Chapter 12
Quantum Teleportation

"*We report free-space implementation of quantum teleportation over 16 km*"
– Jin et al.[1]

In science fiction stories, teleportation is usually depicted as a routine means of relocating an object by a process of dissociation, information transmission, and reconstitution. When all goes well the original object is scanned and disassembled at one place only to shimmer reassuringly back into existence at another. For dramatic effect, occasional blunders corrupt the object en route or leave it suspended in some nebulous state. Hapless bit-part actors seem especially prone to malfunctions.

Such accounts of teleportation are convenient literary devices for moving action heroes around the Universe and for introducing paradoxes of identity into story lines. But to what extent is teleportation consistent with known physical laws? In particular, does quantum information offer any new possibilities? In this chapter we look at the *scientific* basis for teleportation.

12.1 Uncertainty Principle and "Impossibility" of Teleportation

Until recently no serious attention had been paid to the physical principles on which true teleportation might be based. The presumption of most scientists, if they had any, was that teleportation was impossible because it would require some sort of scanning, or measurement, operation in order to extract a precise description of the state of all the particles in a system. At the very least this would seem to necessitate having to learn, simultaneously, the positions and momenta of all the particles from which the object was made.

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[1]Source: "Experimental Free-space Quantum Teleportation" by Xian-Min Jin, Ji-Gang Ren, Bin Yang, Zhen-Huan Yi, Fei Zhou, Xiao-Fan Xu, Shao-Kai Wang, Dong Yang, Yuan-Feng Hu, Shuo Jiang, Tao Yang, Hao Yin, Kai Chen, Cheng-Zhi Peng & Jian-Wei Pan, Nature Photonics, Volume **4** (2010) pp. 376–381.

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Unfortunately, such a measurement is provably impossible! The Heisenberg Uncertainty Principle shows that whenever we try to measure a pair of observables whose corresponding observable operators do not commute, the product of the uncertainties in the expected values of the two operators is greater than a definite minimum value. This is the case for position and momentum observables, because the position observable, X, does not commute with the momentum observable, P, and in fact $[X, P] = i\hbar$ from which one can deduce (as we will show below) that $\Delta X \Delta P \geq \frac{\hbar}{2}$. Consequently, teleportation seemed doomed to fail because you could never obtain *complete* information about the original object sufficient to resynthesize it perfectly elsewhere.

12.1.1 Heisenberg Uncertainty Principle

To understand where the Heisenberg Uncertainty Principle comes from, consider any pair of observables represented by Hermitian operators, A, and B. We are interested in quantifying the uncertainties with which we can know the values of these observables simultaneously. We characterize these uncertainties via their mean square deviations. Starting with the operators:

$$\begin{aligned} \Delta A &= A - \langle A \rangle \\ \Delta B &= B - \langle B \rangle \end{aligned} \tag{12.1}$$

as the deviations of A and B from their true means, squaring gives us:

$$\begin{aligned} \langle (\Delta A)^2 \rangle &= \langle A^2 \rangle - \langle A \rangle^2 \\ \langle (\Delta B)^2 \rangle &= \langle A^2 \rangle - \langle A \rangle^2 \end{aligned} \tag{12.2}$$

as the mean square deviations. These quantities characterize how uncertain we are in the value of observables A and B.

Next, to obtain our desired formula we can use the Cauchy-Schwarz inequality. For vectors u and v the Cauchy-Schwarz inequality tells us how their inner products are related, namely:

$$\langle u|u\rangle \langle v|v\rangle \geq |\langle u|v\rangle|^2 \tag{12.3}$$

Setting $\langle u| = \langle \psi|(\Delta A)^\dagger$ and $|v\rangle = \Delta A|\psi\rangle$ the Cauchy-Schwarz inequality implies:

$$\underbrace{\langle \psi|(\Delta A)^\dagger}_{\langle u|}\underbrace{(\Delta A)|\psi\rangle}_{|u\rangle}\underbrace{\langle \psi|(\Delta B)^\dagger}_{\langle v|}\underbrace{(\Delta B)|\psi\rangle}_{|v\rangle} \geq \underbrace{|\langle \psi|(\Delta A)^\dagger}_{\langle u|}\underbrace{(\Delta B)|\psi\rangle|^2}_{|v\rangle} \tag{12.4}$$

However, as A is Hermitian and $\langle A \rangle$ is a real number, $\Delta A = A - \langle A \rangle$ must be Hermitian too and so $(\Delta A)^\dagger(\Delta B) = \Delta A \Delta B$. Hence, we obtain

$$\langle (\Delta A)^2 \rangle \langle (\Delta B)^2 \rangle \geq |\langle \Delta A \Delta B \rangle|^2 \tag{12.5}$$

So far so good, but to make progress we now need to say something about $\Delta A \Delta B$. To do so, let us split it into two equal terms and insert zero written in just the right way. Namely,

$$\begin{aligned}\Delta A \Delta B &= \frac{1}{2}\Delta A \Delta B + \frac{1}{2}\Delta A \Delta B \\ &= \frac{1}{2}(\Delta A \Delta B - \Delta B \Delta A) + \frac{1}{2}(\Delta A \Delta B + \Delta B \Delta A) \\ &= \frac{1}{2}[\Delta A, \Delta B] + \frac{1}{2}\{\Delta A, \Delta B\} \end{aligned} \tag{12.6}$$

which shows $\Delta A \Delta B$ can be written as the sum of commutator $[\Delta A, \Delta B]$ and an anti-commutator $\{\Delta A, \Delta B\}$. The significance of this is that the commutator of two Hermitian matrices is itself anti-Hermitian, i.e., $[\Delta A, \Delta B]^\dagger = -[\Delta A, \Delta B]$, and the expectation value of an anti-Hermitian operator is purely *imaginary*. Conversely, the anti-commutator of two Hermitian matrices is itself Hermitian, i.e., $\{\Delta A, \Delta B\}^\dagger = \{\Delta A, \Delta B\}$, and the expectation value of an Hermitian operator is purely *real*. Thus, from (12.5) and (12.6) we see that:

$$\begin{aligned}\langle(\Delta A)^2\rangle\langle(\Delta B)^2\rangle \geq |\langle \Delta A \Delta B\rangle|^2 &\geq \left|\left\langle \frac{1}{2}[\Delta A, \Delta B] + \frac{1}{2}\{\Delta A, \Delta B\}\right\rangle\right|^2 \\ &\geq \frac{1}{4}|\langle[\Delta A, \Delta B]\rangle|^2 + \frac{1}{4}|\langle\{\Delta A, \Delta B\}\rangle|^2 \\ &\geq \frac{1}{4}|\langle[A, B]\rangle|^2 + \frac{1}{4}|\langle\{\Delta A, \Delta B\}\rangle|^2 \geq \frac{1}{4}|\langle[A, B]\rangle|^2 \end{aligned} \tag{12.7}$$

where we have used $[\Delta A, \Delta B] = [A, B]$. Thus, we arrive at the Heisenberg Uncertainty Principle:

Heisenberg's Uncertainty Principle For any two hermitian operators, the product of the uncertainties in their values always satisfies the inequality:

$$\langle(\Delta A)^2\rangle\langle(\Delta B)^2\rangle \geq \frac{1}{4}|\langle[A, B]\rangle|^2 \tag{12.8}$$

Thus, if teleportation requires that an object be scanned to ascertain (say) the position and momentum of all the particles which comprise it, then as the observables for position and momentum do not commute, i.e., as $[X, P] = i\hbar$, we have (setting $A = X$ and $B = P$) $\langle(\Delta X)^2\rangle\langle(\Delta P)^2\rangle \geq \frac{1}{4}|i\hbar|^2$, which implies the more famous Heisenberg Uncertainty Relation $\Delta X \Delta P \geq \frac{\hbar}{2}$. It is therefore, as a matter of physical *principle*, quite impossible to determine, simultaneously, the exact position and exact momentum of all the particles in an object. Hence, the Heisenberg Uncertainty Principle appeared to rule the possibility of true physical teleportation given that the (presumed) scanning step it must involve is physically impossible.

12.2 Principles of True Teleportation

The situation changed in 1993 when, in a paper whose author list reads like a "Who's Who?" of quantum information theory, Charles Bennett, Gilles Brassard, Claude Crepeau, Richard Jozsa, Asher Peres, and William Wootters, showed how to exploit entangled states and non-local influences, to circumvent the limitations of the Heisenberg Uncertainty Principle and teleport an arbitrary—even unknown—quantum state between two locations in such a manner that the state did not traverse the intervening distance [49]. The technique transfers the quantum state of the particle to be teleported to another remote particle without the original particle having to traverse the intervening distance. However, in the process, the quantum state of the original particle is necessarily destroyed and that of the receiving particle becomes a perfect reincarnation of the original. Quantum teleportation is therefore distinct from "faxing", which would leave the original intact and transmit an approximate copy of it over the intervening distance. It is also distinct from "cloning", which would leave the original intact and create a perfect copy. Obviously, the notion of teleporting a quantum state of a simple particle is considerably less ambitious than teleporting an entire human being from one place to another, but it is a start and has been demonstrated experimentally to increasing degrees of sophistication [38, 67, 68, 279, 340, 374, 418, 503].

We emphasize that quantum physics dictates that the state of the original particle has to be destroyed during the teleportation operation, otherwise teleportation would produce a perfect copy of the original (unknown) quantum state and this would violate the "no cloning" theorem (see Sect. 11.6). This marks a slight distinction from science fiction accounts of teleportation wherein defective teleporters are apt to create perfect clones.

Note also, that in quantum teleportation it is not the *particle* that is teleported, but rather its quantum *state*. However, if the original particle holding the state, is of exactly the same type as the particle onto which that state will be teleported then, as elementary particles such as electrons have identical properties, the net effect of transferring a quantum state from one electron (say) to another remote electron will appear, to all intents as purposes *as if* the electron itself had been teleported. Charles Bennett, one of the inventors of quantum teleportation, made the humorous collage shown in Fig. 12.1 of himself with co-inventor Richard Jozsa using photographs taken as they passed through the (real) Tokyo Teleport station.

12.2.1 Local Versus Non-local Interactions

As we shall see shortly, quantum teleportation is very much dependent on certain so-called non-local physical effects. So we need to take a brief detour to consider what this means.

A *local interaction* is one that involves direct contact, or employs an intermediary that is in direct contact. The forces with which we are familiar in everyday life, such

Fig. 12.1 Photographic collage courtesy of Claude Crepeau showing Richard Jozsa and Charles Bennett at the Tokyo Teleport station. Crepeau, Jozsa, and Bennett were three of the inventors of quantum teleportation. Photograph provided courtesy of Charles Bennett

as friction and gravity, are local interactions. With friction, the physical contact between two bodies is really mediated by an electromagnetic field, which in turn comes about by the action of an intermediary, the carrier of the electromagnetic force, called the photon. Photons travel at the speed of light, which although fast is still finite. Consequently, electromagnetic influences cannot propagate faster than the speed of light in a vacuum. Moreover, electromagnetic forces tend to weaken the farther you go from the source.

Locality does not necessarily imply "nearby," however. Gravity, for example, is a force that exerts its influence over astronomically large distances. Nevertheless, gravity is still regarded as a local interaction because it is mediated by particles, called gravitons, which travel between gravitating objects. It too drops off in strength as the distance between the gravitating objects increases and cannot travel faster than the speed of light.

An important corollary of local interactions is the following: if two events occur in regions of spacetime such that no signal, not even one traveling at the speed of light, could ever reach one region from the other, these two events ought to be completely independent of one another. Why? Because if no signal could ever travel from one region to the other, how could what happens in one region ever be communicated to the other? In fact, special relativity has a special name for two such regions: it says that they are "spacelike separated."

In short, local interactions can be characterized by three criteria: they are mediated by another entity, such as a particle or field; they propagate no faster than the speed of light; and their strength drops off with distance. Thus the assumption of "locality" allows one to infer that events in spacelike separated regions ought to be independent of one another.

Scientists have shown that all the known forces in the Universe, the electromagnetic, the gravitational, the strong, and the weak forces are all *local*, in this sense. One might think, therefore, that is an end to it, and that reality must be local. After all, if *all* the known forces are local, what is left to be non-local?

Well, what is left is the "collapse of the state vector." State vectors, as we discussed earlier, provide the mathematical description of quantum systems. When we make measurements, the state vectors collapse into eigenstates, at least according to the Copenhagen interpretation of quantum theory. Now the intriguing point is that there is nothing in quantum theory that explains, mediates, or determines the exact mechanism of the collapse. In particular, the collapse of a state vector involves no *forces* of any kind. This lack of reliance upon a force of any kind, provides quantum theory with an "out"; a way to evade the strictures of locality.

How exactly would a non-local influence be defined? We can just negate each criterion for a local interactionto say that a *non-local interaction* is an interaction that is *not* mediated by anything, is *not* limited to acting at the speed of light, and does *not* drop off in strength with distance. Thus non-local interactions would appear to be magic! The question is—do they exist?

12.2.2 Non-locality: Einstein's "Spooky Action at a Distance"

"*That one body may act upon another at a distance through a vacuum without the mediation of anything else … is to me so great an absurdity, that I believe no man, who has in philosophical matters a competent faculty for thinking, can ever fall into.*"
– Isaac Newton[2]

Many scientists have an instinctive distaste for non-local interactions. Certainly, they would seem to be in direct conflict with Einstein's Theory of Special Relativity which says that nothing can travel faster than the speed of light. Indeed, it was the discrepancy between the predictions of relativity and quantum theory concerning the correlations between events in spacelike separated regions that led Albert Einstein, Boris Podolsky, and Nathan Rosen to point out an effect (thereafter known as the EPR effect) whereby one part of an entangled quantum system appears to instantaneously influence another.

To Einstein, Podolsky and Rosen such non-local influences seemed implausible, and they sought to use their seeming absurdity to prove that quantum mechanics gave only an incomplete account of physical reality. In particular, as Special Relativity held that nothing could travel faster than light, they believed that the correlations in measurement outcomes of experiments measuring both members of greatly separated entangled particles were more plausibly explained by hypothesizing that the pairs of particles were not really entangled at all but rather had fixed values of all their measurable attributes from the outset. Thus the experimental outcomes were really being determined by "hidden variables". It was out ignorance of these hidden

[2]Source: [369].

variables that made it appear that the states became definite upon being measured rather than the existence of any instantaneous, unmediated, arbitrarily far separated, "non-local" interactions.

12.2.3 Bell's Inequality

Now here comes the twist. It could be argued that it is simply a matter of philosophical *taste* as to whether you believe the quantum account or the hidden variable account of how the two entangled photons come to have correlated polarization states upon being measured. But what if there were some experimentally testable difference between the predictions of the two theories—then perhaps a physical experiment could resolve a philosophical question?

In the 1960s John Bell, an Irish physicist on leave from CERN (the European Center for Nuclear Research) showed that there was an empirically testable difference between the predictions of any hidden variable theory and the predictions of quantum mechanics. The test relies upon the statistics obtained when collecting data on the outcomes of pairs of polarization measurements on spacelike separated entangled particles when the polarizers are oriented at certain angles to one another.

Just what would we see if we performed a set of pairs of polarization measurements? For clarity let us suppose that the pair of photons exist in an entangled state such that both polarizations are guaranteed to be the same but are otherwise indefinite until they are measured.

Let's call our experimenters Alice and Bob, and let's suppose that they agree to orient their polarizers in the same direction. Thus the angle between their polarizers is 0°. What would Alice and Bob discover? Well, since the entangled particles we are dealing with are perfectly correlated, every time Alice observes a "vertical" Bob also observes a "vertical". And every time Alice observes a "horizontal", Bob also observes a "horizontal". The fraction of times that they agree on the measurement outcomes is 1, i.e., always.

Now let's imagine what would happen if Bob rotated his polarizer through 90°. Now what looks like "vertical" to Bob is actually seen as "horizontal" by Alice. So now when Alice and Bob perform polarization measurements on respective pairs of correlated photons, their results will be perfectly *anti*-correlated. Every time Alice sees "vertical" Bob sees "horizontal" and vice versa. The fraction of times that they agree on the outcomes will be 0, i.e., never.

So far so good. Now suppose Bob rotates his polarizer back towards Alice's vertical so that Bob's polarizer now makes an angle of θ_{12} to Alice's vertical. This is where things get interesting. Suppose Alice measures her photon to be "vertical". Thus the twin photon will be "vertical" (in Alice's basis) too. To Bob however, the photon he receives will appear to be a superposition of his "horizontal" and "vertical" orientations. As a result the outcome of Bob's polarization measurement is not certain: sometimes when Bob measures a photon that Alice sees as vertical Bob will obtain "vertical" too. But at other times when Bob measures a photon

Alice sees as "vertical" Bob will see "horizontal". The net effect is that the fraction of times Alice and Bob agree is now somewhere between 0 (never) and 1 (always) the exact number being dependent on the angle, θ_{12}, between Bob's and Alice's polarizers.

The question is, what degree of correlation would we expect to see in the outcomes of the polarization measurements made by Alice and Bob? To answer this quantitatively, suppose that Alice and Bob's polarizers are oriented in parallel planes so that what Alice thinks of as "vertical" is at angle θ_1 degrees with respect to some reference line, and what Bob thinks of as "vertical" is at θ_2 degrees with respect to the same reference line. Hence, the angle between Alice and Bob's vertical axes is $\theta_{12} = \theta_2 - \theta_1$.

When Alice and Bob make polarization measurements on successive pairs of entangled photons they each obtain either "vertical" or "horizontal" in their respective frames. This means that the quantum state of the joint system can be written as a superposition over product states of polarization outcomes, i.e., $\{|\psi_{xy}\rangle\} \equiv \{|\psi_x\rangle \otimes |\psi_y\rangle\}$, where x and y are vertical or horizontal polarizations as perceived by Alice and Bob respectively. Using geometric arguments we can determine the projection of Bob's basis vectors onto Alice's basis vectors allowing us to write:

$$\begin{aligned}|\Psi\rangle = \frac{1}{2}\cos^2\theta_{12}|\psi_{v_1 v_2}\rangle + \frac{1}{2}\sin^2\theta_{12}|\psi_{v_1 h_2}\rangle \\ + \frac{1}{2}\sin^2\theta_{12}|\psi_{h_1 v_2}\rangle + \frac{1}{2}\cos^2\theta_{12}|\psi_{h_1 h_2}\rangle\end{aligned} \tag{12.9}$$

Hence the probabilities, P_{xy}, of Alice finding photon 1 in polarization x, and Bob finding photon 2 in polarization y are:

$$P_{v_1 v_2} = |\langle\psi_{v_1 v_2}|\Psi\rangle|^2 = \frac{1}{2}\cos^2\theta_{12} \tag{12.10}$$

$$P_{v_1 h_2} = |\langle\psi_{v_1 h_2}|\Psi\rangle|^2 = \frac{1}{2}\sin^2\theta_{12} \tag{12.11}$$

$$P_{h_1 v_2} = |\langle\psi_{h_1 v_2}|\Psi\rangle|^2 = \frac{1}{2}\sin^2\theta_{12} \tag{12.12}$$

$$P_{h_1 h_2} = |\langle\psi_{h_1 h_2}|\Psi\rangle|^2 = \frac{1}{2}\cos^2\theta_{12} \tag{12.13}$$

where $\cos^2\theta_{12} = \cos^2(\theta_2 - \theta_1)$. Notice that the probabilities for the possible outcomes add up to 1.

A particularly interesting situation arises when Alice and Bob are so far apart that no signal, even one traveling at the speed of light, can possibly reach Bob from Alice and vice versa in the time taken for Alice and Bob to complete their measurements of the polarization orientations of their respective photons. On commonsense grounds (as Einstein, Podolsky and Rosen would see it) the fact that Alice and Bob are spacelike separated means that outcome of Alice's measurement should not affect the outcome of Bob's measurement. Based on this assumption, *which amounts to assuming reality is local*, it is possible to derive an inequality that says how the pairs of measurement outcomes Alice and Bob see should be related to one another when Alice and Bob set their polarizers at various pairs of orientations.

To obtain the inequality let us introduce a third polarizer having its polarization axes oriented along v_3 and h_3 rotated through angle θ_3 with respect to the same common reference frame as the first two polarizers. Using a classical viewpoint, in which reality is assumed to be local, in which case Alice and Bob's measurements ought not to affect one another whenever they are spacelike separated) standard probability arguments would predict:

$$P_{v_1 h_2} = P_{v_1 h_2 \underline{v_3}} + P_{v_1 h_2 \underline{h_3}} \tag{12.14}$$

where the right hand side has taken into account the two possible outcomes for the third polarization measurement. Similarly, for other combinations of measurement outcomes we have:

$$P_{v_2 h_3} = P_{\underline{v_1} v_2 h_3} + P_{\underline{h_1} v_2 h_3} \tag{12.15}$$

and

$$P_{v_1 h_3} = P_{v_1 \underline{v_2} h_3} + P_{v_1 \underline{h_2} h_3} \tag{12.16}$$

From these relations it follows that:

$$P_{v_1 h_2} \geq P_{v_1 h_2 h_3} \tag{12.17}$$

and

$$P_{v_2 h_3} \geq P_{v_1 v_2 h_3} \tag{12.18}$$

from which it follows

$$P_{v_1 h_2} + P_{v_2 h_3} \geq P_{v_1 \underline{h_2} h_3} + P_{v_1 \underline{v_2} h_3} \tag{12.19}$$

or more simply

$$P_{v_1 h_2} + P_{v_2 h_3} \geq P_{v_1 h_3} \tag{12.20}$$

which is Bell's inequality. Said more plainly in words:

Bell's Inequality The fraction of times that Alice observes "vertical" and Bob observes "horizontal" when Alice's polarizer is at θ_1 and Bob's polarizer is at θ_2 plus the fraction of times that Alice observes "vertical" and Bob observes "horizontal" when Alice's polarizer is at θ_2 and Bob's polarizer is at θ_3 *must be greater than or equal to* the fraction of times that Alice observes "vertical" and Bob observes "horizontal" when Alice's polarizer is at θ_1 and Bob's polarizer is at θ_3.

Thus, Bell's inequality is a statement about the correlations between probabilities (and hence frequencies of outcomes) of various polarization results when we perform such an experiment that depends upon the orientations of the three polarization detectors. The inequality is derived on the assumption that if Alice and Bob are sufficiently well separated so that no signal, not even one traveling at the speed of light, could propagate between Alice and Bob within the time-frame of the experiment, then nothing that Alice does can affect Bob and vice versa.

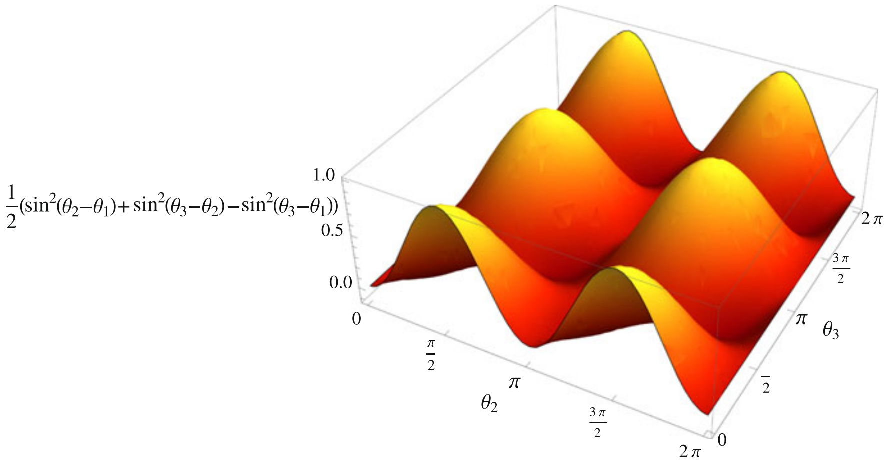

Fig. 12.2 Graphical illustration of Bell's Inequality

Upon expanding out the definitions of these probabilities explicitly, Bell's inequality (12.20) becomes:

$$\frac{1}{2}\sin^2(\theta_2-\theta_1)+\frac{1}{2}\sin^2(\theta_3-\theta_2)\geq\frac{1}{2}\sin^2(\theta_3-\theta_1) \tag{12.21}$$

If reality is "local", Bell's inequality should always hold regardless of the angles at which we set the polarization detectors. In this case the left hand side ought always to be greater than or equal to the right hand side. However, if we fix (say) $\theta_1 = 0°$ and plot the difference between the left and right hand sides of Bell's inequality, we obtain the surface shown in Fig. 12.2. If Bell's inequality holds, this surface ought to touching or above the (θ_2, θ_3)-plane at height 0, but never below it. However, by introducing a plane that cuts the surface at height 0, and then rotating the surface so we can view it from below, we see that indeed there are portions of the surface that are below zero. This means that quantum mechanical reasoning implies that there are values at which the polarizer orientations can be set that will cause a violation of Bell's inequality! So which theory is right—classical reasoning based on pure logic and the (reasonable-sounding) assumption of locality, or quantum mechanics?

12.3 Experimental Tests of Bell's Inequality

Although John Bell derived his inequality in 1964 it was not until 1972 that anyone attempted to check it experimentally [188]. Part of the delay was due to the inability to build perfect polarization detectors and to coordinate sufficiently closely-timed measurements that no speed of light information could make it from one photon to the other within the duration of pair of measurements. In addition, there was very little interest in "reality" research at the time.

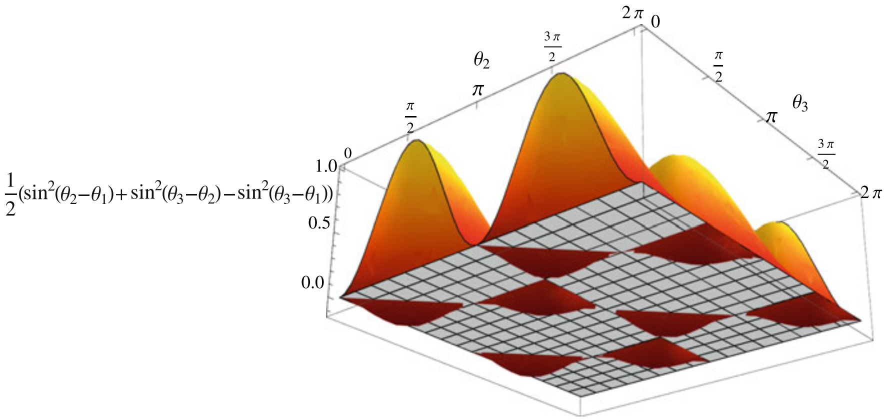

Fig. 12.3 Graphical illustration of violations of Bell's Inequality. In this figure the *vertical axis* is the function

John Clauser, a young researcher at Columbia University was different. Clauser took the reality question seriously. To him, and a growing number of physicists since, it really does matter what is going on behind the mathematical veneer of quantum mechanics. Calculational adequacy alone doesn't cut it. Most physicists became physicists precisely because they wanted to understand how the Universe worked. Comprehension, rather than calculation, was their overriding motivation. However, a physicist's training discourages philosophical musings in favor of prowess in calculation. This is partly cultural as the eminent Austrian physicist Anton Zeilinger has observed:

"[...] *there was and still is a tradition in Europe of philosophical thinking among physicists. I saw that in 1977 when I went to America for the first time. Already after a couple of weeks I started to miss philosophical discussion. Here we're more ready to ask really fundamental questions. In Europe it's important to question things. In America it's important to be able to build something. I don't mean that at all negatively*."
– Anton Zeilinger[3]

The results of Clauser's experiment [188], and even more convincing versions performed later by Alain Aspect, Philippe Grangier, Gérard Roger, and Jean Dalibard [22–24] confirmed the result shown in Fig. 12.3. When one does the experiment one finds that there are indeed certain settings of the angles of the polarizers at which Bell's Inequality is *violated*. Thus the inequality is *wrong*. This means that there must be a mistake in the reasoning under which the inequality was derived. However, the only assumption that was used was the assumption of locality, i.e., events in spacelike separated regions ought not to be able to influence one another. Hence, the assumption of locality must be wrong.

Thus the Clauser and Aspect experiments provide strong *experimental* evidence that reality is non-local. In fact, rather than non-local influences being rare and es-

[3]Source: [561].

oteric events, quite the contrary, every time particles interact with one another their quantum states tend to entangle. Subsequently, when one member of the pair is "measured" the other member behaves as if it too had been measured, and acquires a definite quantum state also. Thus, non-local influences are not the exception they are the rule. We don't notice them in our macroscopic world because we never have occasion in the everyday world to deal with spacelike separated events. But if we could scale the quantum world up to larger proportions these exotic quantum states should be quite evident.

Remarkably such a scaling up has been performed since the original Clauser and Aspect experiments and the phenomenon of non-locality has been shown to persist over much greater distances [493] and the potential so-called "locality" and "detector" loopholes in the original experiment have been closed [21, 346, 426, 526]. Thus, it does indeed appear that Nature *is* non-local and the parts of an entangled system can display correlations that are much stronger than can be accounted for by assuming they always had some definite values from the outset, i.e., were classically correlated.

12.3.1 Speed of Non-local Influences

Strictly speaking the experimental tests proving violations of Bell's inequality only prove that no influence traveling the speed of light (or less) could be responsible for enforcing the observed non-local correlations. However, a philosophical possibility (if not a physical possibility), is the possibility that something (let us call it a "non-local influence") could be traveling faster than the speed of light between the spacelike separated polarization measurements, and these explain how one part of a system can affect the other. How can we test that? Can we place a lower bound on the speed of propagation of such hypothetical influences (assuming they exist)?

Any hypothetical non-local influence has its speed defined in some preferred frame of reference, which is different from the local latitude/longitude frame of the rotating Earth. Thus an experiment that appears fixed with respect to the Earth's surface (i.e., in a latitude/longitude frame) would not be in a fixed orientation with respect to this hypothetical preferred frame. As the Earth rotated during the course of a day the two frames could not possibly be aligned at all times. When the Earth frame was not aligned with the preferred frame, then events that would be simultaneous in the preferred frame would not be simultaneous in the Earth frame. Thus, if a Bell inequality would be violated at simultaneous polarization measurement events in the preferred frame it would not be violated in the Earth frame too as those polarization measurement events would not be simultaneous in the Earth frame, and so the visibility of the interference fringes (in the Earth frame) should disappear.

In 2008, in an experimental tour de force, Swiss physicists Daniel Salart, Augustin Bass, Cyril Branciard, Nicolas Gisin and Hugo Zbinden performed such an extended two photon interface experiment over a 24 hour period between two villages in Switzerland that were oriented along a roughly east-west route, as shown

Fig. 12.4 Experiment to place a lower bound on the speed of quantum information. Pairs of correlated photons were created in Geneva. One member of each pair was sent to the Satigny and the other to Jussy—a pair of villages aligned in an East-West direction—over fiber-optic links of exactly equal length. Tests of Bell's inequality violations were performed continuously over a 24 hours period as the whole experiment rotated with the Earth with respect to a hypothetical reference frame in Space. The observation of persistent Bell inequality violations regardless of the time of day confirms both that "standard" result that the quantum correlations are greater than any classical correlation could be (and so the correlations cannot be accounted for as merely arising from a common cause) but, more importantly, that if there is any nonlocal influence passing between the two receiving stations then, given the timing resolution of the experimental equipment, its speed must be greater than ten thousand times the speed of light. However, quantum mechanics does not predict such nonlocal influences propagate between the receivers (because nonlocal effects are unmediated), and this experiment does nothing to confirm that they exist. Rather the experiment proves that *if* such nonlocal influences to travel between the receivers they would have to travel much faster than light, if not instantaneously

in Fig. 12.4. The east-west alignment meant that as the Earth rotated the experiment essentially scanned through all possible orientations for the hypothetical preferred reference frame within a 24 hour period. If there is a preferred frame, then it should be revealed by seeing (in the rotating Earth frame) periodic times when Bell's inequality is violated and times when it is not violated over any continuous 24 hour period.

However, when Salart et al. performed their experiment two-photon interference fringes were observed throughout the full 24 hour period with a visibility at all times far exceeding the threshold set by Bell's inequality. This implies that there is no preferred frame for non-local effects.

Moreover, assuming non-local influences propagated at all (which is neither predicted by quantum mechanics nor implied by the Salart et al. experiment) then the experimental results showed that their speed must be at least ten-thousand times the speed of light! So 10,000 c can be regarded as a lower bound for the speed of propagation of these hypothetical non-local influences.

Last but not least, the Salart et al. experiment provided yet another confirmation violations of a Bell inequality—and hence non-local effects—over a distance of approximately 18 km.

Given the apparent reality of long distance entanglement, and non-local effects, can we put these phenomena to use? In the next section we show that the answer is a resounding YES!

12.4 Quantum Teleportation Protocol

The basic idea is that Alice wishes to send Bob a qubit that is in a state unknown to her, but she does not want to transmit it through the medium between herself and Bob. If Alice and Bob had met face to face previously and had each retained one member of an entangled pair of particles, Alice can accomplish her desired state transfer by the process of quantum teleportation.

Thus quantum teleportation depends crucially on Alice and Bob each having possession of one end of an entangled pair of particles. Such shared prior entanglement could take many forms. For example, Alice and Bob might each be in possession of any of the following maximally entangled Bell pairs:

$$\begin{aligned}
|\beta_{00}\rangle &= \tfrac{1}{\sqrt{2}}(|00\rangle + |11\rangle)\\
|\beta_{01}\rangle &= \tfrac{1}{\sqrt{2}}(|01\rangle + |10\rangle)\\
|\beta_{10}\rangle &= \tfrac{1}{\sqrt{2}}(|00\rangle - |11\rangle)\\
|\beta_{11}\rangle &= \tfrac{1}{\sqrt{2}}(|01\rangle - |10\rangle)
\end{aligned} \tag{12.22}$$

These states can be summarized in a single equation as:

$$|\beta_{xy}\rangle = \frac{1}{\sqrt{2}}(|0, y\rangle + (-1)^x|1, 1-y\rangle) \tag{12.23}$$

Such a state can be synthesized using a quantum circuit such as that shown in Fig. 12.5. To obtain different Bell states, one need only input different combinations of computational basis states, $|x\rangle|y\rangle$, in order to obtain $|\beta_{xy}\rangle$.

Let us suppose that Alice and Bob each possess one particle from the Bell state pair $|\beta_{11}\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$. This state is known as a "singlet" state and has a net spin of zero.[4] It is an especially interesting Bell state because it retains the same basic form under any unitary transformation.

The qubit Alice wishes to teleport to Bob may be assumed to be in state $|\psi\rangle_1 = a|0\rangle_1 + b|1\rangle_1$ such that $|a|^2 + |b|^2 = 1$, but we assume Alice is ignorant of the values of a and b. Hence, we can say $|\psi\rangle_1$ is "unknown" to Alice. This pre-

[4] The "singlet" name refers to the fact that the quantum number M_S can only take on a single value $M_S = 0$ when the net spin is $S = 0$ as it is for $|\beta_{11}\rangle$. Contrast this with the Bell state $|\beta_{01}\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$, which is known as a triplet state because the quantum number M_S can take on three values, namely, $M_S = -1, 0, +1$, when the net spin is $S = 1$ as it is for $|\beta_{01}\rangle$.

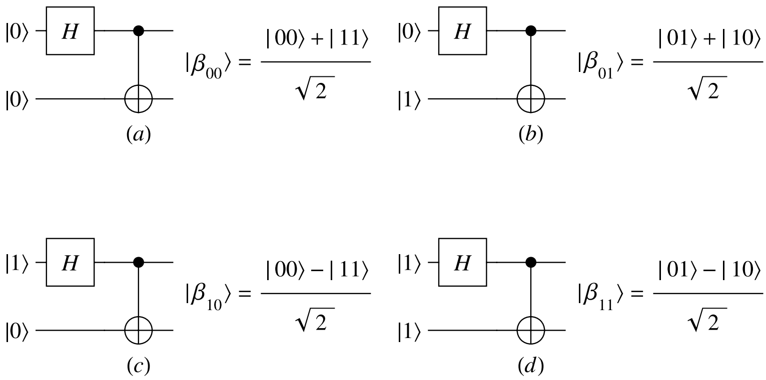

Fig. 12.5 Quantum circuit for synthesizing each of the four Bell states starting from different combinations of computational basis states

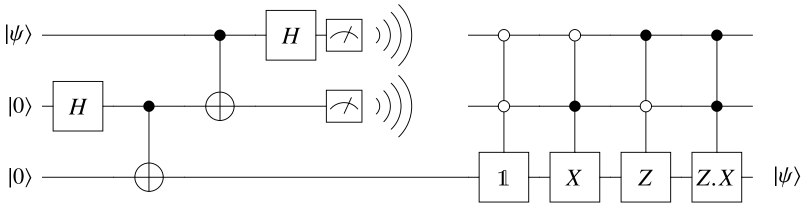

Fig. 12.6 Quantum circuit for teleporting an unknown quantum state from Alice to Bob. The protocol begins with Alice creating an entangled pair of particles in the Bell state $|\beta_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$. She retains one of these qubits and sends the other to Bob. Next Alice performs a Bell basis measurement between the qubit she wishes to teleport and the particle she retained which is entangled with a particle in Bob's possession. After the measurement Alice obtains two classical bit values that she passes to Bob. Upon receipt Bob performs a rotation of the particle he obtained from Alice conditional on the values of the two bits he received from Alice. This conditional rotation transforms Bob's particle into an exact replica of the state Alice wished to teleport. In the process Alice's state has been destroyed locally due to the Bell basis measurement Alice made. Hence, Bob obtains the state that was originally in Alice's possession without that state traveling through the intervening space between Alice and Bob

vents Alice from measuring the state to confirm its identity, and if she attempts to measure the state without choosing the right basis, her attempt will perturb the state dramatically.

So instead, Alice transmits the quantum information defining the unknown state to Bob using the non-local correlations established by the shared Bell state, and two bits of classical communication. The scheme, which is illustrated in Fig. 12.6, works as follows: Initially Alice possesses the state $|\psi\rangle_1 = a|0\rangle_1 + b|1\rangle_1$ and Alice and Bob each hold one particle from a singlet state $|\beta_{11}\rangle = \frac{1}{\sqrt{2}}(|01\rangle_{23} - |10\rangle_{23})$. Here we have used subscripts to keep track of which particles we are discussing. Thus there are three particles in all, labeled 1, 2, and 3. Initially, as particle 1 is not

Table 12.1 Alice's measured states and Bob's corresponding corrective actions. N.B. the 1-qubit gates $\mathbb{1}$, X, and Z are all Pauli operators

Alice's state	Alice's measurement	Bob's state	Corrective action	Operators
$\lvert\beta_{00}\rangle_{12}$	00	$a\lvert 1\rangle - b\lvert 0\rangle$	$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	$Z.X$
$\lvert\beta_{01}\rangle_{12}$	01	$a\lvert 0\rangle - b\lvert 1\rangle$	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	Z
$\lvert\beta_{10}\rangle_{12}$	10	$a\lvert 1\rangle + b\lvert 0\rangle$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	X
$\lvert\beta_{11}\rangle_{12}$	11	$a\lvert 0\rangle + b\lvert 1\rangle$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\mathbb{1}$

entangled with particles 2 and 3, the 3-qubit input state, $|\Psi_{\text{init}}\rangle$, is:

$$\begin{aligned}|\Psi_{\text{init}}\rangle &= |\psi\rangle_1 \otimes |\beta_{11}\rangle_{23} \\ &= (a|0\rangle_1 + b|1\rangle_1) \otimes \frac{1}{\sqrt{2}}(|01\rangle_{23} - |10\rangle_{23}) \\ &= \frac{a}{\sqrt{2}}|001\rangle_{123} - \frac{a}{\sqrt{2}}|010\rangle_{123} + \frac{b}{\sqrt{2}}|101\rangle_{123} - \frac{b}{\sqrt{2}}|110\rangle_{123} \end{aligned} \tag{12.24}$$

Without applying any further physical operation, this state can simply be re-written as:

$$\begin{aligned}|\Psi_{\text{init}}\rangle_{123} = \frac{1}{2}[&|\beta_{11}\rangle_{12}(a|0\rangle_3 + b|1\rangle_3) + |\beta_{01}\rangle_{12}(a|0\rangle_3 - b|1\rangle_3) \\ &+ |\beta_{10}\rangle_{12}(a|1\rangle_3 + b|0\rangle_3) + |\beta_{00}\rangle_{12}(a|1\rangle_3 - b|0\rangle_3)] \end{aligned} \tag{12.25}$$

Thus if Alice measures particles 1 and 2 in the Bell basis (for which the four states $\{|\beta_{00}\rangle_{12}, |\beta_{01}\rangle_{12}, |\beta_{10}\rangle_{12}, |\beta_{1}\rangle_{12}\}$ are all orthogonal to one another) the state of particle 3 will be projected into a state that bears a simple relationship to the (unknown) quantum state being teleported, i.e., $|\psi\rangle = a|0\rangle + b|1\rangle$.

Specifically, if Alice finds particles 1 and 2 to be in the Bell state $|\beta_{11}\rangle_{12}$, particle 3 will then be in state $a|0\rangle_3 + b|1\rangle_3$. Likewise, if Alice finds particles 1 and 2 to be in state $|\beta_{01}\rangle_{12}$, particle 3 will then be in state $a|0\rangle_3 - b|1\rangle_3$ etc. If Alice communicates the results of her Bell basis measurements to Bob, Bob will then be able to determine what operation to apply to his qubit in order to place it in the (unknown) state $|\psi\rangle$. Table 12.1 lists the operations Bob must perform on his qubit depending on the joint state Alice determines particles 1 and 2 to be in.

The aforementioned steps are summarized is the quantum teleportation protocol:

Quantum Teleportation Protocol

1. Alice wishes to teleport to Bob a single qubit in a pure quantum state, $|\psi\rangle_1 = a|0\rangle_1 + b|1\rangle_1$, which is unknown to her.
2. To do so, Alice creates an entangled pair of particles shared between herself and Bob by feeding the state $|00\rangle_{23}$ into the quantum circuit shown in Fig. 12.5. The net state in Alice's possession is then $|\psi\rangle_1|\beta_{00}\rangle_{23}$

3. Next Alice performs a "Bell basis measurement" on qubits 1 and 2. This is equivalent to applying a CNOT and Hadamard gate to qubits 1 and 2 and then measuring their values in the computational basis to obtain, in output, two classical bits.
4. Alice then transmits these classical bits to Bob using any classical channel of he choosing.
5. Upon receipt, Bob uses the two classical bit values to determine which one of four possible actions he is to perform on the qubit he already received from Alice. The four possible actions are 00 $\rightarrow$ no action, 01 $\rightarrow$ apply an X rotation, 10 $\rightarrow$ apply a Z rotation, or 11 $\rightarrow$ apply an $Z \cdot X$ rotation.

12.4.1 Teleportation Does Not Imply Superluminal Communication

It is important to realize that quantum teleportation does not imply superluminal communications. This is perhaps best understood by redrawing the teleportation decoding circuit as in Fig. 12.7 to expose its reliance on *classical* bit values. The teleportation protocol requires two bits of classical information to be sent from Alice to Bob and these bits cannot be transmitted faster than the speed of light. Moreover, non-local effects between entangled pairs of particles cannot be used for super-luminal communications either, because although the non-local influence is conveyed instantaneously (or at least at speeds in excess of 10,000 c—in accordance with Sect. 12.3.1) such links *cannot* be used for communicating an information bearing message. Instead they can only communicate random bits.

Thus, quantum teleportation is a sound physical procedure and does not violate any known law of physics.

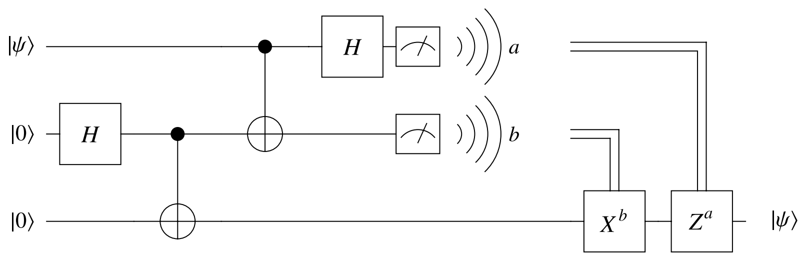

Fig. 12.7 Quantum circuit for teleporting an unknown quantum state from Alice to Bob. This circuit is functionally equivalent to that shown in Fig. 12.6 but emphasizes the fact that the teleportation decoding procedure relies on classical bit values, a and b, to control a pair of quantum gates, which collectively implement the operation $Z^a \cdot X^b$ (X first then Z)

12.5 Working Prototypes

In the late 1990's several working prototypes of quantum teleportation devices were demonstrated. One was built by Dirk Bouwmeester, J.-W. Pan, K. Mattle, Anton Zeilinger, M. Eibl, and H. Weinfurter, in Innsbruck [69] and another was built by Francesco De Martini and collaborators D. Boschi, S. Branca, L. Hardy and S. Popescu in Rome [66], and a third by Jeff Kimble's team at Caltech [194]. There is a little rivalry between the researchers as to which machine constitutes the first *genuine* demonstration of quantum teleportation. But all three schemes are similar in using bench optics components such as beam splitters, parametric down converters, mirrors and photon detectors.

A sketch of Bouwmeester et al.'s set-up is shown in Fig. 12.8. On the left side of Fig. 12.8, Alice sends her "message" photon M, which is prepared in a 45° polarization state, towards a beam splitter. with a specific state, 45 degrees polarization. That is Alice intends to send the quantum state $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ to Bob. Simultaneously, two entangled photons, A (shown as "Photon to Alice") and B (shown as "Photon to Bob"), are created and travel in opposite directions: photon A goes to Alice's beam splitter and photon B to Bob's beam splitter. The timings are arranged so that one of the entangled photons arrives at Alice's beam splitter at just the same instant as Alice's message photon M. Some of the time the two photons emerge from Alice's beam splitter in different directions but Alice is unable to distinguish which photon is which. As a result of this indistinguishability, Alice's message photon becomes entangled with photon A. Now neither M nor A has a definite polarization state but they must be opposite since they went to different detectors when they emerged from Alice's beam splitter. However, photon B also had the opposite polarization state to photon A. Therefore, photon B must acquire the same polarization state

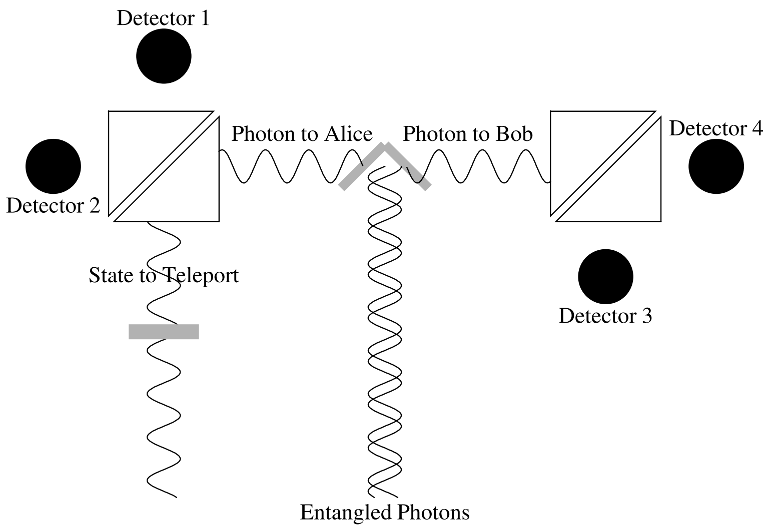

Fig. 12.8 The Innsbruck quantum teleportation experiment

as photon M (the message photon). Hence teleportation is complete and Bob sees photon B has a polarization of 45°.

It was quite a surprise that there was a physical basis for teleportation and an even bigger surprise that the process evolved from a concept to a working prototype in just four years. Who knows what potential this technology has over the coming decades.

12.6 Teleporting Larger Objects

From a technological perspective quantum teleportation is much simpler than even the most rudimentary quantum computation. In fact, in 1997 two groups reported optical schemes in which they successfully teleported an unknown quantum state across a laboratory bench [67, 68]. Scaling quantum teleportation up to the level of an entire human being however, is quite unrealistic at this point. Samuel Braunstein has estimated how much information you would need to transmit in order to perform such a feat. Starting from the observation that the visible human project, sponsored by the American National Institute of Health, requires about 10 Gigabytes of bits (about 10 CD-ROMs) to hold the information needed to describe the full three-dimensional structure of a human to a 1 mm^3 resolution, Braunstein estimates that an entire human could be described, down to the atomic level, using roughly 10^{32} bits. With current communication channel capacities, Braunstein estimates that it would take about a hundred million centuries to transmit this information down a single channel!

However, there have been some interesting advances in quantum teleportation recently, that push it in interesting new directions. Of special note is an experiment by Qiang Zhang, Alexander Goebel, Claudia Wagenknecht, Yu-Ao Chen, Bo Zhao, Tao Yang, Alois Mair, Jörg Schmiedmayer, and Jain-Wei Pan, showing that it is possible to teleport a multi-particle entangled state [563]. The experiment basically doubled the complexity of the regular quantum teleportation circuit, requiring a 6-photon interferometer to transfer the joint polarization state of a pair of photons in such a manner that their entanglement was preserved under the teleportation operation. This shows that it is possible to teleport the quantum state of objects that are more complex than single qubits. This is a step in the direction of teleporting the state of a complete molecule.

Another key advance is an experiment by Jacob Sherson, Hanna Krauter, Rasmus Olsson, Brian Julsgaard, Klemens Hammerer, Ignacio Cirac, and Eugene Polzik, showing teleportation of a quantum state between light and matter, i.e., objects of dissimilar type [454]. The significance of this is that there is currently much interest in using photons to convey quantum information over long distances, and in using the long-lived collective spin states of ensembles of alkalai atoms to store quantum information over relatively long times. Quantum teleportation could be useful in transferring quantum information from flying qubits into stationary qubits by controlled light-matter interactions. If such interfaces can be perfected they could enable true quantum repeaters that would greatly extend the range of quantum communications in fiber optic cables [83].

Finally, it is worth mentioning, insofar as demonstrations of quantum teleportation rely upon quantum interferometry, that there have been many exciting developments on demonstrating quantum interference using objects as complex as fullerenes, i.e., molecules consisting of a cage of 60 carbon atoms [18, 224]. These experiments are quite remarkable given the relative complexity of the molecules and consist of multi-level quantum systems.

12.7 Summary

This chapter has examined the physical basis for true teleportation—the transmission of a quantum state from A to B without it having to pass through the intervening medium. In the process the quantum state is necessarily destroyed at the source location and is re-incarnated at the receiving station. The scheme requires shared prior entanglement between the source and receiver, and a classical communications channel over which to pass the two bit result obtained by making a complete Bell basis measurement. For the latter reason, quantum teleportation cannot be achieved super-luminally as the transmission of the classical message through the medium is limited to traveling at the speed of light.

Notice that quantum teleportation teleports the quantum *state* of an object, not the object itself. This is slightly different from the usual science fiction view of teleporting an object. Consequently, we cannot use this scheme to teleport an electron in its entirety from one place to another. Rather, we can teleport the *spin* state of one electron at a particular location to another electron at a different location (or indeed a different kind of particle entirely). The net effect, however, is similar: A particle in a specific state at the source location has its state destroyed and reincarnated on another particle at the destination without the original particle traversing the intervening distance.

We emphasize that the non-local effects that underpin quantum teleportation cannot be used to transmit a content bearing message super-luminally either. At best they would be limited to transmitting random bits. Quantum mechanics neither requires nor predicts these non-local influences propagate through the medium between A and B. However, a Swiss team showed recently established experimentally that *if they did so propagate* (which is not proven and is frankly unlikely) then they would have to travel in excess of ten thousand times the speed of light. The quantum mechanical prediction that non-local effects should exist between the parts of a spacelike-separated entangled quantum system regardless of the distance between them, the nature of the intervening medium, and without the need for the mediation of any influence of any kind, was established experimentally by Freedman and Clauser [188] and Aspect, Grangier and Roger in [22, 24], and substantiated with much improved experiments later [346, 426, 493, 526]. These results show the quantum mechanical predictions are correct and contrary to the expectations of Einstein, Podolsky and Rosen: reality is, as far as we can tell, non-local.

Thus, quantum teleportation is a physically sound protocol, has been demonstrated many times experimentally, and researchers are inching forward to more teleporting more complex entities. The protocol has been demonstrated in photonic systems, atomic systems and even between the two. However, a teleportation machine of the complexity envisioned by science fiction writers is utterly impossible given current know-how. But that does not mean teleportation is not useful. Indeed it could prove to be key to making practical quantum repeaters, and has been proposed as a primitive operation for quantum computation. More on that in Chap. 15.

12.8 Exercises

12.1 Consider two observables represented by Hermitian matrices, A and B. Prove that their commutator $[A, B] = A \cdot B - B \cdot A$ is anti-Hermitian, and that their anti-commutator, $\{A, B\} = A \cdot B + B \cdot A$, is Hermitian. A matrix, M, is Hermitian iff $M = M^\dagger$ and anti-Hermitian iff $M = -M^\dagger$.

12.2 Consider the two Bell states $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ and $\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$. Transform each of these states to a new basis. What is the form of each of these states in the U basis? Do you notice any difference?

12.3 Recall the definition of the Bell basis states, $|\beta_{00}\rangle$, $|\beta_{01}\rangle$, $|\beta_{10}\rangle$, and $|\beta_{11}\rangle$ defined by:

$$|\beta_{xy}\rangle = \frac{1}{\sqrt{2}}(|0, y\rangle + (-1)^x|1, 1-y\rangle)$$

and consider an *arbitrary* single qubit pure state defined by:

$$|\psi\rangle = \alpha|0\rangle + \sqrt{1 - |\alpha|^2}|1\rangle$$

such that $|\alpha| \leq 1$.

(a) Write down a state, $|\psi^\perp\rangle$, which is orthonormal to $|\psi\rangle$, i.e., a state for which $\langle\psi^\perp|\psi\rangle = 0$.
(b) If α is *purely real* with $-1 \leq \alpha \leq 1$, but otherwise arbitrary, prove that $\frac{1}{\sqrt{2}}(|\psi\psi\rangle + |\psi^\perp\psi^\perp\rangle) = |\beta_{00}\rangle$.
(c) If α is *complex* with $|\alpha| \leq 1$, but otherwise arbitrary, prove that $\frac{1}{\sqrt{2}}(|\psi^\perp\psi\rangle - |\psi\psi^\perp\rangle) = |\beta_{11}\rangle$.

12.4 In this chapter, we developed the quantum teleportation protocol using a source of entangled pairs of particles with each pair in the state $\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$. But this form of entanglement is not essential. Modify the quantum teleportation scheme to use a source of entanglement that produces pairs of particles each in the state $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$.

12.5 The states $|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$ and $|W\rangle = \frac{1}{\sqrt{2}}(|001\rangle + |010\rangle + |100\rangle)$ are fundamentally *inequivalent* types of entangled states.

(a) Can you devise a quantum teleportation scheme that uses $|\mathrm{GHZ}\rangle$ as the source of entanglement?
(b) Can you devise a quantum teleportation scheme that uses $|W\rangle$ as the source of entanglement?

12.6 In many quantum information processing tasks it is useful to "measure a state in the Bell basis", i.e., a basis consisting of 2-qubit entangled states, $\{|\beta_{00}\rangle, |\beta_{01}\rangle, |\beta_{10}\rangle, |\beta_{11}\rangle\}$. However, we often find the state represented, initially, in the computational basis. Thus, it is worthwhile knowing how to switch from the computational basis to the Bell basis. Practice this by rewriting the following states in the Bell-basis:

(a) An entangled state: $\frac{1}{\sqrt{2}}(|00\rangle + i|11\rangle)$
(b) An unentangled state: $\frac{1}{\sqrt{2}}(|01\rangle + |11\rangle)$
(c) A state whose entanglement is unknown $a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$

12.7 In many quantum information processing tasks it is useful to perform operations on a state represented in the Bell-basis, i.e., a basis consisting of 2-qubit entangled states, $\{|\beta_{00}\rangle, |\beta_{01}\rangle, |\beta_{10}\rangle, |\beta_{11}\rangle\}$. However, we often find the operator is given, initially, in the computational basis. Thus, it is worthwhile knowing how to map gate specified in the computational basis into the equivalent gate in the Bell basis. Practice this by rewriting the following states in the Bell-basis:

(a) CNOT
(b) SWAP
(c) iSWAP
(d) Berkeley B

12.8 Suppose Alice and Bob are a pair of Space-faring astronauts who desire to stay in touch with one another over long spaceflights in opposite directions deep into the cosmos. Realizing that speed of light signal delays will pose a challenge they take a crash course in quantum information theory in the hopes of devising a way to use entanglement to overcome speed-of-light signal delays, and thereby keep in contact. Based on their limited understanding of quantum information theory, Alice and Bob believe that the following communication protocol will allow them to communicate superluminally over arbitrarily great distances! They suppose they each start out with an inexhaustible supply of marching EPR particles each in the state $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$. Before leaving Alice, and Bob agreed on an order and timing pattern in which to measure their respective EPR particles. Alice promises to measure an EPR particle in her possession (let's call it her "message ebit") on the stroke of each minute. If the answer she obtains is the bit she wishes to send to Bob, Alice subsequently measures the next 29 ebits (her "check" bits) in the agreed upon order on every *even* numbered second $(2, 4, 6, \ldots, 58)$ making 29 additional measurements in all.

Likewise, Bob promises to measure the EPR particle matching Alice's "message ebit" at precisely one second after the start of each minute. In addition, Bob promises to apply a Walsh-Hadamard gate, H, to each of the next 29 ebits in the agreed upon order and then to measure their bit values on every *odd* numbered second $(3, 5, 7, \ldots, 59)$ making 29 additional measurements in all.

Alice and Bob believe that this scheme will allow them to communicate one message bit per minute over arbitrarily great distances by reasoning as follows: Each of their EPR pairs begins in the state $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$. When Alice makes her measurement and obtains a 0 or a 1, Bob's matching EPR particle acquires the *same* bit value *instantaneously* and so reveals the bit Alice obtained. However, Bob does not know whether this was the bit Alice intended to send or just a random bit. To communicate a real message Alice must do something to allow Bob to tell whether the "message ebit" he measured at one second after the start of the minute is the bit Alice *intended* to send. This is the reason for the subsequent 29 measurements. Before Bob makes his measurements, and if Alice has not measured, each of Bob's particles are equally likely to be measured as "0" or "1", and hence will be in the state $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$. But after applying the Walsh-Hadamard transform, Bob's particle is rotated into the state $|0\rangle$. Hence, when Bob makes his measurements on the remaining 29 particles he will always obtain a "0" if Alice has not measured her corresponding EPR particles. However, Bob will only obtain a "0" 50% of the time if Alice has measured her 29 "check" particles. Hence, by observing the number of "0" 's he obtains, Bob can determine (with probability $1 - \frac{1}{2^{29}}$) whether or not Alice measured her 29 "check" particles. Hence, Bob learns whether the "message ebit" he measured (at one second after the start of the minute) is or is not the bit Alice intended to send. As this scheme requires no classical communication Alice and Bob can be arbitrarily far apart and be able to communicate classical information at the rate of one bit per minute!

Alas, the aforementioned scheme contains a crucial flaw. Your job is to find the flaw and explain why it prohibits superluminal communication of classical information. The following facts about Walsh = Hadamard gates may be helpful:

$$
\begin{aligned}
H &= \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \\
H|0\rangle &= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \\
H|1\rangle &= \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)
\end{aligned}
\tag{12.26}
$$