

Chapter 17

Models for Effective Deployment and Redistribution of Shared Bicycles with Location Choices



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Abstract We develop practical OR models to support decision making in the design and management of public car-sharing or bicycle-sharing systems. We develop a network flow model with proportionality constraints to estimate the flow of bicycles within the network, and to estimate the number of trips supported by the system given an initial allocation of bicycles at each station. Furthermore, the number of docks needed at each station, to support the flow, can also be estimated. We also examine the impact of periodic redistribution of bicycles in the network to support more flows, and the location choices of bicycle stations. We conduct our numerical analysis using transit data from the train and bus operators in Singapore. Given that a substantial proportion of the passengers in the train system commute short distance – more than 16% of the passengers alight within 2 stops from the start station – this forms a latent segment of demand for the bicycle-sharing program. We argue that for the bicycle-sharing system to be most effective for this customer segment, the system must deploy the right number of bicycles at the right place, as this affects the utilization rate of the bicycles, how the bicycles circulate within the system, and also the effectiveness of any redistribution strategy. The same approach can be extended to incorporate the issue of station location choices, by incorporating the proportional flow constraints into the MIP formulation. Using a set of bus transit data, we implemented this approach to identify the ideal locations for the bicycle stations in a new town in Singapore, to support the movement of passengers from residential areas to the train station.

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17.1 Introduction

In recent years, the sharing economy has morphed into a major part of the global economy, impacting various aspects of consumers' lives. Among many other prominent examples, bike sharing which has been around since 1965 has developed dramatically in the 2000s with the introduction of new information technology. The number of bicycle-sharing systems (BSSs) around the world increased from around 200 in 2010 (MetroBike LLC 2011) to approximately 1200 in operation (MetroBike LLC 2017) and more than 350 under construction or being planned for the near future (Russell and DeMaio 2017) at the end of 2016.

With heightened concerns about global oil prices, carbon emissions, and traffic congestion, governments around the world are exploring ways to “nudge” urban residents to commute using public transport instead of private automobiles. Many cities have set up public BSSs to facilitate short trips within the city. As of August 2017, there were approximately 172,700 bikes shared worldwide in 132 cities.

17.1.1 *Review of the Bicycle-Sharing Systems*

Bicycle-Sharing System (BSS) is perceived as a green, healthy and sustainable mode of public transport, which helps decrease greenhouse gas emissions through reducing road congestion and fuel consumption, and improves the first/last mile connection to other modes of transport. The citizens of Amsterdam in the Netherlands initiated arguably the world's first generation of bicycle-sharing program with “White Bicycles” on July 28, 1965. This system operated on a rather ad hoc basis, i.e., one could locate a white bicycle on the street, ride it to the destination, and leave it there for the next trip. Unfortunately, due to theft and abuse, the program only survived for several days. The second generation of the BSS came with improved product and system design features. For example, the bicycle was specifically designed for urban use, its components were not usable on other types of bicycles, and public bicycle-sharing stations were equipped with coin-deposit machines. The citizens of Copenhagen in Denmark launched such a system in 1995. However, this system still faced the problem of bicycle theft since the system was not able to track the identity of the users. This gave rise to the impetus to develop the third generation of BSSs with user tracking abilities – Smart Bicycles, equipped with electronically-locking racks, telecommunication systems, magnetic stripe cards or smart cards, and mobile phone access. The first third generation BSS took off in 2005, with the launch of the Vélo'v in Lyon. In a typical third generation BSS, base stations are located around the city with pre-determined number of bicycles at the beginning of each day. Due to the unbalanced usage of bikes in the system, there is either congestion (dock unavailability) or starvation (bicycle unavailability) of bicycles at the stations each day, which results in a lot of unmet demands (Ghosh et al. 2017). Through analyzing social media data for the BSSs in Spain, Serna et al.

(2017) concluded that 16.4% of demand would be lost due to system unavailability that was caused by uneven usage. As such, managing the redistribution activities to increase system availability is crucial to any successful implementation of such BSSs (The Economist 2011). Given that potential dock unavailability is a major issue, recently a tech-on-bike/dockless system, in which the locking and rental technology is located on the bike itself, was developed.

Bikes are now the new frontier to on-demand transportation. The number of BSSs is expected to continue to grow as new technology is utilized to improve the current systems. Currently, there are a few main types of Bicycle-Sharing Systems (BSSs) in use, namely Ad-hoc, Kiosk-based, Tech-on and Managed Fleet system.

Ad-hoc Bicycle-Sharing System An ad-hoc system involves the operator purchasing and distributing marked bicycles across the community without any locking technology or bike stations. Such systems are usually informal by design and rely on the integrity of individuals to use the bikes in an appropriate manner. The ad-hoc bicycle-sharing system typically works in closed campuses, such as universities, or companies located in Silicon Valley, where the area tends to be large, and the usage of public transport is minimal.

Kiosk-Based System In a kiosk system, bikes are secured to and rented from tech-enabled docking stations. These stations range in sophistication from simple bike racks with key lockboxes to digital automatic locking kiosks with integrated rental systems. Kiosk systems are more common in the public BSSs around the world. Public BSSs can either be introduced under the government, or under joint efforts made by a company and the government. Such examples include Taiwan's YouBike (collaboration between the government and a local company) and Korea's Seoul Bike (managed by the city government).

Tech-on Bike/Dockless System Technology advancement in recent years have enabled tech-on-bike systems, in which the locking and rental technology is located on the bike itself. Riders check out bikes using a smartphone app which allows them to release the lock that secures the bike to the rack. Several Asian countries such as China and Singapore (of0 and Mobike) (Forbes 2017) have adopted this system. A challenge in this sharing system is that it usually results in a large number of discarded bikes. Development of new technology which enables better tracking of the whereabouts of the bikes is needed to handle this problem.

Managed Fleet System In a managed fleet system, bikes are stored in a central location and managed by an employee. Typically, such bikes are equipped with locks, and bike users would have to obtain the keys from the employee at a centralized location, such as a student center on a campus, to unlock the bike for usage.

While Kiosk-Based Systems are still the most commonly used systems around the world at present, future technology development which enables better tracking capability may increase the adoption rate for the Tech-on Bike/Dockless Systems. However, dockless or not, managing the redistribution activities to increase system availability is still crucial since a system without dock unavailability problems is likely to still face bike unavailability problems.

17.1.2 *Research Issues and Structure of the Chapter*

Most of the earlier work on bike redistribution focused on the operational problem of moving bikes using special vehicles deployed for this purpose. Benchimol et al. (2011) studied the static bike repositioning problem (SBRP) and developed a station balancing technique based on the traveling salesman problem (TSP) to reposition bikes by a single vehicle. Raviv et al. (2013) developed four mixed integer linear program formulations to solve large instances with the objective of minimizing the user dissatisfaction in face of stochastic demand. Angeloudis et al. (2014) introduced a novel strategic repositioning algorithm to tackle SBRP, addressing both routing and assignment problems.

Li et al. (2016) considered multiple classes of bikes in SBRP, whereas Kloimüller and Raidl (2017) considered only fully loaded vehicles for movement among the rental stations in SBRP. Schuijbroek et al. (2017) solved SBRP by combining inventory and vehicle routing issues in the BSSs, and proposed a new cluster-first route-second heuristic to search for the optimal solution.

Although the aforementioned techniques are effective in reducing the repositioning cost to some extent, these solutions could not incorporate the real-time demand of the users in their approach. To tackle this issue, Shu et al. (2013) proposed a model on bicycle deployment and flow in BSSs and used the model to address various pertinent issues in managing bicycle-sharing networks. In the rest of this chapter, we address the following questions by introducing the work in Shu et al. (2013) and extending the model and discussion to incorporate location decisions in Sect. 17.2.

- Given the location of the stations, what is the appropriate number of bicycles to deploy in the network? The availability of bicycles affects the number of bicycle trips made and also the bicycle utilization rate. The former measures how much of existing demand can be captured, whereas the latter affects the economic viability of the system. Given the demand pattern, we need an optimal number of bicycles, appropriately located, to make effective use of the resources available to meet demand.
- Impact of redistribution: The flow of the bicycles is dictated by the travel patterns of the commuters. To deal with flow imbalances and to improve bicycle utilization, we may have to do periodic redistribution of bicycles within the system. However, if the bicycles are already heavily (or under) utilized, the periodic redistribution strategy may have only a limited impact on the performance of the system (measured by the additional number of bicycle trips supported). Given the high operational cost associated with redistribution in the BSS, it is thus crucial to estimate the improvement in performance prior to its adoption in actual operation.
- The number of bicycle docks to be installed at each station: To make the bicycle-sharing program implementable, we need to consider how many bicycle docks to install at each station so that commuters can return their bicycles upon arrival at the destination station. Clearly, the number of docks needed at each station

depends on the utilization rate of the bicycles and how the flows are supported in the system, and whether periodic redistribution is used to match supply with demand. In fact, redistribution has the potential to reduce the number of docks needed at each station. The dock design of the bicycle-sharing network is thus intimately tied to the operational decision on bicycle redistribution. It will be useful to have a system to help estimate the number of docks to be installed at each station to support the flow in the system.

In this chapter, we introduce the work in Shu et al. (2013) about a simple proportional network flow model to help to address the above issues. We discuss the theory and intuition for this model in the next section. To validate the findings from the model, a set of commuter data in a Singapore mass rapid transit system to develop the demand model is used for the BSS in Sect. 17.3. By focusing on this segment of the market, and through comparison with extensive simulation results, we demonstrate that the proposed model can be used to approximate the flow of bicycles in the system to a reasonable level of accuracy. In Sect. 17.2.2, we describe how the model can be adopted for general bicycle-sharing network design to incorporate the selection choices of bicycle stations in the network. This leads to a mixed integer proportional network flow model with 0–1 decision variables to denote station choices. We validate the findings in Sect. 17.4 using transit data on bus trips in a new town in Singapore. Finally, we conclude the chapter in Sect. 17.5.

17.2 The Stochastic Network Flow Model

We assume that there are an initial allotment of bicycles at each train station. For each time period, passengers arrive randomly at the station to use the bicycles to travel to their destinations. Data from existing BBSs shows that bike trips are normally within short distance. Our data also shows that in Singapore more than 16% of the train system commuters alight within two stops from the start station. Therefore our study focus on origin-destination demands within two stops in the transit network. The goal is to analyze and estimate the number of such trips that can be supported and substituted by the public bicycle-sharing system, based on the initial allotment of bicycles and the passenger arrival process. This is a technically challenging problem.

Formally, let \mathbf{S} denote the set of stations in the network. In each time period t , the number of passengers who arrive with plan to travel from station i to station j follows a Poisson process, with rate $r_{ij}(t)$. The total number of passengers arriving to use bicycles at station i is thus given by a Poisson process with rate $\sum_{j:j \neq i} r_{ij}(t)$. Within each time period, let $D_{ij}(t)$ and $D_i(t)$ denote the number of arrivals traveling on each link and into each station, respectively. We assume that all rides can be completed within a single time period. Note that bicycles are allocated to the passengers on a first-come-first-serve basis, so that whenever the initial stock of bicycles at a station is depleted, the late comers will not be able to ride to their

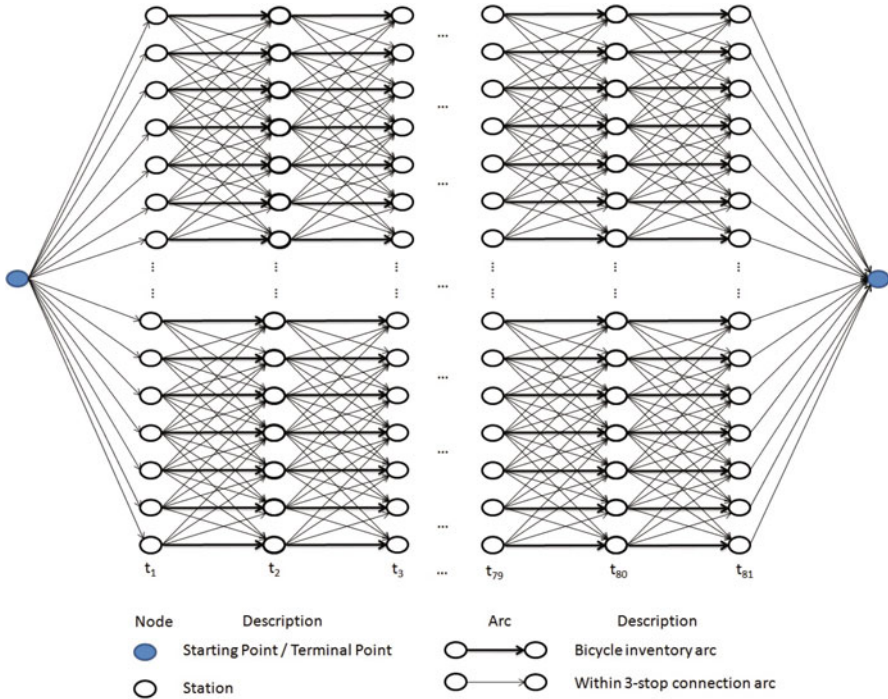


Fig. 17.1 Bicycle flow network: time expanded graph

destinations using bicycles and such demands are considered lost. Figure 17.1 shows the time expanded view of the entire network, where the flow on each arc depends on the realization of the number of passengers arriving in each period to each station, the order of arrivals, and also the number of bicycles available at the station.

To gain better insights into this problem, we consider an initial allotment of bicycles $x_i(t)$ at station i in time period t . The number of bicycle trips that will materialize in time period t will be $\min(x_i(t), D_i(t))$. However, the number of bicycles flowing from i to j will depend on the order of arrivals of passengers at station i , and is more complicated to track. For $0 < p < 1$, let $D_i(t)[p]$ denote the number of tagged passengers where each passenger is tagged with probability p upon arrival. More formally, let $\{\eta_i(p)\}$ denote a sequence of independent Bernoulli r.v.s with mean p , then

$$D_i(t)[p] = \sum_{k=1}^{D_i(t)} \eta_k(p).$$

By the well known Poisson Thinning Lemma, $D_i(t)[p]$ is Poisson with rate $p \times (\sum_{j:j \neq i} r_{ij}(t))$. Let $p_{ij}(t) = r_{ij}(t) / \sum_{k:k \neq i} r_{ik}(t)$. Hence

$$D_{ij}(t) \sim D_i(t)[p_{ij}(t)].$$

By a slight abuse of notation, for some number of bicycles $x_i(t)$, let

$$\min(x_i(t), D_i(t))[p] = \sum_{k=1}^{\min(x_i(t), D_i(t))} \eta_k(p).$$

If there are $x_i(t)$ bicycles at station i , the number of bicycles leaving station i at time t is clearly $\min(x_i(t), D_i(t))$. The number of bicycles traveling from i to j however depends on the order of arrival of the customers traveling to different destinations. In particular, the number of passengers traveling to j follows the distribution of

$$\min(D_i(t), x_i(t)) [p_{ij}(t)].$$

The number of bicycles at station i at the end of the time period is given by

$$\begin{aligned} x_i(t+1) &= x_i(t) - \underbrace{\min(D_i(t), x_i(t))}_{\text{total departures}} + \underbrace{\sum_{j:j \neq i} (\min(D_j(t), x_j(t)) [p_{ji}(t)])}_{\text{total arrivals}} \\ &= x_i(t) - \sum_{j:j \neq i} (\min(D_i(t), x_i(t)) [p_{ij}(t)]) \\ &\quad + \sum_{j:j \neq i} (\min(D_j(t), x_j(t)) [p_{ji}(t)]) \end{aligned} \quad (17.1)$$

The expected number of trips traversed using bicycles is given by

$$\sum_{t=0}^N \sum_{i \in S} \sum_{j:j \neq i} E(\min(D_i(t), x_i(t)) [p_{ij}(t)]).$$

Let $y_i(t) = E(x_i(t))$, and

$$\begin{cases} y_{ij}(t) = E(\min(D_i(t), x_i(t)) [p_{ij}(t)]), \\ y_{ii}(t) = y_i(t) - \sum_{j:j \neq i} y_{ij}(t). \end{cases}$$

By the above definition, $y_{ij}(t)$ stands for the expected number of bicycles traveling from station i to station j during time period t . We next describe some simple structural properties of $y_{ij}(t)$.

Lemma 1 $y_{ij}(t) : y_{il}(t) = r_{ij}(t) : r_{il}(t)$.

Lemma 2 $y_{ij}(t) \leq r_{ij}(t)$.

Lemma 3 $y_i(t+1) = y_i(t) - \sum_{j:j \neq i} y_{ij}(t) + \sum_{j:j \neq i} y_{ji}(t)$.

Let Z^* denote the optimal objective value to the following linear programming problem:

$$\begin{aligned}
 Z^* &= \max \sum_{t=0}^N \sum_{i \in S} \sum_{j:j \neq i} y_{ij}(t) \\
 \text{subject to} \quad &y_i(t+1) = y_i(t) - \sum_{j:j \neq i} y_{ij}(t) + \sum_{j:j \neq i} y_{ji}(t), \quad \forall i, t; \\
 &y_i(t) = y_{ii}(t) + \sum_{j:j \neq i} y_{ij}(t), \quad \forall i, t; \\
 &\frac{y_{ij}(t)}{y_{il}(t)} = \frac{r_{ij}(t)}{r_{il}(t)}, \quad \forall i, j, l, t; \\
 &y_i(0) = x_i(0), \quad \forall i; \\
 &0 \leq y_{ij}(t) \leq r_{ij}(t), \quad \forall t, i \neq j.
 \end{aligned}$$

The second constraint depicts, for each station i , the number of bicycles which are available at the beginning of period t equals to the number of bicycles which remain at station i and the number of bicycles which leave station i during period t . Given an initial allotment of bicycles at station i denoted by $x_i(0)$, the mean number of bicycle trips supported in the BSS on each link is a feasible solution to the above LP. Hence we have:

Theorem 1 Z^* denotes an upper bound to the expected number of bicycle trips in the system when the initial allotment of bicycles to station i is given by $x_i(0)$.

The above LP is surprisingly effective in providing a simple estimate on the performance (based on the number of bicycle trips that the system can support) of the BSS with an initial bicycle inventory position $x_i(0)$. We will use this model extensively in the next section to examine the issues of bicycle utilization and the value of bicycle redistribution.

Example 1 To see that the above LP is not exact, consider a 3-station example where there are 2 bicycles at station 3 initially, and none at the other 2 stations. Suppose $r_{31}(0) = r_{32}(0) = 1$, $r_{23}(t) = r_{32}(t) = 1$ for all $t > 1$, and $r_{ij}(t) = 0$ otherwise. In this case, to support the maximum number of flow in the network, the optimal LP solution suppress the flow of bicycles from station 3 to station 1 and 2 in the first period, so that 2 bicycles will remain in station 3 from period 1 onwards to serve the flow between station 2 and station 3, without parking any bicycle at station 1. This LP solution dominates the expected number of trips in the stochastic network flow model.

17.2.1 Equilibrium State in Time Invariant System

In the rest of this section, we further analyze the properties of this formulation (simple network flow with proportionality constraints) to gain insight on the problem.

Suppose the Poisson arrival in each time period is stationary with rate r_{ij} . Is there a way to characterize the number of bicycles in the equilibrium state of the bicycle-sharing network? We modify the LP to provide a glimpse to the answer to this problem.

In the equilibrium state, we expect $y_i(t+1) = y_i(t)$ as $t \rightarrow \infty$. Let

$$y_{ij} = \lim_{t \rightarrow \infty} y_{ij}(t).$$

The total number of bicycles in the system is denoted by N . Let y_{ij}^* denote the optimal solution to the following LP.

$$\begin{aligned} Z^*(\infty) &= \max \sum_{i,j \in \mathbf{S}: j \neq i} y_{ij} \\ \text{subject to} \quad & \sum_{j:j \neq i} y_{ij} = \sum_{j:j \neq i} y_{ji}, \quad \forall i; \\ & \frac{y_{ij}}{y_{il}} = \frac{r_{ij}}{r_{il}}, \quad \forall i, j, l; \\ & 0 \leq y_{ij} \leq r_{ij}, \quad \forall i, j; \\ & \sum_i \left(y_{ii} + \sum_{j:j \neq i} y_{ij} \right) = N. \end{aligned}$$

It can be seen easily that there exists i^* such that $y_{i^*j} = r_{i^*j}$ for all $j \neq i^*$, otherwise we could scale the solution to improve the objective value. We call such nodes the *sink* stations. Furthermore, if there exists i such that $y_{ii}^* > 0$ but $y_{ij}^* < r_{ij}$ for all $j \neq i$, then we could modify the solution by shifting y_{ii}^* to the station i^* , without affecting the feasibility and quality of the solution. i.e.,

$$y_{ii}^* \leftarrow 0, \quad y_{i^*i^*}^* \leftarrow y_{i^*i^*}^* + y_{ii}^*.$$

We call such nodes where $y_{ij}^* < r_{ij}$ the *transient* stations. Note that WLOG, we can assume that $y_{ii}^* = 0$ when i is transient.

Let $z_i^* = \sum_{j:j \neq i} y_{ij}^*$. By the proportionality constraints, it is easy to see that

$$y_{ij}^* = \frac{r_{ij}}{\sum_{k:k \neq i} r_{ik}} z_i^*.$$

Note that z_i^* is a solution to the following system of linear equations:

$$z_i = \sum_{j:j \neq i} \frac{r_{ji}}{\sum_{k:k \neq j} r_{jk}} z_j, \quad i = 1, \dots, n. \tag{17.2}$$

If the transition probability matrix constructed using $r_{ji}/\sum_{k:k \neq j} r_{jk}$ is irreducible, then the above system of equations has essentially a unique solution scaled to a constant. Note that $z_i^* \leq \sum_{k:k \neq i} r_{ik}$ since $y_{ij}^* \leq r_{ij}$, and $\sum_i z_i^* \leq N$. Since our objective is to maximize $\sum_i z_i^*$, the solution to the linear system (17.2) is scaled in such a way that either (i) $\exists \mathbf{S}$ such that $z_i^* = \sum_{k:k \neq i} r_{ik}$ for all $i \in \mathbf{S}$, and $z_i^* < \sum_{k:k \neq i} r_{ik}$ otherwise; or (ii) $\sum_i z_i^* = N$ and $z_i^* < \sum_{k:k \neq i} r_{ik}$ for all i . \mathbf{S} corresponds to the set of sink nodes in the system. In case (i), the surplus $N - \sum_i z_i^*$ can be distributed to any of the y_{ii}^* variables for $i \in \mathbf{S}$ without affecting the optimality of the solution.

Theorem 2 *The linear program $Z^*(\infty)$ may have multiple optimal solutions, but the flow solution $y_{ij}^*, i \neq j$, is uniquely determined by the rates r_{ij} , if the transition probability matrix is irreducible. The “surplus” denoted by y_{ii}^* for the sink nodes are however non-determined and can be distributed across different sink nodes.*

Since the surplus y_{ii}^* have zero weights in the objective function, having large surplus does not help improve the quality of the solution. This result indicates that given the rates r_{ij} 's, there is a limit N^* such that any number of bicycles beyond this limit N^* will not help to improve the performance of the system.

Example 2 As shown in Fig. 17.2, we have three stations which are connected to each other. The number beside each direct arc (i, j) stands for the arrival rate r_{ij} . Station 1 has a net outflow of three passengers per unit, whereas stations 2 and 3 have net inflow of two and one passenger, respectively. Naively, we expect the average number of bicycles at station 1 to drain down to 0 quickly, with the bulk of bicycles building up at stations 2 and 3. However, note that once the bicycle at station 1 drains down to 0, stations 2 and 3 immediately receive less inflow and in fact station 3 will now have a net outflow of 2 passengers per unit.

Fig. 17.2 Numerical example with 3 stations

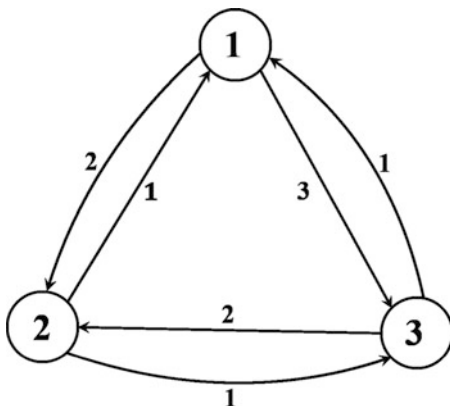


Table 17.1 Computational results of the numerical example with three stations

	Simulation		Deterministic	Gap %
	$t = 0$	$t = 50$		
Avg no. of bicycles at station 1	5	1.656600000	1.65666651648	0.004015241
Avg no. of bicycles at station 2	2	5.506000000	5.50643297152	0.007863631
Avg no. of bicycles at station 3	3	2.837400000	2.83690051200	-0.017603722

We use the outputs from the simulation model to plot the time average level of bicycles at each station over 2,000 periods and summarize the computational results in Table 17.1. In particular, the gap is calculated as $100\% \times (\text{output of the deterministic model} - \text{output of the simulation model (average of 2,000 simulations)}) / \text{output of the simulation model}$. We observe that the time average number of bicycles at each station stabilizes after 10 time periods, and the LP model gives very accurate prediction to the time average level of bicycles in the stochastic system.

17.2.2 Bicycle-Sharing System Design with Location Choice

The previous model assumes that the stations are fixed. This is reasonable when the location choices are obvious – like the train stations in the C-Bike system. It is however inevitable, just like the C-bike system in Kaohsiung, for the network to extend its reach into hot spots and residential areas to capture new passengers who would otherwise not use the public train system for their transport needs. It is therefore essential that we incorporate the location decision of the bicycle-sharing stations as one of the key decisions in our model.

The difficulty in the modeling approach is to incorporate the proportionality constraints into the formulation. We have seen in the earlier section that this class of constraints is crucial for the LP model to approximate the performance of the stochastic network flow model. To see that this is hard to incorporate into the formulation, suppose station i , j , and k have been set up, but l has been omitted in the model. Then we would need to incorporate the proportionality constraint on the flow from i into j and k , i.e., $y_{ij}(t)/y_{ik}(t) = r_{ij}(t)/r_{ik}(t)$, but not for flow from i to l , since the flow $r_{il}(t)$ would be lost as l has not been selected as a node in the bicycle-sharing network.

To deal with complications in the location choice formulation, we need to introduce 0–1 decision variables into our formulation. With a slight abuse of notation, we can redefine S as the set of potential bicycle dock stations which includes MRT stations and neighborhood locations. Let f_i be the setup cost of installing bicycle docks at location i , q_{ij} the environmental benefit/amount charged by the operator of trip ij , and N the number of planning periods.

We also let z_i be a binary variable which represents the presence of bicycle station at location i . We can modify the Linear Program developed in the earlier section to

account for location choice as follows: Let $u_i(t)$ be a decision variable to denote the effective rate of demand substitution from station i at time t . All positive flows out of i , to any other station j at time t , normalized over the demand rate $r_{ij}(t)$, must be identical to this ratio $u_i(t)$. Then, the model of the BSS design with location choice can be formulated as:

$$Z_L^*(\beta) = \max_{x_i(0), y_{ij}(t)} \left(\sum_{t=0}^N \sum_{i \in S} \sum_{j: j \neq i} q_{ij} y_{ij}(t) - \sum_{i \in S} f_i z_i \right)$$

$$\text{subject to } y_i(t+1) = y_i(t) - \sum_{j: j \neq i} y_{ij}(t) + \sum_{j: j \neq i} y_{ji}(t), \quad \forall i, t; \quad (17.3)$$

$$y_i(t) = y_{ii}(t) + \sum_{j: j \neq i} y_{ij}(t), \quad \forall i, t; \quad (17.4)$$

$$u_i(t) - (1 - z_j) \leq \frac{y_{ij}(t)}{r_{ij}(t)} \leq u_i(t) + (1 - z_j), \quad \forall i, j, t; \quad (17.5)$$

$$y_i(0) = x_i(0), \quad \forall i; \quad (17.6)$$

$$y_{ij}(t) \leq r_{ij}(t) z_i, \quad \forall i, j, t; \quad (17.7)$$

$$y_{ij}(t) \leq r_{ij}(t) z_j, \quad \forall i, j, t; \quad (17.8)$$

$$0 \leq u_i(t) \leq 1, \quad \forall i, t; \quad (17.9)$$

$$z_i \in \{0, 1\}, \quad \forall i. \quad (17.10)$$

Constraint (17.5) is crucial for this formulation – when there is a station setup at location j , then $z_j = 1$ and $y_{ij}(t) = u_i(t)r_{ij}(t)$. This forces all flow from i to other locations with stations setup to follow the proportionality constraint and the effective rate of demand substitution for all trips leaving i will equal to $u_i(t)$. Otherwise, the flow from i to j is zero. Constraints (17.7) and (17.8) model the fact that if there are bicycle stations at both location i and j , the flow between the stations will be no more than the demand rate. If either location i or j does not have a station, the flow between i and j will be 0. Constraint (17.9) forces the effective rate of demand substitution to be between 0 and 1. All the other constraints follow from the models derived in the previous sections.

17.3 Bicycle Sharing as Substitute for Train Rides

The Mass Rapid Transit (MRT) system of Singapore operates around 5.00 a.m. to 01.00 a.m. each day, with morning peak hour traffic occurring at around 7.30 a.m. to 9.30 a.m., and evening peak at around 5.30 p.m. to 7.30 p.m. To construct a numerical example for our model, we use a one-week sample of train service passenger-flow data (covering more than 10 million trips) to construct our demand

model. Interestingly, we found that about 16% of the trips are short trips, i.e., with passengers leaving the train system within two stops from their starting stations. The longest trip can take up to 33 stops, but the average number of stops traversed is only around 7.7 stops. Note that except for a handful of stations, commute time between neighboring stations are around two to three minutes. The statistics thus show that a significant proportion of passengers (around 16%) commute up to at most six minutes on the train on a daily basis. An alternate public transport system such as a public bicycle-sharing service, located at the MRT stations, is an attractive alternate for such commuters, especially during the morning and evening peak hours. The challenge however is to determine the right level of bicycles to deploy at each station, and how the utilization rates are affected by the demand pattern.

We compare next the proposed proportional network flow model with a simulation model to identify the operational characteristics of the bicycle-sharing service.

17.3.1 Bicycle Deployment and Utilization

We split the horizon into 15-min intervals, starting from 05:00 am, to collect passengers data on those alighting within two stations. There are 80 time intervals for each day and 560 time intervals for a week. We use a directed time-expanded network to model each MRT station at each time interval on each day. Let \mathcal{A} denote the arc set in the time-expanded network. There are two types of arcs in \mathcal{A} . The first is the one which links station i in time t to station j in time $t + 1$, for all t in which station j is within two stops away from station i . The other arcs are inventory arcs, joining the same station across two consecutive time periods.

We also adopt the following notations:

- We define the system bicycle utilization rate $\alpha(t)$ for each time period t as follows:

$$\alpha(t) \equiv \frac{\sum_{i,j:i \neq j} y_{ij}(t)}{\sum_i x_i(0)},$$

where $\sum_i x_i(0)$ represents the total number of bicycles positioned at all the stations at the beginning of the planning horizon. Hence $\alpha(t)$ is the proportion of bicycles in use at time t .

- Similarly, since the number of bicycles in the system is a constant,

$$\beta = \sum_t \alpha(t)$$

measures the total number of rides in the system, divided by the total number of bicycles available, i.e., the (average) number of times each bicycle is being used.

To support a larger number of trips on bicycles, we might have to deploy more bicycles in the system, but the average bicycle utilization rate might decrease in this case. On the other hand, if we want to enhance the bicycle utilization rate, we could try to deploy relatively fewer bicycles in the system. Therefore, there is a one-to-one relationship between the number of bicycles (optimally) deployed and the utilization rate of each bicycle. To design the BSS, we opt to first determine the desired level of β in the system. Note that β determines the economic viability of the BSS – the bicycle needs to be used more than a threshold value within a stipulated number of years to justify the initial investment in the bicycle.

With the above defined notations, we can modify the linear program developed in the earlier section to account for the utilization rate:

$$Z^*(\beta) = \max_{x_i(0), y_{ij}(t)} \sum_{t=0}^N \sum_{i \in S} \sum_{j: j \neq i} y_{ij}(t)$$

$$\text{subject to } y_i(t+1) = y_i(t) - \sum_{j: j \neq i} y_{ij}(t) + \sum_{j: j \neq i} y_{ji}(t), \quad \forall i, t; \quad (17.11)$$

$$\sum_{t=0}^N \sum_{i \in S} \sum_{j: j \neq i} y_{ij}(t) \geq \beta \sum_i x_i(0); \quad (17.12)$$

$$y_i(t) = y_{ii}(t) + \sum_{j: j \neq i} y_{ij}(t), \quad \forall i, t; \quad (17.13)$$

$$\frac{y_{ij}(t)}{y_{il}(t)} = \frac{r_{ij}(t)}{r_{il}(t)}, \quad \forall i, j, l, t; \quad (17.14)$$

$$y_i(0) = x_i(0), \quad \forall i; \quad (17.15)$$

$$0 \leq y_{ij}(t) \leq r_{ij}(t), \quad \forall t, i \neq j. \quad (17.16)$$

Note that constraint Eq. 17.12 requires the weekly bicycle utilization rate to be at least β . The above LP determines the total number of bicycles and their deployment at the beginning (i.e., $x_i(0)$) of the planning horizon, to attain the desired utilization rate of β for the system. We solve the above model using the CPLEX LP solver. We solve the above program to obtain the maximum number of substituted trips using bicycles, the number of bicycles positioned at each station initially, and bicycle utilization rate $\alpha(t)$ at each time period.

We also compare the solutions obtained from the deterministic model with a simulation model. The detailed steps in implementing the simulation are given as follows.

- We fix a β , solve the deterministic model outlined in this section, and obtain the optimal $x_i(0)$ to be deployed at each station i .

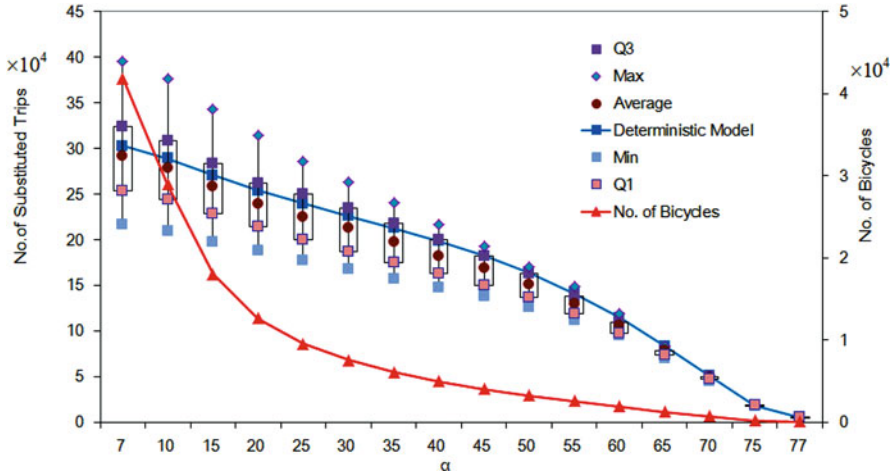


Fig. 17.3 Short trip substitution boxplot

- We use $x_i(0)$ as the input to run the simulation model for stochastic network flow system with Poisson demand at each arc in the network. We run the simulation 100 times for each β to obtain the sample average of the system performance.
- In each simulation, we use the direct time-expanded network and assume the number of passengers arriving at each station during each 15 min time interval follows a Poisson process. In particular, the mean of the inter-arrival time for passengers arriving at Station i with destination Station j at time index t equals to $15/r_{ij}(t)$. We then sort the passengers at each node according to their arrival time at node i and discard those arrivals after 15 min. The bicycles at station i are used by the passengers arriving on a first-come-first-serve basis. We run this for a whole week to obtain the number of trips on bicycles and the utilization rate.

Figure 17.3 shows the performance of the bicycle-sharing network when short trips (within 2 stops) can be completely substituted. In the figure, the x -axis corresponds to the average daily utilization rate (denoted by α , where $\alpha = \beta/7$). The y -axis on the left shows the number of trips using bicycles, and the y -axis on the right shows the number of bicycles deployed in the system. The box plots (obtained via simulation) show that the variations in the number of bicycle trips increase when the daily utilization rate decreases. More amazingly, the numerical results show that the deterministic LP model yields very tight estimate (upper bound) to the average number of trips on bicycles in the stochastic network flow model.

The relationship between the number of bicycle trips supported by the system and the daily utilization rate appears to be almost linear – the number of trips decreases linearly as the targeted utilization rate of the system increases. However, the number of bicycles needed is inversely proportional to the targeted daily utilization rate, in the optimal configuration.

These trade-offs have important implications – it appears that an appropriate targeted utilization rate to operate is around the region $\alpha = 30\text{--}40$ in this test case – at the rate above this level, we need to deploy a significantly many bicycles to support a small increase in the number of supported trips. However, in this range, the service level will not be high as a significant portion of demand for rides cannot be supported. The average total demand within the system is around 308,000 trips, whereas the system operating at $\alpha = 40$ can only support on average 50,000 trips, under the assumption that all short trips can be substituted by bicycle rides if possible.

17.3.2 Number of Bicycle Docks Needed

Technically, we need to set up enough number of bicycle docks at each station so that passengers have space to return the bicycles when they reach their destinations. We calculate the number of docks needed for each station as the maximum bicycle quantities at each station across all time periods. Figures 17.4, 17.5, and 17.6 give the maximum bicycle quantities at each station among all time periods for $\alpha = 10, 40, 70$, for the network flow and the simulation model.

Interestingly, the data extracted from the network flow model is pretty close to the actual peak inventory level at each station obtained from the simulation model. Furthermore, the number of docks needed to support storage of peak inventory decreases with increased utilization (i.e. less number of bicycles deployed).¹

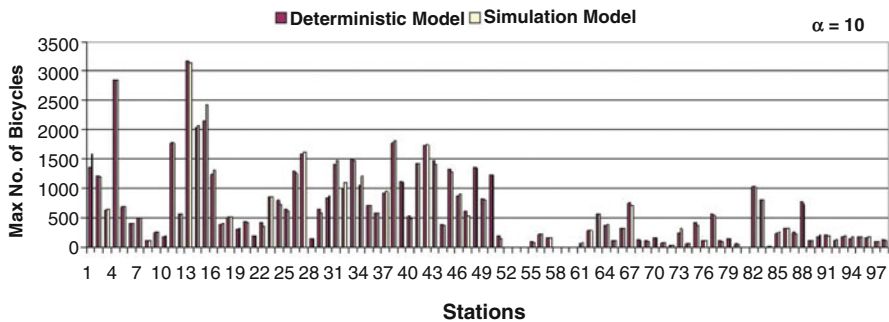


Fig. 17.4 Two-stop no. of docks: deterministic model vs. simulation model

¹Note that we have assumed all passengers will use bicycles to substitute their short distance MRT trips (within 2-Stop), upon the availability of the bicycles. We have thus actually obtained a gross over-estimate on the total volume of trips that can be substituted by bicycles. In reality, only a small percentage of the short distance passengers captured in the data will choose to use bicycles, say 10%. Therefore, all our numbers must be scaled down by a factor of 10 accordingly. In this case, we can see that for $\alpha = 40$, the maximum number of bicycle docks we need to setup among all stations is no more than 80 for our system.

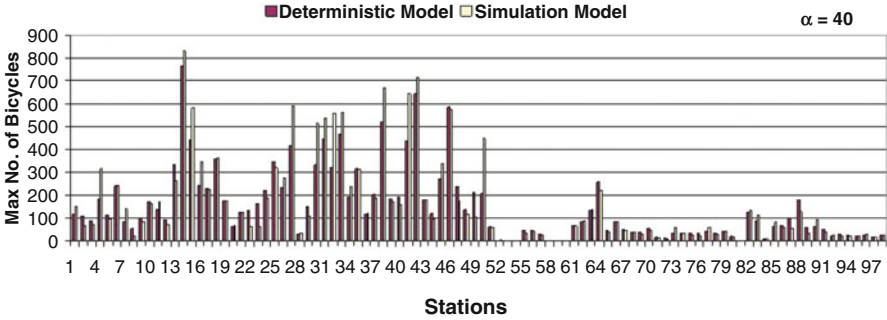


Fig. 17.5 Two-stop no. of docks: deterministic model vs. simulation model

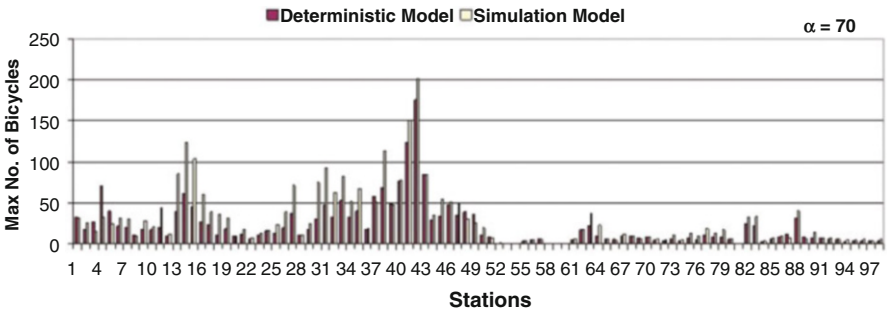


Fig. 17.6 Two-stop no. of docks: deterministic model vs. simulation model

The computational results also show interestingly that with a smaller number of bicycles deployed ($\alpha = 70$), the system should deploy more bicycles near and around the stations in the central business district (Station 30–50 in the chart), leading to a relatively higher number of docks at these stations. However, with more bicycles available ($\alpha = 40$ or $\alpha = 10$), the deployment of the additional bicycles should move towards other congested areas such as the stations near the interchange in the East (station 1–20), leading to a surge in the number of docks there. This suggests that the operators should focus first in the central business district area with a small number of bicycles and stations, to capture the maximum number of trips supported, before branching out into major residential areas as the scale of the system grows.

17.3.3 Effectiveness of Bicycle Redistribution

With a slight abuse of notation, we redefine the time-expanded network to model the passengers flow for each day k . Let N_k denote the time index in the network on day k . We conduct the experiments as follows. We first solve the deterministic model $Z^*(\beta)$ proposed earlier based on the one-week data to obtain the number of

bicycles deployed (denoted by C_β). We then use these as input to run the following program ($Z_k^*(\beta)$) for each day k :

$$\begin{aligned}
 Z_k^*(\beta) &= \max_{x_i(0), y_{ij}(t)} \sum_{l \in N_k} \sum_{i \in S} \sum_{j: j \neq i} y_{ij}(t) \\
 \text{subject to} \quad &y_i(t+1) = y_i(t) - \sum_{j: j \neq i} y_{ij}(t) + \sum_{j: j \neq i} y_{ji}(t), \quad \forall i, t; \\
 &\sum_i y_i(0) = C_\beta; \\
 &y_i(t) = y_{ii}(t) + \sum_{j: j \neq i} y_{ij}(t), \quad \forall i, t; \\
 &\frac{y_{ij}(t)}{y_{il}(t)} = \frac{r_{ij}(t)}{r_{il}(t)}, \quad \forall i, j, l, t; \\
 &0 \leq y_{ij}(t) \leq r_{ij}(t), \quad \forall i, j, t.
 \end{aligned}$$

The above linear program computes the optimal way to locate the C_β bicycles in the system, given the travel patterns of the day. Note that we solve an LP for each β .

The redistribution strategy has impact on the performance of the BSS. Figure 17.7 shows that this strategy prevents surplus bicycles from building up at stations, and thus reduces the need to build large number of docks at each station. For $\alpha = 40$, it reduces the peak docking stations needed from 800 to around 700.²

Although redistribution strategy can enhance the system performance in terms of the number of substituted trips supported in the system and the number of docks needed in each station, it is a very time consuming and expensive task. The concern is how often shall we conduct the redistribution in the system? In the rest of this

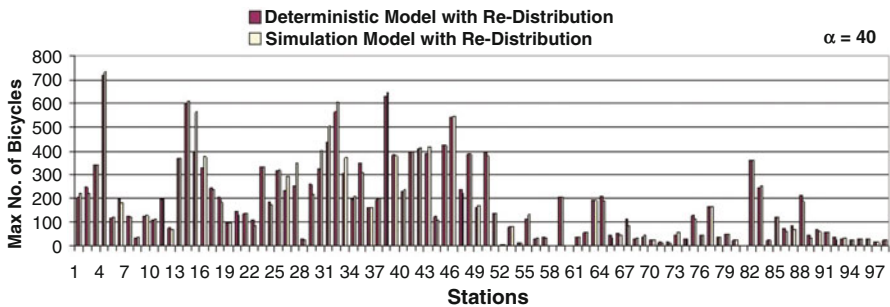


Fig. 17.7 Two-stop no. of docks: deterministic model vs. simulation model with redistribution

²If we assume that the take-up rate for bicycle trip is only 10% of the full demand, then the corresponding number of docks needed will be reduced by 90%, i.e., from 700 to 70 docks.

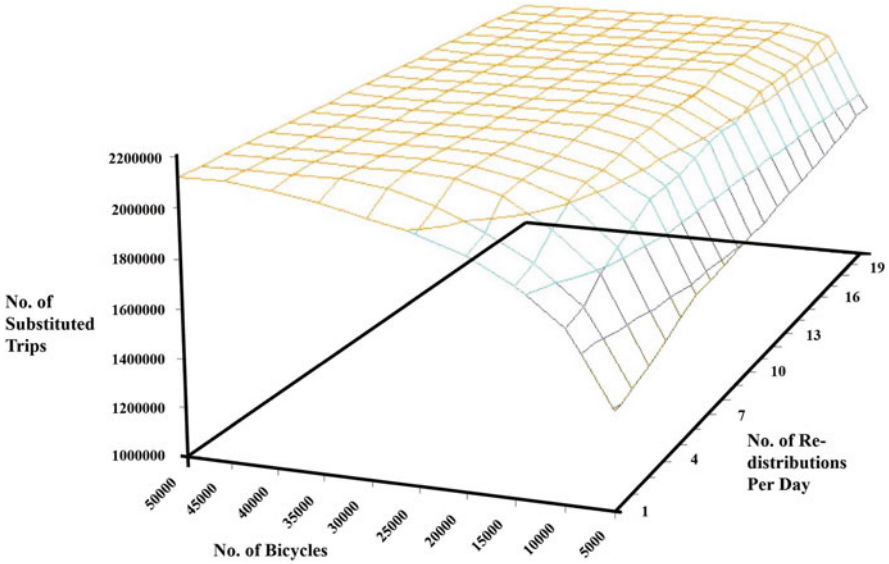


Fig. 17.8 3-D illustration of periodic redistribution

section, we discuss the value of periodic redistribution. Figure 17.8 shows the number of substituted trips supported in the system under given combinations of the total number of bicycles and the number of periodic redistributions per day. In this set of experiments, we subdivide the time horizon evenly into 80 smaller time intervals in a day, and perform periodic redistribution at equal time interval over a day. For certain cases, when 80 time intervals is not divisible by the number of redistributions per day, we keep the remainder in the last time interval of the day. Figure 17.8 shows the end result: when the total number of bicycles invested into the system is more than 30,000, frequent periodic redistribution does not add much to the number of bicycle trips supported by the system. Furthermore, a small number of daily redistribution (says 2–4) suffices, since more frequent redistribution will not add much to the total supported bicycle trips.

17.4 Case Study on Bicycle Sharing with Location Decisions

Punggol is a neighborhood in the northeastern region of Singapore. Initially an area populated by farms, Punggol has been developed into a residential new town. Currently, the district is home to 17,980 HDB³ flats and has an estimated residential

³Housing and Development Board – a statutory board of the Singapore Government responsible for public housing.

population of 59,200. There are plans to develop Punggol as Singapore’s first Eco-Town to enhance the living environment in its estates and encourage residents to do their part for the environment (Housing Development Board, 2010). A BSS would be an appropriate addition to the Punggol landscape as it actively promotes different forms of sustainable transportation.

The Punggol district is served by one Mass Rapid Transit (MRT) station, 29 bus stops, and a Light Rail Transit (LRT) network of eight stations. A total of eight bus services serve the area. The LRT was set up in 2005 as an alternative transportation mode or feeder service within the neighborhood. At present, private bicycles are already being used as a mode of transportation within the Punggol area. Residents were spotted riding their personal bicycles both on the roads as well as on the footpaths. Also, bicycles were seen parked at LRT stations and there is even a designated Bike Park area at the Punggol MRT Station cum Bus Interchange for residents to park their bicycles.

We use a set of commuters data on bus and LRT services to design the bike-sharing network. For this reason, it would be most appropriate for the candidate locations to be at the bus stops, LRT stations, and MRT station. The 16 candidate locations are as shown in Fig. 17.9.

There are two peaks in the travel pattern in this town. Figure 17.10 shows the number of trips during each time period, where each time period corresponds to a 15-min period. Period 1 starts at 05:15 a.m. while Period 79 ends at 01:00 a.m. It can also be concluded that throughout the day, the LRT is the more utilized transportation mode.

The most traversed route is from Location 11 to 5, while the route from Location 5 to 11 is ranked second. Location 5 corresponds to the MRT Station while Location 11 corresponds to the area around a LRT Station named Meridian. It would be

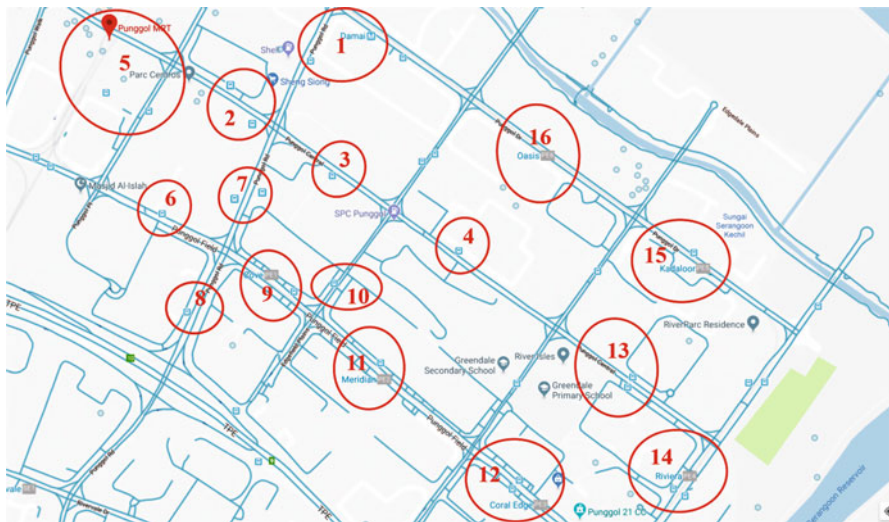


Fig. 17.9 Candidate locations

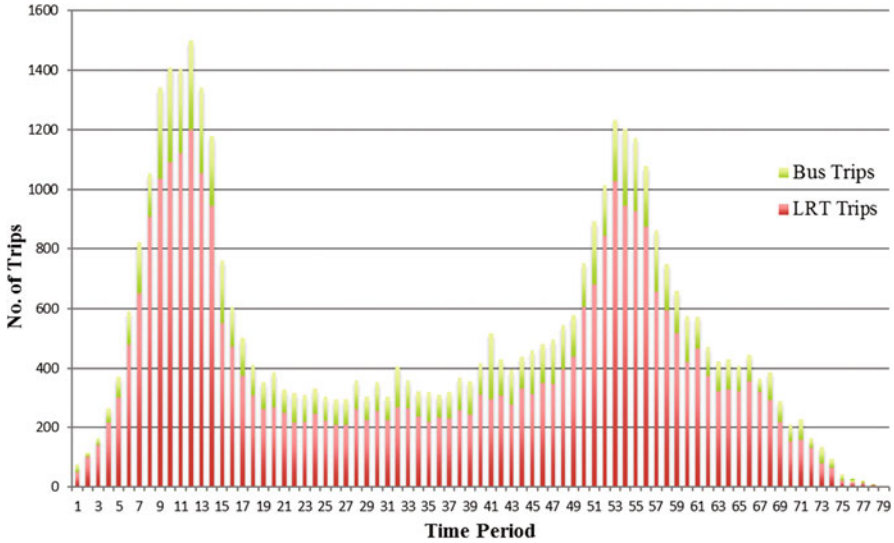


Fig. 17.10 Demand pattern

expected that these 2 routes are the most heavily traversed as Location 11 has the greatest number of residential blocks surrounding it. It should also be noted that some routes see no trips the entire day, hence suggesting that there would be little or no demand for a BSS to cover these routes. Based on the distribution of trips across locations, the frequency at which Location 5 is involved is significantly higher than any of the other locations. This suggests that Location 5 would be a strong candidate location to be chosen for a bike station. However, note that the routes involving Location 5 also suffer more from trip imbalance throughout the day. In the morning, there is a huge demand for bikes to go to the MRT station while in the evening the demand transfers to bike leaving the MRT station. For the rest of the day, the flow of passengers to and from the MRT station is much less, which might result in bikes being stranded at the MRT station in the middle of the day. This would affect the overall utilization of the bikes. Therefore, in solving the model, a case where Location 5 is excluded will be solved in order to see if this results in higher utilization of the bikes.

However, we need to account for the fact that there would not be 100% uptake of the BSS. Based on an informal survey conducted, as well as reference to the average uptake rates predicted for overseas systems (Dector-Vega et al. 2008), it is predicted that the average uptake rate in Singapore should lie between 4 to 6%. Furthermore, this uptake rate is unlikely to be constant throughout the day. It is expected that the uptake would be higher in the mornings and evenings when the weather is cooler. Fewer people are likely to switch to cycling in the afternoon when the weather is hotter. This is taken into consideration by allocating a predicted uptake rate of 6% for the periods between 05:15 a.m.–11:45 a.m. and 05:00 p.m.–01:00 a.m. and an uptake rate of 4% for the period between 11:45 a.m.–05.00 p.m.

Table 17.2 Optimization results

Case	Uptake rate	β	No of locations selected	locations selected	Initial no of bikes at location	Final no of bikes at location
Without accounting for redistribution costs						
1	6% for periods 1–26, 48–79 4% for periods 27–47	12	5	5	0	0
				9	4	1.6
				11	0	4.7
				12	8	11.6
				15	18	12.1
2	4% throughout the day	10	5	5	0	0
				9	6	3.3
				11	0	6.1
				12	10	10.1
				15	14	10.5
3	3% throughout the day	8	5	5	0	0
				9	8	4.4
				11	0	6.6
				12	9	10.1
				15	13	8.9
Accounting for redistribution costs						
4	6% for periods 1–26, 48–79 4% for periods 27–47	12	5	5	0	0
				9	2	2
				11	4	4
				12	12	12
				15	12	12
5	4% throughout the day	10	5	5	0	0
				9	4	4
				11	4	4
				12	11	11
				15	11	11

We use the model introduced in Sect. 17.2.2 to design an optimal Bike-Sharing network given 30 bikes and around four to five bike stations. The model was coded and solved using the CPLEX MIP solver in General Algebraic Modelling System (GAMS). We also account for the inventory imbalance at the start and end of the day, and use that to penalize for redistribution cost. Table 17.2 summarizes the results obtained for several cases where uptake rates were varied. The optimal number of bike stations, optimal locations for the bike stations, and optimal number of bikes to locate at each station initially were determined and tabulated for each case.

The model was also run with an additional term in the objective function to penalize for redistribution costs at the end of the day, in order to determine the effects this had on the solution.

By inspecting the output of our location model, we obtained the following interesting observations:

- Regardless of the uptake rate, it is found that the same number of stations is selected. However, the utilization rate that can be achieved is reduced according to the uptake rate. For an uptake rate of 3% throughout the day, the maximum utilization that can be achieved is 8, compared to 10 for an uptake rate of 4% throughout the day.
- It is interesting to note that regardless of the uptake and utilization rates, and whether redistribution is accounted for or not, the same 5 locations are always chosen. The location decision is thus insensitive to the accuracy of the uptake and utilization rates.
- Even though the same locations for the bike stations are chosen for each case, there are slight differences in the initial number of bikes that should be located at each station. The common feature in all cases is that no bikes should be located at Location 5 initially.
- For cases 1–3 where redistribution costs are not accounted for in the model, it is found that the number of bikes found at each station at the end of the day varies pretty significantly from the initial number of bikes. On average, 7.5 bikes would have to be redistributed at the end of each day.

Take for instance the scenario corresponding to case 4 in our experiment. The model proposed that bike stations should be installed at locations 5, 9, 11, 12, and 15. Also, at the start of the day, there should be 2 bikes at Location 9, 4 bikes at Location 11, 12 bikes at Location 12, 12 bikes at Location 15, and none at Location 5. The model predicted that with this configuration, the bikes should circulate throughout the day such that at the end of each day, the number of bikes at each station will be back to the number at the beginning of the day. We compare the results obtained with a simulation output. The optimal number of bikes that should be located at each station initially, which was determined by the MIP model, was used as input in the simulation model. Again, we assumed that the number of passengers arriving at each station i with destination j during each time period follows a Poisson process. The simulation was run for 100 days. Table 17.3 confirms that the stochastic model behaves more or less as predicted, with utilization rate of 11.22, which is close to the rate of 12 predicted by the MIP model. Based on the maximum number of bikes present at each of the locations over all the time periods, the number of bike docks that should be installed at each station can also be determined.

Table 17.3 Comparison with simulation output

	Station	Initial number	Maximum number	Utilization
Simulation model	5	0	30	
	9	2	4	
	11	4	6	11.22
	12	12	14	
	15	12	17	
MIP model	5	0	25	
	9	2	8	
	11	4	8	12
	12	12	13	
	15	12	13	

17.5 Concluding Remarks

Despite the many problems and success stories of the third generation BSS, there seems to be further advancement to the fourth generation, where more emphasis will be placed on improved efficiency, sustainability, and usability (cf. DeMaio 2009). This can be achieved by focusing on improving the deployment and tracking of bicycles, improving the installation and powering of bicycle stations, creating new business models, and building both intra- and inter-transport system integration (Forbes 2017; MetroBike LLC 2012).

In recent years, BSS vendors have emerged and created their own systems which they sell to local operators. Also, startups like CityRide are converting bike rides into carbon offset that can be sold on the carbon market. The evolution in business strategies and pricing strategies allows the different BSSs to seek out a business model that would be profitable, thus ensure that new BSSs will continue to be set up all around the world, regardless of the goals or scale.

Nevertheless, the fundamental issue of deployment remains a challenge. The deployment can be improved by balancing the supply and demand at each of the bicycle stations, and providing relevant incentives in order to steer demand towards the less popular bicycle stations or routes. In particular, operations research tools can be used to design a network with an improved deployment of bicycles.

In this chapter, we review a novel bicycle-sharing model proposed in Shu et al. (2013) in which passengers use bicycles to substitute their short distance trips. We use a deterministic LP model to approximate the system performance of the stochastic system, and show that the deterministic model can imitate the actual system performance very closely based on actual Singapore MRT ridership data. We use extensive numerical experiments to discuss the important issues such as the bicycle utilization rate, the value of redistribution of the bicycles, and the number of bicycle docks that should be set up at each station.

Our model can be extended to incorporate the scenario of using bicycles to transport between MRT stations and neighborhoods. We implemented our model

using a set of bus transit data in a new town in Singapore, and identified the ideal locations to set up the bicycle stations for the bicycle-sharing network. Our numerical results suggest that the optimal location choices are robust to input errors – for various demand scenarios, the same set of locations are identified as optimal.

Our approach is general enough to incorporate various other features in practice. For example, when the passengers are not able to reach their destination station using bicycles within 15 min time period, we only need to slightly modify the arcs in the time-expanded network defined in the earlier section to allow them to extend across multiple time periods. The same LP based approach can be used to model the flow of commuters in the network. Of course, in the most general case, we need to use queueing network based approach⁴ to model the flow of bicycles in this system. However, the associated optimization problem becomes intractable using this approach, due to the time varying nature of the travel patterns.

The performance of the LP model can also be further enhanced, exploiting recent advances in stochastic optimization (cf. Natarajan et al. 2009, 2011). In particular, a promising direction is to enhance the model further using the constraint that

$$x_i(t) - \underbrace{\min(D_i(t), x_i(t))}_{\text{total departures}} > 0$$

if and only if all passengers arriving in time t to station i can find a bicycle. Thus

$$\left(x_i(t) - \sum_{j:j \neq i} \left(\min(D_i(t), x_i(t)) [p_{ij}(t)] \right) \right) \times \left(D_{ik}(t) - \left(\min(D_i(t), x_i(t)) [p_{ik}(t)] \right) \right) = 0$$

for every OD pair i, k in all realization of the stochastic system. This can be handled by lifting the problem into a higher dimension, and using the copositive cone approach in Natarajan et al. (2011) to deal with the quadratic constraints.

Another interesting direction of research is to explore the usage of incentive schemes to balance the flow. Our approach hinges crucially on the fact that system parameters $r_{ij}(t)$ are given as input. When they are endogenous to the model, i.e., that promotional activities can be used to influence the flow rate between i and j , then the problem is still unsolved.

We leave these and other issues to future research.

Acknowledgements We thank Singapore Mass Rapid Transit and Land Transport Authority for providing the data used in this research. This research was supported in part by NUS Academic Research Fund R-314-000-078-112.

⁴We thank Prof Gideon Weiss for pointing this out.

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