Meshless Methods for 'Gas - Evaporating Droplet' Flow Modelling

Oyuna Rybdylova and Sergei S. Sazhin

Abstract The main ideas of simulation of two-phase flows, based on a combination of the conventional Lagrangian method or Osiptsov method for the dispersed phase and the mesh-free vortex and thermal blob methods for the carrier phase, are summarised. A meshless method for modelling of 2D transient, non-isothermal, gas-droplet flows with phase transitions, based on a combination of the viscous-vortex and thermal-blob methods for the carrier phase with the Lagrangian approach for the dispersed phase, is described. The one-way coupled, two-fluid approach is used in the analysis. The method makes it possible to avoid the 'remeshing' procedure (recalculation of flow parameters from Eulerian to Lagrangian grids) and reduces the problem to the solution of three systems of ordinary differential equations, describing the motion of viscous-vortex blobs, thermal blobs, and evaporating droplets. The gas velocity field is restored using the Biot–Savart integral. The numerical algorithm is verified against the analytical solution for a non-isothermal Lamb vortex. The method is applied to modelling of an impulse two-phase cold jet injected into a quiescent hot gas, taking into account droplet evaporation. Various flow patterns are obtained in the calculations, depending on the initial droplet size: (i) low-inertia droplets, evaporating at a higher rate, form ring-like structures and are accumulated only behind the vortex pair; (ii) large droplets move closer to the jet axis, with their sizes remaining almost unchanged; and (iii) intermediate-size droplets are accumulated in a curved band whose ends trail in the periphery behind the head of the cloud, with larger droplets being collected at the front of the two-phase region.

O. Rybdylova (✉) · S. S. Sazhin
The Sir Harry Ricardo Laboratories, Advanced Engineering Centre, School of Computing Engineering and Mathematics, University of Brighton, Brighton, UK
e-mail: O.Rybdylova@brighton.ac.uk

S. S. Sazhin
e-mail: S.Sazhin@brighton.ac.uk

A. Korobeinikov (ed.), *Extended Abstracts Summer 2016*,
Trends in Mathematics 10, https://doi.org/10.1007/978-3-030-01153-6_12

1 Background

Two-phase flows are widely observed in engineering and environmental conditions (e.g., [1]). In such flows, the admixture sometimes forms high concentration regions with complex structures, see [1]. The Eulerian approaches cannot describe such regions with reasonable accuracy, since these approaches are based on the assumption of single-valued fields of the particle/droplet (hereafter referred to as droplet) concentration and velocities. As demonstrated in [3], the only approach capable of calculating the droplet concentration field, without using excessive computer power, is the one suggested by Osiptsov [6]. The latter approach is commonly known as the Osiptsov method or approach.

Various meshless methods have proved to be efficient tools for investigating complex single-phase flows both with primitive and vorticity-velocity variables; see, e.g., [5]. Lebedeva–Osiptsov–Sazhin [4] proposed a method combining the viscous-vortex method for the carrier phase and Osiptsov's approach [6] for particles/droplets. This approach combined the advantages of both the viscous-vortex and Osiptsov methods.

The approaches mentioned above were focused on hydrodynamic aspects of particle-laden flows. However, in many engineering applications, including automotive applications [8], the effects of heat and mass transfer are significant. In [7], the approach described in [4] was generalised to take into account some of these effects. The phase transition on the droplet surface was described using a simple model based on the assumption that the heat flux reaching the droplet is spent on its evaporation (cf., a similar assumption used for qualitative engineering analysis of droplet evaporation in multiphase flows [2, 8]).

The aim of this abstract is to present a brief summary of the models developed and used in the previous papers [4, 7] (Sect. 2) and the main results obtained there (Sect. 3). The publication of this mini-review can be justified by the fact that the original papers were published in engineering journals which are almost unknown to the mathematical community. At the same time, these papers are focused primarily on the description of new mathematical tools and their engineering applications, which are expected to be relevant to mathematical research in this field.

2 Models

In what follows, the main ideas of the models developed and used for 2D plane flow (Cartesian coordinates) in [4, 7] are briefly summarised.

2.1 Osiptsov Method

In the Osiptsov approach [6], the dispersed phase number density is inferred from the solutions to the following systems of ordinary differential equations along chosen droplet trajectories:

$$n_s\,|J| = n_{s0}, \tag{1}$$

$$\frac{\partial \mathbf{x}_s}{\partial t} = \mathbf{v}_s, \quad \frac{\partial \mathbf{v}_s}{\partial t} = \beta(\mathbf{v} - \mathbf{v}_s)\chi_d + \frac{1}{\mathrm{Fr}^2}\mathbf{e}_g, \tag{2}$$

$$\frac{\partial J_{ij}}{\partial t} = q_{ij}, \tag{3}$$

$$\begin{aligned}
\frac{\partial q_{ij}}{\partial t} &= \beta\left(\frac{\partial v_i}{\partial x}J_{xj} + \frac{\partial v_i}{\partial y}J_{yj} - q_{ij}\right)\chi_d + \beta\,(v_i - v_{si})\,\frac{\partial \chi_d}{\partial x_{j0}}, \\
\frac{\partial \chi_d}{\partial x_{j0}} &= \frac{1}{9}\frac{\mathrm{Re}_{s0}}{\mathrm{Re}_s^{1/3}}\frac{1}{|\mathbf{v} - \mathbf{v}_s|}\left((u - u_s)\left(\frac{\partial u}{\partial x}J_{xj} + \frac{\partial u}{\partial y}J_{yj} - q_{xj}\right) + \right. \\
&\left. + (v - v_s)\left(\frac{\partial v}{\partial x}J_{xj} + \frac{\partial v}{\partial y}J_{yj} - q_{yj}\right)\right),
\end{aligned} \tag{4}$$

where

$$J_{ij} = \frac{\partial x_i}{\partial x_{j0}}, \quad q_{ij} = \frac{\partial v_{si}}{\partial x_{j0}}, \;\; \chi_d = 1 + \mathrm{Re}_s^{2/3}/6,$$

and

$$\beta = \frac{6\pi\sigma\mu R_0^2}{m\Gamma_0}, \quad \mathrm{Fr} = \frac{\Gamma_0}{R_0\sqrt{gR_0}}, \quad \mathrm{Re}_s = \mathrm{Re}_{s0}\,|\mathbf{v} - \mathbf{v}_s|\,, \quad \mathrm{Re}_{s0} = \frac{2\sigma\Gamma_0}{R_0\nu},$$

with indices i and j taking values of x or y in the Cartesian coordinate system; $\mathbf{e}_g$ is the unit vector along the direction of the gravity force; x_{s0}, y_{s0} are the Lagrangian variables (the coordinates of initial particle positions); J is the Jacobian of the transformation from the Eulerian to the Lagrangian coordinates. Equation (1) is the continuity equation rewritten in the Lagrangian variables; Eq. (2) are momentum balance equations along chosen particle trajectories; Eqs. (3) and (4) are additional equations to calculate the Jacobian components; they are derived from Eq. (2) by differentiation with respect to x_{s0} and y_{s0}. These equations are solved subject to standard initial conditions for plane sprays.

2.2 Viscous-Vortex and Thermal Blobs

In the viscous-vortex and thermal-blob methods the dimensionless carrier-phase equations are written in the form [7]:

$$\frac{\partial \omega}{\partial t} + \operatorname{div}(\omega \mathbf{v}) = \frac{1}{\mathrm{R}e} \Delta \omega, \tag{5a}$$

$$\frac{\partial T}{\partial t} + \operatorname{div}(T \mathbf{v}) = \frac{\gamma}{\mathrm{Re\,Pr}} \Delta T, \tag{5b}$$

where $\omega = \nabla \times \mathbf{v}$ is the vorticity; $\mathrm{Re} = \rho L U/\mu$ and $\gamma = c_p/c_v$ are the Reynolds number and the specific heat ratio; ρ, μ and U are density, dynamic viscosity and velocity of the carrier phase (gas). Equation (5a) is the vorticity transport equation which follows from the Navier–Stokes equations for an incompressible fluid. Equation (5b) is the transient heat conduction equation.

Introducing the vortex and thermal diffusion velocities, $\mathbf{v}_{dv}$ and $\mathbf{v}_{dT}$, Eq. (5) can be presented in the divergence forms:

$$\frac{\partial \omega}{\partial t} + \operatorname{div}(\omega(\mathbf{v} + \mathbf{v}_{dv})) = 0,$$

$$\frac{\partial T}{\partial t} + \operatorname{div}(T(\mathbf{v} + \mathbf{v}_{dT})) = 0,$$

where

$$\mathbf{v}_{dv} = -\frac{1}{\mathrm{R}e} \frac{\nabla \omega}{\omega}, \quad \mathbf{v}_{dT} = -\frac{\gamma}{\mathrm{Re}} \mathrm{P}r \frac{\nabla T}{T}.$$

This allows tracking of viscous-vortex and thermal blobs moving with velocities $v + v_{dv}$ and $v + v_{dT}$ respectively by solving ODEs subject to corresponding initial conditions. Then, vorticity and temperature fields are calculated.

In the first method, the domain with a non-zero vorticity is discretised into N elements, with the area of the i-th element equal to Δ_{vi}:

$$\omega(\mathbf{r}, t) \approx \sum_{i=1}^{N} \Gamma_i \zeta_{\varepsilon_i} (\mathbf{r} - \mathbf{r}_{vi}(t)),$$

$$\Gamma_i \approx \omega_0 (\mathbf{r}_{vi}(t_0)) \Delta_{vi} = \mathrm{const},$$

where $\zeta_{\varepsilon_i}(\mathbf{r})$ are the so-called cut-off functions. The elements of discretisation are called blobs.

Similarly, the equations for M thermal blobs, take the form:

$$T\left(\mathbf{r},t\right)\approx\sum_{i=1}^{M}\Theta_i\zeta_{\varepsilon_i}\left(\mathbf{r}-\mathbf{r}_{Ti}\left(t\right)\right),$$
$$\Theta_i=T_0\left(\mathbf{r}_{Ti}\left(t_0\right)\right)\Delta_{Ti}=\text{const},$$

where Θ_i and $\mathbf{r}_{Ti}$ are the strength and position of the i-th thermal blob.

Once the vorticity field is calculated, then the velocity field can be restored using the Biot–Savart integral.

3 Results

The accuracy of calculations depends on a number of parameters used in the discretisation formulas, including the numbers of viscous-vortex and thermal blobs, the initial geometry of the blobs, and the time step used in calculating the systems of ordinary differential equations. To verify the numerical algorithm, the Lamb vortex flow described by an exact analytical solution to the transient Navier–Stokes equations was used. Once the model was verified, it was applied to the simulation of the injection of a cold, two-phase jet into a hot, quiescent gas. In the case of an impulse jet with a step-like velocity distribution, a vortex ring (or vortex pair) is usually formed after the jet injection. The study presented in [4, 7] was focused on the formation and dynamics of a two-phase vortex pair both taking into account and not taking into account thermal effects.

The flows with the finest droplets were shown to demonstrate better mixing: low-inertia droplets were shown to form ring-like structures. Droplets of medium size were shown to collect into narrow bands. The clouds of droplets with the largest inertia were shown to remain close to the jet axis. The latter result was supported by experimental observations [1].

Acknowledgements The authors are grateful to EPSRC (grants EP/K005758/1 and EP/M002608/1) for their financial support.

References

1. S. Begg, F. Kaplanski, S.S. Sazhin, M. Hindle, M. Heikal, Vortex ring-like structures in gasoline fuel sprays under cold-start conditions. Int. J. Eng. Res. **10**(4), 195–214 (2009)
2. I. Goldfarb, V. Goldshtein, G. Kuzmenko, S.S. Sazhin, Thermal radiation effect on thermal explosion in gas containing fuel droplets. Combust. Theory Model. **3**, 769–787 (1999)
3. D. Healy, J. Young, Full Lagrangian methods for calculating the particle concentration and velocity fields in dilute gas-particle flows. Proc. R. Soc. Lond. Math. Phys. Eng. Sci. **461**(2059), 2197–2225 (2005)
4. N.A. Lebedeva, A.N. Osiptsov, S.S. Sazhin, A combined fully Lagrangian approach to mesh-free modelling of transient two-phase flows. At. Sprays **23**(1), 47–69 (2013)

5. J. Monaghan, Smooth particle hydrodynamics and its diverse applications. Annu. Rev. Fluid Mech. **44**, 323–346 (2012)
6. A.N. Osiptsov, Lagrangian modeling of dust admixture in gas flows. Astrophys. Space Sci. **274**, 377–386 (2000)
7. O. Rybdylova, A.N. Osiptsov, S.S. Sazhin, S. Begg, M. Heikal, A combined viscous-vortex, thermal-blob and Lagrangian method for non-isothermal, two-phase flow modelling. Int. J. Heat Fluid Flow **58**, 93–102 (2016)
8. S.S. Sazhin, Droplets and Sprays. Springer (2014)