Constructing an Outranking Relation with Weighted OWA for Multi-criteria Decision Analysis

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Abstract. Some decision aiding methods are based on constructing and exploiting outranking relations. An alternative *a* outranks another *b* if *a* is at least as good as *b (aSb)*. One well known method in this field is ELECTRE. The outranking relation is usually built by means of a weighted average (WA) of the votes given by a set of criterion with respect to the fulfilment of aSb. The value obtained represent the strength of the majority opinion. The WA operator can be observed to have sometimes an undesired compensative effect. In this paper we propose the use of other aggregation operators with different mathematical properties. In particular, we substitute the WA by three operators from the Ordered Weighted Average (OWA) family of operators because it permits to decide the degree of andness/orness that is used during the aggregation. The OWAWA (Ordered Weighted Average Weighted Average), WOWA (Weighted Ordered Weighted Average) and IOWA (Induced Ordered Weighted Average) operators are studied. They are capable to combine the importance given to each criterion with the conjunctive/disjunctive requirement applied in the definition of the outranking relation.

Keywords: Decision support systems · Outranking relations
Ordered Weighted Average

1 Introduction

Multiple Criteria Decision Aiding discipline studies systematic methods for complex decision problems concerning diverse and often contradictory criteria, by analyzing a set of possible alternatives in order to find the best one [1]. One of the most successful approaches nowadays is known as *outranking methods*. It is based on social choice models that copy the human reasoning procedure [4].

MCDA methods take a set of alternatives (i.e. potential solutions) and generate a ranking of the alternatives according to a set of criteria. Criteria are tools constructed for the evaluation of alternatives compared in terms of suitability based on the decision maker's needs. Each criterion corresponds to a point of view considered in the decision process. Outranking methods are characterized by being based on constructing preference relations between the alternatives by means of pairwise comparisons, instead of

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aggregating directly the values given by the criteria. The aim is to build a binary outranking relation *aSb*, which means "*a* is at least as good as *b*" [1]. Each criterion is asked about its contribution to this outranking assertion and it provides a vote in favor or against to *aSb*. Votes must be aggregated in order to associate a value to *aSb* for all possible pairs of alternatives. There are two main methods known as PROMETHEE and ELECTRE. In this study, we focus on ELECTRE method as it strictly applies the concept of veto. Moreover, ELECTRE method has been widely acknowledged as an efficient decision aiding tool with successful applications in many domains [4].

ELECTRE uses a weighted average to merge all the votes supporting *aSb* and then it includes the opposite votes by using a veto procedure. Once the valued outranking relation is constructed, different exploitation procedures exist in order to derive a ranking from it [1]. The contribution of this paper is the use of other aggregation operators for merging the votes in favor of the outranking relation. In particular, we propose the use of OWA-like operators because they enable the definition of conjunctive/disjunctive policies of aggregation that may be more appropriate in some decision problems. The compensation problem of classic weighted average may be solved with the possibility of establishing a more appropriate and-like aggregation (to model simultaneity) or or-like operator (for replaceability). As we do not want to suppress the weights representing the voting power for each criterion, we propose the use of Weighted OWA operators like OWAWA, WOWA and IOWA.

The paper is structured as follows. Section 2 presents the different aggregation operators based on OWA that will be used in the study. Section 3 briefly outlies the ELECTRE method. Section 4 defines the new procedure for calculating the overall concordance. Section 5, makes an empirical analysis and comparison. Finally, Sect. 6 discusses the main conclusions of this study.

2 Weighted OWA Operators

Aggregation operators are mathematical formulations that map a set of n values R^n to a single value R and must satisfy certain properties (idempotency, monotonicity, etc.) [9]. The most popular aggregation operators are averaging operators. The simplest aggregation operator with weights is the weighted average (WA), where the source of the values (i.e. the evaluation criteria) are assigned weights to indicate its trade-off importance. Given a set of arguments $A = (a_1, \ldots, a_n)$ and a weighting vector V with weights $v_j \in [0, 1]$ associated with each argument source (i.e. criterion), such that $\sum_{j=1}^{n} v_j = 1$. The weighted average is defined as:

$$WA(A) = \sum_{j=1}^{n} v_j a_j \quad (1)$$

Differently, OWA [10] uses weights to provide a parameterized family of mean type aggregation operators. The main distinguishing feature of this operator is the reordering of arguments according to their values before weights are assigned. Given a set of arguments $A = (a_1, \ldots, a_n)$ and a weighting vector W with weights $w_j \in [0, 1]$, such that $\sum_{j=1}^{n} w_j = 1$. The ordered weighted average is defined as:

$$OWA(A) = \sum\nolimits_{j=1}^{n} w_j b_j, \tag{2}$$

where b_j is the jth largest of the a_i.

An interesting fact about OWA is that weights are not given to the criteria but to the values. Thus, we can perform different aggregation policies (disjunctive or conjunctive) according to the decision maker (DM) needs. For example, the DM could assign weights in such a way that extreme arguments are regarded less than central arguments. In summary, the weights of OWA shows the importance of arguments in relation to the ordering of the arguments.

In some problems the DM is interested in carefully considering the weighting policies due to is significant impact on the results [3]. The use of OWA weights enables to model the andness/orness, which can be combined with usual WA weights for the different criteria. Next subsections introduce three different ways of combining them in OWA-like operators that exploit the advantages of both OWA and WA approaches.

2.1 OWAWA

In [6] the OWAWA operator is introduced as a generalization of the WA and the OWA operator.

An OWAWA operator is a mapping $A = (a_1, \ldots, a_n) \to R$, having an associated weighting vector V (WA), with $\sum_{i=1}^{n} v_i = 1$ and $v_i \in [0, 1]$ and a weighting vector W (OWA), with $\sum_{j=1}^{n} w_j = 1$ and $w_j \in [0, 1]$, such that:

$$OWAWA_\beta(A) = \beta \sum\nolimits_{j=1}^{n} w_j b_j + (1 - \beta) \sum\nolimits_{i=1}^{n} v_i a_i, \tag{3}$$

where b_j is the jth largest of the a_i and $\beta \in [0, 1]$.

The novel feature of the OWAWA operator is the ability to take into account the degree of importance of WA and OWA in specific situations. This is managed with the parameter β. As $\beta \to 1$, the importance of OWA increases while as $\beta \to 0$, the importance of WA increases. The OWAWA operator is monotonic, idempotent, commutative and bounded.

2.2 WOWA

The WOWA operator was introduced in [8] as a combination of the WA operator and the OWA operator by means of constructing a different weight that integrates the associated weighting system seen in WA, V, with the weighting according to ordering of OWA, W.

A WOWA operator is a mapping $A = (a_1, \ldots, a_n) \rightarrow R$ of dimension n where,

$$WOWA(A) = \sum_{j=1}^{n} \omega_j b_j, \tag{4}$$

where b_j is the jth largest of the a_i and the weight ω_i is defined taking into account the importance of the sources of the arguments and their position after the reordering step, defined as $\omega_i = w^*\left(\sum_{j \le i} v_{\sigma(j)}\right) - w^*\left(\sum_{j < i} v_{\sigma(j)}\right)$ with $\sum_{i=1}^{n} \omega_i = 1$.

w^* is a non-decreasing function that interpolates the points $\{(0,0)\} \cup \left\{(i/n, \sum_{j \le i} w_j)\right\} \forall i = 1, \ldots, n$. w^* is required to be a straight line when the points can be interpolated in this way. Moreover, w^* may be a regular monotonic non-decreasing quantifier $Q(x)$, with $Q(0) = 0, Q(1) = 1$ and if $x > y$ then $Q(x) \ge Q(y)$.

The WOWA operator is defined in such a way that it reduces to the OWA operator when $v_i = 1/n$ and reduces to the WA operator when $w_i = 1/n$. This shows that OWA and WA are special cases of the generalized WOWA operator.

2.3 IOWA

The last method to combine the two different sets of weights is by means of an induced ordered weighted averaging operator (IOWA). IOWA was introduced in [11] to introduce an additional variable that influences the ordering stage of OWA. The IOWA operator rather ordering arguments by their numeric values an ordered inducing variable is used to order the arguments. Then, IOWA operator is defined in terms of arguments in form of a two-tuple, called an OWA pair $\langle u_i, a_i \rangle$, where u_i is the order inducing variable of the i th argument and a_i is the argument variable of the i th argument. In the reordering step a_i is not used but u_i.

Given n arguments to be aggregated denoted as $A = (a_1, \ldots, a_n)$, the ordered arguments are obtained in a way such that b_j^u is the a value of the OWA pair having the j th largest u value. The IOWA operator can then be defined as:

$$IOWA(\langle u_1, a_1 \rangle, \ldots, \langle u_1, a_1 \rangle) = \sum_{j=1}^{n} \omega_j b_j^u \tag{5}$$

In IOWA a tie occurs when two OWA pairs $\langle u_j, a_j \rangle, \langle u_k, a_k \rangle$ have equal order inducing variables, i.e. $u_j = u_k$. In this case, each OWA pair is replaced with an OWA pair having the same order-inducing variable u but an argument variable that is an average of the previous argument variables. This means that having $\langle u_j, a_j \rangle$ and $\langle u_k, a_k \rangle$ *where* $u_j = u_k$ they are replaced by $\langle \left(u_j = u_k\right), \left(a_j + a_k/2\right) \rangle$ in the aggregation process. The IOWA operator is idempotent, communicative, monotonic and bounded.

Using as inducing variable the vector V of importance of the criteria, we have another way of combining V and W.

3 Outranking Relations in the ELECTRE Methodology

The so-called outranking methods in the MCDA literature are based on conducting a pairwise comparison of alternatives with regards to each criterion. The goal of the comparison is to find out if it satisfies an outranking relation *aSb* meaning that alternative *a* is "at least as good as" alternative *b*.

The outranking relation *S* may be binary or valued. In this paper we study the case of valued outranking relations, having then $S = A \rightarrow [0, 1]$. The value assigned to *S* is usually denoted as credibility (on the outranking relation). In the ELECTRE method, to calculate the credibility of *aSb*, two conditions must hold:

1. Concordance condition: After pairwise comparison of *a* and *b* for each criterion, a majority of the criteria must support *aSb*. It accounts for the majority opinion.
2. Non-discordance condition: Ensures that among the minority no criteria strongly refutes *aSb*. It permits the right to veto (i.e. "respect to minorities").

The outranking concept explained above is inspired in voting models used in different theories of social election. It is similar to voting procedures applied United Nations Security Council, where some countries have the right to veto the majority opinion. Following this idea, in ELECTRE methodology, to calculate the credibility value of the outranking relation $\rho(a, b) \in [0, 1]$, the following steps are applied [1]:

1. Calculation of a partial concordance index for each criterion $c_j(a, b) \in [0, 1]$. In each criterion, two discrimination thresholds may be used to model the uncertainty of the decision maker: the indifference and the preference threshold.
2. Calculation of the overall concordance $c(a, b) \in [0, 1]$. It is calculated as a weighted average of $c_j(a, b)$ using as weights the voting power of each criterion. The resulting value represent the strength of the coalition of criteria being in favor of the outranking relation *aSb*.
3. Calculation of a partial discordance index for each criterion $d_j(a, b) \in [0, 1]$. The DM can give to some criteria the right to veto the majority opinion if there are essential reasons to refute it. In this case, the criteria has an associated veto threshold, such that larger differences of this threshold in favor of *b* will eliminate the possibility that option *a* outranks option *b*.
4. Calculation of the final credibility as:

$$\rho(a, b) = \begin{cases} c(a, b) & if\ \forall_j d_j(a, b) \leq c(a, b) \\ c(a, b). \prod\limits_{j \in J(a,b)} \frac{1 - d_j(a,b)}{1 - c(a,b)} & otherwise \end{cases}, \quad (6)$$

where $J(a, b)$ is the set of criteria for which the discordance is larger than the overall concordance.

Once the credibility matrix is obtained, an exploitation procedure is applied in order to establish a preference-based order among the alternatives. A simple ranking technique is known as Net Flow Score (NFS) procedure. NFS is based on the two evidences: strength and weakness. They are measured in the graph corresponding to the valued credibility matrix calculated in step 4. The strength of alternative a is defined as the sum of the credibility values of the output edges to the node a. The weakness of alternative a is defined as the sum of the credibility values of the input edges to the node a. In terms of outranking relations, the net flow score of an alternative a is defined in Eq. 7. A total ranking can be derived from the NFS, being the higher the score, the better.

$$NFS(a) = |b \in A : aSb| - |b \in A : bSa| \tag{7}$$

4 Using Weighted OWA in the Overall Concordance Calculation

Some previous works have considered a modification of the way that overall concordance is calculated in ELECTRE in different situations. The paper [7] looks at a situation where the extent to which a criterion surpasses the preference threshold can be reflected in a change in the importance of that criterion in the concordance calculation. In [2] the concordance index is modified to take into consideration three possible interactions between the criteria that modify each joint importance: mutual strengthening, mutual weakening and antagonistic. In both cases, the weights are modified but the overall *concordance index* for each pair a, b is calculated as the weighted average of the partial concordances indices.

Having $C = \{c_j(a,b)\}, j = 1\ldots n$:

$$c(a,b) = WA(C) \tag{8}$$

In this paper we propose the substitution of the WA operator by a weighted OWA operator, presented in Sect. 2. The first proposal is using OWAWA operator that linearly combines both the result of WA and the result of OWA. In this case, the parameter beta must be defined by the user. This parameter allows to base the result most on the criteria importance weights or on the and/or weights.

$$c(a,b) = OWAWA_{\beta}(C) \tag{9}$$

The second proposal consists in using IOWA operator with the criteria importance V used as order-inducing variable. In this case, the values provided by the most important criteria will the ones assigned to the first weights of the OWA vector W.

$$c(a,b) = IOWA(\langle V, C \rangle) \tag{10}$$

The third approach uses the WOWA operator which generates a new weighting vector from the V and W.

$$c(a,b) = WOWA(C) \tag{11}$$

5 Experiments

The OWA-based outranking construction proposed has been tested with two different datasets. To evaluate the differences produced by the different operators, we compare the ranking obtained using the Net Flow Score. A minimum credibility of 0.8 in the outranking relation is used in this procedure. The correlation between different rankings is calculated to see how new proposals are able to integrate both sets of weights.

5.1 Finding a Hotel

The first case study poses the problem of making lodging arrangements to attend a congress in Jyväskylä (Finland). The DM wishes to make a choice from six hotel alternatives all in proximity to the congress site. The choice will be made based on the criteria and weights listed below. The data used in this case study is given in Tables 1 and 2. Six hotels have been evaluated using 6 criteria: **C01**- Distance to the congress site, **C02**- Distance to the city center, **C03**- Sports facilities, **C04**- Restaurants available, **C05**- Category and **C06**- Services provided (wifi, laundry, etc.). Two first criteria are minimized (−) and the rest are maximized (+).

Table 1. Hotels performance table

	C01−	C02−	C03+	C04+	C05+	C06+
Alexandra	1600.0	300.0	2.0	3.0	4.0	5.0
Sokos	1700.0	400.0	2.0	2.0	4.0	5.0
Cumulus	1700.0	550.0	4.0	0.0	3.0	3.0
Scandic	600.0	350.0	3.0	2.0	4.0	2.0
Kampus	1550.0	610.0	4.0	0.0	3.0	2.0
Alba	110.0	1300.0	1.0	1.0	3.0	4.0

Table 2. Criteria parameters

	C01	C02	C03	C04	C05	C06
Indifference	200.0	100.0	0.0	0.0	0.0	1.0
Preference	700.0	300.0	1.0	1.0	0.0	1.0
Weight	0.1	0.3	0.3	0.05	0.15	0.1

Two sets of OWA weights have been considered for this study: a disjunctive policy with w_c = (0.408, 0.169, 0.130, 0.109, 0.096, 0.088) and a conjunctive policy with weights w_d = (0.028, 0.083, 0.139, 0.194, 0.25, 0.306). These weights were obtained from the use of a regular monotonic non-decreasing quantifier, as proposed in [7].

To establish the disjunctive policy, the quantifier $Q(x) = \sqrt{x}$ is used, while for the conjunctive policy $Q(x) = x^2$.

Next tables show the overall concordance values obtained with the three combined operators proposed in this paper to merge the partial concordance indices (Table 3).

Table 3. Outranking values with weighted average (WA)

	Alexa	Sokos	Cumulus	Scandic	Kampus	Alba
Alexa	1.0	1.0	0.7	0.6	0.7	0.9
Sokos	0.95	1.0	0.7	0.6	0.7	0.9
Cumulus	0.475	0.625	1.0	0.55	1.0	0.85
Scandic	0.85	0.9	0.7	1.0	0.7	0.842
Kampus	0.4	0.535	1.0	0.46	1.0	0.75
Alba	0.2	0.2	0.4	0.2	0.4	1.0

For OWAWA, three values of β have been tested. In orange, concordances higher than 0.8 are highlighted, as they are the ones used in the NFS ranking procedure. We show only some results for the disjunctive version of the operators (and WA as reference) for space limitations (Tables 4, 5, 6 and 7). For conjunctive policies, we have observed that the values of the outranking matrix are much lower than with the rest, finding very few values above the threshold of 0.8 and leading to rankings with many ties.

Table 4. Outranking values with OWA disjunctive (OWAd)

	Alexa	Sokos	Cumulus	Scandic	Kampus	Alba
Alexa	1.0	1.0	0.912	0.816	0.912	0.912
Sokos	0.912	1.0	0.912	0.816	0.912	0.912
Cumulus	0.6095	0.6745	1.0	0.642	1.0	0.816
Scandic	0.816	0.912	0.912	1.0	0.912	0.85632
Kampus	0.577	0.6355	1.0	0.603	1.0	0.707
Alba	0.577	0.577	0.816	0.577	0.816	1.0

Table 5. Outranking values with OWAWA beta = 0.5 disjunctive (OWAWA.5d)

	Alexa	Sokos	Cumulus	Scandic	Kampus	Alba
Alexa	1.0	1.0	0.806	0.708	0.806	0.906
Sokos	0.936	1.0	0.806	0.708	0.806	0.906
Cumulus	0.54225	0.64975	1.0	0.596	1.0	0.833
Scandic	0.833	0.906	0.806	1.0	0.806	0.84916
Kampus	0.4885	0.58525	1.0	0.5315	1.0	0.7285
Alba	0.3885	0.3885	0.608	0.3885	0.608	1.0

Table 6. Outranking values with IOWA disjunctive (IOWAd)

	Alexa	Sokos	Cumulus	Scandic	Kampus	Alba
Alexa	1.0	1.0	0.7115	0.609	0.7115	0.8975
Sokos	0.912	1.0	0.7115	0.609	0.7115	0.8975
Cumulus	0.463125	0.607375	1.0	0.53525	1.0	0.8095
Scandic	0.8095	0.8975	0.7115	1.0	0.7115	0.83805
Kampus	0.391	0.520825	1.0	0.4487	1.0	0.707
Alba	0.205	0.205	0.423	0.205	0.423	1.0

Table 7. Outranking values with WOWA disjunctive (WOWAd)

	Alexa	Sokos	Cumulus	Scandic	Kampus	Alba
Alexa	1.0	1.0	0.83666	0.7746	0.83666	0.94868
Sokos	0.97467	1.0	0.83666	0.7746	0.83666	0.94868
Cumulus	0.68351	0.78561	1.0	0.73455	1.0	0.92195
Scandic	0.92195	0.94869	0.83666	1.0	0.83666	0.9172092
Kampus	0.63246	0.72435	1.0	0.67329	1.0	0.86602
Alba	0.44722	0.44722	0.63245	0.44722	0.63245	1.0

Table 8. Net Flow Score for each hotel and each method

	Alexa	Sokos	Cumulus	Scandic	Kampus	Alba
WA	0	0	1	3	0	−4
OWAd	3	3	−3	3	−4	−2
OWAc	1	−1	0	0	0	0
OWAWA.3d	0	0	1	3	0	−4
OWAWA.3c	1	0	0	1	0	−2
OWAWA.5d	2	2	−2	5	−3	−4
OWAWA.5c	1	1	0	0	0	−2
OWAWA.7d	2	2	−2	5	−3	−4
OWAWA.7c	1	−1	0	0	0	0
IOWAd	0	0	1	3	0	−4
IOWAc	3	0	−4	3	−4	2
WOWAd	2	2	−2	5	−2	−5
WOWAc	1	0	0	1	0	−2

The NFS values (Eq. 7) given to each hotel are shown in Table 8. Their ranking positions (1…6) are given in Table 9, with the hotels in the best positions highlighted.

We can see that in most of the cases Alba is in the worst position, although with a conjunctive policy the worst is Sokos. The best position is given to Scandic or Alexa

Table 9. Rank position of each hotel according to its NFS

	Alexa	Sokos	Cumulus	Scandic	Kampus	Alba
WA	4	4	2	1	4	6
OWAd	2	2	5	2	6	4
OWAc	1	6	3,5	3,5	3,5	3,5
OWAWA.3d	4	4	2	1	4	6
OWAWA.3c	1,5	4	4	1,5	4	6
OWAWA.5d	2,5	2,5	4	1	5	6
OWAWA.5c	1,5	1,5	4	4	4	6
OWAWA.7d	2,5	2,5	4	1	5	6
OWAWA.7c	1	6	3,5	3,5	3,5	3,5
IOWAd	4	4	2	1	4	6
IOWAc	1,5	4	5,5	1,5	5,5	3
WOWAd	2,5	2,5	4,5	1	4,5	6
WOWAc	1,5	4	4	1,5	4	6

(ndra), sometimes in a tie. Sokos is also in the best position when a disjunctive policy is used. The case of Sokos hotel is quite interesting because its position is the least stable of all hotels.

A look at the criteria weights show criteria C02 and C03 to have a combined 60% of the total importance assigned to criteria, as such hotels like Scandic with good performance values on C02 and C03 have better positions in WA. It can also be observed that Alexandra hotel has no low score in any criterion and some high values, thus it is the winner in case of conjunctive policies. It is worth to notice that Sokos is the worst when using OWA disjunctive, but when including the criteria weights it improve its position, as it is good in C02 and C03, as said before. Thus, the operators balance both weighting vectors. Kampus is in an intermediate position with WA because it has bad scores in non-relevant criteria, but when including the and/or weights, it goes to worst positions because of its low score in C04 and C06.

In order to measure the similarity between the rankings, Table 10 gives the Spearman rho correlation between the 3 results obtained using a single set of weights (WA, OWAd and OWAc) with respect to the use of the two sets of weights together. Table 10 also indicates the operator that gives a highest correlation (most similar ranking, with correlation higher than 0.95) for each of the proposed methods.

We can see that the disjunctive policies with OWAWA with low beta and IOWA give similar results to the WA. Rankings similar to OWA with disjunctive weights are obtained with OWAWA also disjunctive and high beta (OWA-like), as expected. Also OWAWA with high beta reproduces the ranking of OWA for the conjunctive case. An interesting observation is that WOWA seems to give a significantly different ranking to all the three basic ones.

Table 10. Correlation between the different rankings obtained in dataset Hotels

	WA	**OWAd**	**OWAc**	**Closest** (>=0.95)
1 **WA**	**1,00**	**0,16**	**0,00**	
2 **OWAd**	**0,16**	**1,00**	**0,00**	
3 **OWAc**	**0,00**	**0,00**	**1,00**	
4 OWAWA.3d	1,00	0,16	0,00	IOWAd
5 OWAWA.3c	0,66	0,56	0,46	WOWAc
6 OWAWA.5d	0,71	0,77	0,00	OWAWA.7d, WOWAd
7 OWAWA.5c	0,16	0,56	0,00	-
8 OWAWA.7d	0,71	0,77	0,00	OWAWA.5c, WOWAd
9 OWAWA.7c	0,00	0,00	1,00	-
10 IOWAd	1,00	0,16	0,00	OWAWA.3d
11 IOWAc	0,06	0,81	0,44	-
12 WOWAd	0,66	0,75	0,00	OWAWA.5d,OWAWA.7d
13 WOWAc	0,66	0,56	0,46	OWAWA.3c

5.2 Generating a Ranking of Universities

The second case study comes from paper [5], with data about British universities from https://www.thecompleteuniversityguide.co.uk/league-tables/rankings a ranking is built. We use the same weights and thresholds than paper [5], but we increased the number of alternatives to 20 universities. Five criteria are taken: **C01**- Academic services spend, **C02**- Completion, **C03**- Entry standards, **C04**- Facilities spend, **C05**- Good honors. Horizontal axis shows the identifier of each method given in Table 10. Again $Q(x) = \sqrt{x}$ and $Q(x) = x^2$ were used to establish the OWA weights (Fig. 1).

Aggregation with IOWA (10 & 11) and with WOWA (11 & 12) is able to change the position of some universities in this dataset. Although the ones in the best and worst positions are robust to the change of agregation operator. For example, U16 and U1 are universities that are sensible to the aggregation policy. U16 is excellent in two criteria (w = 0.2 and 0.1) and very bad in one (w = 0.3). Therefore, when using WA it appears in at rank 11/20, with OWAd it goes to upper positions (6/20). We can also observe that there are many rank reversals between universities in ranks 5 to 15 for IOWAc. A deeper analysis of this operator reveals that using the importance weights V as order inducing variable leads to strange results in some cases.

Correlations table (Table 11) shows that in this case study WA and OWA are initially highly correlated, therefore their combination also leads to high correlation values in most methods. WOWAd is the one that differentiates a bit from the rest. IOWAc is suprisingly similar to OWAd.

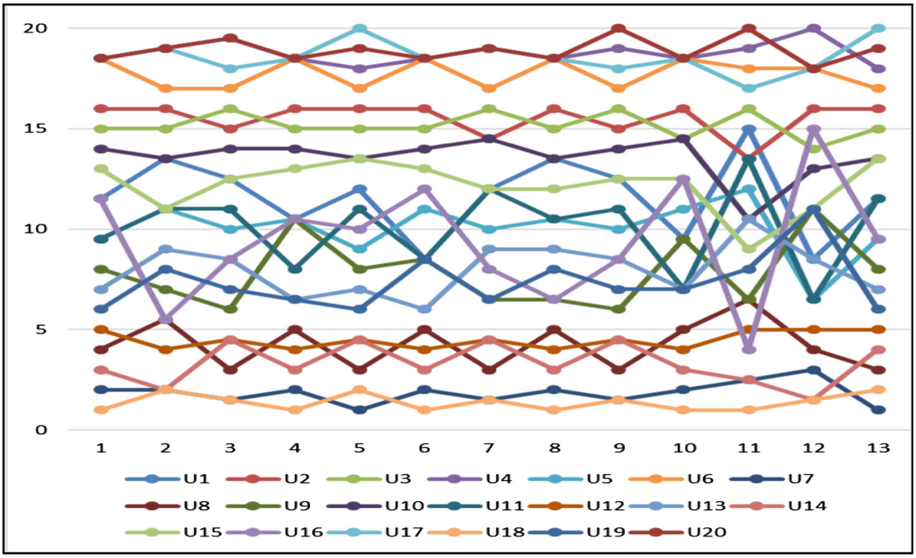

Fig. 1. Rank positions of the 20 universities values with weighted average (WA)

Table 11. Correlation between the different rankings obtained in dataset Universities

	WA	OWAd	OWAc	Closest (>=0.99)
WA	**1,00**	**0,95**	**0,98**	
OWAd	**0,95**	**1,00**	**0,98**	
OWAc	**0,98**	**0,98**	**1,00**	
OWAWA.5d	0,98	0,93	0,95	OWAWA.3d, IOWAd
OWAWA.5c	0,97	0,98	1,00	OWAWA.7c
IOWAd	0,99	0,92	0,95	OWAWA.3d, OWAWA.5d
IOWAc	0,88	0,97	0,93	-
WOWAd	0,93	0,86	0,88	-
WOWAc	0,99	0,96	0,98	OWAWA.5c

6 Conclusions and Future Work

This work presents a new approach to the aggregation of partial concordances in the ELECTRE outranking method. Using weighted averaging may sometimes have an undesired compensative effect between opposite values, as such we look to a family of OWA operators to avoid this effect. OWAWA, WOWA and IOWA which combine WA and OWA may substitute WA, introducing a new way of weighting values.

In the tests we observed that the results of the 3 approaches are different as they model the combination in different ways. An undesired behaviour has been seen in the IOWA conjunctive operator. If a low partial concordance is given by criteria with high importance weight, they will be placed in the first positions during aggregation, so they will receive a low W weight and their contribution is minimized. This seems to go in contrary to common sense. Moreover, WOWA seems to give a significantly different ranking to all the three basic ones. It may indicate that it really combines the information of the two sets of weights in a more suitable way.

Future work concerns the study of the best scenarios for each of these operators. The behaviour of other OWA policies (e.g. Olympic, Balanced) [9] will be studied. Finally, a characterisation of the properties of these operators should be investigated.

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