Chapter 6
The Platform Formation Problem

David Ben-Arieh[1]

David Ben-Arieh
Dept. of IMSE
Kansas State University
Manhattan, KS 66503, USA
davidbe@ksu.edu

Abstract Today's globally competitive world of manufacturing requires participating firms to introduce an increasing number of products with shorter life span, at a lower cost, in an environment where demands are uncertain and with shorter lead times to fulfill those demands. One approach towards meeting these demands is the use of mass customization, specifically the platform based design and production strategy. This chapter presents the platform design problem in which a platform is created with the objective of producing a family of products at a minimum cost. By using the platform every product variant in the family is assembled either directly from its components or from the platform. Three methods for developing such a platform-based strategy are described: design of a single platform, design of multiple platforms, and design of a single platform while considering demand uncertainty.

Abbreviations

BOM	Bill-of-materials
EVPI	Expected value of perfect information
OPL	Optimization programming language
PAR	Part assembly relationship
QFD	Quality function deployment
VSS	Value of the stochastic solution

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[1] Dr. David Ben-Arieh is Professor of Industrial Engineering at Kansas State University, Department of Industrial and Manufacturing Systems Engineering. His research interests include production systems, group decision making, and fuzzy set applications. Dr. Ben-Arieh has worked extensively on product development methodology and holds one patent in this area. His industrial experience includes working for AT&T Bell Laboratories and consulting for the aerospace industry and NASA.

6.1 Introduction

In today's highly volatile market there is a growing concern for fulfilling the individual customer wants and needs. "The customers now have plenty of choice … they have become more aware … they select the product that most closely fulfills their opinion of being the best value for the money …" (Hollins and Pugh 1990). Therefore, "customers can no longer be lumped together in a huge homogeneous market …" (Pine 1993); rather this competitive world of manufacturing requires the manufacturer to introduce an increasing number of products with shorter life span and at a lower cost. This requires the producer to continuously search ways to reduce production costs, while still offering attractive products. In the past, a company could capture the market and enjoy high profits by mass-producing a large volume of the same model. Now, the focus in manufacturing is shifting from mass production to mass customization; a trend no longer limited to high value products. This phenomenon is demonstrated by the fact that from 1973 to 1989, there was a 70% increase in the number of car models produced in the US with a commensurate drop in the volume of production *per* model (McDuffie *et al.* 1996). It is thus important to note that there is a distinction between supporting variety and supporting customization as discussed in Simpson (2004), with the platform technology able to support both concepts.

Toward this end, various strategies have received significant attention in the literature and practice including, but not limited to, the use of the concept of delayed differentiation (Lee 1996, Lee and Tang 1997, Swaminathan and Tayur 1998), exploiting commonality at the product design state (Ulrich and Pearson 1993), the use of lean manufacturing concepts (Womack *et al.* 1990), and the product platform strategy (Meyer and Lehnerd 1997).

Due to its advantages, the platform approach has gained acceptance by many corporations as the means to increase their product count without increasing the cost *per* product. Examples of industrial applications of the platform concept include Black and Decker, which applied this idea to its power tool products (Meyer and Lehnerd 1997). Volkswagen used a platform architecture strategy and reduced development and production costs (Wilhelm 1997). Sony applied this approach to its product development process (Sanderson and Uzumeri 1995). AeroAstro Inc. used platform architecture with their multipurpose radio platform and solved many of the communication problems faced by spacecraft system designers (Caffrey *et al.* 2002). HP's Ink Jet Printer platform architecture is rejuvenated constantly and hence the derivative products are constantly upgraded (Meyer 1997). Other examples of manufacturers that have successfully implemented platform based production strategy include Rolls Royce (Rothwell and Gardiner 1990), Boeing (Sabbagh 1996), and Honda (Naughton et al. 1997).

In this chapter, we discuss a platform based approach for the production of a product family. Using this approach, every product variant in the family may either be assembled directly from its components, or from any platform whose component set resembles those required by the product. The methodology seeks to find an optimal platform that will minimize the overall production costs of the products, which include the costs of production, holding cost of unused platform inventory and shortage cost of lost demands of products (if demand is stochastic), while considering the demand of each product type.

The rest of the chapter is organized as follows: Section 6.2 offers a review of the work related to the platform formation problem. Section 6.3 defines the problem along with the description and formulation of three variants: a single platform system, multiple platform solution, and a single platform problem considering demand uncertainty. Section 6.4 presents an example of the stochastic demand case and Section 6.5 provides the conclusion and some directions for future research.

6.2 Background

Ulrich and Eppinger (2003) define a *platform* as a collection of assets, including component designs, shared by multiple products. It can also be defined as a set of shared functionality, components, subsystems, and manufacturing processes across the product family (Robertson and Ulrich 1998).

Various streams of research in the area of product platforms were greatly influenced by contributions from Pine (1993) in the area of mass customization, Meyer and Lehnerd (1997) in the area of platform concepts, and Sanderson and Uzmeri (1997) in the area of managing product families (Allada and Jiang 2002). Krishnan and Ulrich (2001) provide a literature review of the various decisions that take place during the product realization process including when and how to construct a platform architecture.

Simpson (2004) provides an authoritative definition of the platform as a supporting tool for family based production. He also provides metrics and strategies to support platform based production as well as a review of optimization based approaches for platform design. The author clearly distinguishes between variety and customization, but states that “product platforms play an integral role in facilitating the product customization process” while highlighting web based approaches towards platform based customization.

Jose and Tollenaere (2005) provide a literature review of approaches towards platform design. They describe the concept of standardization and modularization as well as various product architectures that support modularity. Allada *et al.* (2006) provide a review of various problem types related to tactical and strategic platform development as well as a review of platform evaluation techniques. Simpson *et al.* (2006) provide an overview of the platform concept, application areas, and ongoing research in academia and in industry.

Research work on qualitative approaches to the platform problem include Maier and Fadel (2001), Dahmus *et al.* (2001), Shil and Allada (2005), and Wilson and Norton (1989). Such approaches can be exemplified by the work of Martin and Ishii (1997, 2002) who developed a conceptual approach towards developing platform architectures utilizing the quality function deployment (QFD) methodology and describing the design-for-variety approach. Similarly, Kota *et al.* (2000), and Park and Simpson (2005) developed methods to assess the design commonality of a product family and the cost of its production.

Platform development is considered a costly endeavor recovered through consumer willingness to pay for the features provided by the platform. An analysis of platform development cost is provided by Krishnan and Gupta (2001). Similarly an analysis of the optimal set of product configurations termed optimal diversity management problem is addressed by Briant and Naddef (2004). A more engineering based approach towards optimal design of platform features is described in Nelson *et al.* (2001), while a more conceptual description of the platform design process is available in Gonzalez-Zugasti *et al.* (2000).

Various quantitative solution methods to the platform optimization problem looked into finding the optimal design parameters that will satisfy the overall function requirements of the product family. These methods include (but are not limited to) the branch and bound algorithm (Fujita and Yoshida 2001), dynamic programming (Allada and Jiang 2002), agent based techniques (Rai and Allada, 2003), simulated annealing (Fujita *et al.* 1999), and genetic algorithms (Li and Azaram 2002, Simpson and D'Souza 2002, 2004). Similarly, Jiao and Zhang (2005) developed an optimization based approach towards allocating product attributes to a product portfolio considering the consumer utility and preferences, and engineering costs and product life cycle. Clearly the overall analysis of a platform based design can be overwhelming considering issues of component design, performance, and quality, as well as suppliers' management, product life cycle, and demand. Practically, due to its complexity the problem is decomposed into smaller segments – one of which is addressed in this chapter.

Platform based architecture is often utilized towards mass customization and can be defined as "building products to customer specifications using modular components to achieve economies of scale" (Durray *et al.* 2000). Some architectures of mass customization emphasize maximizing commonality in design across internal modules, using a product platform with modules as building blocks (Jiao and Tseng 1999). Such an approach utilizes three views of the product: functional, technical, and physical. Mapping between the technical and the physical views implies considering manufacturing and logistics, important aspects addressed in this chapter. The modular structure and technical modules are realized using physical modules as components and assemblies. This arrangement is similar to the typical bill-of-materials (BOM) – since many products can share the same modules, resulting in a polyhierarchical graph (as suggested in this chapter).

Ross (1996) defines five approaches for utilizing the customer voice towards mass customization. At one end the customer can modify core elements in the product while at the other extreme, known as the "high variety push" the manufacturer provides a high variety of pre-designed products. In addition, MacCarthy *et al.* (2003) identify six processes that are essential to mass customization, one of which is "product validation and manufacturing engineering", which is responsible for generating the manufacturing processes and the bill of materials. The methods presented herein fit into the above mentioned approaches.

The platform approach towards product design

Utilizing platforms to assist in product design can be implemented using three modes:

1. scalable platform formation;
2. module based or configuration based platform formation; and
3. combination of both module based and scalable platform formation.

Scale based product family design is a method by which some of the variables in a product family are kept fixed while other variables such as scaling variables, are "stretched" or "reduced" to generate the variants within the product family. Module based product family design is a method by which a product family member is derived by adding and/or removing modules from the platform. This approach, based on the concept of modularity in product design, is more prominent in practice as it allows the platform to leverage for products from different market segments (Baldwin and Clark 1997, Ulrich and Eppinger 2003).

A combination of both module based and scale based platform formation strategies is considered by Fujita and Yoshida (2001).

Optimization based platform formation methods for various objectives

The platform design and selection concept have been used for various objectives such as reducing cost and simplifying the design effort (Simpson 2004), improving life-cycle design (Ortega *et al.* 1999), optimizing production cost or profit, or reducing time to market (Krishnan and Ulrich 2001). Martin and Ishii (1997) proposed methodologies that can help companies to quantify the costs of providing variety and qualitatively guiding designers in developing products that incur in minimum variety costs. Simpson *et al.* (1999) proposed a model that uses the overall design requirements in generating the product platform and resulting product family that best satisfies the overall design requirements. Farrell and Simpson (2001) try to improve response to customers' requests, reduce design cost, and improve time to market for highly customized products by designing product platforms. Sudjitanto and Otto (2001) use a matrix to group modules for platform determination in order to support multiple brands for platform cost savings as well as revenue enhancing. Nayak *et al.* (2002) propose a variation based method for product family design, which aims to satisfy the range of performance requirements for the whole product family.

6.3 Problem Description

In this section a platform is considered to be a set of shared components among multiple products. A product from a product family is produced using a platform by adding or removing some of the components that are assembled using the platform. Figure 6.1 illustrates a hypothetical product family with four products (P_1,

P_2, P_3, and P_4), each consisting of a different collection of components from the set $\{A, B, C, \ldots, H\}$. Suppose a platform for this set of products is as shown in Figure 6.2. In this case P_1 would be created by using the platform, removing G and adding C, and P_3 would be created by removing D and G and adding C and F.

A platform is only justified if the assembly of the components to the platform can be done efficiently using mass production methods. The platform is not a super-set of all the products in the family, for some products parts will be added to the platform while for other products parts will have to be removed. Thus, adding and removing components from a platform to fit a particular product typically cost more than if the component is included in the platform (*via* mass production) and remains there to be used in the product. However, if the component is not required for a particular product, it can be removed and used in a different product.

Each product's bill of materials is considered to be binary. One complicating factor is that while determining the configuration of the platform, the part family relationship must be maintained.

In order to manage the part assembly relationship constraints, a binary part assembly relationship (PAR) matrix for the product family is determined. An element of PAR, $f_{jl} = 1$ represents that component j precedes component l or component j is needed to be present in the platform for l to be included in it, as component l requires j to be assembled to form a platform. As an example, the PAR for the product family shown in Figure 6.1 is shown in Table 6.1.

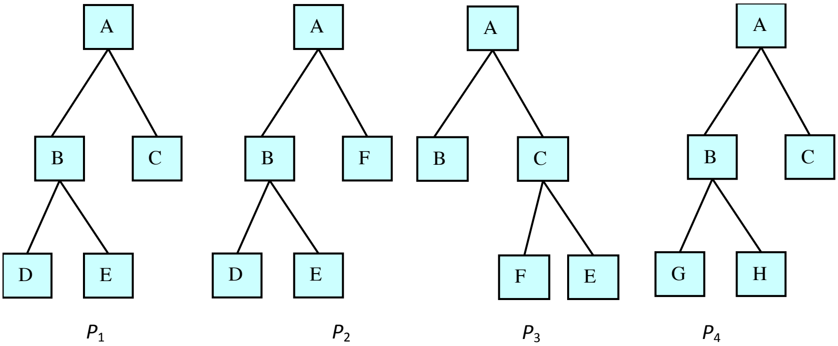

Figure 6.1 Example of a product family

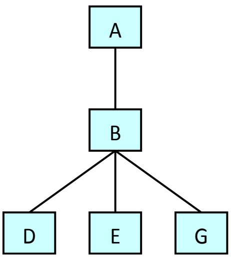

Figure 6.2 A platform for the product family

Table 6.1 PAR for product family in Figure 6.1

	A	B	C	D	E	F	G	H
A		**1**	**1**			**1**		
B				**1**	**1**		**1**	**1**
C					**1**	**1**		
D								
E								
F								
G								
H								

Notations:

K is the set of products in a given product family, $k \in K = \{1,2, \ldots, |K|\}$.
J is the component set, $j \in J = \{1,2, \ldots, |J|\}$.
I represents the platforms, $i \in I = \{1,2, \ldots, |I|\}$; $i = 1$ for the single platform case.
D_k is the demand for the kth product.
C_j is the cost of the jth component (purchasing price).
CP_j is the cost of assembling the j^th component using a platform (mass assembly).
CA_j is the cost of manually adding the jth component to a product ($CA_j > CP_j$).
CR_j is the cost of manually removing the jth component ($CR_j > CP_j$).

V is the product matrix with

$$v_{jk} = \begin{cases} 1 \text{ if product } k \text{ requires component } j \\ \quad 0 \text{ otherwise} \end{cases}$$

f_{ljk} are elements in the PAR such that

$$f_{jlk} = \begin{cases} 1 \text{ if component } j \text{ precedes } l \text{ in product } k \\ \quad 0 \text{ otherwise} \end{cases}$$

A_i is the setup cost to construct platform i.

Decision variables for this model are as follows:

X is a matrix with binary entries describing components included in the platforms, such that:

$$x_{ij} = \begin{cases} 1 \text{ if platform } i \text{ contains component } j \\ \quad 0 \text{ otherwise} \end{cases}$$

Y is a binary matrix that states that product k is made on platform I, with elements:

$$y_{ki} = \begin{cases} 1 \text{ if the product } k \text{ is made using platform } i \\ \quad 0 \text{ otherwise} \end{cases}$$

The following variables are also used:

$$a_{ijk} = \begin{cases} 1 \text{ if the } j^{th} \text{ component is added manually to platform } i \text{ to form product } k \\ 0 \text{ otherwise} \end{cases}$$

$$r_{ijk} = \begin{cases} 1 \text{ if the } j^{th} \text{ component is removed manually from platform } i \text{ to form product } k \\ 0 \text{ otherwise} \end{cases}$$

6.3.1 The Single Platform Design Formulation

Minimize

$$\left(\sum_{j=1}^{|J|} \sum_{k=1}^{|K|} \left(CP_j + C_j \right) D_K X_{1j} \right) + \left(\sum_{j=1}^{|J|} \sum_{k=1}^{|K|} \left(CA_j + C_j \right) D_K a_{1jk} \right) + \left(\sum_{j=1}^{|J|} \sum_{k=1}^{|K|} \left(CR_j - C_j \right) D_k r_{1jk} \right) \tag{6.1}$$

Subject to:

$$a_{1j,k} = (1 - X_{1J}) v_{j,k} \qquad \forall j,k \tag{6.2}$$

$$r_{1,j,k} \le (1 - v_{j,k}) X_{1j} \qquad \forall j,k \tag{6.3}$$

$$X_{1j}, a_{1,j,k}, r_{1,j,k} = \{0,1\} \tag{6.4}$$

The objective (6.1) is to minimize the total production cost, which includes the cost of mass assembly (cost of producing platforms) and the cost of the components (I), the cost of manually adding components to the platform to produce the products (II), and the cost of removing components from the platforms (III) (with allowance to reuse the components). The constraint in (6.2) ensures that a component is added to the platform only if it is required in the product and not present in the platform. The constraint in (6.3) ensures that a component may be removed from the platform only if it is in the platform and it is not required in the product.

6.3.2 The Multiple Platform Problem

Minimize

$$\sum_{i \in I} \sum_{j \in J} \sum_{k \in K} (CP_j + C_j) \cdot x_{ij} \cdot y_{ik} \cdot D_k + \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} (CA_j + C_j) \cdot a_{ijk} \cdot y_{ik} \cdot D_k + \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} (CR_j - C_j) \cdot r_{ijk} \cdot y_{ik} \cdot D_k + \sum_{i \in I} A_i \tag{6.5}$$

Subject to:

$$a_{ijk} = (1 - x_{ij}) \cdot v_{jk} \cdot y_{ki} \qquad \forall i \in I; j \in J; k \in K \tag{6.6}$$

$$r_{ijk} \le (1 - v_{jk}) \cdot x_{ij} \cdot y_{ki} \qquad \forall i \in I; j \in I; k \cup K \tag{6.7}$$

$$\sum_{i=1}^{|I|} y_{ki} = 1 \qquad \forall k \in K \tag{6.8}$$

$$x_{ij} \ge f_{jlk} y_{ki} x_{il} \qquad \forall i \in I; j, l \in J; k \in K \tag{6.9}$$

$$I \in \{1, 2, \ldots, K\} \tag{6.10}$$

$$x_{ij} \in \{0,1\}; y_{ki} \in \{0,1\}; a_{ijk} \in \{0,1\}; r_{ijk} \in \{0,1\} \tag{6.11}$$

The objective (6.5) minimizes the cost, which includes the setup cost for each platform, the optimal set of components to include in each platform, and the optimal assignment of products to platforms. The first term in the objective function represents the cost of production via platforms. The second term represents the cost of adding components manually to the various platforms to form different products. The third term represents the cost of manually removing (and allowing reutilization) excessive components from the platforms to form each product. The last term represents the setup cost of constructing the platforms.

The constraint in (6.6) restricts component *j* to be added to platform *i* to make product *k* only if the component is not already in that platform; thus, component *j* is required for product *k*, and product *k* is assigned to platform *i*. The constraint in (6.7) states that component *j* may be removed from platform *i* if that component is not required in product *k*; thus, the component is assigned to platform *i* and product *k* is built from that platform. The constraint in (6.8) ensures that each product is made from only one platform. The constraint in (6.9) checks the assembly feasibility of each product that uses a platform so that if component *l* precedes component *j* in a product *k* assigned to the platform, and component *l* is assigned to the platform, then component *j* must be in the platform. The constraint in (6.10) states that the optimal number of platforms is an integer, and that the maximum number of platforms is limited by the total number of the products in the family. Finally, the constraint in (6.11) ensures binary decision variables.

6.3.2.1 Improving the Formulation

In the formulation in Section 6.3.2, constraints in (6.6), (6.7), and (6.9) are nonlinear, which makes selecting a solution procedure difficult, at best. The following changes are made to those constraints in order to attain a linear formulation.

Subject to:

$$a_{ijk} + x_{ij} \le 1 \qquad \forall i \in I; j \in J; k \in K \tag{6.12}$$

$$a_{ijk} + x_{ij} \ge v_{jk} \qquad \forall i \in I; j \in J; k \in K \tag{6.13}$$

$$v_{jk} \ge a_{ijk} \qquad \forall i \in I; j \in J; k \in K \tag{6.14}$$

$$x_{ij} \ge r_{ijk} \qquad \forall i \in I; j \in J; k \in K \tag{6.15}$$

$$r_{ijk} + x_{ij} + v_{jk} \le 2 \qquad \forall i \in I; j \in J; k \in K \tag{6.16}$$

$$1 + x_{ij} \ge f_{jlk} \cdot y_{ik} + x_{il} \qquad \forall i \in I; j,l \in J; k \in K \tag{6.17}$$

(6.12)–(6.14) replace the nonlinear constraint in (6.6), (6.15)–(6.16) replace the nonlinear constraint in (6.7); (6.17) replaces the nonlinear constraint in (6.9). The solution space is extremely large, with the total number of possible platform configurations being equal to $2^{|J||I|}$.

To reduce the solution space we introduce some cutting planes. The first cut was added to avoid the symmetrical nature of the problem; *i.e.*, the same solution can be represented in $|I|!$ different ways by merely permuting the platforms. To eliminate such symmetry the following cut has been developed: $\sum_j x_{ij} \ge \sum_j x_{sj} \forall i,s \in I$. A similar cut used is: $\sum_k y_{ik} \ge \sum_k y_{sk} \forall i,s \in I$.

An additional cut that prevents the same component from being added and removed from the same platform is: $a_{ijk} + r_{ijk} \le 1, \forall i \in I; j \in J; k \in K$.

The cuts above are included in the formulation and the model is solved. Adding the first cut reduces the computational time by more than 50%, while the next two cuts have a smaller contribution.

6.3.3 Single Platform Design under Stochastic Demand Problem

Assume that the platform supports *N* types of products. The facility mass-produces *w* units of a single type of platform. The manufacturer experiences stochastic demand for each of the products. If the actual total demand of all product types is higher than the inventory level of the mass produced platforms, sales are lost. On the other hand if the actual total demand of the product is less than the platform's inventory level, demands are satisfied and some holding (or inventory) cost is incurred for the unused platforms.

This problem can be formulated as a two stage stochastic programming model with recourse. The demand for each product is modeled as a set of demand scenarios, each with some probability of occurrence. The probabilities can be assessed during the product customization phase while interacting with the customer. The first stage decision variables are:

1. the configuration (components set of a platform);
2. the number of platforms (inventory level) to be produced.

The second stage decision variables are:

1. The additional components that would be added manually (*i.e.*, without using the mass production methods) to the platform to make a particular product type.
2. The components that would be manually removed from the platform to make a particular product type.
3. The quantity of each product type to be produced for each scenario.

The objective is to minimize the total production cost that includes the platform production cost, the cost of producing the products using the platforms, the holding cost of unused platforms and stock-out cost of lost demands, in addition to the cost of manually adding and removing components.

6.3.3.1 Model Formulation

The following additional notations are used to formulate the stochastic integer program:

1. $k = 1, 2, \ldots, |K|$ index of products
2. $j, l = 1, 2, \ldots, |J|$ index of components
3. $s = 1, 2, \ldots, S$ index of demand scenarios
4. h = platform holding cost, *per* unit
5. q_k = *per* unit stock-out cost for product k
6. ξ_s = vector of demands $(\xi_{1s}, \xi_{ss}, \ldots, \xi_{Ns})$ in scenario s
7. p_s = probability of occurrence of scenario s
8. $$f_{jl} = \begin{cases} 1 \text{ if component } j \text{ precedes component } l \text{ according to the part} \\ \qquad\qquad \text{assembly relationship matrix} \\ \qquad\qquad 0 \text{ otherwise} \end{cases}$$

Decision variables:

$$x_j = \begin{cases} 1 \text{ if the component } j \text{ participates in the platform} \\ \qquad 0 \text{ otherwise} \end{cases}$$

w = number of platforms to be produced

y_{ks} = units of product k to be produced using platforms in scenario s

$$a_{jk} = \begin{cases} 1 \text{ if the component } j \text{ is added manually to the platform to make product } k \\ \qquad 0 \text{ otherwise} \end{cases}$$

$$r_{jk} = \begin{cases} 1 \text{ if the component } j \text{ is removed manually} \\ \quad \text{from the platform to make product } k \\ \qquad 0 \text{ otherwise} \end{cases}$$

u_{ks}^{-} = lost demand of product k in scenario s

v_s^{+} = unused inventory of platforms in scenario s

The following model provides an optimal solution:
minimize

$$w\sum_{j=1}^{|J|}(CP_j + C_j)\cdot x_j + \sum_{s=1}^{S} p_s\left(\sum_{k=1}^{|K|}\sum_{j=1}^{|J|}(CA_j + C_j)\cdot a_{jk}\cdot y_{ks} + \sum_{k=1}^{|K|}\sum_{j=1}^{|J|}(CR_j - C_j)\cdot r_{jk}\cdot y_{ks}\right) + \sum_{s=1}^{S} p_s \sum_{k=1}^{|K|} q_k u_{ks}^{-} + \sum_{s=1}^{S} p_s \cdot h\cdot v_s^{+} \tag{6.18}$$

Subject to:

$$a_{jk} + x_j \le 1 \qquad \forall j \in \{1,2,\ldots,|J|\}, k \in \{1,2,\ldots,|K|\} \tag{6.19}$$

$$a_{jk} + x_j \ge v_{jk} \qquad \forall j \in \{1,2,\ldots,|J|\}, k \in \{1,2,\ldots,|K|\} \tag{6.20}$$

$$v_{jk} \ge a_{jk} \qquad \forall j \in \{1,2,\ldots,|J|\}, k \in \{1,2,\ldots,|K|\} \tag{6.21}$$

$$x_j \ge r_{jk} \qquad \forall j \in \{1,2,\ldots,|J|\}, k \in \{1,2,\ldots,|K|\} \tag{6.22}$$

$$r_{jk} + x_j + v_{jk} \le 2 \qquad \forall j \in \{1,2,\ldots,|J|\}, k \in \{1,2,\ldots,|K|\} \tag{6.23}$$

$$w - v_s^{+} = \sum_{k=1}^{|K|} y_{ks} \qquad \forall s \in \{1,2,\ldots,S\} \tag{6.24}$$

$$y_{ks} + u_{ks}^{-} = \xi_{ks} \qquad \forall s \in \{1,2,\ldots,S\}, k \in \{1,2,\ldots,|K|\} \tag{6.25}$$

$$1 + x_j \ge f_{jl} + x_l \qquad \forall j,l \in \{1,2,\ldots,|J|\} \tag{6.26}$$

$$x_j \in \{0,1\}; a_{jk} \in \{0,1\}; r_{jk} \in \{0,1\}; y_{ks} \ge 0; w \ge 0; u_{ks}^{-} \ge 0; v_s^{+} \ge 0 \tag{6.27}$$

The objective function in (6.18) represents the total production cost and includes the cost of producing the platforms, assembling the products using the platforms, the total stock-out costs, and the total holding cost under all possible scenarios. Constraints in (6.19)–(6.21) state that component *j* must be added to the platform to make product *k* if *j* is not already in the platform (and is required in product *k*). Constraints in (6.22) and (6.23) state that component *j* may be removed from the platform to make product *k* if that component is in the platform and is not required in product *k*. The constraint in (6.24) shows that for any scenario, the total number of products produced cannot exceed the total number of platform in inventory. The constraint in (6.25) limits the total number of units produced of product *k* to be equal to the random demand value of product *k* for any scenario plus the lost demand. The constraint in (6.26) checks the assembly feasibility of the platform while deciding its configuration. This constraint states that if component *l* is in the platform and, according to the part assembly relationship, component *j* precedes *l* ($f_{jl} = 1$), then *j* must also be present in the platform. The constraint in (6.27) ensures the binary and non-negativity nature of the decision variables.

6.4 An Illustrative Example

In this section a small hypothetical example is used to illustrate the solution to the problem presented in Section 6.3.3. The stochastic model is validated by calculating the *stochastic solutions*, *expected value solutions*, and *solutions in case of perfect information*. The model is validated by showing that the *value of stochastic solutions*, VSS (*expected value solution – stochastic solution*) and *expected value of perfect information*, EVPI (*stochastic solution – solution in case of perfect information*) are positive for various instances of the example.

Stochastic solutions are determined by solving the stochastic integer program presented in Section 6.3.3. Expected value solutions are determined by making the value of w (number of platforms to be mass produced) equal to the sum of the expected demand of all products and solving the stochastic integer program with this fixed value of w. The solution in case of perfect information is determined by solving the model by taking one scenario at a time with a given demand value for that scenario; the cost value is then determined for that scenario. Finally the weighted sum of the costs for all scenarios is calculated, where the weight of a scenario is given by the scenario's probability, and that is considered to be the cost in the case of perfect information.

The example uses a family of three products (P1, P2, and P3). The binary bills of materials of the products and the PAR matrix are shown in Tables 6.2 and 6.3, respectively.

Data common for all the scenarios are presented below. There are four distinct components with costs as shown in Table 6.4.

Table 6.2 Material participation matrix (v_{jk}) for the three products

	A	B	C	D
P1	1	1	0	1
P2	1	1	1	0
P3	1	1	0	0

Table 6.3 The product family PAR (f_{jl}) matrix

	A	B	C	D
A		1	1	
B				1
C				
D				

Table 6.4 Various cost used in the example

Component	A	B	C	D
Purchasing cost (US $)	10	11	12	13
Assembly cost (US $)	2	2	2	2
Adding cost (US $)	4	4	4	4
Removal cost (US $)	2	2	2	2

This small example is solved exactly using OPL 3.5 (from ILOG Corporation). The reason for taking such a small size problem was that OPL 3.5 took over 40 h to solve it. That prompted us to develop heuristic based approaches for large, real life problems (see, *e.g.*, Ben-Arieh and Choubey 2008).

Table 6.5 provides the solutions for different demand scenarios, shortage and holding costs, and probabilities of scenario occurrences. Based on the data provided in Table 6.5, the following observations are made:

1. The positive values of VSS and EVPI provide evidence of model correctness; it is obvious that there is an advantage in using the stochastic model over expected solution approaches.
2. When the probability of occurrence of a particular scenario is high, the solutions tend to shift towards that scenario (Cases 1, 2, 3, and 5) except for the case of expected value solutions. For a very symmetric case (Case 4) all three types of solutions are the same, which means that for near symmetric cases using the expected value solution approach would work well.

Table 6.6 provides a sensitivity analysis of the holding cost and shortage cost using various cases. Based on the data presented in Table 6.6, the following can be concluded:

1. When the total demand of products is similar over various scenarios the number of products to be made in each scenario depends solely on the products' shortage costs (Case 1). In addition, the shortage cost should be sufficiently high to justify the production of products as we have not considered the profit obtained by producing items in our model (see Case 2).
2. When there is high variability in total demand over different scenarios, the increase in holding costs encourages lower production for given shortage costs (see Cases 1 and 2).

Figure 6.3 shows the effect of variance in demand and the number of scenarios considered in the stochastic model on the stochastic solution. This figure is a plot of total production cost *vs.* number of demand points considered in the demand distribution for each product *vs.* the standard deviation in a normally distributed demand. The mean demand for each product is kept fixed at 100 units. The figure clearly illustrates that increasing the number of demand points considered in the probability distribution (creating more scenarios) causes the stochastic cost value to decrease; similarly increasing the normal standard deviation leads to an increase in the cost value. These instances of the example are solved using a genetic algorithm approach.

Figure 6.4 shows the advantage of the stochastic model over other models. Figure 6.4 is a plot of objective (cost) value *vs.* standard deviation obtained by using the stochastic model, the expected solution model, and the case of perfect information for the example. All products have the same mean values for their demand distribution and standard deviation is increased for all products. The models are solved using a genetic algorithm method.

Table 6.5 Various solutions for the different instances of the example

Case 1 $h=\text{US }\$5$		q_1 $200	q_2 US $100	q_3 US $100	*Expected value sol.* Obj. val. = 9086 $w=240$			*Stochastic sol.* Obj. val. = 8790 $w=250$			*Perfect information* Obj. val. = 8490 $w1=250$; $w2=200$			VSS	EVPI
Scenarios	Pr.	ξ1	ξ2	ξ3	Y_1	Y_2	Y_3	Y_1	Y_2	Y_3	Y_1	Y_2	Y_3	US $296	US $300
S1	0.8	100	50	100	100	50	90	100	50	100	100	50	100		
S2	0.2	50	100	50	50	100	50	50	100	50	50	100	50		
Case 2 $h=\text{US }\$5$		q_1 US $200	q_2 US $100	q_3 US $1	Obj. val. = 6606 $w=240$			Obj. val. = 6330 $w=150$			Obj. val. = 6014 $w1=200$; $w2=200$			US $276	US $316
Scenarios	Pr.	ξ1	ξ2	ξ3	Y_1	Y_2	Y_3	Y_1	Y_2	Y_3	Y_1	Y_2	Y_3		
S1	0.8	100	50	100	100	50	90	100	50	0	100	50	100		
S2	0.2	50	100	50	50	100	50	50	100	0	50	100	50		
Case 3 $h=\text{US }\$50$		q_1 US $100	q_2 US $100	q_3 US $100	Obj. val. = 12840 $w=220$			Obj. val. = 10365 $w=200$			Obj. val. = 7595 $w1=500$; $w2=200$			US $2475	US $2770
Scenarios	Pr.	ξ1	ξ2	ξ3	Y_1	Y_2	Y_3	Y_1	Y_2	Y_3	Y_1	Y_2	Y_3		
S1	0.1	200	200	100	150	70	0	0	100	100	200	200	100		
S2	0.9	50	100	50	50	100	50	50	100	50	50	100	100		
Case 4 $h=\text{US }\$50/80/100$		q_1 US $100	q_2 US $100	q_3 US $100	Obj. val. = 8725 $w=250$			Obj. val. = 8725 $w=250$			Obj. val. = 8725 $w1=250$; $w2=250$			US $0	US $0
Scenarios	Pr.	ξ1	ξ2	ξ3	Y_1	Y_2	Y_3	Y_1	Y_2	Y_3	Y_1	Y_2	Y_3		
S1	0.5	100	50	100	100	50	100	100	50	100	100	50	100		
S2	0.5	50	100	100	50	100	100	50	100	100	50	100	100		
Case 5 $h=\text{US }\$5$		q_1 US $100	q_2 US $100	q_3 US $100	Obj. val. = 20368 $w=470$			Obj. val. = 18835 $w=500$			Obj. val. = 16265 $w1=500$; $w2=200$			US $1533	US $2570
Scenarios	Pr.	ξ1	ξ2	ξ3	Y_1	Y_2	Y_3	Y_1	Y_2	Y_3	Y_1	Y_2	Y_3		
S1	0.9	200	200	100	200	170	100	200	200	100	200	200	100		
S2	0.1	50	100	50	50	100	50	50	100	50	50	100	50		

Table 6.6 Sensitivity analysis on holding costs and shortage costs

Case # 1 h = US \$50/80/100		q_1 US \$100	q_2 US \$100	q_3 US \$100	*Stochastic sol.* Obj. val. = US \$8790 w = 250		
Scenarios	Pr.	$\xi 1$	$\xi 2$	$\xi 3$	Y_1	Y_2	Y_3
S1	0.8	100	50	100	100	50	100
S2	0.2	50	100	50	50	100	50
Case # 2 h = US \$50/80/100		q_1 US \$20	q_2 US \$20	q_3 US \$20	Obj. val. = 5000 w = 0		
Scenarios	Pr.	$\xi 1$	$\xi 2$	$\xi 3$	Y_1	Y_2	Y_3
S1	0.5	100	50	100	0	0	0
S2	0.5	50	100	50	0	0	0
Case # 3 h = US \$50		q_1 US \$102	q_2 US \$101	q_3 US \$100	Obj. val. = 18825 w = 320		
Scenarios	Pr.	$\xi 1$	$\xi 2$	$\xi 3$	Y_1	Y_2	Y_3
S1	0.5	200	100	100	200	100	20
S2	0.5	100	50	50	100	50	50
Case # 4 h = US \$100		q_1 US \$102	q_2 US \$101	q_3 US \$100	Obj. val. = 27830 w = 200		
Scenarios	Pr.	$\xi 1$	$\xi 2$	$\xi 3$	Y_1	Y_2	Y_3
S1	0.5	200	100	100	200	0	0
S2	0.5	100	50	50	100	50	50

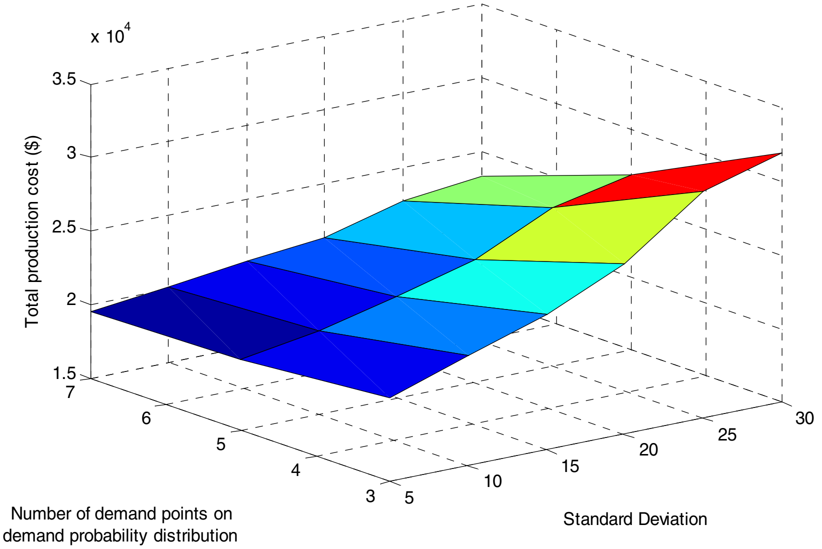

Figure 6.3 Effect of demand variance and the number of scenarios considered in the stochastic model on the stochastic solution

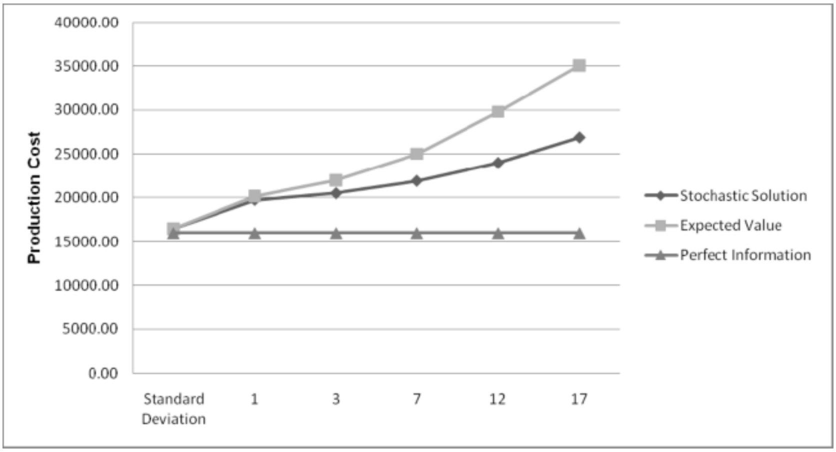

Figure 6.4 Various cost values when increasing the standard deviation of the demand distribution

Based on Figure 6.4 it is possible to conclude that the expected value model is recommended in cases where variance in demand is not very high; otherwise, the stochastic model should be used.

6.5 Conclusion and Recommendations for Future Research

This chapter proposes a platform based optimization approach for the economic production of a product family under different production strategies; namely, using a single or multiple platforms, and considering demand uncertainty.

In the case of stochastic demand the chapter establishes the adequacy of the stochastic model for the platform based production approach, especially when variance in demand is high. The effects of demand variance and various cost components on the optimal platform strategy have also been discussed. The platform based production approach is also explained and illustrated with an example.

Only a very small instance of the problem could be solved by exact approach using OPL 3.5. Therefore, we recommend using heuristic methods that can provide good solutions to large instances of the problem more quickly. One such approach that combines a genetic search process with integer programming provides a near optimal solution for large instances of the problem in reasonable time; yet this approach takes a long time to solve problems with a large number of demand scenarios. Another method – a pure probability based genetic search heuristic – solves problems with a large number of demand scenarios very quickly but with slightly inferior solution quality than the first heuristic approach.

Future work in this area includes consideration of more complex cost structures, and multi-period demand settings with some inventory management policy such as base stock policy. The correlation in demands of the products can be used to capture *cannibalization* effects or to make the problem more tractable for optimization by reducing the number of independent demand scenarios.

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