# Analysis of Control Sensitivity Functions for Power System Frequency Regulation

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**Abstract.** This work studies the behavior of the Control Sensitivity Functions derivated from the frequency regulation structure in power systems. Here, we explore the performance of the sensitivity functions in the presence of changes in the parameters of frequency regulation and power system components. A one-area power system is employed as the simulation benchmark. Results of frequency-domain analysis with Bode plots highlight the more significant parameters for Load Frequency Control and the different changes in sensitivity functions.

**Keywords:** Control sensitivity functions · Power systems
Frequency regulation

## 1 Introduction

The electricity sector is undergoing significant changes worldwide. Factors such as the proliferation of isolated systems with the possibility of connecting to the network and equipped with storage systems (microgrids) [17], the progressive incorporation of electric vehicles, the installation of equipment based on power electronics to control and manage networks (FACTS), and the application of automation and advanced information technologies configure what is usually called the smart grid [2]. These factors, together with the unavoidable integration of unconventional energy sources, pose new challenges for the operation and control of electric power systems, mainly because these factors introduce power and load unbalances that affect the stability on the power system frequency.

Frequency constitutes a significant operational parameter for power systems, establishing a direct relationship with the speed of rotation of conventional synchronous generators. Some electrical phenomena such as power outages, fluctuation in demanded power, variations in renewable energy sources or transmission


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line losses [5] modify the generation and load equilibrium, producing frequency changes in the standard operating values. These phenomena raise the need for control strategies maintaining the system frequency inside the corresponding operational frame. Frequency control scheme for power systems is composed by the damping actions of speed variations of synchronous units by machine inertia, and by primary (Load Frequency Control, LFC) and secondary (Automatic Generation Control, AGC) control structures returning frequency to standard operating ranges in the presence of disturbance events [1].

Frequency control has been a significant topic of research for electrical power systems [3,13,15,16,18]. This work explores the analysis of the so-called Control Sensitivity Functions [20] of the frequency regulation system as a tool for obtaining information useful for controller design. The use of Control Sensitivity Functions for power system analysis has been shown in [22], for the control of an electrically powered steering system through a robust $H_\infty$ controller designed with mixed sensitivity technique. Also, model uncertainties have been studied with complementary sensitivity functions in the development of robust stabilizers for power systems [12]. Robust control is the main topic of application of sensitivity functions in power systems, with reported works in control schemes for doubly fed induction wind generators [11], and solar power generation systems [4]. Also, the use of sensitivity analysis in the assessment of power system stability using small-signal techniques is described in [21].

The specific application of sensitivity functions in frequency control derives from the design of robust controllers using $H_\infty$ methodologies [3]. Related applications with Bode plots for the dynamic study of power system frequency regulation with integration of renewable energy sources (mainly wind farms) are reported in [6,9,10,15]. Also, sensitivity-like expressions and transfer functions were developed in [7,8] for the parametric analysis of the frequency control scheme in power systems. This paper extends the work first reported on [14], where the Control Sensitivity Functions for a single area power system were derived and studied. Here, we explore the behavior of the sensitivity functions in the presence of changes in the parameters of frequency regulation and power system components. Results of frequency-domain analysis with Bode plots highlight the more significant parameters for Load Frequency Control and the different changes in sensitivity functions.

This paper is structured as follows. We begin with a short explanation of the Control Sensitivity Functions extracted from the control systems theory. Next, we describe the frequency regulation for power systems and define the Sensitivity Functions from this structure. Using Bode plots, we analyze the effects of parameter variations in the derived Sensitivity Functions for frequency control in computer simulations for a single area power system with conventional machines. We finish discussing the results and presenting final remarks.

## 2 Sensitivity Functions in Control Systems

Linear time-invariant (LTI) systems are a class of dynamic systems that can be represented by linear, constant-coefficient, differential equations. Transfer

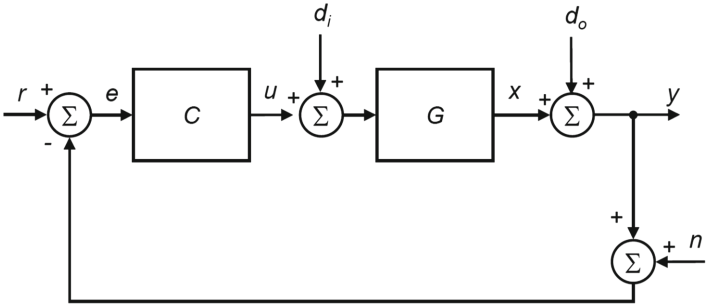


**Fig. 1.** Generalized structure of a control system with a basic feedback loop [20]

functions and block diagrams represent these LTI systems [20]. Consider a control system with a basic feedback loop and without measurement filter, as shown in the block diagram of Fig. 1. Two blocks are representing the system: the controller $C$ and the plant $G$. Four external signals influence the feedback loop: the reference value $r$, the disturbance $d_i$ at the plant input, the output disturbance $d_o$, and the measurement noise $n$. Error signal $e$ is the signal feeding the controller block $C$ for the generation of control action $u$. Finally, signal $y$ denominates the process output. From Fig. 1, the following expressions denoting the signal relationships can be obtained using block algebra:

$$\begin{aligned} U &= \frac{C}{1+GC}R - \frac{GC}{1+GC}D_i - \frac{C}{1+GC}D_o - \frac{C}{1+GC}N \\ Y &= \frac{GC}{1+GC}R + \frac{G}{1+GC}D_i + \frac{1}{1+GC}D_o - \frac{GC}{1+GC}N \end{aligned} \tag{1}$$

In both Fig. 1 and Eq. (1) Laplace variable $s$ has been omitted for simplicity. In Eq. (1) some transfer functions are the same; these are denominated as the *Control Sensitivity Functions* determining system dynamic performance. The four sensitivity functions are given by the following transfer functions [20]:

– Sensitivity S(s), offering an estimation of the disturbance rejection capabilities of the system.

$$S(s) = \frac{1}{1+G(s)C(s)} \tag{2}$$

– Complementary Sensitivity $T(s)$:

$$T(s) = \frac{G(s)C(s)}{1+G(s)C(s)} \tag{3}$$

– Disturbance Sensitivity $S_i(s)$:

$$S_i(s) = \frac{G(s)}{1+G(s)C(s)} \tag{4}$$

– Control Sensitivity $S_o(s)$:

$$S_o(s) = \frac{C(s)}{1 + G(s)C(s)} \tag{5}$$

These Control Sensitivity Functions concentrate characteristical information related to the dynamic behavior of the system, such as disturbance impacts, the performance of the control action and reference signal tracking [20]. Both static (low frequencies) and dynamic (high frequencies) studies can benefit from these descriptions of the system. Moreover, Control Sensitivity Functions are intrinsically related among them through the expressions shown below:

$$S(s) + T(s) = 1 \tag{6}$$

$$S_i(s) = G(s)S(s) = \frac{T(s)}{C(s)} \tag{7}$$

$$S_o(s) = C(s)S(s) = \frac{T(s)}{G(s)} \tag{8}$$

The relationships described by Eqs. (6) to (8) imply that the tuning of a determined controller $C(s)$ is not shaping the Control Sensitivity Functions. Conversely, those restrictions highlight the requirement of trade-off compromises between the different control goals. Designer criteria are of paramount importance for the selection of the proper control scheme concerning the performance objectives.

## 3 Control Sensitivity Functions for Frequency Regulation in Power Systems

### 3.1 Description of Frequency Control System

In electrical systems under normal operating conditions, the generators are rotating in synchronism, and together they generate the total demanded energy [1]. Since electrical energy can not be stored in large quantities, if the power consumed by the load increases but the mechanical power provided by the turbines remains constant, the increase in demand can only be compensated for the stored kinetic energy [1]. This effect supposes a decrease in the rotation speed of the generators and a frequency falls on the system, directly related to said speed. As the demand is continuously changing, a control system is required to automatically adjust the power generated in each generation unit while keeping the frequency within certain operational limits [19]. Frequency control scheme for power systems is formed by the damping effects of speed variations of synchronous machines by the inertia, and by primary (Load Frequency Control, LFC) and secondary (Automatic Generation Control, AGC) control structures returning frequency to standard operating ranges in the presence of disturbance events [1]. A simple representation of the Frequency control for a single area

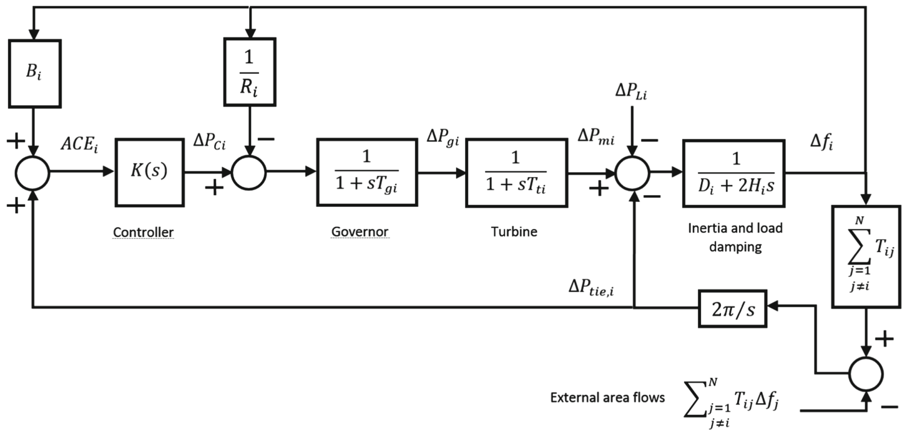


**Fig. 2.** Block diagram for frequency control structure in power system including only one machine for a regulation area $i$ (based on [3])

power system is shown in Fig. 2. The complete derivation of the system can be found in [3].

In Fig. 2, $\Delta P_m(s)$ is the variation $[p.u]$ in the mechanical power of generating units, $\Delta P_l(s)$ represents the load changes $[p.u]$, $\Delta f(s)$ denotes the per-unit frequency deviation, the adjusted inertia characteristic for the area is $H$ $[s]$, and $D$ is known as the load damping constant [3]. $R$ is the speed drop of the machines, $K(s)$ is the secondary control block (usually, and for this work, $K(s) = -k_I/s$) and $B_i = D_i + 1/R_i$ is an adjusted gain denominated as the frequency bias of the area. Finally, $T_{ti}$ and $T_{gi}$ denote the corresponding time constants of the respective turbine and governor first-ordel models.

### 3.2 Derivation of Control Sensitivity Functions for Frequency Regulation Structure

The Frequency regulation structure in Fig. 2 can be expressed as the classic feedback control system of Fig. 1 after the application of certain block operations. Assuming the simplification $r = d_i = n = 0$, the system of Fig. 3 is reached [14]. The output signal $y$ for this resulting control structure is $\Delta f_i$, the change in electrical frequency for a given area $i$ without transferred power to other areas (isolated).

From Fig. 3, we can see how the plant $G$ groups the blocks corresponding to total inertia of the area $i$ and the turbine-governor system. Also, the controller $C$ is combining the effects of both primary and secondary actions in power system frequency control. Moreover, we have to note how the output disturbance $d_o$ shows the load deviations filtered through the load damping and the total inertia of the system. This filtering effect can be seen in Eqs. (9) and (10):

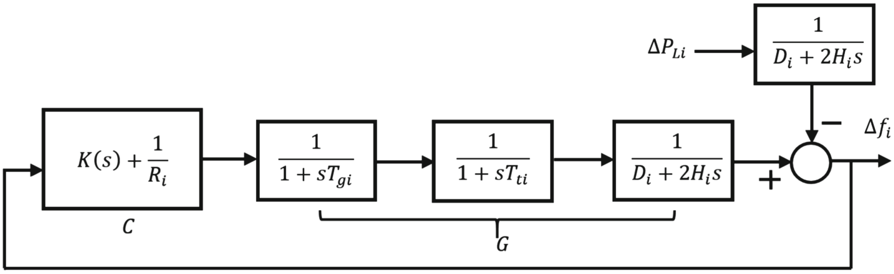


**Fig. 3.** A single area power system frequency regulation scheme seen as a generalized feedback control system [14]

$$C = K(s) + \frac{1}{R_i} = \frac{-k_i B_i}{s} + \frac{1}{R_i} \tag{9}$$

$$d_o = -\Delta P_L \left(\frac{1}{D_i + 2H_i s}\right) \tag{10}$$

The relationship between the process frequency $\Delta f(s)$ and the load disturbance signal $\Delta P_L(s)$ is defined by transfer function $G_f(s)$ in Eq. (11):

$$G_f(s) = \frac{-\frac{1}{2H_i s + D_i}}{1 + \left(K(s) + \frac{1}{R_i}\right) \frac{1}{T_g s+1} \frac{1}{T_t s+1} \frac{1}{2H_i s+D_i}} \tag{11}$$

The control sensitivity functions according to the frequency regulation structure of Fig. 3 are [14]:

$$S(s) = \frac{1}{1 + \left(K(s) + \frac{1}{R_i}\right) \left(\frac{1}{T_g s+1} \frac{1}{T_t s+1} \frac{1}{2H_i s+D_i}\right)} \tag{12}$$

$$T(s) = \frac{\left(K(s) + \frac{1}{R_i}\right) \left(\frac{1}{T_g s+1} \frac{1}{T_t s+1} \frac{1}{2H_i s+D_i}\right)}{1 + \left(K(s) + \frac{1}{R_i}\right) \left(\frac{1}{T_g s+1} \frac{1}{T_t s+1} \frac{1}{2H_i s+D_i}\right)} \tag{13}$$

$$S_i(s) = \frac{\frac{1}{T_g s+1} \frac{1}{T_t s+1} \frac{1}{2H_i s+D_i}}{1 + \left(K(s) + \frac{1}{R_i}\right) \left(\frac{1}{T_g s+1} \frac{1}{T_t s+1} \frac{1}{2H_i s+D_i}\right)} \tag{14}$$

$$S_o(s) = \frac{K(s) + \frac{1}{R_i}}{1 + \left(K(s) + \frac{1}{R_i}\right) \left(\frac{1}{T_g s+1} \frac{1}{T_t s+1} \frac{1}{2H_i s+D_i}\right)} \tag{15}$$

## 4 Effects of Parameter Variations in Control Sensitivity Functions of LFC

We are interested in the behavior of the sensitivity functions in the presence of changes in frequency regulation parameters and power system components. After the parameter variations, sensitivity functions are studied with magnitude Bode plots in absolute units ($abs$). As the simulation benchmark, a power system frequency regulation from a single area one-machine system like the one in Fig. 3 is selected. System parameters for the base case are taken from [19]: $T_g = 0.2$ $sec$, $T_t = 0.5$ $sec$, $H = 5$ $sec$, $R = 0.05$ $p.u$ and $B_i k_i = 7$. System includes an integral secondary controller for the AGC. Then, these values are changed and the control sensitivity functions are obtained for the updated parameters.

### 4.1 Variations in Inertia *H*

The effects of the change of inertia $H$ in the control sensitivity functions of the frequency regulation system are depicted in Fig. 4. As the inertia is decreased gradually, the control sensitivity functions present more prominent peaks of magnitude. These effects signal a performance degradation in the frequency regulation structure and highlight the importance of the inertia for frequency control purposes.

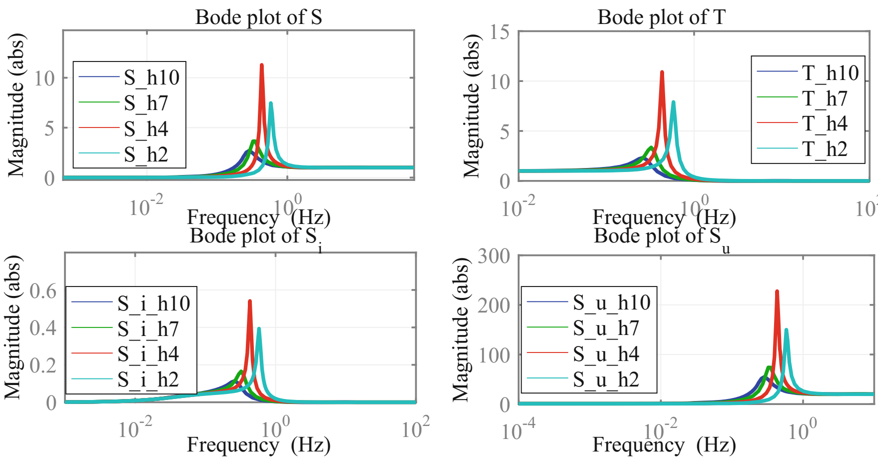


**Fig. 4.** Effects of the variation of inertia $H$ in the control sensitivity functions of frequency regulation system

### 4.2 Variations in Load Damping *D*

The effects of the change of load damping $D$ in the control sensitivity functions of the frequency regulation system are depicted in Fig. 5. Load damping is a parameter representing the impact of the frequency-dependent loads in the frequency regulation structure. The simulations show the minimal effect of $D$ in the sensitivity functions. This behavior is due to the lack of frequency-dependent loads in the power systems, and show the limited value of load-damping for frequency regulation at these penetration levels.

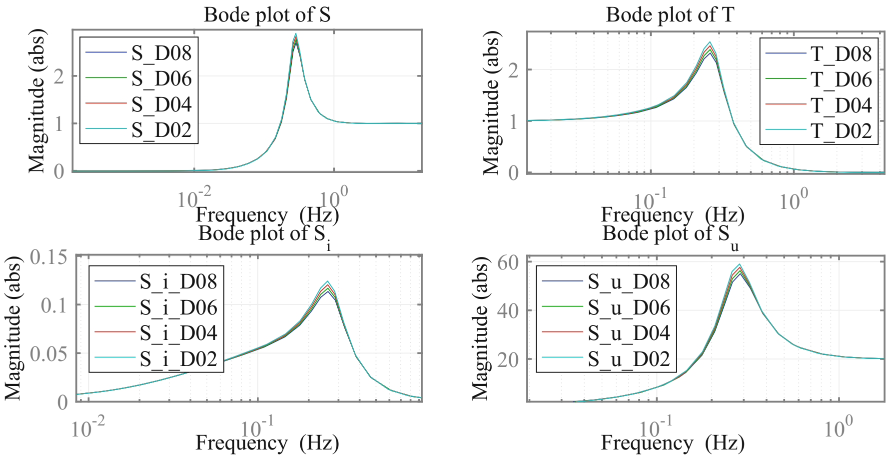


**Fig. 5.** Effects of the variation of load damping $D$ in the control sensitivity functions of frequency regulation system

### 4.3 Variations in Turbine Time-Constant $T$

The effects of the change of turbine time-constant $T$ in the control sensitivity functions of the frequency regulation system are depicted in Fig. 6. The results show how the control sensitivity functions suffer from performance degradation with the increase of time-constant $T$. This behavior causes an increased delay in the response of the system controllers caused by the slower action of the machines.

### 4.4 Variations in Speed Droop $R$

The effects of the change of speed droop $R$ in the control sensitivity functions of the frequency regulation system are depicted in Fig. 7. The parameter $R$ represents the response capability of the machines, the operational margin for changing rotational speed and contribute to primary regulation. In this way, as expected, the performance of the control system improves as the speed drop increases.

### 4.5 Variations in Controller Integral Gain $k_i$

The effects of the change of controller integral gain $k_i$ in the control sensitivity functions of the frequency regulation system are depicted in Fig. 8. As the controller integral gain $k_i$ increases, the only sensitivity function with significant

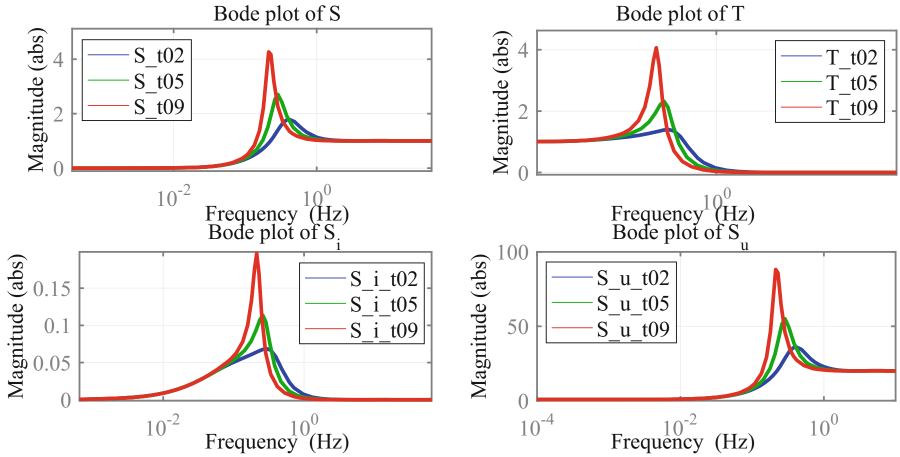


**Fig. 6.** Effects of the variation of the turbine time-constant $T$ in the control sensitivity functions of frequency regulation system

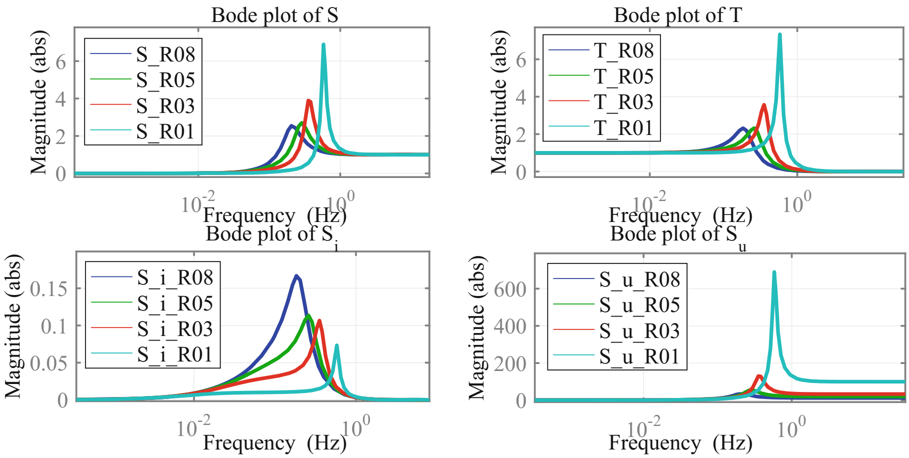


**Fig. 7.** Effects of the variation of the speed droop $R$ in the control sensitivity functions of frequency regulation system

change is $S_i$. This behavior makes sense because the disturbance rejection is improved with the adequate tuning of the secondary controllers for frequency regulation.

## 5 Conclusions

This work analyzed the impacts of parametric variations in the Control Sensitivity Functions of frequency regulation in power systems. The inertia $H$ caused the

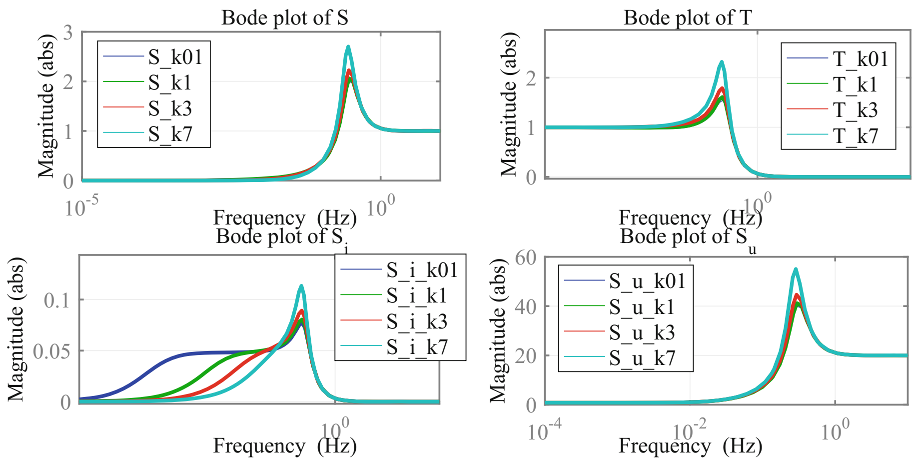


**Fig. 8.** Effects of the variation of the controller integral gain $k_i$ in the control sensitivity functions of frequency regulation system

most significant effect on the performance of frequency control loops. Reduction of this parameter significantly diminishes the system's opposition to frequency variations, even reaching an amplifying effect of power disturbances at maximum value. This fact suggests that in the case of inertia variations for the system, both the integral action and the proportional gain of the controller given by the machine speed *droop* need to be adjusted.

On the other hand, the negligible impact of load damping $D$ suggests the lack of influence of this parameter in frequency regulation. Also, the increase in the time constant of the turbine represented a significant delay in the response of the control system.

The variations in speed droop and integral gain showed that, even when the control parameters were adjusted for the diminishing inertia, disturbance rejection performance decreased as evidenced by control sensitivities. There has to be particular attention to the essential design considerations for the tuning of control parameters, as these determine the disturbance rejection capabilities and the ability for minimizing the steady-state error in frequency deviations.

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