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A Discrete-time Jump Fuzzy System Approach to NCS Design

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Abstract. A discrete-time jump fuzzy system is proposed in this chapter for the modeling and control of a class of nonlinear networked control systems (NCS) with random but bounded communication delays and packets dropout. Above all, a guaranteed cost control with state feedback is developed by constructing a sub-optimal performance controller for the discrete-time jump fuzzy systems in such a way that a piecewise quadratic Lyapunov function (PQLF) can be used to establish the global stability of the resulting closed-loop fuzzy control system. A homotopy-based iterative algorithm solving for linear matrix inequality (LMI) is developed to get the feedback gains. When not all states are available, an output feedback controller is designed. For the NCS based on the mixed networks, a neuro-fuzzy controller is develped, which is composed of three parts: a guaranteed cost state-feedback controller, an adaptive neuro-fuzzy inference system (ANFIS) predictor and a fuzzy controller. The ANFIS predictor is used to improve the performance of the NCS when network delay is longer. Simulation examples are carried out to show the effectiveness of the proposed approaches.

Keywords. Discrete-time jump fuzzy systems, guaranteed cost control, LMI, Markovian jumping parameters, networked control systems.

8.1 Introduction

Over the past five years, networked control systems (NCSs) with feedback loops closed through networks, have received considerable attention in the literature, as illustrated by recent articles [1, 7, 8, 12, 14, 15, 17, 21, 23, 24, 27, 29], due to the enormous advantages, such as low cost, reduced power, simple maintenance and wide applications to novel teleoperating areas.

8.1.1 Fundamental Issues in NCS

An NCS exhibits issues which traditionally have not been taken into account in control system design because control loops are closed through a real-time network. Regardless of the type of network used, these special issues degrade the system dynamic performance and are a source of potential instability. So NCS issues should be investigated.

(i) A network-induced delay occurs while exchanging data among devices connected to the shared medium. The sensor data or control signal arrive at the controller or actuator of the NCS randomly due to network-induced delays.
(ii) The node of the network may discard some of the received packets if it is overloaded. Packets dropout renders the NCS data incomplete.

Compared with traditional control systems, an NCS does not possess data with two different characteristics, namely fixity and integrality. As a result, network delay and packets dropout should be considered simultaneously rather than separately when an NCS is modeled. Most researchers regarded an NCS as a time-delay control system or control system with packet dropout [1, 7, 12, 15, 29]. In addition, most existing literature reports consider only stabilization of linear NCSs whereas nonlinear NCSs have received little attention [1, 8]. Therefore, advanced approaches for nonlinear NCSs are required.

8.1.2 Previous Work

Usually, distributed linear feedback control systems with random network induced delay are modeled as Markovian jump linear control systems [8, 17, 21, 23], in which random variation of system delays corresponds to randomly varying structure of the state-representation. When the Markovian jump system changes abruptly from one mode to another [6, 16, 19, 22, 23, 28], the switching between modes is governed by a Markov process with discrete and finite state space. Markovian jump systems have been studied extensively because jumping systems have been a subject of great practical importance.

Fuzzy systems have been used in recent years for the control of nonlinear processes [5, 10, 11, 18, 20]. Fuzzy system theory enables us to utilize qualitative, linguistic information about a highly complex nonlinear system to construct a mathematical model. And a fuzzy linear model can be used to approximate global behaviors of a highly complex nonlinear system. Local dynamics in different state space regions are represented by local linear systems in this fuzzy linear model. The overall model of the system is obtained by "blending" these linear models through nonlinear fuzzy membership functions. Unlike conventional modeling, which uses a single model to describe the global behavior of a system, fuzzy modeling is essentially a multi-model approach in which simple submodels (a set of linear models) are combined to describe the global behavior of the system. From the middle of the 1980s, there have appeared a number of analysis/synthesis problems for Takagi–Sugeno (T–S) fuzzy systems [18]. Based on the T–S fuzzy systems, Palm and Driankov [16], Choi and Park [6], and Tanaka [19] introduced new switching fuzzy systems for more complicated nonlinear systems.

Motivated by these approaches, a discrete-time jump fuzzy system is proposed to model NCS with random but bounded delay and packet dropout in this chapter. Then new stability theorems and new controller design methods are developed for discrete-time jump fuzzy systems. The chapter is organized as follows. The discrete-time jump fuzzy system and the modeling of NCS are proposed in Section 8.2. In Section 8.3, the LMI-based design of a guaranteed cost state feedback fuzzy controller is presented. The fuzzy output feedback controller is developed in Section 8.4. The neuro-fuzzy controller is provided in Section 8.5. Finally, Section 8.6 summarizes some conclusions.

In this chapter, $\mathcal{Z}$, $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ denote, respectively, the set of integer numbers, the n-dimensional Euclidean space and the set of all $m \times n$ real matrices. As usual, $P > 0$ ($\geqslant$, $<$, $\leqslant$, respectively) will denote that the matrix P is symmetric and positive definite (positive semi-definite, negative definite, negative semi-definite). I_n represents $n \times n$ identity matrix and diag$\{\cdots\}$ represents block diagonal matrix. The symmetric items in symmetric matrices are represented by "$*$". $E[\cdot]$ stands for the mathematical expectation.

8.2 Modeling NCS

The general NCS configuration is illustrated in Fig. 8.1, which is composed of a controller and a remote system containing a physical plant, sensors and actuators. The controller and the plant are physically located at different locations and are directly linked by a data network in order to perform remote closed-loop control. Most networked control methodologies use the discrete-time formulation [22].

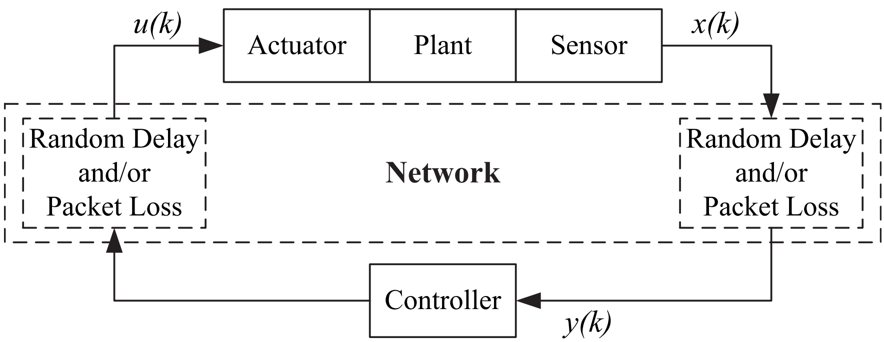

Fig. 8.1. The general NCS configuration

8.2.1 Markov Characteristics of NCS

Suppose that $r(k)$ is the network induced delay at time k with $0 \leqslant r(k) \leqslant d < \infty$, and d is the finite delay bound. When the data is transmitted in turn from sensor to controller or from controller to actuator through the network, the transition probability of $r(k+1)$ is determined only by $r(k)$ and not by $r(0), r(1), \ldots, r(k-1)$ or the time at which it reached the present state. Hence $\{r(k), k \in \mathcal{Z}\}$ is a homogeneous Markov chain. The transition probability is defined as follows:

$$\begin{aligned} pr_{ij} &= \text{Prob}\{r(k+1) = j | r(k) = i\}, \\ pr_i &= \text{Prob}(r(k) = i), \\ i, j &\in \mathcal{S} = \{0, 1, \ldots, d\}. \end{aligned} \tag{8.1}$$

Here $pr_{ij} \geqslant 0$ for $i, j \in \mathcal{S}$, and

$$\sum_{j=0}^{d} pr_{ij} = 1.$$

In real-time control systems, the newest data is the best data [27]. The assumption here means that the controller will always use the most recent data. That is, the data at step k is available for feedback when there is no new information coming in at step $k+1$ (data could be lost or there is a longer delay). So in the model of the NCS, the delay $r(k)$ can increase at most by 1 each step [17]. We develop a new controller for the set $\mathcal{S}$ denoting the possible jump state. In this case, we have

$$\text{Prob}\{r(k+1) > r(k) + 1\} = 0. \tag{8.2}$$

Hence the structured transition probability matrix Pr is

$$Pr = \begin{bmatrix} pr_{00} & pr_{01} & 0 & 0 & \cdots & 0 \\ pr_{10} & pr_{11} & pr_{12} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & pr_{d-1,d} \\ pr_{d0} & pr_{d1} & pr_{d2} & pr_{d3} & \cdots & pr_{d,d} \end{bmatrix}, \quad \text{with } 0 \leqslant pr_{ij} \leqslant 1 \text{ and } \sum_{j=0}^{d} pr_{ij} = 1. \tag{8.3}$$

Each row represents the transition probabilities from a fixed state to all states. The diagonal elements are the probabilities of data coming in sequence with equal delays. The elements above the diagonal indicate that data encounter longer delays, and the elements below the diagonal describe packet dropout.

8.2.2 Discrete-time Jump Fuzzy System

Many nonlinear dynamic systems can be represented by T–S fuzzy models. In fact, it is proved that T–S fuzzy models are universal approximators. So we shall introduce a discrete-time jump fuzzy system to model a class of nonlinear NCSs such as:

$$x_{k+1} = f_{r(k)}(x_k, u_k), \tag{8.4}$$

where $x_k \in \mathbb{R}^n$ is the state vector, $u_k \in \mathbb{R}^m$ is the input vector. Here $f_{r(k)}$ is a local fuzzy function. The models in two-level forms are inferred as follows:

$$\begin{aligned}
&\text{IF } r(k) = i\\
&\text{THEN local plant rule } l:\\
&\qquad \text{IF } z_{k,1} \text{ is } M_{il,1} \text{ and } \cdots \text{ and } z_{k,p} \text{ is } M_{il,p},\\
&\qquad \text{THEN } x_{k+1} = A_{il}x_k + B_{il}u_k,\\
&\qquad\qquad x_0 = x(0),\ l = 1, \ldots, t(i).
\end{aligned} \tag{8.5}$$

Here, $z_{k,1}, \ldots, z_{k,p}$ are the local premise variables, $M_{il,1}, \ldots, M_{il,p}$ are the local fuzzy sets, $t(i)$ is the number of IF-THEN rules when $r(k) = i$, $\{r(k), k \in \mathcal{Z}\}$ is a discrete-time homogeneous Markov chain taking values in a finite set $\mathcal{S} = 0, 1, \ldots, d$, with the transition probability from mode i at time k to mode j at time $k+1, i, j \in \mathcal{S}$, $k \in \mathcal{Z}$.

By the following local fuzzy weighting functions $h_{il}(z_k)$, which are determined by a local premise variable vector $z_k = [z_{k,1}\ z_{k,2}\ \ldots\ z_{k,p}]^T$, the final representation of the discrete-time jump fuzzy system is as follows:

$$x_{k+1} = \sum_{l=1}^{t(i)} h_{il}(z_k)\{A_{il}x_k + B_{il}u_k\}, \tag{8.6}$$

where

$$h_{il}(z_k) = \frac{\prod_{j=1}^{p} M_{il,j}(z_{k,j})}{\sum_{l=1}^{t(i)} \prod_{j=1}^{p} M_{il,j}(z_{k,j})}, \tag{8.7}$$

and $M_{il,j}(z_{k,j})$ is the grade of membership of $z_{k,j}$ in $M_{il,j}$.

To simplify the presentation, the discrete-time jump fuzzy system (8.5) can be represented as follows:

$$x_{k+1} = A_i(H_i(z_k))x_k + B_i(H_i(z_k))u_k, \tag{8.8}$$

where

$$[A_i(H_i(z_k)) \quad B_i(H_i(z_k))] \triangleq \sum_{l=1}^{t(i)} h_{il}(z_k)[A_{il} \quad B_{il}]. \tag{8.9}$$

8.3 State-feedback Controller Design

8.3.1 The Closed-loop Model of an NCS

According to the direction of data transfers, network delays and packets dropout in the NCS can be categorized as sensor-to-controller and controller-to-actuator. When the control or sensor data travel across one type of network, the data has the same transmission characteristic. So the simple NCS configuration in which the network exists only between the sensors and controller is illustrated in Fig. 8.2.

When $r(k) = i$, the mode-dependent jump state feedback control law is:

$$u_k = K_i(H_i(z_k))x_{k-i}, \tag{8.10}$$

where

$$K_i(H_i(z_k)) = \sum_{l=1}^{t(i)} h_{il}(z_k)K_{il}.$$

If we augment the state variable

$$X_k = [x_k^T \ x_{k-1}^T \ \cdots \ x_{k-d}^T]^T, \tag{8.11}$$

where $X_k \in \mathbb{R}^{(d+1)n}$, then the closed-loop system is:

$$\begin{aligned} X_{k+1} &= \left(\tilde{A}_i\left(H_i\left(z_k\right)\right) + \tilde{B}_i\left(H_i\left(z_k\right)\right) K_i\left(H_i\left(z_k\right)\right) \tilde{G}_{r(k)}\right) X_k, \\ X_0 &= [x_0^T \ x_{-1}^T \ \cdots \ x_{-d}^T]^T, \end{aligned} \tag{8.12}$$

where

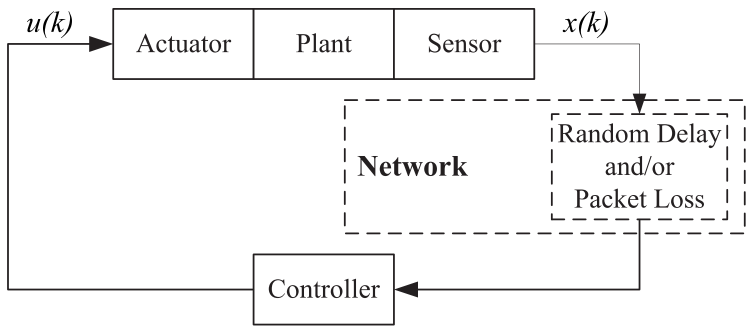

Fig. 8.2. The simple NCS configuration

$$\tilde{A}_i(H_i(z_k)) = \begin{bmatrix} A_i(H_i(z_k)) & 0 & \cdots & 0 & 0 \\ I & 0 & \cdots & 0 & 0 \\ 0 & I & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I & 0 \end{bmatrix},$$

$$\tilde{B}_i(H_i(z_k)) = \begin{bmatrix} B_i(H_i(z_k)) \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

$$\tilde{G}_{r(k)} = \begin{bmatrix} 0 \cdots 0 \; I \; 0 \cdots 0 \end{bmatrix},$$

and $\tilde{G}_r(k)$ has all elements zero except for the $r(k)$th block, which is an identity matrix. Equation (8.12) corresponds to a discrete-time jump fuzzy system.

8.3.2 Guaranteed Cost Controller Design

Now we will consider a guaranteed cost controller. For the performance criterion, an upper bound of LQ cost associated with states and inputs in the global systems called guaranteed cost is described as follows:

$$\min \max_{h_i(z_k)\in\mathcal{H}} \mathcal{E}\left\{\sum_{k=0}^{\infty} (X_k^T Q_{r(k)} X_k + u_k^T R_{r(k)} u_k)\right\}, \tag{8.13}$$

where $Q_{r(k)} > 0$, $R_{r(k)} > 0$, and $\mathcal{H}$ is defined as a set of all possible fuzzy weighting functions. In this chapter, the LQ cost is a function of the grades $h_i(z_k)$.

Definition 1.1 of [3] is extended, and we have the following definitions.

Definition 8.1. For System (8.6) with $u_k \equiv 0$ and $r(k) = i \in \mathcal{S}$, the equilibrium point 0 is said to be stochastically stable, if for every initial state $(X_0, r(0))$, there exists a finite $W > 0$ such that the following holds:

$$\mathcal{E}\left\{\sum_{k=0}^{\infty} \|X_k(X_0, r(0))\|^2 \,\middle|\, X_0, r(0)\right\} < X_0^T W X_0. \tag{8.14}$$

□

Lemma 8.1. The closed-loop system in (8.12) is stochastically stable if and only if there exists a set of symmetric matrices $P_i > 0$, $i \in \mathcal{S}$ satisfying the following coupled matrix inequalities:

$$L_i = \sum_{j=0}^{d} pr_{ij}[\tilde{A}_i(H_i(z_k)) + \tilde{B}_i(H_i(z_k))K_i(H_i(z_k))\tilde{G}_{r(k)}]^T P_j \times [\tilde{A}_i(H_i(z_k)) + \tilde{B}_i(H_i(z_k))K_i(H_i(z_k))\tilde{G}_{r(k)}] - P_i < 0. \tag{8.15}$$

Proof. Sufficiency. For the closed-loop system in (8.6), consider piecewise quadratic Lyapunov stability with the following PQLF candidate $V(X_k)$ mapping from $\mathbb{R}^n$ to $\mathbb{R}$:

$$V(X_k, r(k) = i) = V(X_k, i) = X_k^T P_i X_k > 0. \tag{8.16}$$

The weak infinitesimal operator $\tilde{\mathcal{A}}V(X,i)$ [2, 4] of the stochastic process (X,i) is defined by:

$$\begin{aligned}\tilde{\mathcal{A}}V(X,i) &= \mathcal{E}\{V(X_{k+1}, r(k+1))\,|X_k, r(k) = i\} - V(X_k, i)\\ &= X_k^T\Bigg\{[\tilde{A}_i(H_i(z_k)) + \tilde{B}_i(H_i(z_k))K_i(H_i(z_k))\tilde{G}_{r(k)}]^T\\ &\quad \times \left[\sum_{j=0}^{d} pr_{ij}P_j\right]\Big[\tilde{A}_i(H_i(z_k)) + \tilde{B}_i(H_i(z_k))\\ &\quad \times K_i(H_i(z_k))\tilde{G}_{r(k)}\Big] - P_i\Bigg\}X_k.\end{aligned} \tag{8.17}$$

Thus, if $L_i < 0$, then:

$$\begin{aligned}\tilde{\mathcal{A}}V(X,i) &= \mathcal{E}\{V(X_{k+1}, r(k+1))\,|X_k, r(k) = i\}\\ &\quad - V(X_k, i) \leqslant -\lambda_{\min}(L_i)X_k^T X_k \leqslant -\beta X_k^T X_k\\ &= -\beta\|X_k\|^2,\end{aligned} \tag{8.18}$$

where

$$\beta = \inf\{\lambda_{\min}(-L_i), i \in \mathcal{S}\} > 0.$$

From (8.18), we can see that for any $T \geqslant 1$

$$\mathcal{E}\{V(X_{T+1}, r(T+1))\} - \mathcal{E}\{V(X_0, r(0))\} \leqslant -\beta\mathcal{E}\left\{\sum_{t=0}^{T}\|X_t\|^2\right\}.$$

Then,

$$\begin{aligned}\mathcal{E}\left\{\sum_{t=0}^{T}\|X_t\|^2\right\} &\leqslant \frac{1}{\beta}(\mathcal{E}\{V(X_0, r(0))\} - \mathcal{E}\{V(X_{T+1}, r(T+1))\})\\ &\leqslant \frac{1}{\beta}\mathcal{E}\{V(X_0, r(0))\}.\end{aligned}$$

From Definition 8.1, the stochastic stability is obtained.

Necessity. Let us assume that the closed-loop system in (8.18) is stochastically stable. That is, we have

$$\mathcal{E}\left\{\sum_{k=0}^{\infty}\|X_k(X_0,r(0))\|^2\,|X_0,r(0)\right\} < X_0^T W X_0. \tag{8.19}$$

Consider the following function:

$$X_t^T\tilde{P}_{T-t,r(t)}X_t \triangleq \mathcal{E}\left\{\sum_{k=t}^{T}X_k^T O_{r(k)}X_k\,|X_t,r(t)\right\}, \tag{8.20}$$

with $O_{r(k)} > 0$. Assume that $X_k \neq 0$. Since $O_{r(k)} > 0$, as T increases, either $X_t^T\tilde{P}_{T-t,r(t)}X_t$ is monotonically increasing or it increases monotonically until

$$\mathcal{E}\left\{X_k^T O_{r(k)}X_k\,|X_k,r(k)\right\} = 0$$

for all $k \geqslant k_1 \geqslant t$. It is shown in (8.19) that $X_t^T\tilde{P}_{T-t,r(t)}X_t$ is bounded above, and thus, its limit is given by

$$\begin{aligned} X_t^T P_i X_t &\triangleq \lim_{T\to\infty} X_t^T\tilde{P}_{T-t,r(t)}X_t \\ &\triangleq \lim_{T\to\infty}\mathcal{E}\left\{\sum_{k=t}^{T}X_k^T O_{r(k)}X_k\,|X_t,r(t)=i\right\}. \end{aligned} \tag{8.21}$$

Since this is valid for any X_t, we have

$$P_i = \lim_{T\to\infty}\tilde{P}_{T-t,r(t)}. \tag{8.22}$$

According to (8.21), $P_i > 0$ since $O_{r(k)} > 0$. We get

$$\begin{aligned} &\mathcal{E}\left\{X_t^T\tilde{P}_{T-t,r(t)}X_t - X_{t+1}^T\tilde{P}_{T-t-1,r(t+1)}X_{t+1}\Big|\,X_t,r(t)=i\right\} \\ &= X_t^T O_i X_t. \end{aligned} \tag{8.23}$$

Note that:

$$\begin{aligned} &\mathcal{E}\left\{X_{t+1}^T\tilde{P}_{T-t-1,r(t+1)}X_{t+1}\Big|\,X_t,r(t)=i\right\} \\ &= X_t^T\sum_{j=0}^{d}pr_{ij}\left(\tilde{A}_i(H_i(z_k)) + \tilde{B}_i(H_i(z_k))K_i(H_i(z_k))\tilde{G}_{r(k)}\right)^T \\ &\quad\times\tilde{P}_{T-t-1,j}\left(\tilde{A}_i(H_i(z_k)) + \tilde{B}_i(H_i(z_k))K_i(H_i(z_k))\tilde{G}_{r(k)}\right)X_t. \end{aligned} \tag{8.24}$$

This, together with (8.23), implies that for any X_t,

$$X_t^T \left[\tilde{P}_{T-t,r(t)} - \sum_{j=0}^{d} pr_{ij} \left(\tilde{A}_i(H_i(z_k)) + \tilde{B}_i(H_i(z_k)) K_i(H_i(z_k)) \tilde{G}_{r(k)} \right)^T \times \tilde{P}_{T-t-1,j} \left(\tilde{A}_i(H_i(z_k)) + \tilde{B}_i(H_i(z_k)) K_i(H_i(z_k)) \tilde{G}_{r(k)} \right) \right] X_t$$
$$= X_t^T O_i X_t.$$

Letting $T \to \infty$ and noticing that (8.22) and $O_i > 0$, we obtain:

$$P_i - \sum_{j=0}^{d} pr_{ij} \left[\tilde{A}_i(H_i(z_k)) + \tilde{B}_i(H_i(z_k)) K_i(H_i(z_k)) \tilde{G}_{r(k)} \right]^T \times P_j \left[\tilde{A}_i(H_i(z_k)) + \tilde{B}_i(H_i(z_k)) K_i(H_i(z_k)) \tilde{G}_{r(k)} \right] > 0.$$

□

Lemma 8.2. The closed-loop jump fuzzy system(8.12) is stochastically stable in the large and the cost (8.13) will be bounded by $x_0^T P_i x_0$ for any nonzero initial state $x_0 \in \mathbb{R}_i$, if there exist $P_i > 0$, $i \in \mathcal{S}$, and $K_i(H_i(z_k))$ satisfying the following conditions:

$$\begin{bmatrix} -\bar{P}_i & * & \cdots & * & * & * \\ \begin{pmatrix} \tilde{A}_i(H_i(z_k))\bar{P}_i \\ +\tilde{B}_i(H_i(z_k)) \\ \times K_i(H_i(z_k)) \\ \times \tilde{G}_{r(k)}\bar{P}_i \end{pmatrix} & \begin{pmatrix} -(pr_{i0})^{-1} \\ \times \bar{P}_0 \end{pmatrix} & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \begin{pmatrix} \tilde{A}_i(H_i(z_k))\bar{P}_i \\ +\tilde{B}_i(H_i(z_k)) \\ \times K_i(H_i(z_k)) \\ \times \tilde{G}_{r(k)}\bar{P}_i \end{pmatrix} & 0 & \cdots & \begin{pmatrix} -(pr_{id})^{-1} \\ \times \bar{P}_d \end{pmatrix} & 0 & 0 \\ \bar{P}_i & 0 & \cdots & 0 & -Q_i^{-1} & 0 \\ \begin{pmatrix} K_i(H_i(z_k)) \\ \times \tilde{G}_{r(k)}\bar{P}_i \end{pmatrix} & 0 & \cdots & 0 & 0 & -R_i^{-1} \end{bmatrix} < 0. \quad (8.25)$$

Furthermore, a sub-optimal guaranteed cost controller can be obtained via the following semi-definite programming:

$$\text{Minimize } \gamma \text{ subject to (8.25) and } \begin{bmatrix} \gamma & x_0^T \\ x_0 & \bar{P}_i \end{bmatrix} \leqslant 0. \quad (8.26)$$

Proof. Consider the cost (8.13) associated with states as follows:

$$\min \max_{h_i(z_k)\in\mathcal{H}} \sum_{k=0}^{\infty} \Big\{ X_k^T Q_i X_k + X_k^T (K_i(H_i(z_k))\tilde{G}_{r(k)})^T \times R_i K_i\left(H_i\left(z_k\right)\right)\tilde{G}_{r(k)} X_k \Big\}. \tag{8.27}$$

Then, the closed-loop system is stable via the guaranteed cost controller, if there exists positive-definite symmetric P_i and P_j such that for all X_k and $i, j \in \mathcal{S}$, the following condition holds:

$$\tilde{\mathcal{A}}V(X,i)X_k^T Q_i X_k + X_k^T \left(K_i(H_i(z_k))\tilde{G}_{r(k)}\right)^T \times R_i K_i(H_i(z_k))\tilde{G}_{r(k)} X_k < 0. \tag{8.28}$$

We obtain:

$$[ABK]^T \sum_{j=0}^{d} pr_{ij} P_j \times [ABK] - P_i + Q_i + (K_i(H_i(z_k))\tilde{G}_{r(k)})^T R_i K_i(H_i(z_k))\tilde{G}_{r(k)} X_k < 0,$$

where

$$ABK = \tilde{A}_i(H_i(z_k)) + \tilde{B}_i(H_i(z_k))K_i(H_i(z_k))\tilde{G}_{r(k)}.$$

Using Schur complements, we have the following matrix inequality:

$$\begin{bmatrix} -P_i & * & \cdots & * & * & * \\ ABK & -(pr_{i0}P_0)^{-1} & & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ ABK & 0 & \cdots & -(pr_{id}P_d)^{-1} & 0 & 0 \\ I_n & 0 & \cdots & 0 & -Q_i^{-1} & 0 \\ \begin{pmatrix} K_i(H_i(z_k)) \\ \times \tilde{G}_{r(k)} \end{pmatrix} & 0 & \cdots & 0 & 0 & -R_i^{-1} \end{bmatrix} < 0. \tag{8.29}$$

The left-hand side of the inequality (8.29) can be pre- and post-multiplied by J^T and J, respectively, where

$$J = \text{blockdiag}\left\{P_i^{-1}, I_n, I_n, I_m\right\},$$

which yields the following:

$$\begin{bmatrix} -P_i^{-1} & * & \cdots & * & * & * \\ ABKP & -(pr_{i0}P_0)^{-1} & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ ABKP & 0 & \cdots & -(pr_{id}P_d)^{-1} & 0 & 0 \\ P_i^{-1} & 0 & \cdots & 0 & -Q_i^{-1} & 0 \\ \begin{pmatrix} K_i\left(H_i\left(z_k\right)\right) \\ \times \tilde{G}_{r(k)} P_i^{-1} \end{pmatrix} & 0 & \cdots & 0 & 0 & -R_i^{-1} \end{bmatrix} < 0, \tag{8.30}$$

with

$$ABKP = \tilde{A}_i(H_i(z_k))P_i^{-1} + \tilde{B}_i(H_i(z_k))K_i(H_i(z_k))\tilde{G}_{r(k)}P_i^{-1}.$$

Let $\bar{P}_i \triangleq P_i{}^{-1}$. We obtain:

$$\begin{bmatrix} -\bar{P}_i & * & \cdots & * & * & * \\ \begin{pmatrix} \tilde{A}_i\left(H_i\left(z_k\right)\right)\bar{P}_i \\ +\tilde{B}_i\left(H_i\left(z_k\right)\right) \\ \times K_i\left(H_i\left(z_k\right)\right) \\ \times \tilde{G}_{r(k)}\bar{P}_i \end{pmatrix} & -\left(pr_{i0}\right)^{-1}\bar{P}_0 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \begin{pmatrix} \tilde{A}_i\left(H_i\left(z_k\right)\right)\bar{P}_i \\ +\tilde{B}_i\left(H_i\left(z_k\right)\right) \\ \times K_i\left(H_i\left(z_k\right)\right) \\ \times \tilde{G}_{r(k)}\bar{P}_i \end{pmatrix} & 0 & \cdots & -\left(pr_{id}\right)^{-1}\bar{P}_d & 0 & 0 \\ \bar{P}_i & 0 & \cdots & 0 & -Q_i^{-1} & 0 \\ \begin{pmatrix} K_i\left(H_i\left(z_k\right)\right) \\ \times \tilde{G}_{r(k)}\bar{P}_i \end{pmatrix} & 0 & \cdots & 0 & 0 & -R_i^{-1} \end{bmatrix} < 0. \tag{8.31}$$

So, if the condition (8.25), $\bar{P}_i > 0$, $P_j > 0$ hold for all $i, j \in \mathcal{S}$, then

$$\tilde{\mathcal{A}}V(X,i) < 0 \text{ at } x_k \neq 0.$$

Owing to continuity, there exists $M_i > 0$ such that

$$\tilde{\mathcal{A}}V(X,i) - M_i < 0.$$

Based on Lemma 8.1, the closed-loop jump fuzzy system is stochastically stable.

When the condition (8.25) holds, the cost (8.13) will be bounded for any nonzero initial state $X_0 \in \mathbb{R}_i$:

$$\max_{h_l(z_k)\in\mathcal{H}} \sum_{k=0}^{\infty} \left\{X_k^T Q_i X_k + u_k^T R_i u_k\right\} < X_0^T P_i X_0.$$

Since any feasible solutions γ, $\bar{P}_i$, P_j, and $K_i(H_i(z_k))$ yielding (8.25) will also satisfy

$$\max_{h_l(z_k)\in\mathcal{H}} \sum_{k=0}^{\infty} \{X_k^T Q_i X_k + u_k^T R_i u_k\} < X_0^T P_i X_0 \leq \gamma,$$

for any h_l and nonzero $X_0 \in \mathbb{R}_i^{d+1}$, we can use (8.26) to minimize $X_0^T P_i X_0$ for known nonzero initial states. The proof is completed. □

8.3.3 Homotopy Algorithm

The design problem to determine the state feedback gains K_i for (8.26) can be defined as follows: Find P with the constraints (8.25) and K_i such that (8.26) are satisfied.

However, in general, the inequalities (8.25) cannot be transformed equivalently to LMIs and we will utilize the homotopy method [13] to solve it in an iterative manner.

The homotopy algorithm uses a continuous deformation to embed difficult problems into a family of related problems. As a result, once the solution to an "easy to solve" problem in this family is obtained, a continuous path may be followed in solution space to obtain the desired solution to the original problem. To construct a homotopy path, we introduce a real number λ varying from 0 to 1, and define:

$$\begin{bmatrix} -\bar{P}_i & * & \cdots & * & * & * \\ \left(\tilde{A}_i(H_i(z_k))\bar{P}_i + \tilde{B}_i(H_i(z_k))((1-\lambda)K_0 + \lambda K_i(H_i(z_k)))\tilde{G}_{r(k)}\bar{P}_i\right) & -(pr_{i0})^{-1}\bar{P}_0 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \left(\tilde{A}_i(H_i(z_k))\bar{P}_i + \tilde{B}_i(H_i(z_k))((1-\lambda)K_0 + \lambda K_i(H_i(z_k)))\tilde{G}_{r(k)}\bar{P}_i\right) & 0 & \cdots & -(pr_{id})^{-1} & 0 & 0 \\ \bar{P}_i & 0 & \cdots & 0 & -Q_i^{-1} & 0 \\ ((1-\lambda)K_0 + \lambda K_i(H_i(z_k)))\tilde{G}_{r(k)}\bar{P}_i & 0 & \cdots & 0 & 0 & -R_i^{-1} \end{bmatrix} < 0, \quad i,j \in \mathcal{S}. \tag{8.32}$$

Then the homotopy algorithm can be summarized as follows:

Step 1: Initialization: set $k = 0$, select N and $N_{\max}$. Compute the initial values K_0 and P_0.

Step 2: Set $k = k + 1$ and $k = k/N$, set P to P_{k-1}.

If the LMIs (8.32) are feasible,
Then denote the feasible solution as ${K_i}^k$, set $P_k = P_{k-1}$, and go to Step 4,
Else go to Step 3.

Step 3: Set K_i to ${K_i}^{k-1}$,

If the LMIs (8.32) are feasible,
Then solve the minimization problem:
min trace(P) subject to (8.32),
denote the feasible solutions as P_k, and set $K_i = {K_i}^{k-1}$, then go to Step 4,

Else set $N = 2N$,
If $N > N_{\max}$, then the algorithm fails in giving feasible solution,
Else set $k = 0$, go to Step 2.

Step 4: If $k < N$, go to Step 2. If $k = N$, the obtained solutions ${K_i}^k$ and P_k are a set of feasible solutions of (8.25) and (8.26). □

8.4 Output Feedback Controller Synthesis of an NCS

8.4.1 Fuzzy Observer Design

Suppose not all state variables are available, the following fuzzy observer is considered:

$$\begin{aligned}\hat{x}_{k+1} &= A_i(H_i(z_k))\hat{x}_k + B_i(H_i(z_k))u_k \\ &\quad + \hat{L}_i(H_i(z_k))(\hat{y}_k - y_k), \\ \hat{y}_k &= C_i(H_i(z_k))\hat{x}_k + D_i(H_i(z_k))u_k.\end{aligned} \tag{8.33}$$

We wish to find observer gains $\hat{L}_i\left(H_i\left(z_k\right)\right)$ such that $e_k = \hat{x}_k - x_k \to 0$ asymptotically as $k \to \infty$. Define the fuzzy error system as:

$$e_{k+1} = \hat{x}_{k+1} - x_{k+1} = A_i^{cL}\left(H_i\left(z_k\right)\right)e_k, \tag{8.34}$$

where $A_i^{cL}\left(H_i(z_k)\right) = A_i\left(H_i(z_k)\right) + \hat{L}_i\left(H_i(z_k)\right)C_i\left(H_i(z_k)\right).$

Then we come to the result for piecewise fuzzy jump observer synthesis.

Lemma 8.3. The closed-loop fuzzy error system (8.34) is stochastically stable, if for any given set of matrices $N_i > 0$, $i \in \mathcal{S}$, there exists a set of matrices E_i, F_i, and a set of symmetric matrices $X_i > 0$, $i \in \mathcal{S}$, satisfying the following matrix inequality:

$$\begin{bmatrix} -X_i + N_i & * & \cdots & * \\ O_i & pr_{i0}X_0 - E_i - {E_i}^T & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ O_i & 0 & \cdots & pr_{id}X_d - E_i - {E_i}^T \end{bmatrix} < 0, \tag{8.35}$$

where

$$O_i = E_iA_i\left(H_i\left(z_k\right)\right) + F_iC_i\left(H_i\left(z_k\right)\right),\ i \in \mathcal{S}.$$

In addition, the observer gain for each subspace is given by:

$$\hat{L}_i = E_i^{-1}F_i,\ i \in \mathcal{S}. \tag{8.36}$$

Proof. Based on Definition 8.1 and Lemma 8.1, the closed-loop fuzzy error system (8.34) is stochastically stable if there exists a set of symmetric positive definite matrices $P_i > 0$, satisfying the following inequalities,

$$\sum_{j=0}^{d} pr_{ij} \left[A_i^{cL}\left(H_i\left(z_k\right)\right)\right]^T P_j \times \left[A_i^{cL}\left(H_i\left(z_k\right)\right)\right] - P_i + N_i < 0. \tag{8.37}$$

With $F_i = E_i \hat{L}_i$, the LMI (8.35) is equivalent to:

$$\begin{bmatrix} -X_i + N_i & * & \cdots & * \\ E_i A_i^{cL}\left(H_i\left(z_k\right)\right) & pr_{i0}X_0 - E_i - E_i^T & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ E_i A_i^{cL}\left(H_i\left(z_k\right)\right) & 0 & \dots & pr_{id}X_d - E_i - E_i^T \end{bmatrix} < 0. \tag{8.38}$$

The left-hand side of the inequality (8.38) can be pre-multiplied by T_1 and post-multiplied by $T_2 = T_1^T$ to yield the inequality (8.37), where

$$T_1 = \big[I \quad \underbrace{\left(A_i^{cL}(H_i(z_k))\right)^T \cdots \left(A_i^{cL}\left(H_i\left(z_k\right)\right)\right)^T}_{d+1} \big].$$

Thus, LMI (8.35) implies the inequality (8.37). It can be concluded that the fuzzy error system (8.34) is stochastically stable. □

8.4.2 Output Feedback Controller Design

The output feedback fuzzy controller design presented above with sub-optimal guaranteed cost performance is based on the sub-optimal state feedback fuzzy controller and fuzzy observer in each subspace. When $r(k) = i$, the observer equation is defined in (8.33) and the output feedback jump fuzzy control law is:

$$u_k = \hat{K}_i\left(H_i\left(z_k\right)\right)\hat{x}_{k-i}, \tag{8.39}$$

where

$$\hat{K}_i\left(H_i\left(z_k\right)\right) = \sum_{l=1}^{t(i)} h_{il}\left(z_k\right)\hat{K}_{il}.$$

If we augment the variable as

$$\begin{aligned} \tilde{\hat{x}}_k &= \left[\hat{x}_k^T \;\; \hat{x}_{k-1}^T \cdots \hat{x}_{k-d}^T\right]^T, \;\; \tilde{\hat{x}}\left(k\right) \in^{(d+1)n}, \\ \tilde{e}_k &= \left[e_k^T \; e_{k-1}^T \cdots e_{k-d}^T\right]^T, \end{aligned} \tag{8.40}$$

then the closed-loop system becomes

$$\begin{aligned}\tilde{\hat{x}}_{k+1} &= \left(\tilde{A}_i\left(H_i\left(z_k\right)\right) + \tilde{B}_i\left(H_i\left(z_k\right)\right)\hat{K}_i\left(H_i\left(z_k\right)\right)\tilde{G}_{r(k)}\right)\tilde{\hat{x}}_k \\ &\quad + \hat{L}_i\left(H_i\left(z_k\right)\right)\tilde{C}_i\left(H_i\left(z_k\right)\right)\tilde{e}_k, \\ \tilde{e}_{k+1} &= \left(\tilde{A}_i\left(H_i\left(z_k\right)\right) + \hat{L}_i\left(H_i\left(z_k\right)\right)\tilde{C}_i\left(H_i\left(z_k\right)\right)\right)\tilde{e}_k,\end{aligned} \tag{8.41}$$

where

$$\tilde{C}_i\left(H_i\left(z_k\right)\right) = \left[C_i\left(H_i\left(z_k\right)\right) \ \ 0 \cdots 0\right],$$

$$\tilde{L}_i\left(H_i\left(z_k\right)\right) = \begin{bmatrix} L_i\left(H_i\left(z_k\right)\right) \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}.$$

Here for simplicity, the closed-loop output feedback jump fuzzy system dynamics can be described by

$$\bar{x}_{k+1} = \bar{A}_i\left(H_i\left(z_k\right)\right)\bar{x}_k \tag{8.42}$$

where

$$\bar{x}_k = \left[\tilde{\hat{x}}_k \ \ \tilde{e}_k\right]^T,$$

$$\bar{A}_i\left(H_i\left(z_k\right)\right) = \begin{bmatrix} \tilde{A}_i^{CK} & \hat{L}_i\left(H_i\left(z_k\right)\right)\tilde{C}_i\left(H_i\left(z_k\right)\right) \\ 0 & \tilde{A}_i\left(H_i\left(z_k\right)\right) + \hat{L}_i\left(H_i\left(z_k\right)\right)\tilde{C}_i\left(H_i\left(z_k\right)\right) \end{bmatrix},$$

$$\tilde{A}_i^{CK} = \tilde{A}_i\left(H_i\left(z_k\right)\right) + \tilde{B}_i\left(H_i\left(z_k\right)\right)\hat{K}_i\left(H_i\left(z_k\right)\right)\tilde{G}_{r(k)}.$$

Then the output feedback fuzzy controller is obtained.

Lemma 8.4. The closed-loop output feedback jump fuzzy system (8.39) is stochastically stable if for any given set of symmetric matrices $W_i > 0$, $i \in \mathcal{S}$, there exists a set of symmetric matrices $\tilde{P}_i > 0$, $i \in \mathcal{S}$ satisfying the following matrix inequality:

$$\sum_{j=0}^{d} pr_{ij}\bar{A}_i^T\left(H_i\left(z_k\right)\right)\tilde{P}_j\bar{A}_i\left(H_i\left(z_k\right)\right) - \tilde{P}_i + W_i < 0. \tag{8.43}$$

Proof. The result directly follows from Lemma 8.1. □

Lemma 8.4 is only useful for checking the closed-loop stability of the discrete-time jump fuzzy control system when the output feedback fuzzy controller is already available. Note that the matrix inequality (8.43) contains product terms involving $\hat{K}_i$, $\hat{L}_i$, $\tilde{P}_i$ and W_i. Nonlinear matrix inequality (NMI) technique is required to generate the output feedback jump fuzzy controller. Luckily, we have the following theorem by extending Theorem 5 of [25].

Lemma 8.5. The matrix inequality (8.43) has feasible solutions if LMIs (8.25)–(8.26) and (8.35)–(8.36) do.

Proof. Based on Theorem 5 of [25], we will show that if the feasible solutions to LMIs (8.25)–(8.26) and (8.35)–(8.36) can be found then there always exists a positive scalar α, such that:

$$\tilde{P}_i = \begin{bmatrix} \bar{P}_i & 0 \\ 0 & \alpha X_i \end{bmatrix}, \; i \in \mathcal{S}, \tag{8.44}$$

satisfies (8.43). And α can be obtained from the following inequality:

$$\begin{aligned} \alpha \left[\sum_{j=0}^{d} pr_{ij} \left(A_i^{cL} \right)^T X_i A_i^{cL} - X_i \right] < & \left(B_i \left(H_i \left(z_k \right) \right) \hat{K}_i \left(H_i \left(z_k \right) \right) \right)^T \\ & \times \left[\sum_{j=0}^{d} pr_{ij} \left(A_i^{CK} \right)^T \bar{P}_i^{-1} A_i^{CK} - \bar{P}_i^{-1} \right]^{-1} \\ & \times \left(B_i \left(H_i \left(z_k \right) \right) \hat{K}_i \left(H_i \left(z_k \right) \right) \right), \end{aligned} \tag{8.45}$$

where

$$A_i^{CK} = A_i \left(H_i \left(z_k \right) \right) + B_i \left(H_i \left(z_k \right) \right) \hat{K}_i \left(H_i \left(z_k \right) \right).$$

□

Based on Lemmas 8.4 and 8.5, the output feedback jump fuzzy controller can be designed.

8.4.3 Simulation Example

To illustrate the proposed theoretical results, a numerical example is considered.

Different networks vary in network-induced delay bounds and the rate of data loss. The induced delays and data dropout of typical networks are shown in Figs. 8.3 and 8.4 by simulations using OPNET software.

In our example, local area network (LAN) including Ethernet, token ring, etc., in which the induced delay is low and rate of data dropout is nearly zero, is used as the communication network in the NCS. With the purpose of defining $v(r(k))$, NCS experiments with fixed constant delays bounded by the LAN delay are presented. If the low delay is considered, a good output result is shown in Fig. 8.5 when delay is less than 0.001. On the contrary, if the delay is high, the system is out of control when delay is larger than 0.007.

Based on the experiment, the states $r(k) = 0, 1, 2$ denote that the network induced-delay is low, medium and high, respectively, and have the following transition probability matrix:

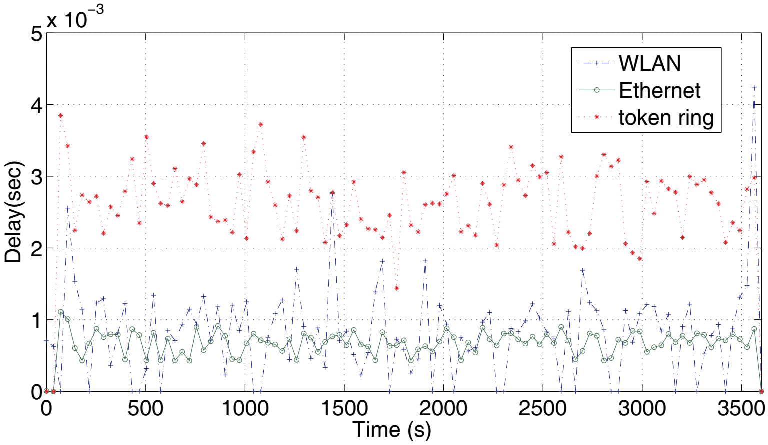

Fig. 8.3. Induced delay of typical networks

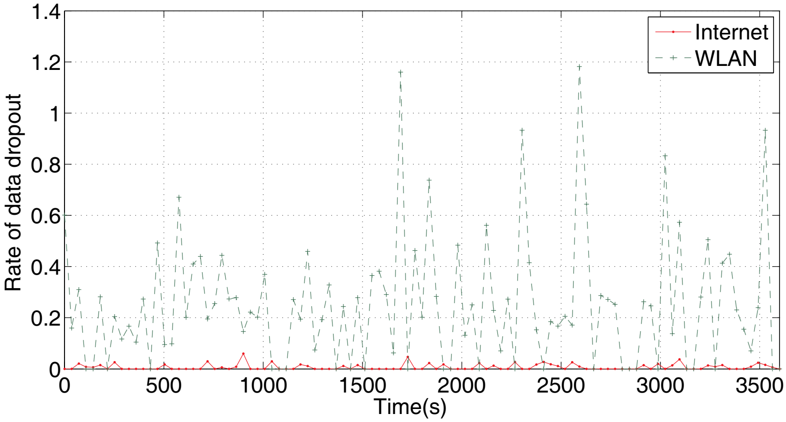

Fig. 8.4. Data dropout rate of typical networks

$$Pr = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.3 & 0.6 & 0.1 \\ 0.3 & 0.6 & 0.1 \end{bmatrix}.$$

The delays $v(r(k))$ corresponding to the three states are:

$$\begin{cases} v(r(k)) \in [0, 0.001], & r(k) = 0, \\ v(r(k)) \in [0.001, 0.007], & r(k) = 1, \\ v(r(k)) \in [0.007, 0.01], & r(k) = 2. \end{cases}$$

Consider the discrete-time jump fuzzy system given by

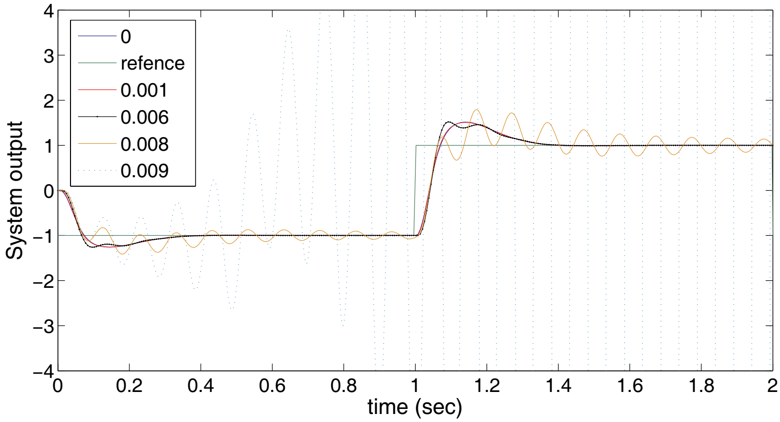

Fig. 8.5. System outputs under different fixed delays

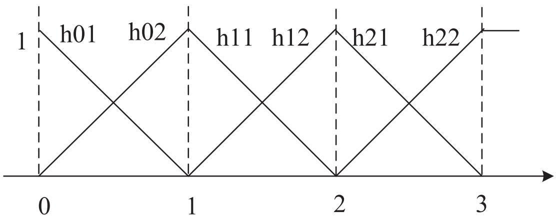

Fig. 8.6. The local fuzzy weighting functions of the example

$$x_{k+1} = \sum_{l=1}^{2} h_{il} \left(\tau_k \left(r\left(k\right)\right)\right) \{A_{il} x_k + B_{il} u_k\},$$

$$y_{k+1} = \sum_{l=1}^{2} h_{il} \left(i + pr_{(i-1)i}\right) \{C_{il} x_k + D_{il} u_k\},$$

with

$$A_{01} = \begin{bmatrix} 0.8 & 0.1 \\ -0.5 & 0.8 \end{bmatrix}, \qquad A_{02} = \begin{bmatrix} 0.5 & 0.1 \\ -0.4 & 0.5 \end{bmatrix}, \qquad A_{11} = \begin{bmatrix} 0.7 & 0.1 \\ -0.3 & 0.7 \end{bmatrix},$$

$$A_{12} = \begin{bmatrix} 0.5 & 0.1 \\ -0.1 & 0.5 \end{bmatrix}, \qquad A_{21} = \begin{bmatrix} 1 & 0.1 \\ -0.4 & 1 \end{bmatrix}, \qquad A_{22} = \begin{bmatrix} 0.5 & 0.1 \\ -0.3 & 0.5 \end{bmatrix},$$

$$B_{01} = \begin{bmatrix} 0 & 0.1 \\ 0.2 & 0 \end{bmatrix}, \qquad B_{02} = \begin{bmatrix} 0 & -0.2 \\ 0.2 & 0 \end{bmatrix}, \qquad B_{11} = \begin{bmatrix} 0 & 0.1 \\ 0.2 & 0 \end{bmatrix},$$

$$B_{12} = \begin{bmatrix} 0 & -0.1 \\ 0.4 & 0 \end{bmatrix}, \quad B_{21} = \begin{bmatrix} 0 & -0.2 \\ -0.5 & 0 \end{bmatrix}, \quad B_{22} = \begin{bmatrix} 0 & 0.3 \\ 0.2 & 0 \end{bmatrix},$$

$$C_{01} = \begin{bmatrix} 0 & 0.1 \\ 0.2 & 0 \end{bmatrix}, \quad C_{02} = \begin{bmatrix} 0 & -0.2 \\ 0.2 & 0 \end{bmatrix}, \quad C_{11} = \begin{bmatrix} 0.1 & 0 \\ 0.2 & 0 \end{bmatrix},$$

$$C_{12} = \begin{bmatrix} 0.2 & 0 \\ 0 & -0.4 \end{bmatrix}, \quad C_{21} = \begin{bmatrix} -0.2 & 0 \\ 0 & 0.3 \end{bmatrix}, \quad C_{22} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix},$$

$$D_{01} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad D_{02} = \begin{bmatrix} 1 & 0.1 \\ 0 & 1 \end{bmatrix}, \quad D_{11} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix},$$

$$D_{12} = \begin{bmatrix} -1 & 0.1 \\ 0 & -1 \end{bmatrix}, \quad D_{21} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad D_{22} = \begin{bmatrix} 1 & 0.12 \\ 0 & 1 \end{bmatrix},$$

where the local fuzzy weighting function $h_{il}(r(k))$ followed the local fuzzy weighting functions of Fig. 8.6. Figure 8.7 shows one simulation run of the Markovian jump delays according to the given transition probability matrix.

By using mincx() in MATLAB® LMI Toolbox, the minimal α for the closed-loop output feedback fuzzy control system to be asymptotically stable is 0.5438.

For the initial condition:

$$x_0 = [3, 2]^T, \quad x_1 = [2.5, 2]^T, \quad x_2 = [1, 1]^T,$$
$$\hat{x}_0 = [0, 0]^T, \quad \hat{x}_1 = [1.5, 0]^T, \quad \hat{x}_2 = [1, 1]^T.$$

The response behaviors of the closed-loop system are presented in Figs. 8.8 and 8.9 using the output feedback fuzzy controller. Fig. 8.8 shows the system state responses and their estimates, while Fig. 8.9 shows outputs and control variables curves and their corresponding estimates. It is shown from these two figures that the estimated variables can converge to the original ones asymptotically such that good system performance can be guaranteed.

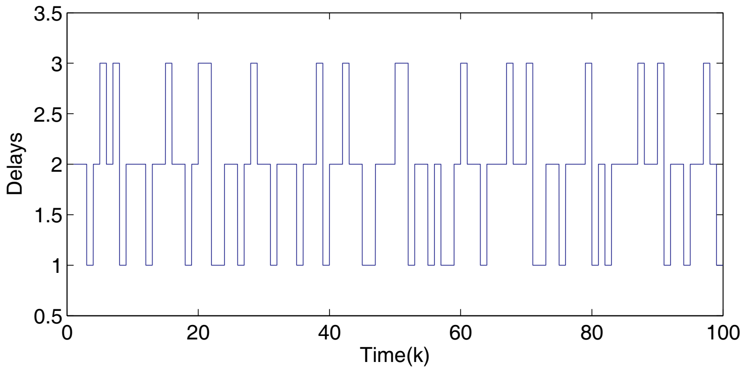

Fig. 8.7. Random delays

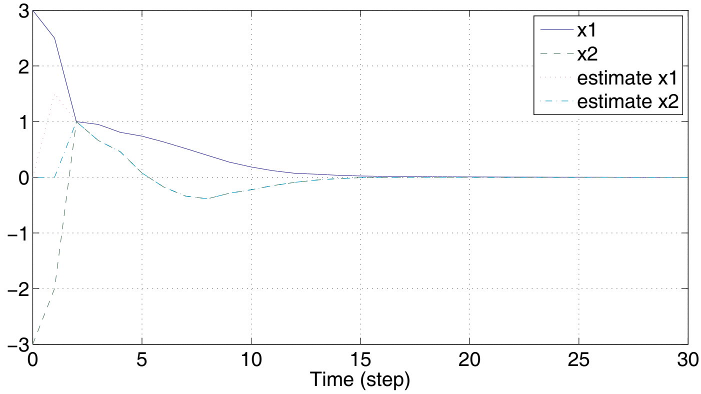

Fig. 8.8. The system state responses and their estimates

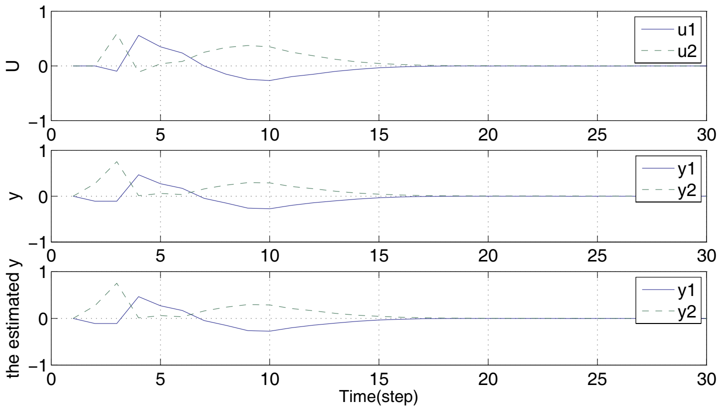

Fig. 8.9. The output and control variables curves and their corresponding estimates

8.5 Neuro-fuzzy Controller Design

Network type has a great effect on the characteristic of NCS. Fig. 8.5 shows the system is out of control when the delay is high. Satellite network [26] is a typical network with longer induced delay shown in Fig. 8.10 by simulation using OPNET software.

A new neuro-fuzzy controller is presented for the NCS based on the mixed network including terrestrial networks and satellite networks. It is constructed of three parts: the guaranteed cost controller, the ANFIS predictor and the fuzzy controller, presented in Fig. 8.11.

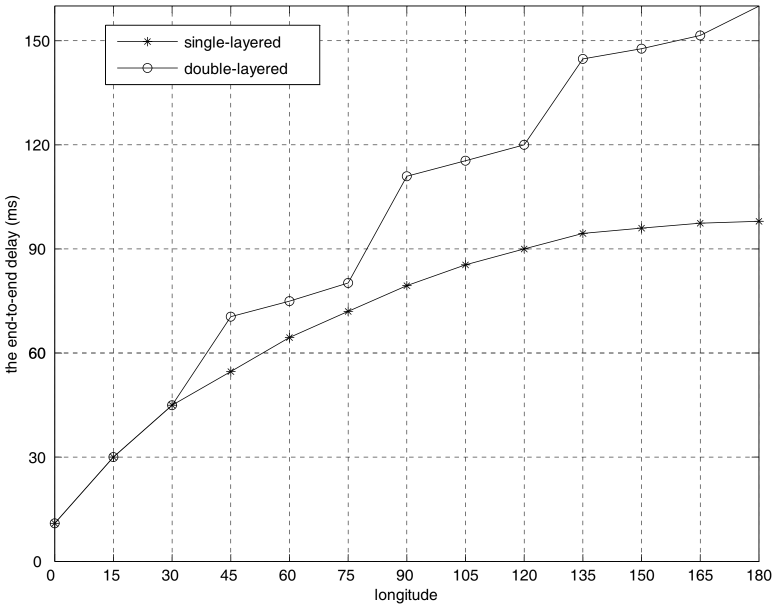

Fig. 8.10. The end-to-end delay when the link usage rate is 90%

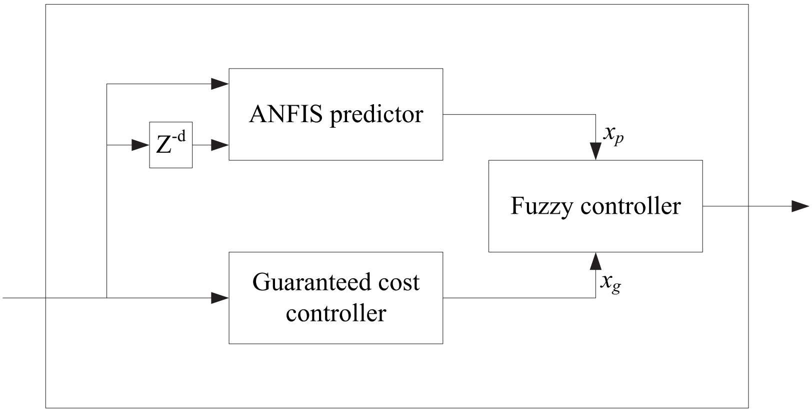

Fig. 8.11. The neuro-fuzzy controller of NCS

8.5.1 Neuro-fuzzy Predictor

In this subsection, a new method is proposed to improve the performance of the NCS by adding a predictor to estimate the plant state. The neuro-fuzzy predictor is computationally straightforward and has shown excellent prediction capabilities. So the decision is made to use the ANFIS [18]. The ANFIS predictor has two inputs: system states at time k and $k-d$, and produce the predicted state value at time $k+d$. The architecture of the ANFIS predictor is shown in Fig. 8.12, where d is the delay bound.

In the following description, ${u_i}^k$ denotes the ith input of a node in the kth layer, ${o_i}^k$ denotes the ith node output in layer k, and there are n input values.

The ANFIS predictor uses Gaussian functions for fuzzy sets. The reason is that a multidimensional Gaussian membership function can easily be decomposed into the product of one-dimensional membership functions. With this choice, the operation performed in this layer is

$$o_{ij}^2 = \exp\left\{-\frac{\left(u_i^2 - m_{ij}^2\right)}{\left(\delta_{ij}^2\right)^2}\right\}, \; i = 1, 2, \; j = 1, 2, \ldots, m, \tag{8.46}$$

where u_i^2 and δ_{ij}^2 are, respectively, the center and the width of the Gaussian membership function. The ANFIS predictor uses Gaussian functions for fuzzy sets, linear functions for the outputs, and Sugeno's inference mechanism [16]. The parameters of the network are the mean and standard deviation of the membership functions and the coefficients of the output linear functions. The ANFIS predictor learning algorithm is then used to obtain these parameters.

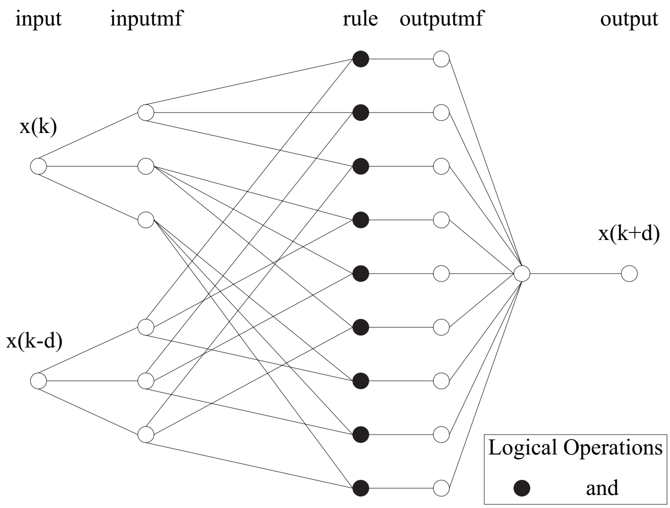

Fig. 8.12. The structure of the ANFIS predictor

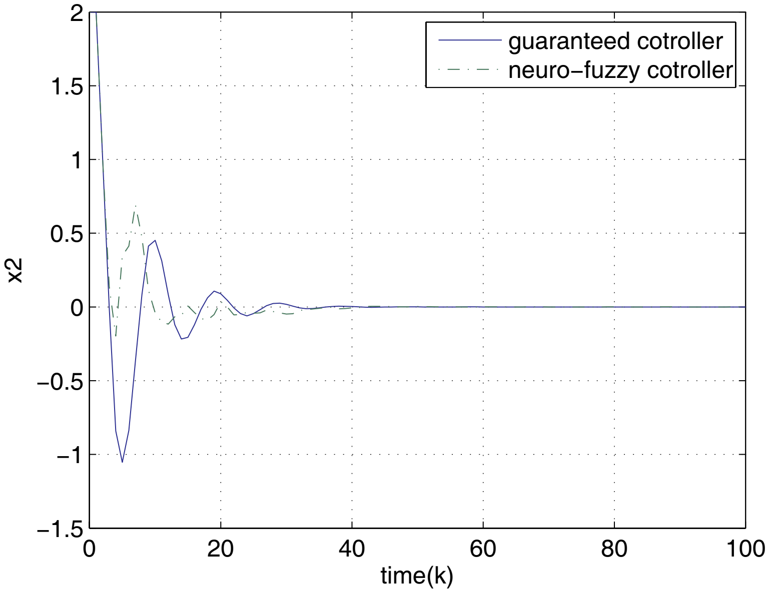

Fig. 8.13. State responses

The learning algorithm is a hybrid algorithm consisting of the gradient descent and the least squares estimate.

8.5.2 Fuzzy Controller

The fuzzy controller has two rules:

$$\text{Rule 1: IF } r(k) < d, \text{ THEN } u_k = K_i(H_i(z_k)) = x_g;$$
$$\text{Rule 2: IF } r(k) = d, \text{ THEN } u_k = K_i(H_i(z_k)) = x_p.$$

When $r(k) < d$, the guaranteed cost controller controls the system. When network delay is longer, the ANFIS predictor provides the predicted state at time $k+d$. In this way, the impact of network's longer delay can be moderated.

With the same example as in Section 8.4, better control performance of the neuro-fuzzy controller is illustrated in Figs. 8.13 and 8.14 for state and control inputs compared with the guaranteed cost controller.

8.6 Conclusions

In this chapter, we studied the problem of modeling and controller design for networked control systems, where a discrete-time jump fuzzy system is developed to model networked control systems with random but bounded delays

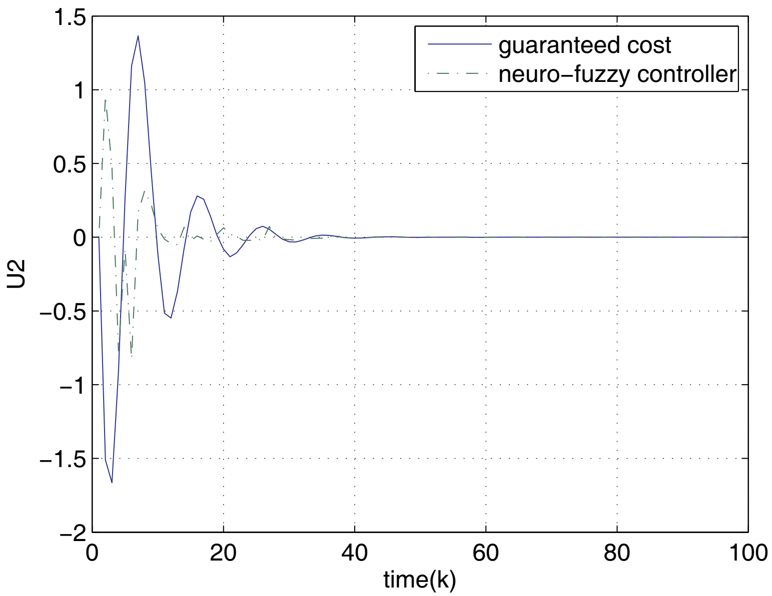

Fig. 8.14. Control inputs

and packets dropout. On the basis of the assumption that all state variables of an NCS are available, a state feedback controller is developed for the discrete-time jump system with sub-optimal guaranteed cost performance based on a piecewise quadratic Lyapunov function. It is shown that the state feedback sub-optimal fuzzy controller can be obtained by solving a set of LMIs using the homotopy approach. Because not all state variables are available in many practical cases, an output feedback fuzzy controller is proposed, which cannot only stabilize the system, but also meet certain desired sub-optimal system performance criteria. The LMI technique is used to effectively minimize the overall cost function and thus achieve the sub-optimal system. When the network is in a poor condition, a novel neuro-fuzzy controller including an ANFIS predictor is designed to deal with the problem. Finally, the effectiveness of the proposed approaches are verified by a numerical example.

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