

# Chapter 3

## Affect as a System: The Case of Sara



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**Abstract** Research in the affective domain has often been restricted to focused attention on a single affective variable. This is ironic given that we know that affective variables tend to cluster. Perhaps the reason for this is that we lack theories for thinking about affective clusters. In this paper I use Green's theory of a belief cluster (1971) as the foundation for looking at a new construct—the affect cluster—and how it functions in an experience-rich environment. This proves to be a useful construct in explaining the case of Sara, a girl whose affect around mathematics has been completely changed.

### 3.1 Introduction

Sara has always found mathematics difficult. Not impossible—just difficult. It is the subject she works the hardest in and receives the least reward. When she was younger she always got A's for her effort, but in grade 8 she slipped to a B and in grade 9 she slipped further to a C+. So in grade 10 she worked harder than she had ever worked in a mathematics class, in any class, before and managed to only get a low B. Now at the start of her grade 11 year she is worried. This is going to be her last year taking a mathematics class and she is worried she might not pass. But that was 3 months ago. Sara is now a full term into her Math 11 course and she is beaming. She is loving mathematics. She is thriving in Ms. Marina's class. She is still getting a B, but she doesn't seem to care. She is even talking about taking Math 12 next year. Over the last 3 months Sara has been completely transformed.

But what exactly is it that has transformed for Sara? Without a doubt, she is happier, less worried, and more optimistic. She is experiencing mathematics differently than she had anticipated, and she is even seeing a different future for herself. But her marks are the same as when she was anxiously entering her Math 11 course.

It is exactly this phenomenon that I am interested in understanding. When students change as radically as Sara has, what exactly is it that has changed and how

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can that change be explained? In the research presented in this paper I pursue this phenomenon through a new construct that, I have come to call, an *affective system*.

### 3.2 Belief Systems

The idea of an affective system is born from Green’s (1971) notion of a belief system—a metaphor for talking about the fact that “beliefs come always in sets or groups, never in complete independence of one another” (p. 41). These systems are organized according to the quasi-logical relations between the beliefs, the psychological strengths with which each belief is held, and the ways in which beliefs cluster, “more or less, in isolation from other clusters” (p. 47).

Green’s idea of a belief system, like all systems (Bánáthy, 1992; Bertalanffy, 1974; Buckley, 1967; Hammond, 2003), can be illustrated as a connected graph (see Fig. 3.1). In this example there are 13 beliefs organized into two (mostly) distinct clusters. Within this example the quasi-logical relationship between beliefs is indicated by an arrow with the tail of the arrow indicating a primary belief (cf. B1, B5, B10) and the head indicating a derived belief (cf. B8, B9, B13). The psychologically strength with which a belief is held is indicated by font size with larger fonts indicating centrally held beliefs (cf. B8, B10) and small font indicating more peripheral beliefs (cf. B4, B6, B11).

Within this framework, Green (1971) sees teaching as “the unending effort to reconstitute the structure of our way of believing”—of changing the belief system. Being a system, changes to one part of the belief system will have an effect on other parts of the system (Bánáthy, 1992; Bertalanffy, 1974; Buckley, 1967; Hammond, 2003). Chapman (2002) used this idea as a framework for looking at teachers’ changing practice. She concluded that we need to attend to teachers’ central and

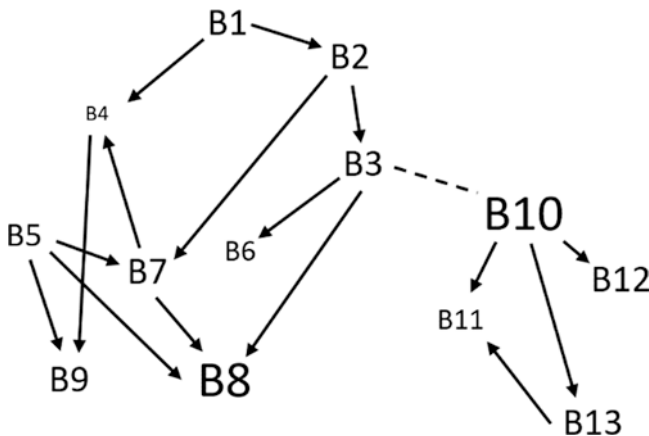


Fig. 3.1 Metaphoric belief system

primary beliefs if we are intending to influence a teacher's belief system, and subsequently, their teaching practice.

The question is this: assuming that we can identify these central and primary beliefs and that we are able to change them, is this going to be enough to change the whole system? In the example below (see Fig. 3.1), B10 is both a centrally held belief and a primary belief and changes to it would likely have a significant impact on the cluster deriving from B10, but no impact on the rest of the beliefs. However, this being a system, small changes to B10 may result in a corrective response from B11, B12, and B13. "In a system, all the features reinforce each other. If one feature is changed, the system will rush to *repair the damage*" (Stigler & Hiebert, 1999). Further, Green (1971) is quite clear about the reality that a "belief may be logically derivative and yet be psychologically central, or it may be logically primary and psychologically peripheral" (p. 46). For example, belief B8 is psychologically central, yet logically derived. Pushing to change that belief may not have the ripple effect that Chapman (2002) is talking about. Even the idea of pushing on central and primary beliefs, is compromised by the changing nature of the relationship between beliefs since "the quasi-logical arrangement of beliefs is distinguished from the fixed and stable relations of the logician, there is no reason a priori to suppose that primary beliefs might not become derivative, and vice-versa" (p. 45). For example, a derived belief such as B10 may then break away from the primary belief of B3 and become a primary belief on its own right.

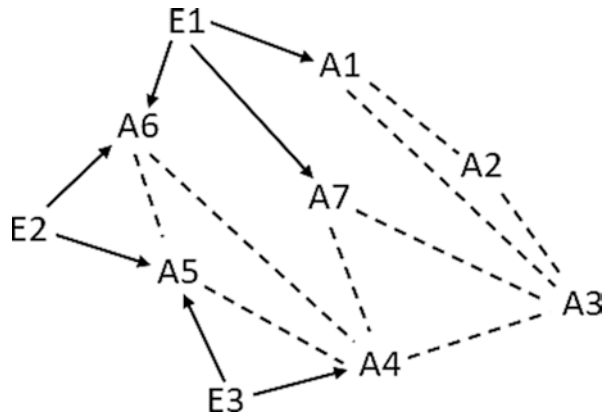
Liljedahl, Rolka and Rösken (2007) present an altogether different mechanism of change. In their work they found, first, that pre-service teachers' beliefs about mathematics were clustered with their beliefs about teaching and learning mathematics. They also found that all of the beliefs within this cluster were radically changed through immersion in a problem solving rich environment. Finally, they found that the new beliefs about mathematics remained clustered with their beliefs about teaching and learning mathematics. In the context of the metaphoric belief system in Fig. 3.1, this type of change could be modelled as a wholesale change to all of beliefs B10, B11, B12, and B13 by pushing directly on each of these beliefs as opposed to relying on the ripple effect caused by pushing on a central and primary belief.

This second method of change is especially important to consider for situations where it is difficult to identify the primary and centrally held beliefs that can trigger a cascading change across an entire system or cluster. It is also useful when systems become complex and the quasi-logical cause-and-effect relationships are no longer clear.

### 3.3 Affect Systems

I propose that a constellation of affective variables—such as beliefs, attitudes, goals, emotions, goals, and efficacy—associated with one individual can be clustered into a metaphoric affective system that, like a belief system, can be represented using a

**Fig. 3.2** Metaphoric affect system



connected graph (see Fig. 3.2). With the exception of beliefs, however, affective variables are not held logically—they are felt not reasoned. As such, neither a quasi-logical nor a psychological organization applies to the relationships between affective variables. This is not to say that a person’s affect arises out of nothing. Within this framework there are two sources of causality: experience–affect causality and affect–affect causality.

Consider, for example, a student who has low self-efficacy about doing mathematics (A5). This student would also, likely, have high anxiety around writing mathematics tests (A6). In this relationship we could say that the anxiety is a logical derivative of the primary affective variable of low self-efficacy (affect–affect causality). But both low self-efficacy and high anxiety may have actually arisen jointly from some negative experience (E2) involving poor performance on a test accompanied by some sort of negative consequence like being scolded by a parent or some form of public shaming (experience–affect causality). The reality is, however, that once established, whether it is derived from a negative test experience or directly from low self-efficacy, this student’s anxiety will quickly become a robust affective variable on its own right. As such, within this framework, and for the purposes of the research presented here, affect–affect relationships are considered to not have a primary-derivative relationship and, as with the belief system above, are represented with dotted lines. However, environment–affect relationships are seen as causal and are, therefore, represented with arrows.

Changes to an affective system, like with a belief system, can then be accomplished either through transforming one experience of the system and relying on the ripple effect to restructure a significant part of the system, or by pushing on many of the experiences all at once. In the research presented here I look at the changes to one student’s affective system through the second of these mechanisms—a massive change in her experiences of mathematics class.

## 3.4 Methodology

The data for this study comes from a larger study in which I look at how students react to their emersion in, what I have come to call, a thinking classroom (Liljedahl, [in press a](#), [b](#); 2010, 2014, 2016a, b).

### 3.4.1 *The Thinking Classroom*

The thinking classroom framework is predicated on a desire to design “a classroom that is not only conducive to thinking but also occasions thinking, a space that is inhabited by thinking individuals as well as individuals thinking collectively, learning together and constructing knowledge and understanding through activity and discussion. It is a space wherein the teacher not only fosters thinking but also expects it, both implicitly and explicitly” (Liljedahl, 2016a, p. 364). My empirical work on the design of such spaces emerged a collection of 14 elements that both describes a thinking classroom and offers a prescriptive framework for teachers to building such a classroom. For the teacher in whose class I was doing this research 11 of these elements are present and can be used to describe her classroom norms (Yackel & Rasmussen, 2002).

1. *The type of tasks used*: Lessons begin with good problem solving tasks. In the beginning these tasks were highly engaging, non-curricular, collaborative tasks. After a period of time these were gradually replaced with curricular problem solving tasks that permeate the entirety of the lesson.
2. *How tasks are given to students*: As much as possible, tasks are given orally. If there are data, diagrams, or long expressions needed these are provided on paper or projected on the wall, but the instructions pertaining to the activity of the task are still given orally.
3. *How groups are formed*: At the beginning of every class a visibly random method is used to create groups of 2–3 students who will work together for the duration of the class.
4. *Student work space*: Groups stand and work on vertical non-permanent surfaces such as whiteboards, blackboards, or windows. This makes the work visible to the teacher and other groups.
5. *Room organization*: The classroom is de-fronted. The desks are placed in a random configuration around the room, and away from the walls, and the teacher addresses the class from a variety of locations within the room.
6. *How questions are answered*: It turns out that students only ask three types of questions: (1) proximity questions—asked when the teacher is close; (2) stop thinking questions—most often of the form “is this right” or “will this be on the test”; and (3) keep thinking questions—questions that students ask so they can get back to work. The teacher answers only the third type of questions.

7. *How hints and extensions are used*: The teacher maintains student engagement through a judicious and timely use of hints and extensions to maintain a perfect balance between the challenge of the current task and the abilities of the students working on it.
8. *Student autonomy*: Students interact with other groups extensively, both for the purposes of extending their work and getting help. As much as possible, the teacher encourages this interaction by pushing students towards other groups when they are stuck or need an extension.
9. *When and how a teacher levels their classroom*: When every group has passed a minimum threshold the teacher pulls the students together to debrief what they have been doing.
10. *Student notes*: After the levelling has occurred the teacher asks the students to write some notes for themselves. These notes are based on the work that is already existing on the boards and can come from their own work, another group's work, or a combination of work from many groups.
11. *Assessment*: Assessment is mostly about communicating with students about where they are and where they are going in their learning. It honours the activities of a thinking classroom through a focus on the processes of learning more so than the products, and it includes both group work and individual work.

### 3.4.2 Participants

The participants in this study include the teacher, Ms. Marina, and six of her students. Ms. Marina is a high school teacher with 12 years' experience who is fully using the *Thinking Classroom* framework. At the time of the study she was teaching a block of Math 8, two blocks of Math 9, and a block of Pre-Calculus 11<sup>1</sup>. The research presented here took place in her Pre-Calculus 11 class.

The six students were selected for participation prior to the course beginning and were identified based on their incoming grades from their Math 10 class. Among the six students selected there was a student with a high A (95–100%), a low A (86–90%), a high B (80–85%), a low B (73–79%), a C+ (67–72%), and a C (60–66%). For the purposes of parsimony, only the case of Sara (low B) will be presented.

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<sup>1</sup>In the location where the research was conducted there are three different Math 11 courses: Apprenticeship and Workplace 11, Foundations 11, and Pre-Calculus 11. Pre-Calculus 11 is the most academic of the three and credit for this course is a suitable pre-requisite for entry into all post-secondary institutions in the province.

### 3.4.3 Data

Data consist of six semi-structured interviews with each participant. The first interview was during the first week of class and thereafter occurred approximately every 2 weeks. On the days of the interview I would begin by observing the class in general, and the participants' activities in particular. I would then conduct brief semi-structured interviews with each of the six students and a more in-depth interview with Ms. Marina.

Each interview consisted of questions pertaining to what I had observed happen in class, how they were experiencing the class, and how they feel about themselves as a learner (or teacher) in comparison to previous experiences in mathematics. These interviews were audio-recorded and transcribed. For each of the six students these data were taken as a whole and turned into a third-person narrative (Clandinin, 1992) which encapsulated all of what I had learned about that student. This narrative was then shared with the student for feedback and editing. This process was repeated until the student was satisfied that the narrative reflected their experiences. For the purposes of the research presented here, these narratives were then coded for indications of affect as well as changes to affect.

## 3.5 The Story of Sara

There was a time when Sara used to like math. This was back when it came easily to her and her marks were good. All through her elementary education (K-7) she felt like this. During this time she was confident in her ability, "I would answer questions in class and take charge when we were working in groups." In grade 7 she was designated as a peer tutor and tasked with helping some of the struggling students in her class.

Then she moved on to high school (9-12) and into Math 8. The year started out ok for her. "There was a lot of review of stuff that I had mastered last year. So I found this easy." However, for the first time in her life "I had mathematics homework." In elementary school she had always finished her work in class. Now, the teacher was assigning work specifically to be done at home. But the homework was on content from the year before so it went quickly and without difficulty. At the end of all the reviews there was a test. "I aced it—I had an almost perfect score."

The next unit was on fractions. She had been good at fractions in elementary school and she had aced the fraction part of the review test, so she was feeling very confident about this unit as well. "But my mark on the unit test was a shock." For the first time in her life she didn't get an A on a math test. She didn't even get a B. She got 70% (C+). "I was devastated." She knew as soon as the test was over that it had not gone well. In hindsight, she was aware that the content had gotten harder. But she had done all the homework and it all seemed fine, but on the test it was not fine. She got confused when everything was all together.

“Then came algebra.” This was new content for her so she did not start off feeling overly confident. For this unit Sara worked extra hard, doing extra homework questions, rewriting all her notes, and paying extra careful attention in class. She was nervous going to the test. She got 80% (B). She remembers not feeling good about this. “I was working twice as hard as in grade 7 and doing worse.”

This set the pattern for Sara for the next 3 years. She worked harder and harder, but her efforts didn’t outpace the increasing complexity of the topics. In grade 9 she almost failed the rational expressions unit, and she remembers thinking “I’m not going to pass the year”. Her dreams of becoming a doctor were starting to look more and more impossible. If she couldn’t get through mathematics “I am not going to get into university”. Sara finished the year with a C+.

In grade 10 she doubled her efforts again and her parents hired a tutor to meet with her weekly and before every test. “I finished the year with a B but remember feeling completely wrung out.” She had lost all her self-confidence in mathematics, it had been 2 years since she volunteered an answer in class, and during group work she just listened. But none of that compared to the devastation when her school councillor offered her the possibility of moving to the Apprenticeship and Workplace (A&W) mathematics course for grade 11, “that way you can finish math and ensure you will graduate.” Sara had always known that A&W was for “dummies and burn-outs”. “He thought I was one of them. There was no way I was going into that class.” By this point she had downgraded her dream of becoming a doctor to becoming a nurse and if she went into the A&W stream even that would not be possible. So Sara enrolled into Pre-Calculus 11 and landed in Ms. Marina’s class.

The first week was very challenging for Sara. She shied away from group work and here every class was group work. Sara did not like this at all. “The only good thing was that the problems were fun. They didn’t feel like math. And I actually found myself contributing a little bit here and there.” In the second week the problems shifted to curriculum and suddenly they were factoring polynomials. This was one of Sara’s least favourite topics so she became very anxious. “I felt like they were going to figure out that I don’t know anything, that I’m a fraud, and that I shouldn’t be in this class. I wanted to transfer to a different class—to a normal class. But I was afraid to go to my councillor because he would just say ‘I told you so’.” So Sara endured.

Then something interesting happened. Suddenly, towards the end of the second week “I saw that my group had done something wrong. I just sort of said, ‘I don’t think that’s right’. My group mates looked at me and waited for me to explain, but I didn’t know how. So I took the pen and started writing on the board.” And she was right. That was a turning point for Sara. After that she was more willing to offer ideas and even occasionally hold the pen. Before long Sara found herself in a group where she wanted to hold the pen at the start of the problem. “I didn’t actually think I knew what to do, but no one else knew what to do either, and we had to start somewhere. So I grabbed the pen.” This is now the norm for Sara. “It doesn’t matter who grabs the pen first. We need to start. It will work out in the end if we just start.”

At the end of the polynomial unit Sara scored a low B on her test. “I was super happy with that. I mean, I knew I could do it in a group with others, but I wasn’t sure



how that was rubbing off on me. But it seems to have worked just fine.” Sara wasn’t anxious about homework or marks the same. “The learning is happening in class now. I don’t feel like I have to go home and learn it on my own after class, or re-write note, like last year. We do a few minute of notes at the end of class and I do re-do some of the problems when I am studying, but nothing like last year where I was doing every question in the book.”

As the term rolled on Sara started to enjoy herself in the class more and more. “This is now my favourite class. Actually, drama is my favourite, but this is my favourite academic class.” She especially liked when the problems got tough. “Every once in a while we get a really tough one. We usually don’t know that it is a tough one when we start, but we get to this point when you just realize ‘this is tough’. And you look at each other and you grin and you just kind of dig in. And usually it works out. There is no feeling like it and everyone is high fiving each other.”

Sara received a B (75%) on her first report card. “That’s awesome. I mean I would have liked an A, but there were some really tough units in there”. Whereas in grade 10 mathematics class, mathematics learning, notes, and her marks were all one thing, Sara now sees her marks as being related to, but still separate from her learning. “I mean, we come to class and it’s great. We’re learning every day—making it work. But we are in a group and we are stronger together. The mark is like a measure of what part I picked up from the group work.” Sara is even rethinking her future. “If I can get Ms. Marina again next year, I think I’ll take Math 12. Maybe even if I can’t get Ms. Marina I’ll take Math 12. I can do this.”

### 3.6 Results

Looking closely at the narrative about Sara we can see that there have been many changes for her across a wide variety of affective variables. In what follows I summarize these.

*Beliefs:* Coming out of grade 10 Sara had a belief that learning mathematics was about doing all her homework, doing extra questions, taking and re-taking notes, and paying “extra careful” attention in class. Now, three months into grade 11 she believes learning mathematics is “happening in class”, that she “learns every day”, and that this learning comes from “working it out” through the meaning making that is happening in her random groups. Her view of learning has shifted from being about doing all the proxies of learning (Liljedahl, 2017) to doing mathematics. Sara’s beliefs about marks has also changed from being something that is continuous and synonymous with mathematics class, mathematics learning, and notes to something that is an occasional measure of how much she is learning from the group work.

*Attitude:* These changes in her beliefs are accompanied by changes in her attitude towards mathematics, mathematics class, and assessment. Sara has moved from an attitude of pessimism, disdain, and dislike for mathematics in grade 10 to one of

optimism, enjoyment, and liking in grade 11. And her attitude towards her marks has shifted from a B as a negative and an indicator of poor performance to a B as “awesome” and an indicator of what group work has rubbed off on her.

*Emotions:* For Sara, all her emotions in Math 10 were negative—she was sad, disappointed, fearful, often devastated, and in the end, wrung out. By the end of the first term in grade 11 her emotions are all positive—“super happy”, “that’s awesome”, and “great”.

*Enjoyment:* Many of these emotions are linked to a burgeoning enjoyment of mathematics class. Although she did not like what she was experiencing in the first week of Math 11, this slowly changed to the point where she now considered Math 11 her “favourite academic class”. Sara especially enjoys tough problems “when you just realize ‘this is tough’. And you look at each other and you grin”.

*Efficacy:* By the end of grade 10, Sara had very low self-efficacy. She didn’t believe she was capable of learning mathematics. Now in grade 11 not only is her belief that she is capable of learning mathematics improved, but she is demonstrating a very positive *group-efficacy*. Group-efficacy is a new construct that I am attributing to Sara’s belief that “it will work out in the end if we just start” and “we are stronger together”. It is interesting to note that Sara’s belief in the power of the group is irrespective of who else is in that group. She has a general belief that whatever group she is placed in will be able to solve the problem.

*Confidence:* By the end of grade 10 Sara had lost all self confidence in herself and her ability to do mathematics—she thought she was going to fail. She had stopped contributing ideas in group settings and it had been 2 years since she volunteered an answer in class. Although she felt like a fraud at the beginning of Math 11, 12 weeks in Sara had her spark back. It began slowly at first with a small suggestions, a declaration that her group had made a mistake, and taking hold of the pen out of a need to try to explain something. And then the floodgates opened and now she is ready to jump in and lead the discussion with her group. She has confidence in her group to get through it, and she is even ready to take on Math 12.

*Goals:* Whereas Sara used to see mathematics as a course she had to get through to fulfil her goals (and her downgraded goals), she is now rethinking her future and looking at taking mathematics in grade 12 not for the fulfilment of a goal but because she wants to. Mathematics is no longer a means to an end, but an end unto itself.

### 3.7 Discussion and Conclusion

From an affective perspective Sara has completely changed, been completely changed, from her participation in Ms. Marina’s thinking classroom. She came into Math 11 with a disdain for mathematics and mathematics class, with low self-efficacy and low confidence, with negative emotions and a negative attitude, with a

goal of just getting through, and with beliefs that proxies for learning are the same as learning. Sara now enjoys Math class—loves it, has not only self-efficacy but also group-efficacy, is more confident, has a positive attitude about mathematics and her marks, and has a belief that learning comes from doing mathematics.

Looking at these changes through the lens of an affective system we can see that the massive change to her affective variables has been occasioned by a substantive set of changes to almost all of her experiences with mathematics class. She has gone from being anonymous in her desk to standing and being visible, from working alone to working collaboratively, from having the teacher showing the mathematics to having to negotiate with others to make meaning of the mathematics, from working hard alone at home to working hard collaboratively in class, from being assessed on what she has managed to retain from her own work to being assessed on what she has taken out of the collaborative meaning making process. In fact, almost every part of her experience of what mathematics class has been radically altered. The notable exception to this is her marks. Sara is still performing at the same level (marks wise) as she had been when she was in Math 10. Either this no longer has the primary quality it did before, or its primacy is dwarfed by the wholesale changes she is experiencing within Math 11.

In short, Sara's affective system has been completely re-structured, not through small changes in one experience, but through massive changes to almost all of her experiences.

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