Prokop and Karbowski (2018).

10.4 Evaluation of Firm Behaviour and Performance

Now, we may use the equilibria obtained in the previous two sections to compare the decisions of firms and their performance under the quantity-leadership competition and in the cartelized industry.

First, consider the decisions of companies regarding the investments in research and development when the final products offered by firms have a medium level of differentiation ($s = 0.5$). A cartel member invests less in R&D activities than a firm in the non-cartelized industry for the relatively low levels of technological spillovers. When parameter β is below 0.4, the cartel is expected to generate a smaller amount of R&D investments than the non-cartelized industry. For the values of parameter β not smaller than 0.4, the amount of research investments by a single firm is higher in the cartelized industry. Thus, it can be claimed that cartels speed up technological development for a sufficiently large size of research spillovers. Unfortunately, the prices offered by the cartel members are significantly higher than the price levels expected in the non-cartelized industry.

Next, compare the performance of companies in the cartelized and non-cartelized industries characterized by product differentiation. It can be seen from Tables 10.1 and 10.3 that the profit of cartel members is always higher than the profit of non-colluding firms. Thus, the incentives for cartel formation under the quadratic cost function are the same as under the linear cost functions analyzed by Prokop and Karbowski (2018).

The numerical analysis shows that under the quadratic cost functions for any extent of product differentiation, it is always better for both firms to create a cartel in order to maximize profits rather than compete according to the quantity-leadership pattern. This conclusion is different from the results of Prokop and Karbowski (2018) for the case of linear cost functions. These authors showed that the industry cartelization is better for both firm only when products are differentiated. Under the linear cost functions, when products are homogenous, the Stackelberg leader prefers not to form a cartel.

10.5 Concluding Remarks

In this paper, we considered the relationship between the research and development activities and the behavior of firms in the differentiated product market under the assumption of quadratic cost functions. Two types of firms' conduct in the final product market were investigated: quantity leadership and industry cartel. On the one hand, the effect of different levels of product differentiation was analyzed. And, on the other hand, the impact of the extent of research spillovers was considered.

The numerical analysis led to the conclusion that, under the assumption of quadratic cost functions, it was always beneficial for both firms to form an industry cartel. This conclusion is different from the results of Prokop and Karbowski (2018) for the case of linear cost functions. These authors showed that the threat of industry cartelization was not present when the final products were homogenous (i.e. $s = 1$). Thus, the existence of increasing marginal costs of production generates a serious risk of collusion among the duopolists and creates negative consequences for consumers.

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