Design of Cam Mechanisms with Swinging Roller Follower: The Modern Algorithm-Based Approach

Lyudmilla Chernaya(✉), Andrei Vukolov, and Il'ya Safronoff

Bauman Moscow State Technical University, Moscow, Russian Federation
chernayala@yandex.ru, twdragon@bmstu.ru

Abstract. Modern mechanical engineering and design of the mechanisms and machines are closely connected with development of numerical methods and algorithms. Education is the useful target to apply these methods. Development of the analytical and algorithmic competences is possible only using high technologies through whole course. The modern numerical method on the metric synthesis of cam mechanism is demonstrated in this paper using new algorithmical approach and `MathCAD` realization.

Keywords: `MathCAD` · Cam mechanism · Swinging follower
Numerical methods · Numerical modeling · Roller follower

1 Introduction

In Bauman Moscow State Technical University the generalized methods of design and exploration of machines are expounded within Theory of Mechanisms and Machines (TMM) course [2,12]. In the past the approximate graphical methods [19,20] were mostly used. Now the available calculation resources made it possible to use almost any method [3], including numerical and analytical [16].

The followers of cam mechanisms move under strictly determined trajectory [11] and motion laws [4]. They require precise calculation of the profile's coordinates for successful manufacturing [17,18]. The modern calculation software products like `MathCAD` [14,15] provide such capabilities. They are easy to use, so development and realization of new approaches and algorithms are good challenges for engineering students [7,10]. Here and below the typical cam mechanism with swinging follower and four-phased (rise, return and two dwellings) cycle will being considered as an example.

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2 Target Setting

The mechanism (Fig. 1*a*) is constrained with following parameters:

- Maximal displacement h_B of the follower's contact point B which determines rising angle β;
- Length l_2 of the rocker 2;
- Phase angles: rise angle ϕ_{1y}, return angle ϕ_{1c} and dwelling angle $\phi_{1\partial}$;
- Motion law of the follower defined graphically;
- Upper limit of the pressure angle $[\vartheta]$;
- Rotation direction defined with sign of the ω_1 value.

The whole task of designing of the cam mechanism can be splitted into three stages [8]:

1. Building of kinematical diagrams and calculating of geometrical characteristics;
2. Metrical synthesis constrained by $[\vartheta]$;
3. Kinematical synthesis—building of theoretical cam profile and envelope surface.

The whole motion law of the follower for the considered mechanism is presented on Fig. 1*b*. The motion law here defined with coefficients a_1, a_2, a_3, a_4, scales μ and ratio of their values.

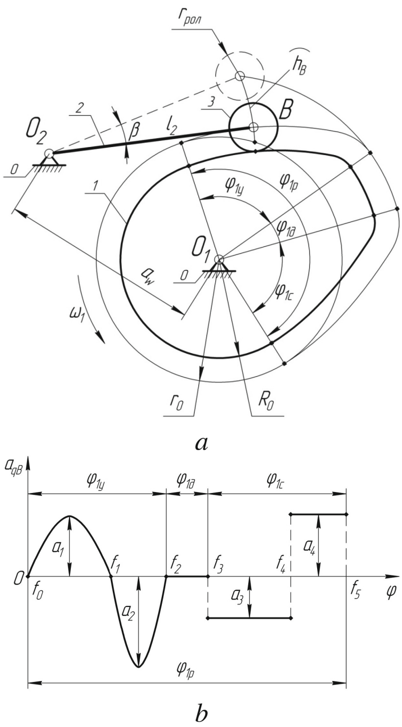


Fig. 1. Cam mechanism and follower's motion law

3 Kinematical Diagrams

According to Fig. 1*b* the acceleration analog function a_{qB} has discontinuity points of type I at f_3, f_4 and f_5. To satisfy requirements of algorithmical approach the Haeviside function $\Phi(x)$ [9] used to determine discontinuities. Now the following expressions can be written for the followers's motion law on the rising phase:

$$a_{qBy}(\varphi) = \begin{cases} \frac{\varphi}{\varphi_1} \sin\left(\pi, \frac{\varphi}{f_1}\right), \text{ where } 0 \le \varphi < f_1 \\ \frac{-\pi}{f_2 - f_1} \sin\left[\pi \frac{f_2 - \varphi}{f_2 - f_1}\right], \text{ where } f_1 \le \varphi \le f_2 \\ 0, \text{ where } f_2 < \varphi \le f_3 \end{cases} \quad (1)$$

... and on returning phase where $f_3 < \varphi \le f_5$ (Fig. 1*b*):

$$a_{qBc}(\varphi) = [-a_3 \cdot \Phi(\varphi - f_3) + (a_3 + a_4) \cdot \Phi(\varphi - f_4) - a_4 \cdot \Phi(\varphi - f_5)] \qquad (2)$$

Now the kinematical characteristics: speed analog function $v_{qB}(\varphi)$ and follower displacement function $S_B(\varphi)$ could be obtained.

4 Metric Synthesis

The main constraint of the metric synthesis task is the pressure angle limitation $[\vartheta]$. The formalities for algorithmical approach are the following:

1. All schemes considered as they are built in right Cartesian coordinates S_{xOy} with origin placed into the fixed point O_1 of the cam (Fig. 1*a*).
2. Rotation angle φ of the cam directed conterclockwise is generalized coordinate;
3. Angular velocity analog function ω_{q1} of the cam determines rotation direction (1 for counterclockwise direction).

Let the cam 1 (Fig. 1*a*) rotate and the follower 2 moves by arbitrary trajectory and vector $\overline{v_B} = \overline{v_2}$ of the contact point B's absolute speed is known. Let the Y axis of the S_{xOy} coordinate system with origin in O_1 is parallel to $\overline{v_2}$. Now the pressure angle can be defined as the following function:

$$\tan\vartheta(\varphi) = \frac{v_{qB}(\varphi) - x_B(\varphi)}{y_B(\varphi)} \qquad (3)$$

Now we reinterpret (3) for case of swinging follower:

$$\tan\vartheta(\varphi) = \frac{\omega_{q1} \cdot v_{qB}(\varphi) - l_2 + a_w \cos(\varphi_{20} + \varphi_2(\varphi))}{a_w \sin(\varphi_{20} + \varphi_2(\varphi))} \qquad (4)$$

where

$$\cos\varphi_{20} = \frac{l_2^2 + a_w^2 - r_0^2}{2l_0 \cdot a_w}$$

and φ_{20} is initial angle [5] between l_2 and O_1O_2 line at the lower dwelling phase (Fig. 1*a*).

With known dependence between ϑ and φ the phase diagram $S_B[v_{qB}(\varphi)]$ can be built [17] in right Cartesian coordinate system S_{xAy} with origin in A point which coincides with center O_2 of the rocker (Fig. 1). On the diagram (Fig. 2) the O_1 and O_2 points demonstrate possible positions of the cam center [5]. The metric synthesis of the cam mechanism includes obtaining values for a_w and r_0 parameters with known l_2 and $[\vartheta]$. With known direction of rotation of the cam the upper limit of the pressure angle [6,12] can be determined on rising phase $[\vartheta_y]$, returning phase $[\vartheta_c]$ or on both phases for reversible mechanism. Now the lines defined by $[\vartheta]$ have to be determined in S_{xAy}. If coordinates of i-point of the diagram are X_i, Y_i the line passing this point is defined as

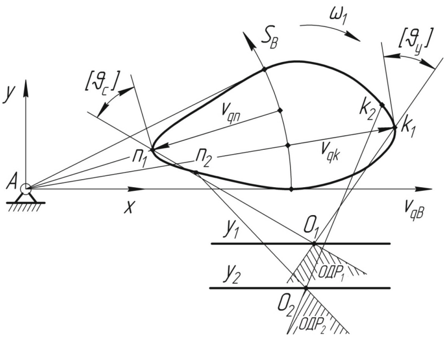


Fig. 2. Phase diagram

$Y_i = k_i \cdot X_i + b_i$ equation. The extreme position of such line is determined on the rising phase with angle $\varphi_2(\varphi) + \pi/2 - [\vartheta_y]$ and on returning phase with angle $\varphi_2(\varphi) + \pi/2 + [\vartheta_c]$. Now:

$$\begin{aligned} k_{yi} &= \tan\left(\varphi_2(\varphi) + \pi/2 + \omega_{q1}\,[\vartheta]\right) \\ k_{ci} &= \tan\left(\varphi_2(\varphi) + \pi/2 - \omega_{q1}\,[\vartheta]\right) \end{aligned} \tag{5}$$

The intersection point O_i between lines on Fig. 2 is defined by equation $k_y \cdot X + b_y = k_c \cdot Y + b_c$. After substitutions we have:

$$\begin{aligned} X &= -\frac{b_c - b_y}{k_c - k_y} \\ Y &= \frac{k_c b_y - k_y b_c}{k_c - k_y} \end{aligned} \tag{6}$$

Now the point O_1 with coordinates X_1, Y_1 is placed where the lines defined by (6), maximal and minimal values of $v_{qB}(\varphi)$ intersect. For each point of the diagram the line could be drawn through O_1:

$$X_i = \frac{Y_1 - b_i}{k_i} - X_1 \tag{7}$$

Now the function $f(x) = X_i(\varphi)$ can be obtained and the lines which constrain the zone of possible location for the cam center are placed above Ax axis for rising phase and below it for returning. The algorithm of cam center point calculation is now determined within the three stages:

- Extremum analysis of the function $X_i(\varphi)$ for coordinates φ_y and φ_c of the most distant lines (Fig. 2);
- Substitution of φ_y and φ_c into (6) for X_2 and Y_2. Now the levels y_1 and y_2 can be placed;
- Obtaining of the pressure angle function $\vartheta(\varphi)$ from (4).

Realization of the considered approach were performed by I. Safronoff using `MathCAD`. Here the fragment of the source code[1] is presented:

$$k_{Vi}(\phi) := \begin{cases} \tan\left(\varphi_2(\phi) + \frac{\pi}{2} + \theta d \cdot \omega_{qk}\right) & \text{if } f0 \le \phi \le f2 \\ \tan\left(\varphi_2(\phi) + \frac{\pi}{2} - \theta d \cdot \omega_{qk}\right) & \text{if } f3 \le \phi \le f5 \\ 0 & \text{otherwise} \end{cases}$$

$$b_{.Vi}(\phi) := Ny(\phi) - k_{.Vi}(\phi) \cdot Nx(\phi)$$

Initial approximations: left extremum

$$\phi_{Vmax} := 100 \cdot deg$$

Given $\quad X_{Vi}(\phi_{Vmax}) = 0.1 \qquad 60 \cdot deg < \phi_{Vmax} < 130 \cdot deg$

$$\varphi_{Vmax} := \text{Minerr}(\phi_{Vmax}) = 100.8 \cdot deg \qquad X_{Vi}(\varphi_{Vmax}) = 3.688 \times 10^{-4}$$

Right extremum:

$$\phi_{Vmin} := 275 \cdot deg$$

Given $\quad X_{Vi}(\phi_{Vmin}) = -0.1 \qquad 250 \cdot deg < \phi_{Vmin} < 300 \cdot deg$

$$\varphi_{Vmin} := \text{Minerr}(\phi_{Vmin}) = 281.933 \cdot deg \qquad X_{Vi}(\varphi_{Vmin}) = -9.843 \times 10^{-3}$$

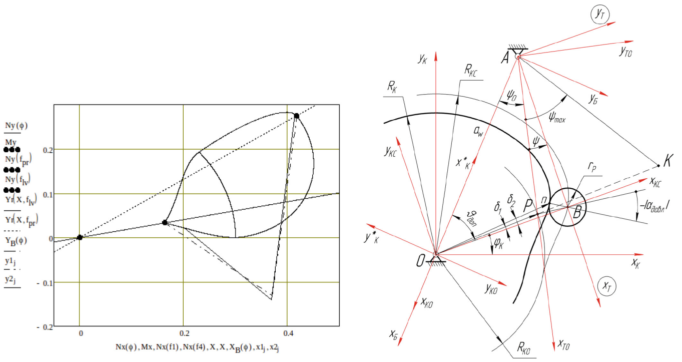
Intersection point:

$$X_2 := -\left[\frac{b_{Vi}(\varphi_{Vmax}) - b_{Vi}(\varphi_{Vmin})}{(k_{Vi}(\varphi_{Vmax}) - k_{Vi}(\varphi_{Vmin}))}\right] \qquad Y_2 := \frac{k_{Vi}(\varphi_{Vmax}) \cdot b_{Vi}(\varphi_{Vmin}) - k_{Vi}(\varphi_{Vmin}) \cdot b_{Vi}(\varphi_{Vmax})}{k_{Vi}(\varphi_{Vmax}) - k_{Vi}(\varphi_{Vmin})}$$

a_w and R_{min}:

$$a_w := \sqrt{X_2^2 + Y_2^2} = 0.395 \qquad R_{min} := \sqrt{(Xk - l_t)^2 + Yk^2}$$

Phase diagram obtained from `MathCAD` is presented on Fig. 3*a*. The coordinate system described in Sect. 4 is presented on Fig. 3*b* as $x_K O y_K$ axis. The coordinate



a *b*

Fig. 3. Phase diagram obtained using MathCAD and cam profile with coordinate systems applied

[1] Here and below all `MathCAD` source code listings are written by L. Chernaya and I. Safronoff.

system $x_T A y_T$ is connected to the follower. Angle α_∂ represents pressure angle ϑ from Fig. 2. From phase diagram the minimal cam radius R_{min} now can be obtained.

5 Kinematical Synthesis

Matrix equation of coordinate transform between cam and follower (Fig. 3) is presented below. The transition performs over coordinate systems $S_{x_T A y_T}$ and $S_{x_{KC} A y_{KC}}$:

$$[A_{KcT}(\varphi)] = [A_{KcK}] \cdot [A_{KK^*}(\varphi)] \cdot [A_{K^*K_0}] \cdot [A_{K_0B}]\,[A_{BT}(\varphi)] \tag{8}$$

The final equation of theoretical cam profile with argument φ looks as:

$$K_{cp}(\varphi) = [A_{KcT}(\varphi)] \cdot \begin{pmatrix} l_2 \\ 0 \\ 1 \end{pmatrix} \tag{9}$$

where l_2 is rocker length (Fig. 1*a*). `MathCAD` realization of matrix calculations described here is obtained by the following code for initial circle R_0:

$$X_{k.0}(\varphi) := R_{min} \cdot \cos(\varphi + \omega_{qk} \cdot \varphi_k) \qquad Y_{k.0}(\varphi) := R_{min} \cdot \sin(\varphi + \omega_{qk} \cdot \varphi_k)$$

theoretical cam profile:

$$\psi_\Sigma(\varphi) := \psi 0 + \phi_2(\varphi) \qquad \varphi_{dop} := \operatorname{acos}\left(\frac{a_w^2 + R_{min}^2 - l_t^2}{2 \cdot a_w \cdot R_{min}}\right) = 42.854\ \mathrm{deg}$$

$$A_{BT}(\varphi) := \begin{pmatrix} \cos(\psi 0 + \phi_2(\varphi)) & -\sin(\psi 0 + \phi_2(\varphi)) & 0 \\ \sin(\psi 0 + \phi_2(\varphi)) & \cos(\psi 0 + \phi_2(\varphi)) & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad A_{KxKo} := \begin{pmatrix} 1 & 0 & a_w \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad A_{KoB} := \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A_{KKx}(\varphi) := \begin{pmatrix} \cos(\varphi + \varphi_{dop}) & -\sin(\varphi + \varphi_{dop}) & 0 \\ \sin(\varphi + \varphi_{dop}) & \cos(\varphi + \varphi_{dop}) & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad A_{KcK} := \begin{pmatrix} \cos(\varphi_k) & -\omega_{qk} \cdot \sin(\varphi_k) & 0 \\ \omega_{qk} \cdot \sin(\varphi_k) & \cos(\varphi_k) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$TR_C(\varphi) := A_{KcK} \cdot A_{KKx}(\varphi) \cdot A_{KxKo} \cdot A_{KoB} \cdot A_{BT}(\varphi) \qquad r_{rol} := 0.3 \cdot R_{min} = 0.047$$

$$REZ_C(\varphi) := TR_C(\varphi) \cdot \begin{pmatrix} l_t \\ 0 \\ 1 \end{pmatrix} \qquad X_{kCM}(\varphi) := REZ_C(\varphi)_0 \qquad Y_{kCM}(\varphi) := REZ_C(\varphi)_1$$

...and for the envelope curve:

$$TURN_v(\varphi) := \begin{pmatrix} \cos(\theta_{dav}(\varphi)) & -\sin(\theta_{dav}(\varphi)) & 0 \\ \sin(\theta_{dav}(\varphi)) & \cos(\theta_{dav}(\varphi)) & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad SHIFT_L := \begin{pmatrix} 1 & 0 & l_t \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad TR(\varphi) := SHIFT_L \cdot TURN_v(\varphi)$$

The final cam profiles calculated and drawn in `MathCAD` are presented on Fig. 4. The dotted lines represent initial circle R_{K0} and theoretical profile from Fig. 3. The solid ones represent envelope curve (manufacturing-ready profile) and minimal radius R_K. The roller radius r_p is fixed [11]. The algorithm developed by

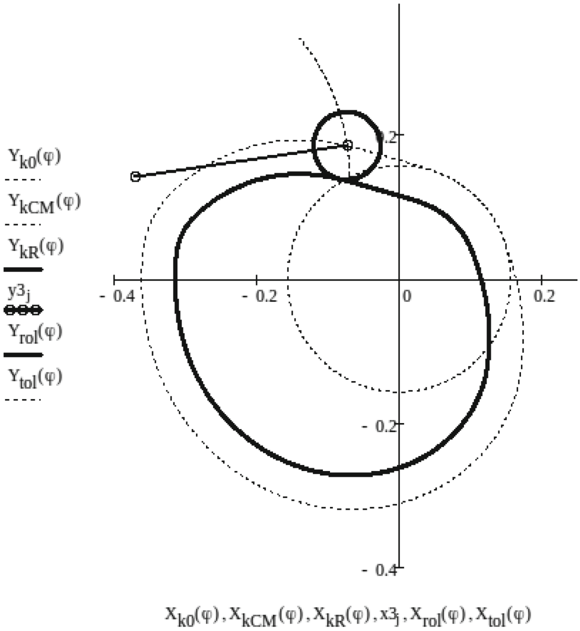


Fig. 4. Cam profiles drawn by MathCAD

I. Safronoff allows to easily connect the numerical methods which `MathCAD` implements to both highly-constrained calculation task of the metric synthesis and geometrical task of profiling.

6 Conclusion

The algorithm-based approach demonstrated above allows the student to use his skills of software development and toolchain building. In Bauman University it links traditional design with new techniques. Theoretical knowledge of the theory of mechanisms and machines provided with lectures could be successfully applied by student using continuous integration with modern software and technologies [1]. The case of `MathCAD` usage quoted in this paper allows to export data into simple format supported by manufacturing solutions using available built-in functions. In the modern system of engineering education the scripts, datasheets and programs developed by students themselves is the way to provide possibility to obtain industrial experience within training. Also the networks and Web-based programming techniques [13] allow university to build open educational space with free distributed workflow.

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