Chapter 6 The Complexity of Basic Number Processing: A Commentary from a Neurocognitive Perspective

Bert De Smedt

Abstract In this commentary, I refect from a neurocognitive perspective on the four chapters on natural number development included in this section. These chapters show that the development of seemingly basic number processing is much more complex than is often portrayed in neurocognitive research. The chapters collectively illustrate that children's development of natural number cannot be reduced to one basic neurocognitive factor, but instead requires a multitude of skills with different developmental trajectories. Specifcally, these contributions highlight that there is much more than the processing of magnitude, or the so-called Approximate Number System, and they elaborate on the roles of subitizing, place value understanding, and children's spontaneous attention to number and relations. They also point out that number is something that needs to be constructed and that number processing is in essence a symbolic activity, which requires the integration of multiple symbolic representations, a focus that has been increasingly emphasized in more recent neurocognitive research. The contributions in this volume provide fresh perspectives that will help to further our understanding of children's natural number development and how it should be fostered. They also offer novel avenues for investigating the origins of atypical mathematical development or dyscalculia.

Keywords Number processing · Neurocognitive factors · Approximate number system · Dyscalculia · Symbolic representations

6.1 Introduction

The four contributions in this section on natural number development in children highlight that the development of seemingly basic number processing is much more complex than is often portrayed in neurocognitive studies in numerical cognition. The section illustrates that basic number processing cannot be reduced to just one core cognitive system or one brain area, such as the intraparietal sulcus (see also

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B. De Smedt (\boxtimes)

Faculty of Psychology and Educational Sciences, University of Leuven, Leuven, Belgium e-mail: Bert.DeSmedt@kuleuven.be

Vanbinst & De Smedt, [2016\)](#page-9-0). This collection of chapters on natural number development highlights that there is more than the so-called Approximate Number System (ANS) and emphasizes the critical roles of subitizing (Clements, Sarama, $\&$ MacDonald, Chap. [2\)](https://doi.org/10.1007/978-3-030-00491-0_2), place value understanding (Mix, Smith, & Crespo, Chap. [5\)](https://doi.org/10.1007/978-3-030-00491-0_5), and children's spontaneous attention to number and relations (McMullen, Chan, Mazzocco, & Hannula-Sormunen, Chap. [4](https://doi.org/10.1007/978-3-030-00491-0_4)). These chapters point out that number is something that needs to be constructed and that number processing is in essence a symbolic activity, which requires the integration of multiple symbolic representations (Ulrich & Norton, Chap. [3](https://doi.org/10.1007/978-3-030-00491-0_3)). Interestingly, this focus on symbolic representations has also been emphasized in more recent neurocognitive research (Merkley & Ansari, [2016;](#page-9-1) Schneider et al., [2017;](#page-9-2) Vanbinst & De Smedt, [2016](#page-9-0)).

After a very brief sketch of the neurocognitive approach to number processing, I discuss, against the background of the chapters in this section, the relevance of the ANS (Ulrich & Norton, Chap. [3](https://doi.org/10.1007/978-3-030-00491-0_3)) and I illustrate that there is more than the processing of magnitude, by pointing to the roles of subitizing (Clements et al., Chap. [2\)](https://doi.org/10.1007/978-3-030-00491-0_2), place value understanding (Mix et al., Chap. [5](https://doi.org/10.1007/978-3-030-00491-0_5)), and spontaneous focusing on number and relations (McMullen et al., Chap. [4\)](https://doi.org/10.1007/978-3-030-00491-0_4). I end this commentary with some concluding thoughts and avenues for future research, inspired by the four contributions in the current section on natural number development.

6.2 A Neurocognitive Perspective on Number Processing

Neurocognitive research on number processing in children is a young but rapidly expanding feld of inquiry, with nearly all studies published in the last decade (Schneider et al., [2017\)](#page-9-2). A key aim in these neurocognitive studies has been to understand why there are large individual differences in the way children acquire mathematical skills (Dowker, [2005\)](#page-8-0) and why learning mathematics is so easy for some but so diffcult for others (Berch, Geary, & Mann-Koepke, [2016\)](#page-8-1). It is assumed that by understanding the very basic cognitive processes that underlie these individual differences, learners' profles can be identifed. These profles then allow one to develop educational interventions and diagnostic approaches that are optimally tailored to the needs of the individual learner. A particular focus in this research has been the study of children with atypical mathematical development, a condition also known as dyscalculia or mathematical learning disability, which is a persistent and specifc disorder in learning mathematics that is not explained merely by uncorrected sensory problems, intellectual disabilities, other mental disorders or inadequate instruction (American Psychiatric Association, [2013](#page-8-2)).

Dyscalculia has been categorized as a *neuro*developmental disorder (American Psychiatric Association, [2013](#page-8-2)), suggesting that the origin of these diffculties lies at the neurobiological level (De Smedt, Peters, & Ghesquiere, [in press\)](#page-8-3). It is important to emphasize that only a handful of brain imaging studies have investigated these neurobiological factors, i.e., brain function and/or structure (De Smedt et al., [in](#page-8-3) [press](#page-8-3); Peters & De Smedt, [2018\)](#page-9-3) and the same applies to the study of typical

development (Merkley & Ansari, [2016\)](#page-9-1). Most research has focused on the study of neurocognitive variables, as these are on a theoretical level closer to the study of neurobiological factors (Hulme & Snowling, [2009](#page-8-4)). These behavioral studies do not involve collecting neurobiological data, but rather they consist of investigating cognitive variables whose roles can be predicted on the basis of (developmental) brain imaging data on the processing of number and arithmetic (Arsalidou, Pawliw-Levac, Sadeghi, & Pascual-Leone, [2018](#page-8-5); Peters & De Smedt, [2018\)](#page-9-3). These cognitive variables can be characterized as domain-specifc skills, i.e., skills that are exclusively relevant for learning mathematics (e.g., numerical magnitude processing, De Smedt, Noel, Gilmore, & Ansari, [2013\)](#page-8-6), or domain-general skills that are also relevant for learning in other academic domains, of which working memory has been the most extensively studied (Peng, Namkung, Barnes, & Sun, [2016](#page-9-4)).

This neurocognitive body of evidence originally focused on non-symbolic numerical magnitude processing as a domain-specifc core factor of individual differences in mathematics (e.g., Piazza, [2010\)](#page-9-5) and of dyscalculia (Wilson & Dehaene, [2007\)](#page-9-6). Likewise, neuroimaging studies have narrowed their focus to activity in the intraparietal sulcus (IPS) during mathematical tasks, highlighting it as a key and specifc area for processing number (e.g., Nieder & Dehaene, [2009\)](#page-9-7). This narrow focus on one core factor has been seriously criticized and challenged by both behavioral and neuroimaging data.

Several studies have failed to observe an association between non-symbolic number processing and mathematics achievement (De Smedt et al., [2013\)](#page-8-6) and metaanalytic data indicate that this association is small $(r = 0.24$, Schneider et al., [2017\)](#page-9-2). Neuroimaging studies have revealed that many more brain regions other than the IPS show specifc increases in activity when children engage in processing number (Arsalidou et al., [2018;](#page-8-5) Peters & De Smedt, [2018\)](#page-9-3). The increases in brain activity in the IPS during the processing of number have been interpreted to refect not only numerical processing but also other general cognitive functions, such as spatial working memory, serial order processing, or visual attention (see Fias, [2016](#page-8-7), for a discussion). The chapters in this volume collectively align with these criticisms, as they indicate that the understanding of natural number represents a much more complex endeavor that cannot be reduced to one factor. Instead, this understanding builds on a variety of learning mechanisms that are domain-specifc as well as domain-general.

6.3 The Approximate Number System: Is It Relevant for Understanding Number Development?

One central concept in many neurocognitive studies on children's number development has been the so-called ANS, or the ability to process non-symbolically presented numerical magnitudes (Dehaene, [1997](#page-8-8); Gebuis, Kadosh, & Gevers, [2016;](#page-8-9) Leibovich, Katzin, Harel, & Henik, [2017](#page-8-10)). This system has been suggested to be

innate as well as to be the foundation of understanding symbolic number and mathematical development (Feigenson, Dehaene, & Spelke, [2004;](#page-8-11) Piazza et al., [2010](#page-9-8)) and individual differences therein (Halberda, Mazzocco, & Feigenson, [2008\)](#page-8-12). It has been proposed that the etiology of dyscalculia is best explained by a deficit in this ANS (Wilson & Dehaene, [2007](#page-9-6)). The existence of an ANS and its role in mathematical development continues to be the most debated topic in the feld of numerical cognition (Ansari, [2016;](#page-8-13) Gebuis et al., [2016;](#page-8-9) Leibovich et al., [2017](#page-8-10)). Increasing evidence suggests that the ANS might not be numerical (Gebuis et al., [2016;](#page-8-9) Leibovich et al., [2017](#page-8-10)) and that it even may not be the ground onto which the understanding of number, which is in essence a symbolic activity, is built (Leibovich $\&$ Ansari, [2016](#page-8-14)). These neurocognitive studies have been executed without much contact with the relevant work in mathematics education research. The contribution by Ulrich and Norton (Chap. [3\)](https://doi.org/10.1007/978-3-030-00491-0_3), focusing on children's construction of number, nicely illustrates how mathematics education research might help to constrain theories of the ANS and its role in children's understanding of natural number.

Ulrich and Norton (Chap. [3](https://doi.org/10.1007/978-3-030-00491-0_3)) aptly point to the critical difference between magnitude and number. They indicate that the ANS deals with magnitude but not with number. Number entails the measurement of a magnitude; it needs to be constructed and it necessitates the understanding of a countable unit (see also Clements et al., Chap. [2](https://doi.org/10.1007/978-3-030-00491-0_2)). This points to the critical role of understanding counting, which requires learning number words and symbolic representations and which takes years of mathematical experience to develop.

The contribution of Ulrich and Norton (Chap. [3\)](https://doi.org/10.1007/978-3-030-00491-0_3) nicely echoes recent discussions in the neurocognitive feld on the extent to which the ANS is numerical (Gebuis et al., [2016;](#page-8-9) Leibovich et al., [2017\)](#page-8-10) and to which it provides a ground for learning symbolic number (Leibovich & Ansari, [2016\)](#page-8-14). For example, Gebuis et al. [\(2016](#page-8-9)) contend that the ANS merely refects the integration of different sensory cues, such as area and/or density, rather than something numerical. These authors argue that a sense of magnitude, based on area or density, rather than a sense of number, enables the discrimination between two magnitudes, as is also suggested by Ulrich and Norton (Chap. [3](https://doi.org/10.1007/978-3-030-00491-0_3)). Lyons, Bugden, Zheng, De Jesus, and Ansari [\(2018](#page-9-9)) recently coined the term Approximate Magnitude System (AMS), as an alternative to ANS. In line with the reasoning of Ulrich and Norton (Chap. [3](https://doi.org/10.1007/978-3-030-00491-0_3)), AMS might be a better term to denote this cognitive ability.

Another important conundrum in neurocognitive research, touched upon by Ulrich and Norton (Chap. [3\)](https://doi.org/10.1007/978-3-030-00491-0_3), is the extent to which the ANS provides a ground for learning symbolic number (Leibovich & Ansari, [2016\)](#page-8-14). While the dominant theory assumes that the ANS provides the ground for children's symbolic representations of number (Piazza, [2010\)](#page-9-5), this has been seriously challenged by developmental and brain imaging data (Ansari, [2016](#page-8-13); Leibovich & Ansari, [2016](#page-8-14)). For example, Lyons et al. ([2018\)](#page-9-9) showed that in kindergartners, symbolic comparison abilities predicted subsequent non-symbolic comparison but not vice versa. This suggests that it is the acquisition of exact number that facilitates growth in the ANS, rather than vice versa. This aligns with the critical role of unitizing and measurement in the

development of number, as discussed by Ulrich and Norton (Chap. [3](https://doi.org/10.1007/978-3-030-00491-0_3); see also Clements et al., Chap. [2\)](https://doi.org/10.1007/978-3-030-00491-0_2).

The contribution by Ulrich and Norton (Chap. [3\)](https://doi.org/10.1007/978-3-030-00491-0_3) provides new avenues for further study that can beneft from collaborations between researchers in mathematics education and cognitive psychology. These studies should clarify how the awareness of magnitude and the development of number are related. Even though infants may have a sense of magnitude, it may not be critical to learning number. On the other hand, we need to understand how the development of number affects the awareness of magnitude (see Lyons et al., [2018\)](#page-9-9).

This view of the ANS as relevant to magnitude rather than number also offers a fresh perspective on understanding dyscalculia. De Smedt et al. [\(2013](#page-8-6)) observed in their review of the literature that impairments on ANS-tasks were only observed in older (starting from age 10) children with dyscalculia (when compared to typically developing children). It might be that children with dyscalculia have a preserved awareness of magnitude, but that they do not beneft as much as typically developing children from their understanding of number or their ability to measure magnitude that allows them to fuently execute the dot comparison task. As suggested by Ulrich and Norton (Chap. [3\)](https://doi.org/10.1007/978-3-030-00491-0_3), children could use different strategies to solve a seemingly basic dot comparison task. Children with dyscalculia might rely more on their perceptual sense of magnitude to perform this task, while typically developing children might rely more on their understanding of (symbolic) number and quantity, leading to differences in performance. Future studies are needed to verify this conjecture. They will require the consideration of different strategies that children use during comparison tasks, and these are not necessarily the same as the ones used by adults, as pointed out by Ulrich and Norton (Chap. [3](https://doi.org/10.1007/978-3-030-00491-0_3)).

6.4 More than Magnitude: The Roles of Subitizing, Place Value, and Spontaneous Focusing

Subitizing—the immediate apprehension and identifcation of the exact number of items in small sets up to four items—has been studied for a long time in cognitive psychological research, yet it has been relatively neglected in mathematics education research (Clements et al., Chap. [2\)](https://doi.org/10.1007/978-3-030-00491-0_2). It needs to be emphasized that it is not so easy to measure subitizing reliably, as subitizing is typically a very accurate process that occurs within a timeframe of less than 1 s. Clements et al. (Chap. [2](https://doi.org/10.1007/978-3-030-00491-0_2)) aptly point out that the basic process of subitizing has a much more complex and protracted developmental course than is assumed in cognitive psychological research. They argue that, during this development, a perceptual process that is in essence nonnumerical has to be linked with an exact (symbolic) concept of number, echoing Ulrich and Norton's (Chap. [3\)](https://doi.org/10.1007/978-3-030-00491-0_3) discussion of the ANS. Fully functional subitizing requires the understanding of a countable unit as well as the number words to construct an exact cardinal representation of a collection (Clements et al., Chap. [2\)](https://doi.org/10.1007/978-3-030-00491-0_2), but

the critical question remains when in development this happens. This again emphasizes that number processing is in essence a symbolic activity, which requires the integration of multiple symbolic representations (Merkley $\&$ Ansari, [2016](#page-9-1)), the developmental trajectories of which remain to be further understood.

Clements et al. (Chap. [2\)](https://doi.org/10.1007/978-3-030-00491-0_2) also discuss a more complex type of subitizing, *conceptual subitizing,* which has a high educational relevance. Conceptual subitizing refers to the child's ability to organize a set of items via partitioning, decomposing, and composing to quickly determine its number. This conceptual subitizing provides children experiences with additive situations, and it fosters their understanding of part-whole relations, which are a critical scaffold for learning arithmetic operations. This discussion of Clements et al. (Chap. [2\)](https://doi.org/10.1007/978-3-030-00491-0_2) provides a nice example of how elementary numerical activities can act as a stepping-stone for learning more complex arithmetic and mathematics. This type of theorizing on the mechanisms of why basic number processing correlates with more advanced mathematical achievement has been somewhat lacking in neurocognitive studies. These latter studies have typically focused on what predicts mathematics achievement but not on why it predicts this achievement (De Smedt et al., [2013\)](#page-8-6). The combination of perspectives from mathematics education with psychological research might be a fruitful avenue to further understand these mechanisms. Such research is needed to further elucidate when conceptual subitizing develops and how it is related to children's learning of arithmetic and its individual differences.

The large majority of neurocognitive studies on (symbolic) number processing have narrowed their focus to single-digit numbers, but to fully "crack the code" of Arabic numerals, children need to learn place value and multi-digit number meanings, which are concepts that are diffcult to master for many of them (Mix et al., Chap. [5](https://doi.org/10.1007/978-3-030-00491-0_5)). Mix et al. elaborate on this learning of place value and how it can be fostered, through the domain-general lens of relational learning mechanisms, such as statistical learning and structure mapping. Their chapter nicely illustrates that the development of symbolic number is much more complex than the simple mapping between a symbol and the quantity it represents, as has often been assumed in neurocognitive studies. Their chapter offers a key to the solution of the symbolgrounding problem in numerical cognition (Leibovich & Ansari, [2016](#page-8-14)). More specifically, Mix et al. (Chap. [5](https://doi.org/10.1007/978-3-030-00491-0_5)) highlight that, in addition to domain-specific numerical mechanisms, domain-general relational learning mechanisms, which play a role in the acquisition of language, particularly the learning of syntax (Ullman, [2004\)](#page-9-10), also need to be investigated. These investigations have the potential to further explain the strong associations between measures of language and mathematics (LeFevre et al., [2010\)](#page-8-15) and to elucidate the comorbidity of dyscalculia with language disorders (Evans & Ullman, [2016\)](#page-8-16).

It is important to emphasize that the learning of place value depends on the transparency of the language in which children learn number. Some languages, such as Chinese, have a very regular alignment between the structure of their number words and their numerals $(23 = two \times 1)$ times ten and three) whereas other languages, such as Dutch, do not $(23 =$ three-and-twenty). It is evident that the learning of place value will be much harder in the latter languages than in the former and that different types of instruction might be needed in these different languages. In all, this highlights that contextual factors moderate children's understanding of number (see also Clements et al., Chap. [2](https://doi.org/10.1007/978-3-030-00491-0_2), and McMullen et al., Chap. [4](https://doi.org/10.1007/978-3-030-00491-0_4)), an issue that has been central in educational research but that has been often ignored in neurocognitive studies.

McMullen et al. (Chap. [4\)](https://doi.org/10.1007/978-3-030-00491-0_4) guide our attention to children's spontaneous focusing tendencies on number (SFON) and relations (SFOR) and highlight that these are key elements of children's understanding of number and its individual differences. McMullen et al. emphasize that children differ in their attention to mathematical elements of everyday situations outside the formal learning context. Children who are more attentive to the numerical and mathematical aspects of an everyday situation will have more (self-initiated) practice with it and, consequently, develop better mathematical skills. This again points to the critical role of the environment, including both the home and school environment, and the contexts in which children are confronted with number as powerful moderators of children's numerical development. It remains, however, as yet unclear what aspects of the environment trigger children's attention to number. On the other hand, it is clear that children's understanding of number and numerical relations and their spontaneous focus on it develop in an iterative way (McMullen et al., Chap. [4](https://doi.org/10.1007/978-3-030-00491-0_4)).

6.5 Concluding Thoughts

The chapters in this volume collectively indicate that children's development of natural number cannot be reduced to one basic neurocognitive ability but instead requires a multitude of skills that have different developmental trajectories. These chapters also suggest that these skills develop in a bidirectional way although their precise interactions and their developmental timing need further investigation.

It is also important to point out that the use of the term "neurocognitive" sometimes mistakenly suggests a direction of associations, such that neurocognitive variables are more easily perceived as predictive or causal in learning, in this case, natural number. However, it also might be that learning natural number itself changes related neurocognitive processes. It is the research design and not the type of data (i.e., either neurocognitive or brain imaging data) that determines predictive value or causality. This should be kept in mind when evaluating the existing neurocognitive data. Intervention studies that manipulate a given factor are needed to further determine which factors are causal and which are not. Carefully controlled longitudinal studies (i.e., cross-lagged designs) can also test the directions of associations between these skills (see McMullen et al., Chap. [4,](https://doi.org/10.1007/978-3-030-00491-0_4) for an example).

The idea that the so-called basic processing of number consists of a multitude of skills also opens opportunities for understanding the origins of dyscalculia, which has been characterized in neurocognitive studies as a disorder that originates from a

deficit in processing number (De Smedt et al., [in press](#page-8-3)). Against the background of the chapters in this volume, it seems unlikely that such a deficit in number processing can be reduced to one single defcit in one numerical ability. This echoes recent models of other neurodevelopmental disorders, such as dyslexia or ADHD, which have posited that multiple deficits rather than one single deficit account for their emergence (Peterson & Pennington, [2015](#page-9-11)). The numerical abilities highlighted in this section might all constitute risk factors for developing defcits in learning to calculate and consequently, future studies on atypical development should consider the relative contribution of each of these risk factors. As has been illustrated throughout the chapters in this volume, these numerical skills are also related to domaingeneral learning mechanisms, such as statistical learning (Mix et al., Chap. [5\)](https://doi.org/10.1007/978-3-030-00491-0_5), perceptual abilities (Clements et al., Chap. [2](https://doi.org/10.1007/978-3-030-00491-0_2)), or sensorimotor abilities (Ulrich & Norton, Chap. [3](https://doi.org/10.1007/978-3-030-00491-0_3)), which also require additional consideration when studying the origins of atypical mathematical development.

The current collection of chapters also reveals that children's learning of natural number will require specifc instruction. Clements et al. (Chap. [2\)](https://doi.org/10.1007/978-3-030-00491-0_2) and Mix et al. (Chap. [5\)](https://doi.org/10.1007/978-3-030-00491-0_5) nicely illustrate that cognitive models of different types of numerical skills can help to inform the design of educational programs. Outlining the developmental trajectories of a given numerical ability, such as subitizing, provides a ground for designing activities that can be optimally tailored to support students at various points in these different trajectories (Clements et al., Chap. [2](https://doi.org/10.1007/978-3-030-00491-0_2)). Similarly, general psychological learning mechanisms, such as statistical learning or structure mapping (Mix et al., Chap. [5](https://doi.org/10.1007/978-3-030-00491-0_5)) can provide insight into ways to improve educational programs. It needs to be acknowledged that there will be individual differences in both these domain-specifc and domain-general components that are critical to understanding number (Vanbinst & De Smedt, 2016). A cognitive analysis of these components will allow educators to verify which abilities require more scaffolding (weaknesses) and which abilities can be used as compensatory factors (strengths) (see Mix et al., Chap. [5](https://doi.org/10.1007/978-3-030-00491-0_5)). For example, children who are less likely to spontaneously attend to number and relations might require more guided instruction compared with others (McMullen et al., Chap. [4\)](https://doi.org/10.1007/978-3-030-00491-0_4).

To conclude, the contributions in the current volume clearly show that children's understanding of number cannot be reduced to one neurocognitive factor, such as the ANS, but instead represents a complex development of different types of abilities that become gradually connected over development. It is clear that this development involves domain-specifc as well as domain-general learning mechanisms. The contributions in this volume provide fresh perspectives that will help to further our understanding of children's natural number development in both the mathematics education and neurocognitive research communities. It is clear that both disciplines can learn from each other and that these chapters are a starting point for further inquiry on the cognitive mechanisms of children's understanding of number as well as on the design and evaluation of educational interventions that aim to support this understanding.

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