

Ultimate Bearing Capacity of Eccentrically Inclined Loaded Circular Foundation on Sand Layer of Limited Thickness Using ANN

B. P. Sethy^{1(\Box)}, C. R. Patra¹, K. Sobhan², and B. M. Das³

¹ National Institute of Technology, Rourkela, Rourkela, India barada. jeetu@gmail.com, crpatral9@yahoo.co.in ² Florida Atlantic University, Boca Raton, FL, USA ksobhan@fau.edu ³ California State University, Sacramento, USA brajamdas@gmail.com

Abstract. Laboratory model tests were conducted for the ultimate bearing capacity of shallow rough circular surface foundation resting over a sand layer of limited thickness subjected to an eccentrically inclined load. Based on the laboratory model test results, a neural network model is developed to estimate the reduction factor (*RF*). The reduction factor can be used to estimate the ultimate eccentrically inclined load per unit area of the foundation supported by a sand layer of limited thickness from the ultimate bearing capacity of a foundation on a sand layer extending to a great depth under an eccentrically inclined load. A thorough sensitivity analysis was carried out to determine the important parameters affecting the reduction factor. Importance was given on the construction of neural interpretation diagram. Based on this neural interpretation diagram, the direct or inverse relationships that exists between the input and output parameters were determined. Results from the artificial neural network (ANN) were compared with the laboratory model test results and these results are well matched.

Keywords: Eccentrically inclined load \cdot Sand \cdot Limited thickness ANN \cdot Ultimate bearing capacity \cdot Reduction factor

1 Introduction

During the last three decades, a number of laboratory model test results and few field test results have been published that are related to the ultimate bearing capacity of shallow foundation resting over homogeneous sand bed and clay extending to a great depth. The presence of a hard layer within a certain depth below the foundation base can significantly influence the ultimate bearing capacity supported by the soil. In many practical problems, the thin soil layer may be underlain by rigid rock base. In these situations, the ultimate bearing capacity of shallow foundation resting on soil layer of limited thickness is influenced by the lower rigid boundary. Most of the experimental studies were related to centric vertical loading. However, none of the published studies address the effect of load eccentricity, load inclination, and the effect of rigid base located at a limited depth on the ultimate bearing capacity of circular foundation on limited thickness of sand layer using ANN. The purpose of this study is to develop a neural network model from the results of laboratory model tests to estimate the reduction factor. Artificial neural network (ANN) is an artificial intelligence system inspired by the behavior of human brain and nervous system. In the present study a feed forward back propagation neural network model has been used to predict the reduction factor of eccentrically inclined loaded circular foundation. Backpropagation neural network is most suitable for prediction problems and Levenberg-Marquadrt algorithm is adopted as it is efficient in comparison to gradient descent backpropagation algorithm (Goh et al. 2005; Hornik et al. 1989). By drawing a neural interpretation diagram relationship between input and output are found out. A prediction model is developed based on the weights of the ANN model. The developed reduction factor is compared with the experimental reduction factor.

2 Analysis and Data

All the laboratory model tests were conducted using a poorly graded sand with effective grain size $D_{10} = 0.325$ mm, uniformity coefficient, $C_u = 1.45$, and coefficient of gradation, $C_c = 1.15$. Relative density D_r of sand is 69% and angle of internal friction, $\phi = 40.9^{\circ}$. Model foundations used for the tests had dimensions of 100 mm diameter. Mild steel plate 30-mm thick was used to make the model foundations. The bottom of the foundation was made rough by applying glue and rolling the steel plate over sand.

One hundred laboratory model tests were conducted. Three parameters H/B, α/ϕ and e/B are used as inputs in the ANN model, and the output is the reduction factor *RF* given by

$$RF = \frac{q_{u(H/B, \alpha/\phi, e/B)}}{q_{u(\alpha/\phi, e/B)}} \tag{1}$$

where $q_{u(H/B, \alpha/\phi, e/B)}$ is the ultimate eccentrically inclined load per unit area of a circular foundation on the surface of a sand layer of limited thickness (Table 1) and $q_{u(\alpha/\phi, e/B)}$ is the ultimate eccentrically inclined load per unit area on a sand layer extending to a great depth (Table 2).

Out of 100 tests, 75 tests are considered for training and the remaining 25 are considered for testing. All the inputs and output are normalized in the range of [-1, 1] before training. A feed-forward back-propagation neural network is used with hyperbolic tangent sigmoid function and linear function as the transfer function. The network is trained with Levenberg-Marquardt (LM) algorithm as it is efficient in comparison to gradient descent back-propagation algorithm. The ANN has been implemented using MATLAB V 7.11.0(R2015b).

Data	Expt.	H/B	α/ϕ	e/B	$q_{u(H/B, \alpha/\phi, e/B)}$	RF _{expt.}	<i>RF</i> _{ANN}	Deviation
type	No.				kN/m ²	_		(%)
Training	1	0.3	0.000	0	880	7.59	7.91	-4.21
	2	0.3	0.000	0.05	810	7.79	7.84	-0.62
	3	0.3	0.000	0.1	690	7.84	7.77	0.94
	4	0.3	0.122	0	810	7.79	7.44	4.45
	5	0.3	0.122	0.05	745	7.76	7.37	4.97
	6	0.3	0.122	0.15	510	7.29	7.24	0.62
	7	0.3	0.244	0	680	7.08	6.99	1.30
	8	0.3	0.244	0.1	515	6.78	6.86	-1.24
	9	0.3	0.244	0.15	420	6.67	6.80	-1.93
	10	0.3	0.367	0.05	508	6.51	6.48	0.55
	11	0.3	0.367	0.1	427	6.47	6.41	0.95
	12	0.3	0.367	0.15	350	6.60	6.34	4.06
	13	0.3	0.489	0	420	5.92	5.96	-0.77
	14	0.3	0.489	0.05	372	5.90	5.86	0.82
	15	0.3	0.489	0.1	300	5.66	5.75	-1.65
	16	0.5	0.000	0	425	3.66	3.72	-1.52
	17	0.5	0.000	0.05	390	3.75	3.68	1.75
	18	0.5	0.000	0.15	270	3.51	3.62	-3.11
	19	0.5	0.122	0	385	3.70	3.49	5.69
	20	0.5	0.122	0.1	295	3.51	3.43	2.41
	21	0.5	0.122	0.15	245	3.50	3.40	2.96
	22	0.5	0.244	0.05	290	3.30	3.25	1.32
	23	0.5	0.244	0.1	240	3.16	3.22	-2.03
	24	0.5	0.244	0.15	200	3.17	3.19	-0.55
	25	0.5	0.367	0	247	2.98	3.05	-2.45
	26	0.5	0.367	0.05	220	2.82	3.00	-6.42
	27	0.5	0.367	0.1	190	2.88	2.95	-2.38
	28	0.5	0.489	0	186	2.62	2.59	1.00
	29	0.5	0.489	0.05	160	2.54	2.55	-0.28
	30	0.5	0.489	0.15	108	2.54	2.47	2.68
	31	1	0.000	0	194	1.67	1.57	5.84
	32	1	0.000	0.1	144	1.64	1.57	3.93
	33	1	0.000	0.15	110	1.43	1.57	-9.95
	34	1	0.122	0.05	156	1.63	1.56	3.94
	35	1	0.122	0.1	132	1.57	1.56	0.89
	36	1	0.122	0.15	103	1.47	1.55	-5.50
	37	1	0.244	0	140	1.46	1.47	-1.05
	38	1	0.244	0.05	130	1.48	1.44	2.63
	39	1	0.244	0.1	112	1.47	1.40	5.24

Table 1. Database used for ANN model and compared with experimental results

(continued)

43

Data type	Expt. No.	H/B	α/φ	e/B	$q_{u(H/B, \alpha/\phi, e/B)}$ kN/m ²	$RF_{expt.}$	RF _{ANN}	Deviation (%)
	40	1	0.367	0	112	1.35	1.19	11.95
	41	1	0.367	0.05	101	1.29	1.18	9.23
	42	1	0.367	0.15	70.5	1.33	1.16	12.75
	43	1	0.489	0	90	1.27	1.15	9.35
	44	1	0.489	0.1	62	1.17	1.15	1.95
	45	1	0.489	0.15	54	1.27	1.15	9.80
	46	2	0.000	0.05	120	1.15	1.23	-6.55
	47	2	0.000	0.1	104	1.18	1.19	-1.08
	48	2	0.000	0.15	92	1.19	1.17	2.29
	49	2	0.122	0	120	1.15	1.11	3.46
	50	2	0.122	0.05	110	1.15	1.11	3.01
	51	2	0.122	0.1	95	1.13	1.11	1.89
	52	2	0.244	0	112	1.17	1.11	5.13
	53	2	0.244	0.05	99	1.13	1.11	1.63
	54	2	0.244	0.15	74	1.17	1.11	5.79
	55	2	0.367	0	94	1.13	1.11	2.29
	56	2	0.367	0.1	72	1.09	1.11	-1.44
	57	2	0.367	0.15	62	1.17	1.11	5.40
	58	2	0.489	0.05	66	1.05	1.11	-5.63
	59	2	0.489	0.1	54	1.02	1.11	-8.61
	60	2	0.489	0.15	44	1.04	1.11	-6.89
	61	3	0.000	0	119	1.03	1.11	-7.87
	62	3	0.000	0.05	111	1.07	1.11	-3.68
	63	3	0.000	0.1	94	1.07	1.11	-3.60
	64	3	0.122	0	110	1.06	1.11	-4.62
	65	3	0.122	0.05	102	1.06	1.11	-4.15
	66	3	0.122	0.15	74	1.06	1.11	-4.68
	67	3	0.244	0	102	1.06	1.11	-4.15
	68	3	0.244	0.1	82	1.08	1.11	-2.56
	69	3	0.244	0.15	67	1.06	1.11	-4.05
	70	3	0.367	0.05	83	1.06	1.11	-3.99
	71	3	0.367	0.1	70	1.06	1.11	-4.34
	72	3	0.367	0.15	57	1.08	1.11	-2.89
	73	3	0.489	0	74	1.04	1.11	-6.17
	74	3	0.489	0.05	65	1.03	1.11	-7.26
	75	3	0.489	0.1	54	1.02	1.11	-8.61
Testing	76	0.3	0.000	0.15	565	7.34	7.70	-4.92
	77	0.3	0.122	0.1	645	7.68	7.31	4.83
	78	0.3	0.244	0.05	610	6.93	6.93	0.09

 Table 1. (continued)

(continued)

Data	Expt.	H/B	α/φ	e/B	$q_{u(H/B, \alpha/\phi, e/B)}$	RF _{expt.}	<i>RF</i> _{ANN}	Deviation
type	No.				kN/m ²			(%)
	79	0.3	0.367	0	550	6.63	6.54	1.24
	80	0.3	0.489	0.15	245	5.76	5.66	1.86
	81	0.5	0.000	0.1	330	3.75	3.65	2.67
	82	0.5	0.122	0.05	360	3.75	3.46	7.76
	83	0.5	0.244	0	323	3.36	3.28	2.46
	84	0.5	0.367	0.15	150	2.83	2.89	-1.96
	85	0.5	0.489	0.1	135	2.55	2.51	1.57
	86	1	0.000	0.05	170	1.63	1.57	3.74
	87	1	0.122	0	168	1.62	1.56	3.20
	88	1	0.244	0.15	83	1.32	1.35	-2.54
	89	1	0.367	0.1	82	1.24	1.17	6.10
	90	1	0.489	0.05	81	1.29	1.15	10.72
	91	2	0.000	0	128	1.10	1.27	-15.17
	92	2	0.122	0.15	84	1.20	1.11	7.62
	93	2	0.244	0.1	86	1.13	1.11	2.20
	94	2	0.367	0.05	83	1.06	1.11	-3.99
	95	2	0.489	0	75	1.06	1.11	-4.76
	96	3	0.000	0.15	83	1.08	1.11	-2.66
	97	3	0.122	0.1	89	1.06	1.11	-4.44
	98	3	0.244	0.05	95	1.08	1.11	-2.51
	99	3	0.367	0	88	1.06	1.11	-4.37
	100	3	0.489	0.15	43	1.01	1.11	-9.37

Table 1. (continued)

3 Results and Discussion

Three inputs and one output parameters were considered in the ANN model. The schematic diagram of the ANN architecture is shown in Fig. 1, which was computed from the database. The number of neurons in hidden layer is varied and the optimum number was taken based on mean square error (mse) value which was maintained at 0.001. In this ANN model there were two neurons evaluated in hidden layer as shown in Fig. 2. Therefore the final ANN architecture as 3-2-1[i.e. 3 (input) – 2 (hidden layer neuron) – 1 (output)].

Mean square error (MSE) is defined as

$$MSE = \frac{\sum_{i=1}^{n} \left(RF_i - RF_p \right)^2}{n} \tag{2}$$

Coefficient of efficiency, R^2 is defined as

α/ϕ	e/B	$q_{u \ (\alpha/\varphi, \ e/B)} \over (\mathrm{kN/m}^2)$
0	0	116
0	0.05	104
0	0.10	88
0	0.15	77
0.122	0	104
0.122	0.05	96
0.122	0.10	84
0.122	0.15	70
0.244	0	96
0.244	0.05	88
0.244	0.10	76
0.244	0.15	63
0.367	0	83
0.367	0.05	78
0.367	0.10	66
0.367	0.15	53
0.489	0	71
0.489	0.05	63
0.489	0.10	53
0.489	0.15	42.5

 Table 2.
 Database used for ANN model







Fig. 2. Variation of hidden layer neuron with mean square error (mse)

$$R^2 = \frac{E_1 - E_2}{E_1} \tag{3}$$

where,

$$E_1 = \sum_{i=1}^{n} \left(RF_i - \overline{RF} \right)^2 \tag{4}$$

and

$$E_2 = \sum_{i=1}^{n} \left(RF_p - RF_i \right)^2 \tag{5}$$

where, RF_i , \overline{RF} and RF_p are the experimental, average experimental, predicted RF values respectively; and n = number of training data.

The coefficient of efficiency (R^2) is found to be 0.997 for training and 0.996 for testing as shown in Figs. 3, and 4. The weights and biases of the network are presented in Table 3. These weights and biases can be utilized for interpretation of relationship in between the inputs and output, sensitivity analysis and framing an ANN model in the form of an equation. The residual analysis was carried out by calculating the residuals in between experimental reduction factor and predicted reduction factor for training data. Residuals can be defined as the difference between the experimental and predicted *RF* value and is given by

$$e_r = RF_i - RF_p \tag{6}$$



Fig. 3. Correlation between predicted reduction factors with experimental reduction factor for training data



Fig. 4. Correlation between predicted reduction factors with experimental reduction factor for testing data

The residuals are plotted with the experiment number as shown in Fig. 5. It is observed that the residuals are evenly distributed along the horizontal axis of the plot. Therefore it can be said that the network is well trained and can be used for prediction with reasonable accuracy.

Neuron		Weight				
		W _{ik}		Wk		Bias
	$(H/B)_n$	$(\alpha/\phi)_n$	$(e/B)_n$	RF	b_{hk}	b_0
Hidden neuron $1(k = 1)$	-4.8926	-0.1266	-0.014	2.233	-5.1941	1.317
Hidden neuron $2(k = 2)$	-6.0138	-3.6338	-0.3389	0.0586	-2.5935	

Table 3. Values of connection weights and biases



Fig. 5. Residual distribution of training data

4 Sensitivity Analysis

Sensitivity analysis was carried out for selection of important input variables. Different approaches have been suggested to select the important input variables. The Pearson correlation coefficient is one of them in selecting proper inputs for the ANN model. It was approached by Guyon and Elisseeff (2003) and Wilby et al. (2003). Goh (1994), Shahin et al. (2002), Behera et al. (2013), Sahu et al. (2017, 2018), Sethy et al. (2017) have used Garson's algorithm (Garson 1991) in which the input-hidden and hidden-output weights of trained ANN model are partitioned. In Garson's algorithm the absolute values of weights are taken to select the important input variables. It does not provide information on the effect of input variables in terms of direct or inverse relation to the output. Olden et al. (2004) proposed a connection weights approach based on the neural interpretation diagram (NID), in which the actual values of input-hidden and hidden-output weights are taken. Table 4 shows the cross-correlation of the three input parameters with the reduction factor (*RF*) value. From the Table 4 it can be seen that *RF* is highly correlated to *H/B* with a values of -0.71 followed by $\alpha/\phi = -0.11$ and

49

e/B = -0.01. The relative importance, *quantified through the parameter* S_i of three input parameters as per Garson's algorithm is presented in Table 5. The *H/B* is found to be the most important input parameters with relative importance value being 78.71% followed by 19.45% for α/ϕ and 1.84% for e/B. As per the connection weight approach (Olden et al. 2004) the relative importance of the present input variables is also presented in Table 5. *H/B* is also the most important input parameter ($S_i = -11.28$) followed by α/ϕ ($S_i = -0.5$) and e/B ($S_i = -0.05$). The S_i values being negative imply that *H/B*, α/ϕ , and e/B are indirectly related to *RF*. In other words increase in *H/B*, α/ϕ , and e/B leads to decrease in *RF* and leads to decrease in ultimate bearing capacity.

Cross-correlation Matrix						
	(H/B)	(α/φ)	(e/B)	RF _{expt}		
(<i>H</i> / <i>B</i>)	1	0	0	-0.71		
(α/φ)		1	0	-0.11		
(<i>e</i> / <i>B</i>)			1	-0.01		
RF _{expt}				1		

Table 4. Cross-correlation matrix of input and output for reduction factor

 Table 5. Relative importance of different inputs as per Garson's algorithm and connection weight approach

Parameters	Garson's algo	rithm	Connection weight approach		
	Relative importance	Ranking of input as per relative importance	S_i values as per connection weight approach	Ranking of input as per relative importance	
H/B	78.71	1	-11.28	1	
α/ϕ	19.45	2	-0.5	2	
e/B	1.84	3	-0.05	3	

5 Neural Interpretation Diagram (NID)

Ozesmi and Ozesmi (1999) proposed neural interpretation diagram for visual interpretation of the connection weight among the neurons. For the present study with the weights as obtained and shown in Table 1, an NID is presented in Fig. 6. The lines joining the input-hidden and hidden output neurons represent the weights. The positive weights are represented by solid lines and negative weights by dashed lines and the thickness of the line is proportional to its magnitude.

It is seen from Table 5 that S_i values for parameters H/B, α/ϕ , and e/B are negative indicating that the parameters are indirectly related to *RF* values, whereas S_i values for. This is shown in Fig. 6. Therefore, the developed ANN model is not a black box and could explain the physical effect of input parameters on the output.



Fig. 6. Neural interpretation diagram (NID) showing lines representing connection weights and effects of inputs on reduction factor (RF)

6 ANN Model Equation for Reduction Factor Based on Trained Neural Network

A model equation is developed using the weights obtained from trained neural network model (Goh et al. 2005). The mathematical equation relating input parameters (*H*/*B*, α / ϕ , and *e*/*B*) to output given by

$$RF_n = f_n \left\{ b_0 + \sum_{k=1}^h \left[w_k f_n \left(b_{hk} + \sum_{i=1}^m w_{ik} X_i \right) \right] \right\}$$
(7)

where RF_n is the normalized value of RF in the range [-1, 1], f_n is the transfer function, h is the number of neurons in the hidden layer, X_i is the normalized value of inputs in the range [-1, 1], m is the number of input variables, w_{ik} is the connection weight between the i^{th} layer of input and k^{th} neuron of hidden layer, w_k is the connection weight between the k^{th} neuron of hidden layer and single output neuron, b_{hk} is the bias at the k^{th} neuron of hidden layer and b_0 is the bias at the output layer.

The model equation of RF of shallow circular foundations on sand layer of limited thickness subjected to eccentrically inclined load was formulated using the values of the weights and biases shown in Table 3 as per the following steps. Step 1

The input parameters were normalized in the range [-1, 1] by the following expressions

$$X_n = 2 \left(\frac{X_n - X_{\min}}{X_{\max} - X_{\min}} \right)$$
(8)

Step 2

Calculate the normalized value of reduction factor (RF_n) using the following expressions

$$A_1 = -4.89 \left(\frac{H}{B}\right)_n - 0.13 \left(\frac{\alpha}{\phi}\right)_n - 0.01 \left(\frac{D_f}{B}\right)_n - 5.19 \tag{9}$$

$$A_{2} = -6.01 \left(\frac{B}{L}\right)_{n} -3.63 \left(\frac{e}{B}\right)_{n} -0.34 \left(\frac{D_{f}}{B}\right)_{n} -2.59$$
(10)

$$B_1 = 2.23 \left(\frac{e^{A_1} - e^{-A_1}}{e^{A_1} + e^{-A_1}} \right)$$
(11)

$$B_2 = 0.06 \left(\frac{e^{A_1} - e^{-A_1}}{e^{A_1} + e^{-A_1}} \right) \tag{12}$$

$$C_1 = B_1 + B_2 + 1.32 \tag{13}$$

$$RF_n = C_1 \tag{14}$$

Denormalize the RF_n value obtained from Eq. 14 to actual RF as

$$RF = 0.5(RF_n + 1)(RF_{\max} - RF_{\min}) + RF_{\min}$$
(15)

$$RF = 0.5(RF_n + 1)(7.84 - 1.01) + 1.01$$
(16)

Figure 7 Shows the comparison of reduction factor (*RF*) obtained from Eqs. 16 and 1. It can be seen that the results predicted by using artificial neural network (ANN) are closer to the results obtained from model tests. The deviation between the *RF* obtained from model tests and those predicted *RF* is within $\pm 10\%$ except two values as shown in Table 1. The proposed ANN model can be used as an effective tool in predicting the reduction factor (*RF*) and hence, the ultimate bearing capacity shallow circular foundation on sand layer of limited thickness underlain by rigid rough base subjected to an eccentrically inclined load.

51



Fig. 7. Comparison of ANN results with experimental RF

7 Conclusion

Based on developed neural network model, the following conclusions may be drawn.

- 1. The errors are distributed evenly along the centerline as per residual analysis. It can be concluded that the network was well trained and can predict the reduction factor (RF).
- 2. Based on Pearson correlation coefficient and Garson's algorithm, it was observed that H/B is the most important input parameter followed by α/ϕ and e/B.
- 3. The developed ANN model could explain the physical effect of inputs on the output, as described in NID. It has been observed that H/B, α/ϕ , and e/B is inversely related to *RF*.
- 4. A model equation is developed based on the trained weights of ANN to calculate the ultimate bearing capacity of circular foundation on sand layer of limited thickness.
- 5. The deviation between the *RF* obtained from model tests and those predicted from ANN model is within $\pm 10\%$

References

- Behera, R.N., et al.: Prediction of ultimate bearing capacity of eccentrically inclined loaded strip footing by ANN part 1. Int. J. Geotech. Eng. (2013). https://doi.org/10.1179/1938636212z. 00000000012
- Garson, G.D.: Interpreting neural-network connection weights. Artif. Intell. Expert 6(7), 47–51 (1991)

- Guyon, I., Elisseeff, A.: An introduction to variable and feature selection. J. Mach. Learn. Res. 3, 1157–1182 (2003)
- Goh, A.T.C., et al.: Bayesian neural network analysis of undrained side resistance of drilled shafts. J. Geotech. Geoenviron. Eng. (2005). https://doi.org/10.1061/(asce)10900241(2005) 131:1(84)
- Hornik, K., et al.: Multilayer feed forward networks are universal approximators. Neural Netw. 2, 359–366 (1989)
- Olden, J.D., et al.: An accurate comparison of methods for quantifying variable importance in artificial neural networks using simulated data. Ecol Model. (2004). https://doi.org/10.1016/j. ecolmodel.2004.03.0.013
- Ozesmi, S.L., Ozesmi, U. An artificial neural network approach to spatial modeling with inter specific interactions. Ecol. Model. (1999). https://doi.org/10.1016/S0304-3800(98)00149-5
- Sahu, R., et al.: Use of ANN and Neuro fuzzy model to predict bearing capacity factor of strip footing resting on reinforced sand and subjected to inclined loading. Int. J. Geosynthetics Ground Eng. **3**(3), 29 (2017)
- Sahu, R., et al.: Bearing capacity prediction of inclined loaded strip footing on reinforced sand by ANN. In: International Congress and Exhibition "Sustainable Civil Infrastructures: Innovative Infrastructure Geotechnology", pp. 97–109. Springer, Cham (2018)
- Sethy, B.P., et al.: Application of ANN and ANFIS for predicting the ultimate bearing capacity of eccentrically loaded rectangular foundations. Int. J. Geosynthetics Ground Eng. **3**(4), 35 (2017)
- Shahin, M.A., et al.: Predicting settlement of shallow foundations using neural network. J. Geotech. Geoenviron. Eng. (2002). https://doi.org/10.1061/(asce)1090-0241(2002)128:9 (785)