Benjamin Rott · Günter Törner
Joyce Peters-Dasdemir · Anne Möller
Safrudiannur *Editors*

Views and Beliefs in Mathematics Education

The Role of Beliefs in the Classroom

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Preface

It is always a pleasure for editors to finalize a new book by writing a preface. In particular, we are happy to have the 23rd international conference series on Mathematical Views (MAVI). In 1995, the first MAVI conference was held at the University of Duisburg in Germany, organized by Erkki Pehkonen (Helsinki) and Günter Törner (Duisburg). In the proceedings, the editors of this first MAVI conference stated: "The aim of this research group […] is to study and examine the mathematical-didactic questions that arise through research on mathematical beliefs and mathematics-education."

In all these years, MAVI conferences have remained manageable conferences with 40–50 attendants from several (mostly European) countries; this time, there were participants even from Thailand, Japan, Indonesia, and Canada. The atmosphere and the discussions are always very cooperative and friendly, which makes MAVI conferences particularly successful in attracting younger scientists.

From October 4 to 6, 2017, the conference returned to the University of Duisburg-Essen. The theme of the 23rd MAVI was "Views and Beliefs in Mathematics Education." Compared to the 1990s, the landscape of views and beliefs has changed significantly. Today, beliefs are not a neglected and largely unexplored field of research anymore. Instead, they are non-neglecting variables which are omnipresent in contemporary research in mathematics education. However, there is still a lot of work to be done, as this volume shows.

The papers presented in this volume provide a good entry into contemporary research on beliefs, values, affect, and other related constructs.
Meanwhile, a new homepage http://www.mathematical-views.org/ has been started where MAVI documents and information regarding upcoming conferences will be compiled. With young researchers joining this group, we wish that there will be further MAVI conferences and volumes following up in the research tradition of the previous ones.

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Chapter 1
Are Researchers in Educational Theory Free of Beliefs: In Contrast to Students and Teachers?—Is There an Overseen Research Problem or Are There "Blank Spots"?

Günter Törner

Abstract In this article, the author gives an overview of current research on the topic of beliefs and raises the question whether beliefs of researchers themselves have been overlooked.

1.1 Belief Research in Mathematics Didactics—Anno 2018

By now, the amount of research articles dealing with the role of beliefs in mathematical teaching and learning processes has become almost unmanageable. It is questionable what exactly the respective researchers refer to when using the term "belief," only very few of them explicitly explain the terminology underlying their works. More so, a unification of terms, as recommended by the author, has only reached a couple of inclined readers (Törner, 2002). Eventually, every author uses his personal definition and these subjective definitions of beliefs have become excellent examples for actual beliefs.

However, there has been a significant change since 2002, as in those days beliefs had still been described as "hidden variables" by Leder, Pehkonen, and Törner (2002). By now, beliefs—however defined—have proven to be a multifaceted and important factor of explanation and already in Goldin, Rösken and Törner (2009) we have been able to announce: Beliefs are no longer a hidden variable!

Given that, within the frame of scientific publications, beliefs are only seldom further defined as being constructs, a functional understanding of beliefs seems to offer a complementary frame of research. The doctoral dissertation by Rolka (2006) has made a major contribution in this respect. Already in Abelson (1986) we can find corresponding approaches. Very often, beliefs disclose learning impediments and barriers within learning processes. The failure of the curricular

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Problem-Solving-Initiative is an excellent example since its implementation failed due to inadequate beliefs (cf. Frank, 1988, 1990; Schoenfeld, 1985). Schoenfeld (2010) follows a similar approach when using the term "orientation" instead of "beliefs" in order to refer to the often unreflected "personal subjective theories" of the active players in question. This is especially true for decision-making processes, as emphasized by Schoenfeld.

Furthermore, it has become apparent by now that we should not diametrically oppose beliefs to what we consider as "knowledge" (Abelson, 1986). The author modifies a metaphor deriving from the field of history and being attributed to the renowned German historian Nipperdey (1927–1992); we formulate analogously:

> The colors symbolizing knowledge and beliefs are not those of a chessboard, namely black and white, instead they are constituted by infinite nuances and shades of gray.

Such a view helps us get rid of what is occasionally suggested when knowledge is grated as being good and beliefs as being bad. At this point we also need to recall the title of a book by Lakoff and Johnson (1980) dealing with the role of metaphors: Metaphors We Live By. We have come to realize: Yes indeed, we all live with beliefs. In the end this is both inevitable and very normal. Alan Schoenfeld has personally stated to the author: We are all victims of our beliefs structures which are shaped by both our experience and our communities. Very often we do not reflect on this circumstance.

Very often it seems—and the author has been able to pin this insight in his surveys—as if beliefs simply prevent us from having a cognitive vacuum. Elements of unknowingness in our knowledge networks are compensated by beliefs, whereby the respective networks undergo stabilization. In those subject-specific mathematical contexts in which we are not able to store reliable elements of knowledge, the resulting gaps are filled by beliefs. It happened once that in an interview the author tried to explain the aspect of exponential growth in further detail, when the interviewee answered by pointing out that during World War I, the North Sea could not be fished heavily due to military ships which resulted in an exponential growth of populations.

Even though we often speak of a so-called "body of knowledge," it appears beneficial to also include the numerous beliefs in these considerations instead of separating them. Apparently, it seems likely that beliefs and elements of knowledge can coexist "peacefully," and that even very contradictory and dissenting beliefs do not necessarily need to cause conflicts.

1.2 Bearers of Beliefs: The Case of Researchers

Lately, the author has often been dealing with a lack of discussion with regard to beliefs in specific areas of research literature. As already pointed out in an article included in the book by Leder et al. (2002), beliefs can initially be differentiated by the objects they refer to (their beliefs' objects), meaning the context of the specific belief. According to the author, a further coordinate axis is constituted by the specific bearers of beliefs.

In literature (and also during congresses) the differentiation of beliefs very often only goes as far as "beliefs of teachers" on the one hand, and "beliefs of students" on the other hand. Occasionally also outsiders experience discussion: parents, political stakeholders, or any people of a given society. If we take a closer look into our investigations, we will find that in the literature of mathematics education, there are hardly any articles dealing with the beliefs of (mathematics education) researchers. They seem to have been neglected. Why so?

Is this due to the fact that beliefs are not considered being as noble as knowledge and that we consequently should not assume researchers to have such inferior beliefs? Are beliefs parts of a fake-news-reality? Is not the sole presence of knowledge considered the manifestation of researchers' rationality?

So far, the author's database includes exactly three articles discussing the beliefs of mathematics teacher educators (Aydin, Baki, Kögce, & Yildiz, 2009; Aydin, Baki, Yildiz, & Kögce, 2010; Nathan & Koedinger, 2000). These works are definitely interesting; however, they do not primarily focus the differentiation of teachers' and researchers' beliefs. Instead, they focus the confirmation of slightly differing perceptions. The works cited do not answer the posed question. This much being said as an introduction. A first answer will be dealt with in the following section.

1.3 Beliefs as Myths

In the following we will deal with the question whether in relevant literature there is proof for researchers having beliefs after all, eventually just referred to by using different terms.

Given our reference to the terminology, orientation, preferred by Schoenfeld, it becomes apparent that the term belief may be worn and unclear. In German research literature the term "belief" has experienced untranslated establishment in order to underline its status of being a specialist term. All possible Germanizations of the term are unclear and in parts contextually fraught.

The author repeats himself when emphasizing that beliefs are multifaceted fuzzy constructs appearing in different coverings. There is no denying about Pajares' (1992, p. 308) comment that: "... the most fruitful concepts are those to which it is impossible to attach a well-defined meaning. The respective terms may vary, but the functional patterns and modes of action only differ slightly."

This being said, in some educational scientific contexts, beliefs are often referred to as myths. Oser (2014) explains this by the fact that our understanding of the variables and their optimal combination in teaching and learning processes within the classroom is not yet satisfying (see also Rauin (2004)). Oser continues (p. 764):

> The search for the optimal combination of those variables, enabling subjectively and objectively successful teaching and learning processes, resembles the search for the Holy Grail: There is something we keep looking for and even though it is selectively apparent in single elements, we cannot really get hold of it.

This search for the Holy Grail encourages subjective theories—beliefs in the end—to grow and to get out of control. At this point we need to mention the example of *empirical myths*.

The author does not want to deal with these empirical myths in further detail; however, please note: Empirical myths arise from educational sciences being divided into an empirical and a non-empirical branch, as well as from an often detectable incorrect mutual interpretation of the different theoretical principles. It happens that explorative models are reinterpreted as loadable theory statements, so that we need to assume specific and mostly unreflected beliefs on the parts of some researchers. These are the beliefs we keep looking for.

In a 2006 talk, the well-known (German) educationalist Helmke touched upon the so-called *method-myths*. He listed a couple of examples and spoke of the following method-myths:

- Confusion of quantity and quality: Researchers equate the so-called "innovative methods" (such as open classroom instruction, activity-oriented lessons, project teaching, and learning cycles) with good teaching.
- The same group is convinced that teacher-centered instruction necessarily results in receptive and superficial learning.
- Often, we can come across representatives of a faction of educationalists who propagate that especially weaker students could benefit from open classroom instructions (or the so-called extended forms of learning).
- During classroom observations, the author has come to notice that currently active teachers and maybe even the mentor himself live by the thesis: The more various the methods, the better…

Surely the reader can confirm having come across such statements (beliefs). The examples given should have highlighted that there are convictions in the different factions among researchers which are, upon closer examination, nothing but beliefs. In literature, however, they are only seldom discussed under this specific headline even though they do have about the same effects.

At this point we could surely mention numerous beliefs—on mathematics and on the teaching and learning of it—being stated by mathematical researchers with full conviction of their propriety. However, we are eager to deliberately restrict our considerations to researchers in the field of mathematics education.

In the following we will mention three further areas of beliefs' objects by mathematics educationalists which the author refers to as "blank spots" since they show stereotypical standard statements. In fact, these are nothing but beliefs.

1.3.1 Blank Spots in Beliefs Research?

Numerous papers by researchers deal with teachers, the institution school and the belonging students.

1.4 Teachers, School, Students

Surely, numerous didactical research papers address school reality. They give the impression that the newly gained insights are of relevance for school practice and that they should consequently be implemented. However, which idea of teachers is implicitly rooted in the statements of the researchers involved?

Teachers are the immediate addressees of researchers. They are always open-minded, interested and thankful for being able to gain new insights based on current research. Why should experience from different cultural environments not be rewarded and thus exploited for our own practical application?

Eventually, at this point researchers inadmissibly project their own self-perception onto other people. We imply that researchers are constant learners, that they have time for this process at their disposal, that they are diversely interested and curious about others' actions in the process of teaching and learning at schools. These features constitute the ideal of any scientific profession. However, these features only seldom apply to teachers working at school.

Initially we have to remark that teachers do not merely concentrate on teaching, instead they have to cover numerous duties accompanying the teaching processes at school: consultations with parents, correction of class tests, preparation of lessons, cooperation among colleagues, and many more. Other features include administrative tasks like curriculum or teacher conferences. The time of actual teaching may consume about 26 h per week. Roughly estimated this covers about 60% of the total working hours at best. With other words: There is only little time for autonomous and freely organized learning.

It is quickly neglected that only very few teachers are able to take note of the articles in research journals. Given the number of journals this is already tough for researchers who are usually confined to one specific area of research. We cherish an illusion in believing that teachers go sit in the library of the nearby university in order to go through the latest publications. How should they even take note of them?

Even if we assume that (some) highly interested teachers were fond of falling back upon external suggestions from the research sector, do not such teachers need to struggle with the belief that researcher often lack broad practical teaching experience? Following the author's observations, teachers are often skeptical towards well-meant recommendations by researchers. A renowned scientist from the USA has confirmed to the author:

> … but they resonate with my experience in the US—there is, in my opinion (and as a gross abstraction) a gulf between content-focused researchers and policy-related researchers/practitioners.

Those content-focused researchers who have "lived" in schools for some time may be somewhat realistic (I hope to count myself among them), but for the most part, the content-focused and policy communities seem to live separate realities. This causes difficulties in both directions—a neglect of school realities on the part of content-focused researchers, which is as you describe, and a neglect of content-based necessities on the part of most policy people.

Further arguments cannot be neglected: Are not teachers closely bound to the curricular teaching guidelines in most countries? Besides, in most schools (recommended) consultations take place among the group of colleagues when specific contents in parallel courses are taught by different teachers. How should one single teacher step out of line just because he or she has been recommended a modification of lessons by a researcher?

1.5 Research and Practice

This conceptual couple highlights a central task being in store for research: Influencing the practice with newly gained insights. Admittedly, this conceptual couple raises a lot of questions which are not answered easily. Berliner (2002) refers to this dilemma when describing *Education Research as the Hardest Science of All*.

Many colleagues agree with the author in admitting that answering a research question is far easier than using the gained insights as implications for actual teaching. We have not realized this only yesterday, but this insight is in fact about as old as the attempt to improve teaching. Writing about this in further detail would surely fill dozens of pages. At this point we refer to a recently published special issue of the Journal for Research in Mathematics Education and the article (Cai et al., 2017):

> In our May editorial (Cai et al., 2017), we argued that a promising way of closing the gap between research and practice is for researchers to develop and test sequences of learning opportunities, at a grain size useful to teachers, that help students move toward well-defined learning goals. We wish to take this argument one step further. If researchers choose to focus on learning opportunities as a way to produce usable knowledge for teachers, we argue that they could increase their impact on practice even further by integrating the implementation of these learning opportunities into their research.

1.6 Continuous Professional Development of Teachers

The author believes in having found a further "blank spot" in relation to researchers. This topic, however, can only shortly be touched on. It is to be judged favorably that this obligation for teachers is becoming more evident and indisputable within the scientific community. It is B. Rösken's (2011) credit who, in her PhD thesis, highlighted the fact that continuous professional development of teachers is loaded with various beliefs of which adequacy often needs to be questioned. Furthermore, the author points to the work by Timperley, Wilson, Barrar, and Fung (2008) which underlines that in order to be successful it is necessary to question and contrast many of the beliefs uttered by the teacher clientele.

Especially the political side and sometimes also the research side occasionally make the suggestion that it would merely (?) take an investment in further education in order to liberate the tedious deficits in greater areas of teaching methodology.

In doing so, they ignore that there are various conditional factors that need to be influenced positively in order to guarantee change. But how does an averagely engaged teacher learn? When it comes to adult learning, the respective individuals are often occupied with the question: Does this expenditure of energy and time really pay off? It takes massive efforts of motivation from the parts of teacher educators. We have to keep in mind that the introduction of a new curriculum results—among other expenses—in the fact that many of the teachers' teaching transcripts become outdated. Many of the documents designed for teaching lessons need massive revision or have simply become invalid. Do researchers have this in mind when propagating ad hoc curricular changes? Are the teachers who need to be taught ready for this?

1.7 Final Remarks

It should have been pointed out that in research literature dealing with beliefs, researchers' beliefs are often neglected. This may be due to the assumption that researchers should not be accused of having beliefs in the first place. Beliefs are regarded as features of subordinate teachers, students, parents, educational administrators and further stakeholders, but not as features of researchers. In research literature, this lack of self-reflection is hardly ever mentioned. We believe that this can be regarded as a "blank spot." This circumstance is tragic since researchers have to be seen as important players in terms of educational change. Especially the school sector requires the important educational agents to cooperate on equal terms. Given this background, this work is supposed to encourage a detailed stocktaking. The author believes that it appears inevitable to refer to the work by Abelson (1979, 1986). Despite its year of publication, it is still a good read as it describes beliefs as possessions and warns that the costs associated with the adoption of beliefs should not be lost sight of.

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Part I
Pupils' and Students' Views and Beliefs of Mathematics

Chapter 2
Engagement in Mathematics MOOC Forums

Chiara Andrà and Elisabetta Repossi

Abstract The research focuses on mathematics MOOC discussion forums, how affective instances emerge from written interactions and how they can be measured. Interactionist research, as well as the intertwining of affective and cognitive components in students' interactions, represents the theoretical background of our investigations. In particular, we refer to engagement as the main affective element in discussion forums. The affective lens is paired with network analysis to examine how and to what extent forums may represent an occasion for a deeper understanding of mathematics for the students. This paper reports on a pilot phase of the research and considers two examples of discussion forums that involved around ten students each. The findings from a small scale analysis serve as a basis for first, general conclusions.

2.1 Introduction

Interactional research does not only postulate the intrinsically social nature of learning (e.g. Ernest, 1998) but also provide evidence that both *cognitive* and *affective* aspects of students' interaction play a role in mathematical understanding. Lave (1988) maintains that "developing an *identity* as a member of a community and becoming *knowledgeably skilful* are parts of the same process" (p. 65). Goos (2004) observes that community is essential to both the development of *a sense of belonging* and to the students' *active participation*. Roth and Radford (2011) further stress that every *idea* contains an *affective attitude* towards the piece of reality the idea refers to, and hence propose that each activity is made of both the conscious awareness and the emotion of each individual engaged in it.

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When we are engaged with others in social interactions, we do not share our ideas only through utterances, but we also share our emotions: simulation theories (e.g. Gallese, Eagle, & Migone, 2007) refer to mirror neuronal circuits to suggest that, in order to recognise an interlocutor's emotion, we experience that emotion ourselves. Vertegaal, van der Veer, and Vons (2000) make a strong link between the amount of eye contact people give and receive to their degree of participation in group communications. Hence, Goos' (2004) sense of belonging and active participation of the students in a group can be further characterised by exchange of glances, mirroring gestures and echoing emotions. Furthermore, with Roth and Radford (2011), we can say that the students' identity develops during the interaction as part of the emotionally intense and embodied process of understanding, and the flow of glances contributes both to the development of their identities and their becoming knowledgeably skilful.

To transfer all these considerations into the context of MOOC is all but straightforward: if we maintain that mathematical understanding is unavoidably interactional, Naidu (in press) observed that most contemporary MOOCs have tended to adopt a predominantly content-specific approach to teaching and learning with little or no regard to the value of promoting and supporting a rich set of interactions between and among students and their teachers about the subject matter. If we maintain that learning is made of an amalgam of cognitive, social and affective components, and that for learning to take place the interlocutors should establish a sense of belonging at cognitive, social and emotional levels by sharing not only the ideas, but also the emotions that come with these ideas, and if eye-contact plays a crucial role in such a sharing, we can question how all this is possible in MOOCs. Many MOOCs, however, provide discussion forums parallel to the video contents and one of their major purposes is to allow the students to engage in an exploration of their ideas to develop their knowledge and understanding of the subject (Zhang, Skryabin, & Song, 2016). A promising approach for the analysis of the dynamics of such free-flowing discussion forums is network analysis, which enables insights into the different roles the interactors can take, namely creating, maintaining or terminating ties (Snijders, van de Bunt, & Steglich, 2010). Our understanding of Snijders et al.'s roles is as follows: in a creative tie, a student poses a new question or problem in the forum. In a maintaining tie, a student replies and opens the possibility to be replied, while in a terminating tie a student posts an answer which does not prompt the others to intervene.

In this paper, we focus on how students develop their *knowledge* and deepen their *understanding* in mathematics, in relation with their *engagement* in discussion forums by first building and then analysing the network of their interactions. Our theoretical framework, thus, consists in Goldin's (2017) understanding of engagement, while our methodology is built around the construction of a network in order to resort to standard mathematical tools for network analysis, paired with an analysis of the affective dimension (engagement). The research question reads as follows: what does the intertwining of network analysis and engagement structure add to our understanding of MOOC discussion forums?

2.2 Engagement

Engagement is considered as fundamental to learning outcomes in general and to students' interactions in particular: Davis (1996), for example, argues that for a true dialogue to take place the interlocutors need to be *willing to engage* in the conversation. According to Goldin (2017), engagement can be characterised by *motivating desires*, namely by *the reasons for engagement*. Gerald Goldin and his colleagues identify a list of desires, but in this paper we recall and adapt the ones that emerged in discussion forums: *Get The Job Done* (the desire to complete an assigned task), *Look How Smart I Am* (the desire to exhibit one's mathematical ability, and have it recognised or acknowledged), *Check This Out* (the desire to control whether a computation is correct), *I'm Really Into This* (the desire to enter and maintain the experience of doing mathematics), *Let Me Teach You* (the desire to explain a mathematical procedure or concept to another student), *Help Me* (the desire to obtain help or support in solving a mathematical problem or understanding the mathematics), *Value Me* (the desire to be held highly in the opinion or caring of other students or teacher), and *Stop The Class* (the desire to interrupt the ongoing mathematical activity of others in the class).

According to Goldin (2017) an engagement structure consists not only of a motivating desire, but also of behaviours and social interactions, thoughts, emotions, which interact dynamically. Most of the motivating desires identified have some explicit *social* aspect (e.g. belonging, recognition, respect, equity, generosity). Some of the motivating desires involve *approach* goals, while others involve *avoidance* goals. Most importantly for a discussion forum, many of the motivating desires tend to *productive mathematical* engagement (Goldin, 2017).

Goldin observes that to infer a student's motivating desire is all but simple and different tools entail different limitations. In analysing a MOOC forum, the limitations seem to be even more, given that we have to resort only to written words. Moreover, Goldin argues that not always a unique motivating desire guides a student's response, given the complexity of engagement. Hence, a student's post seems to be susceptible to more than one interpretation about its motivating desire. However, we claim that some clues in the statements may help us revealing the main motivating desire that is guiding a student's response in the discussion forum.

2.3 Methodology

As stated in the previous session, we try to infer the motivating desires that move the students in interaction forums, and we plug this lens of analysis onto a network that is built from the discussion flow of two forums.

2.3.1 The Tasks, the Participants and the Context

The data for this study come from a blended course that has taken place on January–February 2017. It involved 30 students from grades 12 and 13 (16–18 years old), who attended a math course aimed at strengthening their mathematical knowledge that is necessary for the transition to university mathematics. The students attended six traditional math lessons at the Polytechnic of Milan, on a weekly basis: the lessons paid specific attention to the conceptual understanding of mathematics, how the main mathematical ideas arose historically and how these connect to the most common algorithms in calculus. Between one lesson and the following one, the students had to attend a "week" on a MOOC course, which recaps the main concepts and focuses on the procedural aspects of the mathematical ideas the students have been exposed to in the traditional lessons. Parallel to this, every evening a tutor (the second author of this paper) posted a task on the MOOC discussion forum, intended to enhance the students' conceptual understanding. The students were invited to interact in solving the task. Among the 30 tasks posted, we select the following two ones.

Task A: compute the perimeter and the area of the triangle ABC, where A(2,0), B(8,1) and C(4,5).

Task B: consider the points A(3,2) and B(9,2). The point C varies on the straight line $y = 5$. How does the area of the triangle ABC varies with C? How does the perimeter?

As regards task A, we can see that it is rather a routine exercise and we expect that the students' interactions would be on the results and/or the way to compute them.

As regards task B, the points A and B lie on the horizontal line $y = 2$, hence the area of ABC does not change when C varies on the horizontal line $y = 5$, since its basis remains AB and it height remains equal to 3. The perimeter, indeed, changes. We can notice that task B has a conceptual nature, since it prompts the students to reason, discuss and generalise about the properties of areas and perimeters.

We analyse the motivating desires that drive the students' comments and in particular which ones lead to creating/maintaining and which ones lead to terminating ties.

2.3.2 Network Analysis

Network analysis is a mathematical tool that features a network as made of *nodes* and *links* between two nodes. In case of MOOC forums, the nodes can be thought of as the participants and a link as a participant's reply to another one's post. If a person replies more than once to another person, the link can be counted more than once, namely the network can be *weighted*. If we want to distinguish the case when A replies to B to the case when it is B that replies to A, the network can be *directed*.

In our situation, a network represents the interactions between participants within a forum discussion. In order to recognise the role played by each person inside the discussion, or better, its *centrality* inside the network, it is possible to analyse a node's *degree,* that is, the more links arrive and depart from a node, the higher its *degree*. In our study, we represent the degree with the radius of the circle: the bigger the radius, the higher the degree. The colour of the nodes denotes the *in-degree,* that is the number of links that reach this node: the lighter the colour, the higher the *in-degree*. So a big node in light blue means that the person receives many replies to her posts. A big node in dark blue means that the person makes many comments. The colours of the links correspond to the colour of the node the comment is made to.

From network analysis, we draw on Zhang et al.'s (2016) study, which focuses on reciprocity, transitivity and preferential attachment in a MOOC discussion forum, and aims at explaining how these three network-effects could be used as metrics to inform the design of a better social learning environment.

Reciprocity refers to a communicative relationship in which a conversation is paired up with a returned flow. Research has shown that it is important that participants use the forum not only to express their own ideas and thoughts but also to interact with others by responding to their messages (Arvaja, Rasku-Puttonen, Häkkinen, & Eteläpelto, 2003). Reciprocal interaction is considered as a vitally important part of sharing the cognitive processes at a social level (Resnick, Levine, & Teasley, 1993). The network of the discussion forum can, thus, be characterised by the number of reciprocal interactions.

A *transitive relationship*, in which A connects to B, B connects to C, and A also connects to C, may be more conducive to social learning, as participants are more likely to receive stimuli from multiple peers as the desired information diffuses through a network (Centola, 2010; Todo, Matous, & Mojo, 2015). Hence, the network can be characterised by the number of transitive interactions.

Preferential attachment represents the tendency of heavily connected nodes to receive more connections in a network. That is, if a new participant contributes to the forum, the probability of replying to or being replied by another participant would be proportional to her degree. Initially random variations, such as a participant having started to contribute earlier than others, are increasingly enlarged, thus greatly amplifying differences among participants. Network centralisation is a measure of how unevenly centrality is distributed in a network (Scott, 2000). Centrality relates to the importance or power of a participant in a network. Highly centralised networks appear to be conducive to the efficient transmission of information (Crona & Bodin, 2006), as the central participants play an important role in delivering messages. But central participants can manipulate the communications in networks, and thus, centralised networks are not likely to enable optimum levels of intellectual exchange because of the high imbalances of power in such settings (Leavitt, 1951). Furthermore, learning processes are more likely to collapse if a central participant leaves the networks (Nicolini & Ocenasek, 1998). Hence, the network can be characterised by its even centrality, and in particular we can focus on the degree and in-degree of each participant.

To build the networks and to compute the measures of centrality we have used the open source software Gephi (Bastian, Heymann, & Jacomy, 2009).

2.4 Data Analysis

Figure 2.1 (left) shows the discussion network around task A, which unfolds as follows:

SD	Distance between two points in the Cartesian plane: square root of [(x2−x1)^2 + (y2−y1)^2]. So AB = square root of 37 = 6.1 BC = square root of 16 = 4 AC = square root of 5 = 2.2 Perimeter: 6.1 + 2.2 + 4 = 12.3. Area = square root of [P/2 × (P/2-AB) × (P/2-BC) × (P/2-AC)] = square root of [6.15 × (6.15–6.1) × (6.15–4) × (6.15–2.2)] = 1.6
AJ	Why do you say that BC is the square root of 16? If you compute better, you find out that it is the square root of (16 + 16), that is the square root of 32.
ALC	To find AB: square root of [(2–8)^2 + (0–1)^2] = 6.1 To find AC: square root of [(2–4)^2 + (0–5)^2] = 5.3 To find BC: square root of [(8–4)^2 + (1–5)^2] = 5.6 2p = 6.1 + 5.3 + 5.6 = 17 cm. To find the area when the sides are known: 1/2 × 17 = 8.5 cm square root of [8.5(8.5–6.1) (8.5–5.3)(8.5–5.6)] = 13.7 cm^2
IC	I got different results. AB = 6.1, BC = 16 and CA = 5.3 …I have computed them putting always before x2 and y2. As a consequence, *p* = 27.4 and *A* = 13.22. Before computing the area I have found AB's median point and then the height CH = 4.33 with Pythagora's theorem then I have used the results to compute the area …Why we got different results?
FI	Isn't that you have confused the median with the height: to pass through the median point is a property of the median, not of the height. For the perimeter you have put BC = 16 when actually is it 4 times the square root of 2

SD opens the conversation and recalls the general formula to compute the distance between two points on the Cartesian plane, then she applies the formulas to the given points and computes the area and the perimeter. AJ replies to her, correcting a computation: the length of BC is not the square root of 16, but the square root of 32. We infer that her motivating desire is *Let me teach you*. ALC posts an independent post with his computations. While AJ's comment can be seen as a maintaining tie, ALC's one can be seen as a terminating tie and his motivating desire can be inferred to be *Get the job done*. IC intervenes and says that her results are different

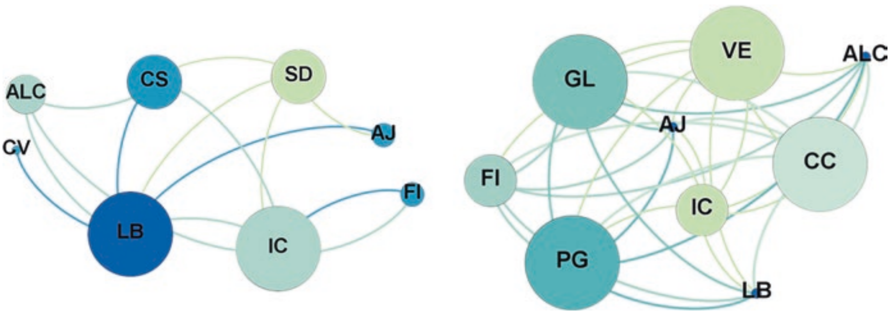

Fig. 2.1 The network for the forum discussion about task A (left) and B (right)

from her mates' ones, hence a link is established from IC to SD and to ALC in the network. IC's post has a maintaining purpose and we also infer that her motivating desire is *Help me*. FI provides her with an explanation, in a way that reveals *Check this out* as motivating desire, and a terminating tie.

Why from AJ's post we infer that her motivating desire is *Let me teach you*, and from FI's one we infer *Check this out*? AJ writes: "if you make the computations accurately, you'll find out that it is the square root of 16 + 16, not 16". AJ seems to be willing to teach SD. FI, instead, writes: "isn't that you have confused the median with the height?". FI's post has a dubitative nature, suggesting IC to check her results but also being quite sure that he is right.

IC replies to FI with a terminating tie, saying: "You're right, thanks!" We interpret her motivating desire as *Get the job done*. We can also see that a reciprocal interaction is established between FI and IC, since they reply to each other. Furthermore, given that IC posts a question to SD and to ALC, and given that FI replies to IC's question, we can also say that a transitive relationship is established from FI to IC to SD and ALC. The discussion goes on:

CS	I got a different result for the area. The sides are the same AC = sqrt29, CB = sqrt32, AB = sqrt37. To find the height CH, I have used the formula to find the distance between a point and a straight line on the Cartesian plane. The straight line on which the segment AB lays is −1/6x + y + 1/3 = 0. The distance between C and the straight line is |1/6 × 4 + 1 × 5 + 1/3|/sqrt 1/6^2 + 1^2 = 36/sqrt37. Hence the area is sqrt37 × 36/37 × 1/2 = 18

CS's post establishes a link to IC, to ALC and to SD by replying to their posts. We infer that her motivating desire is *Help me*, and hers is a maintaining tie, but nobody replies. Instead, AJ and CV post their solutions with no reference to the previous posts. These look like terminating ties. The motivating desires of these two students seem to be: *Look how smart I am* for AJ, and *Get the job done* for CV. Finally, LB's post seems to be a terminating tie and her motivating desire seems to be *Value me*.

The network in Fig. 2.1 (left) has eight nodes: the highest degree is associated to nodes IC and LB, but the former's one is given by many links towards the node, while the latter one is the result of many links going out from the node. IC, in fact, appears in the discussion quite early and poses a question, hence she got responded by some students; LB's post, conversely, is the last one in the discussion: she mentions and replies to the posts of her mates, but she gets no answer. We interpret this phenomenon as a case of *preferential attachment*: participants having started to contribute earlier than others receive more comments to their posts. The same holds for the other three nodes that have a quite high degree: SD and ALC, who show up in the first two interventions, receive many links, while CS's degree is determined by going-out-from-the-node links. A relationship of *reciprocity* is established between the nodes IC and FI, and *transitivity* for FI -> IC -> SD and for FI -> IC -> ALC. We can also see that in this network there are three maintaining and six terminating ties. The motivating desires associated to the maintaining ties are: *Let me teach you* in one case, and *Help me* in the other two cases. We have further observed, however, that only one of these maintaining ties receives a reply: IC's one. Why? We notice that her post comes quite early in the conversation and her desire is to get help. Coming late in the discussion with a desire of getting helped, or coming early with a desire to teach seems not to attract a reply in this discussion. For the terminating ties, *Get the job done* is the motivating desire associated to three cases, while *Check this out, Look how smart I am* and *Value me* characterise the other three cases.

Figure 2.1 (right) shows the network of the discussion around task B. Nine students intervene in the discussion and the network seems much more connected.

IC	The area remains constant because basis and height remain constant. The perimeter varies with a symmetry around $x = 6$. Right?
VE	I agree: the area is constant because the basis is so (the segment AB remains fixed) and the height is so (because, even if C varies, it is always a point on the straight line that is parallel to the segment AB). The perimeter varies and increases as C gets far and far (either to the left or to the right) from the position (6, 5). I was thinking that, if the point C tends to infinity, the area would remain the same, but would the perimeter tend to infinity?

IC's opening is quite different from the opening of the previous discussion: while SD is assertive, IC here ends with a question. Also in the previous discussion, however, IC intervened with a question and it is possible that her style of being into a discussion entails being interrogative rather than assertive. VE's post results to be a creative tie since she poses a new question: "if the point C tended to infinity, the area would remain the same, but would the perimeter tend to infinity?" The motivating desire seem to be *I am really into this*. CC replies to the first post saying that she agrees, and to the second post saying that to her the perimeter cannot tend to infinity since it is a geometrical object. The motivating desire seem to be *I am really into this*, but this is a maintaining tie. The discussion goes on, with FI that writes a long post to provide an argumentation for CC's observation, and it links to all the previous posts. It ends with a question ("how can the sides of a triangle be infinite?"), hence it is a maintaining tie and the motivating desire seems to be *I am really into this*. GL and PG intervene, saying that they agree: these are terminating ties and the former one is characterised by *Value me* as motivating desire, since it shortly explains why there's agreement and then it goes on saying "one can notice that the triangle's shape will be more and more stretched when C goes further and the angle in A will get closer to 180°, never reaching this value". The latter one can be seen as another case of *I am really into this*, since PG provides a long argumentation to sustain the other students' point of view. A terminating tie comes from LB's comment: "I do not know what to add to the discussion" and her motivating desire seems to be *Stop the class.* The same features can be assigned to ALC's post, which says "I think that the given responses are exhaustive". The last post comes from AJ, who says that she believes there's not so much to add to the others' posts, but she proposes to prove that the point C (6, 5) makes the perimeter the smallest possible. This is a creating tie, but nobody replies. Her desire seems to be *Value me*.

Since the network is strongly connected, all the nodes have high degree except three ones: LB, ALC and AJ. The first two ones are terminating ties and want to stop the discussion, hence they do not receive any comment, while the latter is a creative tie but comes late in the discussion. Between the first comment and these three last comments, we can see a really interesting flow of posts: one creative, two maintaining and two terminating ties. The motivating desire *I am really into this* characterises four of these links, while *Value me* does the remaining one. Many transitive relationships are established, even if the number of posts are quite few. If we compare the two discussions, the one about task A has 10 posts for 8 nodes, while this discussion has 9 posts for 9 nodes: each student internees only once (and no reciprocal relationship is established), but the nature of the intervention is like building on the mates' ideas in order to better understand the task. Task B's network has also a more evenly centrality than task A's one.

2.5 Discussion and Conclusion

In this paper we have modelled a mathematics MOOC forum discussion as a network and we have resorted to network analysis and to engagement structures to analyse the data. The aim of this study, with a limited number of cases, is not to draw general conclusions about the relationships between each participant's desire in the discussion and the kind of network that results from them. Instead, we aim at discussing within the MAVI community the viability of applying such lenses of analysis to a discussion forum and show possible, provisional conclusions about the kind of insight we can get from this. For example, the network for task B is more connected and has two creative ties and two maintaining ties. At the same time, the motivating desire that is somehow predominant is *I am really into this*. While some students want to go on with the conversation, others intervene to *Stop the class*. This does not happen for the task A's discussion forum, where the students' desire was rather to *Get the job done*. Which of the many differences should be related to the different nature of the two tasks, or to the different students intervening in the discussion, or to the different desires moving the same students in the two discussions is almost impossible to infer at this stage of the study: we need to analyse more discussions, but we also believe that a first step towards better understanding of the dynamics of a discussion forum should be to establish a sound methodology, which should put affective issues at one of the most important focuses. Among them, we would like to stress that our two examples confirm that there is a strong relationship between engagement and understanding: some motivating desires seem not to lead to a deeper understanding (task A), while others drive the students to pose new questions, digging deeper and provide long and detailed argumentations for their claims (task B).

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Chapter 3
Affect as a System: The Case of Sara

Peter Liljedahl

Abstract Research in the affective domain has often been restricted to focused attention on a single affective variable. This is ironic given that we know that affective variables tend to cluster. Perhaps the reason for this is that we lack theories for thinking about affective clusters. In this paper I use Green's theory of a belief cluster (1971) as the foundation for looking at a new construct—the affect cluster—and how it functions in an experience-rich environment. This proves to be a useful construct in explaining the case of Sara, a girl whose affect around mathematics has been completely changed.

3.1 Introduction

> Sara has always found mathematics difficult. Not impossible—just difficult. It is the subject she works the hardest in and receives the least reward. When she was younger she always got A's for her effort, but in grade 8 she slipped to a B and in grade 9 she slipped further to a C+. So in grade 10 she worked harder than she had ever worked in a mathematics class, in any class, before and managed to only get a low B. Now at the start of her grade 11 year she is worried. This is going to be her last year taking a mathematics class and she is worried she might not pass. But that was 3 months ago. Sara is now a full term into her Math 11 course and she is beaming. She is loving mathematics. She is thriving in Ms. Marina's class. She is still getting a B, but she doesn't seem to care. She is even talking about taking Math 12 next year. Over the last 3 months Sara has been completely transformed.

But what exactly is it that has transformed for Sara? Without a doubt, she is happier, less worried, and more optimistic. She is experiencing mathematics differently than she had anticipated, and she is even seeing a different future for herself. But her marks are the same as when she was anxiously entering her Math 11 course.

It is exactly this phenomenon that I am interested in understanding. When students change as radically as Sara has, what exactly is it that has changed and how

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can that change be explained? In the research presented in this paper I pursue this phenomenon through a new construct that, I have come to call, an *affective system*.

3.2 Belief Systems

The idea of an affective system is born from Green's (1971) notion of a belief system—a metaphor for talking about the fact that "beliefs come always in sets or groups, never in complete independence of one another" (p. 41). These systems are organized according to the quasi-logical relations between the beliefs, the psychological strengths with which each belief is held, and the ways in which beliefs cluster, "more or less, in isolation from other clusters" (p. 47).

Green's idea of a belief system, like all systems (Bánáthy, 1992; Bertalanffy, 1974; Buckley, 1967; Hammond, 2003), can be illustrated as a connected graph (see Fig. 3.1). In this example there are 13 beliefs organized into two (mostly) distinct clusters. Within this example the quasi-logical relationship between beliefs is indicated by an arrow with the tail of the arrow indicating a primary belief (cf. B1, B5, B10) and the head indicating a derived belief (cf. B8, B9, B13). The psychologically strength with which a belief is held is indicated by font size with larger fonts indicating centrally held beliefs (cf. B8, B10) and small font indicating more peripheral beliefs (cf. B4, B6, B11).

Within this framework, Green (1971) sees teaching as "the unending effort to reconstitute the structure of our way of believing"—of changing the belief system. Being a system, changes to one part of the belief system will have an effect on other parts of the system (Bánáthy, 1992; Bertalanffy, 1974; Buckley, 1967; Hammond, 2003). Chapman (2002) used this idea as a framework for looking at teachers' changing practice. She concluded that we need to attend to teachers' central and

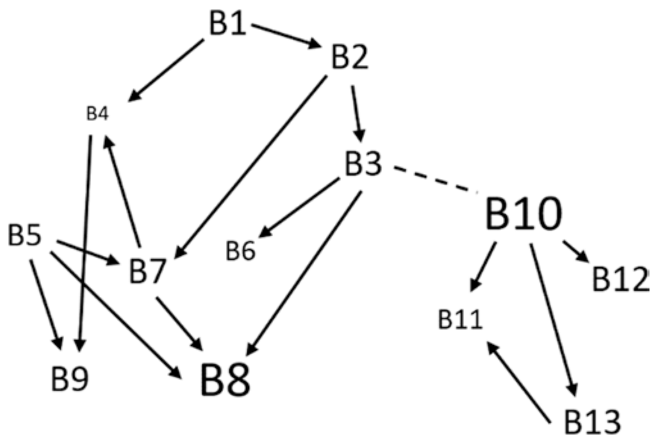

Fig. 3.1 Metaphoric belief system

primary beliefs if we are intending to influence a teacher's belief system, and subsequently, their teaching practice.

The question is this: assuming that we can identify these central and primary beliefs and that we are able to change them, is this going to be enough to change the whole system? In the example below (see Fig. 3.1), B10 is both a centrally held belief and a primary belief and changes to it would likely have a significant impact on the cluster deriving from B10, but no impact on the rest of the beliefs. However, this being a system, small changes to B10 may result in a corrective response from B11, B12, and B13. "In a system, all the features reinforce each other. If one feature is changed, the system will rush to *repair the damage"* (Stigler & Hiebert, 1999). Further, Green (1971) is quite clear about the reality that a "belief may be logically derivative and yet be psychologically central, or it may be logically primary and psychologically peripheral" (p. 46). For example, belief B8 is psychologically central, yet logically derived. Pushing to change that belief may not have the ripple effect that Chapman (2002) is talking about. Even the idea of pushing on central and primary beliefs, is compromised by the changing nature of the relationship between beliefs since "the quasi-logical arrangement of beliefs is distinguished from the fixed and stable relations of the logician, there is no reason a priori to suppose that primary beliefs might not become derivative, and vice-versa" (p. 45). For example, a derived belief such as B10 may then break away from the primary belief of B3 and become a primary belief on its own right.

Liljedahl, Rolka and Rösken (2007) present an altogether different mechanism of change. In their work they found, first, that pre-service teachers' beliefs about mathematics were clustered with their beliefs about teaching and learning mathematics. They also found that all of the beliefs within this cluster were radically changed through immersion in a problem solving rich environment. Finally, they found that the new beliefs about mathematics remained clustered with their beliefs about teaching and learning mathematics. In the context of the metaphoric belief system in Fig. 3.1, this type of change could be modelled as a wholesale change to all of beliefs B10, B11, B12, and B13 by pushing directly on each of these beliefs as opposed to relying on the ripple effect caused by pushing on a central and primary belief.

This second method of change is especially important to consider for situations where it is difficult to identify the primary and centrally held beliefs that can trigger a cascading change across an entire system or cluster. It is also useful when systems become complex and the quasi-logical cause-and-effect relationships are no longer clear.

3.3 Affect Systems

I propose that a constellation of affective variables—such as beliefs, attitudes, goals, emotions, goals, and efficacy—associated with one individual can be clustered into a metaphoric affective system that, like a belief system, can be represented using a

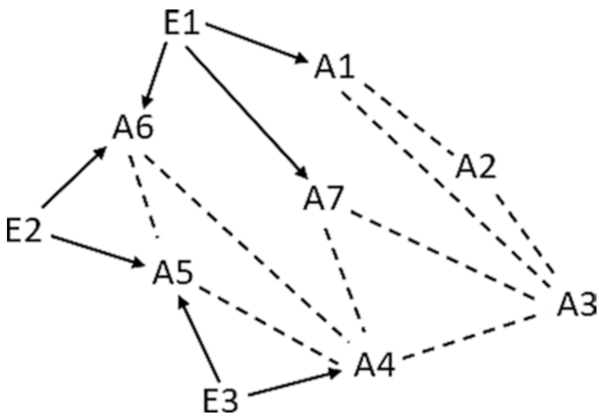

Fig. 3.2 Metaphoric affect system

connected graph (see Fig. 3.2). With the exception of beliefs, however, affective variables are not held logically—they are felt not reasoned. As such, neither a quasi-logical nor a psychological organization applies to the relationships between affective variables. This is not to say that a person's affect arises out of nothing. Within this framework there are two sources of causality: experience–affect causality and affect–affect causality.

Consider, for example, a student who has low self-efficacy about doing mathematics (A5). This student would also, likely, have high anxiety around writing mathematics tests (A6). In this relationship we could say that the anxiety is a logical derivative of the primary affective variable of low self-efficacy (affect–affect causality). But both low self-efficacy and high anxiety may have actually arisen jointly from some negative experience (E2) involving poor performance on a test accompanied by some sort of negative consequence like being scolded by a parent or some form of public shaming (experience–affect causality). The reality is, however, that once established, whether it is derived from a negative test experience or directly from low self-efficacy, this student's anxiety will quickly become a robust affective variable on its own right. As such, within this framework, and for the purposes of the research presented here, affect–affect relationships are considered to not have a primary-derivative relationship and, as with the belief system above, are represented with dotted lines. However, environment–affect relationships are seen as causal and are, therefore, represented with arrows.

Changes to an affective system, like with a belief system, can then be accomplished either through transforming one experience of the system and relying on the ripple effect to restructure a significant part of the system, or by pushing on many of the experiences all at once. In the research presented here I look at the changes to one student's affective system through the second of these mechanisms—a massive change in her experiences of mathematics class.

3.4 Methodology

The data for this study comes from a larger study in which I look at how students react to their emersion in, what I have come to call, a thinking classroom (Liljedahl, in press a, b; 2010, 2014, 2016a, b).

3.4.1 The Thinking Classroom

The thinking classroom framework is predicated on a desire to design "a classroom that is not only conducive to thinking but also occasions thinking, a space that is inhabited by thinking individuals as well as individuals thinking collectively, learning together and constructing knowledge and understanding through activity and discussion. It is a space wherein the teacher not only fosters thinking but also expects it, both implicitly and explicitly" (Liljedahl, 2016a, p. 364). My empirical work on the design of such spaces emerged a collection of 14 elements that both describes a thinking classroom and offers a prescriptive framework for teachers to building such a classroom. For the teacher in whose class I was doing this research 11 of these elements are present and can be used to describe her classroom norms (Yackel & Rasmussen, 2002).

1. *The type of tasks used*: Lessons begin with good problem solving tasks. In the beginning these tasks were highly engaging, non-curricular, collaborative tasks. After a period of time these were gradually replaced with curricular problem solving tasks that permeate the entirety of the lesson.
2. *How tasks are given to students*: As much as possible, tasks are given orally. If there are data, diagrams, or long expressions needed these are provided on paper or projected on the wall, but the instructions pertaining to the activity of the task are still given orally.
3. *How groups are formed*: At the beginning of every class a visibly random method is used to create groups of 2–3 students who will work together for the duration of the class.
4. *Student work space*: Groups stand and work on vertical non-permanent surfaces such as whiteboards, blackboards, or windows. This makes the work visible to the teacher and other groups.
5. *Room organization*: The classroom is de-fronted. The desks are placed in a random configuration around the room, and away from the walls, and the teacher addresses the class from a variety of locations within the room.
6. *How questions are answered*: It turns out that students only ask three types of questions: (1) proximity questions—asked when the teacher is close; (2) stop thinking questions—most often of the form "is this right" or "will this be on the test"; and (3) keep thinking questions—questions that students ask so they can get back to work. The teacher answers only the third type of questions.

7. *How hints and extensions are used*: The teacher maintains student engagement through a judicious and timely use of hints and extensions to maintain a perfect balance between the challenge of the current task and the abilities of the students working on it.
8. *Student autonomy*: Students interact with other groups extensively, both for the purposes of extending their work and getting help. As much as possible, the teacher encourages this interaction by pushing students towards other groups when they are stuck or need an extension.
9. *When and how a teacher levels their classroom*: When every group has passed a minimum threshold the teacher pulls the students together to debrief what they have been doing.
10. *Student notes*: After the levelling has occurred the teacher asks the students to write some notes for themselves. These notes are based on the work that is already existing on the boards and can come from their own work, another group's work, or a combination of work from many groups.
11. *Assessment*: Assessment is mostly about communicating with students about where they are and where they are going in their learning. It honours the activities of a thinking classroom through a focus on the processes of learning more so than the products, and it includes both group work and individual work.

3.4.2 Participants

The participants in this study include the teacher, Ms. Marina, and six of her students. Ms. Marina is a high school teacher with 12 years' experience who is fully using the *Thinking Classroom* framework. At the time of the study she was teaching a block of Math 8, two blocks of Math 9, and a block of Pre-Calculus 11[1]. The research presented here took place in her Pre-Calculus 11 class.

The six students were selected for participation prior to the course beginning and were identified based on their incoming grades from their Math 10 class. Among the six students selected there was a student with a high A (95–100%), a low A (86–90%), a high B (80–85%), a low B (73–79%), a C+ (67–72%), and a C (60–66%). For the purposes of parsimony, only the case of Sara (low B) will be presented.

[1] In the location where the research was conducted there are three different Math 11 courses: Apprenticeship and Workplace 11, Foundations 11, and Pre-Calculus 11. Pre-Calculus 11 is the most academic of the three and credit for this course is a suitable pre-requisite for entry into all post-secondary institutions in the province.

3.4.3 Data

Data consist of six semi-structured interviews with each participant. The first interview was during the first week of class and thereafter occurred approximately every 2 weeks. On the days of the interview I would begin by observing the class in general, and the participants' activities in particular. I would then conduct brief semi-structured interviews with each of the six students and a more in-depth interview with Ms. Marina.

Each interview consisted of questions pertaining to what I had observed happen in class, how they were experiencing the class, and how they feel about themselves as a learner (or teacher) in comparison to previous experiences in mathematics. These interviews were audio-recorded and transcribed. For each of the six students these data were taken as a whole and turned into a third-person narrative (Clandinin, 1992) which encapsulated all of what I had learned about that student. This narrative was then shared with the student for feedback and editing. This process was repeated until the student was satisfied that the narrative reflected their experiences. For the purposes of the research presented here, these narratives were then coded for indications of affect as well as changes to affect.

3.5 The Story of Sara

There was a time when Sara used to like math. This was back when it came easily to her and her marks were good. All through her elementary education (K-7) she felt like this. During this time she was confident in her ability, "I would answer questions in class and take charge when we were working in groups." In grade 7 she was designated as a peer tutor and tasked with helping some of the struggling students in her class.

Then she moved on to high school (9–12) and into Math 8. The year started out ok for her. "There was a lot of review of stuff that I had mastered last year. So I found this easy." However, for the first time in her life "I had mathematics homework." In elementary school she had always finished her work in class. Now, the teacher was assigning work specifically to be done at home. But the homework was on content from the year before so it went quickly and without difficulty. At the end of all the reviews there was a test. "I aced it—I had an almost perfect score."

The next unit was on fractions. She had been good at fractions in elementary school and she had aced the fraction part of the review test, so she was feeling very confident about this unit as well. "But my mark on the unit test was a shock." For the first time in her life she didn't get an A on a math test. She didn't even get a B. She got 70% (C+). "I was devastated." She knew as soon as the test was over that it had not gone well. In hindsight, she was aware that the content had gotten harder. But she had done all the homework and it all seemed fine, but on the test it was not fine. She got confused when everything was all together.

"Then came algebra." This was new content for her so she did not start off feeling overly confident. For this unit Sara worked extra hard, doing extra homework questions, rewriting all her notes, and paying extra careful attention in class. She was nervous going to the test. She got 80% (B). She remembers not feeling good about this. "I was working twice as hard as in grade 7 and doing worse."

This set the pattern for Sara for the next 3 years. She worked harder and harder, but her efforts didn't outpace the increasing complexity of the topics. In grade 9 she almost failed the rational expressions unit, and she remembers thinking "I'm not going to pass the year". Her dreams of becoming a doctor were starting to look more and more impossible. If she couldn't get through mathematics "I am not going to get into university". Sara finished the year with a C+.

In grade 10 she doubled her efforts again and her parents hired a tutor to meet with her weekly and before every test. "I finished the year with a B but remember feeling completely wrung out." She had lost all her self-confidence in mathematics, it had been 2 years since she volunteered an answer in class, and during group work she just listened. But none of that compared to the devastation when her school councillor offered her the possibility of moving to the Apprenticeship and Workplace (A&W) mathematics course for grade 11, "that way you can finish math and ensure you will graduate." Sara had always known that A&W was for "dummies and burnouts". "He thought I was one of them. There was no way I was going into that class." By this point she had downgraded her dream of becoming a doctor to becoming a nurse and if she went into the A&W stream even that would not be possible. So Sara enrolled into Pre-Calculus 11 and landed in Ms. Marina's class.

The first week was very challenging for Sara. She shied away from group work and here every class was group work. Sara did not like this at all. "The only good thing was that the problems were fun. They didn't feel like math. And I actually found myself contributing a little bit here and there." In the second week the problems shifted to curriculum and suddenly they were factoring polynomials. This was one of Sara's least favourite topics so she became very anxious. "I felt like they were going to figure out that I don't know anything, that I'm a fraud, and that I shouldn't be in this class. I wanted to transfer to a different class—to a normal class. But I was afraid to go to my councillor because he would just say 'I told you so'." So Sara endured.

Then something interesting happened. Suddenly, towards the end of the second week "I saw that my group had done something wrong. I just sort of said, 'I don't think that's right'. My group mates looked at me and waited for me to explain, but I didn't know how. So I took the pen and started writing on the board." And she was right. That was a turning point for Sara. After that she was more willing to offer ideas and even occasionally hold the pen. Before long Sara found herself in a group where she wanted to hold the pen at the start of the problem. "I didn't actually think I knew what to do, but no one else knew what to do either, and we had to start somewhere. So I grabbed the pen." This is now the norm for Sara. "It doesn't matter who grabs the pen first. We need to start. It will work out in the end if we just start."

At the end of the polynomial unit Sara scored a low B on her test. "I was super happy with that. I mean, I knew I could do it in a group with others, but I wasn't sure how that was rubbing off on me. But it seems to have worked just fine." Sara wasn't anxious about homework or marks the same. "The learning is happening in class now. I don't feel like I have to go home and learn it on my own after class, or re-write note, like last year. We do a few minute of notes at the end of class and I do re-do some of the problems when I am studying, but nothing like last year where I was doing every question in the book."

As the term rolled on Sara started to enjoy herself in the class more and more. "This is now my favourite class. Actually, drama is my favourite, but this is my favourite academic class." She especially liked when the problems got tough. "Every once in a while we get a really tough one. We usually don't know that it is a tough one when we start, but we get to this point when you just realize 'this is tough'. And you look at each other and you grin and you just kind of dig in. And usually it works out. There is no feeling like it and everyone is high fiving each other."

Sara received a B (75%) on her first report card. "That's awesome. I mean I would have liked an A, but there were some really tough units in there". Whereas in grade 10 mathematics class, mathematics learning, notes, and her marks were all one thing, Sara now sees her marks as being related to, but still separate from her learning. "I mean, we come to class and it's great. We're learning every day—making it work. But we are in a group and we are stronger together. The mark is like a measure of what part I picked up from the group work." Sara is even rethinking her future. "If I can get Ms. Marina again next year, I think I'll take Math 12. Maybe even if I can't get Ms. Marina I'll take Math 12. I can do this."

3.6 Results

Looking closely at the narrative about Sara we can see that there have been many changes for her across a wide variety of affective variables. In what follows I summarize these.

Beliefs: Coming out of grade 10 Sara had a belief that learning mathematics was about doing all her homework, doing extra questions, taking and re-taking notes, and paying "extra careful" atten.tion in class. Now, three months into grade 11 she believes learning mathematics is "happening in class", that she "learns every day", and that this learning comes from "working it out" through the meaning making that is happening in her random groups. Her view of learning has shifted from being about doing all the proxies of learning (Liljedahl, 2017) to doing mathematics. Sara's beliefs about marks has also changed from being something that is continuous and synonymous with mathematics class, mathematics learning, and notes to something that is an occasional measure of how much she is learning from the group work.

Attitude: These changes in her beliefs are accompanied by changes in her attitude towards mathematics, mathematics class, and assessment. Sara has moved from an attitude of pessimism, disdain, and dislike for mathematics in grade 10 to one of optimism, enjoyment, and liking in grade 11. And her attitude towards her marks has shifted from a B as a negative and an indicator of poor performance to a B as "awesome" and an indicator of what group work has rubbed off on her.

Emotions: For Sara, all her emotions in Math 10 were negative—she was sad, disappointed, fearful, often devastated, and in the end, wrung out. By the end of the first term in grade 11 her emotions are all positive—"super happy", "that's awesome", and "great".

Enjoyment: Many of these emotions are linked to a burgeoning enjoyment of mathematics class. Although she did not like what she was experiencing in the first week of Math 11, this slowly changed to the point where she now considered Math 11 her "favourite academic class". Sara especially enjoys tough problems "when you just realize 'this is tough'. And you look at each other and you grin".

Efficacy: By the end of grade 10, Sara had very low self-efficacy. She didn't believe she was capable of learning mathematics. Now in grade 11 not only is her belief that she is capable of learning mathematics improved, but she is demonstrating a very positive *group-efficacy*. Group-efficacy is a new construct that I am attributing to Sara's belief that "it will work out in the end if we just start" and "we are stronger together". It is interesting to note that Sara's belief in the power of the group is irrespective of who else is in that group. She has a general belief that whatever group she is placed in will be able to solve the problem.

Confidence: By the end of grade 10 Sara had lost all self confidence in herself and her ability to do mathematics—she thought she was going to fail. She had stopped contributing ideas in group settings and it had been 2 years since she volunteered an answer in class. Although she felt like a fraud at the beginning of Math 11, 12 weeks in Sara had her spark back. It began slowly at first with a small suggestions, a declaration that her group had made a mistake, and taking hold of the pen out of a need to try to explain something. And then the floodgates opened and now she is ready to jump in and lead the discussion with her group. She has confidence in her group to get through it, and she is even ready to take on Math 12.

Goals: Whereas Sara used to see mathematics as a course she had to get through to fulfil her goals (and her downgraded goals), she is now rethinking her future and looking at taking mathematics in grade 12 not for the fulfilment of a goal but because she wants to. Mathematics is no longer a means to an end, but an end unto itself.

3.7 Discussion and Conclusion

From an affective perspective Sara has completely changed, been completely changed, from her participation in Ms. Marina's thinking classroom. She came into Math 11 with a disdain for mathematics and mathematics class, with low self-efficacy and low confidence, with negative emotions and a negative attitude, with a goal of just getting through, and with beliefs that proxies for learning are the same as learning. Sara now enjoys Math class—loves it, has not only self-efficacy but also group-efficacy, is more confident, has a positive attitude about mathematics and her marks, and has a belief that learning comes from doing mathematics.

Looking at these changes through the lens of an affective system we can see that the massive change to her affective variables has been occasioned by a substantive set of changes to almost all of her experiences with mathematics class. She has gone from being anonymous in her desk to standing and being visible, from working alone to working collaboratively, from having the teacher showing the mathematics to having to negotiate with others to make meaning of the mathematics, from working hard alone at home to working hard collaboratively in class, from being assessed on what she has managed to retain from her own work to being assessed on what she has taken out of the collaborative meaning making process. In fact, almost every part of her experience of what mathematics class has been radically altered. The notable exception to this is her marks. Sara is still performing at the same level (marks wise) as she had been when she was in Math 10. Either this no longer has the primary quality it did before, or its primacy is dwarfed by the wholesale changes she is experiencing within Math 11.

In short, Sara's affective system has been completely re-structured, not through small changes in one experience, but through massive changes to almost all of her experiences.

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Chapter 4
The Roles of Teacher and Parent Attitudes and Some Student Characteristics on Confidence in Learning Mathematics

Özge Gün

Abstract This study mainly investigates the roles of students' perceptions of their mother's, father's and teacher's attitudes toward them as learners of mathematics, and gender and mathematics achievement on their confidence in learning mathematics. Moreover, in this study, students' confidence is investigated according to gender and mathematics achievement. Data was collected from 1960 seventh grade students enrolled in 19 public elementary schools in a big city of Turkey. Results revealed that the difference between male and female students' confidence in learning mathematics was not significant; but it was significant among achievement groups. On the other hand, the results of multiple regression analysis yielded that the model composed of five variables significantly predicted the confidence scores of students. In addition, it was found that mathematics achievement and students' perceptions of their teachers' attitudes toward them as learners of mathematics were the strongest predictors of their confidence in learning mathematics.

4.1 Introduction

Confidence is an important affective variable in mathematics education and it is one of the most studied types of mathematical beliefs (Nurmi, Hannula, Maijala, & Pehkonen, 2003). In general, it has to do with one's belief about his/her competence in mathematics (McLeod, 1992). Considerable research about confidence in learning mathematics indicates the importance of this variable in relation to mathematics achievement, gender-related differences, election of optional mathematics courses, and classroom processes. However, to clarify these relationships more knowledge must be gained about what factors explain the level of confidence in learning mathematics (Reyes, 1984).

Several studies have shown that the relationship between confidence and achievement in mathematics is generally quite strong. Although the causal direction of the

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relationship is problematic (Hannula, Maijala, & Pehkonen, 2004), there was evidence for causality from achievement to self-concept during the first school years (Chapman, Tunmer, & Prochnow, 2000).

One specific field of research on mathematics-related affect that has accumulated very strong evidence over the years is the role of gender (Hannula, 2011). Related to self-confidence, across age and performance levels, female students tend to have lower self-confidence in mathematics than male students (e.g., Hannula, Maijala, Pehkonen, & Nurmi, 2005; Leder, 1995; Nurmi et al., 2003). However, there is evidence that sex appropriateness of academic learning changes from one socioeconomic status (SES) class to another. It was found that in lower SES classes learning tends to be sex-typed as female and in upper SES classes learning tends to be sex-typed as male (Sherman, 1971).

An individual's mathematical beliefs originate from his personal experiences in school and outside of school (Nurmi et al., 2003). For example, there is evidence that female students' lack of confidence in mathematics is consistent with their teachers' beliefs (Li, 1999; Soro, 2002; Sumpter, 2009). That is, mathematics teachers do likely hold the belief that their male students often have a hidden talent, but due to being lazy and careless they underperform, while female students tend to reach their performance due to diligence and hard work even if they are not very talented. This is one of the reasons for differences between the sexes in mathematics confidence. Another reason related to the differences in confidence can be students' perceptions of their mother's and father's attitudes about their learning of mathematics. Therefore, in this study student's perceptions of his/her mother's, father's, and teacher's attitudes toward him/her as learner of mathematics, gender, and mathematics achievement have been hypothesized as factors affecting his/her confidence in learning mathematics. The specific research questions of the study were:

- Are there significant mean differences in seventh grade students' confidence in learning mathematics scores with respect to gender and mathematics achievement?
- How well do the students' perceptions of their mother's, father's, and teacher's attitudes toward them as learners of mathematics, gender, and mathematics achievement predict confidence in learning mathematics of seventh grade students?

4.2 Method

4.2.1 Design

Quantitative research methods were used in the study. In particular, two associational research types, causal-comparative and correlational research design were used. For the first research question, independent samples t-test and one-way ANOVA were performed and for the second research question, standard multiple regression analysis was done.

4.2.2 Sample

The sample of the study consisted of 1960 seventh grade students enrolled in 19 different public elementary schools in one of the districts of Istanbul, Turkey. Convenience-sampling was used to select the subjects. The distribution of the subjects with respect to demographic characteristics was presented in Table 4.1.

4.2.3 Instruments

In order to measure students' perceptions of their mother's, father's, and teacher's attitudes toward them as learners of mathematics, and their confidence in learning mathematics, Mother, Father, Teacher, and Confidence in Learning Mathematics scales of Fennema–Sherman Mathematics Attitudes Scales (Fennema & Sherman, 1976) were used. There were 12 items in each, 6 of them positively stated and 6 of them negatively stated scaled on a five-point Likert Type Scale: Strongly Agree, Agree, Undecided, Disagree, and Strongly Disagree. For example, one of the items for Mother scale is "My mother would not encourage me to plan a career which includes mathematics." A sample item for Father scale is "My father has always been interested in my progress in mathematics." Another item for Teacher scale is "Mathematics teachers think I'm the kind of person who could do well in

Table 4.1 The distribution of the subjects with respect to demographic characteristics

	f	%	*f*	%
Gender				
Female	1001	51.1		
Male	959	48.9		
School mark in mathematics				
1	258	13.2		
2	348	17.8		
3	543	27.7		
4	463	23.6		
5	348	17.8		
Education level	Mother		Father	
Illiterate	192	9.8	41	2.1
Literate	123	6.3	85	4.3
Primary school graduate	1063	54.2	869	44.3
Middle school graduate	320	16.3	487	24.9
High school graduate or equivalent	191	9.7	338	17.3
Higher education graduate	11	0.6	30	1.5
University graduate	50	2.6	88	4.5
Higher degree	10	0.5	22	1.1
Total	1960	100	1960	100

mathematics." Lastly, a sample item for Confidence scale is "I am sure that I can learn mathematics." The negatively worded items were scored starting from strongly agree as 5, to strongly disagree as 1, and positively worded items were reversed to a negative direction for scoring purposes.

Data of the present study were first analyzed for the reliability and validity by conducting reliability analysis and principle components analysis, respectively. To test the internal consistency of each scale, Cronbach's alpha coefficients were calculated (Pallant, 2007). Initially, the alpha reliability coefficients of the Mother, Father, Teacher, and Confidence scales were found 0.833, 0.817, 0.690, and 0.879, respectively. To test the construct validity of each scale and to determine whether or not they have sub-dimensions, principle components analysis using principle components as factor extraction technique was done (Pallant, 2007). According to the initial principal factor solution with iterations, it was found that all scales were one-dimensional. Using factor loadings of 0.4 or greater as a criterion, item "My mother wouldn't encourage me to plan a career which includes mathematics" and "My father would not encourage me to plan a career which includes mathematics" were excluded from Mother and Father scales. After omitting those items, the final form of the Mother and Father scales with 11 items (6 positive and 5 negative) had alpha reliability coefficients of 0.840 and 0.843, respectively. After varimax rotation their eigen-values remained the same, negatively stated items came together under the first factor and positively stated items came together under the second factor, which indicated that the scales have no sub-dimensions.

In this study, students' achievement in mathematics was measured by using their latest year's school marks in mathematics course (self-reported). In Turkish schools a typical mark in any course ranges from 1 (unsuccessful) to 5 (very good). According to school marks in mathematics, students were divided into three achievement groups: low (having school mark 1), moderate (2 and 3) and high (4 and 5).

4.3 Results

Results are summarized into two sections; descriptive and inferential statistics.

4.3.1 Descriptive Statistics

Descriptive statistics regarding the Confidence in learning mathematics scale were given. The descriptive statistics such as mean scores and standard deviations related to the confidence in learning mathematics with respect to gender and mathematics achievement group are presented in Table 4.2.

The analysis for Confidence scale was done with total scores of the items to obtain a confidence in learning mathematics level score for each student. In Table 4.2, the mean score of Confidence scale for total were reported as above the

Table 4.2 Mean scores of confidence in learning mathematics with respect to gender and mathematics achievement group

	M	SD	*N*
Gender			
Female	43.26	9.003	1001
Male	43.62	9.005	959
Mathematics achievement			
Low	37.40	7.520	258
Moderate	40.79	7.754	891
High	48.27	8.323	811
Total	43.44	9.004	1960

midpoint score (36 out of 60). This indicates that the participants of the study had relatively moderate levels of confidence in learning mathematics. When gender variable was investigated, it was observed that males' mean confidence scores were slightly higher than that of females. In terms of mathematics achievement, the results yielded that students from the high achievement group had higher confidence in learning mathematics compared to students from the moderate and low achievement groups. In a similar way, the confidence of students from the moderate achievement group was higher than that of students from the low achievement group.

4.3.2 Inferential Statistics

In order to examine the significance of the differences in confidence in learning mathematics in terms of different gender types and achievement groups, independent samples *t*-test and one-way ANOVA were performed at 0.05 significance level. Moreover, multiple regression analysis was run to investigate the role of students' perceptions of their mother's, father's, and teacher's attitudes toward them as learners of mathematics, gender, and mathematics achievement on predicting confidence in learning mathematics of seventh grade students.

Before performing independent samples *t*-test and one-way ANOVA, the assumptions level of measurement, independence of observations, normality and homogeneity of variance were checked (Pallant, 2007). It was seen that all the assumptions except for equal distribution of variance within each population of mathematics achievement ($p < 0.01$), were assured for investigating the difference in mean confidence in learning mathematics scores with respect to gender and mathematics achievement.

In order to investigate the difference in confidence in learning mathematics scores of students with respect to gender, an independent samples *t*-test was performed. According to the analysis results, no statistically significant difference was found in the confidence scores for females and males [$t(1958) = 0.879, p > 0.05$].

To investigate the difference in confidence in learning mathematics scores of students with respect to mathematics achievement, the Robust Tests for Means results were reported since the homogeneity of variance assumption was violated (Pallant, 2007). According to the analysis results, a statistically significant difference was found among three achievement groups' confidence scores [$F(2,1957) = 273.144, p = 0.000$].

In order to reveal the difference among achievement groups, the post-hoc analysis was performed with Dunnett C analysis. Post-hoc comparison indicated that the mean score for high achievement group students was significantly different from that of moderate and low achievement group students. In the same way, the mean score for moderate group students was also significantly different from that of low achievement group students.

In order to investigate the role of students' perceptions of their mother's, father's, and teacher's attitudes toward them as learners of mathematics, gender, and mathematics achievement on predicting confidence in learning mathematics of seventh grade students, multiple regression analysis was performed. According to Tabachnik and Fidell (2007), multiple regression analysis is one of the fussier of statistical techniques, and seven major assumptions of multiple regression analysis, sample size, multicollinearity and singularity, outliers, normality, linearity, and homoscedasticity, have to be satisfied for performing a multiple regression analysis. It was seen that all the assumptions were assured for the investigation. Multiple regression analysis results revealed that the linear combination of three variables (students' perceptions of their mother's, father's, and teacher's attitudes toward them as learners of mathematics) and two demographics (gender and mathematics achievement) was significantly related to confidence in learning mathematics scores, [$F(5, 1954) = 252.418, p = 0.000$]. That is, the provided model consisted of five variables that significantly predicted the confidence in learning mathematics scores. Moreover, the sample multiple correlation coefficient was 0.626 and R-square = 0.392. That is, approximately 40% of the variance of confidence scores in the sample can be accounted for by the linear combination of three variables and two demographics of interest. According to Tabachnik and Fidell, r-square value below 0.4 indicates poor regression fit, between 0.4 and 0.7 moderate fit and above 0.7 strong fit (2007). Indeed, the regression was found a relatively moderate fit for this study. In addition, to investigate which of the variables included in the model contributed to the prediction of the confidence scores and reflect the relative strengths of individual predictors, all the variables made a statistically significant unique contribution to the prediction of confidence scores.

From standardized Beta Values, it was found that students' perceptions of their mother's ($Beta = 0.142, p = 0.000$), father's ($Beta = 0.095, p = 0.000$), and teacher's ($Beta = 0.312, p = 0.000$) attitudes toward them as learners of mathematics, gender ($Beta = -0.103, p = 0.000$), and mathematics achievement ($Beta = 0.296, p = .000$) significantly predicted confidence scores. For gender, negative beta indicates males' confidence scores were higher than that of females. Besides, Unstandardized B Values reflect the weights associated with the regression equation. The regression equation with three predictors (students' perceptions of their mother's, father's, and teacher's attitudes toward them as learners of mathematics) and two demographics (gender and mathematics achievement) are significantly related to confidence in learning mathematics. According to these B weights, the regression equation is as follows:

$$\text{Confidence} = 0.162_{\text{mother}} + 0.106_{\text{father}} + 0.387_{\text{teacher}} - 1.848_{\text{gender}} + 3.904_{\text{achievement}} + 8.019$$

The square of *Part-R* indicates unique contribution of the variable to the total *R*-square. That is, "how much of the total variance in the dependent variable is uniquely explained by the variable and how much *R* square change if it was not included in the model" (Tabachnik & Fidell, 2007, p. 145). It was found that mathematics achievement was recorded the highest part correlation coefficient, (*Part-R* = 0.275, $p < 0.001$), indicating mathematics achievement uniquely explains 7.5% of the variance in confidence scores. Moreover, students' perceptions of their teacher's attitudes toward them as learners of mathematics had a moderate part correlation coefficient, (*Part-R* = 0.268, $p < 0.001$), indicating 7.1% unique contribution of total variance in confidence scores. Similarly, gender was recorded the significant part correlation coefficient, (*Part-R* = −0.101, $p < 0.001$), meaning 1% of variance in confidence scores could be explained uniquely by this variable. Besides, students' perceptions of their mother's attitudes toward them as learners of mathematics was recorded the significant part correlation coefficient, (*Part-R* = 0.096, $p < 0.001$), indicating 0.9% of variance in confidence scores could be explained uniquely by this variable. Lastly, students' perceptions of their father's attitudes toward them as learners of mathematics reported the significant part correlation coefficient, (*Part-R* = 0.065, $p < 0.001$), meaning 0.4% unique contribution to total variance.

4.4 Discussion

Confident students tend to learn more, feel better about themselves, and be more interested in pursuing mathematical ideas than students who lack of confidence (Reyes, 1984). Hence, we believe that the findings of the present study could present some clues about the influence of perceived attitudes of important others together with some student characteristics on confidence in learning mathematics.

One concern regarding gender for this study was to investigate mean difference in seventh grade boys' and girls' confidence in learning mathematics scores. In this study, it was observed that this difference was not significant. However, this result contradicts with earlier studies that reported males' superiority in confidence scores; there is evidence that sex appropriateness of academic learning changes from one socioeconomic status class to another. On the contrary, there exist some studies reporting that girls also had a significantly more positive career interests related to mathematics than boys (Savas & Duru, 2005).

The other concern was to investigate the mean difference in confidence in learning mathematics of seventh grade students in different mathematics achievement groups. The findings of the study reveal that the students' confidences in learning mathematics are significantly differentiated in different achievement groups. As we expected, students from the high achievement group had higher confidence than students from the moderate and low achievement group. Similarly, students from the moderate achievement group had higher confidence than students from the low achievement group. The result was consistent with the former studies where it was found that the weak pupils had the weakest self-confidence, the good pupils other way round, and the average pupils were between these (Nurmi et al., 2003).

Lastly, the results of the multiple regression analysis revealed that the provided model significantly predicted the confidence in learning mathematics of students. Moreover, each variable made significant unique contribution in explaining confidence scores of students. This result is consistent with former researches supporting the influence of students' perceptions of their parent's (Eccles, Wigfield, & Schiefele, 1998) and teacher's (Pintrich & Schunk, 2002) attitudes toward them as learners of mathematics, gender (Hyde, 2004; Wigfield & Eccles, 1992) and achievement (Hannula & Malmivuori, 1996; Malmivuori & Pehkonen, 1996; Tartre & Fennema, 1995) in predicting mathematics confidence. Moreover, in this study it was found that mathematics achievement made the highest unique contribution. An interesting finding of the study was that students' perceptions of their teacher's attitudes toward them as learners of mathematics has contributed as much as mathematics achievement while explaining confidence in learning mathematics. However, gender, students' perceptions of their mother's and father's attitudes toward them as learners of mathematics have contributed far less than that of mathematics achievement and students' perceptions of their teacher's attitudes toward them as learners of mathematics.

The design of this study was a kind of causal comparative and correlation research design. Indeed, the purpose of the design was to explain and predict the existing relationship and differences among variables. However, finding a significant relation among variables did not mean the reasons of differences in confidence scores only due to predictor variables. Therefore, experimental studies might be conducted in order to investigate the likelihood of causal connections among these variables. In addition, other personal constructs (i.e., self-concept, self-regulation) and demographics (grade level) can be inserted in future studies. On the other hand, this study was a typical quantitative study, which means that the study was limited to inferences of the numeric data collected from questionnaires. However, the inference made from these numeric data might not reflect in depth results among the variables. Therefore, future studies could be supported by qualitative data. That is, the students are asked to write self reports or interviews are conducted so as to describe the complete picture of the relationship in given constructs.

Finally, based on the results of this study, some implications for mathematics teachers, educators, counsellors, and mathematics curriculum developers could be stated. Determination of students' personal constructs was of great importance in predicting confidence and understanding the differences in confidences in learning mathematics of elementary students. This study also revealed that students' perceptions of their teacher's attitudes about themselves were very influential while explaining their confidence in learning mathematics among the variables of the study. Therefore, mathematics teachers should be aware of their influence in shaping their students' future mathematics trajectories.

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Chapter 5
Valuing from Student's Perspectives as a Lens to Understand Mathematics Learning: The Case of Hong Kong

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Abstract Values, as a culturally specific notion, have a vital role to play in classroom mathematics learning. In this chapter, we argue that valuing, especially from the student's perspective, serves as a lens for us to better understand how they perform in mathematics learning. To develop such a lens, we investigated the componential structure of Hong Kong mathematics students by utilising a values questionnaire as a lens to better understand what the students find important in mathematics learning. A principal component analysis has been used in this study with the objective to interrogate the data. The analysis of the data led to the identification of nine components valued by Hong Kong students in their mathematics learning. We discuss the findings in terms of the Hong Kong culture and context and the value categories and we suggest implications for future research studies.

5.1 Introduction

It is common knowledge that mathematics as well as its learning and teaching are not value-free (Bishop, 1988, 1996). Pedagogically, what this means is that not only are values espoused and taught through mathematics lessons, but that values might also be useful for facilitating students' understanding and engagement in mathematics learning.

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From the socio-cultural perspective, in which learning is a function of participation in social practices, valuing refers to an individual's embrace of convictions, which are considered to be of importance and worth. It provides the individual with the will and grit to maintain any "I want to" mindset in the learning and teaching of mathematics. In the process, this conative variable shapes the manner in which the individual's reasoning, emotions, and actions relating to mathematics pedagogy develop and establish values are the convictions which an individual has internalised as being the things of importance and worth. What an individual values defines for her/him a window through which s/he views the world around her/him. Valuing provides the individual with the will and determination to maintain any course of action chosen in the learning and teaching of mathematics. They regulate the ways in which a learner's/teacher's cognitive skills and emotional dispositions are aligned to learning/teaching in any given educational context (Seah, 2018, p. 575).

Earlier definitions of values in mathematics education, however, reflect a tradition of psychological perspectives. Bloom's Taxonomy of Educational Objectives (Krathwohl, Bloom, & Masia, 1964) considered attitudes and beliefs to be progressively internalized into values. Bishop's (1999) definition of "the deep affective qualities which education fosters through the school subject of mathematics" (p. 2) also reflects such a perspective. Seah's (2005) investigation of what immigrant teachers valued in mathematics education perceived these as having been "inculcated through the nature of mathematics and through the individual's experience" (p. 43). In other words, then, the mathematics classroom represents a place where the values of the teacher and his/her students meet as they interact with one another, reflecting what each person considers to be important (and thus not so important) in the mathematics discipline, in the learning and teaching of mathematics, in the educational nature of the schooling experience, and in the wider society. The expert teacher, according to Seah and Andersson (2015), is one who is adept not only at teaching cognitive skills and structuring affective climate, but also one who navigates the myriad of values coming together to bring about a values alignment.

It is reasonable to argue that a successful values alignment is dependent on a teacher's valid knowledge of what his/her students in their mathematics class value. A teacher's professional experience in the class would provide such information to himself/herself. Yet, to the extent that such values identification is accomplished accurately and early enough in the school year, the potential value conflicts, which result from what teachers and their students invariable value differently, can compromise all the teacher's efforts in the cognitive and affective aspects of planning for learning. It is thus important for the identification and mapping of students' values to be made efficiently and accurately.

This chapter reports on Hong Kong students' participation in an international research study, entitled "***W***hat ***F***ind ***I***mportant (in mathematics learning)" (WIFI), in which a values questionnaire was constructed and validated for use across 21 countries in four continents (Seah & Wong, 2012). It is the WIFI study researchers' contention that such a survey instrument will not only allow for efficient and valid assessment of what a student values in mathematics learning, but its cross-cultural reliability will contribute towards comparative studies of the valuing that takes place in mathematics classrooms in different parts of the world.

5.2 Relationship Between Beliefs and Values

Attitudes, beliefs and values are constructs which are closely related. Krathwohl et al.'s (1964) taxonomy positions values as having developed from beliefs, and beliefs from attitudes. Clarkson, Bishop, FitzSimons, and Seah (2000) expressed the relationship in terms of the volitional aspect of values:

> 'Values are beliefs in action'. That is, the values that teachers are teaching in the mathematics classroom are not only beliefs the teacher holds, but their behaviour in the classroom actually point to these beliefs. (p. 191)

However, one may ask if values are necessarily expressed as actions. Might it be that in some cultures, what is valued may not be expressed as an action since there are more important values, which are prioritized and acted upon instead?

Yet another perspective emphasizes the difference in nature between beliefs and values, although each affects the development of the other within an individual. According to Seah, Atweh, Clarkson, and Ellerton (2008), beliefs relate to what is considered to be true (or false), whereas values relate to what is considered important (or unimportant). Thus, two teachers may value *information and communication technology* (ICT) but their beliefs can be very different. One teacher may value the use of four-function calculators in the early years so as to free more time for students to think about the solution process. The other teacher who also values ICT, however, may feel very strongly against this belief, and could instead subscribe to another belief that the adoption of data-loggers facilitates the collection of authentic data. Certainly, some different values are represented by these two beliefs as well. While the teachers' common valuing of ICT might have helped develop the two different beliefs, it can be observed that the belief statements support the valuing of *authenticity* in the second teacher. Thus, through this example, we demonstrate the interactional relationship between values and beliefs. In other words, they are interconnected.

5.3 Value Categories

Bishop (1996) proposed that the values present in school mathematics classrooms might be divided into three overlapping groups, namely, mathematical, mathematics educational, and general educational. Mathematical values relate to the extent to which aspects of Western mathematics are valued. Earlier, Bishop (1988) had theorised three pairs of complementary mathematical values, which are *rationalism* and *objectivism*; *control* and *progress*; and *mystery* and *openness* (see Bishop, 1988 for details). Thus, we acknowledge that different individuals will value, for example, *mystery* and *openness* to different degrees, and this can be thought of as being one of three continua of mathematical values, where the values at the end of the continuum are *mystery* and *openness*.

Mathematics educational values, on the other hand, express the extent to which we value different aspects of classroom norms and practices that relate to the teaching/learning of school mathematics. For example, different teachers value *exposition* (versus *exploration*) to different degrees according to the importance and worth they each attach to teachers teaching content explicitly and directly. Likewise, these values can be conceptualised to be occupying extreme nodes of a values continuum. Regardless of where each of their valuing is located along this continuum, it reflects what the teacher regards as important in his/her pedagogical practices. These values have been investigated through several studies coordinated by the Third Wave Project (Seah & Wong, 2012), a consortium of international research groups comprising 11 countries/regions, including Australia, Hong Kong, Malaysia, Singapore, and Sweden. These studies have investigated how values and valuing shape mathematics pedagogy (Law, Wong, & Lee, 2012; Seah, 2011). The underlying values of such "moments of effective learning" were examined with the students through interviews. Thus, through the identification of the following continua, we have developed some examples of mathematics educational values:

Ability	*Effort*
Process	*Product*
Application	*Computation*
Facts	*Ideas*
Exposition	*Exploration*
Recalling	*Creating*
ICT	*Paper-and-pencil*

Bishop's (1996) third category, general educational values, refers to those values, which are inculcated to the young through the business of school education. These might include civic values such as *honesty*, and in some schools, religious values.

Seah (2005) has further suggested that a societal category be added as a fourth category of values in the mathematics classroom, to allow us to fully account for the principles and convictions that are valued and co-valued amongst the players within the classroom. Geert Hofstede's notion of cultural dimensions may be useful here. Hofstede's analysis of data collected across many different countries in the 1970s had led to his assertion that each culture (which he defined generally to include classroom cultures as well) can be uniquely defined in a five-dimensional space (Hofstede, 1997). There are five cultural dimensions, namely *power distance*, *collectivism/individualism*, *femininity/ masculinity*, *uncertainty avoidance*, and *life* *orientation*. *Power distance,* for example, shows the extent to which subordinates and the less powerful members of a community expect and accept that power is shared unequally.

5.4 Research Questions

The aims of this current study were to investigate how mathematics learning has been valued through Hong Kong students' lenses, and to make sense of values and valuing as a culturally specific notion in the context of Hong Kong's mathematics classrooms. The results of this study would further contribute to a better understanding of how values in mathematics learning would be perceived by other equivalent student groups through cross-regional comparisons. As such, this study aims at (a) assessing what students in Hong Kong value or what they have learnt to value in mathematics learning and (b) validating the design of a culturally-sensitive, user-friendly and efficient online instrument with which Hong Kong students' values relating to mathematics and mathematics education can be assessed.

The present study addressed the following question:

What values relating to mathematics and to mathematics learning are associated with upper primary and lower secondary school students in Hong Kong?

5.5 Research Design

This research study was conducted as part of an international collaborative research. We adopt a stance that the role of values in facilitating effective mathematics pedagogy is universal and as such it is desirable to conduct this study in a cross-cultural setting for exploring how this plays out (Lee, 2009). The research collaboration provided us with theoretical and practical information regarding how values might be interpreted in the different cultural settings (Klassen, 2004). In exploring the values as defining features of (mathematics) education systems across different cultures, we do not make any assertion based on a "simple, common pattern" of data sets but instead make attempts to characterise the mathematics lessons as inherent in the implicit pedagogical principles as adopted for delineating any particular lesson pattern or prevalent instructional activity (Clarke, 2003).

This study has adopted a quantitative approach using factor analytic methods to develop interpretable groups of students' values. The draft of the student questionnaire has been discussed and examined by the Australian, Hong Kong, Malaysian, and Swedish research teams, which had initially come together to conduct this study. Such a discussion had not only enhanced the methodological validity and reliability, but also showed respect to the cultural sovereignty of all participating research teams, and the cultures they represent. An online version of the questionnaire can be accessed via: https://www.surveymonkey.com/r/WIFI_maths.

5.5.1 Participants

The participants were 1080 upper primary school (242 Grade 5 and 125 Grade 6) and lower secondary school (290 Grade 8 and 423 Grade 9) students (M = 486, F = 594) from various metropolitan Hong Kong schools and from a variety of contexts, which were considered relevant to student or school characteristics.

5.5.2 Instrument

A Likert-type scoring format was used for the first 64 items (Section A)—students were asked to indicate the extent of importance of each statement presented. A five point scoring system was used—absolutely important (AI) to absolutely unimportant (AU). A score of 1 was assigned to the AI response and a score of 5 to AU. Students had the option to provide comments on this section. Section B consists of ten continua dimensions against two bipolar statements each, such as, "How the answer to a problem was obtained" vs "What the answer to a problem is". Students were asked to indicate which of the two statements was more important to their mathematics learning. Section C consists of four scenario-stimulated items; and Section D consists of students' demographics. The focus of this chapter is on analysing the data in Section A of the survey.

5.6 Data Analysis

A principal component analysis (PCA) was conducted in order to interrogate the various mathematics and mathematics education value dimensions of the survey items using SPSSwin®. Given the exploratory nature of the study and guided by the scree plot and the interpretability of the components, a nine components orthogonal solution was accepted after the extraction of principal components and a Varimax rotation. The significance level was set at 0.05, while a cut-off criterion for component loadings of at least 0.45 was used in interpreting the solution. Given that the structure could vary, four PCAs—one for each of the possible combinations between the gender categories (male and female) and the 2 Year Levels (primary—ages 11 and 12 combined and secondary—ages 14 and 15 combined)—were performed in order to investigate possible differences between year levels and gender. No differences were observed in the four initial analyses. A final PCA was carried out, following the elimination of psychometrically poor items, resulting in nine components (each with eigenvalue greater than one), explaining 57.20% of the variance, with almost 12.32% attributed to the first component (C1), namely, *Valuing the problem solving process with mathematical understanding*.

A matrix that is factorable should include several sizable correlations. For this reason (Tabachnick & Fidell, 1996), it is helpful to examine matrices for partial correlations where pairwise correlations are adjusted for effects of all other variables. Further, if the Kaiser–Meyer–Olkin (KMO) measure of sampling adequacy is greater than 0.6 and the Bartlett's test of sphericity (BTS) is significant, then factorability of the correlation matrix may be assumed. The KMO is 0.96 and Bartlett's test of sphericity (BTS) is significant at the 0.001 level and so factorability of the correlation matrix is assumed. The resulting model was further tested and validated as follows. The total sample ($N = 1080$) was randomly split into two equal samples of 540 participants and the component models were re-estimated to test for comparability. The two Varimax rotations were almost identical in terms of both components and loadings for all nine components. We also tested the resulting model by gender. The total sample ($N = 1080$) was split by gender (M = 486, F = 594) and we re-estimated the component models to test for comparability. The two Varimax rotations were nearly identical for all components.

Missing data in large-scale studies could potentially be the cause of problems in multivariate research (Graham & Hofer, 2000). In the current study, there are no variables with 5% or more missing values, and the Expectation Maximization (EM) algorithm was implemented for all samples, in conjunction with pairwise deletion methods for missing data. As expected with so few missing data, the results of the EM algorithm were very similar to the pairwise deletion methods. An initial data screening was carried out to test for univariate normality (Tabachnick & Fidell, 1996). Descriptive statistics normality tests (normal probability plot, detrended normal, skewness, and kurtosis) showed that assumptions of univariate normality were not violated.

The naming of the nine components (C1–C9) was discussed amongst the authors, guided by the nature of the items associated with each component and the relevant research literature. Component 1 (C1)—Valuing the problem solving process with mathematical understanding, loaded on the item, "Knowing the steps of the solution"; C2—Valuing control through linkage with mathematics outside the classroom, loaded on the item "Stories about mathematics"; C3—Valuing effort through mathematics practice and assessment, loaded on the item "Doing a lot of mathematics work"; C4—Valuing ideas through mathematical discourse, loaded on the item "Alternative solutions"; C5—Recalling known facts and routine manipulation, loaded on the item "Knowing the times tables"; C6—Using ICT in mathematics, loaded on the item "Using the calculator to check the answer"; Feedback, dialogue, and interaction (C7), loaded on the item, "Feedback from my friends"; C8—Broadening of mathematical vision, loaded on the item "Relating mathematics to other subjects in school" C9 and Learning approach, loaded on the item "Explaining by the teacher".

Reliability analysis yield satisfactory Cronbach's alpha values for each component: C1, 0.91; C2, 0.85; C3, 0.86; C4, 0.79; C5, 0.79; C6, 0.79 (after removing item 27: "Being lucky at getting the correct answer", to enhance the reliability of the subscale); C7, 0.82; C8, 0.72; and C9, 0.70, indicating a strong or acceptable degree of internal consistency in each subscale.

5.7 Discussion

The results of the present study have identified nine key components valued by Hong Kong students in their mathematics learning. The significance of these findings is to establish an empirical research tool for testing a conceptualisation of values in mathematics education. Such a tool is believed to bear significance for our understanding of how students "choose to engage (or not engage) with mathematics" (Bishop, Clarke, Corrigan, & Gunstone, 2006, p. 7).

Through negotiation by examining in detail the items for each component, we did recast some of our initial labels for the nine components. For instance, we have included "process", in the labelling of the first component (C1) and for the second component (C2) we have incorporated the term "control" with "mathematics outside the classroom". These components reflect learners viewing mathematical knowledge as something meaningful to them. As such, the students value the exercising of "prediction mastery over environment knowing" (Bishop, 1988) by linking with the thinking tools such as mathematical puzzles and stories about recent development in mathematics to the life experiences outside the mathematics classroom. Recasting through negotiation can thus be seen as a necessary and powerful iterative mechanism through which we build up the value/valuing model of each cultural region by accommodating the educational approach valued in the mathematics classroom of that particular region under study.

An interpretation of *meaningfulness, autonomy*, and *positive attitude* (Law et al., 2012) can be further enhanced by an explicit consideration of the nine components as labelled in this study. With meaningfulness, students prefer to have relaxing atmosphere in which the classroom learning can have a feeling of "enjoyment" through *Valuing the problem solving process with mathematical understanding (C1); Recalling known facts and routine manipulation (C5); and Broadening of mathematical vision (C8).* A combination of C1 and C5 provides a clue that "practice with understanding" enables students "to have fun and have something to learn". Nonetheless, component C8, could mean that meaningful mathematical learning requires the learners to go beyond mastery of technical proficiency in mathematics in order to develop a vision of their own learning, viz. practice and effort is treasured, at the same time mathematics should be linked with real life contexts. This is in line with earlier findings among Hong Kong and mainland Chinese mathematics students (Wong, 2004; Wong, Wong, Lam, & Zhang, 2009).

A view, which may be unique when considering mainland Chinese and Hong Kong mathematics learning, is that the students consider obtaining the correct answer as a major indicator of understanding (Wong & Watkins, 2001) and they treasure deep procedural understanding (Baroody, Feil, & Johnson, 2007) as a means of achieving successful problem solving (Cai & Wong, 2012). This is essentially what the first component (C1) reflects. On the other hand, the students develop their social values of learning autonomy by *Valuing control through linkage with mathematics outside the classroom (C2); Valuing ideas through mathematical discourse (C4); and Using ICT in mathematics (C6).* Also, students engage positively in classroom activity by *Valuing effort through mathematics practice and assess-*

ment (C3); Feedback, dialogue and interaction (C7); and Learning approach (C9). All the values identified in this study have been mathematics educational (Bishop, 1996) in nature, suggesting that Hong Kong students are pretty pragmatic. In fact, none of the societal values, including those reflecting Hoftsede's cultural dimensions, were valued highly by the respondents in this study. It could be argued that through teacher-led monitoring and teacher support, students can have a better chance of getting the incentives or rewards required for their learning.

5.8 Conclusion

In the context of responding to the research question, the outcomes of a factor analysis on the questionnaire data have suggested that students in Hong Kong value the following nine attributes of mathematics learning: *problem solving, control, effort, ideas, basic facts, ICT, feedback, mathematical vision,* and *learning approach*.

The establishment of the componential structure as reported in this article allows us to compare value structures amongst different cultures and to investigate relationships between value priorities and learning components like gender, age, and outcomes. It forms a strong basis for developing a reliable and robust survey for future research. The present study will help classroom teachers to develop a better understanding of what their students find important in the learning of mathematics. The validation of the questionnaire has meant that it remains a reliable instrument with which values can be evaluated in a multicultural environment. This is not just useful for teachers to evaluate the values held by their students who are increasingly multicultural within individual classes or schools, but it is also relevant for the use of the questionnaire across different cultures and beyond the context of Hong Kong.

Further recasting of the values components through accommodation of the approaches as identified in the subsequent cross-regional comparisons will be made. In the meantime, the present study has established an empirical tool for identifying values in mathematics education, and will facilitate further collaborations on investigating the cross-cultural aspects of values in mathematics education.

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Chapter 6
Value-Focused Thinking in the Mathematics Classroom: Engaging Students in Decision-Making Through Socially Open-Ended Problem Solving

Orlando González, Takuya Baba, and Isao Shimada

Abstract Value-focused th3inking (VFT) is a methodology originally from management research, in which values, rather than pre-established alternatives, are the primary focus of any decision-making process. Socially open-ended problem solving is a methodology to find resolution to problems embedded in a real-life context and purposively designed to elicit students' mathematical, social and personal values through modeling and argumentation. Starting from the premise that, under VFT, a decision-making problem is intrinsically an open-ended one, the authors examine in this article the possibility of educationally applying VFT via socially open-ended problem solving. This idea is then illustrated by analyzing the implementation of a socially open-ended problem in a Grade 4 mathematics classroom in Japan.

6.1 Introduction

In today's value-pluralistic society, awareness of own and others' values is considered a necessary skill, because values—defined as deeply held beliefs through which people act upon, associated with a desirable/undesirable dichotomy (Philipp, 2007, p. 259)—are used to filter information, as well as to carry out problem solving and decision-making (Keeney, 1988, 1992). In fact, the assessment and judgement of value statements made by others, and thinking about these statements in relation to one's own values, are important functions of education and critical thinking

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(Ernest, 2001). On this regard, the school is seen to have a *value-clarifying* function, through which school education helps students to find their own values through seven *valuing process* skills (Rokeach, 1979, pp. 265–267): (1) seeking alternatives by prizing and cherishing, when faced with a choice, (2) looking ahead to probable consequences before choosing, (3) making choices on one's own without depending on others, (4) being aware of one's preferences and valuations, (5) being willing to affirm one's choices and preferences publicly, (6) acting in ways that are consistent with choices and preferences, and (7) acting in these ways repeatedly. As the reader can see, most of these skills are explicitly related to choosing and decision-making, and they are fundamental for the highly systematic decision-making managerial approach known as *value-focused thinking* (hereafter VFT, Keeney, 1988, 1992).

Many curriculum developers and international agencies around the world have recently paid special attention, implicitly or explicitly, to the role of values and decision-making in compulsory education, particularly in mathematics education. For example, in the case of Japan, the 2008–2009 reform to the Japanese Mathematics Course of Study (MEXT, 2008, 2009) emphasized, as overall objective at secondary school level, the importance of nurturing students' decision-making attitudes—with attitudes being defined as an organization of several beliefs and values focused on a specific situation or object, predisposing one to respond in some preferential manner (Rokeach, 1968, p. 159). However, despite this emphasis, how to promote such attitudes is not explained in the Mathematics curricula or teaching guides.

Under this scenario, socially open-ended problems (Shimada & Baba, 2015) seem to be useful instructional tools to nurture students' valuing process skills and decision-making attitudes. This is because socially open-ended problems extend the traditional open-ended problem—mathematical situations in which students are given freedom to formulate the problem and to choose different, but equally valid, solutions and argumentations to solve it (Pehkonen, 2009)—, by intentionally eliciting students' values through mathematical modeling and argumentation.

This paper aims to explore, from the perspective of an educational implementation of VFT—originally developed in the context of management theory—, the emergence of students' values from solving socially open-ended problems. For this purpose, two research questions are addressed: (1) What similarities and differences between VFT and socially open-ended problem solving should be considered to develop an educational implementation of VFT? (2) How could an educational implementation of VFT be used to explain, in a more systematic way, socially open-ended problem solving? For the first question, a theoretical analysis of VFT and socially open-ended problem solving will be carried out. For the latter question, we will use the answer to the former one to analyze the implementation, in a Grade 4 mathematics classroom in Japan, of a socially open-ended problem engaging students in decision-making.

6.2 Theoretical Background

6.2.1 Socially Open-Ended Problems: Addressing Multiple Values in Mathematics Education

Socially open-ended problems are open-ended problems embedded in a real-life context, purposively developed to elicit students' mathematical, social and personal values through mathematical modeling and argumentation (Shimada & Baba, 2015). By using this kind of problem, students are expected to develop the ability to address multiple values (Shimada, 2015, pp. 11–12), which requires the following threefold skill-set: (1) The ability to build mathematical models based on values, which is usually manifested in the first-half of the mathematics lesson; (2) the ability to regard the diversity of mathematical models based on values, which is usually manifested in the middle of the mathematics lesson; and (3) the ability to critically examine mathematical models based on students' own or others' values, which is usually manifested in the last-half of the mathematics lesson. In the classroom, socially open-ended problem solving follows four stages (Shimada & Baba, 2015): problem provision; individual resolution of the problem (which is an opportunity for students to develop their ability to build mathematical models based on values); whole-class presentation and discussion of the mathematical models and reasons for choice (which allows students to develop their ability to regard the diversity of mathematical models based on values); and final individual selection of one model and its respective reason for choice (which is an opportunity for students to develop their ability to critically examine mathematical models based on their own or others' values).

As for the processes of elicitation, clarification, discussion, and selection of students' values and mathematical models, socially open-ended problem solving may have flexibility but lacks systematicity. On this point, we believe that VFT, due to its highly systematic nature, can provide a more refined way to explain socially open-ended problem solving.

6.2.2 Value-Focused Thinking: Definition and Components

Traditional decisions analysis methods mostly emphasize *alternative-focused thinking* (AFT), which is based on the exploration of pre-established alternatives. AFT may not work in all decision-making situations because (1) alternatives can be misleading the decision, and (2) the available alternatives may not reflect what the decision-maker really wants: what he/she values (Keeney, 1988, 1992). Then, by stressing the importance of the decision-maker's values in any decision process, Keeney (1988, 1992) proposed *value-focused thinking* (VFT) as a different paradigm for addressing decision problems from the traditional AFT (a comparison of the two approaches is shown in Table 6.1). VFT would provide, among other things,

Table 6.1 Comparison of AFT and VFT approaches (Keeney, 1992, p. 49)

Alternative-focused thinking (AFT)	Value-focused thinking (VFT)
1. Recognize a decision problem	1. Recognize a decision problem
2. Identify alternatives	2. Specify values
3. Specify values	3. Create alternatives
4. Evaluate alternatives	4. Evaluate alternatives
5. Select an alternative	5. Select an alternative

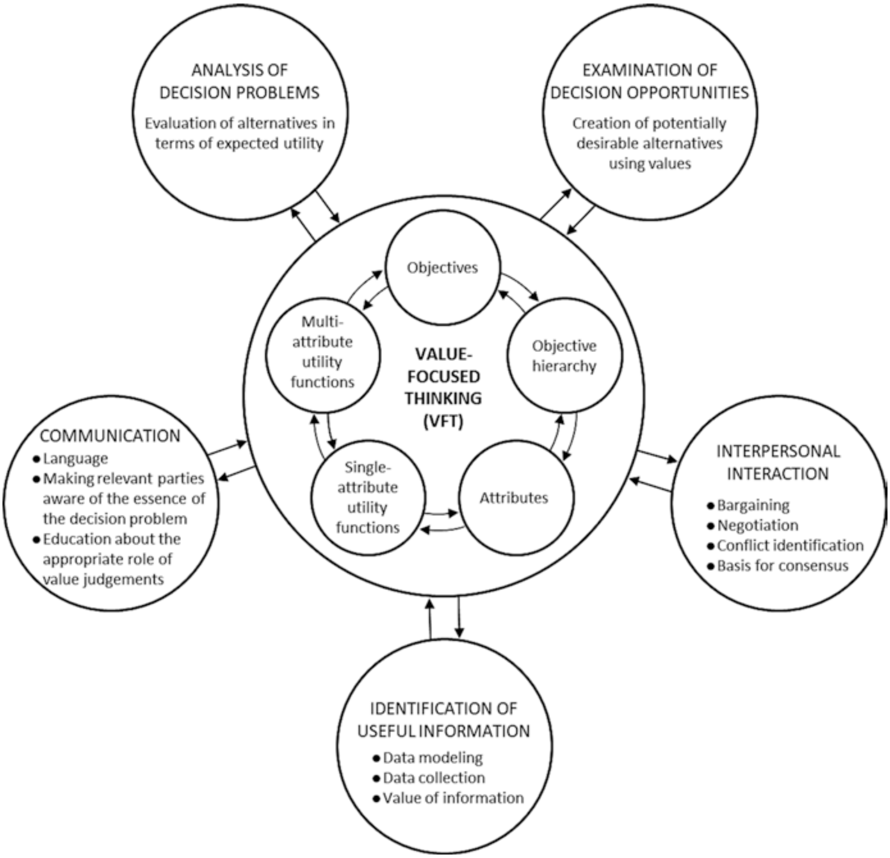

Fig. 6.1 Overview of value-focused thinking (adapted from Keeney, 1988). An arrow should be read as "influences"

(1) the identification of creative alternatives to better achieve what is desired, (2) a wider range of alternatives, and (3) articulated values, by explicitly stating them in the form of alternatives and objectives (Keeney, 1988).

According to Keeney (1988, pp. 466–468), the systematized thinking about values involved in VFT can be categorized into the following five core components (Fig. 6.1):

1. *Objectives*: Objectives tell what decision-makers want or consider important from a particular decision—that is, what decision-makers value (Keeney, 1992, pp. 5, 71). There is an *overall objective* (which is extremely broad and indicates the reason for being interested in the decision problem), and *lower-level objectives* (which break the overall objective into operational parts, defining more specifically what is meant by that overall objective and what are its important aspects). So, lower-level objectives will help decision-makers to articulate their values as statements of desired states or preferred directions (von Winterfeldt & Edwards, 1986, p. 38).
2. *Objective hierarchy*: It is a top-down approach to arrange the previously identified overall and lower-level objectives in a hierarchical structure, also known as *value tree* (von Winterfeldt & Edwards, 1986). In an objective hierarchy, the top layer of the tree contains very general, and sometimes vague, values, articulated as overall objectives. The values become more specific and operational in the lower layers of the tree (Keeney, 1992; von Winterfeldt & Edwards, 1986, p. 36).
3. *Attributes*: In order to specify the meaning of the objectives, and the value judgments associated with them, we need to specify attributes (or *value dimensions*, as they are called by von Winterfeldt & Edwards, 1986, p. 38). Attributes are measurable value-relevant preferences of physical or abstract properties of the entity in consideration (Keeney, 1988, p. 470; Keeney & Raiffa, 1993, p. 32).
4. *Single-attribute utility functions*: To evaluate alternatives, decision-makers need to build mathematical models from the identified attributes, known as *utility functions*. If only a single attribute is considered in building the model, then a *single-attribute utility function* is created, which is a univariate mathematical model.
5. *Multi-attribute utility functions*: Another way of evaluating alternatives is through building a mathematical model combining single attributes into a *multi-attribute utility function*, which is multivariate in nature.

These five core components of VFT are influenced by, and influence, the five peripheral components identified in Fig. 6.1.

6.2.3 From VFT in the Management Field to VFT in the Mathematics Classroom

VFT was developed, and has been mainly used, to guide decision-making in problems regarding corporate management, governmental policy, environmental studies, and risk analyses. Nevertheless, the educational applicability of VFT is by no means far-fetched, because of the many similarities between the VFT approach and the socially open-ended one—for example in building mathematical models as alternatives based on values (Keeney, 1988, 1992; Shimada & Baba, 2015). In fact, the five major activities associated with VFT (see Table 6.1) can be mapped onto the four stages that a classroom implementation of a socially open-ended problem follow:

the "problem provision" stage is an opportunity for students to recognize a decision problem; the "individual resolution of the problem" stage is an opportunity for students to specify values and create alternatives; the "whole-class presentation and discussion of the mathematical models and reasons for choice" stage will allow students to specify values developed by others, which might be similar or different to their own, and to possibly create more alternatives; and the "final individual selection of one model and its respective reason for choice" stage represents an opportunity for students to select an alternative after evaluating all the available ones. Furthermore, socially open-ended problem solving provides students with opportunities to engage in value exploration and clarification (Rokeach, 1979), discussion, argumentation and sustained exchange of ideas (Becker & Shimada, 1997; Ernest, 2001), activities strongly related to the peripheral components of the VFT approach depicted as outer circles in Fig. 6.1.

Despite the similarities, there are some striking differences in decision-making through VFT in a managerial setting and through socially open-ended problem solving. Due to these differences, a potential application of VFT in the mathematics classroom is not a straightforward process, and then some adjustments to the original VFT approach will be required in order to address such differences. To address this scenario, we were able to identify, from a theoretical analysis, the following six main differences between components of VFT in a managerial setting and their corresponding meanings in decision-making in the classroom through using socially open-ended problems.

1. In a managerial setting, overall objectives are posed by decision-makers themselves. In the case of a classroom setting, the decision-makers (i.e., the students) will not be the ones posing the overall objective of the problem at hand; instead, the teacher will do it beforehand, through his or her lesson plan.
2. In the case of a managerial setting, the overall objective is mainly defined in terms of maximizing performance indicators (such as profit or customer satisfaction) or minimizing costs, losses and risks. In a classroom setting, the overall objective is defined in terms of developing mathematical (e.g., Ernest, 2001) and valuing process (e.g., Rokeach, 1979) skills, related to the decision-making process.
3. In a managerial setting, *utility* is defined in the traditional economic sense, as the maximization of corporate profits. In a classroom setting, utility is understood as *experience utility* (Adler, 2013, pp. 1518–1520), which is a measure of a person's happiness, positive affects, or feelings of satisfaction.
4. In a managerial setting, modeling single- and multiple-attribute utility functions is used to derive optimal decision algorithms, in order to help the decision-maker reach an optimal decision, generally in terms of performance indicators. In a classroom setting, the main concern of function modeling should be the promotion of creativity and self-expression (Ernest, 2001, p. 283), as well as value awareness and clarification through decision-making (Rokeach, 1979; Shimada, 2015), rather than reaching optimal decision algorithms or mathematical sophistication.

5. In a managerial setting, attributes such as cost and efficiency are usually considered, independently of the context. In a classroom setting, attributes in decision-making problems are context-dependent, with basically no usually considered attributes.
6. In a managerial setting, making a final decision requires negotiation, compromise and consensus among decision-makers (Keeney, 1992, p. 307). In a classroom setting, students, as decision-makers, do not necessarily have to reach consensus about the final decision to be made. In fact, in the open-ended approach, students are expected to find as many methods as possible to solve a particular problem (Becker & Shimada, 1997, pp. 53–60). Nevertheless, students still have to reach consensus in socially open-ended problem solving: consensus on accepting that there are many values as a result of engaging in the process; consensus on learning to accept that some classmates have different, and equally correct, understandings to their own (E. Pehkonen, personal communication, October 9, 2017).

Understanding these differences will allow us to redefine particular components of VFT in terms of mathematics classroom practice, and then achieve the development of an educational implementation of VFT.

6.3 Using the Perspective of VFT to Analyze a Socially Open-Ended Problem in the Mathematics Classroom

In this section, it is illustrated how a practical educational implementation of VFT can contribute to systematically describe the way students engage in decision-making through socially open-ended problem solving. In order to achieve that aim, the socially open-ended problem "Hitting the target" (Shimada & Baba, 2015, Fig. 6.2) was chosen. This problem requires from students to make a decision by building mathematical models based on values (Keeney, 1988, 1992; Shimada &

At a school cultural festival, your class offers a game of hitting a target with three balls. If the total score is more than 13 points, you can choose three favorite gifts. If you score 10 to 12 points, you get to choose two prizes, and if you score 3 to 9 points, you get to choose only one prize. A first grader threw a ball three times and hit the target in the 5-point area once, in the 3-point area once, and on the borderline between the 3-point and 1-point areas. How would you score this student? Please write your ideas.

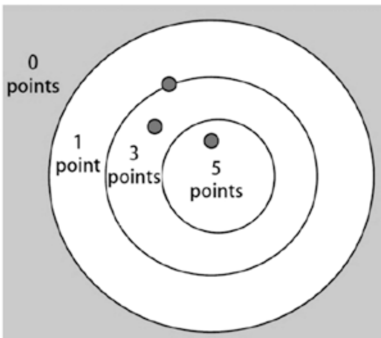

Fig. 6.2 The "Hitting the target" problem (Shimada & Baba, 2015)

Baba, 2015), which can be achieved through making use of the ability to address multiple values (Shimada, 2015) and the previously introduced seven valuing process skills (Rokeach, 1979).

The third author, who was a school mathematics teacher for 40 years, implemented the problem on March 12, 2013, with a group of 38 Grade 4 students (19 boys, 19 girls) in a private elementary school in Tokyo, Japan. The lesson followed the four stages identified by Shimada and Baba (2015). The whole class was video-recorded, and the initial and final models chosen by each of the students were collected.

The focus of this section is to illustrate how to use, from an educational standpoint, each of the five core components comprising the VFT framework (see Fig. 6.1) to systematically describe and explain the aforementioned classroom implementation of the socially open-ended problem "Hitting the target." For this purpose, any redefinition of particular components of VFT in terms of mathematics classroom practice will be highlighted when relevant, in order to address the previously identified six differences between decision-making through VFT and socially open-ended problem solving, as well as to clarify the educational implementation of VFT.

6.3.1 Objectives

In the case of the "Hitting the target" problem, the overall objective is the following one: Students will develop a mathematical model for scoring the "Hitting the target" game situation given, by taking into account their own and others' values. As it was previously mentioned, the "poser" of the overall objective is the teacher, whereas the "doers" of the overall objective are the decision-makers, the students.

As for lower-level objectives, we can pose the following ones, based on the three skills identified by Shimada (2015) in relation to develop, through using socially open-ended problems, the ability to address multiple values in the mathematics classroom:

To build mathematical models based on values: Students will create their own scoring alternatives for the problem, by articulating their values as mathematical models. This is strongly related to the peripheral VFT component decision opportunities (Fig. 6.1).

To regard the diversity of mathematical models based on values: Students will consider the alternatives created by their peers—that is, their peers' value-laden mathematical models. In order to achieve this, students, by engaging in communication and interpersonal interaction, will be given the opportunity to present to the rest of the class the models they came up with. In our example, this step is very important, because students not only affirmed their choices and preferences publicly, but also may have become aware of the existence of new value-laden alternatives that they did not consider during the individual resolution stage (Rokeach, 1979). This is strongly related to the peripheral VFT components decision opportunities, communication, personal interaction and useful information (Fig. 6.1).

To critically examine mathematical models based on students' own or others' values: After a whole-class discussion and personal interaction with peers, decision-makers may have expanded their set of alternatives, because now they are aware of the ones developed by others, which might be similar or different to their own. By assessing and judging such set of alternatives—which are value statements—in relation to their own values, students will be engaging in critical thinking (Ernest, 2001). Then, students will be critically evaluating each available alternative, or mathematical model, by appraising such alternatives (Keeney, 1988, pp. 466–467, 484). It is crucial in this endeavor to ask many questions about other students' thinking and valuations, such as "Why was Model A preferred to Model B?" "Why is Attribute X important at all?" "Why isn't Attribute Y included?" Then, after considering their own and others' arguments, students will go back to their own individuality and make a final choice. This is strongly related to the peripheral VFT component *decision problems* (Fig. 6.1).

6.3.2 Objective Hierarchy

In an objective hierarchy, the lower-level objectives might be divided into mutually exclusive lower-level twigs, helpful in setting up measurement procedures of the objectives they descend from (von Winterfeldt & Edwards, 1986, p. 45). Figure 6.3 depicts possible lower-level twigs coming from the three lower-level objectives listed before.

An objective hierarchy can be easily connected to Rokeach's (1979) seven valuing process skills. For instance, there is a connection between "seeking alternatives by prizing and cherishing, when faced with a choice" and "to develop a mathematical

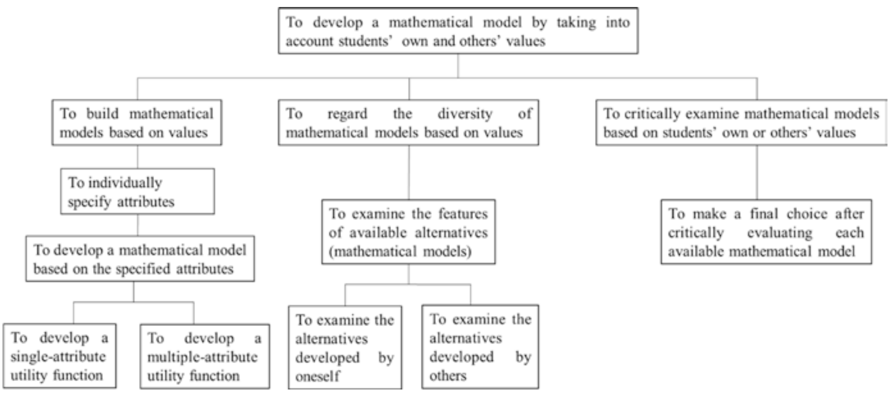

Fig. 6.3 Value tree representation of the objective hierarchy related to a socially open-ended problem, using the "Hitting the target" problem as illustrative example

model based on the articulated evaluation attributes"; between "being aware of one's preferences and valuations" and "to examine the alternatives developed by oneself"; and between "looking ahead to probable consequences before choosing" and "to critically examine mathematical models based on students' own or others' values."

6.3.3 Attributes

As it was explained before, for each lowest-level objective, we might need to associate value-relevant attributes. In the case of the "Hitting the target" problem, based on the initial and final models chosen by each of the students collected by Shimada and Baba (2015) and Shimada (2015), some attributes valued by students were the following:

1. *Physical location of the ball on the target board*: The score for the borderline shot was set based on the location where it landed (e.g., a student named I.K. answered "2 points, because it's between the 1-point and 3-point areas").
2. *Age of the player*: The scoring criterion was set based on how old the player was (e.g., a student named K.R. said "I'll give 3 points to the first grader, but 2 points to a third or fourth grader, and 1 point to a fifth or sixth grader").
3. *Fairness*: Before assigning a score to the borderline shot, the decision- maker considered what would be "fair," and to whom (e.g., a student named S.R. claimed having chosen a model "because it's impartial to everybody; because, if looking at this, a high schooler won't say it's unfair").
4. *"Motivating" scoring rules*: Before assigning a score to the borderline shot, the decision-maker must consider how to make the game appealing, so players would like to try the game again (e.g., a student named T.Y. said that "more and more first graders will come to play the game").
5. *Mathematical beauty*: some students made a decision driven by the beauty they perceived in a mathematical model (e.g., a student named M.H. said to have chosen a model "because the friend's formula is beautiful").

6.3.4 Single-Attribute Utility Functions

In order to evaluate alternatives, the next element in systematizing thinking about values is to build a mathematical model from the identified attributes. If only a single attribute is considered in the model, then the decision-maker created a single-attribute utility function. Based on the data collected by Shimada and Baba (2015) and Shimada (2015), an example of a single-attribute utility function in the case of the "Hitting the target" problem is 5 + 3 + 2 points, just considering the physical location of the ball (the borderline shot was given 2 points, since is not on either on the 1- or 3-point area).

6.3.5 Multi-Attribute Utility Functions

Another way of evaluating alternatives is by combining single attributes into a multi-attribute utility function. Based on the data collected by Shimada and Baba (2015) and Shimada (2015), an example of a multi-attribute utility function in the case of the given problem is 5 + 3 + (3 + 1) points, considering both the physical location of the ball (5 + 3 points) and the age of the player (because of kindness to the first grader, the player was granted the score from both areas separated by the borderline).

6.4 Conclusions

In this paper, an effort has been made to briefly examine and determine the educational applicability of VFT, as well as to use the perspective of VFT to describe, in a systematic way (see Fig. 6.1), the socially open-ended problem solving process. From the classroom implementation of the "Hitting the target" problem, each of the five core elements comprising the inner circle of the VFT framework (Fig. 6.1) was identified, and the differences between the "traditional" and the "classroom" VFT approaches were highlighted (e.g., overall objectives being defined beforehand by the teacher, not by the decision-makers, aiming to develop mathematical and valuing process skills rather than maximizing performance indicators or minimizing losses). Also, the identification by students of a number of value-laden attributes (e.g., the physical location of the ball on the target board, the age of the player, fairness and mathematical beauty), and the use of a single or multiple attributes in their scoring mathematical models, were clear from implementing the "Hitting the target" problem.

This classroom implementation also substantiated the value-clarifying function of solving socially open-ended problems, by empirically illustrating the enactment of Rokeach's (1979) valuing process skills: students seeking different alternatives on their own in order to make a decision for a scoring function; students affirming their preferences publicly through a whole-class discussion; students becoming aware of their own and others' valuations; students cherishing their own and others' models to make a final choice, acting consistently with their preferences. By the way, Rokeach's (1979) valuing process skills related to value clarification do not explicitly encompass consideration of other people's preferences and valuations, as VFT and socially open-ended problem solving do.

By experiencing engagement in socially open-ended problem solving, many values held by the students—such as fairness, kindness to the first grader, ball's landing location, and mathematical beauty—were clarified, articulated, and made explicit in the form of scoring functions (i.e., mathematical models, see Keeney, 1988; Rokeach, 1979) and reasons for choice (i.e., students articulated their values as preferred directions, see von Winterfeldt & Edwards, 1986, p. 38). This seems to confirm the mutually influencing interrelation between the five core components of VFT and the five peripheral components identified in Fig. 6.1 (Keeney, 1988). Thus, engaging in VFT should influence the creation of alternatives in examining decision opportunities; evaluation of alternatives in analyzing decision problems; identification of useful information during the decision-making process; communication; and interpersonal interaction in circumstances such as students consenting to accept, respect, and appreciate that other students are thinking differently.

Lastly, using VFT from an educational perspective to explain the implementation of the socially open-ended problem "Hitting the target" seems to clarify, in a more systematic way, how engaging students in decision-making provide them with opportunities to develop many desirable skills for their participation and informed decision-making in today's society, such as the threefold ability to address multiple values (Shimada, 2015), awareness of and critical reflection on their own and other people's values, interpersonal skills, and mathematical skills. The possibility to develop these skills in our students can be attained under an atmosphere of high degree of student autonomy, due to the nature of the open-ended approach.

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Chapter 7
Young Students' Feelings Towards Problem-Solving Tasks: What Does "Success" Imply?

Hanna Palmér and Jorryt van Bommel

Abstract According to the Swedish curriculum, problem-solving is to be part of mathematics teaching from preschool continuing throughout all grades in school. However, little is known about young students' feelings towards problem-solving tasks. This paper reports on an educational design research study investigating the potential in teaching problem-solving in preschool classes (6-year-olds). Two examples are presented showing how the students evaluate their feelings towards the problem-solving tasks they have been working on. The results show that understanding a task from the beginning or being able to solve it quickly are not necessary prerequisites for young students to experience enjoyment when working with the tasks. Quite the opposite, the majority of the students evaluated the tasks as fun and accessible, even though their initial solutions were often incorrect and they had to struggle a lot to solve the problems.

7.1 Introduction

In the Swedish national curriculum, mathematics is described as a "creative, reflective, problem-solving activity" (National Agency for Education, 2011, p. 62), and problem-solving is emphasized as both a purpose (an ability to formulate and solve problems) and a strategy (a way to acquire mathematical knowledge). Studies have shown that students who meet challenging tasks can be excited about doing mathematics and persistent in their attempts to solve problems (Brown, 2017; Carlson, 1999; Cobb, Yackel, & Wood, 1989). However, other studies have shown that teachers often reduce "the problem" in problem-solving tasks, for example, by giving the students hints or instructions for how to solve the tasks (Smith & Stein, 2014). This is done in good faith to enable students to succeed with the tasks and thereby to

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develop positive feelings towards mathematics and problem-solving (Mason & Johnston-Wilder, 2006; Smith & Stein, 2014). However, reducing "the problem" reduces students' opportunities to develop their ability to complete problem-solving tasks and to acquire mathematical knowledge.

This paper reports on an educational design research study investigating the potential in teaching problem-solving in preschool classes (6-year-olds). Both cognitive and non-cognitive factors are significant while learning mathematics (Schoenfeld, 1985). Previous reports of our study have shown that it is both possible and desirable to use problem-solving in preschool with regard to cognitive factors focused on students' learning (Palmér & van Bommel, 2016; van Bommel & Palmér, 2016). In this paper, we instead focused on a non-cognitive factor, namely, the students' feelings towards the problem-solving tasks they had been working on. Feelings such as frustration, anxiety, confidence, surprise, and curiosity have shown to influence the process of solving non-routine mathematical tasks (Hannula, 2016). The question elaborated on in this paper is:

How do the students themselves describe their feelings when working with problem-solving tasks?

First, we elaborate on problem-solving in mathematics and students' feelings towards mathematics. Then we present the study, followed by results, and finally, conclusions and implications for teaching problem-solving to young students.

7.2 Problem-Solving in Mathematics

According to Lesh and Zawojewski (2007) and Cai (2010), a task becomes a problem-solving task when the individual who is to solve the task has to develop strategies and/or knowledge not yet obtained in order to be able to solve it. Thus, a problem-solving task is challenging, as the student does not know in advance how to proceed to solve the task. Instead, the student has to develop new (for him or her) strategies, methods, and/or models to be able to solve the task.

The emphasis on problem-solving in mathematics in the Swedish curriculum has increased throughout the years, and a similar tendency can be seen in several other countries. There seems to be an international consensus that students should be educated to become competent problem-solvers and that they will develop important mathematical ideas and competences through working with problem-solving tasks (Csapó & Funke, 2017; Lesh & Zawojewski, 2007; Schoenfeld, 1992). In Sweden, the emphasis regarding how and why students are to be taught problem-solving has changed over the years. The emphasis has slowly shifted from a view in which students first need to learn mathematics in order to become problem-solvers, to a view in which problem-solving is to be taught as content itself, towards today's view that problem-solving is a strategy for acquiring new mathematical knowledge (Boesen et al., 2014; Wyndhamn, Riesbeck, & Schoultz, 2000).

However, at the same time as studies have shown that all students benefit from being challenged by an advanced mathematical content (Claessens, Engel, & Curran, 2014), research also shows that early childhood education programs more often include routine tasks than challenging problem-solving tasks (Cross, Woods, & Schweingruber, 2009; Perry & Dockett, 2008). Yet, if students are to become successful problem-solvers, problem-solving needs to be an integral part of early childhood education and not something to be added after other concepts and skills have been taught (Cai, 2010).

The study presented here focused on problem-solving as both a purpose and a strategy; students worked with problem-solving tasks they did not know in advance how to solve, and by working with these tasks they acquired new mathematical knowledge.

7.3 Feelings Towards Mathematics

Teachers wanting their students to develop positive feelings towards mathematics is reasonable, since there are studies indicating correlations between feelings and performance in mathematics, where students' feelings towards mathematics and problem-solving are often connected to mathematics anxiety and mathematics difficulties (Dowker, Bennet, & Smith, 2012). However, while some studies show that students' feelings towards mathematics and problem-solving have a major influence on their learning (Hannula, 2016; Mason & Johnston-Wilder, 2006; Schoenfeld, 1992) and their interest in the subject (Clements & Sarama, 2016), other studies show no such correlations (Dowker et al., 2012; Pinxten, Marsh, De Fraine, Van Den Noortgate, & Van Damme, 2013).

According to Clements and Sarama (2016), young students' interest and feelings in mathematics and science predicts not only their achievement in science, technology, engineering, and mathematics, but also their reading achievement. Students' interest in and feelings towards mathematics are also connected to their mathematical mindset; differences between successful and unsuccessful students are less about the learned content and more about their mindsets (Boaler, 2016). Students can develop static or dynamic mindsets towards mathematics. A static mindset implies a view on intelligence as given and non-changeable, while a dynamic mindset implies a view on intelligence as dynamic and changeable. Further a dynamic mindset implies a view of oneself as a mathematician and a view of learning mathematics as struggling with mathematical tasks. Teaching mathematics through problem-solving is one way to help students develop such a dynamic mindset, which has shown to be connected to both positive feelings and to learning of mathematics.

For different reasons we found it important to investigate the young students' feelings towards problem-solving tasks. For instance, previous studies have shown that students' feelings influence not only the process of solving non-routine mathematical tasks (Hannula, 2016) but also students' views of themselves as learners of mathematics (Boaler, 2016; Clements & Sarama, 2016), and their views of themselves as learners of other subjects (Clements & Sarama, 2016). However, only a few studies on problem-solving in mathematics have involved younger students. Therefore, we included these young students' feelings as part of our study on the potential in teaching problem-solving in preschool classes (6-year-olds). Students' feelings can be probed based on expressions and/or actions, of which none are static but rather are local and context-embedded and thus changeable between different mathematical activities (Debellis & Goldin, 2006). In this study, we focused on students' expressed feelings, asking them to describe their feelings regarding just-finished problem-solving lessons. As affective and cognitive systems are considered to be related (Debellis & Goldin, 2006; Op't Eynde, Corte, & Verschaffel, 2001), we connected these expressed feelings to how the students evaluated the difficulty of the problem-solving task they had been working on during the lesson.

7.4 The Study

The study was conducted through educational design research, which implies designing, testing, and refining interventions (McKenney & Reeves, 2012). These interventions were not about an implementation of ready-made and researched lessons, but an exploration in which we learned in and from the setting. One part of this learning was students' evaluations of the interventions. In the intervention each preschool class worked with six or seven different problem-solving tasks. The lessons were conducted in the students' usual classrooms, and for the lessons, the students were divided into groups of approximately 12. All lessons in the study were designed in line with the previously described research on problem-solving: the mathematical ideas were to be understandable, but the students should not have previously been taught a method for solving the tasks.

The age of the students and the fact that they, in line with the Swedish school system, had just started reading and writing, were important factors to take into consideration in the design of this evaluation. Writing would not be possible for most of the students, and drawings could get away from the focus of the exercise, leaving too much room for interpretation during the analysis. In accordance with other studies (e.g., Chapman, 2003), we offered explicit choices to avoid such diversity. After each lesson, the students evaluated the task they had worked on individually. On paper, they first were to evaluate the difficulty of the task and were also asked to draw a happy, neutral, or sad mouth indicating how they felt during the lesson. As for their judgment of the difficulty of the task, they could choose among three levels: very easy (X); hard at first, but now I understand its solution (XX); hard, and I still do not understand its solution (XXX). In this way, the alternatives could be distinguished, allowing us to connect students' evaluations of their feelings to their evaluations of the difficulty of the tasks.

7.4.1 Selection of Students

Preschool classes are non-compulsory education that serves to facilitate a smooth transition between the optional preschool with its traditions of play and the obligatory school with its traditions of learning. Eight preschool classes (a total of around 150 students) were part of the overall intervention. These preschool classes were selected based on their teachers' previous interest in developing their mathematical teaching. Three of these preschool classes from two different schools, with a total of 49 students, took part in the problem-solving lessons reported in this paper.

The ethical regulations for research provided by the Swedish Research Council (2011) were followed, where both guardians and students approved the participation. The students were given verbal information about the intervention and the interest of the researchers. The students' guardians were given written information about the study and approved their children's participation in line with the ethical guidelines.

7.5 Results

Below are two examples of lessons from the intervention, followed by the students' evaluations. The intention is to provide information on how the lessons were conducted and how the students worked on the tasks, and then to connect this to the student's evaluations of their feelings when working on the tasks.

7.5.1 Example 1: The "Tower" Task

In this lesson, the students were first handed a picture of a tower (Fig. 7.1). The question asked was, "How many blocks will you need to build the tower?" They were to write or draw the number of blocks needed on their paper.

Of the 49 students, 44 were present when working with this task. One student initially wrote that he would need ten blocks to build the tower. Three students initially wrote that they would need eight blocks to build the tower. The remaining 40 students initially wrote that they would need six blocks to build the tower. After discussing their solutions in pairs, the same pairs of students were to build the tower

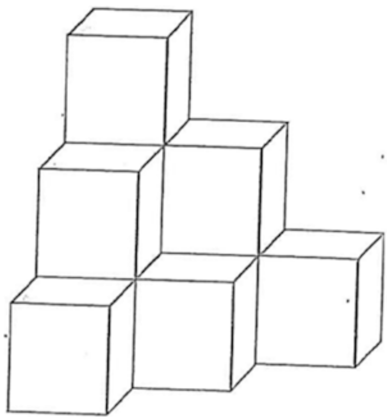

Fig. 7.1 The picture of the tower given to the students. The task is taken from http://ncm.gu.se/kangaru

Table 7.1 The students' evaluations of the tower task

	The task was very easy	The task was hard at the beginning, but now I understand its solution	The task was hard, and I do not understand its solution
Happy mouth	18	21	2
Neutral mouth		3	
Sad mouth			

using blocks. In most cases, it took quite a long time for the students to build the tower, since they wanted to use only six blocks, in line with their first solution. When building the tower with blocks, the students found the need for (at least) ten blocks, and discussions were held about which blocks were "being hidden" in the picture. After all groups had built the tower they were gathered for the whole-class discussion. As such, the students discussed the task and their solutions, both in pairs and as a whole class together. After this, the students evaluated the activity individually. On the back of their papers they evaluated the difficulty of the task and drew a mouth indicating how they felt during the activity. Their evaluations of the tower task are presented below (Table 7.1).

7.5.2 *Example 2: The "Pull Out of the Bag" Task*

In this lesson, the students were to guess, and then to investigate, which pair would win if two marbles were taken at once, 20 or 30 times from a bag with two red and two yellow marbles. The task was introduced orally, where the sample space with all three events (red–red, red–yellow, and yellow–yellow) was explored with the students at the board. After this, the students voted for the event they thought would "win." They were each to draw their individual vote on a small paper before making a summary of the votes by creating a bar chart of the small papers on the table. The outcome was assumed not to be obvious to the students, and the students' predictions confirmed that the mathematical idea had not yet been grasped. Next, the students were asked to document the outcome of the 20 or 30 draws. While marbles were drawn from the bag, students were given their own choice of method for documenting, with which they struggled a lot. Without instructions for how to organize their documentation, a large diversity of strategies and representations appeared, indicating both creativity and understanding of the task (Fig. 7.2).

At the end of the lesson a whole-class discussion was held where the sample space was explored and the students were to look back at their prediction and make a reflection on their documentation. Finally, the students evaluated the activity individually on the back of their papers (Table 7.2).

Fig. 7.2 Examples of students' documentation, illustrating diversity

Table 7.2 The students' evaluations of the "pull out of the bag" task

	The task was very easy	The task was hard at the beginning, but now I understand its solution	The task was hard, and I do not understand its solution
Happy mouth	27	8	4
Neutral mouth	3	1	3
Sad mouth	1	1	1

7.6 Conclusion and Implications

In this study, the young students were exposed to (for them) demanding problem-solving tasks that were solved by only a few from the start. The two lessons presented in the previous section are to be understood as examples of the problem-solving tasks and the layout of the lessons in the intervention and not as the totality from which conclusions and implications are discussed in the last section of the paper.

In the intervention, students' initial ideas about the solution were often shown to be wrong and in need of modification. One reasonable question is how this influenced these students' feelings towards mathematics in general and problem-solving in particular. There could be a risk of them developing negative feelings towards mathematics based on struggling with demanding problem-solving tasks. Accordingly, we focused on how the students themselves described their feelings when working with demanding problem-solving tasks.

As mentioned, students' feelings have shown to have influence on the process of solving non-routine mathematical tasks (Hannula, 2016), on students' views of themselves as learners of mathematics (Boaler, 2016; Clements & Sarama, 2016), and on their views of themselves as learners of other subjects (Clements & Sarama, 2016). The students' evaluations show that struggling with problem-solving tasks was mainly described as positive (happy mouth) by the students. Thus, demanding and difficult can be fun, and actually, the students' evaluations show that even tasks that one does not understand can be perceived as accessible. Very few of the students evaluated the tasks as a negative experience, by drawing a sad face. Further, the students' evaluations indicate that initial failure with a task or working for a long time with the same task did not prevent the students from evaluating the task as very easy and fun. Thus, a reduction of "the problem" in problem-solving tasks is not necessary to enable students to enjoy mathematics and problem-solving, and working on problem-solving tasks may positively influence students' interest in mathematics.

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Chapter 8
Beliefs and Values in Upper Secondary School Students' Mathematical Reasoning

Åke Hestner and Lovisa Sumpter

Abstract This study focuses on upper secondary school students' mathematical reasoning when in pairs solving a task where values are part of the context. In particular, the focus is on arguments for decisions students put forward during their solution attempts and explanations and descriptions in stimulated recall interviews. Three themes of beliefs were identified: expectations, motivation, and emotions. Similar expectations were indicated as in previous studies (e.g. there should be an algorithm to solve the task). The main differences found were about motivation and emotion. Here, the students were more positive compared to previous studies saying such types of mathematical problems including values add a new dimension to problem-solving.

8.1 Introduction

Previous research has stressed the important role of beliefs when students are trying to solve mathematical tasks (e.g. Lester, Garofalo, & Kroll, 1989; Philippou & Christou, 1998; Schoenfeld, 1992). For instance, students can be constrained by their beliefs (Schoenfeld, 1992; Wong, Marton, Wong, & Lam, 2002), such that students would continue with unsuccessful strategies when working with non-routine tasks based on the idea that certain tasks are connected to certain algorithms (Lerch, 2004). But studies also show that beliefs can assist a student to be persistent (Carlson, 1999), or that a student who express confidence and control is more likely to continue and therefore succeed (Hannula, 2006). Previous studies have looked at different types of beliefs that was indicated in student's arguments for the choices made when solving different types of mathematical tasks (Jäder, Sidenvall, & Sumpter, 2017; Sumpter, 2013). Independent of the task, a routine task or a non-routine task, similar themes of beliefs were indicated: expectations, motivations, and one specific emotional belief, security. Also, the indicated

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beliefs seemed to interplay and this in a negative way (e.g. the only way for me to solve this task is to find the algorithm since that is the safest way). In addition, the students tried to solve the tasks with imitative reasoning, this independent if it was a routine task or a non-routine task. Such behaviour seems to reoccur in different countries (Dìaz-Obando, Plasencia-Cruz, & Solano-Alvarado, 2003; Furinghetti & Morselli, 2009), even though one could argue that beliefs are contextually bound (Francisco, 2013): students expect mathematical tasks in school to be solvable by memorised algorithms.

In this present study, we would like to study upper secondary school students when working with a task very different from what normally can be found in textbooks and other materials. We will use values as a mean to see if different themes of beliefs can be generated or different variations of beliefs within these themes. The research questions posed are as follows: (1) What characterises the reasoning students use when solving a problem-solving task addressing societal values? and (2) what are the indicated beliefs in students' mathematical reasoning when solving such a task?

8.2 Background

Beliefs research has provided several definitions of beliefs (Furinghetti & Pehkonen, 2002), which means that there is no consensus. Instead, depending on the research question different theoretical frameworks will provide different sets of analytical tools. This study builds upon previous research and therefore the same definition will be used. Beliefs are here defined as "an individual's understandings that shape the ways that the individual conceptualizes and engages in mathematical behaviour generating and appearing as thoughts in mind." (Sumpter, 2013, p. 1118). In order to study students' beliefs but also acknowledging that beliefs are attributed (Speer, 2005), we use *Beliefs Indications* (BI). BI is "a theoretical concept and part of a model aiming to describe a specific phenomenon, i.e. the type of arguments given by students when solving school tasks in a lab setting." (Sumpter, 2013, p. 1116). We would here like to extend this definition of BI to go beyond "arguments given in task solving situations" and therefore we propose BI to include arguments and explanations given by students in other situations too such as interview sessions. We argue that although beliefs are here seen as something indicated, the results are interesting if they can help predict and explain behaviour.

In the present study, values will be used as a tool to possibly generate different set of indicated beliefs about problem-solving. Values can be seen as mediated from beliefs and attitudes and are expressed when an individual is doing active choices between different alternative with different values attached to them (Clarkson, Bishop, FitzSimons, & Seah, 2000). The choices are related to something being right or wrong (McLeod, 1992), and often closely related to motivation (Hannula, 2012). Although values are part of both the individual actors within the educational system (e.g. a student or a teacher), they can also be manifested in other ways (Bishop, 2012). We will here adopt the values written in the political texts that govern Swedish mathematics education. In the curriculum for upper secondary school, the first chapter is called "Fundamental values and tasks of the school" stipulating that:

> The national school system is based on democratic foundations. The Education Act (2010:800) stipulates that education in the school system aims at students acquiring and developing knowledge and values. It should promote the development and learning of students, and a lifelong desire to learn. Education should impart and establish respect for human rights and the fundamental democratic values on which Swedish society is based. (Skolverket, 2013, p. 4)

These values are listed further down in the texts such as equality and equity. In the syllabus for mathematics, we can read that regarding problem-solving the students should be able to use mathematical models to solve problems concerning both situations connected to future potential profession but also everyday life. This is here linked to the ability to follow and perform mathematical reasoning.

In most reasoning research, reasoning is thought of as high quality thinking but is seldom defined (Lithner, 2008; Sumpter, 2013). Since we would like to talk about reasoning that includes non-mathematical arguments, we need to use a broad definition that goes beyond logical thinking and therefore reasoning is defined as:

> […] the line of thought adopted to produce assertions and reach conclusions in task-solving. It is not necessarily based on formal logic, thus not restricted to proof, and may even be incorrect as long as there are some kind of sensible (to the reasoner) reasons backing it. (Lithner, 2008, p. 257)

Reasoning is a sequence, correct or incorrect, that starts with a task and results with a conclusion including, potentially, the result "no conclusion". We see this sequence having the following four steps (Lithner, 2008): (1) A (sub-)task is met, which is denoted task situation (TS); (2) A strategy choice (SC) is made where "choice" is seen in a wide sense (choose, recall, construct, discover, guess, etc.); (3) The strategy is implemented (SI); and, (4) A conclusion (C) is obtained. The characterisation of reasoning types is the results of the analysis of explicit or implicit arguments for strategy choice, implementation (Lithner, 2008) and conclusion (Hedefalk & Sumpter, 2017). There are two main categories of reasoning: Imitative Reasoning (IR) which means that the task solver applies a recalled or externally provided solution method, and Creative Mathematical founded Reasoning (CMR) where a solution method is constructed by the solver (Lithner, 2008). Dependent how the solver express different types of arguments for the solution, the CMR can be identified as Local CMR or Global CMR where the latter includes verification and control for the whole task situation. IR is a family of different types of reasoning (see Bergqvist, Lithner, and Sumpter (2008) and Lithner (2008) for a longer description and discussion).

8.3 Methods

The task was designed with three aspects in mind. First, the task aims to stimulate CMR, hence a non-routine task/problem-solving task. Secondly, the mathematical resources needed to be an area that has already been covered in the first course of upper secondary mathematics and most likely also at lower secondary level so that the chosen students should be able to solve the task although not know a certain procedure and/or algorithm. Here, the mathematical properties required to solve the task, besides problem-solving, were proportional reasoning and the understanding of natural numbers and divisors of such. Thirdly, we wanted to tap into values. In this case, we decided upon a realistic problem (c.f. Gutstein, 2006). The students were given a statement from an official source, UN-UNDP, which states how money is distributed between individuals: the richest one fifth of the world population shares 75% of the world's resources and the poorest one-fifth shares 2% of the resources. The students were asked to describe this with numbers where they were able to choose any optional number of people and things. They were also encouraged to draw a picture. As a second step, the students were asked to formulate a statement of a distribution they would prefer to see.

Three pairs of students participated, all from an upper secondary school studying a programme with a medium-intensity mathematical course, the Arts Programme. It is a higher education preparatory programme, although only taking one course in mathematics called Mathematics 1b. In this sense, it is not a mathematically intense programme. The decision to allow students working in pairs was made since it worked well in previous studies in order to stimulate the mathematical talk during the problem-solving session (e.g. Jäder et al., 2017); this is compared to when the students were alone in the lab setting (e.g. Sumpter, 2013). The analysis focus on the arguments for the decisions that were made during the problem-solving sessions and therefore the students were filmed in a lab-setting, but the films were also used for stimulated recalled interviews. Each couple worked for about 50 min. A few days later, an interview was made with each of the students individually (about 20 min). In this study, we follow the ethical guidelines and rules given by CODEX. A fourth couple was first asked to participate, but decided to withdraw before the data collection. A decision was made to not replace this couple since the data from the first three couples were rich.

The data both from the problem-solving sessions and the interviews were transcribed, and a description and an interpretation of the problem-solving sessions were made. The task situations (TSs) were identified using an appropriate grain size (c.f. Bergqvist et al., 2008). For each sequence starting with a TS, the central decisions were identified together with the argumentation for these decisions. To be able to identify, analyse and report patterns within the data, we used thematic analysis (c.f. Braun & Clarke, 2006), where the focus was on Beliefs Indication (BI). BI:s could be explicit statements in the transcripts but also hidden in students' behaviour (for a longer discussion about BI, see Jäder et al. (2017) or Sumpter (2013)). The analysis was made by the first author using the second author as a validator of the analysis. A third person functions as an additional validator when needed. Passages when the BI was not clear were left out. The three themes of BI:s from Sumpter (2013), security, motivation and expectation, were used as a basis for deductive analysis, but the analysis was also inductive in order to explore new themes. The themes were checked against each other and back to the original data, this since themes had to "cohere together meaningfully, while there should be clear and identifiable distinctions between themes" (Braun & Clarke, 2006, p. 91). As a last step of the analysis, the reasoning for each TS was analysed using Lithner's (2008) framework. Here, just as

Jäder et al. (2017) the analysis only aimed to characterise the TS using the main type of reasoning, that is, CMR and IR. But compared to Jäder et al. (2017), we also study whether the IR and/or CMR were local (i.e. a separate sequence or global).

All together, we employed the same analysis structure as Sumpter (2013): (1) BI is used as a general initial coding scheme; (2) the four steps of reasoning function as a representational scheme; and, (3) two tools for conducting two different types of analyses (i.e. thematic analysis of BI:s and Lithner's (2008) framework about reasoning).

8.4 Results

Here, we will present some of the results of the analysis. First, we have the results of the analysis of the reasoning, see Table 8.1:

As we can see in Table 8.1, all three couples attempted to use CMR when solving the task: they did not only try IR. However, the three couples differ in the global strategy: A and C looked for algorithms, although producing local CMR, whereas B used CMR as a global strategy as well after starting with IR.

All three pairs started with IR and all three pairs also expressed arguments which could indicate a belief about expectation: there should be an algorithm. This is here exemplified with couple B:

Student 1:	23 times… there is probably a really super clever way of calculating this
Student 2:	Yes, but if we…
Student 1:	Or we can just take 23 times 6
Student 2:	But that can be…?
Student 1:	But does everyone has to have the same? Why should it be equal, [I] think that is stupid. You know…
Student 2:	But if we write a… you know, an equation
Student 1:	Yes
Student 2:	That must be the easiest way to solve it

Although this pair expressed expectations in line with "there should be an algorithm", here a "super clever way", the pair did not stay in the search. Instead, the mathematical talk moved to the context and about values. This could be contrasted with student 1 from couple C who explained the strategy choice in the interview:

Table 8.1 Different types of reasoning used by students when solving the given task

Student pair	IR	CMR	Global CMR	Arrive with correct conclusion
A	Yes	Yes	No	No
B	Initially	Yes	Yes	Yes
C	Yes	Yes	No	No

Table 8.2 Sub-themes of BI connected to emotions regarding mathematical task; *n*

Theme (*n*)	Example
The problem was fun because it was open (3)	I thought it was fun… It was like, you know, as long as you were in the boundaries. A bit like that, that's how it felt. But it was almost more fun 'cause… uhm…. [You] Could do a bit what you wanted [Interview Student 2; couple B]
Creative problems are fun and inspiring (1)	You have to be creative too… like if you… [laughter] you could something fun of it, like… we, I could sit [working] forever [Interview Student 2; couple B]
Fun task within range (1)	It was mainly a fun task, I reckon. Not too easy and not too difficult [Interview Student 1; couple C]
It is important it is fun (1)	It was fun… 'cause that is important [Interview Student 2; couple B]
To work with problem-solving is fun (1)	Like, I think it is fun with problem-solving in general, I think it is fun [Interview Student 1; couple B]

Table 8.3 Sub-themes of motivational BI; *n*

Theme (*n*)	Example
You can learn about social issues at the same time you learn mathematics (3)	Yes, it is like good… you remember it… and you get a picture of [things]… how… you learn something while you learn… calculating in math and then you get it double [knowledge] it feels like. It feels good anyway. You remember it [Int K Par A]
A realistic problem (with source) is motivational (3)	I think it is pretty… fun because you can see it like… a small number… can be about the whole world… that you can calculate and write [about] and so… when you solve the problem [Int A Par A]
The context of the task can encourage learning in other subjects/social science (4)	But then I think it will be easier in life… if you talk about other subjects… that if you have it in other subjects you can get a perception in/of mathematics and then [it is] easier in social science when you talk about the world. So when you have the next subject [in line] you can make connections… how everything is related [Int W Par A]

> Well… there is always a trick, surely, because there always is one, but… the only thing is to remember it. But I don't know... uhm… I got stuck concerning that everyone should have the same amount. Because that is not how it always is. [Interview Student 1; couple C]

However, also mentioning the context of the task, the values, the expectation of "the trick", the expected algorithm, and to try to remember which "trick" it should be was stressed by these students.

If we instead look at the BI:s connected to emotions, it is in the interview responses we find most data. We will here focus on the BI expressed about the mathematical task and problem-solving, see Table 8.2:

In Table 8.2, we see that the most common emotion is fun: no other emotions were mentioned regarding the nature of the task. In this section, we also find some motivational beliefs (e.g. "It is important it is fun" and "To work with problem-solving is fun"). Looking at motivational beliefs, the results indicated three sub-themes, see Table 8.3:

The three sub-themes mainly concern how bringing in social aspects into mathematical tasks and connect it to reality means you learn about other thing while learning mathematics. This is, by the students, considered helpful regarding motivation.

As a summary, our analysis resulted in three themes of beliefs, expectations, motivations and emotions, where the latter two are connected through positive emotions. Expectations were mainly about the strategy choice: “finding the right algorithm”.

8.5 Discussion

Looking at reasoning, compared to previous studies (Bergqvist et al., 2008;Jäder et al., 2017; Sumpter, 2013), in the present study there were more instances of CMR. One pair even had a global approach in their solution attempt. But it is in the analysis of strategy choices in relation to BI:s, we see some different patterns which might be due to the nature of the task. Although all three couples expressed an expectation that mathematical tasks in school should be solvable by memorised algorithms (c.f. Dìaz-Obando et al., 2003; Furinghetti & Morselli, 2009; Jäder et al., 2017, Sumpter, 2013), one couple had a different approach: global CMR. They did not get stuck in the search of “the algorithm” (c.f. Lerch, 2004). Whether this is indeed a result of the nature of the task or due to other causes such as previous training in problem-solving, we can only speculate. We suggest that this needs to be explored and confirmed by further research. However, one conclusion is, just as Jäder et al. (2017), that it is not enough just to give students a non-routine task and leave it to them to try to learn mathematical problem-solving and mathematical reasoning skills.

Regarding BI:s about motivation and emotion, we can see a few differences compared to what was observed in Jäder et al. (2017) and Sumpter (2013). If we focus on BI:s with emotional attributions, here they were more positive (e.g. students talking primarily about “fun”). In previous studies (Jäder et al., 2017; Sumpter, 2013), the emotions mentioned were more about what was considered safe, and more specific, to choose algorithms that feel safe. Here, the emotional BI:s to some degree also encompassed motivational beliefs signalling a relationship between these two affective factors (c.f. Hannula, 2012). Compared to Jäder et al. (2017) and Sumpter (2013), the motivational BI:s were also more positive. In this sense, the students participating in this study, although not all three were successful in their solution attempts, appeared as constrained in their task solving at least from emotional and motivational point of view (c.f. Sumpter, 2013; Wong et al., 2002). We did not observe the same negative interplay. One possible conclusion is that the proposed task does generate different types of indicated beliefs, but expectations appear more strongly held compared to the other two themes. This also needs to be further investigated. An implication is that in mathematics teaching focusing on problem-solving is not enough to give non-routine tasks without being aware of, and potentially addressing, affective factors.

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Chapter 9
Attributional Beliefs During Problem-Solving

Thomas Gawlick

Abstract Drawing on research on attributional and efficacy beliefs, we sketch the development of a category system to investigate their influence on effort and outcome in "think aloud" problem-solving processes. Anchor examples from our sample suggest an influence of attribution styles (mastery vs. self-worth orientation, learned helplessness).

Identifying predictors of students' academic success is an ongoing issue of educational research to. The role of beliefs, (e.g. causal attributions of success or failure) is under scrutiny since the 1970s, with interest renewed by cross-national achievement differences in studies like PISA. The recent result that students' attribution style explains up to 8% of the national variance in PISA mathematics scores (Kozina and Mlekuž, Šolsko Polje 25:101–120, 2014) indicates their predictive relevance. However, there is a lack of studies that directly investigate how attributions influence effort and outcome during task processing.

9.1 Theoretical Framework

9.1.1 Problem-Solving

A mathematical problem is a task for which one lacks "ready access to a solution schema" (Schoenfeld, 1985, 74), hence the transformation from the given state to the goal state is hindered by a barrier (Dörner, 1976, 10). In order to overcome the barrier, the solver has to "combine previously known data in a way that is new (to him)" (Pehkonen 2004, 55) by making use of suitable heuristic and self-regulatory activities.

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9.1.2 Beliefs

Various conceptualizations of belief are extant in the literature. Some researchers (e.g. Grigutsch, Raatz, & Törner, 1998), view beliefs as a kind of attitudes, others (e.g. Griffin & Ohlsson, 2001), distinguish both: "Whereas attitudes refer to subjective evaluations of objects as "positive" or "negative", beliefs refer to the acceptance or rejection of propositions." This view will suit our purpose best. Following Kloosterman (1996), one may distinguish beliefs about mathematics (K1) and beliefs about learning mathematics (K2), which can be differentiated into three subcategories: beliefs about oneself as a learner of mathematics (K21); beliefs about the role of the teacher (K22), and other beliefs about learning mathematics (K23).

9.1.3 Beliefs in Problem-Solving

Schoenfeld (1985) posits that success or failure in problem-solving is determined by four variables: knowledge, heuristic strategies, self-regulation and belief-system ("one's mathematical world view") of the solver. Schoenfeld (1985, 1992) exhibits some typical counterproductive beliefs influencing students' problem-solving behaviour, as became apparent by analyzing verbal protocols, classroom observations and students' questionnaires. Despite the seminal role of Schoenfeld (1985), there seem to be only few studies directly investigating the role of beliefs in problem-solving, and most of them are from general education research. One of the exceptions is the study by Kloosterman and Gorman (1990) who found that by the middle grades, many students begin to perceive mathematics as a domain in which smart students succeed and other students merely "get by" or fail. They begin to believe that success and failure are attributable to ability and that effort rarely results in a significant change in their success patterns. This deserves further study, but according to Kloosterman (2002, 248), motivational theories like Weiner's attribution theory and Bandura's self-efficacy theory have rarely been applied to mathematics education.

9.1.4 Attribution Theory

As the title suggests, we conceptualize attributions as beliefs about the causes of success and failure. As far as learning is concerned, they mostly fall into Kloosterman's category (K21). Weiner's theory of attributions deals with individuals' causal interpretations of events and their effect on thinking and behaviour. Weiner (1985) distinguishes causal factors for one's success or failure by three causal dimensions:

1. Locus of causality (external versus internal);

2. Stability (stable versus unstable);
3. Controllability (controllable versus uncontrollable).

These causal dimensions influence outcome expectancy and thence actual behaviour. According to Weiner, the stability dimension is most closely related to expectancy for success. Esteem related affects are associated with the locus dimension, social related affects to the controllability dimensions (see Table 9.1).

Weiner (1985) posits that people use situational cues to form attributions: Cues for ability are ease, speed or frequency of success; a cue for effort is mental exertion, cues for the difficulty of a task are its features; cues for luck are outcomes that are random and lack relation to effort (Schunk & Zimmerman, 2006, 355). Note that attributions are causes ascribed by the individual and may differ from real causes. Also, the dimensionality may be viewed differently, (e.g. task difficulty may be construed as externally controllable by the teacher). But according to Pintrich and Schunk (2002), the accuracy of an attribution is not important for it having behavioural consequences.

The attribution literature is replete with studies on the relationship between students' attributions and their achievement, especially in mathematics. In particular, Georgiou (1999) investigated the relationship between sixth-graders' performance attributions and attainment in mathematics. Internal attributions (to effort or to ability) correlated positively to achievement, whilst external attributions (to luck and to circumstances) correlated negatively to achievement. Furthermore, according to Weiner's theory, attributions to unstable, controllable causes such as effort increase motivation and perseverance, whilst attributions to stable, uncontrollable causes such as ability weakens motivation and may finally lead to *learned helplessness*: This denotes the belief that one's situation cannot be altered by conscious effort, due to inadequate earlier reinforcement of such effort. Hence, helpless students show performance decrements under failure, whereas mastery-oriented students tend to enhance performance. By analyzing the verbalizations of children who were failing on a cognitive task while thinking loud (cf. Table 9.2), Diener and Dweck (1978) found that helpless children attributed failure to lack of ability, whereas mastery-oriented children made only few attributions but engaged more in self-monitoring and self-instructions. This supports the view of attribution theory that learned

Table 9.1 Classification scheme for causal attributions after Weiner (1985)

Attributions		Dimensions	
Attribution	Locus	Stability	Controllability
Ability	Internal	Stable	Uncontrollable
Effort	Internal	Unstable	Controllable
Strategy	Internal	Unstable	Controllable
Interest	Internal	Unstable	Controllable
Task difficulty	External	Stable	Uncontrollable
Luck	External	Unstable	Uncontrollable
Family influence	External	Stable	Uncontrollable
Teacher influence	External	Stable	Uncontrollable

Table 9.2 Verbalization categories of Diener and Dweck (1978, 455)

1.	*Statements of useful task strategy*. These were statements of a plan or system that under normal conditions would eventually lead to a solution
2.	*Statements of ineffectual approach to task*
3.	*Attributions*, especially to lack of ability (e.g. not having a good memory) or loss of ability (e.g. inability to think)
4.	*Self-instructions*. These statements referred to instructions the child gave to him/herself that, if followed, would improve performance
5.	*Self-monitoring*. Statements concerning the child's solution-oriented behaviour other than task strategy, such as monitoring his or her own effort expenditure or concentration
6.	*Statements of positive affect*. These indicate that the task was enjoyable or a challenge
7.	*Statements of negative affect*. This category included statements that indicated boredom, anxiety, or a desire to terminate the task or to escape from the situation
8.	*Positive prognostic statements*. These express a child's high expectancy of success or indicating a belief that he or she would solve the problem if given sufficient opportunity
9.	*Solution-irrelevant statements*

Antecedent Conditions →	**Perceived Causes** →	**Causal Dimensions** →	**Psychological Consequences** →	**Behavioural Consequences**
Environmental factors	*Attributions for*			
Specific information	Ability			
Social norms	Effort	Stability	Expectancy for success	Choice
Situational features	Luck			Persistence
	Task difficulty	Locus	Self-efficacy	Level of effort
Personal factors	Teacher			Achievement
Causal schemas	Mood	Control	Affect	
Attributional bias	Health			
Prior knowledge	Fatigue, etc.			
Individual differences				

Fig. 9.1 Overview of the internal attributional model of Weiner (1985)

helplessness results from a lack of successes, thus failure is attributed to lack of ability. Consequently, success is viewed as unattainable and the level of effort reduced—a vicious cycle. This exemplifies Weiner's view (Fig. 9.1) on how attributions effect behavioural consequences. Even so, further studies (e.g. Relich, 1984) show that their influence is mediated by self-efficacy.

9.1.5 Self-Efficacy

In social cognitive theory, *self-efficacy* is defined as the belief in one's ability to succeed in specific situations or accomplish a task (Bandura 1986, 391). Perceived self-efficacy is seen as affecting behaviour by influencing the choice of activities as well as raising the expenditure of effort and the persistence in case of difficulties (Bandura 1986). The intricate interplay of self-efficacy, attributional beliefs and achievement has been disputed in the literature, especially concerning the direction of causality. Schunk and Gunn (1986) investigated the relation between achievement, success attributions and self-efficacy and showed that children who attributed success to ability showed enhanced perceptions of self-efficacy, which in turn correlated to higher achievement. Roeser, Midgley, and Urdan (1996) found by sequential regression analyses that perceiving a task goal structure in middle school was positively related to academic self-efficacy and that this relation was mediated through personal task goals.

Whilst attributions refer to *past* performance, self-efficacy estimate *future* performance. Fig. 9.1 shows its place in the cyclic interplay between attribution and behaviour. (Weiner's original model contains instead the less specific concept of self-esteem.) Note that the situational specificity of self-efficacy beliefs is decisive for their mediating role: Whether the present task is construed as similar to a previous one interacts with the estimation whether the certainty to accomplish it is comparable.

9.1.6 Self-Regulation

From the plethora of approaches we choose one that provides a frame to investigate the interplay of regulation strategies, beliefs, problem-solving effort and outcome: Zimmerman and Campillo (2003) analyzed how motivation and personal resourcefulness influence problem-solving. By *self-regulation* they denote self-generated thoughts, feelings, and actions that are planned and cyclically adapted to attain a goal. These activities can be subsumed under three phases (Fig. 9.2): "Forethought processes precede efforts to solve a problem and set the stage for it. Performance phase processes occur during solution efforts and influence attention and action, and self-reflection processes occur after solution performance efforts and influence a person's response to them. These self-reflections, in turn, influence forethought

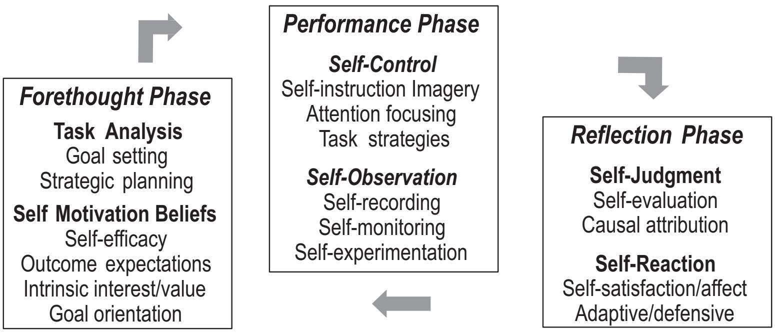

Fig. 9.2 Phases and subprocesses of self-regulation (Zimmerman & Campillo, 2003)

regarding subsequent solution efforts, thus completing a self-regulatory cycle." (ibid, p. 239). Self-efficacy, self-instruction and attributions can be distinguished by the phase in which they occur.

9.2 The Study

9.2.1 Research Questions

Studies about beliefs in problem-solving generally aim to determine beliefs by means of questionnaires and to examine their dependence on covariates and their change after time or intervention (Kloosterman & Stage, 1992). These methods are economic, but apt to various kinds of response bias. Most notably it is an ongoing issue how accurate stated beliefs fit to actual beliefs and to performance in task processing. An exception is Schoenfeld (1985), but unfortunately he does not detail how he derived the reported beliefs from the analyzed problem-solving protocols. Hence it might be worthwhile to find indicators for beliefs directly in problem-solving processes, in which subjects are prompted to "think aloud" in order to elicit belief verbalizations. Based on the considerations in our theoretical framework we set out to investigate:

(a) Can problem-solving protocols be parsed into categories in such a way that indicators for attributional and efficacy beliefs can be found in students' verbalizations?
(b) What is their possible influence on effort, persistence and outcome?

9.2.2 Method

Qualitative Content Analysis (QCA) provides several procedures to methodically categorize text by content-based rules, from which we chose deductive category assignment (Mayring, 2000). After defining theory-based *structuring dimensions*, one has to split them into categories and define coding-rules to ensure the concordance between theoretical concepts and their intended realizations in the data. In the pilot phase, the rules are applied to a sample of the data and refined if necessary to ensure unambiguous category assignments. The revised system of categories, rules and examples is fixed in coding guidelines and then applied to the whole corpus of data.

9.2.3 Data

This study and the conceptual framework pertaining to it emerged from project HeuRekAP (dynamische-geometrie.de/heuristik/HeuReKaP/index.htm), in which we are currently engaged in investigating the efficacy of a problem-solving training

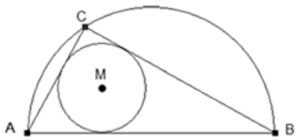

Fig. 9.3 Opened-up version of item K10 from the 1985 TIMS study

based on heuristically reconstructed worksheets. To evaluate its outcome, we administered an opened-up version of the item K10 from the 1985 TIMS study (cf. Fig. 9.3) and obtained 119 written solutions by ninth graders from one grammar school (Gymnasium), half of which obtained our training beforehand. Three months later, 46 of them solved K10 again, thinking aloud, which we taped, transcribed and analyzed (Gawlick & Lucyga, 2016). The current study is a reanalysis of this data.

9.3 Results

To answer question (a), we set out to obtain a suitable category system by refining the verbalization categories of Diener and Dweck (1978), cf. Table 9.2. They were "derived from the data by the authors" (ibid, 455), who unfortunately did not further detail their approach. To address (a), we thus adopted coding rules and anchor examples from the literature cited above for the pertinent categories according to Zimmerman's process model, cf. Fig. 9.2. The resulting system is illustrated by examples below. (For the sake of brevity, we give examples of indicators for beliefs and likewise for their possible influence, thereby addressing also question (b).)

Categories (6) and (7) were omitted since they did not occur in the coded material, category (8) was amended, since it occurred repeatedly. Category (9) was replaced by negative prognostic statements, which are specified as respective counterparts of (8) (Table 9.3).

The resulting coding system seems apt to tackle our research questions: The codes for causal attributions in (3) identify episodes in the process, where previously created beliefs possibly influence students' behaviour in the protocols. Drawing on Zimmerman's phases of self-regulation (cf. Fig. 9.2), we elaborated the further categories to methodically address the question whether subsequent behaviour in the performance phase (parsed as (1) or (2)) is consistent with or made plausible by the assumed attributions (coded by (3)), as brought to effect by self-instructions (4) and self-monitoring (5). The presence or absence of codes for self-efficacy from (8) and (9) may shed some light on its mediating role. These mechanisms of action are already present in Fig. 9.1. By use of the new category (10), this model could be augmented by the forming and/or fostering of attributional and efficacy beliefs during the reflection phase of Fig. 9.2.

Note that in applying the system, we found relatively few *direct* causal attributions. This is not surprising since students directed their attention towards the problem at hand. To adapt our coding system to this circumstance, we augmented the coding rules as to provide for *indirect* indicators. This is explained below by means of examples.

Table 9.3 Refined verbalization categories

1 and 2. *(In)effectual approach to task*: Specification derived from task analysis and related to students' problem-solving processes as in the study by Gawlick and Lucyga (2015)
3. *Attributions*: Subcategories according to Weiner's classification scheme (cf. Table 9.1), operationalized utilizing the situational cues of Schunk and Zimmerman (2006)
4. *Self-instructions*: Pre-actional statements "overtly or covertly describing how to proceed as one executes a task" (Zimmerman & Campillo, 2003, 242)
5. *Self-monitoring*: Post-actional statements "to judge the adequacy of one's solution efforts" (Zimmerman & Campillo, 2003, 243)
8. *Positive prognostic statements*: Anchor examples adapt the statements from the Academic Self-Efficacy Scale (Roeser et al., 1996) to solving problem tasks:
– I'm certain I can master the upcoming scholastic tasks
– I can do even the hardest scholastic tasks at school if I try
– If I have enough time, I can do a good job on all the problem tasks in school
– I can do almost all the problem tasks in school if I don't give up
– Even if the problem tasks in school are hard, I can learn how to solve them
– I'm certain I can figure out how to do the most difficult scholastic tasks
9. *Negative prognostic statements*: As above
10. S*elf-evaluation*: Statements "comparing self-monitored outcomes with a standard or goal" (Zimmerman & Campillo, 2003, 243)

9.3.1 The Case of C21: Attribution, Task-Strategy, Self-Monitoring and -Evaluation

After reading the task, C21 makes clear that it is known to him in process line no. 5:

5	C21:	I think about the, I think it's called Pythagorean theorem… ahem was it… Yes, but I also think we had this in a test and I didn't process the task (*smiles*) because I was unable to

Several verbalization categories apply to different parts of this line: Student C21 first mobilizes a helpful theorem (later stated correctly), which is an example for category (1). He ponders whether the theorem he has in mind is really Pythagoraean theorem, thereby exemplifying *self-monitoring* (5). Then C21 remembers his failure in a previous attempt to solve K10 and *attributes* this *to a lack of ability* (3). A possible influence of this attribution on his process is the repeated occurrence of hesitancy in statements of category (5), like in line no. 22:

22	C21:	I read again (4)… I have the feeling to overlook something simple (5)… something I could actually handle easily, but I don't know what (5)

The penultimate clause also gives a hint towards C21' perceived locus of causality: The "simple" is elaborated by him as "something I could actually handle easily", so its simplicity is rooted in himself—since ease is a situational cue for ability according to Weiner, it could indicate an *ability attribution* (3).

Despite his difficulties C21 develops a *useful task strategy* (1) “that under normal conditions would eventually lead to a solution” (cf. Fig. 9.3):

25	C21:	Ahem if C is 90° I ponder if I could somehow calculate the angles at A and B… I don’t know… I think… perhaps I should look first how exactly I have to halve the angles to obtain here ahem the angles of AMB and then I knew automatically the angle at M
26	C21:	But I have no idea how to calculate them

This *self-monitoring* (5) boils down to what makes K10 a problem: In routine tasks the angles at A and B would be calculated to determine $\mu = \angle AMB$, but here they vary! This the *α-β-barrier* that occurs in many K10 processes. Though C21 finally fails to overcome it, he tries to for further 27 lines, but then resorts to measure angles, though reckoning that thereby μ is not determined correctly (*self-evaluation*) in line no. 53.

9.3.2 The Case of C01: Attributional Indicators from Self-Instruction and Self-Monitoring

Here we find no direct hints towards attributions, but can infer their possible direction: *All* of C01’s self-instructions and self-monitorings are concerned with help. For brevity’s sake, we just give the code numbers of attributional verbalizations in brackets:

2	C01:	Okay, well I think at first I consider what theorems could help me (4)
18	C01:	Now I draw some angle bisectors, perhaps that helps me *draws it* (1,5)
27	C01:	This triangle contains no 90° angle so that doesn’t help me along now (1,5)
28	C01:	(*Looks questioningly at the interviewer. Silence*)
29	C01:	But there’s just no help to find it out so it’s a bit difficult (5)

C01’s attention is focused on mobilizing help, shifting from the figure to the interviewer—this indirectly indicates a possibly previous *external attribution* (3): It seems that for C01success in a task depends on whether help is sufficiently available. Help is obviously something external (in the task or in her counterpart) that is unstable and only externally controllable. So after her tacit appeal does not elicit any help from the interviewer, she contents herself to suppose an improbable derivation of μ from the only mobilized help (Thales’ theorem): In 34, it occurs to here that μ might be 45°, since γ is 90° and it might be the half of it. She decides to stick to that after considering other angles in vein and finishes the task. (During stimulated recall, she recognizes that this cannot be true since μ “is much more ample than 90°.”) We may hypothesize that C01’s line of thought stems from the belief that her effort or ability does not suffice to solve tasks on her own, so she needs support in the instruction and from others. This may be due to previous failure attributions and hence contributes to her relative underperformance—another rotation of the above vicious cycle.

9.3.3 The Case of A25: Effort and Perseverance Due to Internal Attribution?

The process of A25, is one the longest in the sample: it lasts over 167 lines. A25 finally manages to overcome the α-β-barrier as one of few students. His efforts are accompanied by eight positive and nine negative statements that might fit into our belief categories:

33	A25:	I'm just trying to find any solution… any solution possibility
72	A25:	After all, one can make it
75	A25:	There must be a solution, but where. Somewhere one must make progress
85	A25:	An arbitrary point (*points to C*) one can go on working from the 90°
97	A25:	How to make progress?
99	A25:	45 (*points to the bisected angle at C*)
107	A25:	How to determine α? One does not accomplish β either
119	A25:	How only to accomplish something like that?
134	A25:	α and β together would yield 90° and how does that help me on?
136	A25:	How does one accomplish to make progress? 90° … α and β together

Some of these verbalizations are difficultly categorizable: 97 may be (4), the negative part of 107 might count as (9), 85 as (5) or (8), but what about the rest? Yet it is noteworthy that with one exception (134), these statements all focus on "make it" [es schaffen] rather than "can do it" [es können] or on "trying". That this wording remarkably coincides with the effort and perseverance displayed by A25 gives rise to propose an extension of the theoretical framework: These statements can be construed as instances of a new category that may be called "*attribution-in-action*"; like attributions, they relate an outcome to a causal-factor, but not in *retro*spect, but *pro*spectively—so that like statements of self-efficacy, they mediate the subsequent choice of activities, but not in *forethought*, but during *performance*. A25's "attribution-in-action" is to effort—and it plausibly explains that A25 does not give up on the verge of failure (107,119), but takes pains to solve K10—until he finally makes it.

9.4 Discussion

Question (a) was answered in the affirmative: In analyzing a sample of our data, we were able to define coding-rules that are theoretically based and applicable to the data; thence we obtain indeed indicators for the presence of attributional and efficacy beliefs in problem-solving processes and can hypothesize on their possible influence on effort and outcome (see (b)). However, due to the circumstance that our study is a reanalysis of process data collected previously with a different aim, we could identify only a few direct indicators for causal attribution, but more indirect ones that we tentatively inferred from self-regulatory activities. The latter ones were more easily found in our data, and hence in a future application of our coding-system one will amend interview sections to directly survey causal attributions as in Kloosterman's study (1996). Given the issues raised by our case analysis, one will especially want to ask students:

- in *advance*: "What do you think was influential for your success or failure in previous problem-solving?" (*causal attributions*),
- before *task-processing*: "What do you think does it depend on whether you solve this problem or not?" (*self-efficacy beliefs*),
- during "*stimulated recall*": "What do you think influences at that moment whether you are going to succeed or fail?" (*attribution-in-action*),
- in *retrospect*: "What do you think has been decisive or your success or failure in solving this very problem?" (*revisiting causal attributions*).

This also underlines that how attributions-in-action distinguish themselves from self-efficacy beliefs: the former are the latter's link to past experiences, cf. Fig. 9.1. This point of view is corroborated by an interview with an experienced problem-solver who elegantly solved a Pythagoras-like task drawing on Ptolemy's theorem. Asked what let him bring this unusual theorem into play, he answered "Since I was previously successful with it in a similar situation", relating his decision to past experience all on his own. This exemplifies the rationale for our conceptualization of attribution-in-action.

Insofar our indirect indicators to causal attributions are only hypothetical, the answer to (b) from our case studies remains provisional. How does it fit to the literature? Earlier claims (cf. Diener & Dweck 1978) that attribution to effort is generally more favourable than to ability and all the more than external attribution are supported by A25 doing better than C21 and both better than C01. Since we found no hints to beliefs that could explain C21's willingness to spend that much effort despite his previous inability attribution, one may wonder if C21 (like some authors) does not view ability as a stable trait. Likewise, his attribution-in-action may differ from his stated belief, since C21 is ready to retry solving K10, otherwise he would have declined his participation in the interview study. Hence we deem it worthy to consider the newly proposed concept of attribution-in-action as further mediating factor to resolve the disputed issue of in what way the interplay of attributional and efficacy beliefs influences task performance. Especially, Galloway, Leo, Rogers, and Armstrong (1996) showed that attributional styles were closely related to students' self-efficacy. Their questionnaire analyses dovetail nicely to our case studies of problem-solving processes: That C21 does not try as hard as A25 may be due to a "*self-worth orientation"* that lets him limit his efforts lest he risks losing self-esteem (ibid, 199). In contrast, A25 may be seen as "*mastery oriented*" (ibid, 198), that is demonstrating persistence to overcome difficulties for the sake of further learning. C01 also exemplifies a well-known attribution style ("*learned helplessness*"); in addition, she illustrates that for best results, a problem-solving training should address also students' attribution style: Namely, in C01's solution attempt she mobilizes just two elements of our problem-solving training: She tries to find helpful theorems and she draws an auxiliary line (the German "Hilfslinie" literally translates to "helpful line"!), stating "perhaps that helps me". Both heuristics promise help verbatim—thus they fit nicely in her presumed belief that she needs help to succeed in solving such a task. But finally this belief turns out to be not all that helpful to her.

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Chapter 10
Evaluation of an Approach of Professional Role Reflection in Mathematics Education

Katharina Manderfeld and Hans-Stefan Siller

Abstract By giving pre-service teachers feedback based on empirical findings, it is intended to stimulate dealing with the role and tasks of a mathematics teacher and one's own professional identity. The following paper focuses on the approach and findings regarding the reflection of own beliefs, motivational orientations and self-regulation. The results show that especially in regard to beliefs towards mathematics, beliefs about teaching and learning of mathematics and self-regulation, potential critical trajectories can be found. Having a look at the impact of the feedback, the participants report a positive reinforcement concerning their desired career to become a mathematics teacher and an impact of promoting further development.

10.1 Introduction

Recent research has emphasized the necessity and relevance of reflective factors in the professionalization of teachers (e.g. Abels, 2011). At the beginning and during the course of studies, reflection can stimulate the emergence and further development of an individual's professional identity. The research field of mathematics education has paid attention to the topic of reflection for example by understanding reflection as a category of metacognitive actions (Kaune, 2006). Within this understanding, reflection is retrospective, which means that it follows a completed action, which is then the object of reflection (Kaune, 2006). This differs within our approach, because there is no explicit action to which the reflection process refers to. Instead, it refers to the beliefs pre-service teachers have regarding the profession of a mathematics teacher. They are encouraged to discuss their own professional identity concerning requirements for a mathematics teacher and to think about what it means to work as such. Korthagen (2002) mentions that reflection moments are a first step to become

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aware of mental structures, critically question and, if necessary, reconstruct them. In this research, mental structures will be operationalized by beliefs, motivational orientations and self-regulation. The participants are asked to individually work on a questionnaire, within this step the process of becoming aware of those mental structures is initiated. In a second step, the participants are confronted with empirical findings and receive feedback on the answers they gave in the questionnaire. This is meant to initiate an individual critical debate. Changing the mental structures lies beyond the possibilities of this work and should be viewed as a possible further step.

10.2 Theoretical Background

> Active, persistent and careful considerations of any belief or supposed form of knowledge in the light of the grounds that support it and the further conclusions to which it tends constitutes reflective thought (Dewey, 1933).

Dewey's definition shows that there is a general difficulty to initiate a reflective thought. Reflection is an inner process that can only be stimulated externally. It is the responsibility of the pre-service teachers to conscientiously deal with the reflection devices they are facing. But although it is difficult to initiate reflection it is nevertheless important. Frey and Haußer (1987) define identity as a dynamic construct and a self-reflective process and stress the meaning of reflection regarding (further) development of a professional identity, which can be defined as a combination of beliefs, attitudes, emotions, cognitive capacity and one's own life story (Bosse & Törner, 2015). Besides the knowledge and competences of mathematics teachers, also motivational-affective aspects frame the professional identity. This is why motivational orientations, beliefs and self-regulation are addressed within the model of teachers' professional competence by COACTIV (Kunter et al., 2013). They form and determine the own professional identity and are of particular importance for the performance of the profession (Abels, 2011).

In his definition of reflective thoughts, Dewey mentions two objects of reflection: beliefs and supposed forms of knowledge. Beliefs as they are part of the professional role reflection "are mental constructs that represent the codifications of people's experiences and understandings" (Schoenfeld, 1998). In context of mathematics education research, current definitions of beliefs focus primarily on how teachers think about the nature of mathematics and the teaching and learning of it (Aguirre & Speer, 2000). Within research, those are often associated with transmissive or constructivist orientations. A transmissive orientation regarding the nature of mathematics is indicated by beliefs, where mathematics is seen as "a given body of knowledge and standard procedures that has to be 'covered'" (Swan, 2005). Learning mathematics is from this point of view receptive, while teaching goes along with correcting misunderstandings, structuring a linear curriculum and giving explanations before dealing with problems. In contrast, seeing mathematics as a dynamic system filled with ideas and reasoning processes that are interconnected is an indicator for a constructivist orientation. From this view, learning is considered as a collaborative activity, where the teacher is a designer of a learning environment (Kunter et al., 2013; Swan, 2005). The COACTIV study reports that their participants did not refuse statements that show a transmissive orientation, although it is associated negatively with the development of performance of students in mathematics. This is why it is recommended to reduce a transmissive orientation (Kunter et al., 2013). Swan (2005) mentions transmissive orientations coming along with explanations, examples and exercises during lessons, which do not promote robust, transferrable learning that endures over time or that may be used in non-routine situations. Other studies emphasize the positive effects of strong constructivist orientation of a teacher, like a more frequent use of cognitively challenging tasks during class (e.g. Decker, Kunter, & Voss, 2015).

In addition to beliefs, the discharge of the tasks of a mathematics teacher depends also on motivational orientations and self-regulation (Kunter et al., 2013). When reflecting the professional role by pre-service teachers, motivations of becoming a teacher—especially a mathematics teacher—can be emerged. According to that, there is often the distinction between intrinsic and extrinsic motivation. Intrinsic motivated behaviours can be defined as being determined by interests without the necessity of any rewards. They are "engaged in for their own sake" (Deci, Vallerand, Pelletier, & Ryan, 1991). In contrast, extrinsic motivation can be seen in actions, which are done because of an instrumental intention, to achieve a consequence that can be separated from the action itself (Deci et al., 1991). Research has dealt with intrinsic and extrinsic motivation regarding the occupational choice a lot, but the findings were not consistent as—for example—some studies do not consistently find benefits of intrinsic motivation. Nevertheless, cross-sectional researchers have shown that a higher satisfaction with the profession and also a higher tie to the profession go along with intrinsic motivation (Kunter et al., 2013). In addition, enthusiasm for mathematics and perceived self-efficacy can be seen as important parts of the motivational orientation. The latter is defined as the individual certainty to support students learning and behaviour even if those are supposed to be "difficult" or "unmotivated" (Kunter et al., 2013).

Self-regulation as the last-mentioned point of the motivational-affective aspects is defined as the responsible dealing with own resources (Kunter et al., 2013). Personal commitment and resilience frame the centre of reflection regarding to this aspect. Although the pre-service teachers who are addressed in the reflective work are not yet working as teachers, it is assumed that they can state their behaviour due to other experiences they have gained during their education at university or daily life.

10.3 Research Questions

Working on a questionnaire and being confronted with a following feedback based on empirical findings is the approach of professional role reflection used in this research. The feedback is intended to initiate a critical debate regarding beliefs, motivational orientations and self-regulation of each pre-service teacher. In order to evaluate the benefit of the feedback the following questions need to be answered:

1. Do the results of working with the questionnaire help to find out potential manifestations of mental structures which should be critically questioned?
2. What impact of this reflective work do the participating pre-service teachers report?

Within the first question, critical trajectories or manifestations are called “potential”, because they are only recognizable throughout the answers of participants in the questionnaire. The pre-service teachers need to consider on their own whether the feedback fits to their personality or not.

10.4 Methods and Sample

The type of reflective device was defined as an online questionnaire consisting of closed questions. Because of implementing the reflective work in a lecture, a huge number of pre-service teachers (100–200) is integrated into the reflective work at the same time. This is to be seen as one reason for choosing a questionnaire as reflective device. Within this questionnaire the participants have to state their agreement or disagreement to each item by filling in a six-point Likert scale ranging from 1 = totally disagree to 6 = totally agree, except of the questions regarding self-regulation, where a five-point Likert scale was used as it is in the original scales. The questionnaire is worked on individually within a period of 10 days.

The epistemological beliefs about mathematics as well as beliefs about teaching and learning of mathematics are operationalized using scales developed within the framework of the COACTIV study (Baumert et al., 2008). Because of the fact that teachers can have both transmissive and constructivist beliefs (Decker et al., 2015), those beliefs are not conceptualized in a clear dichotomy, but rather two different scales are used. Regarding beliefs towards the nature of mathematics, the two scales “mathematics as a toolbox” and “mathematics as a process” are called upon. The first scale is matched with a transmissive whereas the latter one with a constructivist orientation (Kunter et al., 2013). Beliefs regarding teaching and learning of mathematics are operationalized with the help of five scales: “uniqueness of solution process”, “receptive learning”, “practice of technical knowledge”, “independent and insightful learning” and “trust in mathematical autonomy of students”. The first three scales are matched with the measuring of transmissive and the last two of constructivist orientation. All of the used scales regarding beliefs show an adequate internal consistency with Cronbach’s α reaching from 0.66 to 0.88.

Dealing with motivational orientations, the participants were asked to answer questions regarding their motivations to become a teacher and especially a mathematics teacher. Therefore, scales that have been adopted for the use by pre-service teachers by Cramer (2012) are used. Important for the feedback are the subscales of “intrinsic-pedagogical motivations” and “extrinsic motivations” that exist for the motivation to become a teacher in general and for choosing mathematics as a teaching subject. Also, "enthusiasm for mathematics" (Baumert et al., 2008) and "perceived self-efficacy" (Cramer, 2012) are operationalized by using existing measuring instruments. All of the subscales show an adequate internal consistency, with Cronbach's α above 0.67.

With the help of the AVEM ("Work-related behavioural and experience pattern") (Schaarschmidt & Fischer, 2008), self-regulation is operationalized by collecting self-assessments regarding 11 dimensions (see Fig. 10.1). The relations of those dimensions are expressed within four patterns of behaviour and experience (see Fig. 10.1). Again, all of the scales show an adequate internal consistency with Cronbach's α reaching from 0.65 to 0.84.

The *health-pattern* connects a high but not excessive commitment with a high resilience and a positive life satisfaction. This pattern is a hint for a healthy relation towards work. Therefore it is the most desirable pattern. The *pattern of conservation* can be hint for an obstacle during teacher training, because of the high values regarding the ability to distance oneself going along with a low subjective significance of work, professional ambition and willingness to outspend oneself. If belonging to this pattern the personal motivation should be questioned. The last two patterns are called risk-patterns, because they can be an advance warning to burn-out. *Risk type A* is related to an excessive commitment, a decreased resilience and a restricted life satisfaction. A person who can be matched to *risk type B* has an overall experience of overextension, exhaustion and resignation (Schaarschmidt & Fischer, 2008). Due to their answers, the participants are matched either sure or tending to one of these patterns. A sure match means an agreement with the pattern to more than 95%.

In a second step, the participants receive feedback to each of the aspects, including a summary of scales, a description of necessary terms as well as an explanation of criteria for critical trajectories and a table where the participants are able to see their mean agreement to each scale. Also, the advice is given, that the feedback should not be seen as the absolute truth but rather as a reflection device. According

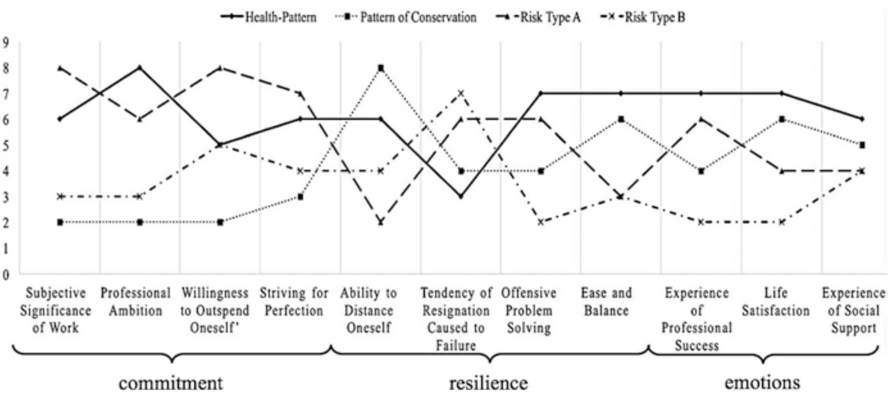

Fig. 10.1 Work-related behavioural and experience patterns (Stanine-scale) (Schaarschmidt & Fischer, 2008)

to the theoretical background, the pre-service teachers receive the feedback that it would be desirable from the research point of view to agree with a constructivist orientation and to disagree with a transmissive one. Therefore, their mean value of agreement is interpreted as a potential critical trajectory, if it is higher than the theoretical mean ($M = 3.5$) regarding transmissive orientation or lower than 3.5 regarding constructivist orientation. Regarding intrinsic-pedagogical motivations critical trajectories are marked when disagreeing ($M < 3.5$) or in the case that the agreement regarding extrinsic motivations is higher than the agreement in context of intrinsic motivations. Dealing with enthusiasm for mathematics and perceived self-efficacy, critical trajectories are mentioned in the feedback, if the pre-service teachers do not expect to be self-efficient ($M < 3.5$) or if they do not show enthusiasm for mathematics ($M < 3.5$). Except of the health-pattern, all other patterns of self-regulation are seen as potential critical trajectories, because of being an advance warning towards burn-out or because of high conservation regarding professional demands.

To get to know the impact of the feedback, a questionnaire is used which is founded within the scope of CCT (Career Counselling for Teachers) and consists of five scales of whom only three will be reported (Nieskens, 2013). The others show a low value of Cronbach's α (<0.60), so that the internal consistency of those scales should be questioned. The questionnaire comprises items regarding "an encouraging impact regarding reflection processes" ("*The feedback made me question if I do own skills that are required for teaching profession*."; Cronbach's $\alpha = 0.74$), "an encouraging impact regarding further explorations" ("*The feedback encouraged me to get further information about the teaching profession*."; Cronbach's $\alpha = 0.63$) and regarding "an impact of promoting further development" ("*The feedback showed me, which competences I can expand*."; Cronbach's $\alpha = 0.85$). Finally, regarding the impact on their desired career the pre-service teachers are asked whether the feedback did confirm them to become a mathematics teacher or not (Nieskens, 2013). Each item is to be answered by a four-point Likert scale reaching from 1 = no to 4 = yes (2 = rather no, 3 = rather yes).

The pilot study took place in April 2017, in which $N = 146$ pre-service teachers dealt with the instrument (age: $M = 22.25$, $SD = 2.38$, range = 19–32; semester: $M = 5.09$, $SD = 2.186$, sex: female = 76%). Due to the fact that all lectures of mathematics education during the bachelor program address future elementary and secondary teachers, both groups were participating in the pilot study. 54.8% of all participants want to become elementary teacher, while the rest aims to become a teacher for secondary school. $N = 49$ of the participants returned the additional questionnaire measuring the impact of the feedback. The percentage of people who returned the questionnaire and show a critical trajectory regarding one aspect is similar to the percentage of all participants showing a critical trajectory regarding this aspect. This is why in context of critical trajectories the group who returned the additional questionnaire is representative.

10.5 Results

Having a look at the results of beliefs about the nature of mathematics, the descriptive statistics show that the pre-service teachers do on average agree with all beliefs that were operationalized. The highest mean is reached by the scale dealing with "mathematics as a process" ($M = 4.88$, $SD = 0.68$), while "mathematics as a toolbox" ($M = 3.75$, $SD = 0.89$) reaches in contrast lower values of agreement. Many of the participants agreed with both transmissive and constructivist orientation regarding the nature of mathematics. Considering empirical findings mentioned in the theoretical background, 119 pre-service teachers received the feedback that they might reduce their transmissive orientation because their value of agreement was above the theoretical mean of 3.5. Three participants did not agree with the realized constructivist orientation regarding beliefs about the nature of mathematics. Within these results, there was no significant difference between the statements of future elementary or secondary teachers. Similar findings can be reported by looking at the beliefs regarding teaching and learning of mathematics. There is a general disagreement with the concept of "uniqueness of solution process" ($M = 2.92$, $SD = 1.03$), whereas means regarding "receptive learning" ($M = 3.46$, $SD = 0.75$) are close to the theoretical mean of 3.5. On average, most of the participants agree with the importance of "practicing technical knowledge" ($M = 4.20$, $SD = 0.72$) within mathematics lessons, an "independent and insightful learning" ($M = 4.85$, $SD = 0.53$), and they "trust in mathematical autonomy of students" ($M = 4.43$, $SD = 0.67$). In total, 72 participants agreed with a transmissive orientation regarding learning and teaching of mathematics and one participant disagreed with the constructivist orientation. Again, there is no significant difference between the statements of future elementary or secondary teachers. Dealing with two different overarching orientations, the constructivist and transmissive one, is stressed by having a look at the correlations. A significant ($p = 0.000$; 2-tailed) correlation with a medium effect (*Pearson* $r = 0.47$) between transmissive beliefs about the nature of mathematics and those about the teaching and learning can be found. Analogously, a significant ($p = 0.000$; 2-tailed) correlation is found between the constructivist beliefs about the nature of mathematics and those about the teaching and learning (*Pearson* $r = 0.54$).

Regarding motivational orientations, in general all participants agreed with intrinsic and disagreed with extrinsic motivations (see Table 10.1). When talking about the choice to become a teacher in general, one person did not agree with the

Table 10.1 Mean and standard deviation regarding motivations to become a teacher in general and especially a mathematics teacher, 1 = totally disagree to 6 = totally agree

	Profession choice				Subject choice			
	Intrinsic-pedagogical		Extrinsic		Intrinsic-pedagogical		Extrinsic	
	M	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
Future elementary teachers ($n = 80$)	5.31	0.54	2.81	0.66	4.58	0.66	2.65	1.12
Future secondary teachers ($n = 66$)	4.80	0.58	2.98	0.60	4.78	0.62	2.40	0.96

presented intrinsic motivations and two had a higher agreement with extrinsic than intrinsic motivations. Within the results regarding motivations to choose mathematics as subject, four people did not agree with intrinsic motivations and seven had a higher agreement with extrinsic than intrinsic motivations. The data shows a significant difference with a large effect size between intrinsic motivations of future elementary and secondary teacher. Intrinsic motivations to become a teacher are significantly higher in the answers of future elementary teachers ($t[144] = -5.501$, 2-tailed, $p = 0.00$, *Cohens* $|d| = 0.89$). But in contrast, the agreement regarding intrinsic motivations to choose mathematics as teaching subject is significantly higher with a small effect size in the answers of future secondary teachers ($t[144] = 2.053$, 2-tailed, $p = 0.04$, *Cohens* $|d| = 0.34$).

Most of the participating pre-service teachers report that they are enthusiastic towards mathematics ($M = 5.06$, $SD = 0.64$) and expect themselves to be self-efficient ($M = 4.56$, $SD = 0.49$). There are no differences regarding the type of school. The results of three participants show a mean that can be interpreted as if not being enthusiastic towards mathematics and regarding the perceived self-efficacy no potential critical trajectories can be found.

In Table 10.2, correlations are shown based on manifest variables. Means across the respective items for each scale are calculated and then correlated to each other. Looking at these correlations, it can be stressed that extrinsic and intrinsic motivations seem to be overarching structures, because each of them requires both profession and subject selection. Furthermore, enthusiasm for mathematics and perceived self-efficacy both correlate with each other and intrinsic motivations regarding profession and subject selection.

Within the scope of the AVEM again, there is no difference within the answers of future elementary or secondary teachers. Thirteen participants are sure matched to the *health pattern*, while 54 are matched there in parts (in total 45.9% matches). The *pattern of conservation* is matched sure to seven people and partly to 45 participants. *Risk type A* has no sure match, but 20 partial matches (13.7%) and *risk type B* has one sure match and six partial ones (4.8%). Considering these results a potential critical trajectory regarding the self-regulation of 79 persons (54.1%) can be pointed out. While *risk type A and B* are neither matched nor partly matched to many participants, the *pattern of conservation* is matched to 52 participants, which represents more than one third of the sample (35.6%).

Table 10.2 Correlations between different aspects of motivational orientations

	(1)	(2)	(3)	(4)	(5)	(6)
Extrinsic motivations: profession choice (1)						
Intrinsic motivations: profession choice (2)	0.065					
Extrinsic motivations: subject choice (3)	0.584*	0.098				
Intrinsic motivations: subject choice (4)	0.062	0.450*	−0.083			
Perceived self-efficacy (5)	0.142	0.441*	0.155	0.487*		
Enthusiasm for mathematics (6)	−0.098	0.287*	−0.329*	0.502*	0.289*	

*$p < 0.001$

Having a look at the impact that the participants, who returned the additional questionnaire (*N* = 49), reported, it is to be seen that 75.5% of them answered the question, if they feel encouraged to become a mathematics teacher after being confronted with the feedback by saying “rather yes” (3) or “yes” (4). Furthermore, 40.8% mention that they feel encouraged to talk to others about their professional aim throughout the work with the feedback. But in general, the mean of the encouraging impact regarding further explorations is *M* = 2.18 (*SD* = 0.65), which is close to the theoretical mean of 2. This is why no encouraging impact regarding further explorations through the feedback can be reported. Analogue is the reported encouraging impact regarding reflection processes with a mean of *M* = 1.78 (*SD* = 0.74). Finally, the feedback had an impact of promoting further development (*M* = 2.82, *SD* = 0.74). For example, 79.6% of the participants say that the feedback showed which competences they can expand. Within these findings, there was no significant correlation between the number of critical trajectories that were mentioned in the feedback of a participant and the impact he or she reported.

10.6 Discussion

As the results show, it is possible to give feedback based on empirical findings and to inform the participants about potential critical trajectories. Especially critical trajectories regarding beliefs and self-regulation can be found. With this approach, we are able to point out that many of the participating pre-service teachers own beliefs referring to a transmissive orientation (*M* > 3.5). The results of our study are similar to those of the COACTIV group, who found a higher agreement with constructivist beliefs but also agreements with transmissive ones within the answers of their participants (Kunter et al., 2013). Within the field of motivational orientations, only a few critical trajectories were found. Cramer (2012) also found intrinsic-pedagogical motivations as being the most important motivations for young people to become a teacher in general. But there was a difference between future elementary and secondary teachers regarding their motivations. The results show that in general future secondary teachers generally have higher intrinsic motivations to teach mathematics, while having lower intrinsic-pedagogical motivations to become a teacher. Ziegler (2009) found future secondary teachers showing more subject-oriented motivations to become a teacher, while for future elementary teachers pedagogical motivations seem to give more weight. The mentioned correlations within the field of motivational orientations show that perceived self-efficacy and enthusiasm for mathematics are linked to intrinsic-pedagogical motivations. This finding is interesting regarding the inconsistent findings within the research of intrinsic motivations mentioned in the methods section. The results of the AVEM show a highly frequented match to the pattern of conservation, which might be a reason for other problems within education, like refusing deal with the work sheets within mathematical classes. Within the calibration sample 29% of 972 participating German pre-service teachers were matched to the *health-pattern,* 31% to the *pattern of conservation,* 15% to *risk type A* and 24% to *risk type B* (Schaarschmidt & Fischer, 2008). In comparison to these results it is to be said that in our study there were more participants matched to the *health-pattern* (46%) as well as to the *pattern of conservation* (36%). Regarding *risk type A* the results are quite similar (14% in our study). But there were less matches regarding *risk type B* in our study (5%). The fact that within the calibration sample specifications of the participants like the number of passed semesters are not available interfere searching for reasons regarding the differences.

The aim of this approach was to create a reflective device for professional role reflection. Therefore, the feedback is especially used to critically question the own beliefs, motivational orientations and self-regulation. Still, the participants did not report an encouraging impact regarding the reflection process. Reasons for that might be found in the phrasing of the corresponding items, which all go along with questioning the career choice. It might be that pre-service teachers, who are rather sure about their professional choice, refuse statements that are expressed within the items of this scale. As a next step for the assessment of this approach, thoughts of the pre-service teachers while dealing with the instrument and the feedback need to be uncovered so that the impact can be evaluated more deeply. Furthermore, as identity is defined as a dynamic construct and a self-reflective process (Frey & Haußer, 1987), it is stressed to initiate not only one moment of reflection, but to show the development of pre-service teachers during their education at university by means of several moments of reflection. With the help of those, changes of beliefs can be visualized, which would be of particular importance for the ongoing discussion about the change of beliefs within the MaVi community.

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Chapter 11
It's All About Motivation?—A Case Study Concerning Dropout and Persistence in University Mathematics

Sebastian Geisler

Abstract University dropout rates in mathematics are still high. In this explorative study semi-structured interviews with students, reporting their personal experiences during their first months of their studies of mathematics and their reasons for quitting or continuing their studies, were analyzed. Especially motivational aspects (e.g., motives to choose mathematics, learning motivation) and their connection with beliefs concerning the nature of mathematics seem to have an important impact on dropout and persistence.

11.1 Introduction

University dropout in the STEM subjects (Science, Technology, Engineering, and Mathematics) and especially in mathematics is still a big issue for German universities, regardless of several supporting projects (e.g., preparation courses, mentoring, tutorials). The dropout and subject-change quotas in mathematics add to nearly 80% (Dieter & Törner, 2012). This is not an isolated German phenomenon. U.S. universities have dropout and change quotas from about 78% for associate's degree students in mathematics, and 38% for bachelor's degree students (Chen, 2013). In Germany, most students (34% of the male and 45% of the female students) quit their mathematics studies (or change their subject) during their first year at university (Dieter & Törner, 2012). Most German universities start with the two courses of real analysis and linear algebra during the first year. These two, like most German mathematics courses, are rather theoretical as well as oriented on proof and therefore differ much from general school mathematics.

Dropout and subject-change lead to the same effect, that the student quits his studies in mathematics without a grade. Therefore we will not distinguish between dropout and the change of subjects in this paper.

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11.2 Theoretical Background

Most recent research focuses on dropout in the STEM-subjects (compared with other fields) or dropout in general. The conditions for dropout in mathematics, especially at German universities, are less known. Because of the difference in dropout rates between mathematics and other subjects (Heublein, Hutzsch, Schreiber, Sommer, & Besuch, 2009) it does not seem to be appropriate to transfer results, found in general or in other subjects, to mathematics directly. However, these results can provide insights which are interesting for research on dropout in mathematics, too.

Before illustrating the situation of university dropout in mathematics and the characteristics of mathematics at German universities, we will give a brief overview about dropout in general.

11.2.1 The HIS Dropout Model and Dropout Predictors

Most models describe dropout as a complex process influenced by many conditions. We focus on the HIS (Hochschul-Informations-System) dropout model (Heublein et al., 2009), which has been specially designed for the German university-system. This model distinguishes between conditions referring to the preuniversity phase (e.g., socioeconomic and school background) and conditions of the present study situation (e.g., study conditions, psychological resources of the student).

These conditions, called dropout predictors, do not lead automatically to an exit from tertiary education but they can increase the probability of leaving university and influence the decision to drop out. In this paper we focus on the following three motivational aspects: motives to choose a certain subject, learning motivation and self-efficacy.

The motives that influenced the choice to study a certain subject are an important dropout predictor. Following the HIS dropout model, this condition refers to the preuniversity phase (Heublein et al., 2009). It is assumed that highly intrinsic motives (e.g., interest in the subject) increase the probability of success in one's studies, whereas extrinsic motives (e.g., high income) are associated with dropout (Schiefele, Streblow, & Brinkmann, 2007). It is, therefore, not surprising that students who go on with their studies show more intrinsic interest than those who change their subject (Ulriksen, Madsen, & Holmegaard, 2010). Concerning the choice of subjects, the HIS model names (in addition to intrinsic and extrinsic motives) social motives, uncertain study wishes, stable job wishes, and influenced choices (Heublein et al., 2009).

A major aspect concerning dropout and persistence at university is learning motivation. Students who dropped out show less motivation during their studies than students who go on with their studies (Schiefele et al., 2007). Similar to the choice of subjects, learning can be influenced by intrinsic and extrinsic motivation. Ryan and Deci (2000) state that there are three so-called psychological basic needs, which are essential for the mental health of humans and to generate motivation. These are competence, autonomy, and social relatedness. Humans are motivated to reach certain aims because they can satisfy their basic needs by doing so. Competence and autonomy are suitable to generate intrinsic motivation, whereas extrinsic motivation is connected to all three basic needs (Ryan & Deci, 2000).

The motivation to study for a certain exam or the subject in general can also be influenced by the student's beliefs that he or she is able to succeed. In this context self-efficacy is an important concept. Self-efficacy means the strength of a person's belief in his or her ability to reach a certain goal or solve problems by his or her own competences (Luszczynska et al., 2005). If students believe that they are able to reach their goals, they are more motivated to put effort into learning.

11.2.2 Dropout Reasons in Mathematics

Dropout reasons are the individually recognized conditions that lead to the decision to leave university or change the subject (Heublein et al., 2009). Not every predictor that can influence the decision to quit a study is recognized by the students. For example: the school leaving examination marks are considered to be good dropout predictors (Burton & Ramist, 2001). Students who leave their subject normally do not name their school marks as a reason. Their reasons could be: "I have not gained enough knowledge at school" or "I felt poorly prepared for university." Following the HIS dropout model, the reasons named by students can be summarized in the following groups. The numbers in brackets give the percentage of German mathematics students, who name the aspect as their main reason for quitting their studies of mathematics (Heublein et al., 2009):

- Reasons related to the requirements (e.g., work overload) (33%)
- Reasons related to lack of motivation (25%)
- Reasons related to the study conditions (13%)
- Reasons related to the financial situation or health (12%)
- Personal reasons and those related to family issues (10%)
- New dream job (6%)

Especially the requirements and the students' motivation seem to play an important role.

11.2.3 Mathematics at German Universities

Studies in the USA have shown that in addition to the personal attributes of the students, the characteristics and culture of the subjects at university have a big impact on dropout and persistence in the STEM subjects (Ulriksen et al., 2010).

Rach (2014) summarized the characteristics of mathematics at German universities and compared them to the characteristics of mathematics in school. The mathematics courses at German universities are traditionally very theoretical and proof oriented. One important aim of these courses is to provide the skills to do scientific mathematics on an abstract level. Students should learn logical thinking and gain methods and strategies to understand and develop mathematical proofs. In German schools mathematics is less theoretical and more practical. Definitions and theorems are rather illustrated with examples then presented in a formal way. Compared with tasks at university, tasks in school are more focused on the intelligent use of mathematical methods in (authentic) real life situations (problem solving and modelling) than on proofing theorems. These differences can lead students to have false expectations concerning mathematics at university at the beginning of their studies. However, Rach (2014) found no effect of the students' expectations at the beginning of their studies of mathematics on their success in the examinations at the end of the first semester. Furthermore, many students were able to adapt their expectations during the semester. It seems possible, that students who are not able to revise their expectations drop out during the first semester. These students do not attend the examinations at the end of the semester and therefore were not captured by Rach's study. Due to these considerations, Rach's (2014) results leave uncertain, which impact the students' expectations have on dropout and persistence.

In addition to their expectations, the students' learning of mathematics at university can be influenced by their beliefs concerning the nature of mathematics (Andrà, Magnano, & Morselli, 2011). These beliefs refer to mathematics itself as a discipline, whereas the students' expectations relate to the contents of the lectures and the learning during their studies. Some students view mathematics as a collection of rules, facts, and methods, whereas others see a connected but static structure or experience it as a dynamic and creative field of research (Ernest, 1991). The differences between school and university mathematics can lead to the situation that students establish beliefs concerning the nature of mathematics that do not fit to the way in which mathematics is done and learned at university.

11.3 Methodology

11.3.1 Research Questions

Following the HIS model we state that dropout is a complex matter influenced by many conditions. Several reasons come together and lead to a student's decision to leave mathematics behind. We want to look at the phenomenon dropout from two points of view: in addition to the reasons and predictors which lead to dropout, we want to learn which ones influence students to go on with their studies, even though they were in a similar position compared with students who dropped out. This leads to the following two research questions:

1. Which explanations do the interviewed students give for their decision to drop out or stay?
2. Which role do motivational aspects (motives to choose mathematics, learning motivation) play for their decision to drop out or stay?

11.3.2 Semi-structured Interviews

We decided to use semi-structured interviews because of the explorative character of the study. Furthermore we wanted the interviewees to have the chance to talk very openly about their experiences and reasons, highlighting their own priorities instead of answering fixed questions.

We structured the interviews into four topics, all starting with a very open call for the students to explain their individual experiences. Firstly, we talked about their decision to study mathematics (e.g., How did you decide to study mathematics?) and their experiences and ways of learning during the first semester (e.g., Let us talk about your study, how did you learn?). After that, we talked about their reasons for leaving or staying in the mathematical studies (e.g., Which reasons led to your dropout?) and the process of decision (e.g., How did you come to this decision? What was the process like?).

11.3.3 Sample

The three students whose responses are presented in this paper were interviewed after their first year at Ruhr-University Bochum. All of them went to the same secondary school (German Gymnasium) and therefore knew each other before their transition to university. They even decided together to study mathematics in Bochum. That is why they spend most of their learning-time with each other during their first months at university. At the time of the interviews they all were aged 19. They were successful within the linear algebra exam but failed analysis. We chose this sample because these students share similarities regarding their school background, age and success in the first exams, but chose different ways after their first semester. John (names have been changed) decided to quit his studies of mathematics and left Ruhr-University. Anna decided to repeat the course again after failing analysis. Tom considered leaving mathematics behind after failing analysis but decided to give it another try.

11.3.4 Analysis

The interviews were recorded on tape and transcribed. The resulting transcripts have been examined using structuring qualitative content analysis (Mayring, 2010). For this purpose, certain categories have been defined deductively on the basis of the theoretical background presented before. After a first preanalysis of the material, new categories have been added inductively. This led to a coding-guide (Mayring, 2010) for the analysis of the interviews. The coding-guide was discussed and revised in cooperation with an interrater to increase reliability.

11.4 Results

We present the results beginning with the description of the motives that influenced the choice of subject and followed by the students' experiences of the basic needs during their studies. Finally, we present the reasons for dropping out or staying, which were named by the interviewees.

11.4.1 John's Story

John chose to study mathematics because of his interest in this subject in school and his good marks:

> Mathematics has always been my favourite subject since primary school. I was good in the other subjects, so I could have done quite much, but I chose mathematics because I had the most fun with that in school.

So John's motives for studying mathematics were highly intrinsic and not extrinsic or guided by a special job wish. He did not inform himself about the studies before entering university, so he had little knowledge about the character of university mathematics.

He started his studies with three other students (including Anna and Tom), whom he had known from school. He learned together with this group on weekends. During the week he spent less time for learning at the university.

> This was maybe a little bit curious because I never really found my way into another group. […] That disappointed me a bit, that you somehow held too less contact with others in this time.

These quotes show that John's feeling of social relatedness was rather low. This was one reason for him to quit his studies of mathematics: "I couldn't really get new contacts, that was another point which led me to think again about starting over again somewhere else." In addition, he missed out on engaging himself: "I always had fun being involved at school […] that hasn't been the case during the studies of mathematics, that I had to say something." Carefully interpreted John also had a rather low feeling of autonomy. During the semester, John felt less competence. He described the weekly working sheets, which accompany the lectures, as "frustrating" and explained that he was not able to solve most of the tasks on these sheets on his own. His self-efficacy regarding the exams at the end of the semester was low:

> And then I also thought I wouldn't do well in the exams, because I really worked less on my own on the sheets, copied more and more from the others, because it simply wasn't possible for us to solve them. But the exams have been quite good. But somehow this couldn't convince me to go on.

John earned about 95% in his linear algebra exam and narrowly failed the analysis exam. "My achievements weren't the reason to drop out, but rather my motivation for the studies in general." He often said that he had no motivation to learn at all. John also stressed the change of character of mathematics between school and university:

> At school, it was like this, that from the content it was like, you learned theory and then you had to apply the theory. […] But during the studies it was different then, now you had to prove things for example and I had no fun at all, doing these purely theoretical proofs […]."

John decided to quit after his first semester, but stayed enrolled until the end of semester two. After that, he began to study engineering.

11.4.2 Anna's Story

Originally Anna planned to study business administration. She realized that she needed a second subject in addition to her favored one: "I could decide between physics and maths, because both are things which interested me very much and then I simply saw better job-chances in maths and then started with maths." Choosing to study mathematics, Anna had intrinsic and extrinsic motives, in contrast to John. Her marks in school had been quite good. She did not inform herself about the content of the subject of mathematics at university.

Anna worked in the same group of students as John. In contrast to him, she was able to get in contact with new persons during her studies:

> Yes much, very much, as I said, with those from school, which I already knew, I'm still learning together […] in addition in my actual learning group there are twelve people, which I meet regularly […] so our social relationship is very pronounced.

Anna's social relatedness was rather high. At least she felt some competence regarding her learning in her first semester: "I tried to understand it, that mostly wasn't successful, so it didn't work, but at least I tried it, then I revised it and then it was okay actually." Anna described herself as very motivated to learn "I'm very focussed and have got a very high motivation even after holidays and on Monday mornings always." In her eyes, this is also the biggest difference between John and herself:

> I had the feeling, that John's motivation was very low […] and with me that was very different, right from the beginning, so I'm the one who sits in the library four hours a day, cause I have fun with it, and actually he was never there […].

She thought about quitting her studies for a short time after failing in the analysis exam: "At the end I just thought to myself, okay, the professor was not so good […] and then I will just do it again." So she did not blame herself and decided to repeat the analysis course.

11.4.3 Tom's Story

Tom was sure about his wish to become a teacher and "always liked maths, French at school." So he enrolled in both subjects. His motive to choose mathematics was a stable job-wish. Tom's grades in mathematics had always been good in school. He knew about the workload of the studies of mathematics before the enrolment but underestimated it: "before starting to study, I believe, you nevertheless always think: it won't be that hard."

Tom exclusively learned with the students whom he knew from school at the beginning of his studies:

> We all started to study here together and we were such a group at the beginning. Honestly we didn't try to get contact with others at the beginning […]. At the end of the semester we found other people with whom we still learn together today […].

Tom's social relatedness was rather low at the beginning, but grew through the first semester. He drew a similar picture of his competence. During the first semester he described the learning as frustrating: "Nevertheless I think it is a bit frustrating if you sit there and have no success at all and, I don't know, it's just a little bit sad." But his feeling of competence grew during the second semester: "I did it a little bit better than I used to, so the exams where better then."

Tom considered quitting his studies several times:

> So I really considered two or three times, that it makes no sense anymore, hm, so the first time was really after the first exam, after the first semester […]. If you study mathematics here, it is completely different from school, you have the feeling, that you never did maths before […]. That wasn't the maths that I actually wanted to study or which I decided for.

Like John, Tom recognized a big difference between school and university mathematics. He decided to go on due to his goal to become a teacher:

> Actually my job-wish, I still want to become a teacher, and I think I find it interesting to teach math, and I think to myself, I don't know whether this is true, I tell myself, if I have done it sometime and I'm a math teacher, that it will be again what I want to do.

It seems that his long-term wish to become a teacher guided Tom even through hard times at university. Like Anna, he decided to repeat analysis and looks carefully optimistic into the future, which indicates at least some self-efficacy: "These methods you use, I believe this solidifies itself sometime, I believe, hope, it will become better."

11.5 Conclusion

In contrast to John and Tom, Anna was the only student in this case study who mentioned that she had fun doing mathematics at university, which indicates an intrinsic learning-motivation. In addition, she felt the most satisfaction of the basic needs. Tom and John said that they were interested in school mathematics but had no fun with the mathematics at university. They did not feel much satisfaction of their basic needs. Especially John felt low social relatedness and competence. This feeling of low competence and his lack of self-efficacy stand in contrast to his achievements in the examinations at the end of the semester, but they negatively affected his motivation through the whole semester. John and Tom both stressed the change in the character of mathematics between school and university. It seems that the mathematics at university did not fit to their established beliefs concerning the nature of mathematics. Choosing to study mathematics, John had highly intrinsic motives in contrast to Tom and Anna, who chose mathematics because of professional considerations.

We conclude that the combination of a low feeling of satisfaction of the basic needs and intrinsic motives to choose mathematics, based on interest for the subject, colliding with a change in the character of mathematics led to John's dropout. Tom also felt a difference between his beliefs concerning the nature of mathematics and the university mathematics, but his stable job-wish guided him even through this.

This leads to the hypothesis that students who decide to study mathematics only because of intrinsic motives are more affected by the change of character of mathematics than students who choose mathematics because of extrinsic motives or stable job wishes as well as intrinsic motives.

11.6 Discussion and Outlook

On the first view John does not seem to be a typical dropped out student, because he had been rather successful during his first semester and 70% of the dropped out students in Germany name the requirements at least as one important reason (Heublein et al., 2009). But John did not feel successful through the whole semester and therefore, thought that he could not fulfill the requirements. The change in the character of mathematics between school and university, mentioned by John, is described by many dropped out students and even successfull undergraduate mathematics students decide to leave mathematics behind for this issue (Ward-Penny, Johnston-Wilder, & Lee, 2011).

The openness of semi-structured interviews gives the chance to the interviewees to set their own priorities. But on the other hand this openness makes the interviews less comparable.

Our ongoing research will now focus on quantifying the impact of motivational aspects on success and dropout in mathematics during the first year at university and specially the link between the motives to choose mathematics and the beliefs concerning the nature of mathematics.

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Part II
Teachers' Views and Beliefs of Mathematics

Chapter 12
How to Understand Changes in Novice Mathematics Teachers' Talk About Good Mathematics Teaching?

Hanna Palmér

Abstract This paper focuses on how novice primary-school mathematics teachers talk about (good) mathematics teaching in general and mathematics textbooks in particular at the time of their graduation from university and a year later. The changes in their talk are discussed first in terms of beliefs research and second from a participatory perspective on identity formation. A comparison of findings with the two approaches shows that what beliefs research often explains as changes in belief, inconsistency, or hidden beliefs can be understood as identity formation in communities of practice from a participatory perspective.

12.1 Introduction

The shift from teacher education to actual teaching has long attracted interest from the community of mathematics educators. As an extension of that interest, studies on student teachers and novice teachers has been conducted from the perspectives of beliefs research (e.g., Phillip, 2007; Wilson & Cooney, 2002), mathematical knowledge for teaching (e.g., Hill, Sleep, Lewis, & Ball, 2007; Ponte & Chapman, 2008), and identity (e.g., Beijaard, Meijer, & Verloop, 2004; Skott, 2015). In beliefs research, several studies have reported that new methodological and pedagogical ideas learned in teacher education tend to regress when novice teachers begin working as teachers. Other studies have reported that novice teachers are inconsistent in their beliefs (Phillip, 2007; Wilson & Cooney, 2002).

In response to those findings, this paper focuses on how three novice mathematics teachers discuss (good) mathematics teaching in general and mathematics textbooks in particular at the time of their graduation from university and a year later. Experiences often explained in beliefs research as regression or inconsistencies are elaborated upon from a participatory perspective focusing on identity formation. The purpose of the study is to put novice teachers' talk in relation to their contexts

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and thereby be able to reconcile contradictions or inconsistencies found in earlier studies. The research questions considered are twofold:

- How do novice teachers talk about (good) mathematics teaching in general and mathematics textbooks in particular change during the year following their graduation from university?
- How can those changes, if any, be understood from a participatory perspective on identity formation?

As an introduction, some issues about the national (Swedish) context regarding mathematics teaching and textbooks will be described, however, such focus does not imply that the results are valid only within the Swedish context. Although a national evaluation in 2009 (Swedish Schools Inspectorate, 2009) indicated that most mathematics lessons in Sweden involved students' individual work with textbooks, Sweden has no national regulations regarding how or even whether to use textbooks, or which textbooks to use, if any. At the same time, textbooks are often discussed in negative terms by Swedish mathematics teachers, and having students work individually with textbooks is sometimes cited to explain students' declining performance in mathematics (Neuman, Hemmi, Ryve, & Wiberg, 2014). In many ways, the debate aligns with international reform proposals made in 1991 (NCTM, 1991) that advised spending less time on "paper-and-pencil drills" (p. 19) and more time on group work, discussions, and applications in real-world contexts to connect mathematics to other areas of the curriculum.

12.2 Becoming a Mathematics Teacher

Several studies have reported that teacher education has limited impact on student teachers, because, following graduation from university, they tend to exhibit regression in what they learned in teacher education once they start to teach. By contrast, their individualschooling prior to teacher education is often considered to be an important value in relation to how they think about teaching and how they teach, as the overview in Wang, Odell, and Schwille (2008) shows.

Several studies on the process of becoming a mathematics teacher have focused on the beliefs of student teachers or novice teachers, if not both, and thereby raised fundamental questions about whether beliefs change or remain static and, if the former, then how best to change beliefs. Several such studies have indicated that novice teachers appear to be inconsistent in their beliefs or act inconsistently in relation to their beliefs (Phillip, 2007; Speer, 2005). Such inconsistency has been explained in various ways—for example, by arguing that different beliefs dominate in different situations, that individuals have unconscious beliefs, that beliefs are situated, that individuals are actually being inconsistent, or that researchers and participating teachers have different interpretations of concepts (Phillip, 2007; Speer, 2005).

According to Wilson and Cooney (2002), inconsistency observed in teachers' beliefs and actions could occur for several reasons. Researchers and participating teachers may have different interpretations of concepts, or a teacher might not intentionally act in line with his or her beliefs in certain situations due to practical or logistical circumstances. It is also possible that the specific beliefs studied are peripheral and that more central beliefs are the ones being expressed in the actions. Phillip (2007) and Speer (2005) have highlighted the problem among researchers of claiming that teachers are being inconsistent. According to Speer (2005), researchers attribute all beliefs, and therefore to say that teachers' actions do not align with their beliefs expresses an opinion of the researchers, not the participating teachers. In response to that problem, Phillip (2007) implies that inconsistency ceases to exist when researchers better understand teachers in relation to their social environments.

12.3 Theoretical Framing

To avoid attributing beliefs to teachers that later indicate false regression or inconsistency, Skott (2015) has advocated using participatory perspectives that focus on processes. Participatory perspectives imply theoretical perspectives and lines of research that conceptualize learning as changes in participation in social practices (Borko, 2004). To *participate* means both to absorb and to be absorbed in a community. From a participatory perspective, the physical and social context in which an activity occurs is integral to the activity, and, in turn, the activity is integral to the learning that takes place within it. Sfard (2006) has described this duality as the "individualization of the collective" and the "collectivization of the individual" (p. 158).

From a participatory perspective, becoming a teacher is a process of increased participation in the practice of teaching and, by way of that participation, becoming knowledgeable in and about teaching. To be understandable, teacher-learning needs to be studied in the multiple contexts in which teachers perform their jobs and by taking into account both individual teachers and the social systems in which they participate (Borko, 2004).

The participatory perspective used in this paper is Wenger's (1998) social theory of communities of practice. According to Wenger (1998), individuals are constantly involved in dual process of identity formation, in which one half involves identifying with communities of practice, whereas the other involves negotiating the meaning of mutual engagement, shared repertoire, and joint enterprise in the communities of practice. Mutual engagement encompasses the relationships among members in a community of practice, whose mutual engagement allows them to build a shared repertoire based on collective stories, artifacts, notions, and actions. By extension, mutual engagement fosters mutual accountability—a joint enterprise—that the members feel in relation to the community of practice.

Membership in communities of practice can be focused on regarding individual's identification and/or negotiation within or between communities of practice, based on the learning trajectories within and between communities of practice. Individuals can identify with and negotiate within communities of practice by way of engagement, imagination, and alignment, each of which involves different approaches and conditions and does not necessarily require or necessarily exclude the others. Since imagination and alignment expand participation in communities of practice beyond time and space, individuals can be members of and feel a sense of belonging to communities of practice despite the absence of visible shared practice. Within and between communities of practice, individuals' learning trajectories can be peripheral, inbound, inside, on the boundary, or outbound.

12.4 The Study

The study was a case study of seven novice primary-school teachers in Sweden that lasted from their graduation from university until 2 years later (Palmér, 2013). Their teacher education comprised 3½ years of university study, during which all seven participants specialized in mathematics, hence their inclusion in the study. In Sweden, teacher education integrates professional and subject-specific study at the same time. The structure of teacher education in Sweden at the present time allowed students to choose the number of courses—for at least 15 credits and 52.5 credits at most—in mathematics education that they completed during their teacher education. Participants were contacted during the last semester of their teacher education, and all requirements regarding information, approval, confidentiality, and appliance advocated by the Swedish Research Council (2008) were followed during their recruitment and participation.

Three of the participants, Barbro, Nina, and Helena, were selected for this paper because their cases illustrated the phenomenon under study. Barbro was 22 years old at graduation and had earned 22.5 course credits in mathematics education; Nina was 24 years old at graduation and had earned 37.5 course credits in mathematics education; and Helena was 41 years old at graduation and had earned 45 course credits in mathematics education. At the time of the study, Sweden did not offer any national or local teacher induction and because there were more qualified teachers than positions available, it was difficult for novice primary-school teachers to secure teaching jobs.

An ethnographic approach was followed to make the process of identity formation in communities of practice visible (Aspers, 2007; Hammersley & Atkinson, 2007). Empirical material was collected from self-recordings made by the participants on mp3 players, as well as from observations and interviews. For the self-recordings, participants were instructed to decide what was important for the researcher to know about starting to work as a primary-school teacher of mathematics and to therefore record whatever they wanted at any time and for as long as they wanted. In line with the ethnographic approach (Aspers, 2007), both formal (with template during lessons) and informal (between lessons) observations were made, and the interviews included both spontaneous conversations during observations and formal interviews that followed thematic interview guides.

In the analysis, the ideas and thoughts expressed by participants in self-recordings, interviews, and observations were treated as narratives. According to Cortazzi (2001), collecting and analyzing several narratives makes it possible to distinguish participants' perspectives on particular themes and processes. The joint theme in focus during the study was how participants talked about (good) mathematics teaching in general and textbooks in particular. Coupled with observations, the narratives were interpreted in relation to communities of practice in which the participants seemed to negotiate or identify with, if not both, and examined for how they were influenced by those communities, if at all.

12.5 Results

The results are presented in three subsections: one addressing the participants at the time of their graduation from university, another that presents their individual experiences a year after their graduation, and the last that discusses the analysis of the three cases in terms of the theory of communities of practice.

12.5.1 The Time of Graduation

Interviewed individually at the time of their graduation, Barbro, Nina, and Helena said that they had discovered a "new approach" to mathematics teaching during their teacher education and expressed a clear desire to "reform mathematics teaching." When asked to give examples of the approach, they discussed lessons "outside the frame" of the textbook. In describing what constitutes good mathematics teaching, they emphasized teaching in which students do not realize that they are being taught mathematics, and the examples that they gave can be summarized as varied, laboratory-based, concrete, reality-related, and problem-oriented mathematics lessons.

Helena:	I believe good mathematics teaching is when students have access to learning materials. […] I love these multiplication games we made. […] I also like the games and the problem cards. I like them.
Researcher:	What makes them good?
Helena:	The games are fun. Partly because many of them [the students] do not think of themselves doing math [while playing the game] at the same time as they actually get practice. Often, they do these things with someone else. There can be two or more [students working together]. Then they learn from each other. You hear their dialogues and they check on and inform each other. […] That also increases their understanding.

As Barbro, Nina, and Helena discussed the "new approach" to mathematics teaching, they referred to themselves and their fellow students from the teacher education program as "we." They also distinguished the "new approach" and their experiences as students in school from the teaching that they encountered during their preservice teaching.

> [I have] been at two different schools quite a long time and it feels like many teachers are very controlled by the textbook and that is what counts. (Nina)

The participants discussed experiences from their schooling and preservice teaching as "old-fashioned" and "traditional" and as mathematics lessons that followed a "patterned scheme" within the "frames" of the textbook.

> […] you are that closely tied to the text book that you don't dare leave it. But then maybe I had both the advantage and disadvantage of having a very experienced placement supervisor who had been at the same school for forty years and who probably had been teaching the same way these forty years. So, she was very controlled by the text book. (Barbro)

12.6 The First Year After Graduation

It was difficult for participants as novice primary-school teachers to secure teaching positions at the time of their graduation. Barbro began working as a substitute teacher, meaning that on a given day, if she were fortunate, she would receive a call in the morning and have a job as a substitute teacher for at least that day, if not the days that followed as well. Barbro combined those temporary teaching jobs with other kinds of temporary work. Because she worked at several schools, often for no more than a few days at a time, she did not develop any close relationships with other teachers during her first year after graduation. Working as a substitute teacher meant that she did not create lesson plans but taught mathematics lessons planned by the regular teachers and given to her as notes:

> It [the lessons] is so much the textbooks. It is the textbooks all the time. I have to adjust to that right now, I have to. […] it is mostly the textbooks and I feel like that is not really me. As I probably said the last time, it's more hands-on things, I want to pick and potter and get them to understand in that way. (Barbro)

Nina starts to work as a teaching assistant for a boy with attention deficit hyperactivity disorder. She likes the school where she works, although her work as a teaching assistant prevents her from developing any close relationships with other teachers at the school apart from Diana, the teacher whom she assists. Nina describes spending lessons, breaks, and afternoons with the boy. About Diana, Nina says that she is like a "tutor" to her and that they are "very close." When she talks about the mathematics lessons in the boy's class, Nina refers to them as "our mathematics teaching" and "our class." Since Nina has no time for planning, Diana is the one planning the mathematics lessons, based on a textbook that Nina reports "actually" liking, partly because it differs from "the ordinary ones she used when she was little." She stresses that every chapter of the textbook starts with the goals for that chapter, followed by a "math lab" in which students work with "practical material" in pairs.

Helena represents one of the few graduates to secure a full-time teaching position immediately after graduating from university. She works as a classroom teacher in an upper primary school (Grades 4–6) where she develops close relationships with several of the other teachers. Aside from her teaching colleagues at the school, she has also met with a group of teachers from other schools in the municipality several times each month to create common goals for teaching science. Since most of the teachers in the group also teach mathematics, they often discuss mathematics teaching as well. According to Helena, teachers in the group from lower secondary school (Grades 7–9) complain that students who have not reached the goals of Grade 5 spend all of Grade 6 working to accomplish them and consequently do not learn the mathematics content of Grade 6. In response, the group has often discussed the importance of acquainting all children in the upper primary school with the content in the mathematics text book for Grade 6. In the second term of that year, Helena buy copies of a new textbook for her class in Grade 6, a reform-inspired textbook that she evaluated as part of an assignment during her education as a teacher. She plans her mathematics teaching for the upcoming term around the goal of having students complete the textbook throughout the year, with the chapters in the textbook set to regulate what happens in her mathematics lessons and when.

12.6.1 How to Understand the Three Cases?

When comparing how the respondents talk about (good) mathematics teaching and textbooks at the time of their graduation and then one year later, there are both similarities and differences. Shortly before graduation, they all described mathematics teaching based on a textbook in negative terms and as "old-fashioned" and "traditional." A year later, Nina reported "actually" liking the textbook used in the class where she has worked as a teaching assistant, whereas Helena has bought copies of a new textbook for her class and based the plans of all of her mathematics teaching on the textbook. Conversely, Barbro showed no changes in her discussion of textbooks after a year's time. Thus, one could say that Barbro's beliefs about textbooks are consistent but that Nina's and Helena's beliefs have changed. Or, setting aside the question of changing beliefs, one could argue that Nina and Helena are inconsistent or that they, at the time of graduation, had other hidden or unconscious beliefs that became more influential once they started actually teaching mathematics. In any case, Barbro, who had earned the fewest course credits in mathematics education, did not change the beliefs that she expressed at the time of graduation, whereas Helena, who had taken twice as many credits as Barbro, changed the most.

As shown, schooling prior to attending teacher education often has important value in relation to how novice teachers conceive teaching and actually teach. In that sense, Nina's and Helena's "change" could have stemmed from their schooling and experiences as students. However, Nina says that the textbook that she has used differs from "the ordinary ones she used when she was little," and Helena bought copies of a reform-inspired textbook that she evaluated during her education as a teacher.

How can the cases be understood from the participatory perspective of identity formation in communities of practice? Barbro, Nina, and Helena expressed highly similar ideas about good mathematics teaching and textbooks at the time of graduation, which can be understood as they being members of a community of reform mathematics teaching. The core of that community seems to be within teacher education involving a shared repertoire and joint enterprise regarding textbooks and good mathematics teaching. At the time of graduation, all three respondents participate in this community through engagement and imagination, as they imagine themselves as teachers. As for engagement, they do not talk about being a part of the negotiation of the shared repertoire, but they have been engaged in its teaching during their teacher education. Since the core of the community of reform mathematics teaching seems to be within teacher education, their membership may have been mandatory to pass their exams; however, none of the participants discussed aligning themselves with anything against their wishes.

In the case of Barbro, no changes are visible the year after graduation; she talks similarly about her views on what constitutes good mathematics teaching and uses words such as “old-fashioned” and “traditional” when talking about textbooks. Even though she is not engaged in the community of reform mathematics teaching (not teaching in line with its shared repertoire), she imagines herself teaching in line with its shared repertoire in the future. Her work as a substitute teacher has prevented her from working closely with other teachers, and therefore she has not joined any new communities of practice with shared repertoires regarding mathematics teaching that could have influenced her ideas about good mathematics teaching and textbooks.

Nina uses the word “actually” when she says that she likes the textbook used in the class where she works, which indicates that she is aware that she is talking differently about textbooks now than before. However, the words she uses to describe why she likes the textbook are similar to the words she used to describe good mathematics teaching at the time of graduation. Then, she expressed good mathematics teaching as varied, laboratory-based, concrete, reality-related, and problem-oriented. One year later she describes the textbook as good because it includes the use of practical material, math labs, and students working in pairs and showing their different solutions. Thus, even though Nina has not had opportunities to participate by engagement in the community of reform mathematics teaching since she graduated from teacher education and started to work as a teacher assistant, she still seems to align with its shared repertoire. The choice to use a textbook in “our class” was made by the only teacher Nina cooperates with and she says “our mathematics teaching.” Thus, her cooperation with the class teacher can be understood as an emergent community of practice where her talk about good mathematics teaching and textbooks 1 year after graduation seems to be a merger of the shared repertoires in these two communities.

Helena is the one of the three participants who has changed the most since graduation. During the year, she cooperates with teachers in what can be understood as a community of teachers working with common goals for science teaching. These teachers have a mutual engagement in negotiating a shared repertoire regarding how best to teach mathematics and science. This shared repertoire involves the importance of all students in grade 6 getting through all of the content in the textbook. Her membership in this community through engagement changes how Helena talks about good mathematics teaching and textbooks. The goal of her mathematics teaching becomes to ensure that all of her students work through the entire textbook before the end of the school year. The textbook is reform-oriented, but the way she uses it indicates that she is an outbound member of the community of reform mathematics teaching (mainly participating by imagination) and an inbound member of the community of teachers working with common goals for science teaching (participating by engagement). This shift is understandable since the mutual engagement and negotiation in the community of teachers working with common goals for science teaching is much more intense than her imagined membership in the community of reform mathematics teaching.

12.7 Conclusion

The findings describe the talk of three novice primary-school teachers regarding (good) mathematics teaching in general and textbooks in particular at the time of their graduation from university and a year later. The aim of this paper was to illustrate how changes during a year's time can be understood from a participatory perspective on identity formation. As mentioned in the introduction, Speer (2005) as well as Wilson and Cooney (2002) has emphasized that observed inconsistency between teachers' beliefs and actions can have several explanations not connected to inconsistency. Similarly, Phillip (2007) has stressed that inconsistency ceases to exist when researchers better understand teachers in relation to their social environment. The analysis of the three cases represents an example of better understanding removing inconsistency. By using the participatory perspective of Wenger's (1998) social theory of communities of practice, what at first glance could have been understood as latent or unconscious beliefs, or even as inconsistency or regression, can be understood as changes in participation in social practices. The three cases illustrate how becoming a teacher is a process of increased participation in the practice of teaching and how novice teachers, via their different forms of participation (i.e., engagement, imagination, and alignment) in different communities of practice, develop their identities as teachers differently. Altogether, to understand the talk and teaching of (novice) mathematics teachers, a participatory perspective on identity formation has much to offer, and to make teacher learning understandable, the multiple contexts in which they participate and perform their jobs should be considered.

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Chapter 13
Domain Specificity of Mathematics Teachers' Beliefs and Goals

Andreas Eichler and Angela Schmitz

Abstract In this paper we investigate whether secondary teachers' beliefs and goals regarding the use of visualisation for the learning and teaching of mathematics differ between different mathematical domains. We investigate this issue based on the following domains: fractions, algebra, functions and calculus. The results are part of a qualitative interview study with five secondary teachers. The findings imply the hypothesis that teachers' beliefs and goals regarding the use of visualisation are consistent for different mathematical domains. The findings are discussed in the context of the more general question to which extent teachers' beliefs and goals regarding overarching mathematics related processes differ between different mathematical domains.

13.1 Introduction

Is it possible to identify teachers' mathematics related beliefs that are similar or even identical for every mathematical domain? On the one hand, well-known reviews concerning teachers' beliefs imply the existence of beliefs regarding mathematics in general, for example beliefs referring to the teaching and learning of mathematics, or beliefs about the nature of mathematics (cf. e.g. Hannula, 2012; Philipp, 2007; Skott, 2015; Thompson, 1992). On the other hand, and similar to the theoretical considerations of Törner (2002) about domain-specific beliefs, some results of our own research on teachers' beliefs in the domains of calculus, geometry, and statistics gave strong evidence that teachers hold considerably different beliefs regarding different mathematical domains (cf. Eichler & Erens, 2015).

In the cited research, beliefs were collected primarily as beliefs regarding mathematical content. However, research could be enhanced by taking into consideration

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a process oriented view investigating beliefs regarding overarching mathematics related processes. For example, learning proofs, modelling, use of technology, and visualisation are some of such overarching mathematics related processes. These processes are meaningful for every mathematical domain in school mathematics. For example, as to the process of modelling, we found considerable differences in teachers' beliefs between mathematical domains: Whereas modelling played at most a peripheral role in their geometry teaching, modelling was a central part in statistics teaching (cf. Girnat & Eichler, 2011).

In this paper, we continue research from a process oriented perspective about teachers' beliefs and goals in different mathematical domains, focusing on visualisation. Our research is a contribution in a field where teachers' beliefs regarding visualisation mostly have a focus on only one mathematical domain like fractions (e.g. Izsák, 2008) or on several mathematical domains like fractions and algebra, yet not focusing on a comparison between the domains (e.g. Stylianou, 2010). The main research question of this paper is:

To which extent are there similarities or differences in mathematics teachers' beliefs and goals regarding the use of visualisation in different mathematics domains?

The research is embedded in the overall research question to which extent there are similarities or differences in mathematics teachers' beliefs and goals when looking at overarching processes in different mathematical domains. We investigate the research question for teachers' beliefs and goals regarding visualisation exemplarily on the basis of the mathematical domains fractions, algebra, functions, and calculus.

In the following, we point out our understanding of the constructs of teachers' beliefs and goals. We further define the construct of visualisation. After describing the method of our research, we outline our results for three teachers out of our sample. We conclude this with a discussion in the context of the more general question to which extent teachers' beliefs and goals differ between different mathematical domains.

13.2 Beliefs

The model of a person's mathematics related affect provided by Hannula (2012) is appropriate to explain the understanding of teachers' beliefs and teachers' goals in our study (Fig. 13.1).

Referring to this model, we regard teachers' beliefs from a psychological perspective as the cognitive part of teachers' mathematics related affect. Further, we regard teachers' beliefs as a disposition for the teaching practice from a long-term perspective. Thus, we regard teachers' beliefs as a trait. Finally, we follow Philipp (2007) defining beliefs as "propositions about the world that are thought to be true". For example, a statement like "visualisations are mandatory to understand mathematics" could be assigned with a logical value.

Teaching goals can be understood either as a state or as a trait (Hannula, 2012). For example, (Schoenfeld, 2011, p. 460) understands goals as a state, and

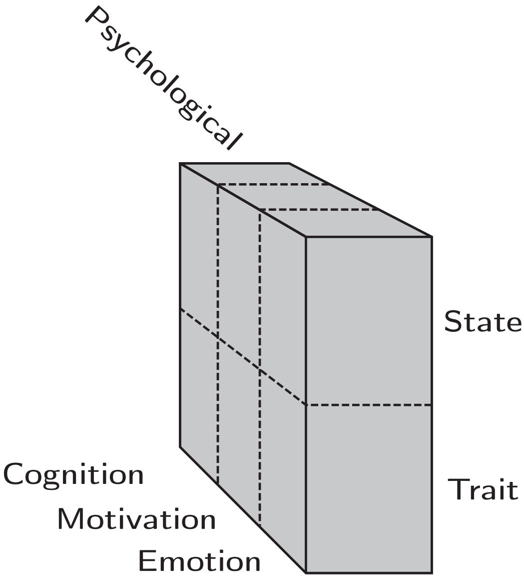

Fig. 13.1 The psychological perspective in a model of mathematics related affect (Hannula, 2012, p. 144)

respectively as "decision making during teaching". In contrast, in our research we understand teaching goals according to Heckhausen and Gollwitzer (1987) as decision-making from a long-term perspective, that is, as decisions influencing the planning of future teaching, and thus as a trait. Following Hannula (2012), we relate goals to the motivational part of mathematics related affect. Finally, a teaching goal is not a proposition with a logical value which is a characteristic of a belief. For example, a teaching goal could be indicated as follows: "I will use visualisations in my teaching". Yet in our understanding beliefs connect different teaching goals: A fictitious teacher could have the goals to use visualisations and to prepare the students' understanding. Both goals are connected to the belief that visualisation promotes mathematical understanding.

In this paper, we primarily refer to results about teachers' goals concerning visualisation. However, as we understand beliefs and goals as connected, we do not focus on a distinction between goals and beliefs when discussing our results.

13.3 Visualisation

We understand visualisation as a "belief object", as it "shares a direct or indirect connection to mathematics" (Törner, 2002, p. 78). Visualisation can be connected to diverse goals as one aspect of the broad definition of Arcavi (2003, p. 217) reveals:

> Visualization is the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds, on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understandings.

For our purpose, that is, investigating mathematics teachers' beliefs and goals regarding visualisation, several aspects indicated in Arcavi's definition are important. Firstly, visualisation is connected to a "purpose" like "advancing understanding" that in our understanding is a teaching goal. Further, visualisation includes diverse kinds of pictures. Thus, we consider everything not being completely symbolic as a visualisation.

For comparing a teacher's goals of visualisation in different mathematical domains, we use a system of goals that is a main result of a larger analysis of mathematics teachers' beliefs regarding visualisation (Schmitz, 2017). This system includes four cognitive goals and one affective goal. Every teacher analysed in our study refers in an individual manner to a subset of these goals:

1. Understanding a mathematical concept: This goal—including for example to understand the concept of fractions—is sometimes an integral part of defining visualisation (e.g. Arcavi, 2003) as well as an integral part of doing mathematics (cf. Giaquinto, 2010).
2. Understanding a mathematical procedure: Although this goal is theoretically close to the first, teachers seem to make a distinction between understanding a concept (e.g. fractions) and a procedure (e.g. multiplying fractions) and the role of visualisation when understanding a concept or a procedure, respectively.
3. Reading and generating: This goal represents the ability to use given visualisations or to generate own visualisations to solve a mathematical problem. This goal has a theoretical basis, since in the tradition of Pólya (1956) visualisation is understood to be a main heuristic in problem-solving (cf. e.g. Duval, 2002).
4. Remembering: This goal describes the use of visualisation for facilitating remembering an idea or procedure. It corresponds to the theoretical aspect that visualisation can serve as a way to mentally store mathematical content (cf. Schnotz, 2010) and facilitate a next step in a mathematical procedure (cf. Arcavi, 2003).
5. Motivation: Some teachers tend to assign to visualisations a motivational effect on students, for example when a visualisation represents an object of the real world.

On the basis of this system of goals, additionally connected by beliefs regarding visualisation (cf. Schmitz, 2017), we compare teachers' beliefs and goals regarding visualisation in different domains. Other goals regarding visualisation like "justifying" or "communicating" that were extracted from literature (cf. Schmitz, 2017) were of little importance for the teachers in our study.

Connecting the constructs of beliefs, goals and visualisation, the main aim of this paper is to compare beliefs and goals towards visualisation for different mathematical subdomains. We see this comparison as a contribution to the more general question whether teachers' beliefs referring to overarching mathematics related processes are different or the same for different mathematical domains.

13.4 Method

Our sample consists of five secondary mathematics teachers (cf. Schmitz & Eichler, 2015) of whom we regard three teachers for this paper. The teachers were selected following a "theoretical sampling" (Glaser & Strauss, 1967) aiming at contrasting cases, for example in respect to the teachers' educational paths, to their types of schools, and to their ages (cf. Schmitz & Eichler, 2015).

The data was collected in semi-structured interviews. The interview questions focused on the use of visualisation in the mathematical domains fractions, algebra, functions, and calculus, with the same questions for every mathematical domain. We included questions such as "please describe how you apply visualisation in the domain of …", and "please describe how your students apply visualisation in the domain of …". The questions for the mathematical domains were complemented by overarching questions regarding the use of visualisation in the context of learning and teaching mathematics in general. The development of the questions and the interaction in the interviews followed the problem-centred interview by Witzel and Reiter (2012). We talked about each mathematical domain for between 20 and 60 min, each interview lasted in total about 2–3 h.

For analysing the interview data, a qualitative coding method was used (e.g. Strübing, 2014). We chose the qualitative method as we aimed at understanding the teachers' own perspectives, and we were affirmed by other studies which revealed manifold aspects of beliefs regarding visualisation by a qualitative method (e.g. Stylianou, 2010). The codes were gained by interpretation of each episode of the verbatim transcribed interviews and indicate beliefs or goals that are connected with visualisation. For example, the five goals of visualisation that we described in the section about visualisation were developed as "core categories" (Strübing, 2014, p. 17) and served as one basis for the comparison of goals between the mathematical domains for each teacher. Furthermore "core categories" describing teachers' beliefs, for example beliefs regarding the learning with visualisation, were developed in the coding process. They also served as a basis for the comparison between the mathematical domains.

13.5 Results

We regard three out of the five teachers for this paper. The selection illustrates exemplary the results for the whole sample regarding in how far teachers' beliefs and goals differ between different mathematical domains.

We firstly consider Mr. A and his teaching goals concerning fractions, algebra and functions. We narrow the focus on the goal of understanding a mathematical procedure. Mr. A describes the goal of understanding a mathematical procedure in the following way (Fig. 13.2):

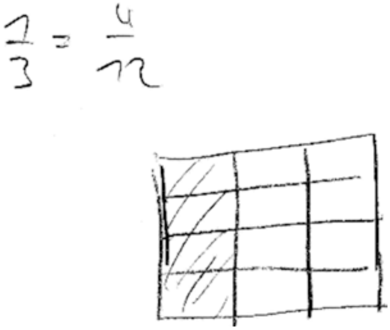

Fig. 13.2 Figures that Mr. A uses for expanding fractions (Schmitz, 2017, p. 188)

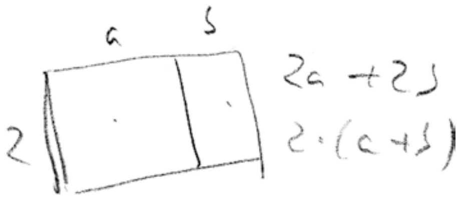

Fig. 13.3 Mr. A's visualisation of the distributive law (Schmitz, 2017, p. 218)

Mr. A: If I expand 1 over 3 with four then I start with such a drawing [he draws the shapes shown in fig. 13.2] […] and that what was 1 over 3—I can see this—Is now 4 over 12. […] thus students can see that expanding means to use a further fragmentation of the shape.

Mr. A: For many, this [addition of fractions with a common denominator] is not clear at first. If you just draw it, they see it immediately.

Mr. A: For multiplying, I use again the rectangles […]. One half of one third. I draw something what represents one third and, then, I draw the half of this.

Actually, with the exception of the procedure of dividing fractions, Mr. A uses visualisation to explain different procedures with fractions and also to legitimise the related formulas. His belief beyond his goal to use visualisation is that it is mandatory to have an image of procedures with fractions for understanding these procedures. For example, the second quote above includes this belief as a proposition that is understood to be true: "If you just draw it, they see it immediately."

However, this goal to use visualisation for understanding mathematical procedures is restricted to introducing these procedures:

Mr. A: Later, they have to deal with that without a drawing. They must be able to calculate without a visualisation.

When Mr. A was asked about his teaching of algebra, he referred, amongst others, to manipulating terms (Fig. 13.3):

Mr. A: To expand the term [2 • (a + b)] I use a visualisation […] Here, they calculate the area of the thing […], i.e. 2a plus 2b […] then they could see, aha, the distributive law.

Again, Mr. A uses visualisation to explain and to legitimise a manipulation of mathematical terms. Even when Mr. A teaches functions, one main aim of visualisation for him is to explain and legitimise the result of mathematical procedures, e.g. the computations of intersection points:

Mr. A: When I have a parabola and a line, what happens? If you have the parabola and the line in your sight it is obvious that it is possible to have no, one or two intersection points.

As mentioned for fractions and algebra, visualisation for Mr. A is a method to introduce procedures whereas the students have to deal with the procedures without visualisation later.

For Mr. A we obtain similar results for other goals and beliefs that are connected with visualisation. Thus, a main result of analysing the goals and beliefs that Mr. A holds about visualisation is the similarity for different mathematical domains.

We obtain similar results for the other teachers in our sample. Similar results in this respect mean that the teachers' perceptions of the five goals could differ considerably, but one teacher's goals are consistent for different mathematical domains. For example, Mrs. B neglects the goal of visualising procedures and, thus, her system of goals is considerably different to the system of goals of Mr. A. However, Mrs. B neglects visualising procedures consistently for teaching fractions, algebra, functions or calculus and, thus, this consistency is similar to the system of goals of Mr. A.

We consider one further example to illustrate the difference between teachers in perceiving a specific goal aspect like "understanding a mathematical procedure" and illustrate the consistency of this teacher's perception of this goal concerning different mathematical domains. For this, we consider the example of Mrs. C. This teacher is similar to the case of Mr. A since Mrs. C acknowledges the facilitating effect of visualisations for understanding mathematical procedures. In contrast to Mr. A, Mrs. C assigns to visualisation a more important role by emphasising that visualisation can have many facets supporting understanding a procedure instead of only legitimising a mathematical procedure. For example, in reference to the topic of functions, Mrs. C states the following:

Mrs. C: Then we have to introduce inverse functions […]. For this purpose, I use slides for an overhead projector that are prepared for keeling over. Doing this, you could see the reflection on the bisecting line.

Consistently to this illustrating example, Mrs. C uses visualisation also in other mathematical domains not only to legitimise a mathematical procedure, but to grasp a deeper understanding of the procedure.

Mrs. C: If the question is about a turning point or a saddle point you have to draw a graph in two columns and to draw the first and the second derivative function. Then you will see the crux of the matter and you are able to develop strategies for calculations.

Similar to the example of visualisation as a facilitator for understanding mathematical procedures, every teacher's goals concerning the other goal aspects (understanding concepts, reading/generating, remembering, motivation, see above) are consistent among the different mathematical domains. We finally illustrate this consistency by considering the case of one teacher, that is, Mrs. B, and referring to a further goal aspect, that is, the use of visualisation in terms of facilitating remembering (the first quote addresses the manipulation of equations, the second quote addresses the derivative).

Mrs. B: To draw these arcs is only an aid for remembering. These arcs could not make clearer why you have to proceed in this way, but this is an aid for remembering the rule.

Mrs. B: It is crucial to know that the derivative is represented by a tangent. I think, this is an aid for remembering how the criteria are, for example, for inflexion points.

13.6 Discussion and Conclusion

In this paper, we have given exemplary evidence that a mathematics teacher holds the same beliefs and goals—with the exception of one example in the results, we have not made a distinction between beliefs and goals in this paper—connected with visualisation in different mathematical domains like fractions, algebra, functions and calculus. We have shown this pattern only for two of five goals, yet we found this pattern for every goal aspect and also referring to a set of beliefs, including for example beliefs about the benefit of visualisation, the students' use of visualisation and even the teachers' preferences in using visualisation when working mathematically (Schmitz, 2017).

Our results are in agreement with the results of Stylianou (2010), who did not report a difference between teachers' beliefs regarding visualisation in the domains of fraction and algebra. Furthermore, our results agree with those of McElvany et al. (2012), who found—for domains different from mathematics—that teachers' beliefs are independent of different subjects when considering using visualisations.

Otherwise, our results concerning teachers' beliefs about visualisation differ considerably from our results concerning teachers' beliefs regarding a specific mathematical domain like calculus, geometry or statistics (cf. Eichler & Erens, 2015). A first explanation of this difference could be that visualisation is a subject that is independent from a specific content and is in some way a possible overarching goal of mathematics teaching. However, modelling could also be seen as an overarching goal of mathematics teaching (cf. Blum, 2011), but teachers seem to hold very different beliefs about modelling when different mathematical domains are regarded (cf. Girnat & Eichler, 2011).

Another explanation of the difference might be the role which an overarching mathematics related process holds in mathematical domains. Modelling can be a detached topic of mathematics teaching; visualisation rather is a means to an end for another goal like understanding a mathematical concept or for remembering a rule. For this reason, it is a hypothesis from our results that beliefs and goals referring to overarching mathematics related processes that could facilitate mathematical learning, like using visualisation or using technology, are perceived by a teacher consistently when different mathematical domains are regarded. In contrast, if a specific content of mathematical domains or content-specific topics like modelling are regarded, different teachers' beliefs for different mathematical domains might be expected. These considerations lead to hypotheses that should be further investigated by regarding teachers' beliefs and goals concerning mathematical processes like the use of technology or problem-solving in different mathematical domains.

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Chapter 14
Teachers' Beliefs About Knowledge of Teaching and Their Impact on Teaching Practices

Vesife Hatisaru

Abstract This study investigated secondary school mathematics teachers' beliefs about knowledge of teaching and its impact on their teaching practices. Two teachers participated in the study. Data were collected through an interview and classroom observations. The results indicated that the teachers' beliefs about the goal of mathematics education and about the importance of teachers' understanding the way students think about certain mathematics subjects had impact on their teaching practices. However, the teachers' teaching practices were also affected by the students.

14.1 Introduction

A growing body of documents suggests that teachers play a crucial role in student learning (e.g. American Council on Education, 1999; Mewborn, 2003) and their knowledge matters (e.g. Ball, Lubienski, & Mewborn, 2001). What teachers should know and understand and their knowledge of learning processes therefore are the focus of interest for educators (e.g. An, Kulm, & Wu, 2004; Even & Tirosh, 2002; Fennema & Franke, 1992; Shulman, 1986). Discussions of teacher knowledge, however, cannot be strictly limited to objective knowledge and should also include teachers' subjective knowledge (Liljedahl, 2008). In fact, "it is the teacher's subjective school related knowledge that determines for the most part what happens in the classroom." (Chapman, 2002, p. 177). Beliefs are accepted as one of the central aspects of subjective knowledge (Op 't Eynde, De Corte, & Verschaffel, 2002) and defined as anything that an individual accepts as true (Beswick, 2007).

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Teachers hold different types of beliefs each that may influence their teaching, including beliefs about mathematics, beliefs about the teaching and learning of mathematics, beliefs about students, etc. (Liljedahl & Oesterle, 2014). Although, for instance, teachers' beliefs about teaching and learning mathematics and their relationship to practice (e.g. Barkatsas & Malone, 2005), and teachers' beliefs about the source and stability of teaching knowledge (Buehl & Fives, 2009) have been discussed in research, much remains unknown about teachers' beliefs about teacher knowledge. This study examined secondary school mathematics teachers' beliefs about teacher knowledge and the impact on their beliefs on their teaching practices. The study focused on the teachers' beliefs about knowledge of teaching, a common denominator of teacher knowledge conceptualizations (Charalambous, 2015).

Philipp (2007) defines beliefs as "psychologically held understandings, premises, or propositions about the world that are thought to be true." (p. 259). The study adapted this definition by replacing "the world" with "knowledge of teaching". The following research questions guided the study: (1) What is the characteristics of teachers' beliefs about knowledge of teaching? (2) How do teachers' beliefs about knowledge of teaching and their teaching practices interrelate? The study is a part of a larger study exploring the interrelations between secondary school mathematics teachers' mathematical knowledge for teaching and students' learning outcomes. The participating teachers' knowledge for teaching and its impact on students' learning outcomes have been described elsewhere (Hatisaru & Erbas, 2017).

14.2 Relationship Between Beliefs and Practice

Teachers' beliefs lie "…at the very heart of teaching" (Kagan, 1992, p. 85) and play a critical role in their teaching practices (Aguirre & Speer, 1999). The relationship between teachers' beliefs and their classroom practices, however, is complex and subtle (Beswick, 2005) and is open to debate. Some research have indicated inconsistencies between teachers' beliefs and their practices (e.g. Raymond, 1997; Skott, 2001). The lack of consistency primary might be due to specific or social conditions of schooling (Ernest, 1989). A wealth of research, on the other hand, has indicated that beliefs held by teachers influence their teaching practices. Ernest (1989), for instance, suggests how a teacher's views of the nature of mathematics provide a basis for his or her mental models of mathematics teaching. Thompson (1992) cites that "…no description of mathematics teaching and learning is adequate and complete unless it includes consideration of the beliefs and intentions of teachers and students (Fenstermacher, 1980)." (p. 142). Pajares (1992) examines the nature of belief structures as outlined by prominent researchers and offers a synthesis of findings about the nature of beliefs. Summing up the research on teachers' beliefs, he states that "their [researchers'] findings suggest a strong relationship between teachers' educational beliefs and their planning, instructional decisions, and classroom practices," (p. 326). Barkatsas and Malone (2005) state that "Mathematics teachers' beliefs have an impact on their classroom practice, on the ways they perceive teaching, learning, and assessment," (p. 71). Liljedahl (2008) underlines that mathematics teachers' teaching practices are guided by what they believe to be true about mathematics and about the teaching and learning of mathematics.

Wilkins (2008) investigates the relationship between 481 in-service elementary teachers' mathematical content knowledge, attitudes toward mathematics, beliefs about the effectiveness of inquiry-based instruction, and use of inquiry-based practices. He finds that the teachers' beliefs have the strongest effect on their practice. Watson and De Geest (2005) report on a project to improve attainment in mathematics among low-attaining secondary students. They implement an action research with ten teachers over 2 years and find that the teachers hold shared beliefs including that all students can learn mathematics and that mathematics is intrinsically interesting, and the teachers' these beliefs guide their decisions and actions. Speer (2008) explores a teaching assistant's beliefs, practices and connections between them. Findings indicate that certain beliefs including beliefs about evidence of student understanding and about how learning happens are useful for investigating connections between beliefs and specific practices.

As cited above, researchers propose different categories of beliefs (e.g. Beswick, 2005; Ernest, 1989; Wilkins, 2008). An, Kulm, Wu, Ma and Wang (2006) categorize teachers' beliefs into four main aspects: "goals of education, primary focus on teaching mathematics, importance of teachers' knowledge, and planning for instruction" (p.452). They state that these aspects relate to each other as follows:

> With a set of goals of teaching in mind, teacher understand their primary focuses in teaching, design and use various approaches in classrooms, and try to find an effective teaching method in their teaching in order to … help all students to learn successfully. To teach effectively at a continuous base, a teacher should enhance… knowledge of students' thinking. Understanding students' thinking can be achieved through many approaches. One of the approaches is to know students' thinking through grading students' homework, in which the teacher can fully assess students' weaknesses and strengths and plan for further instruction according to students' needs (pp. 452–453).

Although many research indicate the impact of beliefs on teachers' instructional practice, much is unknown about how beliefs influence teachers' practice (Aguirre & Speer, 1999; Mansour, 2009; Speer, 2008). Accepting An et al.'s (2006) position, this study aimed to examine teachers' beliefs about knowledge of teaching and its impact on their instructional practices. According to An et al. (2006), knowledge of teaching "consists of knowing students' thinking, preparing instruction, and mastery of modes of delivering instruction." (p. 147). The study therefore focused mainly on certain components of the above mentioned four aspects of teachers' beliefs: beliefs about mathematics education, teacher knowledge, knowledge of students' thinking, approaches to assigning and grading homework, and planning for instruction (for all components, see An et al., 2006, p.453).

14.3 Methodology

Teachers' beliefs can be investigated through various approaches (Mosvold & Fauskanger, 2013). This study investigated teachers' beliefs through interviews. Two mathematics teachers teaching in a technical and industrial vocational high school in Ankara, Turkey voluntarily participated in the study. The teachers were named as Fatma and Ali (pseudonyms). Ali held a bachelor's degree in mathematics. For teaching profession, he got pedagogical formation for four months. He had 14 years' experience in teaching mathematics. Fatma had a bachelor degree in mathematics education. She had 25 years' experience in teaching mathematics. Neither of the teachers had any professional development on mathematics education. Ali and Fatma were teaching a ninth grade secondary school mathematics course in which 33 and 26 students were enrolled, respectively. The students in both groups were low achievers in mathematics.

14.3.1 Data Collection

An interview was conducted to obtain information concerning the participating teachers' beliefs about knowledge of teaching. The interview was framed based on the four aspects of teachers' beliefs addressed by An et al. (2006). The questions were adapted from An (2000). In questions 1 through 4, teachers' beliefs about mathematics education; in questions 5 through 7, teachers' beliefs about teacher knowledge; in questions 8 through 11, teachers' beliefs about knowledge of students' thinking; in questions 12 through 18, teachers' beliefs about approaches assigning and grading homework; and in questions 20 through 23, teachers' beliefs about planning for instruction were addressed (see Table 14.1 in the Appendix).

In building a valid interview, the interview schedule was systematically piloted and modified. Experts in education and in mathematics education were consulted for item clarity. Piloting was carried out with three secondary mathematics teachers from three different technical and industrial vocational high schools with emphasis on the clearness of the questions asked, the quality of the answers given and the time needed to complete the interview. The interviews were held in the school after the teachers' classes. It took them approximately one and half hours to complete the interview.

Classroom observations were critical to identify the influence of teachers' beliefs on teaching practices. A total of 18 classes were observed and audio taped in both teachers' classrooms. During this time, the classes were studying Functions including Defining a Function, the Domain and Range of a Function, Types of Functions, Linear Functions, Inverse Functions, Basic Operations on Functions, Composition of Functions, and Reading Graph of a Function. Throughout the observation, the researcher kept fields notes describing classroom activities. Teachers' interaction with students were also observed and noted.

14.3.2 Data Analysis

A constant comparative method of analysis was used (Glaser, 1965) to analyse the data. To answer the first research question, the interviews were transcribed. The responses of teachers were summarised according to the five components of teachers' beliefs that the study focused on. To answer the second research question, the data obtained from the classroom observations were analysed and compared and integrated with the interview data. The data obtained from the classroom observations were presented in Hatisaru and Erbas (2017). In the present study, the data obtained from the interviews were given.

During the analysis and assessment of the data, participants were contacted face-to-face to confirm their responses on the interviews and observations. The participants were handed a summary of the research findings, and they were asked to give consent to it.

14.4 Results

14.4.1 Beliefs About Mathematics Education

Fatma believed that "the main goal of mathematics education should be to give students different perspectives and to enhance their critical thinking and questioning skills." In this way, students would know that one problem could be solved in several different ways. She thought that vocational high school students have difficulty even in doing the basic arithmetic calculations. One reason for this is that the mathematical content was given in the order of Logic, Sets, Functions and Numbers. Students, however, should first be taught Numbers, and the aim of mathematics education in vocational high schools again should be "to help students develop different perspectives and gain thinking and questioning skills."

Ali stated that "the goal of mathematics education is to help students gain problem-solving skills." The problems that he mentioned, however, were doing numerical calculations or finding out how much change to get back while shopping in daily life. He thought that entering a profession is the priority for most vocational high school students. For this reason, a major objective of vocational high schools should be to teach the basic mathematical concepts and arithmetic operations. He contended that the mathematics curriculum of vocational schools should be different and should include topics such as equations and length measurements. For example, students studying electric and electronic technology need to learn the linear measures to calculate the length of a cable to be laid at a place. For Ali, "to teach students many additional topics does not make much sense. This would be more difficult both for students and teachers."

14.4.2 Beliefs About Teacher Knowledge

Fatma thought that a mathematics teacher should have complete mastery of the field. This is important in “explaining the rationale of the mathematical rules.” However, to have good knowledge of mathematics is not enough for a teacher. A mathematics teacher should also “know how to teach mathematics.” S/he should continuously refresh his/her knowledge and teaching skills. When possible, a mathematics teacher should also know the practical rules about certain content.

Ali thought that a mathematics teacher should have “a complete mastery of his/her field.” This is important in “dealing with questions students may ask in class.” However, there may be some subjects the teacher does not know very well or have forgotten. The teacher should be comfortable at such times. Looking at the subject would suffice to remember the subject. He also believed that a mathematics teacher should be able to “predict which aspects of a subject the students will find difficult or make mistakes at.” If the teacher can do so, s/he can spend more time on these issues in class. Moreover, “a good mathematics teacher should have a good mathematics book resource.” Otherwise, the students may believe that the teacher asks questions from one resource book and may want to obtain that book. He was also in the belief that a mathematics teacher should not behave as if s/he knows everything and can solve any question. There may be some subjects that the teacher does not really know or remember. When this is the case, the teacher should look natural and display a “let’s search this and do it together” kind of attitude.

Fatma believed that teachers can accomplish professional development through in-service training seminars. However, these seminars are mostly about education. Therefore, teachers can develop their mathematics knowledge through mathematics books. For Ali, the teacher can develop his or her mathematics knowledge by solving questions. The teacher’s knowledge of students’ level, however, will develop in the actual class. The questions students frequently ask in class, for example, may help him or her understand what they find difficult.

14.4.3 Beliefs About Knowledge of Students’ Thinking

Fatma believed that “it is very important that a teacher is aware of how his/her students think about a topic and what kind of difficulties they have about it.” If the teacher has such awareness, s/he can know how to teach the subject or the concept. Nevertheless, she thought that it is quite difficult to understand how the students think, because they cannot express clearly their ways of mathematical thinking. Therefore, she pointed out that she has difficulty in understanding the way students think. She said, sometimes she has the students come to the board and ask them to elaborate what exactly they did not understand. She explained that sometimes she spends 10–15 min of class time trying to find out the problem of a single student. Yet she generally finds it difficult to pinpoint the areas of difficulty for students.

Ali also highlighted the importance of knowing the way students think about a certain subject or the difficulties they experience studying this subject. If the teacher has such awareness, s/he can better explain the subject, make the concept more concrete, or have the students solve additional questions. Ali thought that "it is up to the students whether a teacher knows their way of thinking or not." If the students raise questions during class, the teacher can see what they have or they have not understood. He said that he perceives the students' perspectives or what they could or could not learn "from the questions they ask." Especially when the same question is asked by different students, he accepts that the students have problems with that subject. When this is the case, he gives further examples about that content and tries to explain it again making it concrete. When students do not ask questions, he just explains the content and proceeds to the next topic. For him, "another way of following how the students think and how much they have learned is looking at the exam results." If most of students have responded to certain items incorrectly, then it means there is a problem. When this happens, he solves these questions after the exam in the class, or devotes 10–15 min of class time to explain the topic again.

14.4.4 Beliefs About Approaches to Assigning and Grading Homework

Fatma stated that she assigns homework from the course book. For her, the purpose of assigning homework is twofold: "to make students take on responsibility and to have them revise what they have learned in class." She does not apply any restriction when assigning homework; the students are supposed to do the questions that are related to the content covered in the class so far including the things done on that day. She, however, thought that only a minority of students do homework. She said she checks homework during the first few weeks, but later she does not do so even if she believes in the necessity of it. When doing homework check, she looks at whether students did their homework or not. Because it will be too time-consuming, she does not check the accuracy of the answers. She asks the students which questions they could not do, and solve these questions herself on the board.

Ali said he assigns some activities in the book as homework. His aim is to have students open their books at home and study, rather than identify how much they have learned. When selecting the homework material, he prefers the ones that are relatively easy for students. He prefers to solve the questions that are likely to be difficult himself in class. He usually assigns five to ten questions. He never assigns more than ten questions. To make the students take homework seriously, he chooses some exam questions among those homework questions. He checks homework. However, for him, it does not really matter whether students do homework or not. All that matters are the accuracy of the results the students have found and which questions they have generally failed to do.

14.4.5 Beliefs About Planning for Instruction

Fatma pointed out that she made lesson plans regularly in the first 10 years of her teaching profession. However, once she had gained enough experience, she quit making lesson plans. Now she only makes yearly lesson plans, rather than daily lesson plans. For her, making lesson plans is important. When a teacher makes a lesson plan, it is unlikely that s/he accidentally skips any point. For her, however, “A lesson plan only means transferring the things in one’s mind onto paper. What is essentially important is that a teacher should exactly know what s/he will cover in class.”

She stated that “a lesson plan should include the related definitions and questions to be solved.” Yet she decides how a class should develop according to the students, rather than the lesson plan. She decides to move on or not according to how much the students have grasped the topic. For example, sometimes she plans to solve some problems in the class, but does not actually do it because of the students. Sometimes she solves extra questions when students have difficulty understanding a topic. Or sometimes she does not mention a detail about a topic though she was planning to do so, thinking that it might be confusing for students. Sometimes she does the contrary; she mentions a relevant detail thinking that the students need it.

Ali stated that “The course contents are outlined in the annual plans. If teachers feel incompetent about content, they should make daily lesson plans. Otherwise, there is no need to make lesson plans.” He himself generally does not make lesson plans. He plans what to do and which questions to solve in mind. For him, the course book explains the topics anyway, so he does complementary activities for the topics that are not thoroughly understood. However, when he feels incompetent about a certain topic, he makes lesson plans. His lesson plans include “the questions that are to be solved in the class and some practical rules.” Leaving from the idea that the aim of the course is to prepare students for exams, he mostly gives place to possible exam questions in his lesson plans. Besides, he spends some time, little as it is, giving examples from daily life, if possible. For example, when he explains “derivation”, he gives examples from ‘tension’ at bridges. He usually implements his classes as he has planned. Nevertheless, the questions students ask can change the general flow.

14.5 Discussion and Conclusion

This study examined secondary school mathematics teachers’ beliefs about knowledge of teaching and its impact on their instructional practices. The study followed An et al.’s (2006) framework and categorized teachers’ beliefs into four main aspects: goals of education, primary focus on teaching mathematics, importance of teachers’ knowledge, and planning for instruction. The study focused on teachers’ beliefs about knowledge of teaching, and therefore explored certain components of these four aspects: that is, beliefs about mathematics education, teacher knowledge, knowledge of students' thinking, approaches to assigning and grading homework, and planning for instruction. The study found that the teachers' beliefs had an impact on their teaching practices, as has been documented in previous studies (e.g. Barkatsas & Malone, 2005; Speer, 2008; Wilkins, 2008). Ali believed that the goal of mathematics education was to develop students' procedural skills, whereas Fatma believed that the key aim of the mathematics education should be enhancing students' logical and critical thinking skills, as well as procedural skills. In their teaching practices, Ali mostly focused on the procedural aspects of functions. His main goal was to teach students doing operations regarding functions, such as finding the range set of an algebraic function the domain of which is given or evaluating algebraic functions for specific points. Unlike Ali, Fatma tried to achieve both students' conceptual understanding and procedural development in relation to functions. She gave different examples, used analogies, and posed different levels of questions to promote students' ability to think (for more detail, see Hatisaru & Erbas, 2017).

Results revealed, especially for Fatma, it was very important for mathematics teachers to have a profound knowledge of mathematics to be able to explain the reasons of facts, rules, or procedures. Unlike Ali who mostly solved questions on functions, in her instruction, she usually used analogies, provided more detailed and diverse explanations, and made connections among concepts (e.g. between the concept of relation and function). Fatma indicated that teachers can enhance their knowledge through in-service training seminars, which she finds limited to the field of education. Like Ali, therefore, she thought that mathematics teachers can gain knowledge from independent studies, such as using books and/or solving questions.

Teachers' approaches for professional development may vary in different cultures. An et al. (2006) reported that most of their participating teachers in the USA enhance their knowledge through in-service trainings and workshops or from independent studies, and some of them gain knowledge from college study or sharing with colleagues. As to the participating teachers in China, most of them develop their knowledge from independent studies or continuing education in college, and some of them improve by sharing with colleagues or observing each other's instructions. In Turkey, among teachers, there is not a culture of observing each other's classes or sharing. The Ministry of National Education provides in-service training seminars to teachers, but like Fatma indicated, these seminars are limited to specific areas. The 2017–2023 Teacher Strategy Document, published by the Directorate of Teacher Education and Development, supports this result. One of the themes of this strategy document is continuing professional development. The document addresses enhancing the quality of teacher development activities and states that more and varied trainings should be organized by taking teachers' individual needs into account to ensure continuity of teachers' personal and professional development.

Related studies reveal that having the knowledge of students' thinking is essential for planning and teaching (e.g. Even & Tirosh, 2002; Hiebert et al., 2007). It is believed that such knowledge significantly contributes to the teachers' instruction (Even & Tirosh, 2002; Fennema & Franke, 1992) and influences what students learn from the instruction (Fennema & Franke, 1992; Hatisaru & Erbas, 2017). Results of this study showed that the participating teachers believe in the importance of teachers' understanding of the way students think about a certain mathematics subject or the difficulties they experience with it. Ali said he gauges students' thinking from their questions, whereas Fatma knows the same from students' explanations. In their lesson implementations, Ali covered the related concept and proceeded to the next one, each time asking students whether they have any questions or not to check their understanding. However, his students' participation was mostly limited to passive listening and taking notes. To gauge the students' thinking, Fatma dwelled on the concepts through questions, but the disinterest and misbehaviours of some students disrupted the discussion environment many times. Whenever this happened, she covered the respective content superficially. Consequently, many times, the students missed to learn important things, and Fatma missed the opportunities to drive fruitful discussions to understand students' weaknesses regarding functions (See Hatisaru & Erbas, 2017, for a detailed argument for the teachers' classroom practices).

Both the interview and classroom observation results showed that the participating teachers do not believe the necessity of making written lesson plans. Fatma considered students' needs as the basis of lesson implementation, whereas Ali thought course books. Ali mostly followed the course book in his instruction. Unlike most of the Chinese teachers in An et al.'s (2006) study who understand students' thinking by checking students' homework, in this study, the teachers' purpose of assigning and checking homework was to review and practise. If applicable, the teachers graded students' homework by completion. The teachers reported that most of the students in both classes however typically did not do their homework. The teachers therefore did not assign homework in general. As stated in the methodology section, students in both teachers' classes were low achievers in mathematics. Most of them were also not engaged in learning and motivated to learn mathematics. These attitudes did not help them to progress in learning mathematics and nor did it help the teachers to enhance their own learning. These results revealed that the teachers' beliefs had an impact on their instruction, but other factors including classroom situation (Barkatsas & Malone, 2005), the social context (Ernest, 1989; Mansour, 2009) and the students in the context of this study could affect their teaching practices to a greater extent than their beliefs.

Appendix

Table 14.1 The interview questions

Beliefs about mathematics education	1. What is the goal of mathematics education from your point of view? 2. Do you think the goal of mathematics education in technical and industrial vocational high schools is different from other schools? How? 3. What sort of mathematics education do the students in technical and industrial vocational high schools need?
Beliefs about teacher knowledge	4. What types of professional knowledge should a teacher of mathematics have? 5. How important is it for teachers to have this knowledge? 6. How do the teachers continue to enhance their professional knowledge?
Knowledge of students' thinking	7. In what way is it important for a teacher to know how his students approach and understand a particular mathematical content? 8. How does the teacher know about his students' thinking and understanding of this particular content? 9. How do you know about your students' thinking and understanding of this particular content? 10. Do the results of assessment affect your teaching? 11. If yes, how do you reflect your assessment about students' cognition in your teaching?
Teachers' approach to assigning and grading homework	12. Do you assign homework to your students? (If yes) How often? 13. How do you decide what problems to assign to students? 14. How many problems do you assign to your students each time? 15. What is the purpose of assigning homework to your students? 16. Is it important that your students do homework? Why? 17. Do you grade your students' homework? (If yes) How? 18. How do you deal with mistakes in students' homework?
Planning for instruction	19. How do you plan your instruction? Do you write a lesson plan weekly or daily? 20. What is the focus of your lesson plan? 21. How important is it for you to follow your lesson plan? 22. Is there any class in which you do not stick to your plan? 23. If yes, give some examples of when you turn out different from your initial plan.

Note: The interview questions are adapted from An (2000).

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Chapter 15
Positive Education and Teaching for Productive Disposition in Mathematics

Aimee Woodward, Kim Beswick, and Greg Oates

Abstract The Australian Curriculum: Mathematics defines four proficiency strands. The work from which they are drawn includes a fifth proficiency (productive disposition) that relates to students' propensity to persevere and to perceive mathematics as worthwhile. We argue for the importance of productive disposition as reflecting the importance of affect in mathematics learning. We link it with work in positive education, particularly around character strengths, to suggest ways in which mathematics teachers' awareness of the importance of affect might be raised. Positive education may offer a means of putting productive disposition on the agenda in considerations of improving mathematics achievement.

15.1 Mathematical Proficiency

Kilpatrick, Swafford, and Findell (2001), in their seminal work on what it means to be mathematically proficient, described the qualities with respect to mathematics that they believed students should develop as a result of studying mathematics at school. They defined mathematical proficiency in terms of five interdependent aspects: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. Mathematical proficiency requires, in their view, all of these components working together. Crucially, they claimed that mathematical proficiency was as, if not more, important for the teacher of mathematics than for the student, and linked this to the need for teachers to be effective and versatile: Effective in terms of assisting students to learn worthwhile content; and versatile in terms of working effectively with a range of students, environments and content. In this paper we consider how mathematical proficiency is portrayed in the Australian Curriculum: Mathematics (AC: M) (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2016) and elsewhere. The term, disposition is frequently used without explicit definition but implicitly to mean attitude (e.g. Moyer, Robison, & Cai, 2018)

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where attitude refers essentially to a positive or negative assessment of an entity (Ajzen & Fishbein, 1980). We therefore situate productive disposition within the mathematics education research on affect, and consider how ideas from positive education, in particular character strengths, might influence mathematics teaching and assist in the development of students' productive dispositions.

The proficiency strands of the AC: M are based on and similar to the proficiencies described by Kilpatrick et al. (2001). Table 15.1 provides a summary of the four proficiencies common to Kilpatrick et al. (2001) and the AC: M. Problem-solving in the AC: M differs slightly from Kilpatrick et al.'s (2001) strategic competence, with no explicit reference to flexible and novel approaches, but instead calling for the application of existing strategies in seeking solutions. The most obvious difference is the absence of productive disposition among the proficiencies of the AC: M. Kilpatrick et al. (2001) claimed that productive disposition develops as students are engaged in solving problems, reasoning, and developing understanding and fluency, and is also a necessary precursor to the development of the other proficiencies.

Kilpatrick et al. (2001) defined productive disposition as the "habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy" (p. 26). Students with a productive disposition are motivated, confident about their knowledge and ability, see mathematics as sensible, and have a growth mindset concerning their capacity to learn mathematics, believing that effort will lead to success (Kilpatrick et al., 2001). Productive disposition

Table 15.1 Mathematical proficiencies in Kilpatrick et al. (2001) and the Australian Curriculum, v8.3

Proficiencies	Australian curriculum: mathematics	Kilpatrick et al. (2001)
Understanding	Build a robust knowledge of mathematical concepts and be able to adapt, connect and represent this knowledge in familiar and new ways	Conceptual Understanding: develop an integrated and functional comprehension of mathematical content and ideas
Fluency	Develop skills to recall definitions, facts and procedures and to calculate answers efficiently by the selection of appropriate methods	Procedural fluency: develop the "knowledge of procedures … when and how to use them appropriately and [the] skill in performing them flexibly, accurately and efficiently" (p. 121). Knowledge of effective ways to estimate
Problem-solving	Develop skills to make choices, design, interpret, formulate and model familiar and unfamiliar problem situations and to communicate verifiable solutions effectively	Strategic Competence: develop the ability to flexibly formulate, represent and solve mathematical problems. Key focus on the formulation of problems not just solving
Reasoning	Develop logical thought and actions, including analysing, proving, adapting, explaining, inferring, justifying and generalising	Adaptive Reasoning: Capacity to logically consider relationships among concepts and situations, focus on justification of methods and solutions appropriate to the task

concentrates on affective, rather than the cognitive influences on learning, encompassing positive attitudes and beliefs about mathematics, how it is learned, and one's capacity to learn it. It can be regarded as comprising four aspects related to (1) the utility and (2) value of mathematics; (3) self-efficacy and (4) diligence.

Concerns have also been expressed in the United States about the lack of explicit mention of productive disposition (beyond a comment in the introduction) in the Standards for Mathematical practice associated with the Common Core State Standards for mathematics (Grady, 2016). In addition, Andrews (2010) noted that, although the five proficiency strands of Kilpatrick et al. (2001) are reflected in Finnish curricular guidelines, observable evidence of teaching for productive disposition was absent in the four classrooms observed in that country.

15.2 Affect and Mathematics Learning

The attitudes and beliefs (i.e. propositions regarded as true) of both teachers and students have been of interest to mathematics education researchers because of their association with students' mathematics achievement, usually considered in terms of standardised tests or grades (e.g. Ma & Kishor, 1997), and the role they play in teachers' practice. Mathematics educators have struggled to find consensus on the conceptualisation of, and distinctions and relationships among these and other aspects of the affective domain. Hannula (2012) proposed a three-dimensional meta-theory, for organising research on affect in mathematics education. The three dimensions concerned (1) the aspect of affect (e.g. attitude, emotion), (2) whether the affect was considered a trait or state, and (3) whether it was considered a biological, psychological or social phenomenon. The metatheory illustrates the complexity of studying mathematics-related affect. Figure 15.1 shows how the Organisation for Economic Cooperation and Development (OECD) (2016) represented what they considered the most important relationships among affective variables, beliefs, perseverance behaviours, and academic performance. The four aspects of productive disposition can be seen in aspects of the representation. For example, beliefs in the usefulness and of mathematics and that it is worthwhile, as well as beliefs in one's capacity to learn and do mathematics fit in the rightmost box and influence, by way of motivation, the time and effort (i.e. diligence) that students apply and hence academic performance.

In the sections that follow, we provide a brief review of the literature on the relationship of each of student and teacher attitudes and beliefs to mathematics achievement, with a focus on Australian students who are experiencing the AC: M.

15.2.1 Students' Attitudes and Beliefs

Students' attitudes to mathematics positively correlate with achievement (Ma & Kishor, 1997; Thomson, Wernert, O'Grady, & Rodrigues, 2017) but attempts to determine causation have led to the conclusion that affect and achievement interact

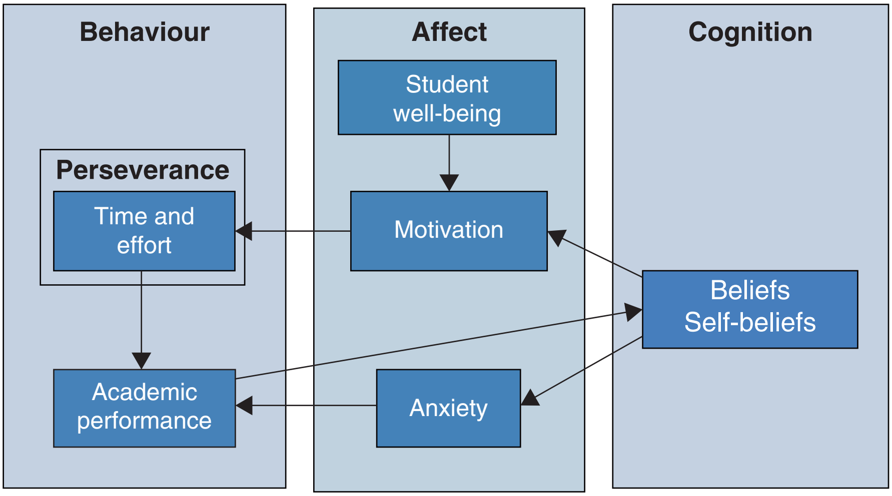

Fig. 15.1 A simplified conceptual map showing the interplay of students' attitudes, beliefs and academic performance (Source: OECD, 2016)

in complex and reciprocal ways (Hannula, 2012). For example, Ma and Kishor (1997) found, from a meta-analysis of 113 studies, weak evidence of causation directed from attitude (encompassing tendencies to like/dislike mathematics, engage with/avoid the subject, consider oneself good/bad at mathematics, and regard mathematics as useful/useless, easy/difficult and important/unimportant) to achievement but the effect sizes were small. Other aspects of affect, including particular dimensions of attitude, self-confidence and beliefs, have also been associated with achievement in mathematics but causal connections are yet to be explored.

The 2012 PISA results for mathematical literacy showed that each of; students' intrinsic motivation, self-concept (i.e. believing that one is good at mathematics), self-efficacy with respect to specific mathematical tasks (i.e. believing that one can succeed with a task), instrumental motivation, (i.e. believing that mathematics is important for such things as finding employment), and the tendency to take responsibility for their own mathematics achievement, were correlated with achievement (Thomson, de Bortoli, & Buckley, 2013). All of these constructs are related to productive disposition: Self-efficacy is a key part of it, instrumental motivation relates to the utility and valuing aspects of productive disposition, and self-concept and taking responsibility are connected to belief in the value of diligence for mathematical achievement. The relationships reported by Thomson et al. (2013) applied to Australian 15-year-olds and across the OECD. Australian students scored similarly to or above the international average on these measures and 90% indicated that they believed that putting in effort (diligence) would result in success in mathematics. Nevertheless, approximately 60% of Australian 15-year-olds reported worrying that their mathematics classes would be too difficult, reflecting a lack of self-efficacy. In addition, mathematics anxiety was negatively associated with achievement (Thomson et al., 2013). The OECD (2014) reported that students who are open to solving problems performed higher on average than other students. Such students believe they "can handle a lot of information, are quick to understand things, seek explanations for things, can easily link facts together, and like to solve complex problems" (OECD, 2014, p. 18). The difference was greater for high achieving students. Nevertheless, in many high performing countries students scored below the OECD average on openness to problem-solving (OECD, 2014). Regardless of achievement, it is a concern that 30% of students in PISA 2012 reported, "that they feel helpless when doing mathematics problems" (OECD, 2014, p. 18) again reflecting low self-efficacy in relation to mathematics.

15.2.2 Teachers' Attitudes and Beliefs

Kilpatrick et al. (2001) emphasised the need for teachers of mathematics to be mathematically proficient themselves. It is established that teachers must know and understand the content that they teach (Ball, Thames, & Phelps, 2008). According to Kilpatrick et al. (2001) other aspects of mathematical proficiency can be understood for teachers in terms of procedural fluency in performing classroom routines, strategic competence in planning and solving problems that arise during teaching, and adaptive reasoning in articulating and reflecting on practice. If teachers are to develop productive dispositions in their students, they must themselves have productive dispositions towards the discipline and its teaching and learning (Kilpatrick et al., 2001). That is, they must believe that they and all their students can learn mathematics and improve in their ability to do so; that mathematics is intelligible, and that they can improve their teaching of mathematics as well as their understanding of the subject through effort.

There is evidence that many teachers of mathematics do not have productive dispositions to the subject and/or its teaching. Primary pre-service teachers commonly exhibit unease with the discipline (Kalder & Lesik, 2011). They often fear and dislike mathematics and are unlikely to have developed adequate understanding (Beswick & Callingham, 2014). Beswick and Callingham (2014) also showed that in-service teachers of mathematics are less likely than mathematics teacher educators to regard problem-solving as inherent to mathematics, but more likely to do so than primary pre-service teachers. Secondary mathematics teachers seem not to regard the proficiency strands that are included in the AC:M, other than fluency, to be teachable, but rather as distinguishing characteristics of capable and struggling students (Beswick, 2017).

Teachers' beliefs and attitudes matter for their students' affective and achievement outcomes. For example, teachers' beliefs about the nature of mathematics, and the teaching and learning of mathematics lead to differences in classroom environment that are discernible to students (Beswick, 2005) and there is evidence that these sorts of differences can have long term impacts on students' perceptions of their mathematical competence (i.e. self-concept and self-efficacy) and how they regard the utility of mathematics (Moyer et al., 2018). Sakiz, Pape, and Hoy (2012) showed that students' perceptions of the affective support provided by their teachers, defined in terms of listening, respect, recognition and fairness, were associated with greater academic enjoyment, self-efficacy and effort. Teachers' beliefs about their students' ability to succeed and their own ability to influence student learning are associated with improved student mathematics achievement (Archambault, Janosz, & Chouinard, 2012). Data from PISA 2012 showed that better teacher–student relationships were associated with better engagement with school and with learning while at school, which in turn were associated with higher performance (OECD, 2014).

15.3 Positive Education

The strong social, emotional and academic components of teaching and learning (Zins, Weissberg, Wang, & Walberg, 2004) have led to international interest in positive education models as evidenced by the International Positive Education Network (IPEN, n.d.). Current research on mental wellbeing has been derived from two general perspectives: the hedonic approach, which focuses on happiness and defines well-being in terms of pleasure attainment and pain avoidance; and the eudaimonic approach, which focuses on meaning and self-realisation and defines well-being in terms of the degree to which a person is fully functioning (Clarke et al., 2011). Key ideas that underpin positive psychology include well-being theory (Seligman, 2011), self-determination theory (Deci & Ryan, 1985), broaden and build theory (Fredrickson, 2001), and growth mindset (Dweck, 2006). Over the past decade, school-based programmes grounded in positive psychology have aimed to cultivate positive states including resilience, optimism, hope, gratitude, mindfulness and perseverance. Well-being curricula have produced positive results for school climate, student autonomy and influence, learning and attainment (Durlak, Weissberg, Dymnicki, Taylor, & Schellinger, 2011). Green (2014) argued that positive education is best when concepts are applied meaningfully and practically to students' academic and personal lives. One strand of positive education that we believe offers potential for assisting mathematics teachers to develop their own and their students' productive dispositions concerns character strengths.

15.3.1 Character Strengths

Peterson and Seligman (2004) defined character strengths as psychological ingredients that define virtues. Virtues are characteristics that have been valued by moral philosophers and religious thinkers, across time and cultures. Neither talents nor abilities are components of character strengths, due to key differences in value across cultures. Park and Peterson (2009) argued that "attention to young people's character is not a luxury for our society but a necessity, and it requires no trade-off with traditional academic goals" (p. 8). Indeed, some curricula, such as that of Australia, contain references to character-related aspects. For example, the Australian Curriculum includes Personal and social responsibility and Ethical understanding among general capabilities that the curriculum is intended to address. The character strengths are not traditional academic areas of success or weakness, such as "your strength lies in English, not mathematics" nor are they at odds with the character-related aims of curricula such as that of Australia. Peterson and Seligman (2001) identified six virtues with which they aligned 24 character strengths—the means by which one achieves virtue. Table 15.2 defines the character strengths aligned with each of the virtues.

Research on relationships between various character strengths and educational outcomes has shown positive connections, although links to academic achievement at the secondary level are rare. For example, Weber, Wagner, and Ruch (2016) found that love of learning, perspective, zest and gratitude all showed a replicable association with school achievement. Shoshani and Slone (2013) found that grade point average could be predicted by the strength of temperance. Madden, Green, and Grant (2011) found that a strengths-based coaching programme was associated with increases in students' self-reported levels of engagement and hope, and Choudhury and Barooah (2016) showed significant correlations of academic achievement with both humour and social intelligence.

15.4 Character Strengths and Productive Disposition

Despite the lack of prominence afforded productive disposition in curriculum documents and practice in many mathematics classrooms (e.g. Andrews, 2010), teachers can explore this area in their own contexts, using research into the development of characteristics that align with increased participation in learning and that facilitate the development of confident, capable and flexible learners. We argue that many of the character strengths align with aspects of productive disposition and that an awareness of students' character strengths can allow teachers to afford opportunities for students to exercise their favoured strengths, and for less-utilised strengths to be addressed. Similarly, teachers' awareness of their own favoured strengths can inform reflection on the extent to which they have a productive disposition towards mathematics, and the ways in which they interact with particular students (those with similar and very different character strengths profiles to their own). Student awareness of teacher strengths could contribute to meaningful, supportive dialogue in the classroom and a powerful way to develop positive teacher–student relationships with consequent benefits for engagement and achievement (OECD, 2014). In the following paragraph we provide initial illustrative examples of how a focus on character strengths could be used to reinforce findings from mathematics education research.

Boaler (2013) discussed the importance of teachers and students having growth mindsets in relation to mathematics learning, the role of open tasks to this end and

Table 15.2 Virtues and character strengths (Peterson & Seligman, 2001)

Virtue	Character strength	Definition
Wisdom and knowledge	Creativity	Novel and productive approaches to activities
	Curiosity	Interest, exploration and discovery of new knowledge and experience
	Open mindedness	Balanced and fair judgements
	love of learning	Seeking new knowledge and skills
	perspective	Wise counsel to oneself and others
Courage	Bravery	Acting upon convictions, not retreating from threat, challenge or difficulty
	Persistence	Finishing what one starts, manoeuvring through obstacles
	Integrity	Being genuine and responsible for one's feelings and actions
	Vitality	Zest, energy
Humanity	Love	Valuing relationships where sharing and caring are mutual
	Kindness	Doing good for others
	Social intelligence	Awareness of the motivations of others and oneself
Justice	Social responsibility	Citizenship; working effectively as a member of a group
	Fairness	Unbiased treatment of others
	Leadership	Encouragement of good relationships within a group
Temperance	Forgiveness	Mercy; acceptance of others' mistakes
	Humility	Humbleness
	Prudence	Self-regulated decision-making
	Self-regulation	Self-disciplined in thought and action
Transcendence	Beauty and excellence	Appreciation of skill and beauty in others and the environment
	Gratitude	Aware and thankful of good things
	Hope	Working to a better future
	Humour	Seeing and sharing the lighter side of life events
	Spirituality	Sense of purpose

the damage that ability grouping can do to students' self-efficacy beliefs. These ideas link to character strengths of creativity, curiosity, open-mindedness, and love of learning. Bravery and persistence are character strengths that align with research by Sullivan and colleagues (e.g. Sullivan & Mornane, 2014) on the use of challenging tasks in teaching mathematics. Metacognition is related to the character strength of self-regulation and has been found to be enhanced when students are tasked with teaching one another (Muis, Psaradellis, Chevrier, DiLeo, & Lajioe, 2016). The act of teaching another draws upon the character strength of perspective. Liking mathematics, motivation, self-efficacy and self-concept in relation to mathematics, and taking responsibility for one's performance are among affective characteristics associated with higher mathematics achievement (OECD, 2016). They have clear connection to the character strengths of love of learning, curiosity, hope, persistence, and bravery. Appreciation for the inherent beauty and value of mathematics is a worthy aim of mathematics education (Romberg & Kaput, 1999) that aligns with the character strength of appreciation of beauty and excellence.

15.5 Conclusion

We have highlighted how within the affective domain, student well-being is identified in the literature as an important component of student attitude and performance, but at the same time such factors are seldom explicit in curriculum documents or their recognition is difficult to identify in classroom practice. When viewed through the lens of positive education, character strengths are linked to students' positive disposition towards mathematics. It could be argued that character strengths could have similar value in promoting productive dispositions towards any school subject and this may well be so, but productive disposition in mathematics, as defined by Kilpatrick et al. (2001), is inherently mathematical: The construct might appear differently in other subjects. In addition, we know that mathematics evokes negative affect that inhibits productive disposition, in many students (OECD, 2014) and so efforts to address productive disposition (in mathematics), including via character strengths is of particular importance. Existing research points to the value of some character strengths for achievement but the potential impacts of others remain unexplored. Little is known about the relationships between character strengths in teachers, the ways in which they teach, and the impacts on students' affective traits (including character strengths), and aspects of attitude and beliefs known to be associated with achievement. These discussions highlight two principle areas that warrant future research and are indeed the subject of a study being undertaken by the first author. These are the extent to which mathematics' teachers are aware of, and seek to build positive disposition within their students, and the value of character strengths in achieving this. As we have argued here, we believe strengthening the understanding and position of productive disposition within the AC: M, and providing teachers with tools by which they might develop this may have real benefits for students' mathematical learning.

Although we have focussed on the Australian context, concern for students' affective responses to mathematics is international, with curricula in many other countries (e.g. USA (Grady, 2016) and Finland (Andrews, 2010)) encompassing in some way Kilpatrick et al.'s (2001) notion of mathematical proficiency. Although further research is needed to examine in detail the ways in which teachers might use aspects of positive education to build their own and their students' productive dispositions in relation to mathematics, we believe that the approach as potential to do so while, at the same time, enhancing students' and teachers' well-being.

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Chapter 16
From Relationships in Affect Towards an Attuned Mathematics Teacher

Manuela Moscucci

Abstract At CERME9 the author and Bibbò presented "relationships", as an interesting construct for affect research and began a 'sort' of characterization of the term "relationship", aimed at setting up a "sort" of implicit definition of the term, borrowing the process from the construction of an axiomatic theory in mathematics. This paper deals with the study that followed, and the correlating presentation of a mathematics teacher figure as an "attuned mathematics teacher". Both the issues refer to the Attachment Theory by Bowlby, and Neuroscience results.

16.1 Introduction

During the last decade, the complexity of the affect research field emerged, in a stronger and stronger way, even if it was clear since the early 1980s. The list of the well-known historical constructs—including beliefs, emotions, attitudes and values—has been extended with new ones such as motivation, self-esteem and anxiety. This trend was supported by many affect researchers who declared the opportunity of expanding and rethinking the whole field, and, in the meantime, highlighted the need of studying the correlations among constructs (Hannula, Evans, Philippou, & Zan, 2004). The proposal to study the construct r*elationships*, made by Moscucci and her collaborator Bibbò in 2015 at CERME9 (Moscucci & Bibbò, 2015), seems to be consistent with those needs and it is absolutely rational inside the affect research, as argued in that paper. On that occasion Bibbò and I declared to have the intention of facing the relationships issue in the nearest future, and with this paper I intend to present some of the following studies.

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16.2 A New Element in the Study of Relationships

The deepening of this issue led me to consider the Attachment Theory (AT). This theory seems to be useful not only for continuing the already begun characterization of the term relationship. Studying AT and linking it to other studies and results of neuroscience and psychology, present two possibilities, that are absolutely correlated and, in my opinion, very interesting. The first is to hypothesize an explanation of a strange phenomenon that I found during my work as a mathematics education teacher. I have met very successful mathematics teachers, without particularly high professional competences but having surprising success. I noticed that those mathematics teachers had common particular personal characteristics. One of them was Rose, a primary mathematics teacher, mentioned also elsewhere (e.g. Moscucci & Bibbò, 2015), who piqued my curiosity in the mid-1980s. As a result of considering her case, the second possibility arose, that is of defining a particular figure of mathematics teacher, as an *attuned mathematics teacher*, a mathematics teacher with a particular educational role, beyond the usual one. This is a first study that will be continued and deepened. However, I chose to present both possibilities in this context, with the aim of sharing with other researchers as soon as possible this opportunity, for a communication focusing on the service of the affect community and the mathematics teacher community.

16.3 About Relationships

As proposed by Moscucci and Bibbò (2015), the problem of the definition of the term relationship may be faced as follows. Current neuroscience is not able to define relationships from a scientific point of view. We therefore assume the term as if it were a sort of primitive term in a somehow axiomatic theory and we care to investigate its nature so as to provide an increasingly articulated description of the qualities of an interpersonal relationship. In this way we intend to bring forward a characterization process of some kind, through which it is possible to contribute to developing an implicit definition of the term relationship in absence of an explicit definition by neuroscientists. Moscucci and Bibbò (2015) presented the first quality of a relationship: a dual nature. The first consists in explicit communication that is established between two human beings through the five physical senses. This type of communication involves people who have full awareness of it and, in part, it is also obvious to any observer. All this has been abundantly studied by psychologists and experts in communication. Moscucci and Bibbò (2015) defined *rapport* the link that is established between two people through this kind of communication. But a relationship is also characterized by a type of communication that cannot be observed by an outside observer and, indeed, even the people directly involved are not aware of its existence, if not only sometimes, and just because of some vague sense they cannot explain. This type of communication was defined as *hidden* *communication* by Moscucci and Bibbò (2015), since it is not perceivable with the five senses. It is due to mirror neurons and perhaps to other neural structures yet to be discovered. So a relationship has two components: one, said *rapport*, based on communication that occurs through the sense organs, and one, that we might call *hidden communication link*, which occurs through mirror neuron systems and perhaps other neural structures. This is the first quality of a relationship, as it has been highlighted during the initial process of characterization.

16.4 A New Step About the Characterization of Relationship

In this paper the study of relationships moves forward, highlighting when and how human beings begin to construct relationships early in life, as a newborn baby, and how all this influences the individuals' way of entering a relationship with others during their life and what that implies. The human being, as it is well known, is an animal that needs a long caring before being autonomous for survival. During this long period, the human being learns to relate to other human beings and to the environment and to everything that is part of it. The mother is the natural caregiver, but in her absence, the role of the caregiver is taken on by another person, sometimes biologically related to the child, or, at other times, a stranger. Therefore, it can be appreciated how useful it is to have appropriate knowledge of the quality of this first human relationship that the human developmental and neuroscience research have demonstrated to be very important for the human being mental health. To understand its importance, a key step is to consider Attachment Theory, AT, (Bowlby, 1980).

Attachment means a natural disposition to guide human actions towards what is useful for survival. Attachment is a symbiotic organized grouping of two human beings: the mother and the child, where the child has a purpose of his own survival and the mother has species survival as her purpose, through the reproductive success. This symbiotic connection seems to be preserved in the evolution of the species through biological mechanisms linked to some genes and some neurobehavioral systems (Polan & Hofer, 1999). However, it is important to highlight what *survival* means. It is not just having food and water. For instance, the well-known US psychologist Harry F. Harlow showed, with his famous Rhesus macaque experiments (e.g. Suomi, 2008), that those animals, when deprived of the natural mother, preferred a surrogate mother that was only comfortable, warm and welcoming, instead of another one that provided only food. Bowlby highlighted the importance of the relationship between a child and his mother regarding the emotional and cognitive, and consequently social, development. Bowlby placed the mother–child relationship in a theoretical framework that can explain how and why this relationship acts on the subsequent development of the infants. However, Bowlby never was an early-attachment "determinist", that is, he believed the way a child relates to others, shortly also called child's *attachment style*, or *primary attachment*, might evolve. AT represented a key step for a paradigm shift: *attachment style* is not a consequence of an internal state of the child, in some sense, "innate", but of one or more patterns built through childhood relationships.

Various types of attachment may be distinguished: secure attachment and insecure-avoidant, insecure-ambivalent or insecure-disorganized attachment (Ainsworth, 1973). In this context, it appears sufficient to distinguish only secure attachment from insecure attachment without going into further detail. A secure attachment arises when the mother or caregiver is available to meet the child's requests and responds consistently to his/her manifested needs. It happens when the mother is careful to feel the child's emotional states, and the child, in turn, feels understood and supported in exploring the world. The child feels loved and worthy of love. Secure attachment is characterized by the serenity of the child. Insecure attachment arises when the mother, or the caregiver, is not available to the child's requests and responds inconsistently, avoiding the child or even being hostile, and it is manifested in a child with insecurity or inability to express his/her emotions, or by restlessness. Although these child characteristics are always and only a result of an insecure attachment, there is a very large scientific literature about the negative consequences related to insecure attachment. For example, the famous Minnesota Longitudinal Study of Parents and Children (Sroufe, Egeland, Carlson, & Collins, 2005) proves the correlation between the quality of attachment and fundamental function of the person, such as cognitive processes, social skills, emotion regulation (e.g. Kochanska & Aksan, 2004; Cicchetti & Rogosch, 2009). In addition researchers support the importance of the quality of attachment in facing all life situations (e.g. Mikulincer & Shaver, 2013).

In summary, the quality of attachment determines a person's relational style and his level of resilience that is resistance to adverse events or difficulties in life. In relation to the problem of the characterization of the term *relationship*, AT allows to highlight a further feature of the relationship: *the nature of a relationship is linked to patterns built during childhood*. This second characteristic is consistent with the first characteristic suggested by Moscucci and Bibbò (2015), namely, the dual nature of the relationship.

16.4.1 Integration, Emotion Regulation and Attachment

16.4.1.1 About Integration

To help readers understand further the importance of attachment, it is useful to provide an account of the essential elements of integration and emotion regulation and their link with attachment. In neuroscience, integration is the connection between differentiated elements: if the linked elements are cerebral circuits (neural areas) it would be more properly called neural integration. If the elements are differentiated parts of the mind, it would be integration. Often we speak of integration, when it concerns mind, brain and body as well. In fact, mind is an entity that emerges from brain activity and is widely considered in neuroscience to be a strongly embodied entity (Varela, Thompson, & Rosh, 1991). Neural integration therefore corresponds to an integration and vice versa. Integration is both a process and a state, whether it is relative to the brain or the mind. Therefore, in this context it is acceptable to speak broadly of integration, because the two types of integration always coexist. Many areas of neuroscience are concerned about integration as one of the cornerstones that underpin the well-being of the person. Interpersonal neurobiology considers integration as a central mechanism for creating a person's overall well-being. In fact, the connection of different elements produces many positive effects including flexibility, adaptability, consistency and stability. The latter does not, however, mean rigidity/cohesion, that is, instead, a prerogative due to the lack of integration, as well as confusion and chaos (Siegel, 2015). Integration is strictly linked to emotion regulation.

16.4.1.2 About Emotion Regulation

There are various approaches to emotions that lead to different classifications (Fosha, Siegel, & Solomon, 2009). The major theoretical dispute lies in considering emotions as internal processes, for instance, in cognitive psychology, or as processes strongly linked to the relationships with the environment. Over the past 10 years, the second location is most often supported, especially since there has been convergence on the claim that emotions, regardless of their quality, are a variation in the state of the brain and mind of which the person concerned may or may not be aware of. That variation involves the entire brain (e.g. Kober et al., 2008). Emotion regulation is about the way we respond to emotions (Gross & Thompson, 2007). The factors, acting in emotion regulation, are internal and external. Among the internal ones, there are, for example, neuroregulatory structures, such as the endocrine system; some cognitive factors, such as beliefs, awareness of need or opportunity of regulation; and some behavioural aspects, such as adaptability and ability to focus. The external ones involve all the relational capacity (Calkins, 1994). So the impossibility of considering separately integration and emotion regulation is apparent. The two constructs are absolutely interdependent and naturally, for their function in the human being, both correlated to resilience. Then, integration and emotion regulation are factors that may simply refer to what is usually called the well-being of the person.

16.4.2 Linking Integration and Emotion Regulation with Attachment

In the following the main links between integration and emotion regulation with AT are presented. Bowlby (1980) claimed that the early caregiver acts as a kind of external regulation system of the child, just as if their brain were a real unit, and later researchers talked about a link due to the emotional communication (Siegel, 2001). In the light of the last 15–20 years results in neurobiology, one can hypothesize that the mirror neuron systems are involved in this (Moscucci & Bibbò, 2015). The child's and the adult's regulatory capacities are linked, and attachment is involved. Schore (2000) observed two facts that are very important in this context: the first is that attachment is, for many reasons, the origin of self-control capacity through up link to emotion regulation, and the second is that a secure attachment contributes to integration. A very large amount of literature exists about these issues, but for the aim of this paper it is sufficient to understand the main links among the three constructs, even if in a very schematic way.

Because integration and emotional regulation play a fundamental role in a person's well-being, and because the type of attachment the person experienced it has a fundamental role in both of them, we may say, from transitiveness, that it is absolutely reasonable to suggest the person's attachment type has a fundamental role in his/her well-being. That is having a secure attachment is a good base for person's well-being. Unfortunately, not every person has secure attachment, and particularly not every student has, and, even more particularly, not every mathematics student has. The problem is critical, because a mathematics student without secure attachment is taught by a teacher who has a fundamental role in his/her relationship with mathematics. For this reason, in this paper, it is proposed that mathematics teachers should confront and address on the context the student–teacher relationship and the problem of mathematics students without a secure attachment. It is clear that everything that can be beneficial for that type of student can be equally beneficial for students who have secure attachment. All this arises from three main considerations, among others: (1) the attachment style, built in childhood, is critical; (2) neuronal plasticity, which means the brain is a lifelong evolving and developing organ, in response to experiences is acknowledged; (3) the power of relationships power in integration, emotional regulation, and attachment style has been equally acknowledged. The last point has been shown by many recent results. One of those is that a careful relationship between adoptive mothers and late-adopted children (aged 4–7 years) can prompt an insecure attachment style to one that is more functional and amenable to children's health (Pace & Zavattini, 2011).

The fundamental/primary attachment pattern built through the relationship with the mother or caregiver at a very early age, can evolve and it can be prompted to do so through establishing particular relationships. In the following sections we consider in what sense is this interesting to a teacher and particularly to a mathematics teacher, beyond the cultural knowledge of this fact? And how can a teacher, and particularly a mathematics teacher, be involved?

16.5 Towards an Improving Student–Teacher Relationship

The importance of the student–teacher relationship is highlighted in literature, particularly in psychology, and has been connected with AT (e.g. Sabol & Pianta, 2012). Mathematics is the cause of the greatest school difficulties for students and so the mathematics teacher is involved with student's well-being much more than every other subject teacher. Ultimately the latent concepts of the whole of affect research are the students' well-being in mathematics learning and teaching and their consequences. Affect research was born because of various types of discomfort, one of them being the student's actual suffering, too often present in mathematics classes. It is no coincidence therefore that affect is a specific field of mathematics education research. Since the student–mathematics teacher relationship is naturally interesting in affect research, its engagement with attachment style, which is so connected with person/student well-being, has to be too.

In my experience, as a mathematics education teacher and creator of many school mathematics projects aimed at addressing the difficulties of mathematics students, I noticed that the mathematics teacher–student relationship presents a very particular aspect. I analysed hundreds of mathematics student stories, mostly in writing, telling the students' school mathematics experiences in relation to their mathematics difficulties, and there it was constantly the presence of a troubled student–mathematics teacher relationship. So the mathematics student–teacher relationship should be treated with carefulness and sensibility, and, as a consequence, it deserves special attention by affect researchers. In addition mathematics teachers can benefit from a deeper study of the issue. In particular, there is a psychotherapeutic intervention model, based on AT, that might be useful to the aim of this paper.

From the 80s to the present, AT has taken on a key role in psychotherapy, and today this role is recognized on two main sides (e.g. Stern, 2004): (1) for the therapist the knowledge of AT is a professional interpretation tool for understanding a client's way of relating to the others; (2) the therapist can take the responsibility of having an active role in the relationship with the client, focusing on promoting developmental experiences of integration and emotion regulation. David J. Walling, the author who perhaps has best interpreted the evolution of AT as a tool in psychotherapy, has highlighted the importance of the quality of the relationship between the therapist and the client, when the therapist makes himself/herself available to the client as a secure reference for the development of the client's attachment patterns (Wallin, 2007). This is known as an "attuned therapist", inspired by "affect attunement" introduced in 1985 by the US psychiatrist and psychoanalyst Daniel Norman Stern to describe the harmony that exists between mother and child. Stern's attuned therapist is focusing on recreating a relationship with his client as similar as possible to that existing between mother and child. Borrowing the figure of attuned therapist, it is possible to introduce a particular figure of mathematics teacher, the *attuned mathematics teacher*.

16.6 The Attuned Mathematics Teacher

The *attuned mathematics teacher* is a mathematics teacher who, in analogy to Point (1) at the end of the previous paragraph, knows AT and, in analogy to Point (2) takes the responsibility to have an active role in the relationship with his/her students. As it is the case for the psychotherapist, the knowledge of AT is a professional tool for every teacher, but particularly for mathematics teachers, due to the learning/teaching difficulties related to this discipline. Indeed, those peculiarities make the mathematics teacher, the more interesting, among teachers, in building productive student–teacher relationships, as relationships inspired by the mother/child attunement. While not a family member the teacher is a person with whom the mathematics student has an important relationship. That relationship, independently from the age of the student, is not only constituted by a rapport but also by hidden communication. The teacher is an affective reference in primary school, and in secondary school too, even if with a different relational role. Although the teacher is not a psychotherapist, as in the case of an attuned therapist, he/she can have an active role in creating a child–adult relationship that is not a replacement of deficient or missing primary secure attachment relationship, but that demonstrates the possible existence of that type of relationship. Just as an attuned therapist can benefit clients, an *attuned mathematics teacher* may be useful to his/her students.

The *attuned mathematics teacher* is firstly a mathematics teacher. The fundamental focus of a mathematics teacher is helping every student to acquire mathematical competencies. A careful approach has as its first aim helping every student to avoid or overcome difficulties in mathematics. Hence, an *attuned mathematics teacher* has this focus, as the primary one. But to achieve this, it is important to be an *attuned mathematics teacher*, because the mathematics teacher has a key role in the quality of the student's relationship with mathematics. However, in the meantime, the mathematics teacher must be aware of the potential of this approach. In fact, beyond teaching mathematics, and indeed, while teaching mathematics, the *attuned mathematics teacher* is committed in building strong relationships with his students, especially those of them with relational problems. It is these problems that prevent the promotion of all students' qualities and a serene and fruitful school life. The *attuned mathematics teacher* may have a role comparable to that of an adoptive mother of a child with insecure attachment of the study mentioned above.

The *attuned mathematics teacher* has exactly the characteristics of particular very successful mathematics teachers I met while teaching mathematics education. The constructs that this paper deals with; that is relationships, AT, integration, emotion regulation and their links, contextualize scientifically that kind of teacher. At the same time, it explains why certain mathematics teachers, such as Rose, are so more successful than others with their students. The following characteristics of the *attuned mathematics teacher* arose from the convergence of the characteristics of the attuned therapist and those of surprisingly excellent mathematics teachers without particular competencies. Some might say that many teachers have the *attuned mathematics teacher*'s characteristics, but the *attuned mathematics teacher* is conscious of the need to, without hesitation or exceptions, to explicitly exhibit the attitudes and claims, and above all the behavioural consistency of the attuned mathematics teacher.

The *attuned mathematics teacher* has a precise, timely, constant care towards every student. He/she tries to show his/her attention, so that the student feels supported. His/her presence in the student's life is tangible, recognisable and recognised by students. He/she explicitly expresses his intention to make available to his students all his personal capacities as well as his competence and acts consistently with those statements. In terms of behaviour the *attuned mathematics teacher* looks into every student's eyes, not just at the students and, of course, with much care, at those whom he knows have greater relational difficulties (it is a teacher's basic professional competence to know the students' different relational abilities); he/she pays attention to every request of the student; accepts every student's disciplinary weaknesses or uncertainties and mistakes, with care so that the student feels accepted and welcomed regardless of his performance. The *attuned mathematics teacher* is particularly open to dialogue, he/she worries explicitly student well-being, and he/she urges students to externalize their feelings with respect to all school activities. He/she promotes the authentic interactions among students, clearly demonstrating the desire to help the student to give his best. The *attuned mathematics teacher* is attentive to all the student's signs and tries to understand every student's circumstances or actions. If he senses discomfort, he gently asks the student for an explanation, eventually about its nature and its origin, encouraging the student to share as a part of processing the discomfort. As well as feeling satisfaction and well-being, he participates with manifested joy.

Importantly the teacher is not acting as a psychologist, but as a person whom is attentive and helpful, leaving, of course, specific issues, from a psychological point of view, to practitioners. These considerations seem appropriate particularly because he is teaching mathematics, a discipline that, too often, creates fear and insecurity. In this position, the mathematics teacher might become a person who leads the students to naturally review their relationship patterns. If the fundamental relational style moves towards a more functional one, integration and emotion regulation benefit from this. All this is consistent with the aim of a mathematics teacher who intends to use mathematics as a tool in support of the potential of the person and not only for the acquisition of competence in mathematics but also mathematical and personal competence.

16.7 Conclusions

Due to the space constraints, I have given an overview about the scientific roots of the issue, rather than extensive details about the attuned mathematics teacher, including the analyses of some mathematics teachers, who are of the type that led me to propose the attuned mathematics teacher. It is my intention, in the future, to describe the attuned mathematics teacher's characteristics more deeply, and to study the efficacy and potential of the attuned mathematics teacher. Also studying in depth, "natural" attuned mathematics teacher cases might be interesting. In my opinion, the issue looks very promising. Every interested researcher, who cares about the student–mathematics teacher relationship, is explicitly asked to participate. Likewise the development of the research about the potential of the attuned mathematics teacher based on the success of the attuned therapist is needed. Some teachers may be naturally an attuned mathematics teacher, but affect researchers have the opportunity of scientifically supporting the relevance of being "purposely" an attuned mathematics teacher. Affect researchers may be promoters of being a more efficient teacher, not only for the results in the discipline but also, and above all, in the integral education of the person. There is potential to be pioneers in this area, perhaps by seeking and promoting collaboration between education and neurosciences researchers.

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Chapter 17
The Role of Mathematics Teachers' Views for Their Competence of Analysing Classroom Situations

Sebastian Kuntze and Marita Friesen

Abstract When teachers analyse mathematics classrooms, it can be expected that they use their professional knowledge, including their instruction-related views. In the case of analysing classroom situations regarding the use of representations, prior research suggests interdependencies. Consequently, when assessing teachers' competence of analysing, the role of teachers' views should be taken into account so as to explore their potential role for the competence construct. This need for research is therefore addressed in this study. For a sample consisting of 31 in-service teachers, interdependencies between instruction-related views and the teachers' analysis were examined by quantitative and qualitative analyses. The results indicate such interdependencies and give insight into possible reasons for these.

17.1 Theoretical Background

A growing number of approaches to teacher expertise use notions such as "noticing" in the sense of "selective attention" (cf. e.g. Seidel, Blomberg, & Renkl, 2013) or in the sense of "knowledge-based reasoning" (Sherin, Jacobs, & Philipp, 2011), "professional vision" (Sherin & van Es, 2009), "usable knowledge" (Kersting, Givvin, Thompson, Santagata, & Stigler, 2012) or "awareness" (Mason, 2002). All of these approaches have in common the emphasis on situation contexts with relevance for the profession of mathematics teachers: research instruments address how teachers attend and/or reflect on situation contexts and how they make use of their professional knowledge against the background of the challenges and demands of those situation contexts. A further notion which takes up key elements of these approaches is the notion of teachers' competence of analysing situation contexts (e.g. Kuntze, Dreher, & Friesen, 2015). By *analysing* we understand an "*awareness-driven, knowledge-based process which connects the subject of analysis with relevant criterion knowledge and is marked by criteria-based explanation and*

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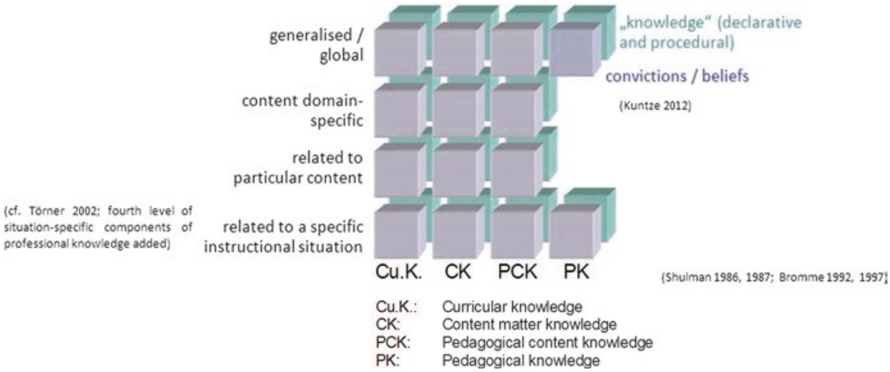

Fig. 17.1 Multilayer model for components of professional knowledge (Kuntze, 2012)

argumentation" (ibid, p. 3214). The subject of analysis is, however, not restricted to classroom situations, but can also be an area of content knowledge, a task, or a piece of student work, for instance. In line with the framework model for professional knowledge we use in our approach (Kuntze, 2012), the knowledge base for analysing includes teachers' views: The model (see Fig. 17.1) combines Shulman's (1986) domains (columns in Fig. 17.1) with the spectrum between knowledge and prescriptive convictions/views, as knowledge and views cannot be separated theoretically ((Pajares, 1992); spectrum shown in Fig. 17.1 by the rear-front dimension). Consequently, teachers' views are considered as individual professional knowledge components as well. As a third dimension, the degree to which professional knowledge is bound to content or classroom situations is used to distinguish different levels of globality (cf. Törner, 2002) or situatedness, respectively (vertically ordered layers in Fig. 17.1). The cells of the model in Fig. 17.1 are not claimed to be strictly separable. The extent to which components of professional knowledge are consistent across different cells may even be interpreted as an indicator of teacher expertise (cf. Doerr & Lerman, 2009). A more detailed description of this theoretical background can be found in Kuntze (2012) and Dreher and Kuntze (2015a).

In line with Weinert (2001), we consider teachers' analysing as a competence, which does not only consist of specific knowledge and abilities but also of the motivational, volitional and social dispositions to make use of this knowledge and to draw on these abilities in order to solve problems in the corresponding domain of expertise. This implies that the teachers' instruction-related views can be expected to play an important role for this competence.

17.1.1 Teachers' Analysis of How Mathematical Representations Are Used in Classroom Situations

A domain-specific competence construct in this sense concentrates on teachers' analysis of how mathematical representations are dealt with in classroom situations. This results from the abstract nature of mathematical objects, which are only accessible through representations, both for experts and for learners (Duval, 2006). Representations can stand for mathematical objects in many different ways (Goldin & Shteingold, 2001) and multiple representations can complement each other by emphasising specific aspects of the corresponding mathematical object (Duval, 2006). Their use may support the development of a rich concept image (Ainsworth, 2006) and problem solving (Lesh, Post, & Behr, 1987). Moreover, the way multiple representations are dealt with in the classroom is crucial, as multiple representations have to be integrated by learners, and changes between different registers of representations have often shown to be a learning obstacle (Duval, 2006; Ainsworth, 2006). As students need to be supported when dealing with multiple representations, teachers need corresponding professional knowledge, and they have to be able to identify and interpret aspects of classroom situations that are relevant for their students' learning support regarding representations. Analysing classroom situations regarding the use of representations can thus be regarded as an important profession-related competence for mathematics teachers. This is also supported by studies showing that such analysing is an important characteristic of teacher expertise (Dreher & Kuntze, 2015a, 2015b) and is learnable in the context of professional teacher development (e.g. Friesen, Dreher, & Kuntze, 2015). In Fig. 17.2, an

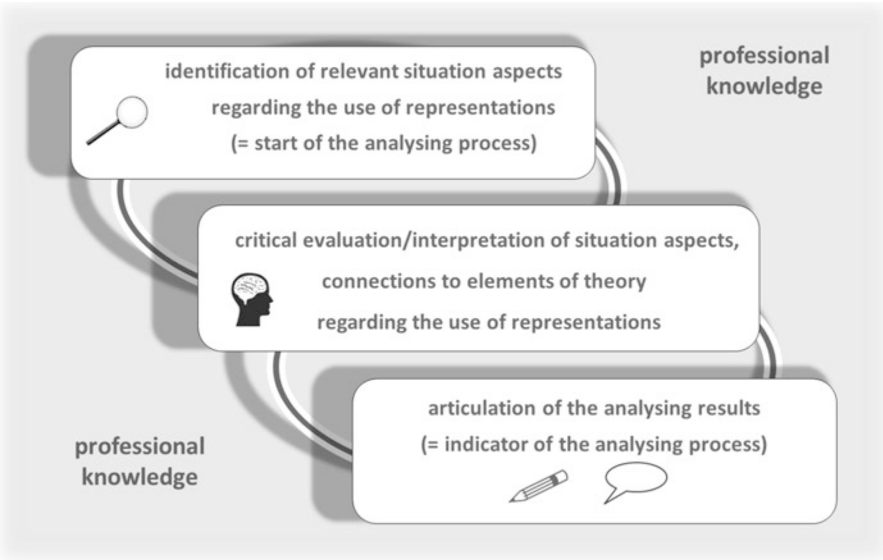

Fig. 17.2 Analysing classroom situations as a knowledge-based process (Friesen & Kuntze, 2016, p. 260)

overview of the analysis process of classroom situations regarding the use of representations is given. This process may be complex and is assumed to take place on the background of professional knowledge: both the identification of relevant situation aspects regarding the use of representations and their interpretation based on related knowledge and views lead to an articulation of individual analysing results. In these, the connection between professional knowledge and situation observations can be considered as a key quality indicator of the analysis (Friesen & Kuntze, 2016).

Professional knowledge including teachers' views related to using multiple representations is thus an important resource (Schoenfeld, 2011) for teachers, which they can draw on when analysing the interaction with students in classroom situations regarding the way representations are dealt with. The role of teachers' views, in particular views related to the use of representations, has been examined in prior studies (Dreher & Kuntze, 2015a, 2015b; Kuntze & Dreher, 2015).

17.1.2 Prior Findings on Teachers' Views Relevant for Dealing with Representations in the Classroom

The findings reported in Kuntze and Dreher (2015) suggest that teachers' knowledge-based analysis can be impeded by a dominance of views related to affective aspects of the mathematics classroom. The evidence from three studies indicates for instance that teachers who put an overemphasis on potentially motivating pictorial representations were less successful in analysing the very limited support provided by these representations for learners in specific tasks used in the study. Moreover, cases in which dominant views about the role of affective aspects hindered the teachers' analysis based on pedagogical content knowledge (PCK) were analysed.

Dreher and Kuntze (2015a, 2015b) investigated teachers' views related to the role of representations for learning mathematics in general and for learning fractions in particular. The data from pre-service and in-service teachers show that both pre-service and in-service teachers held views that multiple representations in the mathematics classroom mainly support students in remembering mathematical facts, that motivation and interest are supported by using multiple representations and that multiple representations help to address different learning types and different input channels of the students. In contrast, the view that multiple representations are necessary for supporting mathematical understanding was shared less by the teachers. The content-bound survey (content area of fractions) revealed differences between pre-service and in-service teachers: In-service teachers mostly approved the views that multiple representations of fractions can foster understanding and that they respond to individual preferences of students. Moreover, the in-service teachers did rather not share the views that there should be only one standard representation for fractions, that students might get confused by the use of multiple representations, and that using multiple representations tends to impede the students' learning of fraction calculation rules. The means of the scale values related to the views of the pre-service teachers showed an inverse pattern. Moreover, cases of teachers' answers presented in Dreher and Kuntze (2015a) suggest that teachers' professional knowledge (including their views) from all different levels of globality/situatedness (see Fig. 17.1) can be used by teachers when analysing classroom situations. This supports the assumption that the teachers' views as described above can play a role for the teachers' analysis.

However, the studies by Dreher and Kuntze (2015a, 2015b) did not examine the role of teachers' global views beyond the use of representations in the mathematics classroom, such as teachers' views related to seeing teaching and learning mathematics according to a direct-transmission paradigm or according to a cognitive constructivist view (Staub & Stern, 2002). In the study by Kuntze (2012), these views have shown interdependencies with teachers' evaluations of videotaped classroom situations. Moreover, scales related to teachers' direct-transmission and cognitive constructivist views, which have initially been used by Fennema and colleagues (e.g. Fennema, Carpenter, & Loef, 1990) have also been used in larger studies such as COACTIV (e.g. Kunter et al., 2013) and TEDS-M (e.g. Blömeke, Kaiser, & Lehmann, 2010). In the following, we will give a short description of these views.

17.1.3 Cognitive Constructivist and Direct-Transmission Views of Teaching and Learning

Cognitive constructivist and direct-transmission views of teaching and learning (Staub & Stern, 2002) can be classified as examples of global components of pedagogical content knowledge (see Fig. 17.1). According to cognitive constructivist views, learners are assumed to play an active role in the learning process. To be effective, mathematics classrooms have to foster this active role by offering students opportunities for connecting with their prior knowledge, discovery and learner-centred interaction. In contrast, views following a direct-transmission paradigm of teaching and learning assume that knowledge is transferred from the teacher to the learners as described in associationist theories (e.g. Gagné, Briggs, & Wager, 1992; Skinner, 1958). This implies that presentations by the teacher are considered as central and learner-centred activities are given less importance. However, even if these views might appear as opposed to each other, they have empirically shown to represent two separate factors which are correlated negatively (e.g. Lipowsky, Thußbas, Klieme, Reusser, & Pauli, 2003).

We will consider these views as they have been shown to have measurable impacts on students' learning outcomes and on observable characteristics of teachers' classroom practice (see Kuntze, 2012, for a summary).

17.2 Research Interest

Even if the views of mathematics teachers described above are likely to play a role for teachers' evaluation and analysis of classroom situations, empirical evidence for such interdependencies is still scarce. Moreover, in cases where first empirical findings suggesting interdependencies are available, the nature of these interdependencies needs further exploration, in order to refine theories about the teachers' competence of analysing.

Consequently, the research interest of this study is to explore whether and how teachers' views related to a cognitive constructivist and related to a direct-transmission paradigm of teaching and learning mathematics play a role for the competence of analysing classroom situations regarding the use of representations. In particular, the research questions are the following: *Are in-service teachers' views as described above associated with the competence scores these teachers reach for the analysis of classroom situations regarding the use of representations? Is there evidence (on the level of cases of teachers' articulation of analysis outcomes), which could explain how such views might have interfered in their analysis of classroom vignettes?*

17.3 Design and Sample

This study uses data which has been gathered using a vignette-based test instrument (e.g. Friesen & Kuntze, 2016). In the core part of the instrument, teachers are asked to analyse six classroom situations (situated in Grade 6) in which dealing with multiple representations plays a key role. All of the six classroom situations follow a similar design pattern: At the start, they show student-centred group work in the content area of fractions—in this setting, the teacher is asked for help by a group of students who have already started to solve a given problem. This means that they are using a certain representation register, in which they have encountered a difficulty. The situations were designed in such a way that the teachers' support of the students is not in line with the theory regarding the use of representations as outlined above: In her or his reaction, the teacher changes to an additional representation register and shifts away from the representation the students have already been using. This change of representations remains unreflected and unexplained and the teacher does not connect the additional representation to the students' representation. Therefore, the teacher's reaction could lead to further problems in the students' understanding rather than support it.

In order to assess the participants' competence of analysing the use of representations, we asked them to evaluate the six classroom situations with respect to the support provided by the teachers through answering the following open-ended question: *How appropriate is the teacher's response in order to help the students? Please evaluate regarding the use of representations and give reasons for your*

Table 17.1 Descriptive information on scales for the teachers' views

Scale	Sample item	Number of items	Cronbach's α	M[a]	SD
Cognitive constructivist view	"Students should be allowed to invent ways to solve problems before the teacher demonstrates how to solve them." (Staub & Stern, 2002)	7	0.82	3.19	0.45
Direct-transmission view	"Students learn maths best by attending to the teacher's explanations." (Staub & Stern, 2002)	7	0.75	2.66	0.55

[a]1: strongly disagree; 4: strongly agree

answer. Moreover, the participants were asked to answer four rating scale items about the helpfulness of the teachers' reaction as far as the use of representations is concerned. In case of a critical evaluation of the teachers' reaction, answers were coded as successful analysis (for details see Friesen & Kuntze, 2016). The corresponding answers collected through rating scale questionnaire sections for each of the six classroom situations yielded competence scores, which could be described empirically by a one-dimensional Rasch model (Friesen & Kuntze, 2016). The way of implementing classroom situations in the instrument followed a format-aware test design: The situations were presented to the participants in video, comic, or text format according to a randomised distribution of test booklets. The videos lasted about 1.5 min each and could be paused or watched several times.

In order to answer the first research question, the teachers were additionally asked to answer the multiple-choice Likert scale items related to the view constructs introduced above from the study by Staub and Stern (2002). Sample items and reliability values are shown in Table 17.1.

The sample of this study consists of $N = 31$ mathematics in-service teachers (67.7% female; $M_{age} = 38.4$, $SD_{age} = 9.8$). They had between two and 31 years of experience in teaching mathematics at secondary school level ($M_{exp} = 9.0$, $SD_{exp} = 6.1$).

In addition and with a focus on the second research question, the participants' answers to the open-ended questions were analysed by bottom-up interpretive methods for answering the second research question. We analysed cases of answers using content analysis (Mayring, 2015) which was based on theoretical criteria referring to the views as introduced above.

17.4 Results

In order to investigate possible associations between the in-service teachers' views and their analysis of the classroom situations, the teachers' answers to the rating scale questionnaire about their views (containing the scales shown in Table 17.1)

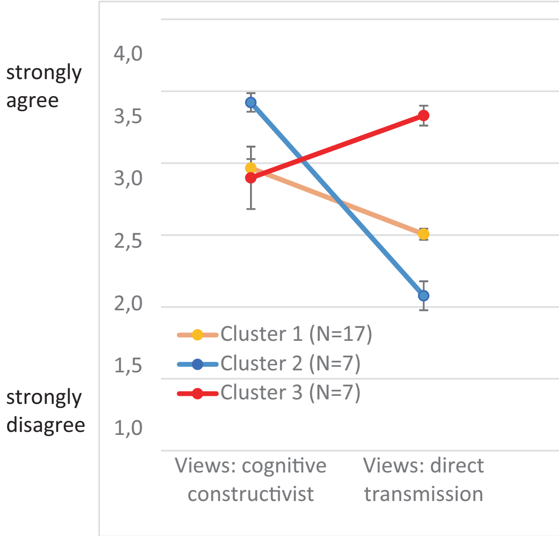

Fig. 17.3 Profiles of in-service teachers' views (resulting from Cluster analysis (Ward method), means and their standard errors)

were first analysed in order to describe the extent to which the teachers held cognitive constructivist or direct-transmission views. As the two scales have to be considered as two factors, a cluster analysis (Ward method) was used to find out about the profiles of the in-service teachers' views. The cluster analysis was carried out on the base of the two variables concerning the views only. The analysis yielded three clusters, the views profiles of the clusters of teachers are shown in Fig. 17.3.

The first cluster contains in-service teachers whose cognitive constructivist views slightly prevail over their direct-transmission views. The in-service teachers in cluster 2 have comparatively strong constructivist views in combination with relatively low direct-transmission views. The third cluster comprises of in-service teachers holding rather strong direct-transmission views, combined with on average lower cognitive constructivist views.

In order to explore whether teachers' views are associated with the in-service teachers' competence of analysing the use of representations, the mean scores were calculated for the three clusters. Figure 17.4 shows corresponding differences. The results indicate that high cognitive constructivist views in combination with comparatively low direct-transmission beliefs were associated with a comparatively higher competence of analysing.

Against the background of these findings, we examined in particular cases of teachers' answers from cluster 2 in contrast with cases from cluster 3 in a corresponding deepening interpretive analysis, as these clusters show an inverse pattern of their views in Fig. 17.3. Figure 17.5 shows an excerpt from a written answer of a teacher from cluster 2.

The teacher's answer was coded as successful analysis, as the teacher focuses on the use of representations and argues that the representations used in the situation

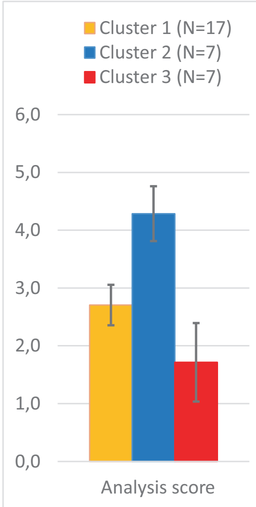

Fig. 17.4 Mean analysis scores for clusters (and their standard errors)

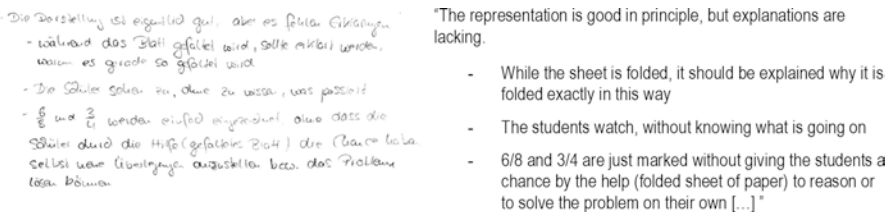

Fig. 17.5 Excerpt from a written answer of a teacher from cluster 2 and its translation

should be connected in a better way and that the vignette teacher's reaction therefore might not be very helpful for the students (e.g. "it should be explained why it is folded exactly in this way", see excerpt in Fig. 17.5). At the same time the teacher appears to focus on the students and in particular on their role in the learning process and in the classroom interaction (e.g. "The students watch, without knowing what is going on"). This comment might suggest that the teacher would have preferred if the students had played a more active role (than simply watching the teacher) and that such an active role might have supported them to better understand (and possibly control) "what is going on". This interpretation is supported by the teacher's next comment, in which the students' reasoning and solving "the problem on their own" appears to be valued. These comments are likely to reflect cognitive constructivist views (cf. sample item in Table 17.1) and at the same time, they express specific analysis steps with respect to the connectedness of different representations appearing in the vignette.

17.5 Discussion

Even though the sample size requires interpreting the evidence with care, the results indicate that teachers with high cognitive constructivist and comparatively low direct-transmission views reached higher analysis scores. A deepening analysis of cases of teachers' answers can provide evidence that might explain how such views may support the teachers' competence of analysing: As outlined above, cognitive constructivist views emphasise an active learner role and a focus on the individual learner. In the example given in Fig. 17.5, such an emphasis might have facilitated the analysis of potential understanding obstacles for the students in the classroom vignette. We may consequently assume that the vignette teacher's reaction could be examined critically by the answering teacher, as the students' role and their understanding were in the foreground according to her or his views.

Beyond providing insight into the competence of analysing for further theory development, these findings are relevant for practice as well, especially for developing professional development programs which are aware of and take into account the teachers' views.

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Chapter 18
Teaching via Problem-Solving or Teacher-Centric Access: Teachers' Views and Beliefs

Anne Möller and Benjamin Rott

Abstract This paper reports on an exploratory study, which investigates teachers' preferences about the implementation of teaching via discovery learning and problem-solving. This discovery way of teaching is compared to receptive learning and a teacher-centric access in mathematics lessons. As most teachers seem to choose the latter way of teaching, we want to get to know, what reasons teachers name for using or not using discovery methods. Therefore, semi-standardized interviews with five teachers were conducted, analysed, and interpreted. This sample encompasses two teachers who prefer a receptive way of teaching, one teacher who uses discovery learning a lot and another two teachers who use both methods. A comparison of two selected teachers reveals that even if teachers prefer a different way of teaching, they may mention the same concerns about discovery learning. They do, however, differ in the number of advantages of discovery learning they mention in the interview. Teaching time and students' performance in mathematics are mentioned as important decision-making factors. As expected, the teacher who uses discovery learning mentions a lot more profits than his colleague.

18.1 Introduction

A problem-solving task, or short: a problem, can be defined as a non-routine task for which no way of solving it is immediately known (cf. Schoenfeld, 1992). While teaching, problem-solving tasks can be used to help students discover new mathematical contents on their own instead of contents being presented by their teachers. Solving problems is an inherent part of doing mathematics. Students can get to know a creative, challenging process of using mathematics. Discovery learning involves students in the process of creating mathematics.

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The use of discovery learning and problem-solving in mathematics lessons requires a changed role for the teachers. This might lead to odd or difficult situations during the lesson. Some processes and actions cannot be anticipated or planned beforehand.

Experience from practice help us to understand the classroom situation better. It is important to get to know how teachers deal with these arguments. In an exploratory interview study, we asked teachers the following questions: Do you use discovery learning in your mathematics lessons? How do you use discovery learning and problem-solving? What advantages and disadvantages have you experienced?

The analysis of the interviews gives first answers; it is divided in two steps: In a first, by analysing the teachers' statements, the frequency of the teacher's use of discovery and problem-solving is collected. In a second step, a category system is built to systematise teachers' views of discovery learning. Especially the advantages and disadvantages they name are categorized. The results of the first analysis show that the sample of teachers differs a lot. In this article, the cases of two teachers with opposing views are described. The analysis of the second evaluation step presented here refers to these two teachers. Therefore, a contrasting comparison of (1) a teacher who does and (2) a teacher who does not use discoveries in their mathematics lessons are depicted.

18.2 Research Questions

Teaching always requires a decision between giving freedom for discoveries and instructions for guided learning, between constructivist and instructional theories as extreme cases. Teachers have to choose a teaching method whenever they teach and that is why every teacher questions the way of teaching at least once in his professional life. After examining many arguments in the literature and research studies, an observation of teaching practice is needed. The aim of this study is to understand teachers' reasons and explanations for their choices of teaching methods. By using interviews, we can identify the reasons teachers have, what advantages they see and use in their daily teaching, and also what disadvantages they name. The following research questions guided the study:

1. Is discovery learning and problem-solving used in mathematics lessons?
2. What are the reasons for mathematics teachers to *use* discovery learning and problem-solving?
3. What are the reasons for mathematics teachers to *not use* discovery learning and problem-solving?

18.3 Theoretical Background

In the following, we will use Winter's (2016) terminology to further describe the opportunities of discovery learning and receptive learning:

18.3.1 Winter's Description of Two Types of Teachers

Considering discovery learning and teaching mathematics, Winter's understanding is referred to. "Though the student grasps the central mathematical ideas by his own discovery, he is guided by the teacher who furnishes the material and hints, refers to similar problems or different aspects of the content and points out the heuristic value of the strategies used by the students" (Lorenz, 1991, p. 88). This implies a form of guided discovery and problem-solving, but also high student-activity, instead of teacher-centred methods. Winter describes the teachers' beliefs and behaviour in the classroom using fifteen statements for both ways of teaching (Winter, 2016, p. 4 f.). For example, he explains that the teacher encourages students to be active solvers and to do mathematics by themselves, if the teacher uses discovery learning. On the other hand, a teacher who uses receptive learning focuses on step-by-step explanations and he tries to prevent the students making mistakes. While the teacher who is using discovery learning approves that teaching should take place in open surroundings to give learners enough time to be creative, to explore the problems by going wrong ways, and to get to know the problem situation, the teacher who uses receptive learning concentrates on accentuating the goal of the lesson mostly via dialogue in class (ibid., p. 4 f.). In this way, Winter pictures two type of teachers, namely those using discovery or receptive learning, which are used for the analysis of the interviews later.

18.3.2 Discovery Learning

Jerome Bruner pioneered discovery learning. Starting from a constructivist view of learning, he saw the advantages of this teaching method, among other things, in the ability to solve problems and the activity of the pupils. Bruner states that it is impossible to prepare a young human for all of those situations and problems he will be facing in his life. Having the ability to solve problems is the best way of being prepared for real life. In addition, the newly acquired knowledge would be better remembered when acquired through independent discoveries. Discovery suggests discovering mathematical situations actively; this process is supposed to lead to long-term knowledge. Contrary, Bruner states that learning of mathematical contents—split into small steps and narrow questions—would lead to oblivion. Learners should be able to solve problems themselves and to acquire knowledge themselves in school. In doing so, students get to know different strategies to solve problems, which is a main learning goal. Necessarily, the learners have to be active in these lessons; teachers keep themselves in the background and perform rather supportive duties than demonstrative ones. It is part of the teacher's work to help learners become self- and spontaneous thinkers (Bruner, 1974). Therefore, students get help from the teachers. Bruner does not want an uncontrolled, arbitrary learning. Today, this idea is connected to the term "guided discovery" (Neber, 1981, p. 49ff.). The main aspects for guided discovery learning are:

1. Learning means much more than the simple reproduction of general principles and learning contents. Every student should make the experience of solving a problem on his or her own.
2. Because only then you reach self-confidence in your own strengths….
3. …A way of knowledge that is meaningful and can be used in thinking.
4. … Get the possibility to build his own point of view of learning, assumption and research.

18.3.3 Receptive Learning

David Paul Ausubel pleaded for a teaching via reception. His central aim was "Meaning and Meaningful Learning" (Ausubel, 1968, p.37ff.), implying that the linkage of new content to existing prior knowledge is the key to long-term memory.

In order to achieve this, a clearly pre-structured instruction (by the teacher) is helpful. The teacher is determined and consistent in planning lessons as well as in the teaching practice itself. For this statement, Ausubel finds various arguments:

> "It may be argued with much justification, of course, that the school is also concerned with developing the student's ability to use acquired knowledge in solving particular problems, that is, with his ability to think systematically, independently, and critically in various fields of inquiry. But this function of the school, although constituting a legitimate objective of education in its own right, is less central than its related transmission-of-knowledge function in terms of the amount of time that can be reasonably allotted to it, in terms of the objectives of education in democratic society, and in terms of what can be reasonably expected from most students." (Ausubel, 1968, p. 23).

In addition, the following three arguments support teaching via reception and lead to a restriction of the use of discovery learning:

1. The general aim of school is knowledge. Ausubel assigns the task to schools to teach a lot of knowledge in a short time. There is not enough time for teaching via discovery. Teaching via reception takes less time.
2. Learning via reception is less complex than learning via discovery. There is no reason for making learning even harder than it already is. Therefore, learning via discovery might be successful for mathematically gifted students, but not for every student.
3. If the mathematical content is more complex, discovery learning supports student's cognitive ideas, which have to be fostered anyway. Discovery learning is less time-consuming teaching complex contents.

Both Ausubel and Bruner are explaining their ideas as a whole concept of teaching which focused on the full development of children and is applied in projects. Talking about teaching and learning mathematics the concepts are transferred in smaller parts.

18.3.4 Arguments and Perspectives from Teachers' Views

There already have been named numerous reasons for using and not using discovery learning and problem-solving from the point of view of educational psychologists (see 18.3.1–18.3.3). Teachers might have another point of view, being experts of teaching and choosing teaching methods very often. With a view to their daily teaching praxis, teachers might centre different aspects. Sawada (1997) wrote the "Developing Lesson Plans", where he listed advantages, but also four disadvantages of using open-ended approaches (which is closely related to teaching via problem-solving):

> 1. It is difficult for teachers to make or prepare meaningful mathematical problem situations. 2. It is difficult for teachers to pose problems successfully. Sometimes students have difficulty understanding how to respond to open problems and thus give answers that are not mathematically significant. 3. Some students with higher ability may experience anxiety about their answers. 4. Students may feel that their learning is unsatisfactory because of their difficulties in summarizing clearly. (Sawada, 1997, 24)

Arguments 1. and 2. are about teachers' concerns about how to implement problem-solving-tasks in maths lessons and about the new role the teacher gets. If students are not used to this kind of tasks, they might not get to the mathematical answer. In situations like that, the teacher needs to know what advises, questions, or hints the students might need. Argument 2. also includes students' inexperience with tasks like that. Following this aspect, students' interests, expectations or even fears are brought into focus in arguments 3. and 4. Here, teachers' worries that they cannot please all students with this way of learning are mentioned as well.

Sawada's list of four arguments can be completed with a fifth argument by Cai. This supplementary reason depends on the teachers' worries focusing on the conceptual understanding of mathematics. Teachers suspect that this would lead to less time to generate content knowledge, which ends in less basic skills (Cai, 2003).

Some of Ausubel's arguments for receptive learning can also be located in this list of arguments; others have a different focal point. For example, Ausubel's first argument—achieving knowledge is the most important goal in school—is similar to Cai's argument. His second argument about complex learning and gifted students is also relevant for Sawada (see argument 3. and 4.). Ausubel's third argument as well as Bruner's four aspects name positive aspects and, therefore, they should be compared with Sawada's advantages of open ended approaches.

The advantages of using open-ended approaches are also used for analysing the data later. For reasons of space, they are not listed here (but in a simplified way in Table 18.3).

18.3.5 Teachers' Beliefs of Mathematics and Teaching

In addition to the curricula change in the USA in the 1980s, in which problem-solving was accentuated, the interest in research studies about beliefs rose. Until today, beliefs are seen as one factor of teaching and learning mathematics. Schoenfeld considers goals of doing mathematics and the way you look at mathematics as follows:

"Goals for mathematics instruction depend on one's conceptualization of what mathematics is, and what it means to understand mathematics." (Schoenfeld, 1992, p. 334) This is a complex connection. He describes two opposite views of mathematics as (1) "a body of facts and procedures dealing with quantities, magnitudes, and forms, and the relationship among them" (1992, p. 334) and (2) as "an almost empirical discipline closely akin to sciences in its emphasis on pattern-seeking on the basis of empirical evidence" (1992, p. 335). Obviously, there are a lot of differentiating views between the two described views.

Research studies about the impact of teachers' beliefs on their teaching practice show differing results: In most studies, a relation between beliefs and teaching practice can be shown (see (Pehkonen, 1994), for an overview). Rott (2016) found a connection between teachers' beliefs on mathematics and their actions in lessons on mathematical problem-solving. Other studies, however, did not find any connection between teachers' beliefs and their teaching practice. Hofer, for example, explains that it is imaginable that beliefs and the way of teaching are connected in many different ways. For example, a teacher that has a static image of mathematics, as a fixed science, can also make students discover and explore contents (Hofer, 2002). It is the same the other way around.

18.4 Methodology

18.4.1 Data Collection

To answer our research questions, an exploratory study has been conducted. To get and understand teachers' professional reasons, a qualitative approach has been chosen. Semi-standardized interviews allow the interviewee to answer freely. The interviewer has a guideline to make sure that none of the relevant aspects are omitted during the conversation. If the conversation pauses, the guideline can be used as well. Otherwise, the interviewees are encouraged to speak freely. The order of aspects, mentioned in the guideline, does not have to be the same in the interview. Besides questions regarding basic personal and professional details, the guideline includes questions about the following four aspects:

i) Description of a typical Mathematics lesson.
ii) Discovery learning in the interviewee's Mathematics lessons.
iii) Mathematical problem-solving in the interviewee's Mathematics lessons.
iv) Beliefs about Mathematics.

At the beginning, the interviewer tries to create a positive and comfortable situation to make the conversation pleasant like it is in a teacher-to-teacher situation. Therefore, the first series of questions are about the structure of a classical math lesson. We want to learn about what methods or goals are important to that teacher and how they are implemented in the lessons. If discovery learning and/or problem-solving are mentioned in this part of the interview, the subject might focus on used tasks, examples, or mathematical contents. Otherwise, the interviewer will pick up the mentioned method and ask for reasons for this choice and if other options are mentioned. The interviewer will ask about discovery learning and the use of problem-solving tasks in the interviewee's lessons later on. This way, we try to talk about practice lessons and not about hypothetical reasons for teaching methods. The information about the actual teaching and classes might help the interviewer to get a precise picture about the use of discovery learning, problem-solving, and receptive teaching. The interview ends with questions about what mathematics is and what doing mathematics does mean to the interviewee.

The analysis follows Mayring's (2015) qualitative content approach and his integrating data analysing. To develop the categories deductively, the theoretical background and research of literature is used; additionally, categories are developed inductively from the data.

18.4.2 Participants of the Study

Five teachers were interviewed on the telephone. That number allows for an overview of different views and arguments, but it is still possible to do a detailed case by case analysis. All participants teach at secondary schools, called *Gymnasium*. Except for teacher D, they have a similar amount teaching experience in years (teacher D he has been teaching twice as long as the others). In Table 18.1, the basic data of all participants is listed.

18.4.3 Data Analysis

Each of the five interviews is analysed and characterised with precise quotes using the answers to the aspects (ii.) and (iii.) of the guideline. In a first comparison, the teachers' positions are compared to each other with regard to using problem-solving and/or discovery learning in their mathematics lessons. Therefore, a category system using Winter's descriptions of teaching via discovery and teaching via reception is used (see Sect. 3.1). This leads to a comparison of all five teachers in the reference of using discoveries and problem-solving (as described in Sect. 5.1).

Because of space restrictions, not all five interviews can be presented. Therefore, the results of the first analysis are used to identify two teachers with opposing

Table 18.1 Basic data of all five interview participants and interview time

	Teacher A	Teacher B	Teacher C	Teacher D	Teacher E
Sex/age	Female/33 y	Male/32 y	Female/31 y	Male/43 y	Female/33 y
Teacher since	2010	2011	2013	2000	2011
Interview time	About 20 min	About 19 min	About 13 min	About 31 min	About 24 min

behaviours: One teacher who is using discoveries and problem-solving and one who is not. The interviews of those two teachers are analysed in detail in a second analysing process.

In this second analysis, statements about advantages and disadvantages for the use of discoveries and problem-solving in mathematic lessons are listed and categorized using the literature presented above. First of all, we compare the teachers´ statements to the five arguments of Sawada and Cai (as described in Sect. 3.4). They used the categories to compare arguments about what is difficult about teaching via discovery learning and problem-solving. In this analysis, some things were noticeable: Some arguments of Sawada and Cai could be found in the teachers' statements like having no time (argument 5 of Sawada and Cai). Other arguments were mentioned by the teacher in a modified way, for example: None of the teachers worried about students with higher abilities and their anxiety (argument 3 of Sawada and Cai.), but most of them mentioned fears or demotivation of students with less abilities. The overall category *students are afraid* was built (see Table 18.2). In such an inductive and deductive process, the final categories were found.

Teachers' statements about students having *no reading skills* fits the second argument of Sawada and Cai *difficulty understanding* and statements about *no motivation* have similar meaning than *students are unsatisfactory* (argument 4 of Sawada and Cai). Argument 1 of Sawada and Cai and the uncertainty of defining problem-solving the teachers mentioned might also be connected.

Only the teacher's statement about *no basic skills in maths* has no analogue in this literature-list of Sawada and Cai. But it is similar to Ausubel's second argument; he therefore supports the use of discovery learning for gifted students (as described in Sect. 3.3).

The same procedure was done for Sawada's arguments of advantages. For reasons of space, the detailed way of working is not explained in this paper.

Table 18.2 Teachers' statements in the interview about their concerns of using discovery learning and problem-solving in class

Teachers' disadvantages (in reference to Sawada and Cai)		Teacher A's statements, she teaches via discovery learning	Teacher E's statements She does not teach via discovery learning
1	No basic skills in maths	Students lack basic skills	Students have no basic skills in mathematics
2	No reading skills	Difficult for students, need reading skills	Students don't know what to do; they have no reading skills
3	Students are afraid	Students feel unsecure	Students are afraid of tasks like this
4	No motivation	Start with a low level, give feeling of success to keep students motivated	Students don't like tasks like that. Students are not motivated
5	No time	No time for discovery learning	There is not enough time for tasks like that

18.5 Results

18.5.1 The Teachers' Statements About the Role of Discovery Learning and Receptive Learning

Teacher A explains different lessons in which discovery learning has taken place. She can recall examples for tasks and illustrate how the diversity of pupils' solutions can make a profit in reaching the learning goal. Asked about her understanding of problem-solving, she gives a definition that is similar to the described definition of Schoenfeld. She also remembers a task in which a bridge is converged by a quadratic function and problem-solving activities were needed to find the quadratic function that describes the bridge best. Overall, one can say, that she is using discovery learning and problem-solving in her mathematic lessons.

As a second case, teacher E claims that in her teaching, she neither includes discovery learning, nor problem-solving. She has the same bridge-task in mind as teacher A and she remembers using this task at another school with different students, years ago. Now she teaches via step-by-step instructions, explaining examples at the board. Overall, she is using reception learning and instruction in her teaching.

When all five teacher-characterisations are compared like this, teacher A is the one that uses discovery learning as well as problem-solving the most. Teacher E uses this teaching method least. The other three teachers are somewhere in between (see Fig. 18.1): Teacher C is very close to teacher E; she prefers a very strict guided way of discovery learning and mentions one task-example for the use of discovery learning. Teacher B uses open access sometimes, sometimes a more guided way. He remembers one problem-solving task he never used himself. The examples he gives yield to calculation exercises. Teacher D has some examples for open approaches, and also for guided discovery tasks. To him, it is really important to lead students in higher classes to being creative and make them connecting algebra and analysis for example. On the other hand, he emphasizes calculation tasks in lower classes a lot.

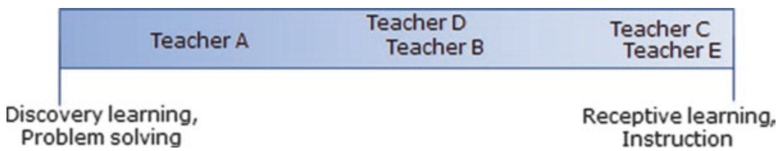

Fig. 18.1 The interpretation of the five teachers' interview statements leads to this relation between the teachers, considering the use of discovery learning and reception learning

Table 18.3 Teachers' statements in the interview about their supports of discovery learning and problem-solving in class

Teachers' advantages (in reference to Sawada and Cai)		Teacher A's statements to the question: What reasons do you have for using discovery learning?	Teacher E's statements to the question: Why did you use discovery learning at your previous school?
1	Higher students ability	The students work in small groups	I did a lot of self-organised learning there
2	Comprehensive use of mathematics	For some students the discovery process is helpful for learning	It is important to demand the students to link knowledge
3	Low achieving students can respond	In this example really every student can get at least one solution	/
4	High student motivation (for proofs)	The higher student motivation is one big advantage	/
5	Approval of fellow	By comparing the results with other students, one gets to the point, that…	/

18.5.2 The Teachers' Statements About Reasons for the Choices of Teaching Method

The statements of teacher A and teacher E are further analysed, because they differ the most according to the use of discovery learning and problem-solving. Their statements about advantages and disadvantages are shown in Tables 18.2 and 18.3.

To conclude, a comparison between the two teachers is made. Similar concerns about teaching via discovery learning and problem-solving were mentioned by both teachers (cf. Table 18.2), even though both teachers decided differently: teacher A is teaching via discovery learning and teacher E is preferring receptive learning. Teacher E explains that the inefficiency of her students made her decide to use step-by-step instructions. That reminds us of Ausubel's second argument: there is no need to make learning harder than it already is. Another reason teacher E mentions is a pressure of time. There are too many mathematical contents to be taught during a school year. Due to the low ability of her students and the limited time to teach a lot of content knowledge, she decides not to use problem-solving or discovery learning at all.

In Table 18.3, Sawada's five advantages are listed in a simplified way. For each of the five aspects by Sawada, teacher A can name at least one advantage of discovery leaning. That is different for teacher E. She can just name two advantages for discovery learning. Maybe that is one reason for not using this was of learning much. It is noticeable, that she does not name category 3, *Low achieving students can respond.* This may be an important factor for using discovery learning or not, especially in comparison to her reasons for using a teacher-centric way of teaching.

According to motivation, there seems to be a contradiction in teacher A's answers. She also gains the experience that students are not motivated to discover mathematics themselves. But she had found a way to deal with it. Her advice is to start with really low level tasks that give the students a sense of achievement. It would be interesting to know if teacher E had tried to use low level tasks.

18.6 Conclusion

As regards for the research questions listed in Sect. 18.2 three conclusions are formulated:

1. Two of the five interviewed teachers state that they use receptive teaching most of the time; two teachers state the use of discovery learning partly and one teacher states using discovery learning more frequently.
2. Even if teachers differ a lot in their use of discovery teaching, their motivations and beliefs about the disadvantages of discovery learning do not need to be very different (cf. teacher A and E). Both teachers name the same disadvantages of discovery learning.
3. A difference between the teachers can be seen in the advantages they list. The teacher with more teaching experience in discovery learning named more advantages than the other teacher. That may be why some disadvantages can be reduced or even changed into positive teaching situations.

The presented literature above was used to build categories for the second step of the interview analysis. This procedure is to be briefly reflected:

The list of Sawada's advantages and disadvantages can be used as a prototype of categories for analysing the teacher interviews. Other complementing categories are also mentioned by Ausubel and Bruner. With an inductive and deductive process final categories could be found to compare the teachers' statements (see Tables 18.2 and 18.3).

Another note: The teachers' answers about their views of mathematics have also been analysed. The statements are not explicit enough to make definite conclusions about their beliefs about mathematics. However, it is interesting to note that all five teachers distinguish between school mathematics and university mathematics.

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Chapter 19
Evaluation of a Questionnaire for Studying Teachers' Beliefs on Their Practice (TBTP)

Safrudiannur and Benjamin Rott

Abstract We developed a beliefs questionnaire named TBTP that allows us to investigate the possible influence of not only beliefs about mathematics but also beliefs about students' math abilities on teachers' actions in teaching mathematics and problem solving. The purpose of this study is to evaluate its reliability and validity. In this study, 43 teachers responded to the TBTP. The evaluation shows that the TBTP is reliable and valid since the analyses of the data confirm our hypotheses: teachers' responses show that they differentiate their style of teaching because of their students' math abilities and there is a correlation between teachers' beliefs about mathematics and their responses about their practice of mathematics and problem-solving.

19.1 Introduction

Several self-report instruments to measure teachers' beliefs and the effects of beliefs on their practices for large samples of teachers have already been developed (cf. Philipp, 2007). Nevertheless, the accuracy of the results of self-report surveys is questioned (Philipp, 2007; Di Martino & Sabena, 2010). Most of them use rating questions (often realized by using a Likert scale). According to Di Martino and Sabena (2010), the use of Likert scales amplifies statistical problems related to the respondents' degree of the tendency for social desirability. Our reviews identified several studies that addressed and discussed difficulties with self-report instruments, especially emerging from the use of Likert scales. For example, Hannula and Oksanen (2016) found that teachers' beliefs had very a small effect, even had no practical significance, on the development of student affect and achievement. They supposed that besides the low reliability, teachers might elicit socially

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appropriate responses on their instrument instead of honest responses (responses and reality might be different).

In another example, we noticed the inconsistency in reported results that also hint at problems with the validity of some self-report instruments. Zakaria and Musiran (2010) conducted a study with 100 teacher trainees. Their data apparently show inconsistencies. For example, 97% of their respondents agree with "learning mathematics must be an active process", but 89% also agree with "mathematics should be learned as sets of algorithms or rules that cover all possibilities", and 94% agree with "to solve math problems you have to be taught the correct procedure".

We argue that such problems appear because the existing instruments give teachers opportunities to respond to them ideally. Whereas there can be social contexts in a classroom which can make the real situation not ideal for teachers. Researchers (e.g., Raymond, 1997) found that social contexts in a classroom (e.g., students' math abilities) can cause the disparities between teachers' beliefs and actual teaching of mathematics and problem-solving.

Instead of using rating questions, some belief instruments use comparative formats (e.g., Raymond, 1997; Liljedahl, 2008; Törner & Pehkonen, 1998). Unlike the rating method that respondents rate items separately, the comparative format groups two or more items and respondents should make a comparative judgment between the items in one group at the same time (Brown & Maydeu-Olivares, 2012). However, Raymond, Törner, and Pehkonen, as well as Liljedahl did not report the reliability and validity of their instruments. Chen and Leung (2013) questioned the accuracy of some belief instruments because their reliability and validity are unknown. Theoretically, the use of the comparative format does not allow us to use conventional reliability and validity statistics (such as Cronbach's alpha), because the correlations of items in the comparative format are negative or close to zero (Brown & Maydeu-Olivares, 2012).

We have developed a quantitative questionnaire for studying **t**eachers' **b**eliefs on **t**heir **p**ractice (TBTP). We considered students' mathematical ability in the TBTP as an important factor influencing teacher behavior. Additionally, we use rank-then-rate questions to overcome the problems emerged from the use of rating and comparative questions. The study conducted by McCarthy and Shrum (1997) revealed that rank-then-rate questions could reduce the tendency of respondents to end-pile (i.e., giving high ratings towards items viewed inherently positive socially) and increase the respondents' willingness to make differentiations. In this paper, we discuss the evaluation of the reliability and validity of the TBTP.

19.2 Theoretical Background

19.2.1 Beliefs, the Nature of Mathematics, and a Mathematical Problem

Philipp (2007) defines beliefs as psychologically held understandings, premises, or propositions about the world that are thought to be true. Regarding beliefs of the nature of mathematics, Ernest (1989a) subsumed the beliefs in three views:

> First of all, there is *the instrumentalist view* that mathematics is an accumulation of facts, rules, and skills to be used in the pursuance of some external end. […] Secondly, there is *the Platonist view* of mathematics as a static but unified body of certain knowledge. […] Thirdly, there is *the problem-solving view* of mathematics as a dynamic, continually expanding field of human creation and invention, cultural product (Ernest, 1989a, p. 250).

Regarding a mathematical problem, Safrudiannur and Rott (2017) found that teachers might have beliefs that a difficult task is a problem although students know procedures to solve it. Some researchers argue that a task should be considered "a routine task (not a problem)" if students straightforwardly know a procedure to solve the task and are able to apply it (Rott, 2011). Pehkonen (2017) suggests that problem-solving should be a process allowing students to struggle to solve a problem. Therefore, we agree that there should be no obvious methods of solution available or no immediate access between the goal and the way to solve a problem (cf. Rott, 2011).

19.2.2 Relationship Between Teachers' Beliefs and Their Practices

Beliefs play an important role in the learning and teaching of mathematics (Philipp, 2007). Ernest (1989a) argues that what teachers believe about the nature of mathematics could be associated with teachers' models of teaching and learning mathematics. Ernest (ibid.) expresses how these three views affect teachers' practices.

> For example, the instrumental view of mathematics is likely to be associated with the instructor model of teaching, and the strict following of a text or scheme. It is also likely to be associated with the child's compliant behaviour and mastery of skills. […] Mathematics as a Platonist unified body of knowledge—the teacher as explainer—learning as the reception of knowledge; Mathematics as problem-solving—the teacher as facilitator—learning as the active construction of understanding, possibly even autonomous problem-posing and problem-solving. (Ernest, 1989a, p. 251–252)

Ernest (1989b) further argues that problem-solving teachers can accept students' own ways in solving a problem, whereas instrumentalist or Platonist teachers can lead students to the fact that there is only one single correct solution for the problem.

19.2.3 Disparities Between Beliefs and Practices and a Factor Causing the Disparities: Students' Mathematical Abilities

Many researchers have found strong indications that teachers' beliefs influence the way in which they teach mathematics and problem-solving (e.g., Anderson, White, & Sullivan, 2005; Stipek, Givvin, Salmon, & MacGyvers, 2001). However, there are also researchers who found no significant interplay of beliefs and actions, even disparities between the beliefs and their real practices (e.g., Raymond, 1997; Cooney, 1985).

Ernest (1989a) argues that social contexts in a classroom can cause the disparities between teachers' beliefs and their practice of teaching mathematics. Similar to Ernest, Raymond (1997) also found that the immediate classroom situation (e.g., students' abilities in a classroom) has strong influences on teachers' practice.

Regarding the influence of students' mathematical abilities, Anderson et al. (2005) reported the following correlation: teachers who express that problems should be presented after students have mastered math facts and skills usually believe that difficult problems are appropriate only for students that are more able. Zohar, Degani, and Vaaknin (2001) found that some teachers believe that higher order thinking such as problem-solving is inappropriate for low-achieving students. Furthermore, Safrudiannur and Rott (2017) also found that students' math abilities can be a barrier for teachers to implement their espoused model of practising problem-solving. Therefore, we argue that students' math abilities can be one of the social contexts in a classroom causing the inconsistency between teachers' beliefs and practice.

19.2.4 Construction of the TBTP (Teachers' Beliefs on Their Practice)

To overcome possible flaws and disadvantages of traditional questionnaires, we use rank-then-rate questions in the TBTP. The rank-then-rate procedure in a questionnaire has been suggested by Munson (1984) and later by McCarthy and Shrum (1997) since the procedure can overcome the weakness of the rate-only method, which we addressed in the introduction.

We group ten rank-then-rate questions in the TBTP into three themes (see Table 19.1). Each theme starts with a note, that is, information to ensure a shared understanding of terms between us as researchers and the respondents. For example, since teachers may have their own understanding about a math problem which may be different from the definition of a problem theoretically (as we pointed out in the theoretical background), we give the definition of a problem in the note for Theme 2 (see Appendix).

Each question has three statements. Each statement of each question is related to one of the views of mathematics described by Ernest (1989a): the first, second, and third statements are always associated with the instrumentalist view, the Platonist view, and the problem-solving view, respectively (cf. Table 19.1 to see how we adopted the views from Ernest into the statements).

To respond to a question, a respondent must order the three statements of each question by assigning a rank 1 (the most important/most agree), 2, or 3 (the least important/least agree) to those statements. After that, s/he must rate them based on her/his ranks. The rating scale is from 1 to 7. Brown and Maydeu-Olivares (2012) argued that if a respondent can assign different ranks between two things, s/he

Table 19.1 Structure of the TBTP

No	Structure of the TBTP	
	General note: Term of an HA and an LA student (see Appendix)	
	Theme 1: Teaching and learning of mathematics Note: The formulas to find the area of a trapezoid (see Appendix)	
1, 2	When you teach the formula, what do you think that is important for you?	
	R1	You demonstrate how to use the formula correctly by giving some examples
	R2	You explain concepts related how to get or to prove the formula
	R3	You let your students discover the formulas in their own ways
3, 4	When you teach the formula, what do you think that is important for the students?	
	S1	They memorize and use the formula correctly
	S2	They understand the concepts underlying the formula from your explanation
	S3	They can draw logical conclusions to deduce the formula
	Theme 2: Practice of problem-solving Note: The definition of a problem and its example and non-example *(see Appendix)*	
5, 6	When you pose a mathematical problem, what do you think that is important for you?	
	T1	You give clues about the right method or formula to solve it
	T2	Besides you assist your students, you ensure that they understand what they write
	T3	Without giving clues, you encourage your students to express their own ideas to solve it
7, 8	When you pose a mathematical problem, what do you think that is important for the students?	
	U1	Through the clues you give, they recall the right method to solve it
	U2	Besides they get the correct answer by your assistance, they can explain what they write
	U3	Without giving clues, they create their own strategies to solve it
	Theme 3: The nature of mathematics Note: The classification of the contents of mathematics in general *(see Appendix)*	
9	In general, what do you think of the contents of mathematics?	
	P1	Mathematics is an accumulation of facts, rules, and skills, which are useful for human life
	P2	The contents of mathematics are interrelated and logically connected within an organizational structure
	P3	Mathematics is a dynamic process of human activities. The contents of mathematics expand and change to accommodate new developments
10	What do you think of the truth of the contents of mathematics?	
	Q1	The truth of mathematics is absolute. They are free of ambiguity and conflicting interpretations
	Q2	Mathematical ideas are preexisting; the contents of mathematics are just discovered by humans. Thus, the truth-value of mathematics is objective, not determined by humans
	Q3	The contents of mathematics are created by human, and therefore their truth-value is also established by humans

Questions 1 = 2, 3 = 4; 5 = 6; 7 = 8, but with a different class (questions 1, 3; 5; 7 for HA class and 2, 4; 6; 8 for LA class). The rating scale for Questions 1–8 is from 1 (not important) to 7 (very important); it for Questions 9 and 10 is from 1 (not agree) to 7 (strongly agree)
P1, Q1, R1, S1, U1, T1 are associated with the instrumentalist view, P2, Q2, R2, S2, U2, T2 are assoc. with the Platonist view; P3, Q3, R3, S3, U3, T3 are assoc. with the problem-solving view

should have a different psychological value (latent) for each of them. Therefore, we determined that the higher rank must have the higher rate.

As we consider students' mathematical abilities as a social context in a classroom (characterized by students' achievement), we divided the questions for Theme 1 and Theme 2 into two conditions: in a class dominated by high-achieving students (HA class) and in a class dominated by low-achieving students (LA class).

To evaluate the TBTP questionnaire, we conduct a quantitative study. The research questions of the study: *(1) How good is the reliability of the TBTP questionnaire? and (2) How good is the validity of the TBTP questionnaire?*

19.3 Study

19.3.1 Participants

The study was conducted on February 16th, 2017, in Samarinda, Indonesia. The participants were mathematics teachers who came to a mathematics competition for students held by the Department of Mathematics Education of the Mulawarman University. The first author gave more than 70 questionnaires to the organizing committee of the competition to give the TBTP questionnaire to teachers. Of these questionnaires, 47 were returned to the organizing committee. However, we excluded the completed questionnaires of four teachers due to incomplete answers. Thus, we only examine the answers from 43 teachers (sex: 33 females, seven males, three unknown; schools: 14 from primary, 16 from lower secondary, 10 from upper secondary, 3 unknown; experience as math teachers: 7 for less than 2 years, 10 for 2–5 years, 9 for 5–10 years, 16 for more than 10 years, 1 unknown).

19.3.2 Method for Evaluation

The evaluation of the reliability. Each item/question in the questionnaire consists of three statements with each statement belonging to either the belief dimensions associated with the instrumentalist view, the Platonist view, or the problem-solving view. We evaluate the internal consistency reliability of the TBTP by calculating the coefficient of Cronbach's alpha of each dimension.

The evaluation of the validity. Bolarinwa (2015) emphasizes that an instrument has a high degree of hypothesis-testing validity if it supports the relationship between the measured concepts (variable) derived from theories. Therefore, to evaluate the hypothesis-testing validity of the TBTP, we propose four hypotheses as below:

1. Theoretically, teachers report that they focus more on guiding students to obtain correct answers but less on developing students' understanding when teaching LA students than when teaching HA students (Zohar et al., 2001; Evan & Kvatinsky, 2009). Thus, the first hypothesis is "Teachers' rates for HA classes are lower than their rates for LA classes in the dimension of the instrumentalist view."
2. Theoretically, teachers generally agree that understanding is important, and they want that their students to understand what they teach (Van de Walle, Bay-Williams, Lovin, & Karp, 2013). Since all statements in the dimension of the Platonist view are related to the assumption that students should understand what they learn, the second hypothesis is "There are no differences between teachers' rates for HA classes and those for LA classes in the dimension of the Platonist view."
3. Theoretically, teachers report that they emphasize teaching for higher order thinking and problem-solving more when they teach HA students than when they teach LA students (Zohar et al., 2001; Evan & Kvatinsky, 2009; Raudenbush, Rowan, & Cheong, 1993). Thus, the third hypothesis is "Teachers' rates for HA classes are higher than those for LA classes in the dimension of the problem-solving view."
4. Theoretically, teachers' beliefs of the nature of mathematics influence teachers' practice. The fourth hypothesis is "Teachers' rates to items related to the beliefs of the nature of mathematics correlate to their rates regarding how they teach mathematics and problem-solving either in HA classes or in LA classes."

Before this study, we evaluated the content validity of the TBTP, that is, to ensure that the TBTP measures what it is intended to measure. The popular way to evaluate the content validity is involving experts for the evaluation (Bolarinwa, 2015; Shepard, 1993). We involved two colleagues from the University of Duisburg-Essen and one colleague from the Mulawarman University as experts (in December 2016) to evaluate the TBTP contents. We asked them to evaluate and criticize whether the statements of each question specifically belong to the concepts of each view of mathematics from Ernest (1989) and whether the note of each theme could help us to capture teachers' view of mathematics. We improved the TBTP based on their critiques and suggestions.

In addition, Bolarinwa (2015) expressed that reviewing the readability, clarity, and comprehensiveness of an instrument is a part of the evaluation of the content validity. We asked five Indonesian secondary teachers (three males, two females) to evaluate the readability, the clarity, and the difficulty of each note, each question, and each statement of the TBTP. We also asked the five teachers to re-express their understanding to the notes, questions, and statements in their own words. We did an extensive discussion with the five teachers until we assessed that they had a similar understanding to the TBTP and it was easy for them to answer it.

19.4 Results and Discussions

19.4.1 Evaluation of Reliability of the TBTP

We computed the Cronbach's alpha coefficient (α) to evaluate the reliability of each beliefs dimension. Table 19.2 shows that the alpha coefficients for each dimension are around 0.8, which implies that they are acceptable (Field, 2005). Regarding items, Field (ibid) argues that any item with an item-total correlation below 0.3 should be dropped. The calculation of the item-total correlation of each item of each dimension varies between 0.35 and 0.68. This indicates that no item needs to be excluded.

19.4.2 Evaluation of Hypothesis-Testing Validity of the TBTP

The first hypothesis: "Teachers' rates to the TBTP for HA classes for the dimension of the instrumentalist view is lower than their rates for LA classes." Table 19.3 shows the results of paired t-tests to evaluate whether teachers differ how they teach mathematics between HA and LA classes. The negative mean differences for all statements (R1, S1, T1, and U1) associated to the instrumentalist view indicate that teachers gave lower rates to the statements for HA classes than those for LA classes. The negative t-values clearly show that teachers' rates for HA classes are significantly lower than those for LA classes in the dimension of the instrumentalist view. Our hypothesis is accepted and, thus, the TBTP confirms the theory that teachers' responses indicate that their style of teaching mathematics in HA classes is less associated to the instrumentalist view than that in LA classes.

The second hypothesis: "There are no differences between teachers' rates to the TBTP for HA classes in the dimension of the Platonist view than those for LA classes." Unlike the dimension of the instrumentalist view, the mean differences for all statements in the dimension of the Platonist view are low. The examination of t-values for this dimension indicates that there are no differences for all statements. Again, our hypothesis is accepted.

The third hypothesis: "Teachers' rates to the TBTP for HA classes in the dimension of the problem-solving view is higher than their rates for LA classes." In contrast to the dimension of the instrumentalist view, mean differences between HA

Table 19.2 Reliability of each dimension of beliefs

Dimension	Statements	α
Instrumentalist	P1, Q1, R1(HA), R1(LA), S1(HA), S1(LA), T1(HA), T1(LA), U1(HA), U1(LA)	0.83
Platonist	P2, Q2, R2(HA), R2(LA), S2(HA), S2(LA), T2(HA), T2(LA), U2(HA), U2(LA)	0.84
Problem-solving	P3, Q3, R3(HA), R3(LA), S3(HA), S3(LA), T3(HA), T3(LA), U3(HA), U3(LA)	0.79

Table 19.3 Results of paired sample t-tests

Dimension	Statements	Teachers' rate HA Mean (sd)	Teachers' rate LA Mean (sd)	Mean differences (sd)	*t*-values (df = 42)	Sig. (2-tailed)
Instrumentalist	R1	4.09 (1.66)	5.65 (1.13)	−1.56 (1.65)	−6.19*	0.000
	S1	4.07 (1.49)	5.35 (1.15)	−1.28 (1.68)	−4.99*	0.000
	T1	4.23 (1.49)	5.40 (1.75)	−1.16 (2.10)	−3.63*	0.001
	U1	4.60 (1.42)	5.88 (1.12)	−1.28 (1.79)	−4.68*	0.000
Platonist	R2	5.02 (1.39)	5.44 (1.42)	−0.42 (1.88)	−1.46	0.152
	S2	5.86 (1.19)	5.47 (1.32)	0.39 (1.65)	1.57	0.124
	T2	4.65 (1.13)	5.14 (1.10)	−0.49 (1.12)	−2.86	0.007
	U2	4.86 (1.28)	5.05 (1.25)	−0.19 (1.39)	−0.88	0.383
Problem-solving	R3	5.33 (1.55)	3.47 (1.24)	1.86 (1.92)	6.35*	0.000
	S3	5.44 (1.05)	3.79 (1.81)	1.65 (1.96)	5.52*	0.000
	T3	6.00 (1.18)	3.88 (1.48)	2.12 (1.71)	8.13*	0.000
	U3	5.60 (1.47)	3.42 (1.31)	2.19 (1.88)	7.62*	0.000

and LA classes for all statements associated with the problem-solving view are positive. Moreover, all positive t-values support our third hypothesis that teachers' rates to the TBTP for HA classes in the dimension of the problem-solving view is very significantly higher than those for LA classes. It means that teachers' responses indicate that their teaching style in HA classes is more associated with the problem-solving view than that in LA classes.

The fourth hypothesis: "Teachers' rates to statements related the beliefs of the nature of mathematics correlate to their rates to statements regarding how they teach mathematics and problem-solving either in HA classes or in LA classes." We expect that the TBTP can predict teachers' actions in teaching mathematics and problem-solving. Therefore, it is necessary for us to evaluate whether the TBTP confirms the theory that beliefs influence practice.

Table 19.4 indicates that teachers' beliefs of the nature of mathematics correlate to their reports about their practice in LA classes for each dimension. All statements of Themes 1 and 2 for LA classes correlate to both or one of two statements related the beliefs of the nature of mathematics of each view (Theme 3), and thus, the correlations confirm the fourth hypothesis.

Table 19.4 Pearson correlation of between items in the Theme 3 and items in the Themes 1 and 2 for each dimension

Dim.	Theme 3	HA				LA			
		Theme 1		Theme 2		Theme 1		Theme 2	
Instrumentalist		R1	S1	T1	U1	R1	S1	T1	U1
	P1	0.296	0.390[a]	0.215	0.274	0.420[a]	0.338*	0.663[a]	0.415[a]
	Q1	0.071	0.288	0.177	0.137	0.443[a]	0.531[a]	0.667[a]	0.532[a]
Platonist		R2	S2	T2	U2	R2	S2	T2	U2
	P2	0.302*	0.281	0.446[a]	0.415[a]	0.181	0.332*	0.475*	0.543[a]
	Q2	0.332*	0.361*	0.185	0.359*	0.319*	0.391[a]	0.220	0.545[a]
Problem solving		R3	S3	T3	U3	R3	S3	T3	U3
	P3	0.278	0.256	0.335*	0.267	0.249	0.202	0.456[a]	−0.086
	Q3	0.099	0.073	0.258	0.182	0.528[a]	0.542[a]	0.384*	0.353*

Mean = mean values; sd = standard deviations, df = degrees of freedom
*significant for $p < 0.004$, df=42 (The adjustment of alpha = 0.05 by Bonferroni's correction for 12 multiple t-test)
*significant for $p < 0.05$; [a]significant for $p < 0.01$

19.5 Concluding Remarks

Our intention of developing a TBTP questionnaire is to provide a reliable and valid quantitative instrument which is not only able to capture the relationship between teachers' beliefs and their models of teaching mathematics and problem-solving but also able to capture the differentiation between the beliefs and the models. Therefore, we designed the TBTP by considering students' mathematical abilities (characterized by students' math achievement) as a social context in a classroom. Raymond (1997) and Ernest (1989a) include students' math abilities as a factor causing the differentiation between teachers' beliefs and practices.

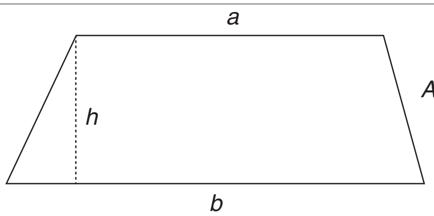
The evaluation of the TBTP questionnaire shows that the reliability related the internal consistency of all dimensions is acceptable. The TBTP also confirms the theory that teachers make a distinction in how they teach mathematics and problem-solving between classes dominated by HA students and classes dominated by LA students. Moreover, our examination confirms that beliefs of the nature of mathematics correlate to their style of teaching of mathematics and problem-solving in LA classes, which we expect that their responses are close to their real practice in the case that they have many LA students in their class. In contrast to other questionnaires focussing on teachers' beliefs, the TBTP allows us to identify that not only beliefs of the nature of mathematics but also beliefs about students' math abilities influence teachers' style of teaching mathematics and problem-solving. Moreover, the TBTP allows us to capture the differentiation of the style of teaching and gives us a better description about the relationship between beliefs about mathematics and teachers' practice.

The limitation of this study is the small size of the sample (<50). Kline (1986) suggested that the minimum sample size to get an accurate Cronbach's alpha for the reliability analysis is 300. However, Yurdugül (2008) has shown that a study with a small sample (n = 30) can also result in an unbiased estimator of alpha.

Further, researchers suggest that validating an instrument in a single study is not enough. Since it is necessary for us to ensure that the TBTP is a valid representation of what it intends to measure (predicting teachers' beliefs and practice), therefore, we use a multimethod-design (interview and observation) in the next study to validate it.

Appendix

Table 19.5 All notes in the TBTP

Types	Contents
General note	As a mathematics teacher, you have experience with high and low achieving students in mathematics. Consider these definitions: **A high achieving (HA)** student is a student who generally shows good understanding of your lessons and regularly has high scores in your tests **A low achieving (LA)** student is a student who generally does not show good understanding of your lessons and often has low scores in your tests To answer all questions, you will be asked to first imagine that you have a class dominated by HA students and then to imagine that you have a class dominated by LA students
Note for Theme 1	You are going to teach a lesson learning the formula to calculate the area of a trapezoid Please imagine this situation to answer items 1 to 4  $A = \frac{1}{2} .(a + b) . h$
Note for Theme 2	In problem-solving, there are several definitions about a mathematical problem. Below is one of the definitions of the mathematical problem **A mathematical problem** is a task for which there is no obvious or straightforward solution method to solve it According to the definition, a task is not a mathematical problem if it can be solved by simply applying methods previously taught. Please see the example to better understand the definition **Example**: You explained how to calculate the average of the following data: 20, 16, 18, 28, 22, and 20. Then you give a task: "*The height of six basketball players is 196 cm, 200 cm, 190 cm, 185 cm, 192 cm, and 200 cm. Find the average height of these six players.*" Although the mathematical task above is related to the real world, the task is not categorized as a mathematical problem according to the definition because your students can simply apply how to calculate the average from what you have taught Now, have a look at the following task: "*The average weight of six futsal players is 65 kg. After a substitution, the new average weight is 63.5 kg. If the weight of the player who left is 64 kg, find the weight of the new player.*" According to the definition, this task can be categorized as a mathematical problem because the students cannot simply apply what you have taught Please use **only** this definition to answer items 5 to 8

(continued)

Table 19.5 (continued)

Types	Contents
Note for Theme 3	Mathematics contents taught at school can be divided into several sub-domains such as numbers, algebra, geometry, measurements, statistics, and probability. The classifications of mathematics contents in general are more complex, for example classical algebra, linear algebra, number theory, differential geometry, calculus, statistics, and probability theory

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Chapter 20
Role of Technology in Calculus Teaching: Beliefs of Novice Secondary Teachers

Ralf Erens and Andreas Eichler

Abstract The appearance of portable technological tools has given rise to a growing body of research at various levels of mathematics education. The entry of these innovations has implications for the teaching and learning of mathematics posing a challenge for all participants in the classroom. With particular attention to graphing and computer-algebra technology, this report focuses on teachers' beliefs and their intended instructional planning towards their teaching of calculus at upper-secondary level. First the theoretical framework and methodology is outlined. Afterwards the focus lies on studying how and why secondary-level teachers actually employ the technological device in the teaching and learning of calculus as the central part of upper-secondary mathematics courses in Germany. Results from a qualitative study of pre-service and trainee teachers will be discussed centred on how their beliefs on the role of technology correlate with beliefs on secondary level calculus teaching.

20.1 Introduction

Teachers' intended instructional design of lessons is represented by their belief systems as a part of mathematics-related affect (Speer, 2005, p. 364). The importance of gaining knowledge towards mathematics teachers' thinking and beliefs has been emphasised by many researchers in mathematics education because teachers' beliefs about the teaching and learning of mathematics have a high impact on their instructional practice (Philipp, 2007). Research about mathematics classrooms suggests that beliefs are one of the significant forces affecting teaching (ibid.). The investigation of teachers' beliefs is thus motivated by their relevance and potential impact as these beliefs represent a "significant determiner of what gets taught [and] how it gets taught" (Wilson & Cooney, 2002, p. 128). With research on mathematics

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teachers' beliefs concerning different mathematical topics insights could be gained that teachers hold distinguishable beliefs about different mathematical objects or topics (Eichler & Erens, 2015). The specificity of secondary teachers' beliefs with a focus on calculus as a central part of the curriculum is connected to considerations concerning technology use at upper secondary level, as the dissemination and use of graphing and computer-algebra technology has received growing attention in teaching and research (cf. Trouche & Drijvers, 2010). As prior beliefs that have been formed from own experiences learning mathematics at school and university serve as a filter for novice teachers' professional learning (Levin, 2015, p. 50), we focus on pre-service and trainee teachers and their beliefs in development phases from university education to working as a professional teacher. As research has shown (Gueudet, Bosch, Disessa, Kwon, & Verschaffel, 2016) this phase of transition from university into teaching includes a cognitive process of adaptation which may be decisive for future meaningful teaching with or without technology. Among many other aspects of teachers' professionalisation, this report concentrates on the explication of our findings concerning the following question: *how* and *why* do prospective secondary teachers intend to use graphing or computer-algebra technology for teaching calculus with respect to different levels of instrumental integration? Before we address the aforementioned reconstruction of beliefs, an outline is given about the theoretical framework and a brief description of those parts of the method being relevant for this paper. Finally, we conclude the paper by reflecting on the main results and discuss possible directions of further research.

20.2 Theoretical Framework

20.2.1 Beliefs and Goals

The main constructs of our theoretical framework are teaching goals and teachers' beliefs. Firstly, according to Pajares (1992), we understand the term beliefs as an individual's personal conviction concerning a specific subject, which shapes an individual's ways of both receiving information about a subject and acting in a specific situation. We further follow Green (1971) referring the internal organisation of beliefs in a belief system. The construct of belief systems involves that beliefs are organised in clusters that are quasi-logically connected, which potentially includes also connections of beliefs that seem contradictory (ibid.). Finally, Green distinguishes primary beliefs and subordinated, derivative beliefs in which enacting derivative beliefs serve as a means to an end for achieving primary beliefs.

According to the framework of Hannula (2012), both belief systems and goals are parts of mathematics-related affect that consists of cognitive, motivational and affective aspects. Hannula (ibid.) further describes beliefs or rather belief systems as a psychological aspect of mathematics-related affect as a trait and, hence representing a disposition. In contrast, he describes goals as a psychological aspect of mathematics-related affect as a state. Goals and beliefs are often seen as different constructs in belief research: on the one hand beliefs can determine decisions for goals (Speer, 2005, p. 365), on the other hand goals can be regarded as part of a belief system (Eichler & Erens, 2014, p. 649). In contrast to the distinction of affect as a trait and affect as a state, we follow the so called Rubicon-model of Heckhausen and Gollwitzer (1987) in which goals are understood in a broader sense constituting a teacher's decision-making (state of awareness referring to the choice of goals) before passing the Rubicon, that is, when a teacher plans his classroom practice, and after passing the Rubicon, that is, the teacher's decision-making during his classroom practice (state of awareness when enacting the goals). Hence in our research the construct of goals is understood as decisions about future teaching in which relevant beliefs connect goals on different levels.

20.2.2 Instrumental Integration

As research has shown, the role of technology in mathematics teaching requires an assiduous distinction between technical and conceptual mathematical activity. Technical activity is primarily concerned with tasks of procedural performance, whereas conceptual activity is concerned with tasks of inquiry, conjectures and justification (Zbiek, Heid, Blume, & Dick, 2007). The function of technology in learning mathematics effectively is a central question for the teaching practice in technology supported classrooms. Although the function is distinguished further (ibid.), we primarily focus on the activities teachers intend to enact in their classrooms. In order to describe the development from technical activities to conceptual activities the construct of *instrumental genesis* seems to be suitable (Artigue, 2002). Central to this theory is the notion of an *instrument*, which is differentiated from an *artefact* (cf. Trouche & Drijvers, 2010). The notion of instrument is a psychological one and not a description of a material artefact (Zbiek et al., 2007; Trouche, 2005). To develop a relationship with the artefact (e.g. a graphing calculator), the user needs to understand the artefact and its capacity. Instrumental genesis is the process of the artefact becoming an instrument and specifically how the artefact becomes a mathematical instrument—a tool that the user can employ for mathematical purposes.

Providing an example of the approach of instrumental genesis relevant for calculus (Goos & Soury-Lavergne, 2010), the construction of a secant or tangent can be seen as an instrument to conceptualise the difference quotient. The students must learn to construct the secant and drag it along the graph of the function up to a given point (*instrumentalisation*). But they also have to learn, why dragging the secant is meaningful (*instrumentation*) and that this process leads to the conceptualisation of a new mathematical definition. Goos and Soury-Lavergne(2010), p. 313), outline that "instrumental integration is a means to describe how the teacher organizes the conditions for instrumental genesis of the technology proposed to the students and to what extent (s)he fosters mathematics learning through instrumental genesis" distinguishing different levels of instrumental integration: on the first level (*instrumental initia-*

tion) the focus lies on making the students familiar with the basic technical aspects of the tool by way of tasks that enable students to use the technology for mathematical activity (tool competence). On the subsequent level, the emphasis lies on students' exploration of the different features the technology offers through mathematical tasks (*instrumental exploration*). If the technology helps the students to overcome (e.g. algebraic) difficulties so that they can concentrate on the mathematical core of the given problem, the level is labelled *instrumental reinforcement*. Finally, if the technology is used by students to solve mathematical tasks with the explicit assistance of the technology so that instrumental integration is necessary to create mathematical content, this is called *instrumental symbiosis*. The question of how a "teacher organizes the conditions for instrumental genesis" (Goos & Soury-Lavergne, 2010, p. 313) is thus closely connected to the teachers' beliefs about what parts of an instrumental genesis facilitate their students learning. Conceptualising teachers' content-specific belief systems about appropriate ways of teaching and learning, a distinction between beliefs about different teaching orientations (transmission view vs. constructivist view, cf. Kunter et al., 2013) is used to reveal findings whether different levels of instrumental integration correlate with such pedagogical beliefs.

20.3 Method

In this paper we refer to a sample of 20 prospective upper secondary teachers divided into two subsamples: ten pre-service teachers, who have just completed their mathematics undergraduate courses at university and ten trainee teachers that have participated in the special teacher education programme between university and the career as a qualified teacher in school. The teachers who participated in our study were recruited from the south-western part of Germany. Although our sample is not a representative but a theoretical sample (Glaser & Strauss, 1967), we tried to vary the characteristics of participants as much as possible (university, second teaching subject, teacher training college, etc.). Since beliefs could be connected to certain communities of practice and social desirability, this design seems to be crucial (Skott, 2015, p.19).

Both subgroups of our sample were interviewed twice. In the first interview data were collected by semi-structured interviews. Topics of these interviews were several clusters of questions that concern the content and goals of calculus teaching, the nature of calculus as a discipline generally and the possible influence of technology on teaching. An exemplary question about technology was:

To what extent have there been changes in calculus courses due to using technological devices such as graphing or computer-algebra technology?

Further, we used prompts to elicit teachers' beliefs, for example, by presenting potential challenges implied by the use of technology, fictive or real statements of teachers concerning instructional objectives and the use of technology in calculus teaching.

In the second interview validation of analysis of the first interview was accomplished by confronting interviewees with key statements from their first interview and eliciting either agreement or further adjusting and explanation. The time span between the two interviews ranged from at least 1 year up to 18 months. The pre-service teachers had then nearly completed their teacher education programme and the trainee teachers worked as teachers for nearly one school year.

For analysing the data, we used a qualitative coding method (Mayring, 2015) that is close to grounded theory (Glaser & Strauss, 1967). The codes gained by interpretation of each episode of the verbatim transcribed interviews indicate goals of calculus teaching. In the process of analysing the data material inductively, notions of teaching practices and beliefs about the role of technology in calculus teaching emerged. Regarding the (dis-)advantages of the use of technology, for example, we developed codes such as "visualisation" describing possible instrumentation reasons to analyse teachers' relevant beliefs. The similarities and differences gained from these inductive data codings were then connected to the construct of instrumental genesis as shown in the results. Codings were conducted by at least two persons and slight differences were solved by discussion and consensus.

20.4 Results

In order to illustrate and categorise novice teachers' beliefs concerning the use of technology in calculus teaching, we organise the discussion of results as follows: First the results from the interviews are described which reveal beliefs about reasons and different ways of technology use. The various features of technology use which emerge from data coding in both interviews are classified according to the relevant level of instrumental integration and inductively gained characteristics of teachers' rationale are reported within these levels. Whether teachers have an affirmative or rather dismissive view about the use of technology in calculus classrooms is then related to associated teaching orientations.

After finishing their university education beginning teachers have to cope with a multidimensional task when they start their first teaching experiments: the transfer of abstract mathematical knowledge to school level is challenging as well as managing complex classroom situations which include the use of technology.

> Mrs A (pre-service teacher, interview 1):
>
> *"Well, before I think of using these calculators, I have to get my lessons structured and organize the content. As I did not grow up with these graphing calculators neither at school or university, my students still have a head start over me."*
>
> Mr B (pre-service teacher, interview 1):
>
> *"Using these calculators would be sensible in my view if students could use the device in their further university education or professional qualification. […] Its use in my calculus lessons is rather scarce because they [students] first need to grasp the concepts."*

Although they mention the use of technology as a visualisation instrument in several parts of the interviews, both pre-service teachers did not encounter tools such as graphing calculators or computer-algebra systems (CAS) in the course of their school and university education, lack a personal experience and also mention critical aspects concerning its integration into their teaching practice. A discontinuity between university learning and school mathematics (Gueudet et al., 2016, p. 11) concerning the use of technology becomes obvious as well as indications that the instrumental integration of technology seems to be very much dependent on previous experiences with such artefacts. The discontinuity between mathematical paradigms at school and university still seems to persist, as the critical reflection of Mr B shows, indicating that his beliefs on technology use in calculus courses go beyond the dimension of the institutional context (i.e. further professional qualification of students). In several parts of both interviews Mr B emphasises the priority of conceptual issues and stresses that the integration of technology is not a key issue of his intended calculus teaching.

Accordingly beliefs on instrumentalisation in connection with teaching calculus can be attributed mainly to the level of instrumental initiation. As the teachers in our sample come from various backgrounds (school and university education) the different levels of technology integration in their intended calculus teaching reveal interesting results *how* and *why* they intend to use a graphing or symbolic calculator. As teachers and students use such calculators following a curricular requirement, it is not surprising that many of the pre-service and trainee teachers at first meet challenges they face on a basic instrumental level. Despite pedagogical and instrumental challenges the majority of teachers in both groups see the key advantage of using the artefact in the possibility of visualising mathematical objects. Occasionally reaching the level of instrumental exploration, the technology is mainly used for a task-based liaison of the concept and a mental picture of functions (➔ visualisation), as the example of Mrs A shows:

Mrs A. (pre-service teacher, interview 2):

"…starting from average speed, slope of the secant, then move on to the slope of the tangent; here I use a lot of visualization enabled by the technology. Despite these visual helps students still have difficulties in grasping the concept."

Very few teachers emphasise the possibility of being able to change between three modes of representation (table, graph and function) neither in the first nor in the second interview. However, effects of using the technology in classroom is seen in further benefits relating to motivational and emotional traits: the possibility of checking results and relieving students from procedural investment:

Mrs C (trainee teacher, interview 2):

"For my students using the calculator has made many steps easier. They can concentrate on the task itself and are not constantly misled by calculation errors."

The avoidance of (algebraic) difficulties by instrumental reinforcement displays further objectives. Economy of time, the functionality as an important control tool and thus a shift of emphasis of cognitive skills in solving conceptual tasks instead of tedious routine calculations are main aspects being mentioned by this trainee teacher in both interviews.

Mrs C (trainee teacher, interview 1):

"Working in pairs with the help of the CAS calculator is particularly effective in my classes. The time gained because they [students] don't have to do these tedious calculations can be used to structure their own approach when they tackle a given problem. […] of course these paper and pencil skills are likely to decrease if they are allowed to use the calculator at all times."

However, Mrs C is aware—like other trainee teachers in our sample—that these phenomena go hand in hand with decreasing student skills with routine tasks. A balancing act is also seen by Mrs D, who saw curve sketching and calculation routines as an essential cornerstone of her calculus course in the first interview. In her second interview she makes an interesting comparison:

Mrs D (trainee teacher, interview 2):

"Of course they [students] do not have to carry out so many routine calculations any more. These skills still have to be practised to some extent but less intensively than in my own calculus courses (…) one can shift the emphasis in tasks more towards problem-solving."

For her the use of technology in the classroom has become a tool to reduce schematic calculations and thus a gain of teaching time. Taking into account that beliefs of novice teachers that emerge from prior experiences learning mathematics at school serve as a filter for their professional learning in teacher education (Richardson, 2003), memorised procedures such as derivation rules and curve sketching are seen as an indispensable part of high-school calculus by some participants. Nevertheless there is a shift of content-related prioritisation induced by the use of graphing or computer-algebra tools. The time gained can be used to foster heuristic strategies in order to develop a deeper insight into fundamental ideas of change, variation, approximation processes and derivatives (➔instrumentation). Regardless of the degree of personal experience and tool proficiency many prospective teachers such as Mrs C and Mrs D quickly overcome the difficulties in bridging the gap between the axiomatic-formal world of calculus concepts (Tall, 2008) and calculus instruction at high-school level. The focus lies on cognitive strategies to help students gain insights into key concepts by visual approaches and using the calculator as a manipulating assistant. Curricular requirements of instrumental integration are rated positively, the growing possibilities of multimodal representations are seen as a benefit and the level of integration can be located between instrumental exploration and reinforcement.

With respect to the teaching of calculus, beliefs on using technology as a learning tool is a derivative belief that serves as a means to an end for most teachers. Validation of key statements on the evaluation of technology use in second interviews and self-perception about technology beliefs after several months of classroom experience reveal further results how and why prospective teachers use a graphing or symbolic calculator. Beyond the advantage of visualisation of mathematical objects, many teachers see the possibility to incorporate more complex modelling tasks as well as the opportunity to further their students´ heuristic competence.

Mrs E (trainee teacher, interview 2):

"…with the help of the calculator, my students are able to quickly generate 20 examples, analyse these and efficiently come to well-founded results – it is just more purposeful for my students in order to grasp the connections and regularities themselves. […] …these more complex real-word applications definitely needed CAS assistance for my students – constantly being able to check mathematical results kept them motivated."

Employing the handheld device as a manipulating assistant, students on the one hand can be additionally supported to overcome algebraic difficulties and on the other hand be given the possibility to dynamically explore conceptual aspects of calculus content with the aim of developing a more advanced mathematical thinking (Tall, 2008). Teachers expressing beliefs in such or a similar way represent characteristics of the instrumental levels of reinforcement and symbiosis as students are encouraged to connect their technical know-how to mathematical knowledge (Goos & Soury-Lavergne, 2010, p. 314). In this way teachers try to maximise instrumental integration. Along with aforementioned possibilities of visualisation as a demonstration tool, many teachers in our sample make remarks on increasingly scaffolding student learning with the technology and specify increasing levels of instrumental integration as a valuable addition to their calculus courses with increasing teaching experience.

Nevertheless there is a substantial variation in our data concerning the approval or repudiation of technology use in calculus courses. Our data yields the emergence of two antithetical belief clusters on the integration of technology in upper secondary calculus courses: a rejectionist and a supportive stance. Teachers with a rejectionist stance disbelieve that technology use in calculus teaching helps students to develop a robust understanding of the relevant content. Visualisations and approximations with graphing technology are seen as counterproductive to learning calculus as students are encouraged to putting an approximate solution on the same level with exact solutions, as the example of this trainee teacher illustrates:

Mr F (trainee teacher, interview 2):

"…these graphical solutions are just misleading to students. They do not see the need any more to find exact solutions by means of algebraic solutions for equations. Or they just take a particular section of the graph and don't think about the full graph of a function. […] Conceptual ideas are not needed as they are satisfied with approximate solutions."

In contrast, those with a supportive stance, such as Mrs D and Mrs E above, believe that the use of technology can provide multiple perspectives on calculus for the students and the instrument can ideally be applied to develop a higher order mathematical thinking.

20.4.1 Technology and Teaching Orientation

As different belief clusters can be quasi-logically connected, causal relations between different clusters have to be considered carefully. It is interesting, however, that teachers' beliefs about concepts of mathematical learning and associated teaching orientations (which have to be distinguished from methodological decisions as well as classroom management) may be related to different levels of instrumental integration and thus to belief clusters about technology use. A teaching orientation that is characterised by a rather unilateral teacher-centred transfer of knowledge (transmission view, cf. Kunter et al., 2013) and provides students with step-by-step instructions to new procedures seems to coincide with a rejectionist stance on technology use und thus a level of instrumental integration that does not go beyond necessary instrumental initiation due to curricular requirements. Being asked for the rationale of such a view, several teachers in our sample with a rather strong transmissive teaching orientation and a negative stance towards the use of technology argue that the complex nature and elevated cognitive content of calculus, in their view, requires a high degree of teacher control.

Mr G (trainee teacher, interview 2):

"…for these challenging tasks in calculus, such as the fundamental theorem of differential and integral calculus, I think the classical lecture format is needed. It needs to be explained otherwise they [students] do not understand its dimension.[…] these graphing tools, I think, are rather counterproductive.[…] So far I haven't seen any meaningful task which required the use of these technologies in any case.[…] For me that is not mathematics."

On the other end of the continuum, teachers favouring an inquiry-oriented, constructivist view on teaching based on a learner-active cognitive activation in constructing new knowledge concur with supportive views on technology use. They embrace a conception of intended calculus teaching by actively engaging students in problem-solving activities and want to assist students in discovering central concepts of calculus by using the possibilities of the artefact.

Mr H (trainee teacher, interview 2):

"…when they [students] use the device as an expert; e.g., to analyse classes of functions and their corresponding derivatives […] and discover derivation rules […]. With the help of the device they can work out rules for new content and we are not restricted to certain functions."

"…continuing to work on a solution by themselves, trying to find a solution […], providing opportunities for learning by discovering phenomena; that's important to me."

This example shows that students were confronted with unknown functions and are encouraged to apply their technical knowledge in order to discover new mathematical knowledge in terms of instrumental symbiosis. In intended phases of discovery tasks, teachers see their role in supporting students' constructive processes with the assistance of technology rather than to transmit rules and knowledge. Depending on the learning step and the suitability of particular activities reported by these teachers, the level of instrumental integration can be located between reinforcement and symbiosis. An accurate distinction between them however is related to particular tasks and content.

20.5 Discussion and Conclusion

The main goal of this report was to focus on beliefs on the integration of technology in secondary calculus courses with particular attention to pre-service and trainee teachers. It has been acknowledged that teachers' beliefs originating from prior experiences serve a filter for beliefs about teaching and learning (Levin, 2015, p. 57). Accordingly, beliefs on the use of technology in connection with calculus can be reconstructed in our data on the basis of prospective teachers' prior or first experiences and can be attributed to different levels of instrumental integration. Assuming that technological tools and mathematical tasks do not automatically lead to learning, the teacher plays a significant role in the design of individual approaches of using the technology in mathematics teaching and learning. In this respect, teachers' beliefs about the meaningful integration of technology are determining factors for the *what*, the *how* and the *why*. By reconstructing teachers' beliefs related to use of technology in calculus as the central part of the curriculum at upper secondary school, our results may be viewed as a basis for understanding teachers' objectives and beliefs in the framework of instrumental genesis. Further we identified two antithetical belief systems referring to the integration of technology ranging from rejection to approval. The data from our research enables to specify reasons for cognitive and motivational traits in teachers' belief systems with regard to some aspects of their intended calculus teaching. Finally, our data seem to confirm presumptions in studying teachers' beliefs that there is a connection between a dynamic view of mathematics and a constructivist approach to teaching referring to a technology-supported approach of teaching calculus.

However, the challenge of a meaningful integration of technology into calculus teaching needs further research in several directions. The identification and understanding of beliefs and their contextual location in a multiple-layered belief system that each teacher tries to make sense of individually is a starting point. Examining and gaining insight in the origin and development of beliefs could be informed by more longitudinal research. Another direction contains the relationship between teachers' beliefs, their actual classroom practice and students' learning. As teachers' beliefs and affect seem to be highly relevant for the instructional choices they make, more research is needed on the relationship between cognitive and affective factors and their impact on teaching and students.

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Chapter 21
Technology-Related Beliefs and the Mathematics Classroom: Development of a Measurement Instrument for Pre-Service and In-Service Teachers

Marcel Klinger, Daniel Thurm, Christos Itsios, and Joyce Peters-Dasdemir

Abstract Beliefs referring to teaching and learning mathematics with technology play an important role when teachers are integrating technology into their classrooms. However, there has been a lack of instruments to measure those beliefs in detail. In this paper, we contribute a detailed inventory to measure technology-related beliefs of in-service and pre-service teachers. This instrument—a questionnaire—is analyzed with data from 246 pre-service and 199 in-service teachers using confirmatory factor analysis. It is found that beliefs of in-service teachers can be measured in more detail than those of pre-service teachers, mainly due to a longer experience and a correspondingly more differentiated system of beliefs.

21.1 Introduction

It is known that the use of technological tools can support the teaching and learning of mathematics (e.g. Barzel, 2012; Zbiek, Heid, Blume, & Dick, 2007). As a consequence, the use of technology in mathematics classrooms is endorsed by politics as well as teacher associations (e.g. for Germany see DMV, GDM, & MNU, 2008).

In realising this claim, it is important to take teachers' beliefs into account, as it is substantiated by the following quote by a teacher commenting on one of his students: "When doing mathematics, he is always using a graphing calculator. The boy is a mathematical fool. When the device is off, he's done, he's totally done."

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(Rögler, 2014, p. 983, translated). It shows that teachers' beliefs have a significant impact on the mathematics classroom.

A review of the literature in the field of technology-related beliefs reveals several qualitative and case studies (e.g. Doerr & Zangor, 2000; Drijvers, Doorman, Boon, Reed, & Gravemeijer, 2010; Pierce, Ball, & Stacey, 2009). However, there is a lack of quantitative studies and field-tested measuring instruments for technology-related beliefs of pre-service *and* in-service teachers.

Therefore, Rögler, Barzel, and Eichler (2013) and Rögler (2014) started the development of a questionnaire, to get a more detailed impression of technological-related beliefs of in-service teachers. The developed set of items had not yet been validated empirically and it was unclear whether it is suitable also for pre-service teachers.

This research paper thus aims at providing an empirically validated quantitative measuring instrument for beliefs referring to the teaching and learning of mathematics with technology based on the preparatory work.

The instrument is administered to 246 pre-service and 199 in-service teachers of mathematics. The data are analyzed with regard to the structure of factors resulting from a factor analysis. The complete instrument can be found in Thurm, Klinger, Barzel, and Rögler (2017).

21.2 Theoretical Background

21.2.1 Mathematics Classroom and Technology

Before discussing the use of technology in mathematics instruction, the term "technology" should be explained. Frequently, "technology" refers to digital media or the use of computers in teaching and learning situations. This is further differentiated between digital learning environments and digital tools for learning (e.g. Barzel, Hußmann, & Leuders, 2005). The term *learning environment* is commonly described as something that instructs the learner from the outside by organizing or structuring learning processes (cf. Barzel et al., 2005, p. 30). *Tools* on the other hand are (to a certain degree) universally applicable aids for solving a wide range of tasks (cf. Barzel et al., 2005, p. 30).

For the purpose of this paper, the term *tool* refers to *digital mathematics tools*, as opposed to general digital tools, like text editing or presentation programs, which can be used in a variety of subjects. Heintz et al. (2014) mention the following tools as significant for mathematics instruction: dynamic geometry software, spreadsheet software, function plotters, computer algebra systems (CAS), as well as multi-representation programs or systems. The latter unify the aforementioned tools in a single program (e.g. GeoGebra) or handheld devices (e.g. TI-Nspire or Casio fx-CG20) (Laakmann, 2008). From now forth, the term *technology* will refer to digital mathematics tools.

Proponents of the use of technology in the mathematics classroom see certain benefits which enable a better understanding of mathematics. Especially multi-representation systems that offer the opportunity of varying representations to the same mathematical object are seen as advantageous for learning, particularly due to the positive consequences for understanding concepts (e.g. Laakmann, 2008). This can mainly be observed in the field of functions and algebra (e.g. Hollar & Norwood, 1999). Additionally, technology-based learning environments can be constructed for the purposes of discovery learning (e.g. Barzel, 2012). Another positive consequence can be found within the principle of shifting (Peschek, 1999), where procedures and operations can be outsourced to a computer instead of being calculated by hand.

Apart from that, disadvantages of the use of technology in learning scenarios can also be found. For instance, concerns are being raised as to whether basic mathematical skills are no longer mastered manually (Handal, Cavanagh, Wood, & Petocz, 2011) or whether working without thinking is being facilitated (Stimmt, 1997). For some authors the thinking capacity of students is at stake, since the principle of shifting allows for thought processes to be conducted with the help of technology, not by students themselves. They state that students' thinking deteriorates as a result. For Mackey (1999), the use of technology in mathematics may lead to a "mindless button pushing" as a substitute for independent thinking.

21.2.2 Beliefs

The use of the term *beliefs* varies among several authors (e.g. Philipp, 2007). Further notions like *subjective theories* (e.g. Baumert & Kunter, 2006) or *mathematical worldviews* (Grigutsch, Raatz, & Törner, 1998) can be found, as well. When using the term belief, we refer to Philipp (2007) who proposed the following definition:

"Psychologically held understandings, premises, or propositions about the world that are **thought to be true**. Beliefs are more **cognitive**, are felt less intensely, and are **harder to change** than attitudes. Beliefs might be thought of as lenses that affect one's view of some aspect of the world or as dispositions toward action. Beliefs, unlike knowledge, may be held with varying degrees of conviction and are **not consensual**. Beliefs are more cognitive than emotions and attitudes." (Philipp, 2007, p. 259, our emphasis).

Based on this definition, *beliefs* are being contrasted to *attitudes*. Beliefs are difficult to change or something that each individual holds to be true for themselves. They do have an affective component, but are mainly cognitive and do not require a consensus regarding other individuals.

Furthermore, beliefs are always attached to certain objects of belief (Goldin, Rösken, & Törner, 2009). Goldin et al. (2009) suggest naming those objects explicitly; otherwise there is a risk of being imprecise. In the present paper, technologies, that is, mathematical tools as described above, take the role of this object. The emphasis is laid explicitly on teachers' beliefs and not on students' beliefs. Hereafter, the term *beliefs* only refers to in-service and pre-service teachers' beliefs.

21.2.3 Technology-Related Beliefs

The beliefs, which refer to the use of technology as an object of beliefs (e.g. Goldin et al., 2009), will be termed in this context as technology-related beliefs. Most studies in educational and classroom research that take technology-related beliefs into account can be classified into two categories: In the first category, the teachers' beliefs are examined using qualitative case study settings with very small samples (e.g. Doerr & Zangor, 2000; Drijvers et al., 2010; Pierce et al., 2009). These studies pinpoint the belief systems of individual teachers as well as the effects of these beliefs on their actions as educators. They reveal a broad bandwidth of different beliefs as well. However, it is not possible to quantify the extent or the intensity of a belief or to generalize the results.

The second category comprises only a few systematic quantitative investigations that survey specific technology-related beliefs (e.g. Pierce et al., 2009; Dewey, Singletary, & Kinzel, 2009). In these works, the beliefs are recorded rather unsystematically, and different aspects of the use of technology are raised with individual items. The developed instruments, therefore, do not correspond to the modern, state-of-the-art psychological tests. At this point, the work of Kuntze and Dreher (2013) can be positively mentioned. In this study, various aspects of the use of technology in mathematics are identified on theoretical principles. These aspects are operationalized and empirically verified in a questionnaire.

21.3 Research Question

The items of the measuring instrument first developed by Rögler et al. (2013) as well as Rögler (2014) are categorized into eight dimensions of technology-related beliefs (shown below in Fig. 21.1). It is unclear, whether these expected dimensions can be measured reliably and validly for pre-service and in-service teachers and how these measured dimensions are interrelated. Based on these considerations, the following research question arises:

What statistical structure of beliefs can be found for pre-service and in-service teachers that answered the questionnaire and do they differ with regard to the eight dimensions?

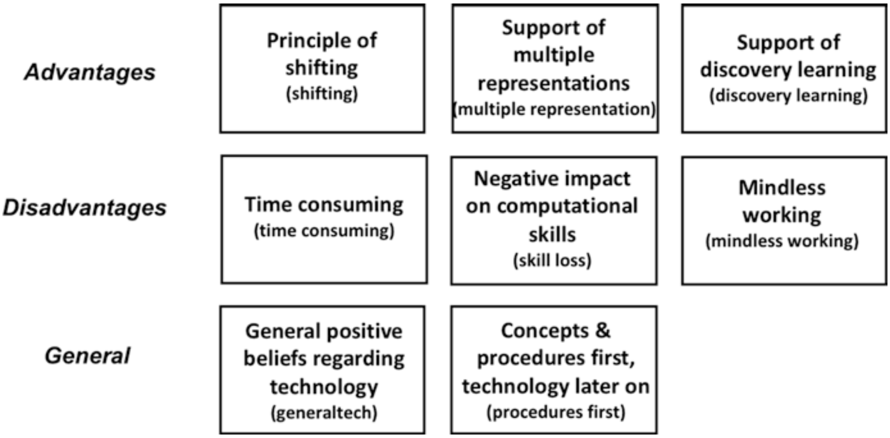

Fig. 21.1 Overview of the eight dimensions and the abbreviations used for them below (the so-called *base model*)

21.4 The Measurement Instrument

In the following section, we introduce the development of the measuring instrument, which was acquired as part of Rögler et al.'s (2013) and Rögler's (2014) research work. Unlike Kuntze and Dreher (2013), who investigate the aspects of technology-related beliefs on a theoretical basis, Rögler developed the dimensions of the beliefs empirically by conducting semi-structured interviews with a total of nine teachers (see Rögler, 2014): The respective transcripts were openly encoded with regard to technology-related beliefs in terms of the *Grounded Theory*. In doing so, 29 categories of beliefs could be found initially. In the next step, items were formulated from those.

In a first pilot study with 300 teachers, eight main categories could be determined using an exploratory factor analysis: "principle of shifting", "support of multiple representations", "support of discovery learning", "time consuming", "negative impact on computational skills", "mindless working", "general positive beliefs regarding technology" and the belief "concepts and procedures first, technology later on".

The first three dimensions connote rather positively regarding the use of digital tools in mathematics instruction and can thus be seen as advantages, whereas "time-consuming", "negative impact on computational skills" and "mindless working" were mentioned as disadvantages at all times. The last two scales derive from statements, which can be classified as neither advantages nor disadvantages. We divided the eight dimensions into categories (advantages, disadvantages and general); for a clearer view (see Fig. 21.1).

In the following, the eight main categories developed from the first pilot-study are described in terms of content and are briefly explained.

21.4.1 Principle of shifting

Technology is used to execute certain procedures in the mathematics classroom, so that they are not done by hand anymore. This offers new possibilities for initiating understanding rather than teaching algorithms and rote procedures (sample item: “Technology should be used to move away from calculation.”).

21.4.2 Support of Multiple Representations

Technology offers a fast and easy switching between representations. For example, a function’s graph can be plotted easily using a graphing calculator (sample item: “Technology supports the connection of multiple representations (e.g., picture, table, term).”).

21.4.3 Support of Discovery Learning

Technology offers new occasions for discovery learning. For instance, structures can be explored by generating diverse examples (sample item: “Technology supports tasks in which students discover new content by themselves.”)

21.4.4 Time-Consuming

The use of technological tools in the mathematics classroom consumes time, especially when being introduced for the first time. Students have to learn how to handle a certain tool (sample item: “One should resign from using technology, you may lose too much time.”).

21.4.5 Negative Impact on Computational Skills

When the use of technological tools is allowed, students may lose certain manual skills, like the derivation of a function (sample item: “When using technology, students may forget how to apply important procedures and algorithms.”)

21.4.6 Mindless Working

This category addresses the concern that technology might lead away from cognitive skills (sample item: "Technology misleads students to solve each task by using the calculator.")

21.4.7 General Positive Beliefs Regarding Technology

This category focuses general positive beliefs regarding technology, which are not tied to a specific part of mathematical instruction (sample item: "I prefer to use technology every time it's possible.").

21.4.8 Concepts and Procedures First, Technology Later On

Pre-service and in-service teachers are asked for an opinion, at which point in the instructional phase technology should be used. For example, technological tools can be used directly when introducing a new topic or omitted at first (sample item: "Students should have gained a certain understanding of mathematical content, before being allowed to use technological tools in the mathematics classroom.").

21.5 Methodology and Analysis

Rögler's questionnaire was specifically developed for in-service teachers. Bühner (2011, pp. 87 ff.) points out the particular importance of specifying the target group for item formulation. Accordingly, it cannot be assumed that an instrument that is valid for in-service teachers is also valid for pre-service teachers. In addition, there are differences among studies carried out with pre-service and in-service teachers, due to separate degrees of teaching experience (Kuntze & Dreher, 2013). Hence, the main aim is to adapt the measuring instrument for pre-service teachers and to ensure its validity.

Cognitive interviews ($n = 5$ pre-service teachers) were used to verify the validity of the pre-service teachers' items as well as to identify problems of validity in their replies (cf. Prüfer & Rexroth, 2005). Techniques like paraphrasing, probing and individual thinking-aloud were used. On the basis of conducted interviews mainly small changes to the items had to be made.

To answer the formulated research question, the base model (Fig. 21.1) is being examined for accuracy of fit using a confirmatory factor analysis in a quantitative

Table 21.1 Overview of the available samples

Sample	Pre-service teachers (#1)	In-service teachers (#2)
Sample size	246	199
Male/female/NA	84/158/0	91/107/1
Description	Teaching students at German universities	Secondary school teachers in the German federal state of North Rhine-Westphalia
Format (gathered by)	Online survey at German universities (Rögler, 2015)	Paper and pencil, GTR NRW research project (Thurm et al., 2015)

setting. Different samples could be provided by two studies (Rögler, 2015; Thurm, Klinger, & Barzel, 2015). They are shown in Table 21.1.

As opposed to the pre-service teachers' sample, the word *technology* was stated more precisely with the words *graphing calculator* throughout the questionnaire in a state-wide study concerning graphing calculators, since it deals with the mandatory introduction of graphing calculators in the federal state of North Rhine-Westphalia (see GTR NRW project, Thurm et al., 2015). The items regarding "General positive beliefs regarding technology" could not be administered in the GTR NRW survey, due to time-efficiency reasons.

All items were equipped with a five-level Likert response pattern ("strongly disagree", "disagree", "neither agree nor disagree", "agree", and "strongly agree").

21.5.1 Statistical Analyses

For all 246 pre-service and 199 in-service teachers, who fully completed the questionnaire, the amount of missing data is less than 5% and less than 10%, respectively. According to Little's MCAR-Test for missing values, items were not systematically omitted. The FIML-method was used to deal with those missing values.

To describe the structure of the measured beliefs, the base model (see Fig. 21.1) was initially used for both samples to see if the items that were theoretically assigned to each category actually belong to those. Good global fit-values could be shown (see Table 21.2). However, better fitting models could be found, as it will be outlined below.

We will begin with the analysis of the pre-service teachers' data. Beside acceptable fit values on the global level, most scales show good indicator reliabilities (>0.4).[1] However, for the scale "principle of shifting" most of the items show bad reliabilities, between 0.16 and 0.36. The validity of the scale for pre-service teachers should thus be doubted. All six items are removed from the data set; the derived model will be called **base model***.

[1] It is not possible to list all reliabilities (as well as correlations) here due to space restriction. However, they can be found in Thurm et al. (2017).

Table 21.2 Overview of the several models and their according global fit indices for both samples (see Thurm et al. (2017) for an explanation of RMSEA, SRMR and CFI)

Model	Sample	χ2	df	p	χ2/df	RMSEA	SRMR	CFI
Thresholds (acceptable resp. good)	–	–	–	>0.05	<3.0 <2.0	<0.08 <0.05	<0.11 <0.05	>0.90 >0.95
Base model	#1	1126	637	0.00	1.767	0.056	0.061	0.899
Base model*	#1	662	443	0.00	1.495	0.045	0.052	0.949
Base model**	#1	308	220	0.00	1.399	0.040	0.042	0.970
Base model	#2	905	474	0.00	1.909	0.068	0.056	0.903
base model#	#2	341	199	0.00	1.711	0.060	0.044	0.953

Looking at correlations between scales, two values are noticeably high. Firstly, the scales “support of multiple representations” and “support of discovery learning” correlate with each other relatively high (0.74). Secondly, “negative impact on computational skills” and “mindless working” show a correlation of 0.83. It must thus be assumed that both pairs form the common scales “advantages” and “disadvantages” of the use of technology, respectively. Therefore, both pairs are merged and the derived model will be called **base model****. As it can be seen in Table 21.2, the model shows the best global fit indices of all compared models.

We will now have a closer look at the in-service teachers’ data. Similar problems to the scale “principle of shifting” appeared in the analysis of the local fit-values, as stated previously in the teaching students’ group. Three of the six items have a very low indicator reliability (<0.5). Additionally, it could not be sufficiently differentiated between the dimensions “principle of shifting” and “time consuming”. The dimension “principle of shifting” will be subsequently excluded, like in the analysis of the teaching students’ data. Moreover, the dimensions “mindless working” and “negative impact on computational skills” once again correlate highly (0.84). It cannot be assumed that they are two separate constructs, therefore they will be merged into a “disadvantages” dimension, as well. The derived model will be called **base model#**. As Table 21.2 shows again, the derived data can be fitted more precisely using this model. It shows very good model fit indices.

21.6 Results and Discussion

The main result of this study is a set of 38 empirically tested items, suitable for both pre-service and in-service teachers. These items have been reviewed in their formulations and wording. Comprehension difficulties are not to be expected. The content validity can be assumed since they are also tested with cognitive techniques such as paraphrasing, probing and individual thinking-aloud.

The research question refers to the structure and interrelations between the items and between the latent constructs, which are measured with these items. The hypothesis that the 38 items could be used to collect eight different latent beliefs had to be partially rejected in the confirmatory factor analysis.

The most striking result is that the category of "principle of shifting" had to be given up completely, both for pre-service and in-service teachers, possibly due to operationalisation difficulties, since it is closely linked to "time consuming". Furthermore, delegating computations to technological tools, such that resources can be made available to other parts of the instruction, are difficult to formulate briefly and concisely and may thus be poorly operationalised. The dimensions "mindless working" and "negative impact on computational skills" could not be measured as two latent dimensions, but as a common construct of "disadvantages".

Differences between in-service and pre-service teachers can be seen in "support of multiple representations" and "support of discovery learning". For pre-service teachers, these two can be merged in a common dimension "advantages", whereas for teachers they are very well distinct. Teachers may have more differentiated beliefs than pre-service teachers who are still in education because of having comparatively less teaching experience. Beliefs of in-service teachers can thus be measured in more detail than those of pre-service teachers.

Even if the representativeness of the present sample cannot be ensured, some considerable points can be observed in the descriptive-statistical analysis. Overall, the results can be summarised as follows: The 23 items used in base model** can be used for a questionnaire that measures five dimensions of technology-related beliefs for students, namely "general positive beliefs regarding technology", "advantages", "time consuming", "disadvantages" and "concepts and procedures first, technology later on". The five dimensions "support of multiple representations", "support of discovery learning", "time consuming", "disadvantages", and "concepts and procedures first, technology later on" of technology-related beliefs for teachers are measured by the 22 items used in base model#. The items of base model** and base model# can be found in Thurm (2017).

For future studies, a basis for a tried-and-tested measuring instrument is available for both pre-service and in-service teachers. It can be particularly helpful to describe changes in technology-related beliefs quantitatively and their interdependencies to other variables (e.g. Thurm, 2018). However, some restrictions in the explanatory power of the present study have to be considered: It is expected that due to the voluntary participation in the two studies that were analyzed, especially highly motivated pre-service and in-service teachers are often represented in the samples. Since the questionnaire for pre-service teachers was offered exclusively online, it must be assumed that, in particular, such students belong to the sample that is in favour of e-mail and internet communication. The sample of the in-service teachers in North Rhine-Westphalia was drawn only a few weeks after graphing calculators were introduced in upper secondary schools. Therefore, they had little experience at the time of the survey. It is known from literature that in-service teachers' beliefs can show a connection to the performance of their respective pupils (Bromme, 2005). Further examination will show whether this connection can be observed as well within the framework of the GTR NRW study with more than 3000 pupils (Klinger, 2018).

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