Chapter 2
Engagement in Mathematics MOOC Forums

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Abstract The research focuses on mathematics MOOC discussion forums, how affective instances emerge from written interactions and how they can be measured. Interactionist research, as well as the intertwining of affective and cognitive components in students' interactions, represents the theoretical background of our investigations. In particular, we refer to engagement as the main affective element in discussion forums. The affective lens is paired with network analysis to examine how and to what extent forums may represent an occasion for a deeper understanding of mathematics for the students. This paper reports on a pilot phase of the research and considers two examples of discussion forums that involved around ten students each. The findings from a small scale analysis serve as a basis for first, general conclusions.

2.1 Introduction

Interactional research does not only postulate the intrinsically social nature of learning (e.g. Ernest, 1998) but also provide evidence that both *cognitive* and *affective* aspects of students' interaction play a role in mathematical understanding. Lave (1988) maintains that "developing an *identity* as a member of a community and becoming *knowledgeably skilful* are parts of the same process" (p. 65). Goos (2004) observes that community is essential to both the development of *a sense of belonging* and to the students' *active participation*. Roth and Radford (2011) further stress that every *idea* contains an *affective attitude* towards the piece of reality the idea refers to, and hence propose that each activity is made of both the conscious awareness and the emotion of each individual engaged in it.

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When we are engaged with others in social interactions, we do not share our ideas only through utterances, but we also share our emotions: simulation theories (e.g. Gallese, Eagle, & Migone, 2007) refer to mirror neuronal circuits to suggest that, in order to recognise an interlocutor's emotion, we experience that emotion ourselves. Vertegaal, van der Veer, and Vons (2000) make a strong link between the amount of eye contact people give and receive to their degree of participation in group communications. Hence, Goos' (2004) sense of belonging and active participation of the students in a group can be further characterised by exchange of glances, mirroring gestures and echoing emotions. Furthermore, with Roth and Radford (2011), we can say that the students' identity develops during the interaction as part of the emotionally intense and embodied process of understanding, and the flow of glances contributes both to the development of their identities and their becoming knowledgeably skilful.

To transfer all these considerations into the context of MOOC is all but straightforward: if we maintain that mathematical understanding is unavoidably interactional, Naidu (in press) observed that most contemporary MOOCs have tended to adopt a predominantly content-specific approach to teaching and learning with little or no regard to the value of promoting and supporting a rich set of interactions between and among students and their teachers about the subject matter. If we maintain that learning is made of an amalgam of cognitive, social and affective components, and that for learning to take place the interlocutors should establish a sense of belonging at cognitive, social and emotional levels by sharing not only the ideas, but also the emotions that come with these ideas, and if eye-contact plays a crucial role in such a sharing, we can question how all this is possible in MOOCs. Many MOOCs, however, provide discussion forums parallel to the video contents and one of their major purposes is to allow the students to engage in an exploration of their ideas to develop their knowledge and understanding of the subject (Zhang, Skryabin, & Song, 2016). A promising approach for the analysis of the dynamics of such free-flowing discussion forums is network analysis, which enables insights into the different roles the interactors can take, namely creating, maintaining or terminating ties (Snijders, van de Bunt, & Steglich, 2010). Our understanding of Snijders et al.'s roles is as follows: in a creative tie, a student poses a new question or problem in the forum. In a maintaining tie, a student replies and opens the possibility to be replied, while in a terminating tie a student posts an answer which does not prompt the others to intervene.

In this paper, we focus on how students develop their *knowledge* and deepen their *understanding* in mathematics, in relation with their *engagement* in discussion forums by first building and then analysing the network of their interactions. Our theoretical framework, thus, consists in Goldin's (2017) understanding of engagement, while our methodology is built around the construction of a network in order to resort to standard mathematical tools for network analysis, paired with an analysis of the affective dimension (engagement). The research question reads as follows: what does the intertwining of network analysis and engagement structure add to our understanding of MOOC discussion forums?

2.2 Engagement

Engagement is considered as fundamental to learning outcomes in general and to students' interactions in particular: Davis (1996), for example, argues that for a true dialogue to take place the interlocutors need to be *willing to engage* in the conversation. According to Goldin (2017), engagement can be characterised by *motivating desires*, namely by *the reasons for engagement*. Gerald Goldin and his colleagues identify a list of desires, but in this paper we recall and adapt the ones that emerged in discussion forums: *Get The Job Done* (the desire to complete an assigned task), *Look How Smart I Am* (the desire to exhibit one's mathematical ability, and have it recognised or acknowledged), *Check This Out* (the desire to control whether a computation is correct), *I'm Really Into This* (the desire to enter and maintain the experience of doing mathematics), *Let Me Teach You* (the desire to explain a mathematical procedure or concept to another student), *Help Me* (the desire to obtain help or support in solving a mathematical problem or understanding the mathematics), *Value Me* (the desire to be held highly in the opinion or caring of other students or teacher), and *Stop The Class* (the desire to interrupt the ongoing mathematical activity of others in the class).

According to Goldin (2017) an engagement structure consists not only of a motivating desire, but also of behaviours and social interactions, thoughts, emotions, which interact dynamically. Most of the motivating desires identified have some explicit *social* aspect (e.g. belonging, recognition, respect, equity, generosity). Some of the motivating desires involve *approach* goals, while others involve *avoidance* goals. Most importantly for a discussion forum, many of the motivating desires tend to *productive mathematical* engagement (Goldin, 2017).

Goldin observes that to infer a student's motivating desire is all but simple and different tools entail different limitations. In analysing a MOOC forum, the limitations seem to be even more, given that we have to resort only to written words. Moreover, Goldin argues that not always a unique motivating desire guides a student's response, given the complexity of engagement. Hence, a student's post seems to be susceptible to more than one interpretation about its motivating desire. However, we claim that some clues in the statements may help us revealing the main motivating desire that is guiding a student's response in the discussion forum.

2.3 Methodology

As stated in the previous session, we try to infer the motivating desires that move the students in interaction forums, and we plug this lens of analysis onto a network that is built from the discussion flow of two forums.

2.3.1 The Tasks, the Participants and the Context

The data for this study come from a blended course that has taken place on January–February 2017. It involved 30 students from grades 12 and 13 (16–18 years old), who attended a math course aimed at strengthening their mathematical knowledge that is necessary for the transition to university mathematics. The students attended six traditional math lessons at the Polytechnic of Milan, on a weekly basis: the lessons paid specific attention to the conceptual understanding of mathematics, how the main mathematical ideas arose historically and how these connect to the most common algorithms in calculus. Between one lesson and the following one, the students had to attend a "week" on a MOOC course, which recaps the main concepts and focuses on the procedural aspects of the mathematical ideas the students have been exposed to in the traditional lessons. Parallel to this, every evening a tutor (the second author of this paper) posted a task on the MOOC discussion forum, intended to enhance the students' conceptual understanding. The students were invited to interact in solving the task. Among the 30 tasks posted, we select the following two ones.

Task A: compute the perimeter and the area of the triangle ABC, where A(2,0), B(8,1) and C(4,5).

Task B: consider the points A(3,2) and B(9,2). The point C varies on the straight line $y = 5$. How does the area of the triangle ABC varies with C? How does the perimeter?

As regards task A, we can see that it is rather a routine exercise and we expect that the students' interactions would be on the results and/or the way to compute them.

As regards task B, the points A and B lie on the horizontal line $y = 2$, hence the area of ABC does not change when C varies on the horizontal line $y = 5$, since its basis remains AB and it height remains equal to 3. The perimeter, indeed, changes. We can notice that task B has a conceptual nature, since it prompts the students to reason, discuss and generalise about the properties of areas and perimeters.

We analyse the motivating desires that drive the students' comments and in particular which ones lead to creating/maintaining and which ones lead to terminating ties.

2.3.2 Network Analysis

Network analysis is a mathematical tool that features a network as made of *nodes* and *links* between two nodes. In case of MOOC forums, the nodes can be thought of as the participants and a link as a participant's reply to another one's post. If a person replies more than once to another person, the link can be counted more than once, namely the network can be *weighted*. If we want to distinguish the case when A replies to B to the case when it is B that replies to A, the network can be *directed*.

In our situation, a network represents the interactions between participants within a forum discussion. In order to recognise the role played by each person inside the discussion, or better, its *centrality* inside the network, it is possible to analyse a node's *degree,* that is, the more links arrive and depart from a node, the higher its *degree*. In our study, we represent the degree with the radius of the circle: the bigger the radius, the higher the degree. The colour of the nodes denotes the *in-degree,* that is the number of links that reach this node: the lighter the colour, the higher the *in-degree*. So a big node in light blue means that the person receives many replies to her posts. A big node in dark blue means that the person makes many comments. The colours of the links correspond to the colour of the node the comment is made to.

From network analysis, we draw on Zhang et al.'s (2016) study, which focuses on reciprocity, transitivity and preferential attachment in a MOOC discussion forum, and aims at explaining how these three network-effects could be used as metrics to inform the design of a better social learning environment.

Reciprocity refers to a communicative relationship in which a conversation is paired up with a returned flow. Research has shown that it is important that participants use the forum not only to express their own ideas and thoughts but also to interact with others by responding to their messages (Arvaja, Rasku-Puttonen, Häkkinen, & Eteläpelto, 2003). Reciprocal interaction is considered as a vitally important part of sharing the cognitive processes at a social level (Resnick, Levine, & Teasley, 1993). The network of the discussion forum can, thus, be characterised by the number of reciprocal interactions.

A *transitive relationship*, in which A connects to B, B connects to C, and A also connects to C, may be more conducive to social learning, as participants are more likely to receive stimuli from multiple peers as the desired information diffuses through a network (Centola, 2010; Todo, Matous, & Mojo, 2015). Hence, the network can be characterised by the number of transitive interactions.

Preferential attachment represents the tendency of heavily connected nodes to receive more connections in a network. That is, if a new participant contributes to the forum, the probability of replying to or being replied by another participant would be proportional to her degree. Initially random variations, such as a participant having started to contribute earlier than others, are increasingly enlarged, thus greatly amplifying differences among participants. Network centralisation is a measure of how unevenly centrality is distributed in a network (Scott, 2000). Centrality relates to the importance or power of a participant in a network. Highly centralised networks appear to be conducive to the efficient transmission of information (Crona & Bodin, 2006), as the central participants play an important role in delivering messages. But central participants can manipulate the communications in networks, and thus, centralised networks are not likely to enable optimum levels of intellectual exchange because of the high imbalances of power in such settings (Leavitt, 1951). Furthermore, learning processes are more likely to collapse if a central participant leaves the networks (Nicolini & Ocenasek, 1998). Hence, the network can be characterised by its even centrality, and in particular we can focus on the degree and in-degree of each participant.

To build the networks and to compute the measures of centrality we have used the open source software Gephi (Bastian, Heymann, & Jacomy, 2009).

2.4 Data Analysis

Figure 2.1 (left) shows the discussion network around task A, which unfolds as follows:

SD	Distance between two points in the Cartesian plane: square root of [(x2−x1)^2 + (y2−y1)^2]. So AB = square root of 37 = 6.1 BC = square root of 16 = 4 AC = square root of 5 = 2.2 Perimeter: 6.1 + 2.2 + 4 = 12.3. Area = square root of [P/2 × (P/2-AB) × (P/2-BC) × (P/2-AC)] = square root of [6.15 × (6.15–6.1) × (6.15–4) × (6.15–2.2)] = 1.6
AJ	Why do you say that BC is the square root of 16? If you compute better, you find out that it is the square root of (16 + 16), that is the square root of 32.
ALC	To find AB: square root of [(2–8)^2 + (0–1)^2] = 6.1 To find AC: square root of [(2–4)^2 + (0–5)^2] = 5.3 To find BC: square root of [(8–4)^2 + (1–5)^2] = 5.6 2p = 6.1 + 5.3 + 5.6 = 17 cm. To find the area when the sides are known: 1/2 × 17 = 8.5 cm square root of [8.5(8.5–6.1)(8.5–5.3)(8.5–5.6)] = 13.7 cm^2
IC	I got different results. AB = 6.1, BC = 16 and CA = 5.3 …I have computed them putting always before x2 and y2. As a consequence, $p = 27.4$ and $A = 13.22$. Before computing the area I have found AB's median point and then the height CH = 4.33 with Pythagora's theorem then I have used the results to compute the area …Why we got different results?
FI	Isn't that you have confused the median with the height: to pass through the median point is a property of the median, not of the height. For the perimeter you have put BC = 16 when actually is it 4 times the square root of 2

SD opens the conversation and recalls the general formula to compute the distance between two points on the Cartesian plane, then she applies the formulas to the given points and computes the area and the perimeter. AJ replies to her, correcting a computation: the length of BC is not the square root of 16, but the square root of 32. We infer that her motivating desire is *Let me teach you*. ALC posts an independent post with his computations. While AJ's comment can be seen as a maintaining tie, ALC's one can be seen as a terminating tie and his motivating desire can be inferred to be *Get the job done*. IC intervenes and says that her results are different

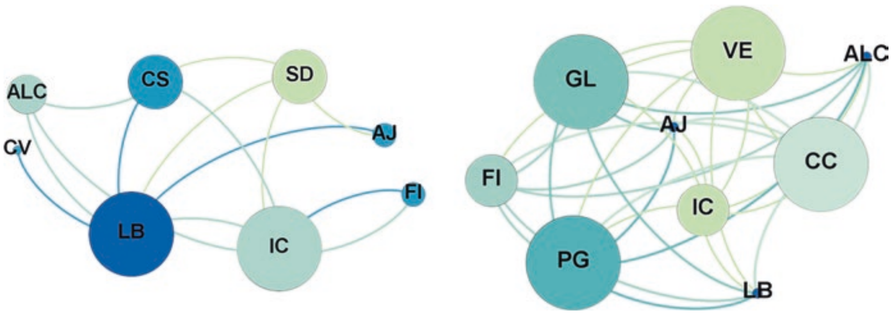

Fig. 2.1 The network for the forum discussion about task A (left) and B (right)

from her mates' ones, hence a link is established from IC to SD and to ALC in the network. IC's post has a maintaining purpose and we also infer that her motivating desire is *Help me*. FI provides her with an explanation, in a way that reveals *Check this out* as motivating desire, and a terminating tie.

Why from AJ's post we infer that her motivating desire is *Let me teach you*, and from FI's one we infer *Check this out*? AJ writes: "if you make the computations accurately, you'll find out that it is the square root of 16 + 16, not 16". AJ seems to be willing to teach SD. FI, instead, writes: "isn't that you have confused the median with the height?". FI's post has a dubitative nature, suggesting IC to check her results but also being quite sure that he is right.

IC replies to FI with a terminating tie, saying: "You're right, thanks!" We interpret her motivating desire as *Get the job done*. We can also see that a reciprocal interaction is established between FI and IC, since they reply to each other. Furthermore, given that IC posts a question to SD and to ALC, and given that FI replies to IC's question, we can also say that a transitive relationship is established from FI to IC to SD and ALC. The discussion goes on:

CS	I got a different result for the area. The sides are the same AC = sqrt29, CB = sqrt32, AB = sqrt37. To find the height CH, I have used the formula to find the distance between a point and a straight line on the Cartesian plane. The straight line on which the segment AB lays is −1/6x + y + 1/3 = 0. The distance between C and the straight line is |1/6 × 4 + 1 × 5 + 1/3|/sqrt 1/6^2 + 1^2 = 36/sqrt37. Hence the area is sqrt37 × 36/37 × 1/2 = 18

CS's post establishes a link to IC, to ALC and to SD by replying to their posts. We infer that her motivating desire is *Help me*, and hers is a maintaining tie, but nobody replies. Instead, AJ and CV post their solutions with no reference to the previous posts. These look like terminating ties. The motivating desires of these two students seem to be: *Look how smart I am* for AJ, and *Get the job done* for CV. Finally, LB's post seems to be a terminating tie and her motivating desire seems to be *Value me*.

The network in Fig. 2.1 (left) has eight nodes: the highest degree is associated to nodes IC and LB, but the former's one is given by many links towards the node, while the latter one is the result of many links going out from the node. IC, in fact, appears in the discussion quite early and poses a question, hence she got responded by some students; LB's post, conversely, is the last one in the discussion: she mentions and replies to the posts of her mates, but she gets no answer. We interpret this phenomenon as a case of *preferential attachment*: participants having started to contribute earlier than others receive more comments to their posts. The same holds for the other three nodes that have a quite high degree: SD and ALC, who show up in the first two interventions, receive many links, while CS's degree is determined by going-out-from-the-node links. A relationship of *reciprocity* is established between the nodes IC and FI, and *transitivity* for FI -> IC -> SD and for FI -> IC -> ALC. We can also see that in this network there are three maintaining and six terminating ties. The motivating desires associated to the maintaining ties are: *Let me teach you* in one case, and *Help me* in the other two cases. We have further observed, however, that only one of these maintaining ties receives a reply: IC's one. Why? We notice that her post comes quite early in the conversation and her desire is to get help. Coming late in the discussion with a desire of getting helped, or coming early with a desire to teach seems not to attract a reply in this discussion. For the terminating ties, *Get the job done* is the motivating desire associated to three cases, while *Check this out, Look how smart I am* and *Value me* characterise the other three cases.

Figure 2.1 (right) shows the network of the discussion around task B. Nine students intervene in the discussion and the network seems much more connected.

IC	The area remains constant because basis and height remain constant. The perimeter varies with a symmetry around $x = 6$. Right?
VE	I agree: the area is constant because the basis is so (the segment AB remains fixed) and the height is so (because, even if C varies, it is always a point on the straight line that is parallel to the segment AB). The perimeter varies and increases as C gets far and far (either to the left or to the right) from the position (6, 5). I was thinking that, if the point C tends to infinity, the area would remain the same, but would the perimeter tend to infinity?

IC's opening is quite different from the opening of the previous discussion: while SD is assertive, IC here ends with a question. Also in the previous discussion, however, IC intervened with a question and it is possible that her style of being into a discussion entails being interrogative rather than assertive. VE's post results to be a creative tie since she poses a new question: "if the point C tended to infinity, the area would remain the same, but would the perimeter tend to infinity?" The motivating desire seem to be *I am really into this*. CC replies to the first post saying that she agrees, and to the second post saying that to her the perimeter cannot tend to infinity since it is a geometrical object. The motivating desire seem to be *I am really into this*, but this is a maintaining tie. The discussion goes on, with FI that writes a long post to provide an argumentation for CC's observation, and it links to all the previous posts. It ends with a question ("how can the sides of a triangle be infinite?"), hence it is a maintaining tie and the motivating desire seems to be *I am really into this*. GL and PG intervene, saying that they agree: these are terminating ties and the former one is characterised by *Value me* as motivating desire, since it shortly explains why there's agreement and then it goes on saying "one can notice that the triangle's shape will be more and more stretched when C goes further and the angle in A will get closer to 180°, never reaching this value". The latter one can be seen as another case of *I am really into this*, since PG provides a long argumentation to sustain the other students' point of view. A terminating tie comes from LB's comment: "I do not know what to add to the discussion" and her motivating desire seems to be *Stop the class.* The same features can be assigned to ALC's post, which says "I think that the given responses are exhaustive". The last post comes from AJ, who says that she believes there's not so much to add to the others' posts, but she proposes to prove that the point C (6, 5) makes the perimeter the smallest possible. This is a creating tie, but nobody replies. Her desire seems to be *Value me*.

Since the network is strongly connected, all the nodes have high degree except three ones: LB, ALC and AJ. The first two ones are terminating ties and want to stop the discussion, hence they do not receive any comment, while the latter is a creative tie but comes late in the discussion. Between the first comment and these three last comments, we can see a really interesting flow of posts: one creative, two maintaining and two terminating ties. The motivating desire *I am really into this* characterises four of these links, while *Value me* does the remaining one. Many transitive relationships are established, even if the number of posts are quite few. If we compare the two discussions, the one about task A has 10 posts for 8 nodes, while this discussion has 9 posts for 9 nodes: each student internees only once (and no reciprocal relationship is established), but the nature of the intervention is like building on the mates' ideas in order to better understand the task. Task B's network has also a more evenly centrality than task A's one.

2.5 Discussion and Conclusion

In this paper we have modelled a mathematics MOOC forum discussion as a network and we have resorted to network analysis and to engagement structures to analyse the data. The aim of this study, with a limited number of cases, is not to draw general conclusions about the relationships between each participant's desire in the discussion and the kind of network that results from them. Instead, we aim at discussing within the MAVI community the viability of applying such lenses of analysis to a discussion forum and show possible, provisional conclusions about the kind of insight we can get from this. For example, the network for task B is more connected and has two creative ties and two maintaining ties. At the same time, the motivating desire that is somehow predominant is *I am really into this*. While some students want to go on with the conversation, others intervene to *Stop the class*. This does not happen for the task A's discussion forum, where the students' desire was rather to *Get the job done*. Which of the many differences should be related to the different nature of the two tasks, or to the different students intervening in the discussion, or to the different desires moving the same students in the two discussions is almost impossible to infer at this stage of the study: we need to analyse more discussions, but we also believe that a first step towards better understanding of the dynamics of a discussion forum should be to establish a sound methodology, which should put affective issues at one of the most important focuses. Among them, we would like to stress that our two examples confirm that there is a strong relationship between engagement and understanding: some motivating desires seem not to lead to a deeper understanding (task A), while others drive the students to pose new questions, digging deeper and provide long and detailed argumentations for their claims (task B).

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