Chapter 1
Early Civilisations

Key Topics

Babylonian Mathematics
Egyptian Civilisation
Greek and Roman Civilisation
Counting and Numbers
Solving Practical Problems
Syllogistic Logic
Algorithms
Early Ciphers

1.1 Introduction

It is difficult to think of western society today without modern technology. The last decades of the twentieth century have witnessed a proliferation of high-tech computers, mobile phones, text-messaging, the internet and the world-wide web. Software is now pervasive and it is an integral part of automobiles, airplanes, televisions, and mobile communication. The pace of change as a result of all this new technology has been extraordinary. Today consumers may book flights over the world-wide web as well as keeping in contact with family members in any part of the world via email or mobile phone. In previous generations, communication often involved writing letters that took months to reach the recipient. Communication improved with the telegrams and the telephone in the late nineteenth century. Communication today is instantaneous with text-messaging, mobile phones and email, and the new generation probably views the world of their parents and grandparents as being old-fashioned.

The new technologies have led to major benefits[1] to society and to improvements in the standard of living for many citizens in the western world. It has also reduced

[1] Of course, while the new technologies are of major benefit to society it is essential that the population of the world moves towards more sustainable development to ensure the long-term survival of the planet for future generations. This involves finding technological and other solutions

the necessity for humans to perform some of the more tedious or dangerous manual tasks, as many of these may now be automated by computers. The increase in productivity due to the more advanced computerized technologies has allowed humans, at least in theory, the freedom to engage in more creative and rewarding tasks.

This chapter considers work done on computation by early civilizations including the work done by our ancestors in providing a primitive foundation for what has become computer science. Early societies are discussed and their contributions to the computing field are considered. There is a close relationship between the technological or computation maturity of a civilization and the sophistication of its language. Clearly, societies that have evolved technically will have invented words to reflect the technology that they use on a daily basis. Clearly, hunter-gatherer or purely agrarian societies will have a more limited technical vocabulary which reflects that technology is not part of their day to day culture. Language evolves with the development of the civilisation, and new words are introduced to describe new inventions in the society. Communities that have a very stable unchanging existence (e.g., hunter gatherer or pastoral societies) have no need to introduce names for complex scientific entities, as these words are outside their day-to-day experience. The language of these communities mirrors the thought processes of these communities.

Early societies had a limited vocabulary for counting: e.g., "one, two, three, many" is associated with some primitive societies, and indicates primitive computation and scientific ability. It suggests that there was no need for more sophisticated arithmetic in the primitive culture as the problems dealt with were elementary. These early societies would typically have employed their fingers for counting, and as humans have 5 fingers on each hand and five toes on each foot then the obvious bases would have been 5, 10 and 20. Traces of the earlier use of the base 20 system are still apparent in modern languages such as English and French. This includes phrases such as "three score" in English and "*quatre vingt*" in French.

The decimal system (base 10) is familiar to most in western society, and it may come as a surprise that the use of base 60 was common in computation *circa* 1500 BC. One example of the use of base 60 today is still evident in the sub-division of hours into 60 minutes, and the sub-division of minutes into 60 seconds. The base 60 system (i.e. the sexagesimal system) is inherited from the Babylonians [Res:84], and the Babylonians were able to represent arbitrarily large numbers or fractions with just two symbols. Other bases that have been used in modern times include binary (base 2) and hexadecimal (base 16). Binary and hexadecimal arithmetic play a key role in computing, as the machine instructions that computing machines understand are in binary code.

The ancient societies considered in this chapter include the Babylonians, the Egyptians, and the Greek and Romans. These early civilizations were concerned with the solution of practical problems such as counting, basic book keeping, the construction of buildings, calendars and elementary astronomy. They used

---

to reduce greenhouse gas emissions as well as moving to a carbon neutral way of life. The solution to the environmental issues will be a major challenge for the twenty first century.

appropriate mathematics to assist them in computation. Early societies had no currency notes like U.S. Dollars or Euros, and trading between communities was conducted by bartering. This involved the exchange of goods for other goods at a negotiated barter rate between the parties. This required elementary computation as the bartering of one cow would require the ability to agree that a cow was worth so many of another animal, crop or good, e.g., sheep, corn, and so on. Once this bartering rate was agreed the two parties then needed to verify that the correct number of goods was received in exchange. Therefore, the ability to count was fundamental.

The achievements of some of these ancient societies were spectacular. The archaeological remains of ancient Egypt are very impressive, and include the pyramids at Giza, the temples of Karnak near Luxor and Abu Simbal on the banks of Lake Nasser. These monuments provide an indication of the engineering sophistication of the ancient Egyptian civilisation. The objects found in the tomb of Tutankamun[2] are now displayed in the Egyptian museum in Cairo, and demonstrate the artistic skill of the Egyptians.

The Greeks made major contributions to western civilization including contributions to Mathematics, Philosophy, Logic, Drama, Architecture, Biology and Democracy.[3] The Greek philosophers considered fundamental questions such as ethics, the nature of being, how to live a good life, and the nature of justice and politics. The Greek philosophers include Parmenides, Heraclitus, Socrates, Plato and Aristotle. The works of Plato and Aristotle remain important in philosophy today, and are studied widely. The Greeks invented democracy and their democracy was radically different from today's representative democracy.[4] The sophistication of

---

[2] Tutankamun was a minor Egyptian pharaoh who reigned after the controversial rule of Akenaten. Tutankamun's tomb was discovered by Howard Carter in the valley of the kings, and the tomb was intact. The quality of the workmanship of the artefacts found in the tomb was extraordinary and a visit to the Egyptian museum in Cairo is memorable.

[3] The origin of the word "democracy" is from demos (δημος) meaning people and kratos (κρατος) meaning rule. That is, it means rule by the people. It was introduced into Athens following the reforms introduced by Cleisthenes. He divided the Athenian city state into thirty areas. Twenty of these areas were inland or along the coast and ten were in Attica itself. Fishermen lived mainly in the ten coastal areas; farmers in the ten inland areas; and various tradesmen in Attica. Cleisthenes introduced ten new clans where the members of each clan came from one coastal area, one inland area on one area in Attica. He then introduced a Boule (or assembly) which consisted of 500 members (50 from each clan). Each clan ruled for $^1/_{10}$th of the year.

[4] The Athenian democracy involved the full participations of the citizens (i.e., the male adult members of the city state who were not slaves) whereas in representative democracy the citizens elect representatives to rule and represent their interests. The Athenian democracy was chaotic and could also be easily influenced by individuals who were skilled in rhetoric. There were teachers (known as the Sophists) who taught wealthy citizens rhetoric in return for a fee. The origin of the word "sophist" is the Greek word σοφος meaning wisdom. One of the most well known of the sophists was Protagorus, and Plato has a dialogue of this name. The problems with the Athenian democracy led philosophers such as Plato to consider alternate solutions such as rule by philosopher kings. This is described in Plato's Republic.

Greek architecture and sculpture is evident from the Parthenon on the Acropolis, and the Elgin marbles[5] that are housed today in the British Museum, London.

The Hellenistic[6] period commenced with Alexander the Great and led to the spread of Greek culture throughout Asia Minor and as far as Egypt. The city of Alexandria was founded by Alexander the Great, and it became a center of learning and knowledge during the Hellenistic period. Among the well-known scholars at Alexandria was Euclid who provided a systematic foundation for geometry. Euclid's work on geometry is known as "The Elements", and it consists of thirteen books. The early books are concerned with the construction of geometric figures, number theory and solid geometry.

There are many words of Greek origin that are part of the English language. These include words such as psychology which is derived from two Greek words: *psyche* (ψυχε) and *logos* (λογος). The Greek word "*psyche*" means mind or soul, and the word "*logos*" means an account or discourse. Other examples are anthropology derived from "*anthropos*" (αντροπος) and "*logos*" (λογος).

The Romans were influenced by the culture of the Greeks, and the Greek language, culture and philosophy was taught in Rome and in the wider Roman Empire. The Romans were great builders and their contributions include the construction of buildings such as aqueducts, viaducts, baths, and amphitheatres. Other achievements include the Julian calendar, the formulation of laws (*lex*), and the maintenance of law and order and peace throughout the Roman Empire. The peace that the Romans brought is known as *pax Romano*. The ruins of Pompeii and Herculaneum (both located near Naples in Italy) demonstrate the engineering maturity of the Roman Empire. The Roman numbering system is still employed today in clocks and for page numbering in documents. However, as a notation it is cumbersome for serious computation. The collapse of the Roman Empire in Western Europe led to a decline in knowledge and learning in Europe. However, the eastern part of the Roman Empire continued at Constantinople (now known as Istanbul in Turkey) until its sacking by the Ottomans in 1453.

1.2 The Babylonians

The Babylonian[7] civilization flourished in Mesopotamia (in modern Iraq) from about 2000 BC until about 300 BC. Various clay cuneiform tablets containing mathematical texts were discovered and later deciphered by Grotefend and Rawlinson in the nineteenth century [Smi:23]. These included tables for multiplication, division, squares, cubes and square roots; measurement of area and length; and the solution

[5] The Elgin marbles are named after Lord Elgin who moved them from the Parthenon in Athens to London in 1806. The marbles show the Pan-Athenaic festival that was held in Athens in honour of the goddess Athena after whom Athens is named.

[6] The origin of the word Hellenistic is from Hellene ("Ελλην") meaning Greek.

[7] The hanging gardens of Babylon were one of the seven wonders of the ancient world.

of linear and quadratic equations. The late Babylonian period (c. 300 BC) includes work on astronomy.

The Babylonians recorded their mathematics on soft clay using a wedge shaped instrument to form impressions of the *cuneiform* numbers. The clay tablets were then baked in an oven or by the heat of the sun. They employed just two symbols (1 and 10) to represent numbers, and these symbols were then combined to form all other numbers. They employed a positional number system[8] and used base 60 system. The symbol representing 1 could also (depending on the context) represent 60, 60^2, 60^3, etc. It could also mean 1/60, 1/3600, and so on. There was no zero employed in the system and there was no decimal point (strictly speaking no "sexagesimal point"), and therefore the context was essential.

The example above illustrates the cuneiform notation and represents the number $60 + 10 + 1 = 71$. The Babylonians used the base 60 system for computation, and this base is still in use today in the division of hours into minutes and the division of minutes into seconds. One possible explanation for the use of base 60 is the ease of dividing 60 into parts as it is divisible by 2, 3, 4, 5, 6, 10, 12, 15, 20, and 30. They were able to represent large and small numbers and had no difficulty in working with fractions (in base 60) and in multiplying fractions. They maintained tables of reciprocals (i.e., $1/\text{n}$, $n = 1, \ldots 59$ apart from numbers like 7, 11, etc., which are not of the form $2^{\alpha}3^{\beta}5^{\gamma}$ and cannot be written as a finite sexagesimal expansion).

Babylonian numbers are represented in the more modern sexagesimal notation developed by Neugebauer (who translated many of the Babylonian cuneiforms) [Res:84]. The approach is as follows: 1;24,51,10 represents the number $1 + 24/60 + 51/3600 + 10/216000 = 1 + 0.4 + 0.0141666 + 0.0000462 = 1.4142129$ and is the Babylonian representation of the square root of 2. The Babylonians performed multiplication as the following calculation of (20) * (1; 24, 51, 10) i.e., 20 * sqrt(2) illustrates:

$$
\begin{aligned}
&20^{*}1 = 20 \\
&20^{*}; 24 = 20^{*}\frac{24}{60} = 8 \\
&20^{*}\frac{51}{3600} = \frac{51}{180} = \frac{17}{60} =; 17 \\
&20^{*}\frac{10}{216000} = \frac{3}{3600} + \frac{20}{216000} =; 0, 3, 20
\end{aligned}
$$

Hence the product 20 * sqrt(2) $= 20; +8; +; 17+; 0, 3, 20 = 28; 17, 3, 20$

The Babylonians appear to have been aware of Pythagoras's Theorem about 1000 years before the time of Pythagoras. The Plimpton 322 tablet records various

[8] A positional numbering system is a number system where each position is related to the next by a constant multiplier. The decimal system is an example: e.g., $546 = 5^{*}10^2 + 4^{*}10^1 + 6$.

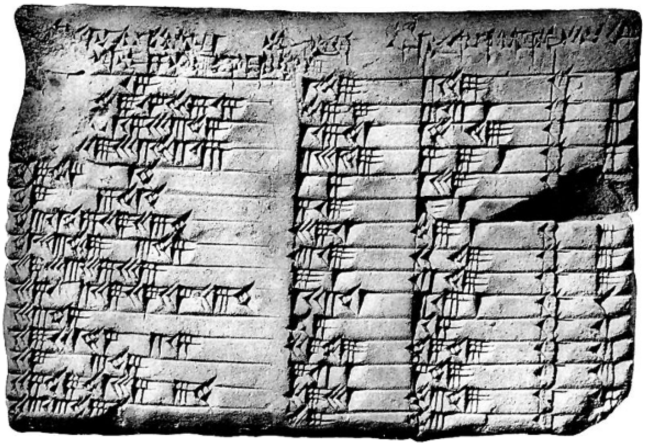

Fig. 1.1 The plimpton 322 tablet

Pythagorean triples, i.e., triples of numbers (a, b, c) where $a^2 + b^2 = c^2$ (Fig. 1.1). It dates from approximately 1700 BC

They developed algebra to assist with problem solving, and their algebra allowed problems involving length, breadth and area to be discussed and solved. They did not employ notation for representation of unknown values (e.g., let x be the length and y be the breadth), and instead they used words like "length" and "breadth". They were familiar with and used square roots in their calculations, and while they were familiar with techniques to solve quadratic equations.

They were familiar with various mathematical identities such as $(a+b)^2 = (a^2 + 2ab + b^2)$ as illustrated geometrically in Fig. 1.2. They also worked on astronomical problems, and they had mathematical theories of the cosmos to make predictions of when eclipses and other astronomical events would occur. They were also interested in astrology, and they associated various deities with the heavenly bodies such as the planets, as well as the sun and moon. Various cluster of stars with associated with familiar creatures such as lions, goats, and so on.

The earliest form of counting by the Babylonians was done using fingers. They improved upon this by developing counting boards to assist with counting and

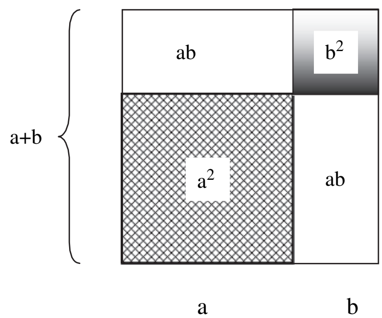

Fig. 1.2 Geometric representation of $(a + b)^2 = (a^2 + 2ab + b^2)$

simple calculations. A counting board is an early version of the abacus, and it was usually made of wood or stone. The counting board contained grooves which allowed beads or stones could be moved along the groove. The abacus differed from counting boards in that the beads in abaci contained holes that enabled them to be placed in a particular rod of the abacus.

1.3 The Egyptians

The Egyptian Civilization developed along the Nile from about 4000 BC and lasted until the Roman Empire. The achievements of the Egyptian civilization are remarkable and their engineers built the gigantic pyramids at Giza near Cairo around 3000 BC.

The Egyptians used mathematics to solve practical problems. This included measuring time, measuring the annual Nile flooding, calculating the area of land, solving cooking and baking problems, book keeping and accounting, and calculating taxes. They developed a calendar circa 4000 BC. It consisted of 12 months, and each month had 30 days. There were then five extra feast days to give 365 days in a year. Egyptians writings were recorded on the walls of temples and tombs[9] and were also recorded on a reed like parchment termed “papyrus”. There are three well-known Egyptian scripts namely the well-known hieroglyphics writing system; the hieratic script; and the demotic script.

The deciphering of the Egyptian hieroglyphics was done by Champollion with his work on the Rosetta stone. The latter was discovered during the Napoleonic campaign in Egypt, and is now in the British Museum in London. It contains three scripts: Hieroglyphics, Demotic script and Greek. The key to the decipherment was that the Rosetta stone contained just one name “Ptolemy” in the Greek text, and this was identified with the hieroglyphic characters in the cartouche[10] of the hieroglyphics. There was just one cartouche on the Rosetta stone, and Champollion inferred that the cartouche represented the name “Ptolemy”. He was familiar with another multi-lingual object which contained two names in the cartouche. One he recognised as Ptolemy and the other he deduced from the Greek text as “Cleopatra”. This led to the breakthrough in translation of the hieroglyphics [Res:84].

The Egyptians writing system is based on hieroglyphs and dates from 3000 BC. Hieroglyphs are little pictures and are used to represent words, alphabetic characters as well as syllables or sounds.

The Rhind Papyrus is one of the most famous Egyptian papyri on mathematics. It was purchased by the Scottish Egyptologist, Henry Rhind, in 1858 and is now in the British museum. The papyrus is a copy and it was created by an Egyptian

[9] The decorations of the tombs in the Valley of the Kings record the life of the pharaoh including his exploits and successes in battle.

[10] The cartouche surrounded a group of hieroglyphic symbols enclosed by an oval shape. Champollion’s insight was that the group of hieroglyphic symbols represented the name of the Ptolemaic pharaoh “Ptolemy”.

scribe called Ahmose.[11] It was originally six meters in length, and it is believed to date to 1832 BC. It contains examples of all kinds of arithmetic and geometric problems, and it was probably intended to be used by students as a textbook to develop their mathematical knowledge. This would allow the students to participate in the pharaoh's building programme. There is another well known papyrus known as the Moscow papryrus.

The Egyptian priests had were familiar with geometry, arithmetic and elementary algebra. They had formulae to find solutions to problems with one or two unknowns. A bases 10 number system was employed separate with symbols for one, ten, a hundred, a thousand, a ten thousand, a hundred thousand, and so on. These hieroglyphic symbols are represented in Fig. 1.3 below:

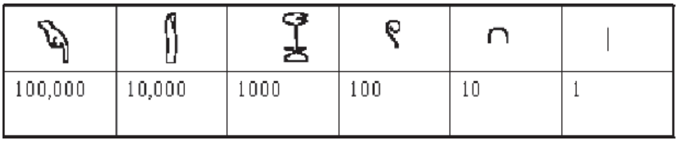

Fig. 1.3 Egyptian numerals

For example, the representation of the number 276 in Egyptian Hieroglyphics is given in Fig. 1.4:

Fig. 1.4 Egyptian representation of a number

The addition of two numerals is straight forward and involves adding the individual symbols, and where there are ten copies of a symbol it is then replaced by a single symbol of the next higher value. The Egyptian employed unit fractions (e.g., $1/n$ where n is an integer). These were represented in hieroglyphs by placing the symbol representing a "mouth" above the number. The symbol "mouth" represents part of. For example, the representation of the number $1/276$ is given in Fig. 1.5:

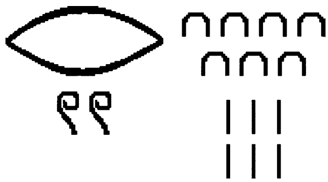

Fig. 1.5 Egyptian representation of a fraction

The problems on the papyrus included the determination of the angle of the slope of the pyramid's face. The Egyptians were familiar with trigonometry including sine, cosine, tangent and cotangent. They knew how to build right angles into

[11] The Rhind papyrus is sometimes referred to as the Ahmes papyrus in honour of the scribe who wrote it in 1832 BC.

their structures and used the ratio 3:4:5. The Rhind Papyrus also considers practical problems such as how many loaves of bread can be baked from a given quantity of grain. Other problems included calculating the number of bricks required for part of a building project. The Egyptians were familiar with addition, subtraction, multiplication and division. However, their multiplication and division was cumbersome as they could only multiply and divide by two.

Suppose they wished to multiply a number n by 7. Then $n * 7$ is determined by $n * 2 + n * 2 + n * 2 + n$. Similarly, if they wished to divide 27 by 7 they would note that $7 * 2 + 7 = 21$ and that $27 - 21 = 6$ and that therefore the answer was $3^6/_7$. Egyptian mathematics was cumbersome and the writing of their mathematics was long and repetitive. For example, they wrote a number such as 22 by $10+10+1+1$.

The Egyptians calculated the approximate area of a circle by calculating the area of a square 8/9 of the diameter of a circle. That is, instead of calculating the area in terms of our familiar πr^2 their approximate calculation yielded $(8/9 * 2r)^2 = 256/81\, r^2$ or $3.16\, r^2$. Their approximation of π was 256/81 or 3.16. They were able to calculate the area of a triangle and volumes. The Moscow papyrus includes a problem to calculate the volume of the frustum. The formula for the volume of a frustum of a square pyramid[12] was given by $V = 1/3\, h(b_1^2 + b_1 b_2 + b_2^2)$ and when b_2 is 0 then the well-known formula for the volume of a pyramid is given: i.e., $1/3\, hb_1^2$.

1.4 The Greeks

The Greeks made major contributions to western civilization including mathematics, logic, astronomy, philosophy, politics, drama, and architecture. The Greek world of 500 BC consisted of several independent city states such as Athens and Sparta, and various city states in Asia Minor. The Greek polis (πολισ) or city state tended to be quite small, and consisted of the Greek city and a certain amount of territory outside the city state. Each city state had political structures for its citizens, and these varied from city state to city state. Some were oligarchs where political power was maintained in the hands of a few individuals or aristocratic families. Others were ruled by tyrants (or sole rulers) who sometimes took power by force, but who often had a lot of support from the public. The tyrants included people such as Solon, Peisistratus and Cleisthenes in Athens.

The reforms by Cleisthenes led to the introduction of the Athenian democracy. This was the world's first democracy, and power was fully placed in the hands of the citizens. The citizens were male members of the population, and women or slaves did not participate. The form of democracy in ancient Athens differed from the representative democracy that we are familiar with today. It was an extremely liberal democracy where citizens voted on all important issues. Often, this led to disastrous results as speakers who were skilled in rhetoric could exert significant influence on the decision making. Philosophers such as Plato were against democracy as a form

[12] The length of a side of the bottom base of the pyramid is b_1 and the length of a side of the top base is b_2

of government for the state, and Plato's later political thinking advocated rule by philosopher kings. These rulers were required to study philosophy and mathematics for many years.

The rise of Macedonia led to the Greek city states being conquered by Philip of Macedonia. This was followed by the conquests of Alexander the Great who was one of the greatest military commanders in history. He defeated the Persian Empire, and extended his empire to include most of the known world. His conquests extended as far east as Afghanistan and India, and as far west as Egypt. This led to the Hellenistic age, where Greek language and culture spread throughout the world. The word Hellenistic derives from "Eλλην" which means Greek. The city of Alexandra was founded by Alexander, and it became a major centre of learning. Alexander the Great received tuition from the philosopher Aristotle. However, Alexander's reign was very short as he died at the young age of 33 in 323 BC.

Early Greek mathematics commenced approximately 500–600 BC with work done by Pythagoras and Thales. Pythagoras was a sixth century philosopher and mathematician who had spent time in Egypt becoming familiar with Egyptian mathematics. He lived on the island of Samoa and formed a sect known as the Pythagoreans. This Pythagoreans were a secret society and included men and women. They believed in the transmigration of souls and believed that number was the fundamental building block for all things. They discovered the mathematics for harmony in music by discovering that the relationship between musical notes could be expressed in numerical ratios of small whole numbers. Pythagoras is credited with the discovery of Pythagoras's Theorem, although this theorem was probably known by the Babylonians about 1000 years before Pythagoras. The Pythagorean society was dealt a major blow[13] by the discovery of the incommensurability of the square root of 2: i.e., there are no numbers p, q such that $\sqrt{2} = p/q$. This dealt a major blow to their philosophy that number is the nature of being.

Thales was a sixth century (BC) philosopher from Miletus in Asia Minor who made contributions to philosophy, geometry and astronomy. His contributions to philosophy are mainly in the area of metaphysics, and he was concerned with questions on the nature of the world. His objective was to give a natural or scientific explanation of the cosmos, rather than relying on the traditional supernatural explanation of creation in Greek mythology. He believed that there was single substance that was the underlying constituent of the world, and he believed that this substance was water. It can only be speculated why he believed water to be the underlying substance but some reasons may be that water is essential to life; when a solid is compressed it is generally transformed to a liquid substance, and so on. Thales also contributed to mathematics [AnL:95] and there is a well-known theorem in Euclidean geometry named after him. It states that if A, B and C are points on a circle, and where the line AC is a diameter of the circle, then the angle $<ABC$ is a right angle.

[13] The Pythagoreans were a secret society and its members took a vow of silence with respect to this discovery. However, one member of the society is said to have shared the secret result with others outside the sect, and the apocryphal account is that he was thrown into a lake for his betrayal and drowned. They obviously took Mathematics seriously back then.

Euclid lived in Alexandria during the early Hellenistic period. He is considered the father of geometry in that he set out a systematic treatment of geometry starting from 5 axioms, 5 postulates and 23 definitions to derive and prove a comprehensive set of theorems. He is therefore the father of the axiomatic method for mathematics. His systematic account was published in the thirteen books of the Elements [Hea:56], and this has been used as a mathematics textbook for over 2000 years. It includes the treatment of geometry and number theory. His method of proof was generally constructive in that as well as demonstrating the truth of a theorem the proof would often include the construction of the required entity. However, he was also familiar with indirect proof as the argument to show that there are an infinite number of primes demonstrates:

1. Suppose there is a finite number of primes (say n primes).
2. Multiply all n primes together and add 1 to form N.

   i. $(\mathrm{N} = p_1 * p_2 * \ldots * p_{\mathrm{n}} + 1)$

3. N is not divisible by $p_1, p_2, \ldots, p_{\mathrm{n}}$ as dividing by any of these gives a remainder of one.
4. Therefore, N must either be prime or divisible by some other prime that was not included in the list.
5. Therefore, there must be at least $n + 1$ primes.
6. This is a contradiction as it was assumed that there was a finite number of primes n.
7. Therefore, the assumption that there is a finite number of primes is false.
8. Therefore, there is an infinite number of primes.

Euclidean geometry included the parallel postulate or Euclid's fifth postulate. This postulate generated interest as many mathematicians believed that it was unnecessary and could be proved as a theorem by using the other axioms and postulates. It states that:

Definition 1.1 (Parallel Postulate) If a line segment intersects two straight lines forming two interior angles on the same side that sum to less than two right angles, then the two lines, if extended indefinitely, meet on that side on which the angles sum to less than two right angles.

This postulate was later proved to be independent of the other postulates. In the nineteenth century other geometries were developed that rejected the fifth postulate as formulated by Euclid. These include the hyperbolic geometry discovered independently by Bolyai and Lobachevsky, and elliptic geometry as developed by Riemann. The standard model of Riemannian geometry is the sphere where lines are great circles. Non-Euclidean geometries became important in the early twentieth century with the work done by Albert Einstein in the Theory of Relativity.

Euclid and contemporary Hellenistic mathematicians aimed to provide constructive solutions to problems. That is, the proof of the existence was generally accompanied by an actual construction of the solution using an unmarked straightedge and compass. The material in the Euclid's Elements is presented logically, and it is a systematic development of geomerty starting from the small set of axioms, postulates and definitions, and it leads to theorems derived logically from the axioms and postulates. Euclid's deductive method has influenced later mathematicians and scientists. There are some jumps in reasoning in The Elements, and Hilbert added extra axioms to Euclidean geometry to make it more complete in the late nineteenth century.

The Elements contains many well-known mathematical results such as:

- Pythagoras's Theorem
- Thales Theorem
- Sum of Angles in a Triangle
- Prime Numbers
- Greatest Common Divisor and Least Common Multiple
- Euclidean Algorithm
- Areas and Volumes
- Tangents to a point
- Algebra

The Euclidean algorithm is one of the oldest known algorithms and is employed to produce the greatest common divisor of two numbers. It is presented in the Elements but was known well before Euclid. The formulation of the gcd algorithm for two natural numbers a and b is as follows:

1. Check if b is zero. If so, then a is the gcd.
2. Otherwise, the gcd (a, b) is given by gcd $(b, a \bmod b)$.

It is also possible to determine integers p and q such that $ap + bq = \gcd(a, b)$.

The proof of the Euclidean algorithm is as follows. Suppose a and b are two positive numbers whose gcd has to be determined, and let r be the remainder when a is divided by b.

1. Clearly $a = qb + r$ where q is the quotient of the division.
2. Any common divisor of a and b is also a divider or r (since $r = a - qb$).
3. Similarly, any common divisor of b and r will also divide a.
4. Therefore, the greatest common divisor of a and b is the same as the greatest common divisor of b and r.
5. The number r is smaller than b and we will reach $r = 0$ in finitely many steps.
6. The process continues until $r = 0$.

Comment 1.1 Algorithms are fundamental in computing as they define the procedure by which a problem is solved. A computer program implements the algorithm in some programming language.

Eratosthenes was a Hellenistic mathematician and scientist who worked as librarian in the famous library in Alexandria. He devised a system of latitude and longitude, and became the first person to estimate of the size of the circumference of the earth. His approach to the calculation was as follows (Fig. 1.6):

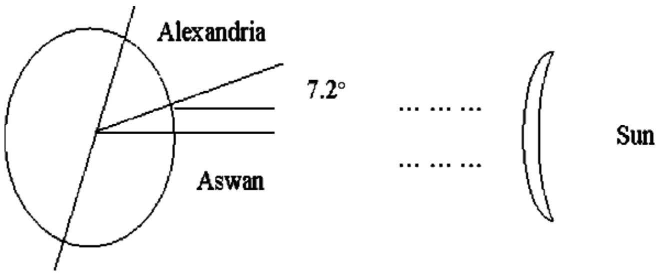

Fig. 1.6 Eratosthenes measurement of the circumference of the earth

1. On the summer solstice at noon in the town of Aswan[14] on the Tropic of Cancer in Egypt the sun appears directly overhead.
2. Eratosthenes believed that the earth was a sphere.
3. He assumed that rays of light came from the sun in parallel beams and reached the earth at the same time.
4. At the same time in Alexandria he had measured that the sun would be 7.2° south of the zenith.
5. He assumed that Alexandria was directly north of Aswan.
6. He concluded that the distance from Alexandria to Aswan was 7.2/360 of the circumference of the earth.
7. Distance between Alexandria and Aswan was 5000 stadia (approximately 800 km).
8. He established a value of 252,000 stadia or approximately 39,6000 km.

Eratosthene's calculation was within 1% of the true value of 40,008 km and was an impressive result for 200 BC. The errors in his calculation were due to:

1. Aswan is not exactly on the Tropic of Cancer but it is actually 55 km north of it.
2. Alexandria is not exactly north of Aswan and there is a difference of 3° longitude.
3. The distance between Aswan and Alexandtia is 729 km not 800 km.
4. Angles in antiquity could not be measured with absolute precision.
5. The angular distance is actually 7.08° and not 7.2°.

Eratosthenes also calculated the approximate distance to the moon and sun and he also produced maps of the known world. He developed a very useful algorithm for determining all of the prime numbers up to a specified integer. The method is known as the Sieve of Eratosthenes and the steps are as follows:

1. Write a list of the numbers from 2 to the largest number that you wish to test for primality. This first list is called A.

[14] The town of Aswan is famous today for the Aswan high dam which was built in the 1960s. There was an older Aswan dam built by the British in the late nineteenth century. The new dam led to a rise in the water level of Lake Nasser and flooding of archaeological sites along the Nile. Several sites such as Abu Simbel and the island of Philae were relocated to higher ground.

2. A second list B is created to list the primes. It is initially empty.
3. The number 2 is the first prime number and is added to the list of primes in B.
4. Strike off (or remove) all multiples of 2 from List A.
5. The first remaining number in List A is a prime number and this prime number is added to List B.
6. Strike off (or remove) this number and all multiples of this number from List A.
7. Repeat steps 5 through 7 until no more numbers are left in List A.

Comment 1.2 The Sieve of Eratosthenes method is a well-known algorithm for determining prime numbers. Computing students often implement this algorithm as an early computer assignment.

Archimedes was a mathematician, astronomer and philosopher who lived in Syracuse. He is famous for his discovery of the law of buoyancy that is known as Archimedes's principle:

> The buoyancy force is equal to the weight of the displaced fluid.

Archimedes is believed to have discovered the principle while sitting in his bath He was so overwhelmed with his discovery that he rushed out onto the streets of Syracuse shouting "Eureka", but forgot to put on his clothes to announce the discovery.

The weight of the displaced liquid will be proportional to the volume of the displaced liquid. Therefore, if two objects have the same mass, the one with greater volume (or smaller density) has greater buoyancy. An object will float if its buoyancy force (i.e., the weight of liquid displaced) exceeds the downward force of gravity (i.e., its weight). If the object has exactly the same density as the liquid, then it will stay still, neither sinking nor floating upwards.

For example, a rock is generally a very dense material and will generally not displace its own weight. Therefore, a rock will sink to the bottom as the downward weight exceeds the buoyancy weight. However, it the weight of the object is less than the liquid it would displace then it floats at a level where it displaces the same weight of liquid as the weight of the object.

Archimedes also made good contributions to mathematics including a good approximation to π, contributions to the positional numbering system, geometric series, and to maths physics. He also solved several interesting problems: e.g., the calculation of the composition of cattle in the herd of the Sun god by solving a number of simultaneous Diophantine equations. The herd consisted of bulls and cows with one part of the herd consisting of white, second part black, third spotted and the fourth brown. Various constraints were then expressed in Diophantine equations and the problem was to determine the precise composition of the herd. Diophantine equations are named after Diophantus worked who worked on number theory in the third century.

Archimedes also worked on another interesting problem to determine the number of grains of sands in the known universe. He challenged the prevailing view that the number of grains of sand was too large to be counted, and in order to provide an upper bound he needed to develop a naming system for large numbers. The largest number in common use at the time was a myriad (100 million) and a myriad is

10,000. Archimedes' numbering system goes up to $8^{*}10^{16}$ and he also developed the laws of exponents: i.e., $10^{a}10^{b} = 10^{a+b}$. His calculation of the upper bound includes not only the grains of sand on each beach but on the earth filled with sand and the known universe filled with sand. His final estimate of the number of grains of sand in a filled universe is an upper bound of 10^{64} for the number of grains of sand in a filled universe.

Is is possible that he may have developed the odometer,[15] and this instrument could calculate the total distance travelled on a journey. An odometer is described by the Roman engineer Vitruvius around 25 BC. It employed a wheel with a diameter of 4 feet, and the wheel turned 400 times in every mile.[16] The device included gears and pebbles and a 400 tooth cogwheel that turned once every mile and caused one pebble to drop into a box. The total distance travelled was determined by counting the pebbles in the box.

Aristotle was born in Macedonia and became a student of Plato in Athens. Plato had founded a school (known as Plato's academy) in Athens in the fourth century BC, and this school remained open until 529 A.D. Aristotle became a famous philosopher in his own right and he founded his own school (known as the Lyceum) in Athens. He was also the teacher of Alexander the Great. Aristotle made contributions to physics, biology, logic, politics, ethics and metaphysics. The work of Plato and Aristotle provided the foundations for Western philosophy (Fig. 1.7).

Aristotle's starting point to the acquisition of knowledge was the senses. He believed that the senses were essential to acquire knowledge. This position is the opposite from Plato who argued that the senses deceive and should not be relied upon. Plato's writings are mainly in dialogues involving his former mentor Socrates.[17] Most of Aristotle's writings are in treatise form although he wrote some dialogues in his early career. Aquinus,[18] a thirteenth century Christian theologian and philosopher, was deeply influenced by Aristotle, and referred to him as the philosopher. Acquinus was an empiricist (i.e., he believed that all knowledge was gained by sense experience), and he used some of Aristotle's arguments to offer five proofs of the existence of God. These arguments included the Cosmological argument and the Design argument. The Cosmological argument used Aristotle's

[15] The origin of the word "odometer" is from the Greek words 'οδοζ (meaning journey) and μετρον meaning (measure).

[16] The figures given here are for the distance of one Roman mile. This is given by $\pi 2^2 * 400 = 12.56 * 400 = 5024$ (which is less than 5280 feet for a standard mile in the Imperial system).

[17] Socrates was a moral philosopher who deeply influenced Plato. His method of enquiry into philosophical problems and ethics was by questioning. Socrates himself maintained that he knew nothing (Socratic ignorance). However, from his questioning it became apparent that those who thought they were clever were not really that clever after all. His approach obviously would not have made him very popular with the citizens of Athens. Socrates had consulted the oracle at Delphi to find out who was the wisest of all men, and he was informed that there was no one wiser than him. Socrates was sentenced to death for allegedly corrupting the youth of Athens, and the sentence was carried out by Socrates being forced to take hemlock (a type of poison). The juice of the hemlock plant was prepared for Socrates to drink.

[18] Aquinus's (or St. Thomas's) most famous work is Sumna Theologicae.

Fig. 1.7 Plato and Aristotle

ideas on the scientific method and causation. Acquius argued that there was a first cause and he deduced that this first cause is God.

1. Every effect has a cause
2. Nothing can cause itself
3. A causal chain cannot be of infinite length
4. Therefore there must be a first cause

Aristotle made important contributions to formal reasoning with his development of syllogistic logic and foundational work in modal logic. His collected works on logic is called the Organon and it was used in his school in Athens. Syllogistic logic (also known as term logic) consists of reasoning with two premises and one conclusion. Each premise consists of two terms and there is a common middle term. The conclusion links the two unrelated terms from the premises. This is best illustrated by an example:

Premise 1	All Greeks are Mortal
Premise 2	Socrates is a Greek.
	. . . .
Conclusion	Socrates is Mortal

In this example the common middle term is "Greek", and this term appears in the two premises. The two unrelated terms from the premises are "Socrates" and "Mortal". The relationship between the terms in the first premise is that of the universal: i.e., anything or any person that is a Greek is mortal. The relationship between the terms in the second premise is that of the particular: i.e., Socrates is a person that is a Greek. The conclusion from the two premises is that Socrates is mortal: i.e., a particular relationship between the two unrelated terms "Socrates" and "Mortal".

The example above is an example of a valid syllogistic argument. Aristotle studied the various possible syllogistic arguments and determined those that were valid and those that were invalid. There are several candidate relationships that may potentially exist between the terms in a premise. These include (Table 1.1):

Table 1.1 Syllogisms: Relationship between terms

Relationship	Abbr.
Universal Affirmation	A
Universal Negation	E
Particular Affirmation	I
Particular Negation	O

In general, a syllogistic argument will be of the form:

S x M
M y P
. . . . .
S z P

where x, y, z may be universal affirmation, universal negation, particular affirmation and particular negation. Syllogistic logic is described in more detail in [ORg:06]. Aristotle's work was highly regarded in classical and medieval times, and was believed to be a fully worked out system. Kant believed that there was nothing else to invent in Logic after the work of Aristotle. There was another competing system of logic proposed by the Stoics in Hellentistic times: i.e., an early form of propositional logic that was developed by Chrysippus[19] in the third century BC. Later work in the nineteenth century by George Boole led to propositional logic, and later work by Frege and others led to predicate calculus. Aristotelian logic is mainly of historical interest today.

The Greeks invented a number of mechanical devices to assist with problem solving, and one of the most famous of these was the Antikythera [Pri:59]. This was an ancient mechanical device designed to calculate astronomical positions. An ancient Antikythera was discovered in 1902 in a week off the Greek island of Antikythera, and dates from about 80 BC. It is one of the oldest known geared devices, and it

[19] Chrysippus was the head of the Stoics in the third century BC.

is believed that it was used for calculating the position of the sun, moon, stars and planets for a particular date entered. The device is comparable in the complexity of its parts and gear arrangement as clocks in the eighteenth century.

The Romans appear to have been aware of a device similar to the Antikythera, as a device that is capable of calculating the position of the planets is mentioned by Cicero. The island of Antikythera was well-known in the Greek and Roman period for its displays of mechanical engineering. A model of how the Antikythera might have worked is available, and according to that model, the front dial shows the annual progress of the Sun and Moon through the zodiac against the Egyptian calendar, with the other rear dials providing specialised information. It is debatable as to how accurate the model is with respect to the actual device. Other models proposed include that the device acted as a planetarium.

1.5 The Romans

Rome is said to have been founded[20] by Romulus and Remus about 750 BC. Early Rome covered a small part of Italy but it gradually expanded in size and importance. Rome destroyed Carthage[21] in 146 BC to become the major power in the Mediterranean. Julius Caesar (Gaius Iulius Caesar) initially conquered the Gauls in 58 BC (Fig. 1.8).

The Gauls consisted of several Celtic[22] tribes who were disunited. Vercingetorix was the leader of the Arverni tribe and he succeeded briefly in uniting the Celts. However, Caesar finally defeated Vercingetorix at the siege of Alesia in 52 BC. Roman merchants needed to develop accounting systems to track their trade across the Roman Empire. The Hellenistic world was colonized by the Romans,[23] and the Romans became familiar with Greek culture and mathematics.

The Romans introduced their own number system where Roman letters represented numbers (Fig. 1.9):

---

[20] The Aenid by Virgil suggests that the Romans were descended from survivors of the Trojan war, and that Aeneas brought surviving Trojans to Rome after the fall of Troy.

[21] Carthage was located in Tunisia, and the wars between Rome and Carthage are known as the Punic wars. Hannibal was one of the great Carthaginan military commanders, and during the second Punic war, he brought his army to Spain, marched through Spain and crossed the Pyrenees. He then marched along southern France and crossed the Alps into Northern Italy. His army also consisted of war elephants. Rome finally defeated Carthage and levelled the city.

[22] The Celtic period commenced around 1000 BC in Hallstaat (near Salzburg in Austria). The Celts were skilled in working with Iron and Bronze, and they gradually expanded into Europe. They eventually reached Britain and Ireland around 600 BC. The early Celtic period was known as the "Hallstaat period" and the later Celtic period is known as "La Téne". The later La Téne period is characterised by the quality of ornamentation produced. The Celtic museum in Hallein in Austria provides valuable information and artefacts on the Celtic period. The Celtic language would have similarities to the Irish language. However, the Celts did not employ writing, and the Ogham writing developed in Ireland was employed later in the early Christian period.

[23] The Romans did not make any major advances on Hellenistic Mathematics.

Fig. 1.8 Julius Caesar

Fig. 1.9 Roman numbers

I = 1
V = 5
X = 10
L = 50
C = 100
D = 500
M = 1000

A Roman number consists of a sequence of Roman letters and there were rules employed in the evaluation. The rules specified that if a number follows a smaller number then the smaller number is subtracted from the large: e.g., IX represents 9 and XL represents 40. Similarly, if a smaller number followed a larger number they were generally added: e.g., MCC represents 1200. They had no zero in their system. Roman numerals are still used today in page numbering for books or on the faces of clocks.

Calculations with Roman numerals was cumbersome, especially operations that involved multiplication or division. In practice, an abacus was often employed to perform the calculation. An abacus consists of several columns in which pebbles are placed. Each column represented powers of 10: i.e., 10^0, 10^1, 10^2, 10^3, etc. The column to the far right represents one; the column to the left 10; next column to the left 100; and so on. Pebbles (calculi) were placed in the columns to represent different numbers: e.g., the number represented by an abacus with 4 pebbles on the far right; 2 pebbles in the column to the left; and 3 pebbles in the next column to the left is 324. Calculations were performed by moving pebbles from column to column. The operator of the abacus needed to be properly trained to be effective.

The Roman merchant needed to perform calculations to keep track of trade within the Roman Empire. They introduced a set of weights and measures (including the *libra* for weights and the *pes* for lengths). The merchants also developed an early banking system to provide loans for business. They commenced minting money about 290 BC. The Romans also made contributions to calendars and the Julian calendar was introduced in 45 BC by Julius Caesar. It has a regular year of 365 days divided into 12 months and a leap day is added to February every four years. It remained in use up to the twentieth century, but has since been replaced by the Gregorian calendar. The problem with the Julian calendar is that too many leap years are added over time. The Gregorian calendar was first introduced in 1582.

The Romans employed the available mathematics that had been developed by the Greeks. Caesar's Cipher was employed by Caesar on his military campaigns in order to safely communicate important messages to his generals. It is one of the simplest and widely known encryption techniques, and involves the substitution of each letter in the plaintext (i.e., the original message) by a letter a fixed number of positions down in the alphabet. For example, a shift of 3 positions cause the letter B to be replaced by E, the letter C by F, and so on. The Caesar cipher is easily broken, as the frequency distribution of letters may be employed to determine the mapping. However, given that Caesar was essentially dealing with Gaulish tribes who were mainly illiterate, and who certainly lacked knowledge of cryptology and frequency distribution of the letters in the alphabet, it is likely to have provided good security. The translation of the Roman letters by the Caesar cipher (with a shift key of 3) can be seen by the following table. It shows each letter of the alphabet and the corresponding cipher symbol that it is mapped on to (Table 1.2):

Table 1.2 Caesar cipher

Alphabet Symbol	abcde	fghij	klmno	pqrst	uvwxyz
Cipher Symbol	dfegh	ijklm	nopqr	stuvw	xyzabc

The process of enciphering a message (i.e., plaintext) simply involves looking up each letter in the plaintext and writing down the corresponding cipher letter. For example, the enciphering of the plaintext message "summer solstice" involves the following:

Plaintext	Summer Solstice
Cipher Text	vxpphu vrovwleh

The process of deciphering a cipher message involves doing the reverse operation: i.e., for each cipher letter the corresponding plaintext letter is identified from the table.

Cipher Text	vxpphu vrovwleh
Plaintext	Summer Solstice

The encryption can also be represented using modular arithmetic by first using the numbers 0–25 to represent the alphabet letters, and then using addition (modula 26) to perform the encryption. That is, the encoding of the plaintext letter represented by the number x is given by:

$$c = x + 3 \pmod{26}$$

Similarly, the decoding of a cipher letter represented by the number c is given by:

$$x = c - 3 \pmod{26}$$

The emperor Augustus[24] employed a similar substitution cipher (with a shift key of 1). The Caesar cipher was still in use up to the early twentieth century. However, by then frequency analysis techniques were available to break the cipher. The Vignère cipher uses a Caesar cipher with a different shift at each position in the text. The value of the shift to be employed with each plaintext letter is defined using a repeating keyword.

The famous library in Alexandria was once the largest library in the world. It was build during the Hellenistic period in the third century BC. Caesar's campaign in Egypt in 48 BC caused damage to the library, and the library was finally destroyed by fire in 391 A.D. The new library in Alexandria was inaugurated in 2003 on the site of the old library.

1.6 Islamic Influence

Islamic mathematics refers to mathematics developed in the Islamic world from the birth of Islam in the early seventh century up until the seventeenth century. The Islamic world commenced with Mohammed in Mecca, and spread throughout the Middle East, North Africa and Spain. Islamic scholars translated the works of

[24] Augustus was the first Roman emperor and his reign ushered in a period of peace and stability following the bitter civil wars. He was the adopted son of Julius Caesar and was called Octavion before he became emperor. The earlier civil wars were between Caesar and Pompey, and following Caesar's assassination civil war broke out between Mark Anthony and Octavion. Octavion defeated Anthony and Cleopatra at the battle of Actium.

the Greeks into Arabic, and this led to the preservation of the Greek texts during the Dark ages in Europe. Further, the Islamic scholars developed the existing mathematics further. The Islamic contribution filled the void that followed the end of the Roman Empire in the sixth century A.D. The Moors[25] invaded Spain in the eighth century A.D., and they ruled large parts of the Iberian Peninsula for several centuries. The Moorish influence[26] in Spain continued until the time of the Catholic Monarchs[27] in the fifteenth century. Ferdinand and Isabella united Spain, defeated the Moors, and expelled them from Spain.

The Islamic mathematicians and scholars were based in several countries including Iran, Iraq, Turkey, North Africa and Spain. Early work commenced in Baghdad, and the mathematicians were influenced by the work of Hindu mathematicians who had introduced the decimal system and decimal numerals. There was a renaissance in European learning and interest in mathematics in the seventeenth century, and the Islamic texts played a key part in the revival.

Many caliphs (Muslim rulers) were enlightened and encouraged scholarship in mathematics and science. The initial work done was the translation of the existing Greek texts, and this led to a centre of translation and research in Baghdad. The translations were done as part of the research effort and included the works of Euclid, Archimedes, Apollonius and Diophantus. Al-Khwarizmi[28] made contributions to early classical algebra, and the word algebra comes from the Arabic word "*al jabr*" that appears in a text book by Al-Khwarizmi.

Early work in algebra had been done by the Babylonians, Egyptians and Greeks. The Babylonians had a general procedure for solving quadratic equations but there was limited use of symbols for unknowns. The Greeks represented quantities as geometrical magnitudes. Later Greek mathematicians such as Diophantus developed algebra for solving Diophantine equations. This included solving equations in several unknowns. The Islamic contribution to algebra was an advance on the achievements of the Greeks. They developed a broader theory that treated rational and irrational numbers as algebraic objects, and moved away from the Greek concept of mathematics as being essentially Geometry. Later Islamic scholars built on Al-Khwarizmi's work, and applied algebra to arithmetic and geometry. This included contributions to reduce geometric problems such as duplicating the cube to algebraic problems. Eventually this led to the use of symbols in the fifteenth century such as:

$$x^n \cdot x^m = x^{m+n}.$$

[25] The origin of the word "Moor" is from the Greek work μυοροζ meaning very dark. It referred to the fact that many of the original Moors who came to Spain were from Egypt, Tunisia and other parts of North Africa.

[26] The Moorish influence includes the construction of various castles (*alcazar*), fortresses (*alcalzaba*) and mosques. One of the most striking Islamic sites in Spain is the palace of Alhambra in Granada, and this site represents the zenith of Islamic art.

[27] The Catholic Monarchs refer to Ferdinand of Aragon and Isabella of Castille who married in 1469. They captured Granada (the last remaining part of Spain controlled by the Moors) in 1492.

[28] The origin of the word algorithm is from the name of the Islamic scholar Al-Khwarizmi.

The poet Omar Khayyam was also a mathematician. He did work on the classification of cubic equations with geometric solutions. Others also applied algebra to geometry, and aimed to study curves by using equations. Other scholars made contributions to the theory of numbers: e.g., a theorem that allows pairs of amicable numbers to be found. Amicable numbers are two numbers such that each is the sum of the proper divisors of the other. They were aware of Wilson's theory in number theory: i.e., for p prime then p divides $(p-1)!+1$. This result was proved formally by Lagrange in the eighteenth century.

Moorish Spain became a centre of learning, and this led to Islamic and other scholars coming to study in the universities in Spain. This includes scholars such as Averros and Avicenna who provided commentaries on the work of Aristotle. Many texts on Islamic mathematics were translated from Arabic into Latin, and this helped to start the renaissance in learning and mathematics in Europe.

1.7 Chinese and Indian Mathematics

The development of mathematics in China was independent of developments in other countries. This was due to the geographical position of China, and its ability to absorb other cultures into its own without changing its own. The development of mathematics in China commenced about 1000 BC. The Chinese approach to mathematics differed from the Greeks, in that its focus was on problem solving rather than on conducting formal proofs. Hellenistic mathematics employed an axiomatic approach with axioms and rules of deduction. Chinese mathematics was pragmatic, and was concerned with finding the solution to practical problems such as the calendar, the prediction of the positions of the heavenly bodies, land measurement, conducting trade, and the calculation of taxes.

The Chinese employed counting boards as mechanical aids for calculation from the fourth century BC. Counting-boards are similar to abaci and are usually made of wood or metal, and contained carved grooves between which beads, pebbles or metal discs were moved. The abacus is a device, usually of wood having a frame that holds rods with freely-sliding beads mounted on them. It is used as a tool to assist calculation, and it is useful for keeping track of the sums, the carrys, and so on of calculations.

Early Chinese mathematics was written on bamboo strips and included work on arithmetic and astronomy. The Chinese method of learning and calculation in mathematics was learning by analogy. This involves a person acquiring knowledge from observation of how a problem is solved, and then applying this knowledge for problem-solving to similar kinds of problems.

The Chinese had their version of Pythagoras's Theorem and applied it to practical problems. One well-knowm Chinese mathematical book is the Mathematical Treatise in Nine Sections. This dates from the thirteenth century and it was used as a textbook for several hundred years. It included the Chinese remainder theorem, the formula for finding the area of a triangle, as well as showing how polynomial equations (up to degree ten) could be solved. Other Chinese mathematicians showed how geometric problems could be solved by algebra, how roots of polynomials could be solved, how quadratic and simultaneous equations could be solved, and how the area of various geometric shapes such as rectangles, trapezia and circles could be computed. Chinese mathematicians were familiar with the formula to calculate the volume of a sphere. The best approximation that the Chinese had of π was 3.14159, and this was obtained by Hui by approximations from inscribing regular polygons with 3×2^n sides in a circle. Hui seems to have been familiar with the idea of a limit, as his approximation to π is achieved using an iterative approach with each iteration achieving a closer approximation to π.

The Chinese made contributions to number theory including the summation of arithmetic series and solving simultaneous congruences. The Chinese remainder theorem deals with finding the solutions to a set of simultaneous congruences in modular arithmetic. Chinese astronomers made accurate observations which were used to produce a new calendar in the sixth century. This was known as the Taming Calendar and it was based on a cycle of 391 years.

Indian mathematicians have made major contributions to the development of mathematics. One key contribution of Indian mathematicians is to the development of the decimal notation for numbers that is now used throughout the world. The decimal system was developed in India sometime between 400 BC and 400 AD. Indian mathematicians also invented zero and negative numbers, and also did early work on the trigonometric functions of sine and cosine The knowledge of the decimal numerals reached Europe through Arabic mathematicians, and the resulting system is known as the Hindu-Arabic numeral system.

The Sulva Sutras is a Hindu text that documents Indian mathematics and it dates from about 400 BC. The Indian mathematicians were familiar with the statement and proof of Pythagoras's theorem, and were familiar with Rational numbers, quadratic equations, as well as the calculation of the square root of 2 to five decimal places.

Panini was a fifth century BC. Indian mathematician and linguist who did pioneering work on phonetics and morphology for the Sanskrit language. His work on grammar allowed sentences to be formed from a set of rules, and is the earliest known work on linguistics and formal grammars.

1.8 Review Questions

1. Describe the number systems employed by the various civilizations discussed in this chapter and discuss the strengths and weaknesses of each system.
2. Describe ciphers used during the Roman civilization and write a program to implement one of these. What were the disadvantages of these ciphers and how would you improve upon them?

3. Discuss the nature of an algorithm and its importance in computing. Describe any algorithm that you are familiar and implement in a programming language of your choice.
4. Discuss the working of an abacus and its application to calculation.
5. Discuss syllogistic logic and identify strengths and weaknesses of the logic. Discuss the similarities and differences between syllogistic logic and propositional and predicate logic.

1.9 Summary

The last decades of the twentieth century have witnessed a proliferation of high-tech computers, mobile phones, and information technology. Software is now pervasive and it is included in automobiles, airplanes, televisions, and mobile communication. It is only in recent decades that technology has become an integral part of the western world, and the pace of change has been extraordinary. It has led to increases in industrial productivity and potentially allows humans the freedom to engage in more creative and rewarding tasks.

This chapter considered the contributions of early civilisations in providing a primitive foundation for what has become computer science. It included a discussion on the Babylonians, the Egyptians, the Greeks and the Romans as well as contributions from Islamic scholars.

The Babylonian civilization flourished from about 2000 BC and they produced clay cuneiform tablets containing mathematical texts. These included tables for multiplication, division, squares, and square roots as well as the calculation of area and the solution of linear and quadratic equations.

The Egyptian Civilization developed along the Nile from about 4000 BC and lasted until the Roman Empire. They used mathematics for practical problem solving, and this included measuring time, measuring the annual Nile flooding, calculating the area of land, solving baking problems. The use of mathematics was essential in constructing pyramids.

The Greeks made major contributions to western civilization including mathematics, logic, philosophy, politics, drama, and architecture. Euclid developed a systematic treatment of geometry starting from a small set of axioms, postulates and definitions to derive and prove a comprehensive set of theorems. Euclid's Elements has been in use as a textbook for over 2000 years.

The Romans developed a cumbersome number system that is still used in clocks today. They also developed the Julian calendar and employed simple ciphers to ensure that information communicated was kept confidential. The Islamic contribution helped to preserve the earlier work of the Greeks, and they also developed mathematics and algebra further. Later Islamic scholars applied algebra to arithmetic and geometry.