14

Statistical Process Control

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Abstract: Statistical process control (SPC) is a tool used for on-line quality control in mass production. Statistical sampling theory is effectively used for this purpose in the form of control charts. Various types of control charts have been developed in industry for controlling different types of quality characteristics. The basic principles of development, design and application of various types of control charts are discussed in this chapter. The state of the art and recent developments in SPC tools are included with references for further research. A separate section on process capability studies is also included.

14.1 Introduction

The concepts of quality are as old as human civilization. It has been a constant endeavor of any society or culture to design and develop finest pieces of quality in all walks of life. This is visible in many of the human made world wonders such as the Taj Mahal of India, the pyramids of Egypt, the high roads and sculptures of the Roman Empire, the finest paintings of Renaissance Europe, or the latest developments such as space shuttles, super computers, or atomic power generation. However, quality as a science or as a formal discipline has developed only during the 20th century. Quality has evolved through a number of stages such as inspection, quality control, quality assurance, and total quality control.

The concepts of specialization, standardization, and interchangeability resulted in mass production during the Second World War. This also changed the traditional concepts of inspection of individual products for quality control. It was found that applications of statistical principles are much more practical and beneficial in mass production. Statistical sampling theory, for instance, helped to minimize the need of resources for quality control with acceptable levels of accuracy and risk. The concept of statistical process control (SPC) has now been accepted as the most efficient tool for on-line quality control in mass production systems. SPC uses control charts as the main tool for process control. The control chart is one of the seven tools for quality control. Fishbone diagrams or Ishikawa diagrams check sheets, histograms, Pareto-diagrams, scatter diagrams, and stem and leaf plots are other tools. They are discussed in detail in [1]. This chapter focuses on SPC using control charts.

14.2 Control Charts

The control chart is a graphical tool for monitoring the activities of a manufacturing process. The numerical value of a quality characteristic is plotted on the Y-axis against the sample number on the X-axis. There are two types of quality characteristics, namely variables and attributes. The diameter of shafts, the strength of steel structures, service times, and the capacitance value of capacitors are examples of variable characteristics. The number of deformities in a unit, and the number of nonconformities in a sample are examples of attribute quality characteristics. A typical control chart is shown in Figure 14.1.

As shown in this figure, there is a centerline to represent the average value of the quality characteristic. It shows where the process is cantered. The upper control limit (UCC) and the lower control limit (LCL) on the control chart are used to control the process. The process is said to be in statistical control if all sample points plot inside these limits. Apart from this, for a process to be in control the control chart should not have any trend or nonrandom pattern.

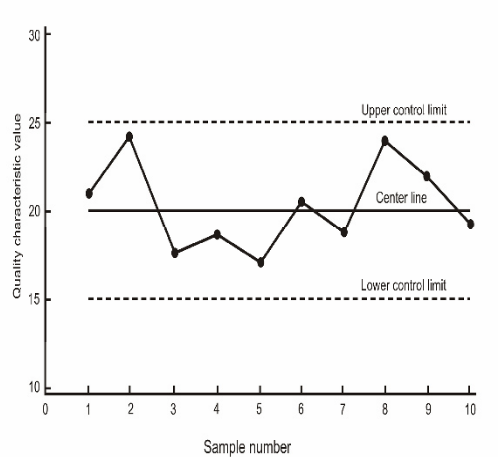

Figure 14.1. A typical control chart

14.2.1 Causes of Process Variation

Many factors influence the manufacturing process, resulting in variability. For example, variation in raw materials, skills of operators, capabilities of machines, methods, management policies, and many other factors including environmental variations affect the performance of a process. The causes of process variability can be broadly classified into two categories, *viz.*, assignable causes and chance causes.

Assignable Causes

If the basic reason for the occurrence of a cause of process variation can be found, then we list it under the category of assignable causes. Improper raw materials, usage of inappropriate cutting tools, carelessness of machine operators, *etc.*, are examples of this. Such causes are also known as special causes. The basic purpose of using control charts is to identify the presence of assignable causes in the process and to eliminate these so as to bring back the process to statistical control.

Chance Causes

These are natural causes inherent in any process. The basic reasons for the occurrence of such causes cannot be correctly established. Elimination of such causes is also not possible in actual practice. A process is said to be out of control if any assignable cause is present in the process. Inherent material variations, operator skills, environmental conditions, and machine vibration are examples of chance causes. These are also known as common or random causes.

It is found that the assignable causes result in large variations in the process parameters whereas chance causes bring only small variations. It is reported that about 15% of the causes of variation are due to assignable causes and the remainder are due to chance causes for which only the management is accountable [2]. It is very important to remember that a process under statistical control will have some variations due to chance causes. In fact, the control limits are designed based on this principle.

Statistical Basis

Control charts are formulated based on the properties of the normal distribution [3]. The central limit theorem [1] states that if we plot the sample average of a process parameter, it will tend to have a normal distribution. The normal distribution is described by its parameters mean

(μ) and standard deviations (σ). For a normal distribution it can be shown that 99.74% of all points fall within the 3σ limits on either side of the mean. The upper and lower control limits of the control chart are determined based on this principle. This means that almost all the data points will fall within 3σ control limits if the process is free from assignable causes.

Errors in Control Charts

Two types of errors are associated with using the control charts. These are type I error and type II error. Type I error is the result of concluding that a process is out of control (based on actual data plotted on the chart) when it is actually in control. For a 3σ control chart this chance (α) is very small (about 0.0026). Type II error is the result of concluding that a process is in control (based on actual data plotted on the chart) when it is actually out of control. This may happen under many situations, such as the process mean changes from its initial setup, but all sample points fall within the control limits. The probability of type II error is generally represented by β and it is evaluated based on the amount of process change and the control limits. A plot of β *versus* the shifting process parameter is known as the operating characteristic (OC) curve of a control chart. The OC curve is a measure of the ability of a control chart to detect the changes in process parameters. A good control chart should have an OC curve as shown in Figure 14.2. For small changes in the process parameter, the probability of nondetection (β) by the control charts is high. For large changes in the process parameter, β should be small so that it is detected and corrected by the control chart.

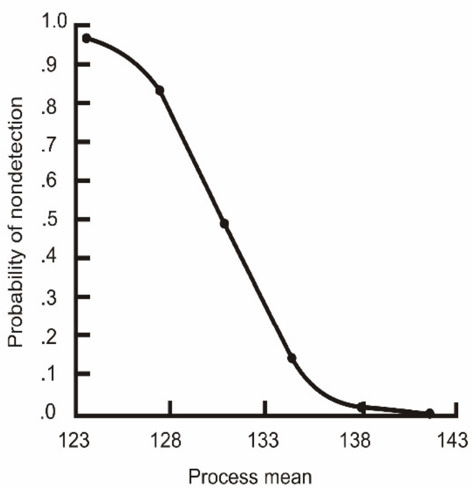

Figure 14.2. Typical OC curve for control charts

Average Run Length (ARL)

The average run length (ARL) is another measure of the performance of a control chart. It is the number of samples required to detect an out-of-control by a control chart. It is measured as reciprocal of type I error α .

$$ARL = \frac{1}{\alpha} .$$

For a 3σ control chart, $ARL = \left(\frac{1}{0.0026}\right) = 385$.

This shows that on an average one sample point is expected to fall out of 385 sample points outside the control limits. A large ARL is preferred since it produces fewer false alarms in a control chart.

Other Considerations

As mentioned earlier, control charts are plotted by taking small samples from the manufacturing process on a regular basis. Therefore, selection of sample size is very important in using the control charts.

Sample Size

It can be shown that a larger sample size results in narrow control limits. Decreasing the sample size makes the control limits wider. A larger sample size is needed if the small shift in the process parameter needs to be detected early. Apart from these factors the selection of sample size is influenced by the availability of resources, the types of tests used for sample evaluation, production rate, *etc*.

14.2.1.8 Frequency of Sampling

Theoretically it is most beneficial if we have more frequent large sample sizes. The type of inspection and the resource constraints are the main factors influencing the selection of these. In most practical situations a small sample size at frequent intervals is preferred.

Decision Rules for Control Charts

Five rules are used to detect when a process is going out of statistical control. These are briefly discussed below:

Rule 1: A process is going out of control if a single point plots outside the control limits.

Rule 2: A process is going out of control if two out of three consecutive points fall outside the 2σ warning limits on the same side of the centerline.

Rule 3: A process is going out of control if four out of five consecutive sample points fall outside the 1σ limits on the same side of the centerline.

Rule 4: A process is going out of control if nine or more consecutive points fall to one side of the centerline.

Rule 5: A process is going out of control if six or more consecutive sample points run up or down.

Applications of Control Charts

Control charts have several applications. This helps us in the following decision making:

1. To decide when to take corrective actions and when to leave the process as it is.
2. They give indications of type of remedial actions necessary to bring the process to control.
3. They help us to estimate the capability of our process to meet certain customer demands or orders.
4. They help us to improve quality.
5. They help us to take decisions such as the need for machine or technology replacement to meet quality standards.

Quality control and improvement are ongoing activities and, therefore, control charts must be maintained or revised as and when changes occur in the process. Installation of a new machine or application of a new technology necessitates the development of new control charts.

As mentioned earlier, the quality characteristics are broadly of two types. These are variables and attributes. Variable characteristics are continuous in their range where as attributes are discrete. Therefore, control charts are broadly classified into two categories, *viz.*, control charts for variables and for attributes.

14.3 Control Charts for Variables

Quality characteristics that can be measured on a numerical scale such as diameter of a shaft, length of a component, strength of a material and weight of a part are known as variables. Process control means controlling the mean as well as the variability of the characteristic. The mean of the variable indicates the central tendency and variability indicates the dispersion of the process. Variability is measured in terms of the range or standard deviation. Various types of control charts are discussed in the following sections.

14.3.1 Control Charts for Mean and Range

These charts are used to control the process mean and its variations. This is because the process control is ensured only if its mean is located correctly and its spread is kept within its natural limits. Therefore, these charts are used in pairs. The following steps are generally used for designing these control charts:

Step I: Decide the sampling scheme (sample size, number of samples, and frequency of sampling) and the quality characteristic to be controlled.

Step II: Collect the samples randomly from the process, measure the quality characteristic, and enter it into a data sheet. Let n be the sample size and X_i be the *i*-th observation, $i = 1...n$.

Step III: For each sample (j) calculate mean and range using the following equations ($j = 1...g$).

$$\overline{X}_j = \frac{\sum_{i=1}^{n} X_i}{n} \quad , \tag{14.1}$$

$$R_j = X_{j\max} - X_{j\min} \quad . \tag{14.2}$$

Step IV: Estimate the centerline (CL) and trial control limits for both mean and range charts using the following equations:

$$CL_{\overline{X}} = \overline{\overline{X}} = \frac{\sum_{j=1}^{g} \overline{X}_j}{g} \quad , \tag{14.3}$$

$$CL_R = \overline{R} = \frac{\sum_{j=1}^{g} R_j}{g} \quad , \tag{14.4}$$

$$\left(UCL_{\overline{X}}, LCL_{\overline{X}}\right) = \overline{\overline{X}} \pm A_2\overline{R} \quad , \tag{14.5}$$

$$\left(UCL_R\right) = D_4\overline{R} \quad , \tag{14.6}$$

$$LCL_R = D_3\overline{R} \quad . \tag{14.7}$$

The values of A_2, D_3, D_4 depend on the sample size and can be taken from Appendix A-7 of [1].

Step V: Plot $\overline{X}_j$ and R_j on the control charts developed as per Step III. Check whether the process is in control as per the decision rules discussed earlier. If so, the control limits in Step III are final. Else revision of control limits by elimination of the out of control points is required. Repeat these steps for revision of control limits until final charts are obtained. The principle of development of other control charts is similar to the above methodology.

If the sample size is not constant from sample to sample, a standardized control chart can be used. The reader is referred to [4] and [5] for more details. Sometimes control charts are to be developed for specified standard or target values of mean and standard deviation. The reader is referred to [1] for the complete procedure for this. If a process is out of control assignable causes are present, which can be identified from the pattern of the control chart. AT & T [6] explains different types of control chart patterns that can be compared with the actual pattern to get an idea about "what" action is to be taken "when".

The effect of measurement error on the performance of $\overline{X}$ and S^2 charts is frequently quantified using gage capability studies [7], which are further investigated using a linear covariate [8]. Their study also identifies conditions under which multiple measurements are desirable and suggests a cost model for selection of an optimal mean. They also suggest taking multiple measurements per item to increase the statistical power of control charts in such cases.

14.3.2 Control Charts for Mean and Standard Deviation $\left(\overline{X}, S\right)$

Both range and standard deviation is used for measuring the variability. Standard deviation is preferred if the sample size is large (say $n > 10$). The procedure for construction of $\overline{X}$ and S charts is similar to that for $\overline{X}$ and R chart. The following formulas are used:

$$CL_s = \overline{S} = \frac{\sum_{j=1}^{g} S_j}{g} \quad , \tag{14.8}$$

$$UCL_s = B_4\overline{S} \quad , \tag{14.9}$$

$$LCL_s = B_3\overline{S} \quad . \tag{14.10}$$

The reader is referred to Appendix A-7 of [1] for the values of B_3 and B_4. $\overline{X}$ and S charts are sometimes also developed for given standard values [1].

14.3.3 Control Charts for Single Units (X chart)

In many practical situations we are required to limit the sample size to as low as unity. In such cases we use an X chart in association with a moving range (MR) chart. The moving range is the absolute value of the difference between successive observations. The assumption of normal distribution may not hold well in many cases of X and MR charts. The following formulas shown in Table 14.1 are used for developing the charts.

Table 14.1. Control limits for X and MR charts

Chart	CL	UCL	LCL
X	$\overline{X}$	$\overline{X}+3\overline{MR}/d_2$	$\overline{X}-3\overline{MR}/d_2$
MR	$\overline{MR}$	$D_4\overline{MR}$	$D_3\overline{MR}$

The values of d_2 depend on the sample size and can be taken from Appendix A-7 of [1]. These charts can also be developed for given standard values.

The control charts discussed so far are initially developed by Walter A Shewhart. Therefore, these charts are also known as Shewhart control charts [9]. Shewhart control charts are very easy to use and are very effective for detecting magnitudes of shifts from 1.5σ to 2σ or larger. However, a major limitation of these charts is their insensitivity to small shifts in process parameters, say about 1.5σ or less. To alleviate this problem a number of special charts have been developed. These are discussed in the following sections.

14.3.4 Cumulative Sum Control Chart (CUSUM)

These control charts are used when information from all previous samples need to be used for controlling the process. CUSUM charts are more effective in detecting small changes in the process mean compared to other charts discussed earlier.

The cumulative sum for a sample m is calculated by

$$S_m = \sum_{j=1}^{m}\left(\overline{X}_i - \mu_o\right) , \tag{14.11}$$

where μ_o is the target mean of the process. In this case CUSUM is plotted on the y-axis. The details of development and implementation of CUSUM charts are discussed in [10]. A V-mark is designed and developed for taking the decision on the process control while using these charts. A methodology to use CUSUM charts for detecting larger changes in process parameters is also available in this reference.

A comparative study of the performance based on the ARL of a moving range chart, a cumulative sum (CUSUM) chart based on moving ranges, a CUSUM chart based on an approximate normalizing transformation, a self-starting CUSUM chart, and an exponentially weighted moving chart based on subgroup variance is discussed in [11, 12]. The CUSUM chart is again compared with several of its alternatives that are based on the likelihood ratio test and on transformations of standardized recursive residual [13]. The authors conclude that the CUSUM chart is not only superior in the detection of linear trend out-of-control conditions, but also in the detection of other out-of-control situations. For an excellent overview of the CUSUM chart techniques the reader is referred to [14].

The adaptive CUSUM (ACUSUM) chart was proposed to detect a broader range of shifts on process mean [15]. A two-dimensional Markov chain model has also been developed to analyze the performance of ACUSUM charts [16]. This improves on the theoretical understanding of the ACUSUM schemes and also allows the analysis without running exclusive simulations. Moreover, a simplified operating function is derived based on an ARL approximation of CUSUM charts [16].

14.3.5 Moving Average Control Charts

These charts are also developed to detect small changes in process parameters. The moving average of width w for a sample number r is defined as:

$$M_r = \frac{\overline{X}_r + \overline{X}_{r-1} + \cdots\cdots + \overline{X}_{r-w+1}}{w} . \tag{14.12}$$

That means M_r is an average of latest w samples starting from the r-th sample. The control limits for this chart will be wider during the initial period and stabilize to the following limits after the first (w-1) samples:

$$CL = \overline{\overline{X}} , \tag{14.13}$$

$$\left(UCL, LCL\right) = \overline{\overline{X}} \pm \frac{3\sigma}{\sqrt{nw}} . \tag{14.14}$$

The initial control limits can be calculated by substituting r in place of w in these equations. Larger values of w should be chosen to detect shifts of small magnitudes. These charts can also be used when the sample size is unity.

14.3.6 EWMA Control Charts

The exponentially weighed moving average (EWMA) control chart was introduced in 1959 [17]. EWMA charts are also used for detecting shifts of small magnitudes in the process characteristics. These are very effective when the sample size is unity. Therefore, these are very useful for controlling chemical and process industries, in discrete part manufacturing with automatic measurement of each part, and in automatic on-line control using micro computers. EWMA is similar to MA, except that it gives higher weighting to the most recent observations. Therefore, the chances of detecting small shifts in process are better compared to the MA chart. These charts are discussed in details in [18–20], and [1]. The control limits of the EWMA chart are

$$CL = \overline{\overline{X}} \ , \tag{14.15}$$

$$(UCL, LCL) = \overline{\overline{X}} \pm 3\sigma\sqrt{\frac{p}{n(2-p)}\left[1-(1-p)^{2r}\right.} \tag{14.16}$$

where p is the weighing constant $(0 < p \leq 1)$, and r is the sample number.

It may be noted that if $p = 1$, EWMA chart reduces to Shewhart chart and for $p = 2/(w+1)$, it reduces to MA chart. Selecting a small value of p (say 0.05) ensures faster detection of small shifts in process. These charts are also known as geometric moving average control charts.

As discussed earlier, violation of the assumption of independent data results in increased number of false alarms and trends on both sides on the centerline. A typical approach followed in the literature to study this phenomenon is to model the autocorrelated structure of the data and use a traditional control chart method to monitor the residuals. See [21–25], for more details. An alternative approach is the exponentially weighted moving average (MCEWMA) chart proposed in [26]. The literature also explores the shift detection capability of the moving centerline exponentially weighted moving average (MCEWMA) chart and recommends enhancements for quicker detection of small process upsets [27].

14.3.7 Trend Charts

In many processes the process average may continuously run either upward or downward after production of every unit of product. This is a natural phenomenon and therefore, it is an acceptable trend. Examples are effects of wearing of the punch, die, cutting tools or drill bits. However, such a trend in the process mean is acceptable only within some upper and lower limits (in most cases the specification limits). The trend charts are developed to monitor and control these types of processes. The centerline of the trend chart will have an upward or downward trend, and the upper and lower control limits will be parallel to the centerline. The intersection of centerline a and the slope b can be evaluated from the observations collected from the process [1]. The equations for the control limits are

$$CL = a + b_i \ , \tag{14.17}$$

$$UCL = (a + b_i) + A_2\overline{R} \ , \tag{14.18}$$

$$LCL = (a + b_i) - A_2\overline{R} \ . \tag{14.19}$$

These charts are useful for detecting changes in the process and also to decide whether or not a tool change is required. These charts are also known as regression control charts and are very helpful in controlling processes in machine shops and other production machines.

14.3.8 Specification Limits on Control Charts

If we want to include specification limits on the control charts, we require modification of the control limits. This is because the specification limits are defined on individual units where as most control charts are developed for sample average values. A simple methodology for finding the modified control limits is discussed in [1].

14.3.9 Multivariate Control Charts

The quality of a product is a function of many characteristics. For example, the length, diameter, strength, and surface finish among others contribute to the quality of a shaft. Therefore controlling of all these variables is required to control the quality of the product. Multivariate control charts are developed to simultaneously control several quality characteristics. The procedure for development and application of multivariate control charts are discussed in detail in [1]. The T^2 distribution is used to develop the control chart and the F-distribution is used for finding the upper control limit [28]. The lower control limit is zero. The probability of type I error for this type of chart is very difficult to establish if the variables are dependent. If all the variables are independent then we can calculate this probability by the equation:

$$\alpha^* = 1 - (1 - \alpha)^p \quad , \tag{14.20}$$

where p is the number of independent variables.

Two phases in constructing multivariate control charts are defined, with phase I divided into two stages [29]. In stage I of phase I, historical observations are studied for determining whether the process was in control and to estimate the in-control parameters of the process. The T^2 chart of Hotelling is used in this stage as proposed in [30], and [31]. Control charts are used in stage II with future observations for detecting possible departures from the parameters estimated in the first stage. In the phase II charts are used for detecting any departures from the parameter estimates, which are considered the true in-control process parameters. A T^2 control chart based on robust estimators of location and dispersion is proposed in [32]. Using simulation studies the author shows that the T^2 control chart using the minimum volume ellipsoid (MVE) estimators is effective in detecting any reasonable number of outliers (multiple outliers).

Multiway principal components analysis (MPCA), a multivariate projection method, has been widely used for monitoring the batch process. A new method is proposed in [33] for predicting the future observation of the batch that is currently being operated (called the new batch). The proposed method, unlike the existing prediction methods, makes extensive use of the past batch trajectories.

The effect of measurement error on the performance of the T^2 chart is studied in [34]. For some multivariate nonnormal distributions, the T^2 chart based on known in-control parameters has an excessive false alarm rate as well as a reduced probability of detecting shifts in the mean vector [35]. The process conditions that lead to the occurrence of certain nonrandom patterns in a T^2 control chart are discussed in [36]. Examples resulting from cycles, mixtures, trends, process shifts, and auto correlated data are identified and presented. Results are applicable to a phase I operation or phase II operation where the T^2 statistics is based on the most common covariance matrix estimator. The authors also discuss the cyclic and trend patterns, effects of mixture of populations, process shifts and autocorrelated data on the performance of the T^2 chart.

A strategy for performing phase I analysis (of the multivariate control charts) for high-dimensional nonlinear profiles is proposed in [37]. This consists of two major components: a data reduction component that projects the original data into a lower dimension subspaces while preserving the data-clustering structure and a data-separation technique that can detect single and multiple shifts as well as outliers in the data. Simulated data sets as well as nonlinear profile signals from a forging process are used to illustrate the effectiveness of the proposed strategy.

Excellent reviews on the T^2 chart are presented in [38, 39]. Several useful properties of the T^2 statistics based on the successive difference estimator which give a more accurate approximate distribution for calculating the upper control limit individual observation in a phase I analysis are demonstrated in [40].The author discusses how to accurately determine the upper control limit for a T^2 control chart based on successive difference of multivariate individual observations.

A multivariate extension of the EWMA chart was proposed in [41]. This chart, known as MEWMA chart, is based on sample means and on the sum of squared deviations from the target. The performance of many of these control charts depends on the direction of the shifts in the mean vector or covariance matrix [42].

14.4 Control Charts for Attributes

Attribute characteristics resemble binary data, which can take only one of two given alternatives. In quality control, the most common attribute characteristics used are "conforming" or "not conforming", "good" or "bad". Attribute data need to be transformed into discrete data to be meaningful.

The types of charts used for attribute data are:

- Control chart for proportion nonconforming items (*p* chart)
- Control chart for number of nonconforming items (*np* chart)
- Control chart for nonconformities (*c* chart)
- Control chart for nonconformities per unit (*u* chart)
- Control chart for demerits per unit (*U* chart)

A comprehensive review of the attribute control charts is presented in [43]. The relative merits of the *c* chart compared to the X chart for the Katz family covering equi-, under-, and over-dispersed distributions relative to the Poisson distribution are investigated in [44]. The Katz family of distributions is discussed in [45]. The need to use an X chart rather than a *c* chart depends upon whether or not the ratio of the in control mean is close to unity. The X chart, which incorporates the information on this ratio, can lead to significant improvements under certain circumstances. The c chart has proven to be useful for monitoring count data in a wide range of application. The idea of using the Katz family of distribution in the robustness study of control charts for count data can be extended to the cumulative sum (CUSUM) and exponentially weighted moving average (EWMA) chart.

The *p* and *np* charts are developed based on binomial distribution, the *c*, *u*, and *U* charts are based on Poisson distribution. These charts are briefly discussed in this section.

14.4.1 The *p* chart

The *p* chart is used when dealing with ratios, proportions or percentages of nonconforming parts in a sample. Inspection of products from a production line is a good example for application of this chart. This fulfils all the properties of binomial distribution. The first step for developing a *p* chart is to calculate the proportion of nonconformity for each sample. If n and m represent the sample size and number of nonconforming items in the sample, then the fraction of nonconforming items p is given by:

$$p = \frac{m}{n} . \tag{14.21}$$

If we take g such samples, then the mean proportion nonconforming $\overline{p}$ is given by:

$$\overline{p} = \frac{p_1 + p_2 + ... + p_g}{g} . \tag{14.22}$$

The centerline and the 3σ limits of this chart are as follows:

$$CL = \overline{p} , \tag{14.23}$$

$$UCL = \overline{p} + 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}} , \tag{14.24}$$

$$LCL = \overline{p} - 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}} . \tag{14.25}$$

In many situations we may require to develop p-charts with variable sample size. In such situations control charts can be developed either for individual samples or for a few representative sample sizes. A more practical approach is to develop a standardized chart. For this a standardized value z of p for each sample is calculated as follows:

$$z_i = \frac{p_i - \overline{p}}{\sqrt{\overline{p}(1-\overline{p})/n_i}} . \tag{14.26}$$

z_i is then plotted on the chart. This chart will have its centerline at zero and the control limits of 3 on either side. A number of rules are developed for decision making on the out-of-control situations. Different types of p-charts and the decision rules are discussed in more detail in [1] and [5]. A p chart has the capability to combine information from many departments, product lines, and work centers and provide an overall rate of product nonconformance.

14.4.2 The *np* chart

The *np* chart is similar to the *p* chart. It plots the number of nonconforming items per sample. Therefore it is easier to develop and use compared to the *p* chart. While the *p* chart tracks the proportion of nonconformities per sample, the *np* chart counts the number of defectives in a sample. The binomial distribution can be used to develop this chart. The mean number of nonconformities in a sample is *np*.

The centerline and the control limits for an *np*-chart are as follows:

$$CL = n\overline{p}\,, \tag{14.27}$$

$$UCL = n\overline{p} + 3\sqrt{n\overline{p}(1-\overline{p})}\,, \tag{14.28}$$

$$LCL = n\overline{p} - 3\sqrt{n\overline{p}(1-\overline{p})}\,. \tag{14.29}$$

np charts are not used when the sample size changes from sample to sample. This is because the centerline as well as the control limits are affected by the sample size. Using and making inferences in such cases are very difficult.

14.4.3 The *c* chart

The *c* chart monitors the total number of nonconformities (or defects) in samples of constant size taken from the process. Here, nonconformance must be distinguished from defective items since there can be several nonconformances on a single defective item. For example a casting may have many defects such as foreign material inclusion, blow holes, hairline cracks, *etc*. Other examples are the number of defects in a given length of cable, or in a given area of fabric. Poisson distribution is used to develop this chart. If the sample size does not change and the defects on the items are fairly easy to count, the *c* chart becomes an effective tool to monitor the quality of the production process.

If $\overline{c}$ is the average number of nonconformities per sample, then the centerline and the 3σ control limits of the *c* chart are:

$$CL = \overline{c}\,, \tag{14.30}$$

$$UCL = \overline{c} + 3\sqrt{\overline{c}}\,, \tag{14.31}$$

$$LCL = \overline{c} - 3\sqrt{\overline{c}}\,, \tag{14.32}$$

14.4.4 The *u* chart

One of the limitations of the *c* chart is that it can be used only when the sample size remains constant. The *u* chart can be used in other cases. It can be effectively used for constant as well as for variable sample size. The first step in creating a *u* chart is to calculate the number of defects per unit for each sample,

$$u_i = \frac{c_i}{n_i}\,, \tag{14.33}$$

where *u* represents the average defect per sample, *c* is the total number of defects, *n* is the sample size and *i* is the index for sample number. Once all the averages are determined, a distribution of the means is created and the next step will be to find the mean of the distribution, in other words, the grand mean.

$$\overline{u} = \frac{\sum_{i=1}^{g} c_i}{\sum_{i=1}^{g} n_i}\,, \tag{14.34}$$

where g is the number of samples. The control limits are determined based on $\overline{u}$ and the mean of the samples *n*,

$$UCL = \overline{u} + 3\sqrt{\overline{u}/n_i}\,, \tag{14.35}$$

$$LCL = \overline{u} - 3\sqrt{\overline{u}/n_i}\,. \tag{14.36}$$

Furthermore, for a *p* chart or an *np* chart the number of nonconformances cannot exceed the number of items on a sample, but for a *u* chart, it is conceivable since what is being addressed is not the number of defective items but the number of defects in the sample.

14.4.5 Control Chart for Demerits *per* Unit (*U* chart)

One of the deficiencies of the *c* and *u* charts is that all types of nonconformities are treated equally. In actual practice there are different types of nonconformities with varying degrees of severity. ANSI/ASQC Standard A3 classifies the nonconformities into four classes, *viz.*, very serious, serious, major, and minor, and proposes a weighing system of 100, 50, 10, and 1, respectively. The total number of demerits (D) for a sample is therefore calculated as the weighed sum of nonconformities of all types as follows:

$$D = w_1 c_1 + w_2 c_2 + w_3 c_3 + w_4 c_4 \ . \quad (14.37)$$

The demerits per sample (U) is defined as $U = D / n$ where n is the sample size. The center line of the control chart is given by:

$$CL = \overline{U} = w_1 \overline{u_1} + w_2 \overline{u_2} + w_3 \overline{u_3} + w_4 \overline{u_4} \ , \quad (14.38)$$

where $\overline{u_i}$ represent the average number of nonconformities per unit in the i-th class. The control limits of the chart are:

$$UCL = \overline{U} + 3\sigma_U \ , \quad (14.39)$$

$$LCL = \overline{U} - 3\sigma_U \ , \quad (14.40)$$

where

$$\sigma_U = \sqrt{\left(w_1^2 \overline{u_1} + w_2^2 \overline{u_2} + w_3^2 \overline{u_3} + w_4^2 \overline{u_4}\right) / n} \ . \quad (14.41)$$

For a detailed discussion on the U hart the reader is referred to [1].

As mentioned earlier, the success of using control charts for process control depends to a great extent on the observed data. Data must be independent of one another to ensure the random phenomenon. If this is not strictly ensured, the data will be autocorrelated and the inferences on process control based on the control charts will be misleading. In actual practice there is a chance of some level of autocorrelation of the data. Therefore, dealing with autocorrelated data has been a research problem in SPC. Many useful ideas have been developed and published on this topic.

A model for correlated quality variables with measurement error is presented in [46]. It is shown that the performance of multivariate control charting methods based on measured covariates is not directionally invariant to shifts in the mean vector of the underlying process variables, even though it may be directionally invariant when no measurement error exists. For further information on the directional invariance of multivariate control charts the reader is referred to [41, 47, 48], and [49].

The traditional control charts become unreliable when the data are autocorrelated [50]. In the literature the reverse moving average control chart is proposed as a new forecast-based monitoring scheme, compare the new control chart to traditional methods applied to various ARMA(1,1), AR(1), MA(1) processes, and make recommendations concerning the most appropriate control chart to use in a variety of situations when charting autocorrected processes [51].

Many new types of control charts have been proposed in the recent literature to handle different types of data. The proportional integral derivative (PID) chart for monitoring autocorrelated processes based on PID predictors and corresponding residuals is introduced in [52]. The PID charting parameter design, the mean shift pattern analysis, and the relationship between the average run length performance and PID parameter selection are also discussed extensively in the literature. Improved design schemes are suggested for different scenarios of autocorrelated processes and verified with Monte Carlo simulation. This study provides useful information for practitioners to effectively apply PID charts. See [53–56] for further discussions on autocorrelation of data in control charts.

The cumulative conformance count (CCC) chart was introduced as a Six Sigma tool to deal with controlling high-yield processes (see [57]). CCC chart was first introduced in [58] and became popular through [59]. It is primarily designed for processes with sequential inspection carried out automatically one at a time. A control scheme that is effective in detecting changes in nonconforming fractions for high yield processes with correlation within each inspection group is followed in [60]. A Markov model is used to analyze the characteristics of the proposed schemes in terms of which the average run length (ARL) and average time signal (ATS) are obtained. The performance of the proposed schemes in terms of ATS is presented along with the comparison with the traditional cumulative conformance count (CCC) chart. Moreover, the effects of correlation and group size are also investigated by the authors. The authors also have proposed a control scheme, the C^{4-}chart for monitoring high-yield high volume production/process under group inspection with consideration of correlation within each group. Circumstances that lead to group inspection include a slower inspection rate than the production rate, economy of scale in group inspection, and strong correlation in the output characteristics.

Many applications and research opportunities available in the use of control charts for health-care related monitoring are reported in [61]. The advantage and disadvantage of the charting methods proposed in health care and public health areas are considered. Some additional contribution in the industrial statistical process control literature relevant to this area are given. Several useful references in the related areas are listed in this paper. This shows that the application of SPC for health care systems has become increasingly popular in recent times.

14.5 Engineering Process Control (EPC)

In recent times EPC has been used to control the continuous processes manufacturing discrete parts. It is also known as automatic process control (APC) in which an appropriate feedback or feedforward control is used to decide when and by how much the process should be adjusted to achieve the quality target. It is an integrated approach in which the concepts of design of experiments and robust design are also effectively used for designing control charts. EPC has been developed to provide an instantaneous response, counteracting changes in the balance of a process and to apply corrective action to bring the output close to the desired target. The approach is to forecast the output deviation from target that would occur if no control action were taken and then to act so as to cancel out this deviation [62].

14.6 Process Capability Analysis

Process capability represents the performance of a process when it is in a state of statistical control. It is measured as the total process variability when only common causes are present in the system. The process spread 6σ is generally taken as a measure of the process capability. 99.74% of all products will be within this spread if the normality assumption is valid. In many situations we are required to check if our existing process is capable of meeting certain product specifications. Such decisions are taken based on the process capability indices (PCI). The following PCI are generally used.

14.6.1 Process Capability Indices

This relates the process spread to the specification spread as follows:

$$C_p = \frac{USL - LSL}{6\sigma}. \tag{14.42}$$

where *USL* and *LSL* are the upper and lower specification limits. From the above equation it can be seen that the process is capable when $C_p > 1$. However, C_p is not a good measure since it does not take care of the location of the center of the process. C_p represents only the process potential. Therefore other PCI such as upper capability index (*CPU*), lower capability index (*CPL*), C_{pk} and C_{pm} are also developed for such studies. They are defined as follows:

$$CPU = \frac{USL - \mu}{3\sigma}, \quad CPL = \frac{\mu - LSL}{3\sigma}, \quad \text{and}$$
$$C_{pk} = Min\{CPU, CPL\}. \tag{14.43}$$

Since C_{pk} also takes into account the position of the centerline of the process (μ), it represents the actual process capability of the process with the present parameter values. Taguchi proposed and used another index, *viz.*, C_{pm} [63, 64]. The author emphasizes the need to reduce the process variability around a target value *T*. C_{pm} is defined as follows.

$$C_{pm} = \frac{USL - LSL}{6\tau}, \tag{14.44}$$

where τ is the standard deviation from the target value and is calculated by

$$\tau^2 = E\left[(X - T)^2\right] . \tag{14.45}$$

Combining the merits of these indices, a more advanced index, C_{pmk}, is proposed that takes into account process variation, process centering, and the proximity to the target value, and has been shown to be a very useful index for manufacturing processes with two-sided specification limits. The behavior of C_{pmk} as a function of process mean and variation is discussed in [65]. If the variation of the process increases, the maximum value of C_{pmk} moves from near the target value to the midpoint of the specification. If the process mean varies inside the specification, C_{pmk} decreases as the variation increases. It is argued that these properties may constitute a sensible behavior of the process capability index. For an extensive study on process capability the reader is referred to [66–68].

In many situations we may require to compare several processes based on process capability. If there are two processes, the classical hypothesis testing theory can be applied as suggested in [69, 70]. A bootstrap method for similar studies is proposed in [71]. When there are more than two processes, the best subset selection method proposed in [72–76] can be effectively used. A solutions to this problem based on permutation testing methodology is proposed in [77]. In the case of two processes, the methodology is based on a simple permutation test of the null hypothesis that the two processes have equal capability. In the case of more than two processes, multiple-comparison techniques are used in conjunction with the proposed permutation test. The advantage of using the permutation methods is that the significance levels of the permutation tests are exact regardless of the distribution of the process data. The methodology is demonstrated using several examples, and the potential performance of the methods are investigated empirically.

References

[1] Mitra A. In: Fundamentals of quality control and improvement. Pearson Education Asia, 2001.

[2] Deming W Edwards. In: Quality, productivity, and competitive position. Cambridge, Mass: Center for Advanced Engineering Study, MIT, 1982.

[3] Duncan AJ. In: Quality control and industrial statistics. 5th Edition, Homewood, III: Richard D Irvin, 1986.

[4] Nelson LS. Standardization of control charts. Journal of Quality Technology 1989; 21(4):287–289.

[5] Nelson LS. Shewart control charts with unequal subgroup sizes. Journal of Quality Technology 1994; 26(1): 64–67.

[6] AT&T, Statistical Quality Control Handbook, 10th printing, 1984.

[7] Montgomery DC, Runger GC. Gauge capability and designed experiments. Part 1: Basic methods. Quality Engineering 1994; 6:115–135.

[8] Linna KW, Woodall WH, Busby KL. The performance of multivariate control charts in the presence of measurement errors. Journal of Quality Technology 2001; 33:349–355.

[9] Nelson LS. The Shewart control chart tests for special causes. Journal of Quality Technology 1984; 16 (4): 237–239.

[10] Lucas JM. The design and use of V-mask control schemes. Journal of Quality Technology 1976; 8(1):1–12.

[11] Cesar A Acosta-Mejia, Joseph J Pignatiello Jr. Monitoring process dispersion without subgrouping. Journal of Quality Technology 2000; 32 (2):89–102.

[12] Klein Moton. Two alternatives to the Shewhart X-bar control chart. Journal of Quality Technology 2000; 32(4):427–431.

[13] Koning Alex J, Does Ronald JMM. CUSUM chart for preliminary analysis of individual observation. Journal of Quality Technology 2000; 32(2):122–132.

[14] Hawkins DM, Olwell DH, Cumulative sum charts and charting for quality improvement. Springer, New York, NY, 1998.

[15] Sparks RS. CUSUM charts for signaling varying location shifts. Journal of Quality Technology 2000;32:157–171.

[16] Shu Lianjie, Wei Jiang. A Markov chain model for the adaptive CUSUM control chart. Journal of Quality Technology 2006; 38(2): 135–147.

[17] Roberts SW. Control chart tests based on geometric moving averages. Technometrics 1959; 1.

[18] Crowder SV. Design of exponentially weighed moving average schemes. Technometrics 1987; 21.
[19] Crowder SV. A simple method for studying run length distributions of exponentially weighed moving average charts. Technometrics 1989;29.
[20] Lucas JM, Saccussi MS. Exponentially weighed moving average control schemes: Properties and enhancements. Technometrics 1990;32.
[21] Alwan LC, Roberts HV. Time series modeling for statistical process control. Journal of Business and Economic Statistics 1988;6 (1):87–95.
[22] Alwan LC. Radson D. Time series investigation of sub sample mean charts. IIE Transactions 1992; 24(5): 66–80.
[23] Montgomery DC, Friedman DJ. Statistical process control in a computer- integrated manufacturing environment. In: Keats JB, Hubele NF, editors. Statistical process control in automated manufacturing. Marcel Dekker, New York, 1989.
[24] Yourstone SA, Montgomery DC. Development of a real time statistical process control algorithm. Quality and Reliability Engineering International 1989; 5:309–317.
[25] Notohardjono BD, Ermer DS. Time Series control charts for correlated and contaminated data. Journal of Engineering for Industry 1996; 108: 219–225.
[26] Montgomery DC, Mastrangelo CM. Some statistical process control methods for autocorrelated data. Journal of Quality Technology 1991; 23: 179–193.
[27] Mastrangelo Christrina M, Brown Evelyn C. Shift detection properties of moving centerline control chart schemes. Journal of Quality Technology 2000; 32 (1):67–74.
[28] Hoteling H. Multivariate quality control. In: Eisenhart C, Hastny MW, Wallis WA, editors. Techniques of statistical analysis. McGraw Hill, New York, 1947.
[29] Alt FB. Multivariate quality control. In: Katz S, .Johnson NL, editors. Encyclopedia of statistical sciences. Wiley, New York, 1985; 6.
[30] Alt FB, Smith ND. Multivariate process control. In: Krishnaiah PR, Rao CR, editors. Handbook of statistics. North-Holland, Amsterdam, 1988; 7: 333–351.
[31] Tracy ND, Young JC, Mason RL. Multivariate control charts for individual observations. Journal of Quality Technology 1992; 24: 88–95.
[32] Vargas Jose Alberto N. Robust estimation in multivariate control charts for individual observations. Journal of Quality Technology 2003; 35(4): 367–376.
[33] Cho Hyun-Woo, Kim Kwang-Jae. A method for predicting future observations in the monitoring of a batch process. Journal of Quality Technology 2003; 35(1): 59–69.
[34] Linna Kenneth W, Woodall William H, Busby Kevin L. The performance of multivariate control charts in the presence of measurement error. Journal of Quality Technology, 2001; 33(3):349–355.
[35] Stoumbos ZG, Sullivan JH. Robustness to nonnormality of the multivariate EWMA control chart. Journal of Quality Technology 2002; 34: 260–276.
[36] Mason Robert L, Chou Youn-Min, Sullivan Joe H, Stoumbos Zachary G, Young John C. Systematic pattern in T^2 chart. Journal of Quality Technology 2003; 35(1):47–58.
[37] Ding Yu, Zeng Li, Zhou Shiyu. Phase I analysis for monitoring nonlinear profiles in manufacturing processes. Journal of Quality Technology 2006; 38(3):199–216.
[38] Fuchs C, Kenett RS. Multivariate quality control: theory and applications. Marcel Dekker, New York, 1998.
[39] Mason RL, Young JC. Multivariate statistical process control with industrial applications. SIAM, Philadelphia, PA, 2002.
[40] Woodall William H. Rejoinder. Journal of Quality Technology 2006; 38(2):133–134.
[41] Lowry CA, Woodall WH, Champ CW, Rigdon SE. A multivariate exponentially weighted moving average control chart. Technometrics 1992; 34: 46–53.
[42] Reynolds Jr. Marion R, Cho Gyo-Young. Multivariate control chart for monitoring the mean vector and covariance matrix. Journal of Quality Technology 2006; 38 (3):230–253.
[43] Woodall WH. Control charting based on attribute data: Bibliography and review. Journal of Quality Technology 1997; 29:172–183.
[44] Fang Yue. C-chart, X-chart, and the Katz family of distributions. Journal of Quality Technology 2003; 35(1):104–114.
[45] Katz L. Unified treatment of a broad class of discrete probability distributions. Proceedings of the International Symposium on Discrete Distributions, Montreal, Canada 1963.
[46] Linna Kenneth W, Woodall William H. Effect of measurement error on Shewhart control charts. Journal of Quality Technology 2001; 33(2):213–222.
[47] Mason, RL, Champ CW, Tracy ND, Wierda SJ, Young, JC. Assessment of multivariate process

control techniques. Journal of quality technology 1997; 29:140–143.
[48] Pignatiello JJ Jr., Runger GC. Comparisons of multivariate CUSUM charts. Journal of Quality Technology 1990; 22:173–186.
[49] Lowry CA, Montgomery DC. A review of multivariate control charts. IIE Transactions 1995; 27:800–810.
[50] Maragah HD, Woodall WH. The effect of autocorrelation on the retrospective X-chart. Journal of Statistical Computation and Simulation, 1992; 40:29–42.
[51] Dyer John N, Benjamin M Adams, Michael D Conerly. The reverse moving average control chart for monitoring autocorrelated processes. Journal of Quality Technology 2003; 35(2):139–152.
[52] Jiang W, Wu H, Tsung F, Nair VN, Tsui KL. Proportional integral derivative charts for process monitoring. Technometrics 2002; 44:205–214.
[53] Shu L, Apley DW, Tsung F. Autocorrelated process monitoring using triggered cuscore charts. Quality and Reliability Engineering International 2002; 18:411–421.
[54] Apley DW, Tsung F. The autoregressive T^2 chart for monitoring univariate autocorrelated processes. Journal of Quality Technology 2002; 34:80–96.
[55] Castagliola P, Tsung, F. Autocorrelated statistical process control for non normal situations. Quality and Reliability Engineering International 2005; 21:131–161.
[56] Tsung Fugee, Zhao Yi, Xiang Liming, Jiang Wei. Improved design of proportional integral derivative charts. Journal of Quality Technology 2006; 38(1):31–44.
[57] Goh TN. Xie M. Statistical control of a Six Sigma process. Quality Engineering 2003; 15:587–592.
[58] Calvin TW. Quality control techniques for 'Zero defects. IEEE Transactions on Components, Hybrids, and Manufacturing Technology CHMT 1983; 6:323–328.
[59] Goh, T.N., A Control Chart for Very High Yield Processes. Quality Assurance, 1987, 13: 18–22.
[60] Tang Loon-Ching, Cheong Wee-Tat. A control scheme for high-yield correlated production under group inspection. Journal of Quality Technology 2006; 38(1):45–55.
[61] Woodball Wiiliam H. The use of control chart in health-care and public-health surveillance. Journal of Quality Technology 2006; 38(2):89–104.
[62] Jin J, Ding Y. Online automatic process control using observable noise factors for discrete-part manufacturing. IIE Transactions 2004; 36:899–911.
[63] Taguchi G. A tutorial on quality control and assurance–the Taguchi methods. ASA Annual meeting, Las Vegas, 1985.
[64] Taguchi G. Introduction to quality engineering. Asian productivity organization, Tokyo, 1986.
[65] Jessenberger Jutta, Weihs Claus. A note on the Behavior of C_{pmk} with asymmetric specification limit. Journal of Quality Technology 2000; 32(4):440–443.
[66] Pignatiello JJ. Process capability indices: Just say 'no!' Annual Quality Congress Transactions 1993; 92–104.
[67] Gunter BH. The use and abuse of Cpk: Parts 1–4. Quality Progress 1989; 22(1):72–73; 22(2):108–109; 22(5):79–80; 86–87.
[68] Polansky AM. Supplier selection based on bootstrap confidence regions of process capability indices. International Journal of Reliability, Quality and Safety Engineering 2003; 10:1–14.
[69] Chou YM, Owen DB. A likelihood ratio test for the equality of proportions of two normal populations. Communications in Statistics Theory and Methods 1991; 20:2357–2374.
[70] Chou YM. Selecting a better supplier by testing process capability indices. Quality Engineering 1994; 6:427–438.
[71] Chen JP, Chen KS. Comparing the capability of two processes using Cpm. Journal of Quality Technology 2004; 36:329–335.
[72] Tseng ST, Wu TY. Selecting the best manufacturing process. Journal of Quality Technology 1991; 23: 53–62.
[73] Huang DY, Lee RF. Selecting the largest capability index from several quality control processes. Journal of Statistical Planning and Inference 1995; 46:335–346.
[74] Polansky AM, Kirmani SNUA. Quantifying the capability of industrial processes. In: Khattree B, Rao CR, editors. Handbook of Statistics. Elsevier Science, Amsterdam, 2003; 22: 625–656.
[75] Daniels L, Edgar B, Burdick RK, Hubele NF. Using confidence intervals to compare process capability indices. Quality Engineering 2005; 17:23–32.
[76] Hubele NF, Bernado A, Gel ES. A Wald test for comparing multiple capability indices. Journal of Quality Technology 2005; 37:304–307.
[77] Polansky Alan M. Permutation method for comparing process capabilities. Journal of Quality Technology 2006; 38(3):254–266.