On the Necessary Accuracy of Representation of Optimal Signals

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Abstract. The way to increase spectral efficiency without significant energy losses by using optimal signals is considered. SDR (software-defined radio) platform is proposed as transceiver prototype. Improvement of its performance may be achieved due to decreasing the number of digits after decimal point and the number of expansion coefficients, which define representation of signals. We used various forms of optimal signals obtained for different restrictions on out-of-band emissions, symbol rate and BER (bit error rate) performance. The influence of the number of digits after decimal point on spectral and energy efficiency of optimal signals is considered. The necessary accuracy of representation providing maximal spectral efficiency is found for different cases.

Keywords: Optimal signals · Accuracy · Energy efficiency
Spectral efficiency · Out-of-band emissions · Optimization problem
SDR-platform

1 Introduction

We can observe active development of next-generation wireless networks (5G). Numerous studies on 5G networks are actively conducted to improve efficiency [1–3]. Many scientific groups are concentrated at tendencies to increase the spectral efficiency in conditions of limited frequency bandwidth [4–7]. Spectral efficiency is calculated as $R/\Delta F$, where R is symbol rate, ΔF – occupied frequency bandwidth. So there are different ways to increase value $R/\Delta F$: increase R or reduce ΔF. Increasing symbol rate is known as Faster-than-Nyquist (FTN) signals [8, 9]. Reducing ΔF can be done by application of optimal signals [4, 6, 10] and by increasing duration of signals. Our approach consists of joint using of optimal signals with increased duration and increasing symbol rate [10–12].

The finite random sequence of N single optimal signals $s_{opt}(t)$ with duration $T_s = LT$ and energy $\xi^2 E_{opt}$ may be written as follows:

$$y(t) = \xi\sqrt{E_{opt}/T_s}\sum_{n=-N/2}^{N/2} c_n s_{opt}(t - n\xi T), \tag{1}$$

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where coefficient ξ defines a symbol rate. This coefficient also is used to keep the average power of random sequence $y(t)$ constant. If we use the binary alphabet, the symbol rate is equal to a bit rate.

Forms of signals $s_{opt}(t)$ for different restrictions on out-of-band emissions, symbol rate, BER performance may be obtained by solving corresponding optimization problem. So we can control all time and spectral characteristics of signals. Application of optimal signals [10–12] allows increasing the symbol rate of data transmission without significant energy losses in BER performance (no more than 0.5 dB) [10, 12].

Next step is to develop the algorithm of formation and processing at the reception. We are planning to use SDR-platform HackRF One [13] to construct prototypes of transceivers. Performance of this SDR-platform is limited. Therefore, we must search ways to simplify algorithms and to increase performance.

Note, that we use limited Fourier series to present optimal signals. Initially the number of significant digits after the decimal point q is not limited and equal to 15. So we must use arithmetic of large numbers when going to integers. If we decrease the number of significant digits, resulting performance will improve.

In this article, it is proposed to consider the influence of the number of expansion coefficients and significant digits on optimal signals with increased duration.

2 Optimization Criteria of Signal Form

An optimality criterion of the signal form is based on the choice of fixed reduction rate of out-of-band emissions. The optimization task may be written in the form of linear functional J minimization [10] for signal $s_{opt}(t)$ with duration T_s and symbol rate $R = 1/\xi T$:

$$\arg\left\{\min_{s_{opt}(t)}(J)\right\},\ J = \int_{-\infty}^{+\infty} g(f)\left|\int_{-\infty}^{+\infty} s_{opt}(t)\exp(-j2\pi ft)dt\right|^2 df, \qquad (2)$$

where $g(f) = f^{2p}$ (p = 1, 2, …) is a weighing function. Choosing $g(f)$ form determines reduction rate of out-of-band emissions.

Restriction on BER performance may be converted to restriction on correlation coefficient between two optimal signals on different time positions [10–12]:

$$\max_{n=1\ldots\lfloor L/\xi\rfloor}\left\{\int\limits_{-(L-2n)T/2}^{LT/2} s_{opt}(t)s_{opt}(t-n\xi T)dt\right\}<K_0. \qquad (3)$$

We have not found analytical solutions of this optimization problem for arbitrary values of T_s, R and K_0. So we switched to numerical solutions. To solve this optimization task numerically we used presentation of $s_{opt}(t)$ in terms of limited Fourier series (m is a number of expansion coefficients).

$$s_{opt}(t) = \frac{s_{opt0}}{2} + \sum_{k=1}^{m-1} s_{optk} \cos\left(\frac{2\pi}{T} kt\right). \quad (4)$$

Then the original optimization task (2) can be transformed into the task of searching for expansion coefficients $\{s_k\}_{k=1}^{m}$, which minimize the function of several variables [10–12]:

$$\min_{\{s_k\}_{k=1}^{m}} J\left(\{s_k\}_{k=1}^{m}\right), \; J\left(\{s_k\}_{k=1}^{m}\right) = T_s/2 \sum\nolimits_{k=1}^{m} (2\pi k/T_s)^{2n} s_k^2. \quad (5)$$

The value of m is determined by the accuracy of representation $s_{opt}(t)$ and complexity of solution (5) caused by an ill-conditioned task. Target functional has a ravine-type shape, i.e. rises sharply along one direction and changes slightly along the other. It is taken into account in this work, therefore, the chosen values of m provide necessary accuracy of $s_{opt}(t)$.

When $s_{opt}(t)$ is obtained, energy spectrum $|S(f)|^2$ of random sequence of signals (1) may be calculated. For statistically independent modulation symbols it is defined in the area of positive frequencies by function $s_{opt}(t)$ and constant value Z [14, 15]:

$$|S(f)|^2 = \lim_{N\to\infty} \frac{1}{NT_s} m_1\left\{\left|S_j(f)\right|^2\right\} = (Z/T_s)\left|\int\limits_{-T_s/2}^{T_s/2} s_{opt}(t)\exp(-j2\pi ft)dt\right|^2, \quad (6)$$

where value Z depends on signal constellation, $S_j(f)$ – spectrum of random sequence of signals (1), $m_1\{\ \}$ – mathematical expectation.

3 Results and Discussion

As $s_{opt}(t)$ we will use results, presented in [10–12] and obtained for next conditions: $p = 2$, $\xi = 0.5$, $K_0 = 0.01$. We can apply these solutions without loss of generality. Envelopes for $T_s = 6T$ and $T_s = 16T$ and corresponding energy spectra are presented on Figs. 1 and 2.

Our aim is to reduce the number of coefficients m. So let us consider the Euclid distance $d_{m,m-1}$ between envelopes formed with the use of m and $m - 1$ coefficients for different m. Firstly we should solve the optimization task for rather high value of m. Then array a is truncated by removing its last value, so we can obtain a new envelope. The Euclid distance between current envelope and previous one is calculated. Here we used $m = 27$ as the initial value and got some interesting results (Fig. 3). We can accept that $d_{m,m-1}$ must be no more than 10^{-3}. The minimal values of m providing such $d_{m,m-1}$ are presented in Table 1.

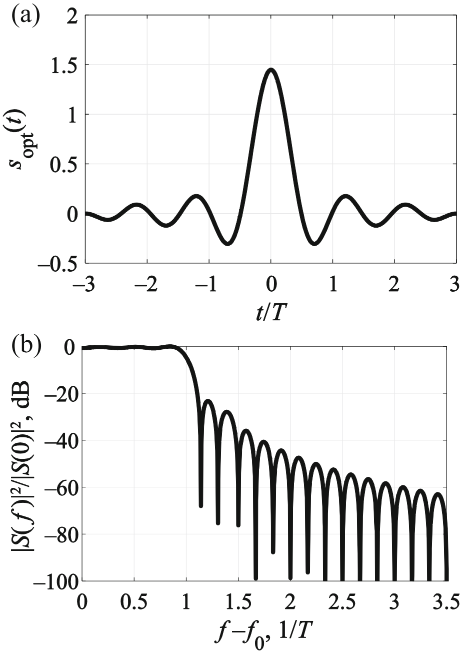

Fig. 1. Envelope (a) of optimal signal $T_s = 6T$ and corresponding energy spectrum (b).

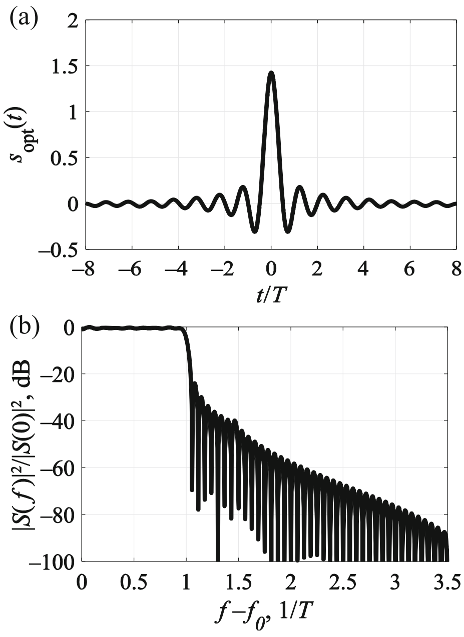

Fig. 2. Envelope (a) of optimal signal $T_s = 16T$ and corresponding energy spectrum (b).

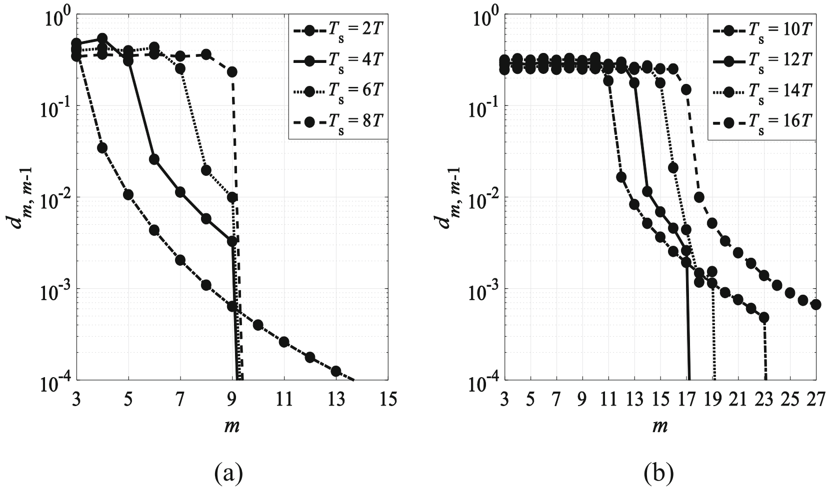

Fig. 3. $d_{m,m-1}$ vs m: (a) $T_s = 2T \ldots 8T$, (b) $T_s = 10T \ldots 16T$.

Table 1. The minimal values of m providing $d_{m,m-1} \le 10^{-3}$.

T_s	m
$2T$	9
$4T$	10
$6T$	10
$8T$	10
$10T$	18
$12T$	18
$14T$	18
$16T$	24

Table 2. Expansion coefficients $\{s_k\}_{k=1}^{m}$ for $T_s = 6T$.

	$s_{opt}(t)$	$s_{opt(3)}(t)$	$s_{opt(2)}(t)$	$s_{opt(1)}(t)$
s_{opt0}	0.230754291462361	0.231	0.23	0.2
s_{opt1}	0.243020058615220	0.243	0.24	0.2
s_{opt2}	0.230277121912867	0.230	0.23	0.2
s_{opt3}	0.244036419329030	0.244	0.24	0.2
s_{opt4}	0.227953096881105	0.228	0.23	0.2
s_{opt5}	0.248713526443278	0.249	0.25	0.3
s_{opt6}	0.145222271533744	0.145	0.15	0.2
s_{opt7}	–0.011250298505672	–0.011	–0.01	0
s_{opt8}	0.005690067650762	0.006	0.01	0
s_{opt9}	–0.000000003319436	0	0	0

Table 3. Expansion coefficients $\{s_k\}_{k=1}^{m}$ for T_s = 16T.

	$s_{opt}(t)$	$s_{opt(3)}(t)$	$s_{opt(2)}(t)$	$s_{opt(1)}(t)$
s_{opt0}	0.083853277988335	0.084	0.08	0.1
s_{opt1}	0.092820789366586	0.093	0.09	0.1
s_{opt2}	0.086660852336211	0.087	0.09	0.1
s_{opt3}	0.089305327068671	0.089	0.09	0.1
s_{opt4}	0.088724099541282	0.089	0.09	0.1
s_{opt5}	0.089673438318048	0.090	0.09	0.1
s_{opt6}	0.087122794017252	0.087	0.09	0.1
s_{opt7}	0.090517166551378	0.091	0.09	0.1
s_{opt8}	0.087930707754474	0.088	0.09	0.1
s_{opt9}	0.088876406957491	0.089	0.09	0.1
s_{opt10}	0.088759244911348	0.089	0.09	0.1
s_{opt11}	0.089758708389713	0.090	0.09	0.1
s_{opt12}	0.087068587535874	0.087	0.09	0.1
s_{opt13}	0.090319412489984	0.090	0.09	0.1
s_{opt14}	0.088453068901215	0.088	0.09	0.1
s_{opt15}	0.088242284193880	0.088	0.09	0.1
s_{opt16}	0.052464281626184	0.052	0.05	0.1
s_{opt17}	–0.003487694515916	–0.003	0	0
s_{opt18}	0.001815455034236	0.002	0	0
s_{opt19}	–0.001160625678960	–0.001	0	0
s_{opt20}	0.000862815650940	0.001	0	0
s_{opt21}	–0.000661057279843	–0.001	0	0
s_{opt22}	0.000486939765318	0	0	0
s_{opt23}	–0.000380730842321	0	0	0

The next step is choosing the envelope with m defined in Table 1 and rounding the expansion coefficients upwards to fewer digits after decimal point. We decided to investigate the results with no more than three digits after decimal point. The expansion coefficients for T_s = 6T and T_s = 16T are presented in Tables 2 and 3 respectively.

To estimate energy and spectral efficiency simulation model was developed in Matlab (Fig. 4). The input data for this model are optimal envelope form $s_{opt}(t)$, its duration, transmission rate R and signal-to-noise ratio E/N_0. When simulation parameters are initialized, the model forms the random sequence of signals by generating random information bits and using BPSK modulation in the block "modulator". After this step, energy spectrum of random sequence of signals (1) may be calculated by averaging on various realizations. Here we used N = 1000 modulation symbols and 200 averages. As a result, we can compute spectral efficiency $R/\Delta F$ knowing ΔF for different level of energy spectra.

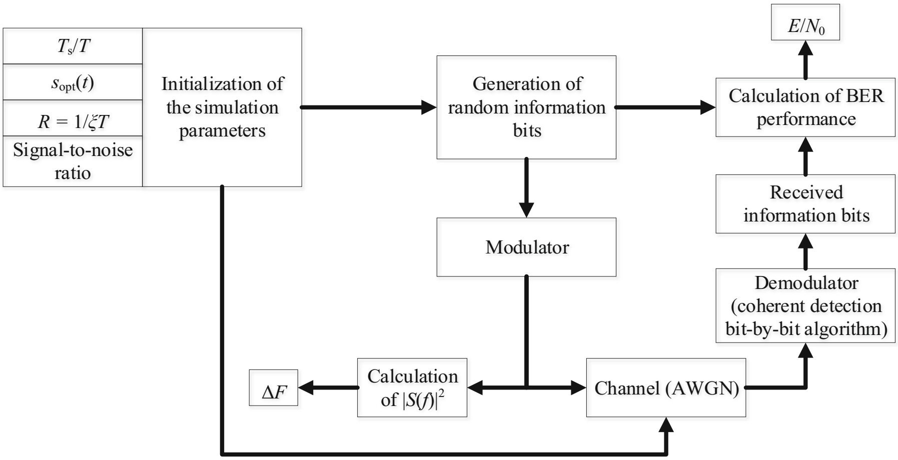

Fig. 4. Block diagram of simulation model.

Another branch of the model includes calculation of BER performance of the random sequence of signals gone through additive white Gaussian noise (AWGN) channel. The block "demodulator" is based on the coherent bit-by-bit detection algorithm, which is simple in realization and provides minimal delay for signal processing. At least 10^6 information bits were transmitted to check BER performance at each signal-to-noise ratio value. The output of this branch is energy efficiency defined as the value of signal-to-noise ratio E/N_0 providing error probability $p_{er} = 10^{-3}$.

Now we should take into account the dependency of energy efficiency on spectral efficiency. Figure 5 shows these relationships for different T_s. The results may be divided into four groups.

The first group includes $T_s = 10T$, $12T$ with maximal spectral efficiency provided by $q = 15$ (Fig. 5*a*). We decided to estimate spectral efficiency relatively to the results with $q = 15$ and energy losses relatively to the theoretical BER performance. Then the loss in spectral efficiency for $q = 3$ is about 12% for $T_s = 10T$ and 3.5% for $T_s = 12T$. Though it is possible in this case to reduce energy losses to the value 0.19 dB for $T_s = 10T$ and 0.26 dB for $T_s = 12T$.

The second group includes $T_s = 4T$, $14T$ with maximal spectral efficiency provided by $q = 3$ (Fig. 5*b*). The energy losses for $q = 3$ vary from 0.19 dB to 0.43 dB for $T_s = 4T$ and $14T$ correspondingly while the gain in spectral efficiency changes from 5.2% to 9.7%.

The third group is composed of $T_s = 2T$, $16T$ with maximal spectral efficiency provided by $q = 2$ (Fig. 5*c*). The energy losses for $q = 2$ are 1.33 dB for $T_s = 2T$ and 0.36 dB for $T_s = 16T$ while spectral efficiency increases by 36% for $T_s = 2T$ and by 34% for $T_s = 16T$.

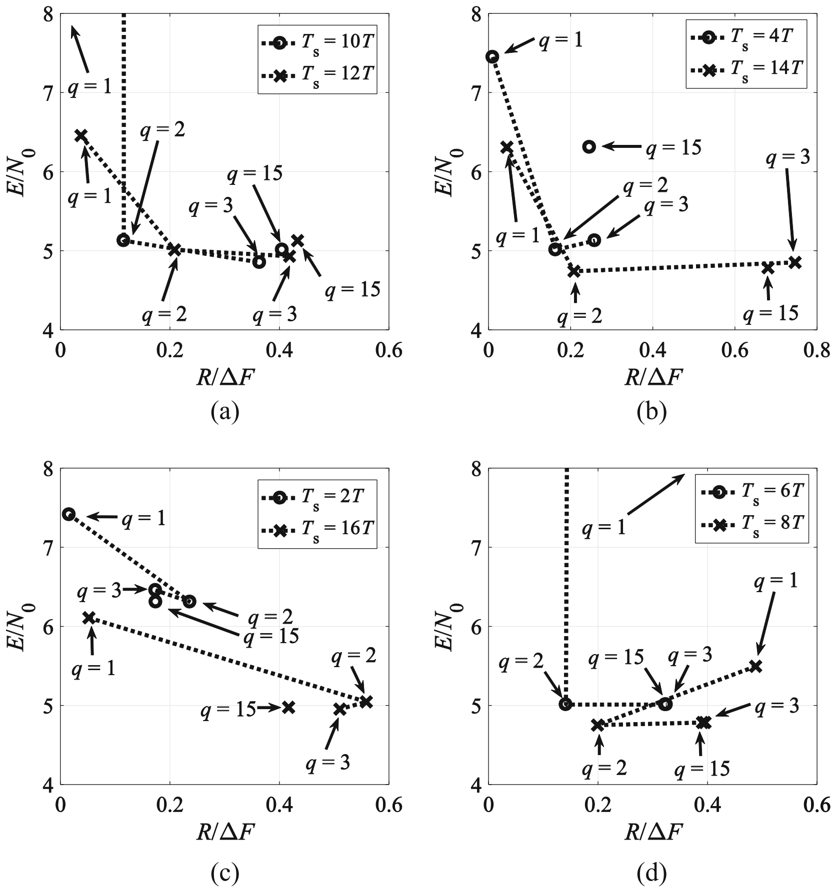

Fig. 5. E/N0 vs $R/\Delta F$: (*a*) T_s = 10T, 12T, (*b*) T_s = 4T, 14T, (*c*) T_s = 2T, 16T, (*d*) T_s = 6T, 8T.

The fourth group unites results with maximal spectral efficiency provided by q = 1 (T_s = 6T, 8T, Fig. 5*d*). Let us start with T_s = 6T. Increasing spectral efficiency by using q = 1 reaches 12.3% relatively to the spectral efficiency of q = 15, but energy losses relatively to the theoretical BER performance are huge (19.3 dB). Using q = 3 allows to reduce energy losses to the value 0.33 dB. However, in this case the increase in spectral efficiency is just 0.65%.

For T_s = 8T the situation is almost the same. If we use q = 1, we can increase spectral efficiency by 25% comparing to q = 15 with energy losses relatively to the theoretical BER performance 0.73 dB. If we use q = 3, energy losses are reduced to the value 0.13 dB, but spectral efficiency increases just by 1.2%.

So we showed the possibility of reducing the number of coefficients and number of significant digits. These results will be applied in the next projects about realization of modem based on spectrally efficient signals.

The results of the work were obtained using computational resources of Peter the Great Saint-Petersburg Polytechnic University Supercomputing Center (http://www.scc.spbstu.ru).

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