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Coordination of Multi-agent Systems Using Adaptive Velocity Strategy

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Abstract. Collective behaviors of biological swarms have attracted significant interest in recent years, but much attention and correlative effort has been focused on constant speed models in which all agents are assumed to move with the same constant speed. One limitation of the constant speed models without attraction functions is that it is quite difficult or even practically impossible for the swarm to form large biological cluster(s) if the speed is relatively fast or the sensory radius is small. In this chapter, we propose an adaptive velocity model with more reasonable assumptions in which every agent not only adjusts its moving direction but also adjusts its speed based on the degree of direction consensus among its local neighbors. It is also a nearest neighbor rule but much easier for swarm agents to form a giant cluster or only one cluster in the adaptive velocity model if each agent moves with a speed that is proportional to its local direction consensus, even though the steady-state speed is still fast. The adaptive velocity strategy also shows that attraction actions or dominant leaders of swarms are not necessities for swarm cohesion. Therefore, the adaptive velocity model provides a powerful mechanism for coordinated motion in biological and technological multi-agent systems.

Keywords. Graph, multi-agent, power-law, swarm.

10.1 Introduction

The emergence of biological swarms is a beauty and wonder of nature [3, 23, 24]. It is common to see huge herds of animals or flocks of birds or schools of fish moving as if they were a single living creature. These swarms often travel in the absence of any leader/leaders or external stimuli, and agents in these swarms usually do not share any global information. How do they form a congregation and move? What collective behaviors and properties do they have? In recent years efforts have been devoted to modeling and exploring the dynamic properties of such systems which can roughly be divided into three approaches: Lagrangian approach [4, 9, 10, 11, 12, 17, 22], Eulerian approach [18, 27, 28, 29], and discrete approach [2, 5, 6, 8, 13, 15, 16, 19, 25, 30].

In 1987, Reynolds introduced three heuristic rules - cohesion, separation and alignment - to create the first computer simulated model of flocking [25]. Later on, Vicsek *et al.* proposed a simplified minimal model, which focused mainly on the emergence of directional alignment in self-driven particle systems [30]. In recent years, the Vicsek model has been one of the most frequently investigated swarm models using nearest neighbor rules to imitate swarming behaviors. For example, effects of noise and scaling behavior of the model were considered in [8]. Intermittency and clustering in self-driven particles [15] and the onset of collective motion [13] were also studied. Stability analysis of swarms revealed the relationship between network connectivity and the stability property [16, 19]. There are some other models that capture the important rule of the directional alignment used in the Vicsek model. For example, Couzin *et al.* showed that the alignment actions together with attraction/repulsion functions between neighboring agents can lead to complex patterns of swarms and revealed the existence of major group-level behavioral transitions [5]. Effective leadership was investigated in [6], which indicated that information owned by a few agents can be transferred within the whole group. Self-driven many-particle systems with general network topologies such as the vectorial network model (VNM) were investigated in [2].

All these researchers assumed that all agents in a swarm move with the same constant speed (i.e., absolute value of the velocity). However, we believe that in natural swarms, it is a more reasonable assumption that agents may not only adjust their moving directions in the swarming evolution but also adjust their speed according to the behavior of their neighbors. Indeed, when an agent finds itself surrounded by scattered moving agents, it may naturally feel at a loss to follow any direction, and may hesitate to move; in this dilemma, it is safer for the agent to move with a slower speed. On the other hand, if a certain moving direction is dominant, the agent may take this direction without hesitation and thus moves relatively fast. Similar analogies are often found in human lives and politics: when several different proposals or choices have nearly the same support or weights, individuals (or organizations) may find themselves embarrassed to decide on and thus little progress will be made in this situation; but when consensus is reached by dominant or all individuals, rapid progress tends to be made immediately. Another human-scale example is the rhythmic clapping in a concert hall after a good performance, which is suggested to be formed by each individual who tend to adjust the natural clapping frequency lower or higher according to his/her hearing [20], just as biological swarms, humans sometimes tend to do what their neighbors do.

In this chapter, we propose an adaptive velocity model in which each agent adjusts its velocity (i.e., both direction and speed) simultaneously according to the behavior of its neighbors. The direction adjustment consideration follows the same rule that used in the Vicsek model. To design our speed adjustment rule, we introduce the concept of local order parameter to measure the local degree of direction consensus (or local polarity) among the neighbors of an agent. At each time step, each agent will move along the average direction of its neighbors with a speed which is taken as the maximum possible speed scaled by a power-law function of the magnitude of its local order parameter. The power-law exponent $\alpha \geq 0$ reflects the willingness of each agent to move faster or slower based on the local degree of direction consensus among its neighbors. If $\alpha = 0$, then the adaptive velocity model reduces to the constant speed Vicsek model and each agent always moves with the maximum constant speed. However, if $\alpha > 0$, then an agent will move with the maximum speed if and only if complete local direction consensus is achieved among its neighbors. A larger value of α implies that an agent will move with a slower speed in the face of a given level of non-complete local direction consensus, which results in higher convergence probability that a group of initially randomly distributed agents will finally move along a global consensus direction.

This chapter is organized as follows. In Section 10.2, we describe briefly the constant speed model proposed by Vicsek *et al.* and compare two order parameters to measure the phase transition phenomena of the swarm. In Section 10.3, we propose an adaptive velocity model with a tunable parameter α based on the concept of local order parameter. Simulation results and discussions are given in Section 10.4. Conclusions are given in Section 10.5.

10.2 The Constant Speed Vicsek Model

We first describe the original constant speed Vicsek model [30]. Consider N agents, labeled from 1 through N, all moving synchronously in a square shaped cell of linear size L with periodic boundary conditions. Each agent has the same absolute velocity v_0 but with different direction at different time steps. Originally, all agents' positions are randomly distributed in the cell with randomly distributed directions in $[0, 2\pi)$. At each time step, agent i adjusts its direction as the average moving direction $\langle\theta_i(k)\rangle_R$ of its neighbors with some random perturbation $\Delta\theta$ added:

$$\theta_i(k+1) = \langle\theta_i(k)\rangle_R + \Delta\theta. \tag{10.1}$$

Here, the neighbors of agent i are defined as those agents who fall in a circle of predefined sensory radius R centered at the current position of agent i. One characteristic of this homogeneous model is that only by local interactions it shows phase transition through spontaneous symmetry breaking of the rotational symmetry. The different pattern behaviors, such as large-scale emergence, convergence and disordered disperse motion, can be observed under different parameters using simulation [30]. This directional rule of local interactions together with constant speed motion of agents has considerable influences [2, 5, 6, 8, 13, 15, 16, 19, 25, 30]. The swarm model in [5] is another important constant speed model that consists of homogeneous agents with directional alignment, attraction and repulsion rules, the emergences are generated by spontaneous symmetry breaking. Certainly, attraction action between agents is another reasonable consideration to form gathering and to have considerable influence. We will show that attraction action is not a necessity for large swarm clusters.

The following order parameter has been widely adopted to measure the phase transition phenomena of the constant speed model from the initial zero net transport to emergence [2, 5, 7, 14, 15, 30]:

$$\Phi_v(k) = \frac{1}{Nv_0}\left|\sum_{i=1}^{N}\vec{v_i}(k)\right|, \ 0 \le \Phi_v(k) \le 1. \tag{10.2}$$

Here $\vec{v_i}(k)$ is the velocity of agent i with direction $\theta_i(k)$ and the constant speed $v_0 = |\vec{v_i}(k)|$ for all $i = 1, 2, \ldots, N$ at all steps k. $\Phi_v(k)$ is a univocal physical parameter by definition – a scaled average momentum of the whole system and emergent behavior can be observed if $\Phi_v(k) \gg 0$. $\Phi_v(k) = 0$ corresponds to the isotropy state of directional distribution and $\Phi_v(k) = 1$ implies convergent or linear coherent motion of all agents only on the prerequisite that all agents have the same fixed speed v_0.

Now suppose that different agents may have generally different speed at different time steps. Let v_0 be the average value of all agents' possible maximum speeds, that is, $v_0 = \frac{1}{N}\left|\sum_{i=1}^{N} v_{i0}\right|$, where v_{i0} is the maximum possible speed of agent i. In this general case, it is possible that $\Phi_v(k) > 1$ even if the moving directions of all agents are isotropic which corresponds to a non-emergence state. And $\Phi_v(k) = 1$ does not necessarily mean linear coherence, unless v_{i0} is the same value for all agents. Thus $\Phi_v(k)$ is not appropriate to measure the level of emergence.

Another order parameter that has been widely adopted, especially for synchronous characteristic in the networked phase oscillators, is defined as follows [13, 26]

$$\Phi_\theta(k) = \frac{1}{N}\left|\sum_{i=1}^{N} e^{i\theta_i(k)}\right|, \quad 0 \le \Phi_\theta(k) \le 1. \tag{10.3}$$

This order parameter eliminates the influence of the agent's speed, but at the expense of having no physical meaning of scaled average momentum. For the constant speed Vicsek model, it is obvious that the two order parameters defined above are the same, i.e., $\Phi_v = \Phi_\theta$.

10.3 The Adaptive Velocity Model

In this section, we propose an adaptive velocity model in which each agent adjusts its direction and speed at different time steps simultaneously. To do so, we first define the complex-valued *local order parameter* of agent i at step $k+1$ as follows:

$$\phi_i(k+1)e^{i\theta_i(k+1)} = \frac{1}{n_i(k+1)} \sum_{j \in \Gamma_i(k+1)} e^{i\theta_j(k)}, \; i = 1, 2, \ldots, N; \; k = 0, 1, \ldots, \tag{10.4}$$

where $e^{i\theta_j(k)}$ is the unit directional vector and $\Gamma_i(k+1)$ is the set of $n_i(k+1)$ neighbors of agent i at step $(k+1)$. Magnitude (or local polarity) $\phi_i(k+1)$ of the local order parameter measures the local degree of direction consensus among the neighbors of agent i at step $(k+1)$. Obviously, $\phi_i(k+1)$ is a local form of the global order parameter (10.3) and $0 \leq \phi_i(k+1) \leq 1$. A larger value of $\phi_i(k+1)$ implies a higher degree of local direction consensus among neighbors of agent i (Fig. 10.1). Angle $\theta_i(k+1)$ is the corresponding moving direction of agent i at step $(k+1)$, which is the average directions of agents in set $\Gamma_i(k)$. Computations using this expression can also avoid some undesired directional problems mentioned in [16].

Denote $\mathbf{X}_i(k)$ as the position of agent i on the complex plane at step k. In our adaptive velocity model, each agent not only adjusts its moving direction, but also adjusts its speed according to the degree of local direction consensus among its neighbors, which is represented by its local polarity. Specifically, the speed of agent i at step k is scaled by a power-law function of its local polarity, i.e.,

$$c(\phi_i(k)) \triangleq \phi_i^{\alpha}(k) \tag{10.5}$$

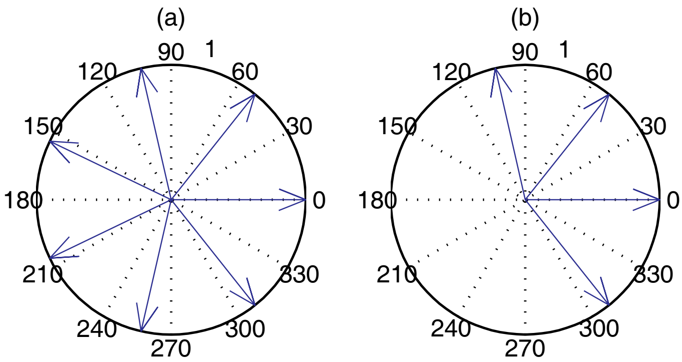

Fig. 10.1. Illustration of local polarity ϕ_i of agent i. The arrows show the moving directional vectors of neighboring agents of agent i. For simplicity, these modular vectors are plotted with the same starting points located in the center of a circle. (a) The collection of agents moving scattered in the plane with no dominant direction, the order parameter $\phi_i \approx 0$ for this situation. (b) The agents with a relatively strong dominant direction, $\phi_i \neq 0$ for this situation. The polarity $\phi_i = 0$ if and only if all the agents in set $\Gamma_i(k)$ move in the same direction.

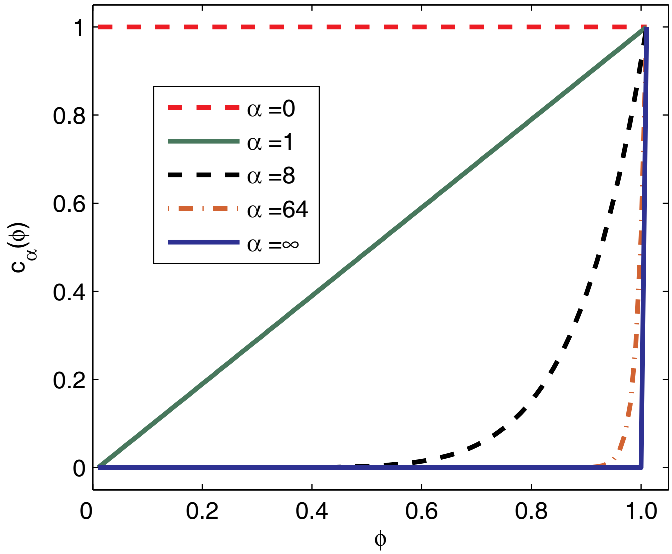

Fig. 10.2. Scaled speed coefficient $c_\alpha(\phi)$ as a power function of local polarity ϕ. For any value of α, $c_\alpha(\phi) = 1$ if $\phi = 1$. For $\alpha = 0$, $c_\alpha(\phi) \equiv 1$. For $0 < \alpha < \infty$, $0 < c_\alpha(\phi) < 1$ if $0 < \phi < 1$. For $\alpha = \infty$, $c_\alpha(\phi) = 0$ if $0 < \phi < 1$.

with an power-law exponent $\alpha \geq 0$ (Fig. 10.2).

The adaptive velocity model can then be described mathematically as follows:

$$\begin{cases} \mathbf{X}_i(k+1) = \mathbf{X}_i(k) + v_0 \times \phi_i^\alpha(k) e^{i\theta_i(k)} \times \Delta t \\ \phi_i(k+1) e^{i\theta_i(k+1)} = \dfrac{1}{n_i(k+1)} \displaystyle\sum_{j \in \Gamma_i(k+1)} e^{i\theta_j(k)} \end{cases} \tag{10.6}$$

$i = 1, 2, \ldots, N$; $k = 0, 1, 2, \ldots$, where Δt is the discrete time interval, and here without loss of generality, we take $\Delta t = 1$. $\vec{v_i}(k) \equiv v_0 \times \phi_i^\alpha(k) e^{i\theta_i(k)}$ represents the velocity of agent i at step k with its moving direction $\theta_i(k)$. Since $0 \leq \phi_i^\alpha(k) \leq 1$ for any value of $\alpha \geq 0$, the corresponding speed $|\vec{v_i}(k)| = v_0 \times \phi_i^\alpha(k)$ satisfies $0 \leq \vec{v_i} \leq v_0$.

This adaptive speed is another important factor that contributes to emergence or swarming clusters that has been previously overlooked, especially for swarms in three or higher dimensions. This adaptive speed model also satisfies fundamental swarm's characteristics: no any leader/leaders, no external stimuli, only homogeneous agents, and only local interactions, but induces more intensified phase transition and symmetry-broken from disordered to ordered state than the constant speed model.

The power-law exponent $\alpha \geq 0$ reflects the willingness of each agent to move faster or lower along the average direction of its neighbors based on the local degree of direction consensus. If $\alpha = 0$, then $c_\alpha(\phi) \equiv 1$. The adaptive velocity model (10.6) reduces to the constant speed Vicsek model and each agent always moves with the maximum constant speed v_0 without any considerations about its local polarity. However, if $\alpha > 0$, then an agent will move with the maximum speed if and only if complete local direction consensus is achieved among its neighbors. In the case of $\alpha = 1$, the local order parameter of agent i is just the direct sum of directional vectors of agent i's neighbors. A larger value of α implies that an agent will move with a slower speed in the face of a given level of local direction consensus. In the limit case that $\alpha = \infty$, we have

$$c_\infty(\phi) = \phi^{+\infty} = \begin{cases} 0, & 0 \leq \phi \leq 1, \\ 1, & \phi = 1. \end{cases} \tag{10.7}$$

It means that each agent will not move unless complete local direction consensus is achieved among its neighbors.

The 2-dimensional adaptive velocity model (10.6) can easily be generalized to general M-dimensional Euclidean space case. Let $P_i = [p_{i1}, p_{i2}, \ldots, p_{iM}]^T$ represent position of agent i, $i = 1, 2, \ldots, N$. The motion direction of agent i is represented by a unitary vector $d_i = [d_{i1}, d_{i2}, \ldots, d_{iM}]^T$ which satisfies

$$\|d_i\| = 1, \ -1 \leq d_{ij} \leq 1, \ j = 1, 2, \ldots, M, \tag{10.8}$$

for all i. Agent i and agent j are neighbors if $\|p_i(k) - p_j(k)\| \leq R$.

Define the order parameter as

$$r_i(k+1) = \frac{1}{n_i(k+1)} \left\| \sum_{j \in \Gamma_i(k+1)} d_j(k) \right\|. \tag{10.9}$$

Of cause, $0 \leq r_i(k+1) \leq 1$. The M-dimensional adaptive velocity model can be described as:

$$P_i(k+1) = P_i(k) + v_0 \times r_i^\alpha(k) \times d_i(k) \times \Delta t, \ k = 0, 1, 2, \ldots, \tag{10.10}$$

$$d_i(k+1) = \left(\sum_{j \in \Gamma_i(k+1)} d_j(k) \right) \Big/ \left\| \sum_{j \in \Gamma_i(k+1)} d_j(k) \right\|, \ k = 0, 1, 2, \ldots. \tag{10.11}$$

10.4 Simulations and Discussions

It is a more natural assumption for swarms moving in the plane to ensure that they can evolve freely and sufficiently. To illustrate the effect of adaptive velocity strategy, we consider N agents moving in the complex plane for simulation instead of in a rectangle of open boundary or periodic boundary

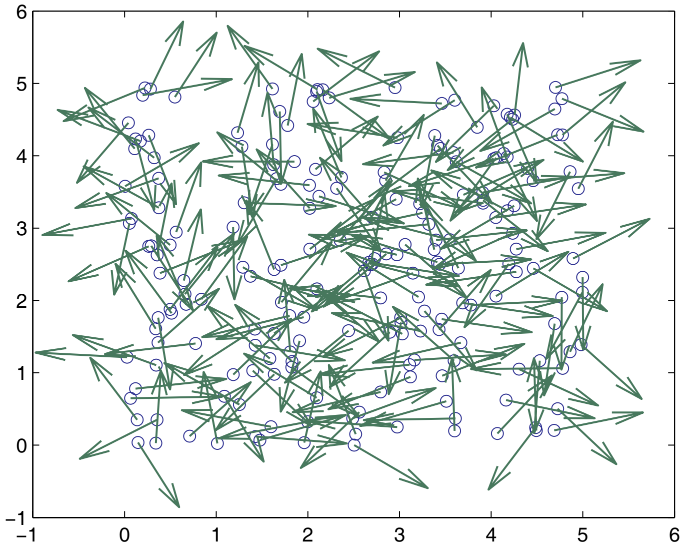

Fig. 10.3. Illustration of initially random distribution of positions and directions of agents, the arrows point to the initial directions of agents, the ends of arrows (denoted by blue circles) are positions of agents. Here the rectangle is 5×5 cell. The swarm evolves in the whole 2D plane.

conditions [30]. The N agents' positions and directions are initially randomly distributed on a rectangle of linear size L (Fig. 10.3). Denote the initially distributed directions and positions of agent i as θ_i and $P_i(0)$, respectively, $i = 1, 2, \ldots, N$. Note that the initial distribution of direction θ_i is not the initial moving direction $\theta_i(0)$. We compute the initial moving direction $\theta_i(0)$ and initial polarity $\phi_i(0)$ of agent i according to local order parameter formula

$$\phi_i(0)e^{i\theta_i(0)} = \frac{1}{n_i(0)} \sum\nolimits_{j \in \Gamma_i(0)} e^{i\theta_j}.$$

This means that each agent moves with adaptive velocity strategy in the very beginning of its evolution. This beginning step is denoted as step $k = 0$ with the corresponding initial speed $v_0 \times \phi_i(0)$.

In simulations, we take the parameters $N = 300$, $L = 5$ and $R = 2$. All estimates are the results of averaging over 400 realizations, if without special mention. We first investigate the influence of power-law exponent α in the adaptive velocity model on the convergence probability p, which is defined as the probability that a group of N initially randomly distributed agents will finally all move along a global consensus direction with the same maximum speed v_0.

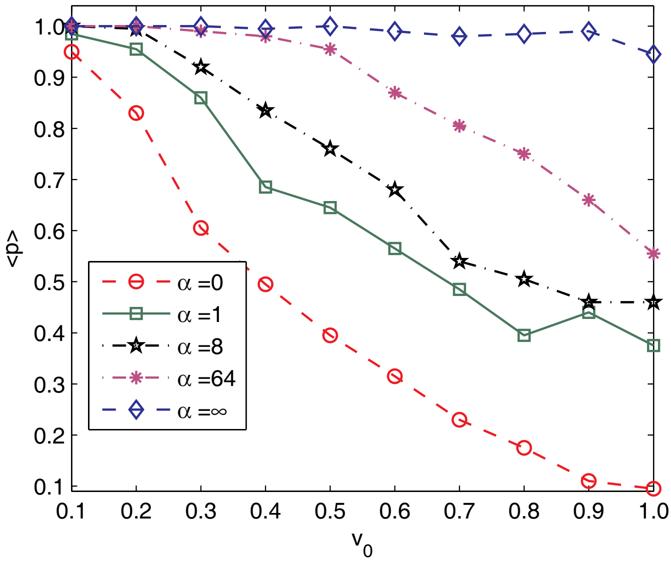

Fig. 10.4. Convergent probability p as a function of the maximum speed v_0 for five different values of α

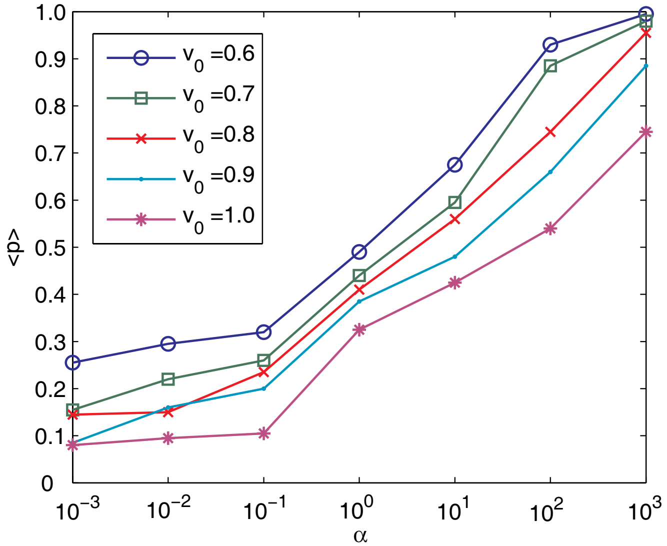

Fig. 10.5. Convergent probability p as a function of the exponent α for five different values of v_0

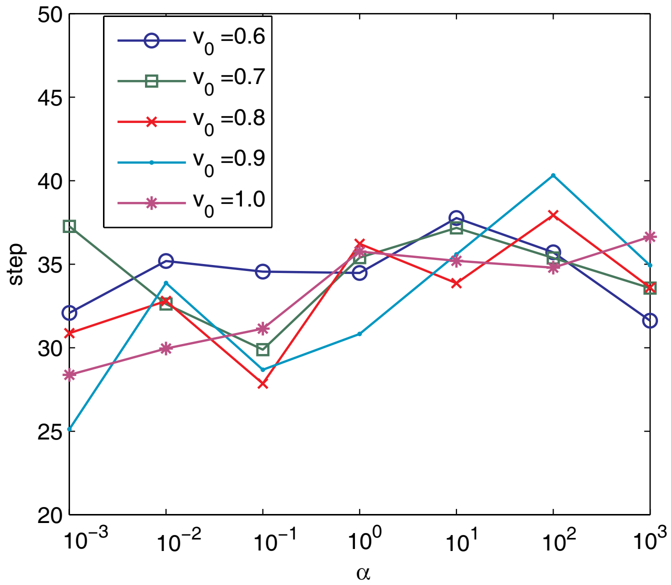

Fig. 10.6. Transient time step as a function of the exponent α. The termination condition for steady state of the swarms is that, the standard deviation of N vectors that consist of the conterminous directional differences of every agent is less than 0.0001. The quantities are averaged over 200 realizations.

Fig. 10.4 shows that for any given value of α, the convergence probability p is a decreasing function of the maximum speed v_0, but it decreases more slowly for larger value of α; while Fig. 10.5 shows that for any given value of v_0, the convergence probability p is an increasing function of the exponent α, and smaller v_0 leads to higher convergence probability. Therefore, if the constant speed v_0 is large enough, even though it is very difficult or even practically impossible to achieve global convergence in the original Vicsek model which corresponds to $\alpha = 0$, the convergence probability can still be high for the adaptive velocity model with a sufficiently large α. In particular, the convergence probability approaches 1 in the case $\alpha = \infty$ for the present system parameters, even without any leader or other global information in the adaptive velocity model.

Note that the dynamic speeds of all agents will always reach the same maximal value v_0 in steady state whether the swarm can finally converge or not; but directions of agents will reach global consensus only under certain conditions. Generally, speeds of agents in the adaptive velocity model are varied over transient time and the average speed $v_{\text{ave}}(k)$ of all agents in the swarm increases monotonically until steady state is achieved. Since a larger value of α implies that an agent will move with a slower speed in the face of a

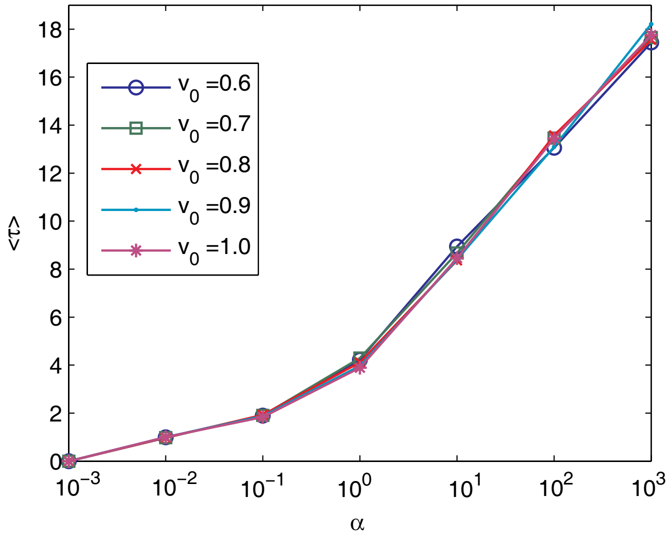

Fig. 10.7. The time step τ required for the average speed of all agents to reach 98% of maximum speed v_0 as a function of the exponent α. All estimates are the results of averaging over 200 realizations.

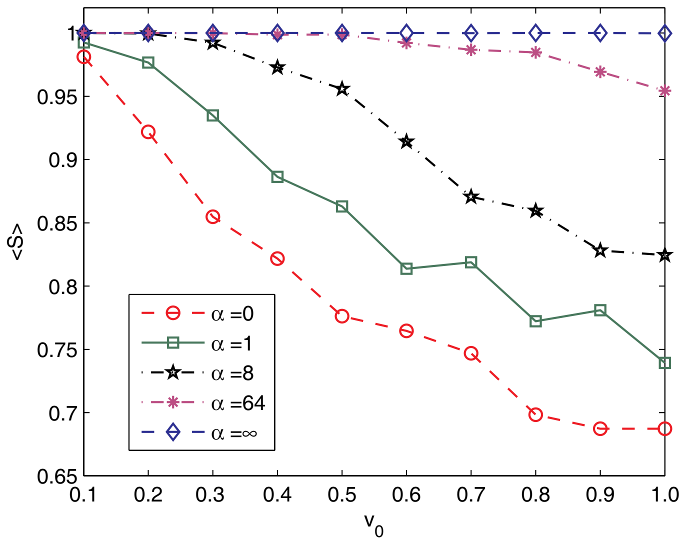

Fig. 10.8. MCSG in steady state as a function of the maximum speed v_0 for five different values of α

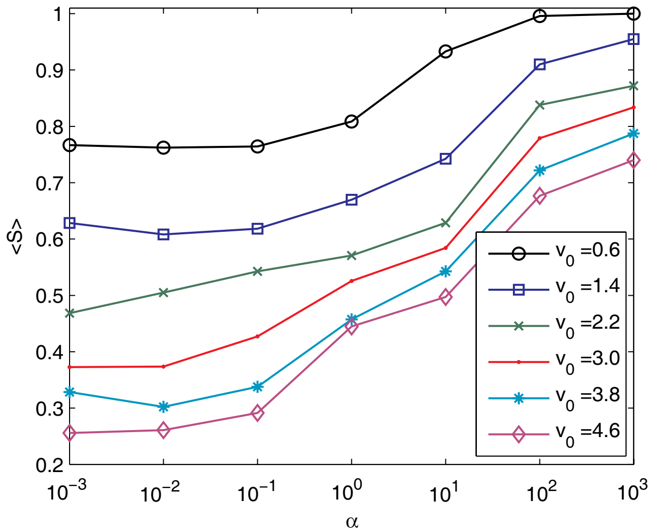

Fig. 10.9. MCSG in steady state as a function of the exponent α for six different values of v_0

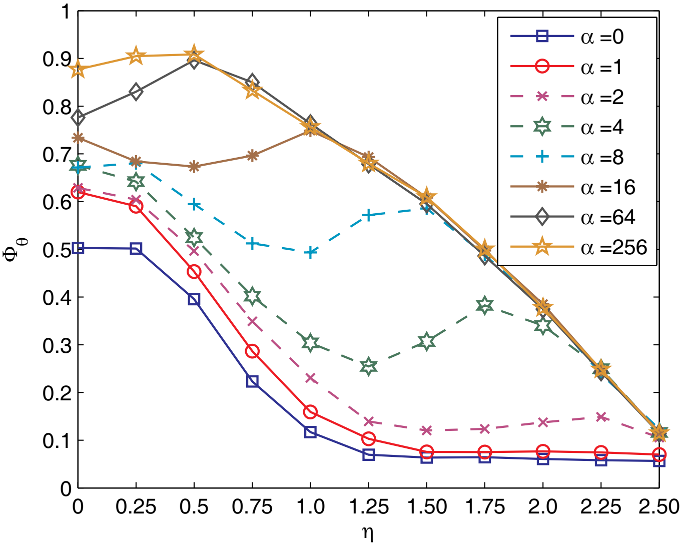

Fig. 10.10. The global order parameter Φ_θ of swarm as a function of noise amplitude η. For large η and α, Φ_θ decreases linearly. All estimates are the results of averaging over 200 realizations.

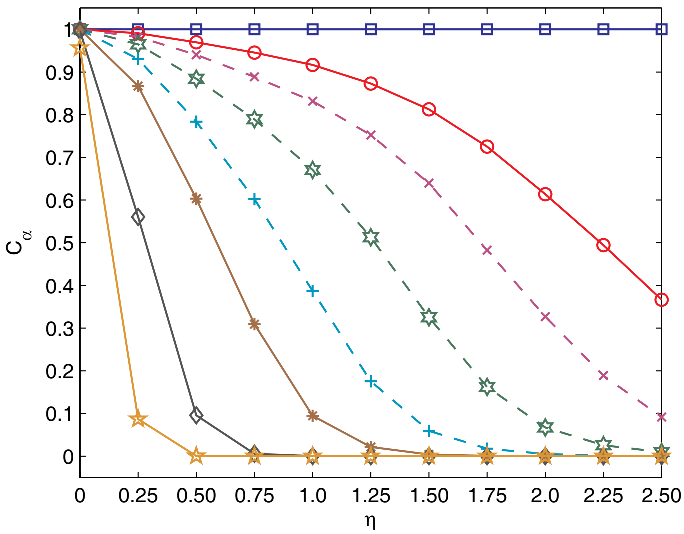

Fig. 10.11. The average speed coefficient C_α decreases to zero as the noise amplitude η increases. All estimates are the results of averaging over 200 realizations.

given level of local direction consensus, one may wonder if the transient time may be longer even though the convergence probability is higher. However, as can be seen from Fig. 10.6, the value of α does not have a significant influence on the transient time. Denote τ as the time step required for the average speed of all agents to reach 98% of maximum speed v_0. We find that τ obeys a simple log scaling law of the form (Fig. 10.7):

$$\tau \approx 4 + \beta \left[\log_{10} \alpha\right], \ \ \alpha \geq 1, \tag{10.12}$$

where $\beta \approx 4.67$. Therefore, even for a high value of $\alpha = 1000$, most agents will move with nearly the maximum speed in just less than 18 steps. This behavior looks somewhat like the applause phenomenon which turns suddenly into synchronized clapping [20].

Why is the convergence probability enhanced as the exponent α increases in the adaptive velocity model? This is because the adaptive velocity strategy with large value of α tends to hold the local agents together to form large cluster. When in the approximate isotropy region, $\phi \approx 0$ which implies agents move in scattered directions, the speeds of agents are relatively small according to adaptive velocity strategy with positive value of α. Even for $0 \ll \phi < 1$, the speeds of agents are still small for large values of α. Thus transformations of those agents' positions are indistinctive, so neighbors tend to be also neighbors in the next step or even later, and communications between them continue to be held, which are beneficial to directional consensus.

From the perspective of the complex network theory [1, 21], swarm topology can also be expressed as a graph $G = (V, E)$: every agent i is represented by a vertex v_i; an undirected edge between agent i and agent j means that they are neighbors and *vice versa*. The component of a graph to which a vertex belongs is that set of vertices that can be reached from it by paths running along edges of the graph [21]. As time evolves, topology of the graph $G(k) = (V, E(k))$ varies. We are interested in the maximal component of the swarm graph (MCSG) in the steady state. Recent analysis shows that for a swarm which moves in the plane instead of in a rectangle of periodic conditions, convergence or emergence is due to the connectivity between agents [16, 19], instead of long-range interactions [30, 31].

Denote S as the ratio of the number of vertices in MCSG in steady state versus the total number of vertices in the whole graph of the underlying swarm. Clearly, $0 < S \leq 1$ and global convergence is achieved if and only if $S = 1$. In this case, the whole graph consists of only one component (swarming cluster). $S \approx 0$ means all the agents disperse without any apparent clusters. For $S \gg 0$, there exists a dominate or giant cluster in the swarm.

For any given value of α, MCSG is understandably a decreasing function of the maximum speed v_0, and it decreases much more slowly for larger values of α (Fig. 10.8); while for any given value of v_0, MCSG is an increasing function of the exponent α and smaller values of v_0 result in higher values of MCSG (Fig. 10.9). Thus, in the case of a large maximum speed v_0, although it is quite difficult or even impossible to form a giant cluster in the constant speed Vicsek model which corresponds to $\alpha = 0$, it is much easier to form a giant cluster for the adaptive velocity model if α is large enough. This also indicates that attractive actions between agents is not a necessity for swarm aggregations.

Figs. 10.10 and 10.11 show the influence of uniformly distributed noise added to the moving direction of each agent with noise amplitude η based on the global order parameter Φ_θ defined in (10.3) and the average speed coefficient C_α defined as:

$$C_\alpha \triangleq \frac{1}{N} \sum_{i=1}^{N} c_\alpha(\phi_i) = \frac{1}{N} \sum_{i=1}^{N} \phi_i^\alpha. \tag{10.13}$$

We can see from Fig. 10.10 that for large noise amplitude η and large exponent α, the global order parameter Φ_θ decreases along the same straight line, which deserves further investigation.

Comparing Figs. 10.10 and 10.11, one finds that the more robust the speed, the less emergence the swarm in the exposure of noise. The value $\alpha = 0$ corresponds to constant speed and it is the least anti-noise case of the swarm.

10.5 Conclusions

We propose an adaptive velocity model in which each agent not only adjusts its moving direction but also adjusts its speed based on the local degree of

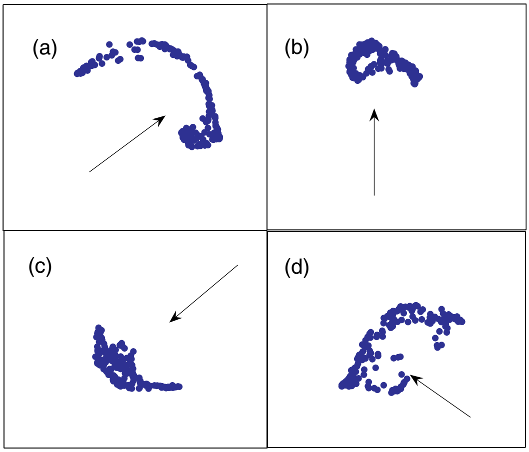

Fig. 10.12. Some interesting shapes that swarms take on. These are all coherent moving cases. The arrows denote the coherent moving direction of swarms. The parameters here are $N = 200, R = 1.2, v_0 = 0.4, \alpha = 0$.

direction consensus among its neighbors at every time step. Each agent takes its moving direction as the average angle of its local order parameter with its speed proportional to the power function of the magnitude of its local complex-valued order parameter at each step. The adaptive velocity model reduces to the constant speed Vicsek model when the power-law exponent $\alpha = 0$. A larger value of α implies that an agent will move with a slower speed in the face of a given level of non-complete local direction consensus, which results in higher convergence probability and larger swarm clusters.

Some difficult yet important problems about the adaptive velocity model remain to be further investigated. For example, under what conditions can we guarantee the existence of a critical value of α such that above the value, a given convergence probability or average MCSG can be guaranteed? Furthermore, stability analysis about the linearized Vicsek's model has been focused on the topology of swarms in the process of evolution [16], but the question of what initial distribution condition of the underlying swarm can guarantee this topology restriction remains unsolved. More practical stability analysis for the adaptive velocity model needs to be explored.

The properties of evolutional graphs of swarms over time may serve as a promising topic for further research. Unlike regular (or quasi-regular) geometric shape the attraction–repulsion models [5, 14, 22] take on (see the figures in the reference papers), what shapes (for example, see Fig. 10.12) the non-attraction-repulsion models, such as adaptive velocity model, will take in the coherent moving state also remains elusive. These questions remain interesting and challenging for further investigation.

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