Chapter 11
Conclusions and Open Problems

The aim of this book is two-fold: to provide an introductory text to max-algebra and to present results on advanced topics. Chapters 1–5 aim to be a guide through basic max-algebra, and possibly to accompany an undergraduate or postgraduate course. Chapters 6–10 are focused on more advanced topics with emphasis on feasibility and reachability.

In the case of feasibility the most important results are: complete resolution of the eigenvalue-eigenvector problem using $O(n^3)$ algorithms; methods for solving two-sided systems of max-linear equations of pseudopolynomial computational complexity; full characterization of strongly regular matrices and the simple image sets of max-linear mappings; $O(n^3)$ algorithms for three presented types of matrix regularity and a polynomial algorithm for finding all essential coefficients of a characteristic maxpolynomial.

Basic reachability problems are solvable in polynomial time. These include the question of reachability of eigenspaces by matrix orbits (for irreducible matrices) and robustness (for irreducible and reducible matrices). Max-linear programs with two-sided constraints can be solved in pseudopolynomial time for problems with integer entries.

There are a number of problems that seem to be unresolved at the time of printing this book. We list some of them:

OP1: Is it possible to multiply out two $n \times n$ matrices in max-algebra in time better than $O(n^3)$?

OP2: Although strong regularity and Gondran–Minoux regularity can be checked in $O(n^3)$ time, it is still not clear whether it is possible to check the strong linear independence or Gondran–Minoux independence in polynomial time.

OP3: Although the question of the existence of permutations of both parities, optimal for the assignment problem for a matrix, is decidable in $O(n^3)$ time, it is not clear whether the best optimal permutations of both parities can be found in polynomial time.

OP4: Although the two-sided max-linear systems with integer entries are solvable in pseudopolynomial time, it is still not clear whether this problem is polynomially solvable or *NP*-complete.

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OP5: Although all essential coefficients of a characteristic maxpolynomial can be found in polynomial time, it is still not clear whether the problem of finding all coefficients is polynomially solvable or *NP*-complete.

OP6: Can the pseudopolynomial algorithms for solving max-linear programs with finite entries be extended to problems with non-finite entries?

OP7: Although it is clear that the greatest corner of a characteristic maxpolynomial is equal to the principal eigenvalue, it is not clear how to interpret the other corners.

OP8: One of the hardest problems in max-algebra seems to be the generalized eigenproblem. Although some progress is presented in Chap. 9, probably no method is available of any kind, exact or approximate (including heuristics), to find at least one generalized eigenvalue for general matrices. In particular, we know that there is at most one generalized eigenvalue if the matrices are symmetric (Theorem 9.1.6), yet there is no clear description of the unique (possible) eigenvalue.