Chapter 15
Approximate Multi-objective Optimization of Medical Foot Support

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Abstract Although splayfoot does not seem to be a serious disease, it can cause fatigue in daily life. Therefore, the solution of this problem can make daily life much more comfortable, particularly for elder people. There are some commercial products to treat splayfoot, but they just add a small amount of support and are not personalized for each patient. In addition, if the support height is not correct for the patient, it can make the condition worse. Physical therapists are able to create foot supports for each patient, but use their experience to make them. There have been many studies made on the structure of the bones of the foot, and a summary of the desired positions has been reported by medical doctors, one of which is called the Mizuno standard. In this chapter, we will describe the design of a medical foot support using approximate multi-objective optimization, in order to position the height of the bones to the Mizuno standard, and show its effectiveness through myoelectric potential tests.

15.1 Introduction

These days, a number of people have problems with their feet, such as splayfoot, flatfoot, bowlegs, and so on. Splayfoot is a phenomenon characterized by a lack of cushioning in the vertical arch, which therefore causes a lot of pressure on the bones in the foot. When too much pressure is placed on the bones, it causes fatigue and sometimes sufferers cannot walk because of the pain. This is not only bad for the bones, but also for the muscles of the leg. One reason for leg fatigue is that the muscles are being used in an unusual way.

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These days, it is possible to buy commercial arch supports (Figure 15.1) through the Internet. However, as these are not designed to heal the patient and sometimes do not fit them properly, they can make the situation worse. In a previous study [1], we examined the effectiveness of support for three different levels of patients through myoelectric potential measurements and showed the different effectiveness for each patient. From these results, we have designed a vertical 2D shape for arch support by comparing the positions of the bones to the Mizuno standard (Figure 15.2 and Table 15.1) using approximate multi-objective optimization [2, 3]. In the previous study, we have shown the effectiveness of the approximate optimization, and the results showed that we can reduce the divergence from the Mizuno standard, and at the same time also reduce the myoelectric potential. In the study, we used experimental data that we had obtained from X-ray results, with various height patterns made from plaster. However, we only designed the vertical shape, and also did not examine the effect of different support materials and plaster on the experiment. In the study described in this chapter, we will try to make a 3D shape using an application of the spline function with control points, and we will use the same material when checking the bone heights. We use the same approximate multi-objective optimization method. As a result, we have had several patterns of the good results, and have validated them with myoelectric potential measurements to show the effectiveness of the results.

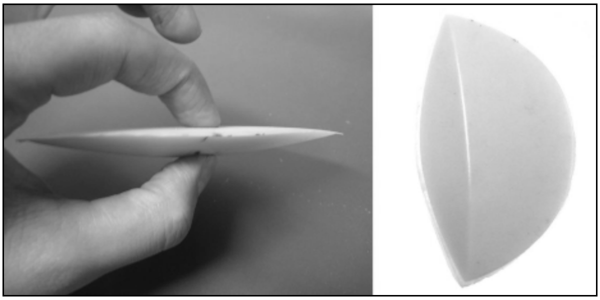

Figure 15.1 Arch support (commercial product)

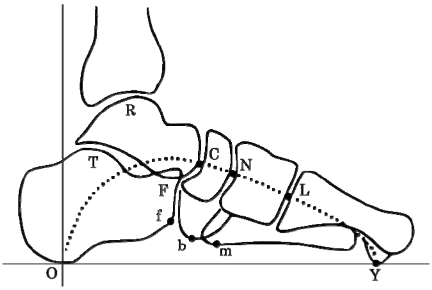

Figure 15.2 Measurement of bone positions in the Mizuno standard [2]

Table 15.1 Mizuno standard foot data

LY	21.66	±0.12%mm
NY	28.36	±0.16%mm
CY	32.71	±0.16%mm
MY	3.02	±0.11%mm
BY	8.11	±0.12%mm
fY	12.08	±0.13%mm

15.2 Approximate Multi-objective Optimization Using Convolute RBF

15.2.1 Convolute RBF

A radial basis function (RBF) network is a multi-layered neural network that gives an output that is a weighted summation of the middle layers. When we use a basis function as a Gaussian distribution with oval form with a different radius for each variable, it becomes:

$$\mathrm{RBF}(\mathbf{x}) = \sum_{i=1}^{m} w_i e^{-\sum_{j}^{n} \frac{(x_j - c_{ij})^2}{r_j^2}} \tag{15.1}$$

where m is the number of the basis function, n is the number of dimensions of variables, r_j is the radius for variable j, and c_{ij} is the ith center of the basis function of variable j. A Gaussian distribution has a character in that its response gets closer to 0 the further it is from its center. Using this property, we propose a convolute RBF as:

$$f_{\mathrm{app}}(\mathbf{x}) = Y(\mathbf{x}) + \sum_{k=1}^{K} \mathrm{RBF}_k(\mathbf{x}), \tag{15.2}$$

where $Y(\mathbf{x})$ is an arbitrary function. With this function the error between the original data set and $Y(\mathbf{x})$ becomes 0 on average. Now, we are going to approximate error between f_{app} and the teaching data. K is the number of convolutions. When there is strong nonlinearity in the original function and/or data, it is impossible to have enough accuracy in a single approximation of RBF. Thus, we need to carry out these approximations several times.

15.2.2 Satisficing Method

A common way of achieving multi-objective optimization is the weighted sum method, which is easy to understand and implement in software. However, even for simple cases, the changes of weights do not coincide with changes in the values of functions, especially when there are more than three objective functions. Therefore, trade-off analysis becomes very difficult to match to the requirements of designers [4].

The satisficing method is an interactive multi-objective optimization method. We set an ideal solution and aspiration level as the desired values of each objective function and obtain a Pareto optimal solution close to the aspiration level. The solution is obtained by minimizing the extended Thebychev scalarization function, as follows:

$$\min_{x\in\Omega}\left\{\max_{i\in N} w_i \frac{f_i(x)-f_i^{\mathrm{asp}}}{f_i^{\mathrm{asp}}-f_i^{\mathrm{ideal}}}+\alpha\sum_{i=1}^{N} w_i \frac{f_i(x)-f_i^{\mathrm{asp}}}{f_i^{\mathrm{asp}}-f_i^{\mathrm{ideal}}}\right\}, \tag{15.3}$$

where Ω is the design space and α is a small value to correct for correct a weak Pareto solution. The superscript "asp" stands for aspiration level, and "ideal" stands for the ideal value that each function can take. w_i is a weight factor that may play a role in distinguishing between objective function and constraints.

15.2.3 Data Addition

In approximate optimization, it is important to indicate where we need to add data for global optimization. If we can indicate appropriately, we can reduce the number of function calls and/or experiments. There are two main reasons for data addition: (1) to obtain a better accuracy around the optimum solution, and (2) to obtain a better approximation for the overall response. The second reason is not only for a better response surface, but also for global optimization to overcome the approximation error in areas with a lower density data distribution. In the multi-objective optimization case, we need to find an approximate scalar function of Equation 15.3 that is strongly nonlinear. When we approximate this function as $\mathrm{RBF}_{\Phi}(\mathbf{x})$ then, to have good response surface, it is important to know the maximum and minimum values of the original $\Phi(x)$. Thus, we would like to find a maximum absolute value of $\mathrm{RBF}_{\Phi}(\mathbf{x})$. In order to estimate the data density, we use +1 for the data that follow the constraints and –1 for those which don't, and we make RBF the function of $\mathrm{RBF}_{\mathrm{ex}}(\mathbf{x})$. We would like to minimize the absolute value of $\mathrm{RBF}_{\mathrm{ex}}(\mathbf{x})$. To make both purposes easier, we introduce the following recommendation function, following the Nash solution as:

$$\mathrm{Rmd}(\mathbf{x})=\left(\left|\mathrm{RBF}_{\Phi}(\mathbf{x})\right|+\alpha\right)\times\left(\beta-\left|\mathrm{RBF}_{\mathrm{ex}}(\mathbf{x})\right|\right), \tag{15.4}$$

where α is included to avoid the approximation error of $\mathrm{RBF}_{\Phi}(\mathbf{x})$, and β is slightly greater than 1. In this recommendation function, the output of $\mathrm{RBF}_{\mathrm{ex}}(\mathbf{x})$ near the active constraints becomes close to 0, so that we can add data both around the area of lower density and surrounding the active constraints. In this paper, we will add data by maximizing the recommendation function using the method that we proposed earlier [5].

15.3 Arch Support

Medical arch support is equipment used for splayfoot patients to raise the arch of the foot and should be designed to return the arch to its normal shape. There are many different kinds of commercial arch support, and one particular example is shown in Figure 15.1.

In our previous study, we examined the effectiveness of commercial arch support for three different subjects – one patient with severe problems, one patient with mild problems, and one with a normal height arch. We asked each subject to walk on a treadmill for a while, and measured the myoelectric potential. We integrated absolute values for them and averaged them over one step and for five trials. Table 15.2 shows the results of the validation. The absolute values differ between each other, so that it does not have any meanings, but the reduction rates show the effectiveness of the support. It is obvious that the support has a postive effect on the severe patient, but a negative effect on the normal subject. There is little affect on the mild patient. From this result, we can see the importance of designing a suitable arch support for each patient.

Table 15.2 Results of effectiveness of foot support

Patient	With support		Without support	
	Big toe	Little toe	Big toe	Little toe
Severe	80.66	81.37	92.38	99.42
Not severe	81.85	55.95	82.79	68.66
None	57.43	44.63	53.90	41.56

15.4 Model of an Arch Support

15.4.1 Spline Curve

A spline curve goes through every control point, and has second-order continuity. In this study, we will use a third-order spline curve. For each interval, it follows the following equation for the interval variable $\tau = x_{i+1} - x_i \ \ (1 \le i \le 3)$:

$$S_j(\tau) = a(\tau)^3 + b(\tau)^2 + c(\tau) + d \ \left(1 \le j \le 3\right), \tag{15.4}$$

where Equation 15.4 has the following conditions:

- it has the same gradient for both sides of control points;
- it has the same changing ratio of gradient for both sides of the control points;
- gradient = 0 for start and end points.

15.4.2 How to Make a Foot Support Model

- Take a picture of a foot as in Figure 15.3 and fit the spline curve using control points, which are denoted by ○ in Figure 15.4. Draw line C. Choose six additional points denoted ●.
- Calculate the spline curve in the vertical direction using four points on lines A and B in Figure 15.4.
- With three spline curves A, B, and C, calculate each start and end point with line C, using two random control points from A and B. With these four points, create a vertical spline curve as shown in Figure 15.5. When four control points are not available, reduce the order of the spline curve and replace those areas.
- From the spline curves shown in Figure 15.5, calculate arbitrary control points to convert the spline curve from vertical to horizontal, as shown in Figure 15.6. As we would like to make an experimental foot support from layers of hard sponge, the shape needs to be in the horizontal plane. Four points (○) in Figure 15.7 become fixed points for the patient's arch, and the height of the other points become the design variables.

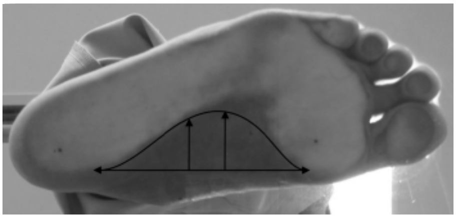

Figure 15.3 Position of the spline curve

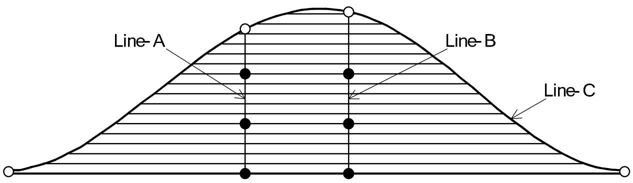

Figure 15.4 Overhead view

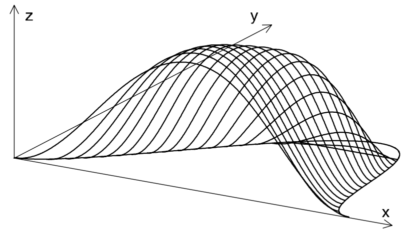

Figure 15.5 Vertical model

15.5 Design of Personal Medical Foot Support

In our previous study, we used plaster to change the height of bones of foot, and we used hard sponge to make the final foot support. Therefore, there may be some difference between experimental result and the actual response of the foot because of the difference in materials. In order to get rid of these differences, we use the same material (hard sponge) in the experiment.

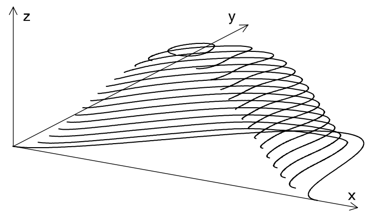

Figure 15.6 Horizontal model

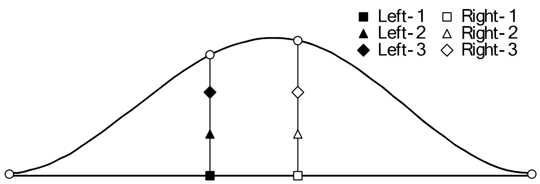

Figure 15.7 Design variable

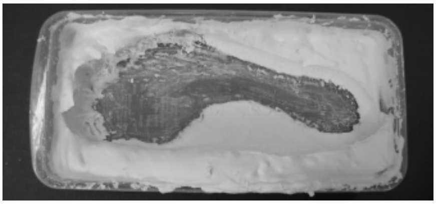

Figure 15.8 Plaster footprint

15.5.1 Initial Pattern

We use plaster to calculate the shape of the patient's foot arch as shown in Figure 15.8. Working from the plaster model, we measure four fixed points as in Figure 15.7.

15.5.2 Ideal Position of Arch

In this study, we assumed that the Mizuno standard from Figure 15.2 and Table 15.1 shows the ideal position of the arch. Table 15.1 shows values for the vertical length from point y to OY, divided by OY in Figure 15.2. We call it % mm.

15.5.3 Detection from X-ray Digital Data

We developed an automatic calculator of each length in Table 15.2, when we indicated the points O, Y, L, N, C, m, b, f on the X-ray digital data as shown in Figure 15.9.

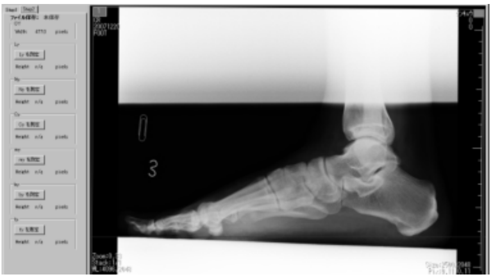

Figure 15.9 X-ray data analysis software

15.5.4 Procedure of the Proposed Method

1. For initial sets, give each design variable randomly.
2. Make arch supports such as shown Figure 15.10. Fit them to the patient, and take X-ray digital data. Measure OY, Ly, Ny, Cy, my, by, and fy.
3. If you are happy with the results, then end. Otherwise go to step 4.
4. Add new data to the database. Make an RBF approximation using the scalar function with:

$$\Phi = \max_{i\in\{L,N,C,m,b,f\}} w_i \frac{100iy/\mathrm{OY} - \mathrm{asp}_i}{1+\mathrm{asp}_i}, \tag{15.5}$$

where asp_i is the square of the error in Table 15.2. Use the average as an arbitrary function. We will use this RBF approximation for data addition. Next, make an RBF approximation for each of six data points (Ly, Ny, Cy, my, by, and fy). Then, using the following scalar function, we will add one data point:

$$\Phi = \max_{i\in\{L,N,C,m,b,f\}} w_i \frac{100\times \mathrm{RBF}_{iy}(\mathbf{x})/\mathrm{OY} - \mathrm{asp}_i}{1+\mathrm{asp}_i}. \tag{15.6}$$

In this case, we had 27 initial data points, and for the initial iteration of the approximate optimization, we gained ten additional data points. We carried out the iteration three times. Table 15.3 shows the complete list of measured data. Figure 15.11 shows the best pattern of arch support.

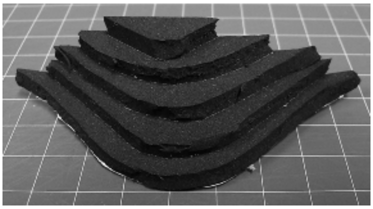

Figure 15.10 Sample pattern of arch support

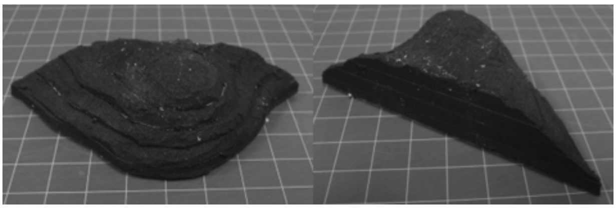

Figure 15.11 Best pattern of arch support

Table 15.3 Results of measurements of all 57 patterns

Pattern	left1	left2	left3	right1	right2	right3	F	F_Lv	F_Nv	F_Cv	F_mv	F_bv	F_fv
1	21.9	14.6	7.31	20.1	13.4	6.69	40.4	0.255	8.07	6.94	14.9	3.34	4.16
2	25.9	17.3	8.64	23.7	15.8	7.91	96.5	1.76	12.3	18.8	9.80	1.18	1.96
3	20.7	13.8	5.80	18.9	12.6	6.30	55.4	1.08	2.94	7.24	33.3	10.3	10.1
4	25.4	17.4	8.70	18.1	11.2	4.13	25.6	0.434	2.77	3.80	25.3	7.91	8.53
5	25.4	17.4	8.70	26.8	14.5	7.25	25.9	0.512	1.54	3.44	26.2	7.97	9.56
6	20.7	13.8	6.88	26.8	14.5	7.25	38.5	0.720	5.17	7.88	19.2	3.93	4.39
7	19.3	12.8	6.42	20.3	10.8	4.73	28.9	0.666	3.11	2.02	29.2	9.35	11.20
8	16.9	10.1	3.38	20.3	10.8	4.73	26.2	1.67	3.56	3.01	18.5	5.77	6.65
9	16.9	10.1	3.38	16.9	8.8	3.38	37.2	0.677	5.70	5.26	10.6	2.98	4.58
10	13.5	8.1	2.03	18.9	12.2	6.08	42.9	1.42	7.29	8.82	8.69	1.84	2.24
11	20.3	12.2	4.73	13.5	7.43	2.70	42.0	1.08	8.39	4.39	12.5	2.54	3.79
12	13.5	6.1	2.03	15.5	7.43	2.70	66.9	1.21	11.2	10.9	10.7	1.51	3.17
13	19.3	12.8	6.42	17.6	11.8	5.88	45.1	0.510	8.26	7.08	9.17	2.04	2.62
14	23.6	16.2	5.41	17.6	11.8	5.88	23.4	1.25	4.83	3.67	7.56	1.62	3.31
15	23.6	18.9	5.41	24.3	20.3	4.73	35.7	0.494	4.10	7.34	12.1	3.42	2.86
16	17.4	14.5	3.62	23.2	19.6	5.07	35.1	0.534	7.23	5.80	14.3	2.88	4.25
17	17.4	14.5	3.62	16.7	12.3	2.90	42.9	0.526	7.69	8.44	5.29	0.761	1.95
18	14.5	11.6	2.17	18.9	14.5	4.35	33.2	0.738	5.37	5.81	6.33	1.08	2.10
19	21.7	15.9	7.25	14.5	10.1	3.62	39.8	2.31	8.19	5.68	5.26	0.636	2.07
20	20.7	15.9	6.52	18.9	9.42	5.07	26.2	1.64	4.16	4.95	10.9	2.75	3.26
21	26.9	23.1	7.69	28.5	23.1	9.23	38.0	1.08	6.70	7.83	5.79	1.21	1.14
22	31.8	23.6	12.7	30.9	20.9	8.18	40.1	1.31	5.66	8.04	4.61	0.91	1.55
23	14.9	11.9	2.38	14.3	12.5	2.98	34.0	0.152	6.56	6.31	14.3	3.66	5.67
24	17.0	13.7	2.98	15.5	12.5	1.79	49.9	0.503	6.26	10.1	6.70	1.33	2.80
25	22.0	20.2	1.79	20.8	17.3	3.57	21.8	1.73	2.69	3.19	12.6	3.10	3.93
26	22.0	20.2	1.79	15.5	10.7	1.79	41.0	1.45	8.07	6.37	8.80	1.47	3.35
27	17.0	16.1	2.98	22.6	19.6	4.17	29.7	1.95	3.40	5.01	11.1	2.95	5.17
28	20.0	19.7	2.49	26.3	19.8	5.68	42.4	1.23	3.32	8.73	6.50	1.35	0.789
29	35.8	12.8	1.50	26.4	17.0	9.74	43.2	0.288	5.14	8.00	6.99	1.10	0.907
30	34.9	25.0	11.8	26.4	17.0	8.57	11.4	1.14	0.694	1.49	8.48	2.63	1.22
31	37.2	11.6	1.50	34.2	28.5	12.8	11.7	0.909	1.64	1.38	7.20	2.51	1.77
32	25.3	37.9	19.9	29.6	16.3	13.6	18.0	1.10	2.66	3.43	9.42	1.96	2.25
33	37.0	24.2	12.1	35.0	28.7	3.38	40.3	0.628	3.51	8.30	16.20	4.05	3.00
34	27.9	15.0	4.72	26.1	12.4	1.50	33.9	0.477	2.27	6.98	9.04	2.32	1.79
35	33.8	43.1	4.60	36.9	19.3	3.71	16.2	1.15	0.531	2.49	4.28	0.819	0.840
36	35.1	18.1	7.75	27.3	17.8	13.9	23.1	3.78	0.305	1.90	10.3	2.60	2.45
37	36.9	18.4	8.71	34.9	29.0	9.24	46.3	0.691	6.10	8.40	9.16	1.04	1.25
38(C)	28.3	16.9	8.33	17.1	11.0	5.85	85.0	0.852	11.9	16.0	11.4	1.42	1.26
39	15.6	25.9	7.53	27.0	16.9	8.94	50.6	0.339	5.28	10.1	10.0	2.10	1.35
40	39.8	24.9	18.3	26.3	16.2	8.55	28.8	1.36	1.44	5.38	9.37	2.75	1.90
41	15.4	11.2	3.07	34.9	28.7	13.0	44.0	1.08	6.31	6.50	10.3	2.57	1.84
42	31.7	12.7	2.93	35.3	27.8	13.7	33.2	0.356	2.70	6.84	3.33	0.690	0.414
43	37.5	43.0	10.8	36.4	19.3	3.86	24.0	2.18	1.12	3.78	5.19	1.60	0.96
44	23.7	43.0	3.38	36.7	19.4	3.77	23.5	2.68	0.28	3.69	3.81	1.18	1.40
45	33.6	12.0	7.36	33.9	28.8	12.5	30.4	1.93	3.42	6.27	5.65	1.47	1.20
46	30.2	42.7	16.5	37.5	19.1	3.70	20.7	0.993	0.670	4.28	0.950	0.261	0.063
47	10.8	13.0	11.8	35.0	28.8	13.0	80.3	0.921	8.90	16.50	6.81	1.23	0.770
48	22.1	16.3	5.63	17.1	11.3	5.68	28.5	0.282	2.78	5.65	7.54	1.18	0.950
49	21.9	17.8	14.4	19.6	11.0	4.26	9.05	0.417	1.12	1.63	4.04	1.05	0.252
50	22.5	17.8	18.9	18.3	11.0	4.42	10.1	1.50	0.340	0.877	4.52	1.24	1.52
51	21.6	16.8	23.9	19.8	11.4	4.49	13.1	0.977	1.11	1.20	13.00	4.02	2.95
52	32.8	43.4	17.1	34.8	18.4	4.05	8.94	1.83	0.0582	0.378	0.909	0.249	0.293
53(best)	22.4	20.4	23.9	20.3	16.7	3.03	4.61	0.462	0.344	0.529	4.68	0.821	1.54
54(A)	27.3	17.3	8.97	29.1	10.7	4.19	10.7	0.143	1.38	2.22	1.73	0.116	0.084
55	25.8	17.4	6.03	36.9	11.2	4.13	28.4	0.659	2.03	5.67	4.16	0.810	1.05
56(B)	34.6	24.2	23.2	25.7	17.5	8.46	38.7	0.033	5.49	6.07	4.59	0.646	0.582
57	22.0	21.5	15.2	18.8	17.6	3.93	57.4	0.417	5.18	11.10	4.74	0.540	0.661

15.6 Validation Test of Personal Medical Foot Support

To validate the model, we made three patterns of arch supports (marked A, B, and C in Table 15.3).

15.6.1 Myoelectric Potential Measurement

The myoelectric potential measurement records the electricity caused by moving a muscle. In the experiments, we used three pairs of electrodes to measure the movement of the calf, big toe, and little toe. In this study, we asked patients to walk on running machines with a speed of 5 km/h, and a slope of 4%.

15.6.2 Experiment

In this validation, a splayfoot patient and a normal person are selected. We prepared a commercial foot support and four kinds of foot supports (A, B, C and Best), as shown in Figures 15.11–15.14. Their data are shown in Table 15.3. To analyze the myoelectric potential measurements, we followed the steps below:

1. Use a low-pass filter to get rid of the landing noise.
2. Calculate a moving average of about four steps.
3. Calculate the absolute moving average difference for each value.
4. Average each value in step 3.
5. Carry out steps 1 to 4 three times and average the results.

The results of the experiment are shown in Table 15.4. Each figure is the amount of movement over 100 μs. It differs from Table 15.2. Therefore, this time we use the moving average as standard following doctors' advice.

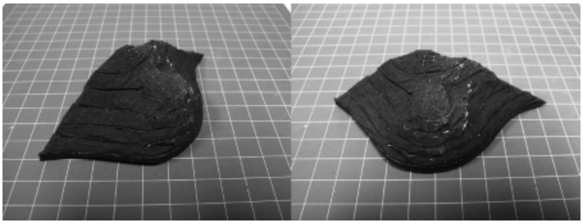

Figure 15.12 Foot support – pattern A

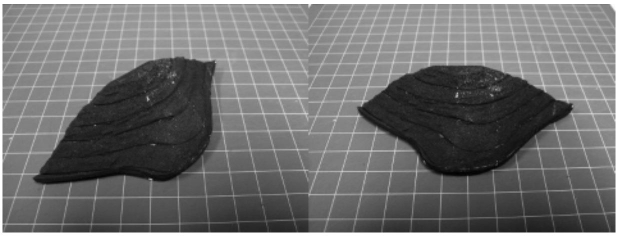

Figure 15.13 Foot support – pattern B

Figure 15.14 Foot support – pattern C

Table 15.4 Results of validation

Pattern	Splay foot			Normal		
	Calf	Big toe	Small toe	Calf	Big toe	Small toe
Bare foot	27.4	96.1	86.3	69.4	83.1	89.0
Comercial	26.9	98.5	84.4	70.9	82.7	80.5
Best	25.0	93.3	80.8	71.6	87.2	80.4
A	25.1	104.0	89.4	71.0	86.7	78.0
B	25.1	99.4	79.4	75.6	86.5	76.5
C	25.3	98.8	86.7	73.6	84.1	81.0

15.6.3 Results

Looking at Table 15.4, we can see that all four foot supports cause a reduction in calf muscle use for the splayfoot patient. However, support patterns A, B, and C have a negative effect, particularly on the big toe. The optimum support is the only support that can simultaneously reduce all three areas of muscle use. Looking at Table 15.3, support pattern A shows better error results for small toes; however, the response on that side is not good at all. It seems that error for the big toe will play a more important role in reducing muscle use. Looking at the results for the normal person, even the commercial support had a negative effect on him, and all the patterns for splayfoot patients show an even worse effect. This result is quite rational, as a foot support should be made personally for each patient if we want to reduce the pressure of muscle use.

15.7 Conclusions

In this chapter, we have used approximate multi-objective optimization using convolute RBF networks in designing made-to-order medical foot supports for splayfoot patients. If we can design the support in the correct way, we can reduce the pressure on bones and muscles for the patients. Of course, it is impossible to make any general models of human beings because we need to identify too many parameters for each individual, and also the measurement must be done in non-destructive way. In this study, we used the Mizuno standard position of bones, and tried to minimize the distance of six positions of bone from them, which gives us six objective functions. Under this condition, we need to carry out approximate multi-objective optimization. In addition, we would like to reduce the number of X-ray inspections, both because of the cost and because of the negative effect on patients. In a previous study, we succeeded in designing a vertical shape using two-dimensional shape optimization. In this study, we extended the model to three-dimensional shape optimization and introduced a 3D spline expression with six control points, and formed a six-variable, six-objective-function optimization problem. Starting from 27 initial data points, we added 30 additional data points to obtain satisfactory results. For validation, we used myoelectronic potential measurement, to show the amount of muscle use. We prepared four different patterns of foot support, with one commercial support. From these validation tests, we have shown that only the best solution can simultaneously reduce all three areas of muscle use – and all four supports have a negative effect on a normal person. These results show the capability of approximate multi-objective optimization: a convolute RBF approximation can calculate an approximate function correctly even in a human model which might have a lot of nonlinearity, and the data addition rule is effective so we only need 57 experiments. Moreover, we have optimized according to the Mizuno standard, but results show that it has a positive effect on the reduction of the pressure on muscles. This means that the effectiveness of the Mizuno standard has been proven from a medical point of view.

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