Chapter 9
Attributional Beliefs During Problem-Solving

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Abstract Drawing on research on attributional and efficacy beliefs, we sketch the development of a category system to investigate their influence on effort and outcome in "think aloud" problem-solving processes. Anchor examples from our sample suggest an influence of attribution styles (mastery vs. self-worth orientation, learned helplessness).

Identifying predictors of students' academic success is an ongoing issue of educational research to. The role of beliefs, (e.g. causal attributions of success or failure) is under scrutiny since the 1970s, with interest renewed by cross-national achievement differences in studies like PISA. The recent result that students' attribution style explains up to 8% of the national variance in PISA mathematics scores (Kozina and Mlekuž, Šolsko Polje 25:101–120, 2014) indicates their predictive relevance. However, there is a lack of studies that directly investigate how attributions influence effort and outcome during task processing.

9.1 Theoretical Framework

9.1.1 Problem-Solving

A mathematical problem is a task for which one lacks "ready access to a solution schema" (Schoenfeld, 1985, 74), hence the transformation from the given state to the goal state is hindered by a barrier (Dörner, 1976, 10). In order to overcome the barrier, the solver has to "combine previously known data in a way that is new (to him)" (Pehkonen 2004, 55) by making use of suitable heuristic and self-regulatory activities.

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9.1.2 Beliefs

Various conceptualizations of belief are extant in the literature. Some researchers (e.g. Grigutsch, Raatz, & Törner, 1998), view beliefs as a kind of attitudes, others (e.g. Griffin & Ohlsson, 2001), distinguish both: "Whereas attitudes refer to subjective evaluations of objects as "positive" or "negative", beliefs refer to the acceptance or rejection of propositions." This view will suit our purpose best. Following Kloosterman (1996), one may distinguish beliefs about mathematics (K1) and beliefs about learning mathematics (K2), which can be differentiated into three subcategories: beliefs about oneself as a learner of mathematics (K21); beliefs about the role of the teacher (K22), and other beliefs about learning mathematics (K23).

9.1.3 Beliefs in Problem-Solving

Schoenfeld (1985) posits that success or failure in problem-solving is determined by four variables: knowledge, heuristic strategies, self-regulation and belief-system ("one's mathematical world view") of the solver. Schoenfeld (1985, 1992) exhibits some typical counterproductive beliefs influencing students' problem-solving behaviour, as became apparent by analyzing verbal protocols, classroom observations and students' questionnaires. Despite the seminal role of Schoenfeld (1985), there seem to be only few studies directly investigating the role of beliefs in problem-solving, and most of them are from general education research. One of the exceptions is the study by Kloosterman and Gorman (1990) who found that by the middle grades, many students begin to perceive mathematics as a domain in which smart students succeed and other students merely "get by" or fail. They begin to believe that success and failure are attributable to ability and that effort rarely results in a significant change in their success patterns. This deserves further study, but according to Kloosterman (2002, 248), motivational theories like Weiner's attribution theory and Bandura's self-efficacy theory have rarely been applied to mathematics education.

9.1.4 Attribution Theory

As the title suggests, we conceptualize attributions as beliefs about the causes of success and failure. As far as learning is concerned, they mostly fall into Kloosterman's category (K21). Weiner's theory of attributions deals with individuals' causal interpretations of events and their effect on thinking and behaviour. Weiner (1985) distinguishes causal factors for one's success or failure by three causal dimensions:

1. Locus of causality (external versus internal);

2. Stability (stable versus unstable);
3. Controllability (controllable versus uncontrollable).

These causal dimensions influence outcome expectancy and thence actual behaviour. According to Weiner, the stability dimension is most closely related to expectancy for success. Esteem related affects are associated with the locus dimension, social related affects to the controllability dimensions (see Table 9.1).

Weiner (1985) posits that people use situational cues to form attributions: Cues for ability are ease, speed or frequency of success; a cue for effort is mental exertion, cues for the difficulty of a task are its features; cues for luck are outcomes that are random and lack relation to effort (Schunk & Zimmerman, 2006, 355). Note that attributions are causes ascribed by the individual and may differ from real causes. Also, the dimensionality may be viewed differently, (e.g. task difficulty may be construed as externally controllable by the teacher). But according to Pintrich and Schunk (2002), the accuracy of an attribution is not important for it having behavioural consequences.

The attribution literature is replete with studies on the relationship between students' attributions and their achievement, especially in mathematics. In particular, Georgiou (1999) investigated the relationship between sixth-graders' performance attributions and attainment in mathematics. Internal attributions (to effort or to ability) correlated positively to achievement, whilst external attributions (to luck and to circumstances) correlated negatively to achievement. Furthermore, according to Weiner's theory, attributions to unstable, controllable causes such as effort increase motivation and perseverance, whilst attributions to stable, uncontrollable causes such as ability weakens motivation and may finally lead to *learned helplessness*: This denotes the belief that one's situation cannot be altered by conscious effort, due to inadequate earlier reinforcement of such effort. Hence, helpless students show performance decrements under failure, whereas mastery-oriented students tend to enhance performance. By analyzing the verbalizations of children who were failing on a cognitive task while thinking loud (cf. Table 9.2), Diener and Dweck (1978) found that helpless children attributed failure to lack of ability, whereas mastery-oriented children made only few attributions but engaged more in self-monitoring and self-instructions. This supports the view of attribution theory that learned

Table 9.1 Classification scheme for causal attributions after Weiner (1985)

Attributions		Dimensions	
Attribution	Locus	Stability	Controllability
Ability	Internal	Stable	Uncontrollable
Effort	Internal	Unstable	Controllable
Strategy	Internal	Unstable	Controllable
Interest	Internal	Unstable	Controllable
Task difficulty	External	Stable	Uncontrollable
Luck	External	Unstable	Uncontrollable
Family influence	External	Stable	Uncontrollable
Teacher influence	External	Stable	Uncontrollable

Table 9.2 Verbalization categories of Diener and Dweck (1978, 455)

1. *Statements of useful task strategy.* These were statements of a plan or system that under normal conditions would eventually lead to a solution
2. *Statements of ineffectual approach to task*
3. *Attributions*, especially to lack of ability (e.g. not having a good memory) or loss of ability (e.g. inability to think)
4. *Self-instructions*. These statements referred to instructions the child gave to him/herself that, if followed, would improve performance
5. *Self-monitoring.* Statements concerning the child's solution-oriented behaviour other than task strategy, such as monitoring his or her own effort expenditure or concentration
6. *Statements of positive affect.* These indicate that the task was enjoyable or a challenge
7. *Statements of negative affect.* This category included statements that indicated boredom, anxiety, or a desire to terminate the task or to escape from the situation
8. *Positive prognostic statements*. These express a child's high expectancy of success or indicating a belief that he or she would solve the problem if given sufficient opportunity
9. *Solution-irrelevant statements*

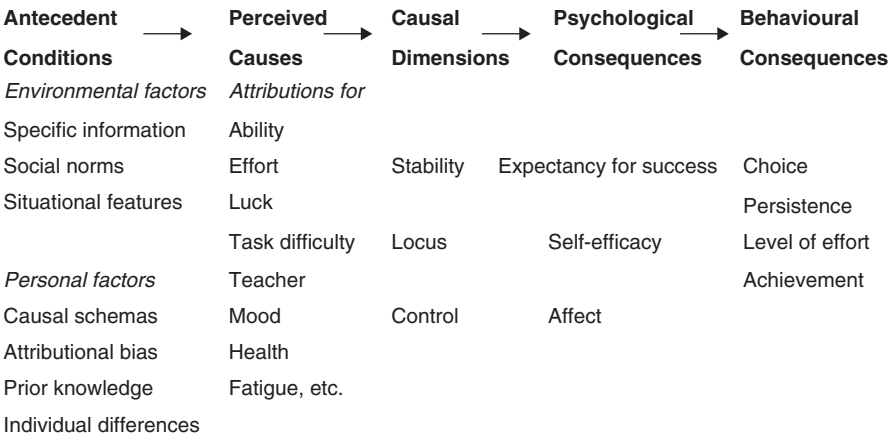


Fig. 9.1 Overview of the internal attributional model of Weiner (1985)

helplessness results from a lack of successes, thus failure is attributed to lack of ability. Consequently, success is viewed as unattainable and the level of effort reduced—a vicious cycle. This exemplifies Weiner's view (Fig. 9.1) on how attributions effect behavioural consequences. Even so, further studies (e.g. Relich, 1984) show that their influence is mediated by self-efficacy.

9.1.5 Self-Efficacy

In social cognitive theory, *self-efficacy* is defined as the belief in one's ability to succeed in specific situations or accomplish a task (Bandura 1986, 391). Perceived self-efficacy is seen as affecting behaviour by influencing the choice of activities as

well as raising the expenditure of effort and the persistence in case of difficulties (Bandura 1986). The intricate interplay of self-efficacy, attributional beliefs and achievement has been disputed in the literature, especially concerning the direction of causality. Schunk and Gunn (1986) investigated the relation between achievement, success attributions and self-efficacy and showed that children who attributed success to ability showed enhanced perceptions of self-efficacy, which in turn correlated to higher achievement. Roeser, Midgley, and Urdan (1996) found by sequential regression analyses that perceiving a task goal structure in middle school was positively related to academic self-efficacy and that this relation was mediated through personal task goals.

Whilst attributions refer to *past* performance, self-efficacy estimate *future* performance. Fig. 9.1 shows its place in the cyclic interplay between attribution and behaviour. (Weiner's original model contains instead the less specific concept of self-esteem.) Note that the situational specificity of self-efficacy beliefs is decisive for their mediating role: Whether the present task is construed as similar to a previous one interacts with the estimation whether the certainty to accomplish it is comparable.

9.1.6 Self-Regulation

From the plethora of approaches we choose one that provides a frame to investigate the interplay of regulation strategies, beliefs, problem-solving effort and outcome: Zimmerman and Campillo (2003) analyzed how motivation and personal resourcefulness influence problem-solving. By *self-regulation* they denote self-generated thoughts, feelings, and actions that are planned and cyclically adapted to attain a goal. These activities can be subsumed under three phases (Fig. 9.2): "Forethought processes precede efforts to solve a problem and set the stage for it. Performance phase processes occur during solution efforts and influence attention and action, and self-reflection processes occur after solution performance efforts and influence a person's response to them. These self-reflections, in turn, influence forethought

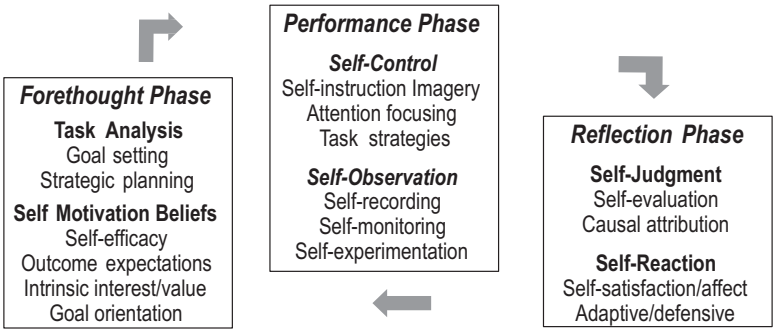


Fig. 9.2 Phases and subprocesses of self-regulation (Zimmerman & Campillo, 2003)

regarding subsequent solution efforts, thus completing a self-regulatory cycle." (ibid, p. 239). Self-efficacy, self-instruction and attributions can be distinguished by the phase in which they occur.

9.2 The Study

9.2.1 Research Questions

Studies about beliefs in problem-solving generally aim to determine beliefs by means of questionnaires and to examine their dependence on covariates and their change after time or intervention (Kloosterman & Stage, 1992). These methods are economic, but apt to various kinds of response bias. Most notably it is an ongoing issue how accurate stated beliefs fit to actual beliefs and to performance in task processing. An exception is Schoenfeld (1985), but unfortunately he does not detail how he derived the reported beliefs from the analyzed problem-solving protocols. Hence it might be worthwhile to find indicators for beliefs directly in problem-solving processes, in which subjects are prompted to "think aloud" in order to elicit belief verbalizations. Based on the considerations in our theoretical framework we set out to investigate:

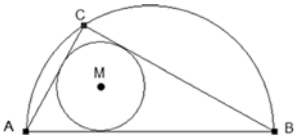
(a) Can problem-solving protocols be parsed into categories in such a way that indicators for attributional and efficacy beliefs can be found in students' verbalizations?
(b) What is their possible influence on effort, persistence and outcome?

9.2.2 Method

Qualitative Content Analysis (QCA) provides several procedures to methodically categorize text by content-based rules, from which we chose deductive category assignment (Mayring, 2000). After defining theory-based *structuring dimensions*, one has to split them into categories and define coding-rules to ensure the concordance between theoretical concepts and their intended realizations in the data. In the pilot phase, the rules are applied to a sample of the data and refined if necessary to ensure unambiguous category assignments. The revised system of categories, rules and examples is fixed in coding guidelines and then applied to the whole corpus of data.

9.2.3 Data

This study and the conceptual framework pertaining to it emerged from project HeuRekAP (dynamische-geometrie.de/heuristik/HeuReKaP/index.htm), in which we are currently engaged in investigating the efficacy of a problem-solving training



AB is the diameter of a semicircle k, C is an arbitrary point of k and M is the incenter of ABC. Determine the measure of ∠AMB.

Fig. 9.3 Opened-up version of item K10 from the 1985 TIMS study

based on heuristically reconstructed worksheets. To evaluate its outcome, we administered an opened-up version of the item K10 from the 1985 TIMS study (cf. Fig. 9.3) and obtained 119 written solutions by ninth graders from one grammar school (Gymnasium), half of which obtained our training beforehand. Three months later, 46 of them solved K10 again, thinking aloud, which we taped, transcribed and analyzed (Gawlick & Lucyga, 2016). The current study is a reanalysis of this data.

9.3 Results

To answer question (a), we set out to obtain a suitable category system by refining the verbalization categories of Diener and Dweck (1978), cf. Table 9.2. They were "derived from the data by the authors" (ibid, 455), who unfortunately did not further detail their approach. To address (a), we thus adopted coding rules and anchor examples from the literature cited above for the pertinent categories according to Zimmerman's process model, cf. Fig. 9.2. The resulting system is illustrated by examples below. (For the sake of brevity, we give examples of indicators for beliefs and likewise for their possible influence, thereby addressing also question (b).)

Categories (6) and (7) were omitted since they did not occur in the coded material, category (8) was amended, since it occurred repeatedly. Category (9) was replaced by negative prognostic statements, which are specified as respective counterparts of (8) (Table 9.3).

The resulting coding system seems apt to tackle our research questions: The codes for causal attributions in (3) identify episodes in the process, where previously created beliefs possibly influence students' behaviour in the protocols. Drawing on Zimmerman's phases of self-regulation (cf. Fig. 9.2), we elaborated the further categories to methodically address the question whether subsequent behaviour in the performance phase (parsed as (1) or (2)) is consistent with or made plausible by the assumed attributions (coded by (3)), as brought to effect by self-instructions (4) and self-monitoring (5). The presence or absence of codes for self-efficacy from (8) and (9) may shed some light on its mediating role. These mechanisms of action are already present in Fig. 9.1. By use of the new category (10), this model could be augmented by the forming and/or fostering of attributional and efficacy beliefs during the reflection phase of Fig. 9.2.

Note that in applying the system, we found relatively few *direct* causal attributions. This is not surprising since students directed their attention towards the problem at hand. To adapt our coding system to this circumstance, we augmented the coding rules as to provide for *indirect* indicators. This is explained below by means of examples.

Table 9.3 Refined verbalization categories

1 and 2. *(In)effectual approach to task*: Specification derived from task analysis and related to students' problem-solving processes as in the study by Gawlick and Lucyga (2015)
3. *Attributions*: Subcategories according to Weiner's classification scheme (cf. Table 9.1), operationalized utilizing the situational cues of Schunk and Zimmerman (2006)
4. *Self-instructions*: Pre-actional statements "overtly or covertly describing how to proceed as one executes a task" (Zimmerman & Campillo, 2003, 242)
5. *Self-monitoring*: Post-actional statements "to judge the adequacy of one's solution efforts" (Zimmerman & Campillo, 2003, 243)
8. *Positive prognostic statements*: Anchor examples adapt the statements from the Academic Self-Efficacy Scale (Roeser et al., 1996) to solving problem tasks:
– I'm certain I can master the upcoming scholastic tasks
– I can do even the hardest scholastic tasks at school if I try
– If I have enough time, I can do a good job on all the problem tasks in school
– I can do almost all the problem tasks in school if I don't give up
– Even if the problem tasks in school are hard, I can learn how to solve them
– I'm certain I can figure out how to do the most difficult scholastic tasks
9. *Negative prognostic statements*: As above
10. S*elf-evaluation*: Statements "comparing self-monitored outcomes with a standard or goal" (Zimmerman & Campillo, 2003, 243)

9.3.1 The Case of C21: Attribution, Task-Strategy, Self-Monitoring and -Evaluation

After reading the task, C21 makes clear that it is known to him in process line no. 5:

5	C21:	I think about the, I think it's called Pythagorean theorem… ahem was it… Yes, but I also think we had this in a test and I didn't process the task (*smiles*) because I was unable to

Several verbalization categories apply to different parts of this line: Student C21 first mobilizes a helpful theorem (later stated correctly), which is an example for category (1). He ponders whether the theorem he has in mind is really Pythagoraean theorem, thereby exemplifying *self-monitoring* (5). Then C21 remembers his failure in a previous attempt to solve K10 and *attributes* this *to a lack of ability* (3). A possible influence of this attribution on his process is the repeated occurrence of hesitancy in statements of category (5), like in line no. 22:

22	C21:	I read again (4)… I have the feeling to overlook something simple (5)… something I could actually handle easily, but I don't know what (5)

The penultimate clause also gives a hint towards C21' perceived locus of causality: The "simple" is elaborated by him as "something I could actually handle easily", so its simplicity is rooted in himself—since ease is a situational cue for ability according to Weiner, it could indicate an *ability attribution* (3).

Despite his difficulties C21 develops a *useful task strategy* (1) "that under normal conditions would eventually lead to a solution" (cf. Fig. 9.3):

25	C21:	Ahem if C is 90° I ponder if I could somehow calculate the angles at A and B… I don't know… I think… perhaps I should look first how exactly I have to halve the angles to obtain here ahem the angles of AMB and then I knew automatically the angle at M
26	C21:	But I have no idea how to calculate them

This *self-monitoring* (5) boils down to what makes K10 a problem: In routine tasks the angles at A and B would be calculated to determine $\mu = \angle AMB$, but here they vary! This the *α-β-barrier* that occurs in many K10 processes. Though C21 finally fails to overcome it, he tries to for further 27 lines, but then resorts to measure angles, though reckoning that thereby μ is not determined correctly (*self-evaluation*) in line no. 53.

9.3.2 The Case of C01: Attributional Indicators from Self-Instruction and Self-Monitoring

Here we find no direct hints towards attributions, but can infer their possible direction: *All* of C01's self-instructions and self-monitorings are concerned with help. For brevity's sake, we just give the code numbers of attributional verbalizations in brackets:

2	C01:	Okay, well I think at first I consider what theorems could help me (4)
18	C01:	Now I draw some angle bisectors, perhaps that helps me *draws it* (1,5)
27	C01:	This triangle contains no 90° angle so that doesn't help me along now (1,5)
28	C01:	(*Looks questioningly at the interviewer. Silence*)
29	C01:	But there's just no help to find it out so it's a bit difficult (5)

C01's attention is focused on mobilizing help, shifting from the figure to the interviewer—this indirectly indicates a possibly previous *external attribution* (3): It seems that for C01success in a task depends on whether help is sufficiently available. Help is obviously something external (in the task or in her counterpart) that is unstable and only externally controllable. So after her tacit appeal does not elicit any help from the interviewer, she contents herself to suppose an improbable derivation of μ from the only mobilized help (Thales' theorem): In 34, it occurs to here that μ might be 45°, since γ is 90° and it might be the half of it. She decides to stick to that after considering other angles in vein and finishes the task. (During stimulated recall, she recognizes that this cannot be true since μ "is much more ample than 90°.") We may hypothesize that C01's line of thought stems from the belief that her effort or ability does not suffice to solve tasks on her own, so she needs support in the instruction and from others. This may be due to previous failure attributions

and hence contributes to her relative underperformance—another rotation of the above vicious cycle.

9.3.3 The Case of A25: Effort and Perseverance Due to Internal Attribution?

The process of A25, is one the longest in the sample: it lasts over 167 lines. A25 finally manages to overcome the α-β-barrier as one of few students. His efforts are accompanied by eight positive and nine negative statements that might fit into our belief categories:

33	A25:	I'm just trying to find any solution… any solution possibility
72	A25:	After all, one can make it
75	A25:	There must be a solution, but where. Somewhere one must make progress
85	A25:	An arbitrary point (*points to C*) one can go on working from the 90°
97	A25:	How to make progress?
99	A25:	45 (*points to the bisected angle at C*)
107	A25:	How to determine α? One does not accomplish β either
119	A25:	How only to accomplish something like that?
134	A25:	α and β together would yield 90° and how does that help me on?
136	A25:	How does one accomplish to make progress? 90° … α and β together

Some of these verbalizations are difficultly categorizable: 97 may be (4), the negative part of 107 might count as (9), 85 as (5) or (8), but what about the rest? Yet it is noteworthy that with one exception (134), these statements all focus on "make it" [es schaffen] rather than "can do it" [es können] or on "trying". That this wording remarkably coincides with the effort and perseverance displayed by A25 gives rise to propose an extension of the theoretical framework: These statements can be construed as instances of a new category that may be called "*attribution-in-action*"; like attributions, they relate an outcome to a causal-factor, but not in *retro*spect, but *pro*spectively—so that like statements of self-efficacy, they mediate the subsequent choice of activities, but not in *forethought*, but during *performance*. A25's "attribution-in-action" is to effort—and it plausibly explains that A25 does not give up on the verge of failure (107,119), but takes pains to solve K10—until he finally makes it.

9.4 Discussion

Question (a) was answered in the affirmative: In analyzing a sample of our data, we were able to define coding-rules that are theoretically based and applicable to the data; thence we obtain indeed indicators for the presence of attributional and efficacy beliefs in problem-solving processes and can hypothesize on their possible influence on effort and outcome (see (b)). However, due to the circumstance that our study is a reanalysis of process data collected previously with a different aim, we could identify only a few direct indicators for causal attribution, but more indirect ones that we tentatively inferred from self-regulatory activities. The latter ones were more easily found in our data, and hence in a future application of our coding-system one will amend interview sections to directly survey causal attributions as in Kloosterman's study (1996). Given the issues raised by our case analysis, one will especially want to ask students:

- in *advance*: "What do you think was influential for your success or failure in previous problem-solving?" (*causal attributions*),
- before *task-processing*: "What do you think does it depend on whether you solve this problem or not?" (*self-efficacy beliefs*),
- during "*stimulated recall*": "What do you think influences at that moment whether you are going to succeed or fail?" (*attribution-in-action*),
- in *retrospect*: "What do you think has been decisive or your success or failure in solving this very problem?" (*revisiting causal attributions*).

This also underlines that how attributions-in-action distinguish themselves from self-efficacy beliefs: the former are the latter's link to past experiences, cf. Fig. 9.1. This point of view is corroborated by an interview with an experienced problem-solver who elegantly solved a Pythagoras-like task drawing on Ptolemy's theorem. Asked what let him bring this unusual theorem into play, he answered "Since I was previously successful with it in a similar situation", relating his decision to past experience all on his own. This exemplifies the rationale for our conceptualization of attribution-in-action.

Insofar our indirect indicators to causal attributions are only hypothetical, the answer to (b) from our case studies remains provisional. How does it fit to the literature? Earlier claims (cf. Diener & Dweck 1978) that attribution to effort is generally more favourable than to ability and all the more than external attribution are supported by A25 doing better than C21 and both better than C01. Since we found no hints to beliefs that could explain C21's willingness to spend that much effort despite his previous inability attribution, one may wonder if C21 (like some authors) does not view ability as a stable trait. Likewise, his attribution-in-action may differ from his stated belief, since C21 is ready to retry solving K10, otherwise he would have declined his participation in the interview study. Hence we deem it worthy to consider the newly proposed concept of attribution-in-action as further mediating factor to resolve the disputed issue of in what way the interplay of attributional and efficacy beliefs influences task performance. Especially, Galloway, Leo, Rogers,

and Armstrong (1996) showed that attributional styles were closely related to students' self-efficacy. Their questionnaire analyses dovetail nicely to our case studies of problem-solving processes: That C21 does not try as hard as A25 may be due to a "*self-worth orientation"* that lets him limit his efforts lest he risks losing self-esteem (ibid, 199). In contrast, A25 may be seen as "*mastery oriented*" (ibid, 198), that is demonstrating persistence to overcome difficulties for the sake of further learning. C01 also exemplifies a well-known attribution style ("*learned helplessness*"); in addition, she illustrates that for best results, a problem-solving training should address also students' attribution style: Namely, in C01's solution attempt she mobilizes just two elements of our problem-solving training: She tries to find helpful theorems and she draws an auxiliary line (the German "Hilfslinie" literally translates to "helpful line"!), stating "perhaps that helps me". Both heuristics promise help verbatim—thus they fit nicely in her presumed belief that she needs help to succeed in solving such a task. But finally this belief turns out to be not all that helpful to her.

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