15. Chain Sampling

A brief introduction to the concept of chain sampling is first presented. The chain sampling plan of type ChSP-1 is first reviewed, and a discussion on the design and application of ChSP-1 plans is then presented in the second section of this chapter. Various extensions of chain sampling plans such as the ChSP-4 plan are discussed in the third part. The representation of the ChSP-1 plan as a two-stage cumulative results criterion plan, and its design are discussed in the fourth part. The fifth section relates to the modification of the ChSP-1 plan. The sixth section of this chapter is on the relationship between chain sampling and deferred sentencing plans. A review of sampling inspection plans that are based on the ideas of chain or dependent sampling or deferred sentencing is also made in this section. The economics of chain sampling when compared to quick switching systems is discussed in the seventh section. The eighth section extends the attribute chain sampling to variables inspection. In the ninth section, chain sampling is then compared with the CUSUM approach. The tenth section gives several other interesting extensions of chain sampling, such as chain sampling for mixed attribute and variables inspection. The final section gives concluding remarks.

Acceptance sampling is the methodology that deals with procedures by which decisions to accept or not accept lots of items are based on the results of the inspection of samples. Special purpose acceptance sampling inspection plans (abbreviated to special purpose plans) are tailored for special applications as against general or universal use. Prof. Harold F. Dodge, who is regarded as the father of acceptance sampling, introduced the idea of chain sampling in his 1959 industrial quality control paper [15.1]. Chain sampling can be viewed as a plan based on a cumulative results criterion (CRC), where related batch information is chained or cumulated. The phrase chain sampling is also used in sample surveys to imply snowball sampling for collection of data. It should be noted that this phrase was originally coined in the acceptance sampling literature, and should be distinguished from its usage in other areas.

Chain sampling is extended to two or more stages of cumulation of inspection results with appropriate acceptance criteria for each stage. The theory of chain sampling is also closely related to the various other methods of sampling inspection such as dependent-deferred sentencing, tightened–normal–tightened (TNT) sampling, quick-switching inspection etc.

In this chapter, we provide an introduction to chain sampling and briefly discuss various generalizations of chain sampling plans. We also review a few sampling plans which are related to or based on the methodology of chain sampling. The selection or design of various chain sampling plans is also briefly presented.

15.1 ChSP-1 Chain Sampling Plan

A single-sampling attributes inspection plan calls for acceptance of a lot under consideration if the number of nonconforming units found in a random sample of size n is less than or equal to the acceptance number Ac. Whenever the operating characteristic (OC) curve of a single-sampling plan is required to pass through a prescribed point, the sample size n will be an increasing function of the acceptance number Ac. This fact can be verified from the table of np or unity values given in *Cameron* [15.2] for various values of the probability of acceptance $P_a(p)$ of the lot under consideration whose fraction of nonconforming units is p. The same result is true when the OC curve has to pass through two predetermined points, usually one at the top and the other at the bottom of the OC curve [15.3]. Thus, for situations where small sample sizes are preferred, only single-sampling plans with $Ac = 0$ are desirable [15.4]. However, as observed by *Dodge* [15.1] and several authors, the $Ac = 0$ plan has a pathological OC curve in that the curve starts to drop rapidly even for a very small increase in the fraction nonconforming. In other words, the OC curve of the $Ac = 0$ plan has no point of inflection. Whenever a sampling plan for costly or destructive testing is required, it is common to force the OC curve to pass through a point, say, (LQL, β) where LQL is the limiting quality level for ensuring consumer protection and β is the associated consumer's risk. All other sampling plans, such as double and multiple sampling plans, will require a larger sample size for a one-point protection such as (LQL, β). Unfortunately the $Ac = 0$ plan has the following two disadvantages:

1. The OC curve of the $Ac = 0$ plan has no point of inflection and hence it starts to drop rapidly even for the smallest increase in the fraction nonconforming p.
2. The producer dislikes an $Ac = 0$ plan since a single occasional nonconformity will call for the rejection of the lot.

The chain sampling plan ChPS-1 by *Dodge* [15.1] is an answer to the question of whether anything can be done to improve the pathological shape of the OC curve of a zero-acceptance-number plan. A production process, when in a state of statistical control, maintains a constant but unknown fraction nonconforming p. If a series of lots formed from such a stable process is submitted for inspection, which is known as a type B situation, then the samples drawn from the submitted lots are simply random samples drawn directly from the production process. So, it is logical to allow a single occasional nonconforming unit in the current sample whenever the evidence of good past quality, as demonstrated by the i preceding samples containing no nonconforming units, is available. Alternatively we can chain or cumulate the results of past lot inspections to take a decision on the current lot without increasing the sample size.

The operating procedure of the chain sampling plan of type ChSP-1 is formally stated below:

1. From each of the lots submitted, draw a random sample of size n and observe the number of nonconforming units d.
2. Accept the lot if d is zero. Reject the lot if $d > 1$. If $d = 1$, the lot is accepted provided all the samples of size n each drawn from the preceding i lots are free from nonconforming units; otherwise reject the lot.

Thus the chain sampling plan has two parameters: n, the sample size, and i, the number of preceding sample results chained for making a decision on the current lot. It is also required that the consumer has confidence in the producer, and the producer will deliberately not pass a poor-quality lot taking advantage of the small samples used and the utilization of preceding samples to take a decision on the current lot.

The ChSP-1 plan always accepts the lot if $d = 0$ and conditionally accepts it if $d = 1$. The probability that the preceding i samples of size n are free from nonconforming units is $P_{0,n}^i$. Hence, the OC function is $P_a(p) = P_{0,n} + P_{1,n} P_{0,n}^i$ where $P_{d,n}$ is the probability of getting d nonconforming units in a sample of size n. Figure 15.1 shows the improvement in the shape of the OC curve of the zero-acceptance-number single-sampling plan by the use of chain sampling. *Clark* [15.5] provided a discussion on the OC curves of chain sampling plans, a modification and some applications. *Liebesman* et al. [15.6] argue in favor of chain sampling as the attribute sampling standards have the deficiency for small or fractional acceptance number sampling plans. The authors also provided the necessary tables and examples for the chain sampling procedures. Most text books on statistical quality control also contain a section on chain sampling, and provide some applications.

Soundararajan [15.7] constructed tables for the selection of chain sampling plans for given acceptable quality level (AQL, denoted as p_1), producer's risk α, LQL (denoted as p_2) and β. The plans found from this source are approximate, and a more accurate procedure that also minimizes the sum of actual producer's and

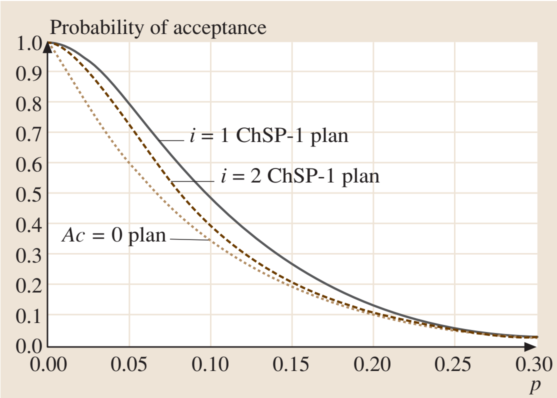

Fig. 15.1 Comparison of OC curves of $Ac = 0$ and ChSP-1 plans

consumer's risks is given by *Govindaraju* [15.8]. Table 15.1, adopted form *Govindaraju* [15.8] is based on the binomial distribution for OC curve of the ChSP-1 plan. This table can also be used to select ChSP-1 plans for given LQL and β only, which may be used in place of zero-acceptance-number plans.

Ohta [15.9] investigated the performance of ChSP-1 plans using the graphical evaluation and review technique (GERT) and derived measures such as OC and average sample number (ASN) for the ChSP-1 plan. *Raju* and *Jothikumar* [15.10] provided a ChSP-1 plan design procedure based on Kullback–Leibler information, and the necessary tables for the selection of the plan. *Govindaraju* [15.11] discussed the design ChSP-1 plan for minimum average total inspection (ATI). There are a number of other sources where the ChSP-1 plan design is discussed. This paper provides additional references on designing chain sampling plans, inter alia, while discussing various extensions and generalizations.

Table 15.1 ChSP-1 plans indexed by AQL and LQL ($\alpha = 0.05$, $\beta = 0.10$) for fraction nonconforming inspection [15.8]. Key $n : i$

LQL	AQL (%)					
(%)	0.1	0.15	0.25	0.40	0.65	1.00
1.5	154:2					
2.0	114:4	124:1				
2.5	91:4	92:2				
3.0	76:3	76:3	82:1			
3.5	65:3	65:3	70:1			
4.0	57:2	57:2	57:2			
4.5	51:2	51:2	51:2			
5.0	45:3	45:3	45:3	49:1		
5.5	41:3	41:3	41:3	45:1		
6.0	38:3	38:2	38:2	38:2		
6.5	35:3	35:2	35:2	35:2		
7.0	32:3	32:3	32:3	32:3		
7.5	30:3	30:3	30:2	30:2		
8.0	28:3	28:3	28:2	28:2	30:1	
8.5	26:3	26:3	26:3	26:3	29:1	
9.0	25:3	25:3	25:2	25:2	27:1	
9.5	24:3	24:3	24:2	24:2	24:2	
10	22:3	22:3	22:3	22:3	22:3	
11	20:3	20:3	20:2	20:2	20:2	
12	19:3	19:3	19:2	19:2	19:2	20:1
13	17:3	17:3	17:3	17:2	17:2	18:1
14	16:3	16:3	16:3	16:2	16:2	16:2
15	15:3	15:3	15:3	15:2	15:2	15:2

15.2 Extended Chain Sampling Plans

Frishman [15.12] extended the ChSP-1 plan and developed ChSP-4 and ChSP-4A plans which incorporate a rejection number greater than 1. Both ChSP-4 and ChSP-4A plans are operated like a traditional double-sampling attributes plan but uses $(k-1)$ past lot results instead of actually taking a second sample from the current lot. The following is a compact tabular representation of Frishman's ChSP-4A plan.

Stage	Sample size	Acceptance number	Rejection number
1	n	a	r
2	$(k\text{-}1)n$	a'	$a'+1$

The ChSP-4 plan restricts r to $a'+1$. The conditional double-sampling plans of *Baker* and *Brobst* [15.13], and the partial and full link-sampling plans of *Harishchandra* and *Srivenkataramana* [15.14] are actually particular cases of the ChSP-4A plan when $k = 2$ and $k = 3$ respectively. However the fact that the OC curves of these plans are the same as the ChSP-4A plan is not reported in both papers [15.15].

Extensive tables for the selection of ChSP-4 and ChSP-4A plans were constructed by *Raju* [15.16, 17] and *Raju* and *Murthy* [15.18–21]. *Raju* and *Jothikumar* [15.22] provided a complete summary of various selection procedures for ChSP-4 and ChSP-4A plans, and also discussed two further types of optimal plans – the first involving minimum risks and the second based on Kullback–Leibler information. Unfortunately, the tables of Raju et al. for the ChSP-4 or ChSP-4A design require the user to specify the acceptance and rejection numbers. This serious design limitation is not an issue with the procedures and computer programs developed by *Vaerst* [15.23] who discussed the design of ChSP-4A plans involving minimum sample sizes for given AQL, α, LQL and β without assuming any specific acceptance numbers. Raju et al. considered a variety of design criteria while *Vaerst* [15.23] discussed only the (AQL, LQL) criterion. The ChSP-4 and ChSP-4A plans obtained from Raju's tables can be used in any type B situation of a series of lots from a stable production process, not necessarily when the product involves costly or destructive testing. This is because the acceptance numbers covered are above zero. The major disadvantage of *Frishman*'s [15.12] extended ChSP-4 and ChSP-4A plans is that the neighboring lot information is not always utilized. Even though ChSP-4 and ChSP-4A plans require smaller sample sizes than the traditional double-sampling plans, these plans may not be economical compared to other conditional sampling plans.

Bagchi [15.24] presented an extension of the ChSP-1 plan, which calls for additional sampling only when one nonconforming unit is found. The operating procedure of Bagchi's plan is given below:

1. At the outset, inspect n_1 units selected randomly from each lot. Accept the lot if all the n_1 units are conforming; otherwise, reject the lot.
2. If i successive lots are accepted, then inspect only $n_2\,(< n_1)$ items from each of the submitted lots. Accept the lot as long as no nonconforming units are found. If two or more nonconforming units are found, reject the lot. In the event of one nonconforming unit being found in n_2 inspected units, then inspect a further sample $(n_1 - n_2)$ units from the same lot. Accept the lot under consideration if no further nonconforming units are found in the additional $(n_1 - n_2)$ inspected units; otherwise reject the lot.

Representing Bagchi's plan as a Markov chain, *Subramani* and *Govindaraju* [15.25] derived the steady-state OC function and a few other performance measures.

15.3 Two-Stage Chain Sampling

Dodge and *Stephens* [15.26] viewed the chain sampling approach as a cumulative results criterion (CRC) applied in two stages and extended it to include larger acceptance numbers. Their approach calls for the first stage of cumulation of a maximum of k_1 consecutive lot results, during which acceptance is allowed if the maximum allowable nonconforming units is c_1 or less. After passing the first stage of cumulation (i.e. when k_1 consecutive lots are accepted), the second stage of cumulation of $k_2 (> k_1)$ lot results begins. In the second stage of cumulation, an acceptance number of $c_2 (> c_1)$ is applied. *Stephens* and *Dodge* [15.27] presented a further generalization of the family of two-stage chain sampling inspection plans by using different sample sizes in the two stages. We state below the complete operating procedure of a generalized two-stage chain sampling plan.

1. At the outset, draw a random sample of n_1 units from the first lot. In general, a sample of size $n_j (j = 1, 2)$ will be taken while operating in the j^{th} stage of cumulation.
2. Record d, the number of nonconforming units in each sample, as well as D, the cumulative number of nonconforming units from the first up to and including the current sample. As long as $D_i \le c_1 (1 \le i \le k_1)$, accept the i^{th} lot.
3. If k_1 consecutive lots are accepted, continue to cumulate the number of nonconforming units D in the k_1 samples plus additional samples up to but no more than k_2 samples. During this second stage of cumulation, accept the lots as long as $D_i \le c_2 (k_1 < i \le k_2)$.
4. After passing the second stage of k_2 lot acceptances, start cumulation as a moving total over k_2 consecutive samples (by adding the current lot result and dropping the k_2^{th} preceding lot result). Continue to accept lots as long as $D_i \le c_2 (i > k_2)$.
5. If, in any stage of sampling, $D_i > c_i$ then reject the lot and return to Step 1 (a fresh restart of the cumulation procedure).

Figure 15.2 shows how the cumulative results criterion is used in a two-stage chain sampling plan when $k_1 = 3$ and $k_2 = 5$.

An important subset of the generalized two-stage chain sampling plan is when $n_1 = n_2$ and this subset is designated as ChSP-(c_1, c_2); there are five parameters:

n, k_1, k_2, c_1, and c_2. The original chain sampling plan ChSP-1 of *Dodge* [15.1] is a further subset of the ChSP-(0, 1) plan with $k_1 = k_2 - 1$. That is, the OC curve of the generalized two-stage chain sampling plan is equivalent to the OC curve of the ChSP-1 plan when $k_1 = k_2 - 1$. *Dodge* and *Stephens* [15.26] derived the OC function of ChSP-(0, 1) plan as

$$P_a(p) = \frac{P_{0,n}\left(1 - P_{0,n}\right) + P_{0,n}^{k_1} P_{1,n}\left(1 - P_{0,n}^{k_2-k_1}\right)}{1 - P_{0,n} + P_{0,n}^{k_1} P_{1,n}\left(1 - P_{0,n}^{k_2-k_1}\right)}, \quad k_2 > k_1 .$$

As achieved by the ChSP-1 plan, the ChSP-(0,1) plan also overcomes the disadvantages of the zero-acceptance-number plan. Its operating procedure can be succinctly stated as follows:

1. A random sample of size n is taken from each successive lot, and the number of nonconforming units in each sample is recorded, as well as the cumulative number of nonconforming units found so far.
2. Accept the lot associated with each new sample as long as no nonconforming units are found.
3. Once k_1 lots have been accepted, accept subsequent lots as long as the cumulative number of nonconforming units is no greater than one.
4. Once $k_2 > k_1$ lots have been accepted, cumulate the number of nonconforming units over at most k_2 lots, and continue to accept as long as this cumulative number of nonconforming units is one or none.
5. If, at any stage, the cumulative number of nonconforming units becomes greater than one, reject the current lot and return to Step 1.

Procedures and tables for the design of ChSP-(0,1) plan are available in *Soundararajan* and *Govindaraju* [15.28], and *Subramani* and *Govindaraju* [15.29]. *Govindaraju* and *Subramani* [15.30] showed that the choice of $k_1 = k_2 - 1$ is always forced on the parameters of the ChSP-(0,1) plan when a plan is selected for given AQL, α, LQL, and β. That is, a ChSP-1 plan will be sufficient, and one need not opt for a two-stage cumulation of nonconforming units.

In various technical reports from the Statistics Center at Rutgers University (see *Stephens* [15.31] for a list), *Stephens* and *Dodge* formulated the two-stage chain sampling plan as a Markov chain and evaluated its performance. The performance measures considered by them include the steady-state OC function, ASN and average run length (ARL) etc. For comparison of chain sampling plans with the traditional or noncumulative plans, two types of ARLs are used. The first type of ARL, say ARL_1, is the average number of samples to the first rejection after a sudden shift in the process level, say from p_0 to $p_s(> p_0)$. The usual ARL, say ARL_2, is the average number of samples to the first rejection given the stable process level p_0. The difference ($\text{ARL}_1 - \text{ARL}_2$) measures the extra lag due to chain sampling. However, this extra lag may be compensated by gains in sampling efficiency, as explained by *Stephens* and *Dodge* [15.32].

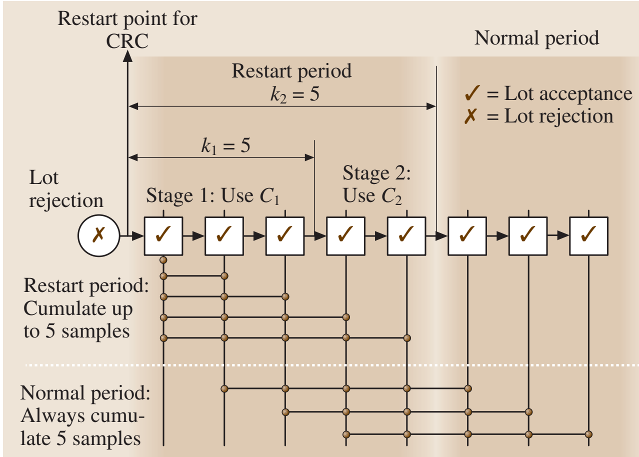

Fig. 15.2 Operation of a two-stage chain sampling plan with $k_1 = 3$ and $k_2 = 5$

Stephens and *Dodge* [15.33] summarized the mathematical approach they have taken to evaluate the performance of some selected two-stage chain sampling plans, while more detailed derivations were published in their technical reports. Based on the expressions for the OC function derived by Stephens and Dodge in their various technical reports (consult *Stephens* [15.31]), *Raju* and *Murthy* [15.34], and *Raju* and *Jothikumar* [15.35] discussed various design procedures for the ChSP-(0,2) and ChSP-(1,2) plans. *Raju* [15.36] extended the two-stage chain sampling to three stages, and evaluated the OC performances of a few selected chain sampling plans, fixing the acceptance numbers for the three stages. The three-stage cumulation procedure becomes very complex, and will play only a limited role for costly or destructive inspections. The three-stage plan will however be useful for general type B lot-by-lot inspections.

15.4 Modified ChSP-1 Plan

In *Dodge*'s [15.1] approach, chaining of past lot results does not always occur. It occurs only when a nonconforming unit is observed in the current sample. This means that the available historical evidence of quality is not fully utilized. *Govindaraju* and *Lai* [15.37] developed a modified chain sampling plan (MChSP-1) that always utilizes the recently available lot-quality history. The operating procedure of the MChSP-1 plan is given below.

1. From each of the submitted lots, draw a random sample of size n. Reject the lot if one or more nonconforming units are found in the sample.
2. Accept the lot if no nonconforming units are found in the sample, provided that the preceding i samples also contained no nonconforming units except in one sample, which may contain at most one nonconforming unit. Otherwise, reject the lot.

A flow chart showing the operation of the MChSP-1 plan is in Fig. 15.3.

The MChSP-1 plan allows a single nonconforming unit in any one of the preceding i samples but the lot under consideration is rejected if the current sample has a nonconforming unit. Thus, the plan gives a psychological protection to the consumer in that it allows acceptance only when all the current sample units are conforming. Allowing one nonconforming unit in any one of the preceding i samples is essential to offer protection to the producer, i.e. to achieve an OC curve with a point of inflection. In the MChSP-1 plan, rejection

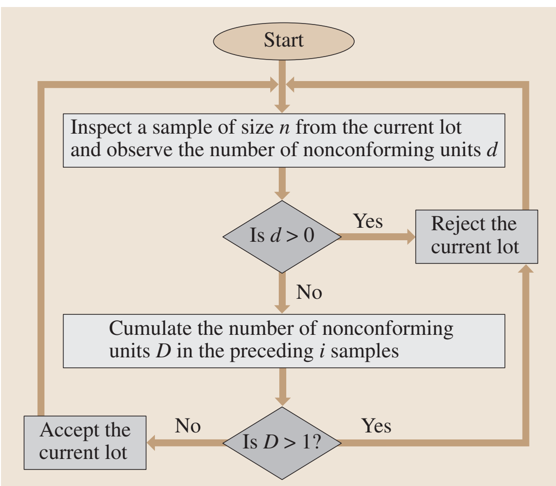

Fig. 15.3 Operation of the MChSP-1 plan

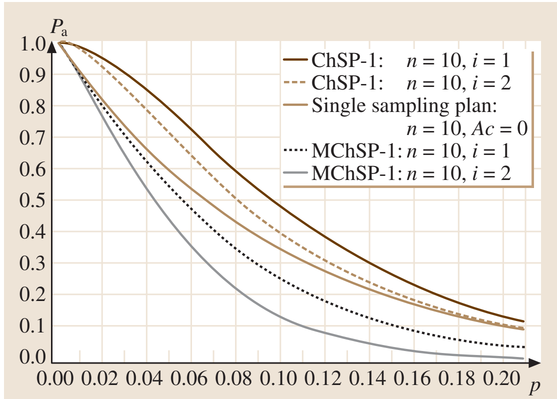

Fig. 15.4 Comparison of OC curves of ChSP-1 and MChSP-1 plans

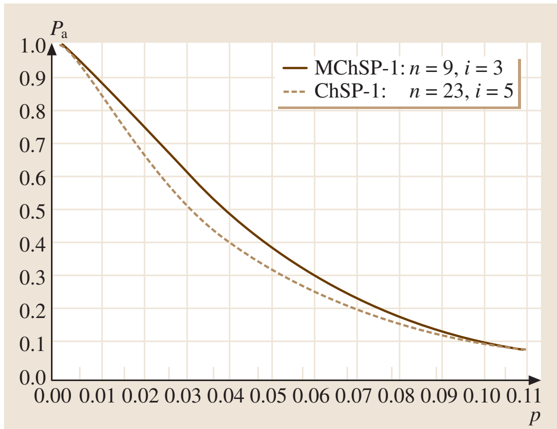

Fig. 15.5 OC curves of matched ChSP-1 and MChSP-1 plans

of lots would occur until the sequence of submissions advances to a stage where two or more nonconforming units were no longer included in the sequence of i samples. In other words, if two or more nonconforming units are found in a single sample, it will result in i subsequent lot rejections. In acceptance sampling, one has to look at the OC curve to have an idea of the protection to the producer as well as to the consumer and what happens in an individual sample or for a few lots is not very important. If two or more nonconforming units are found in a single sample, it does not mean that the subsequent lots need not be inspected since they will be automatically rejected under the proposed plan. It should be noted that results of subsequent lots will be utilized continuously and the producer has to show an improvement in quality with one or none nonconforming units in the subsequent samples to permit future acceptances. This will act as a strong motivating factor for quality improvement.

The OC function $P_a(p)$ of the MChSP-1 plan was derived by *Govindaraju* and *Lai* [15.37] as $P_a(p) = P_{0,n}(P_{0,n}^{i} + iP_{0,n}^{i-1}P_{1,n})$. Figure 15.4 compares the OC curves of the ChSP-1 and MChSP-1 plans. From Fig. 15.4, we observe that the MChSP-1 plan decreases the probability of acceptance at poor quality levels but maintains the probability of acceptance at good quality levels when compared to the OC curve of the zero-acceptance-number single-sampling plan. The ChSP-1 plan, on the other hand, increases the probability of acceptance at good quality levels but maintains the probability of acceptance at poor quality levels. To compare the two sampling plans, we need to match them. That is, we need to design sampling plans whose OC curves pass approximately through two prescribed points such as (AQL, 1-α) and (LQL, β). Figure 15.5 gives such a comparison, and establishes that the MChSP-1 plan is efficient in requiring a very small sample size compared to the ChSP-1 plan. A two-stage chain sampling plan would generally require a sample size equal to or more than the sample size of a zero-acceptance single-sampling plan. The MChSP-1 plan will however require a sample size smaller than the zero-acceptance-number plan.

15.5 Chain Sampling and Deferred Sentencing

Like chain sampling plans, there are other plans that use the results of neighboring lots to take a conditional decision of acceptance or rejection. Plans that make use of past lot results are either called chain or dependent sampling plans. Similarly plans that make use of future lot results are known as deferred sentencing plans. These plans have a strategy of accepting the lots conditionally based on the neighboring lot-quality history and are hence referred to as conditional sampling plans. We will briefly review several such conditional sampling plans available in the literature.

In contrast to chain sampling plans, which make use of past lot results, deferred sentencing plans use future lot results. The idea of deferred sentencing was first published in a paper by *Anscombe* et al. [15.38]. The first and simplest type of deferred sentencing scheme [15.38] requires the produced units to be split into small size lots, and one item is selected from each lot for inspection. The lot-sentencing rule is that whenever Y nonconforming units are found out of X or fewer consecutive lots tested, all such clusters of consecutive lots starting from the lot that resulted in the first nonconforming unit to the lot that resulted in the Y^{th} nonconforming unit are rejected. Lots not rejected by this rule are accepted. This rule is further explained in the following sentences. A run of good lots of length X will be accepted at once. If a nonconforming unit occurs, then the lot sentencing or disposition will be deferred until either a further $(X-1)$ lots have been tested or $(Y-1)$ further nonconforming items are found, whichever occurs sooner. At the outset, if the $(X-1)$ succeeding lots result in fewer than $(Y-1)$ nonconforming units, the lot that resulted in the first nonconforming unit and any succeeding lots clear of nonconforming units will be accepted. As soon as Y nonconforming units occur in no more than X lots, all lots not so far sentenced will be rejected. Thus the lot disposition will sometimes be made at once, and sometimes with a delay not exceeding $(X-1)$ lots. Some of the lots to be rejected according to the sentencing rule may already have been rejected through the operation of the rule on a previous cluster of Y nonconforming units that partially overlaps with the cluster being considered. The actual number of new lots rejected under the deferred sentencing rule can be any number from 1 to X. *Anscombe* et al. [15.38] also considered modifications of the above deferred sentencing rule, including inspection of a sample of size more than one from each lot. *Anscombe* et al. [15.38] originally presented their scheme as an alternative to *Dodge*'s [15.39] continuous sampling plan of type CSP-1, which is primarily intended for the partial screening inspection of produced units directly (when lot formation is difficult).

The deferred sentencing idea was formally tailored into an acceptance sampling plan by *Hill* et al. [15.40]. The operating procedure of *Hill* et al. [15.40] scheme is described below:

1. From each lot, select a sample of size n. These lots are accepted as long as no nonconforming units are found in the samples. If one or more nonconforming unit is found, the disposition of the current lot will be deferred until $(X-1)$ succeeding lots are inspected.
2. If the cumulative number of nonconforming units for X consecutive lots is Y or more, then a second sample of size n is taken from each of the lots (beginning with the first lot and ending with the last batch that

showed a nonconforming unit in the sequence of X nonconforming units). If there are less than Y nonconforming units in the X, accept all lots from the first up to, but not including, the next batch that showed a nonconforming unit. The decision on this batch will be deferred until $(X-1)$ succeeding lots are inspected.

Hill et al. [15.40] also evaluated the OC function of some selected schemes and found them to be very economical compared to the traditional sampling plans, including the sequential attribute sampling plan.

Wortham and *Mogg* [15.41] developed a dependent stage sampling (DSSP) plan (DSSP(r, b)), which is operated under steady state as follows:

1. For each lot, draw a sample of size n and observe the number of nonconforming units d.
2. If $d \le r$, accept the lot; if $d > r+b$, reject the lot. If $r+1 \le d \le r+b$, accept the lot if the $(r+b+1-d)^{\text{th}}$ previous lot was accepted; otherwise reject the current lot.

Govindaraju [15.42] observed that the OC function of DSSP(r, b) is the same as the OC function of the repetitive group sampling (RGS) plan of *Sherman* [15.43]. This means that the existing design procedures for the RGS plan can also be used for the design of DSSP(r, b) plan. The deferred state sampling plan of *Wortham* and *Baker* [15.44] has a similar operating procedure except in step 2 in which, when $r+1 \le d \le r+b$, the current lot is accepted if the forthcoming $(r+b+1-d)^{\text{th}}$ lot is accepted. The steady-state OC function of the dependent (deferred) stage sampling plan DSSP(r, b) is given by

$$P_{\text{a}}(p) = \frac{P_{\text{a},r}(p)}{1 - P_{\text{a},r+b}(p) + P_{\text{a},r}(p)}$$

where $P_{\text{a},r}(p)$ is the OC function of the single-sampling plan with acceptance number r and sample size n. Similarly $P_{\text{a},r+b}(p)$ is the OC function of the single-sampling plan with acceptance number $r+b$ and sample size n. A procedure for the determination of the DSSP(r, b) plan for given AQL, α, LQL, and β was also developed by *Vaerst* [15.23].

Wortham and *Baker* [15.45] extended the dependent (deferred) state sampling into a multiple dependent (deferred) state (MDS) plan MDS(r, b, m). The operating procedure of the MDS(r, b, m) plan is given below:

1. For each lot, draw a sample of size n and observe the number of nonconforming units d.
2. If $d \le r$, accept the lot; if $d > r+b$, reject the lot. If $r+1 \le d \le r+b$, accept the lot if the consecutive m preceding lots were all accepted (the consecutive m succeeding lots must be accepted for the deferred MDS(r, b, m) plan).

The steady-state OC function of the MDS(r, b, m) plan is given by the recursive equation

$$P_{\text{a}}(p) = P_{\text{a},r}(p) + \left[P_{\text{a},r+b}(p) + P_{\text{a},r}(p)\right]\left[P_{\text{a}}(p)\right]^m$$

Vaerst [15.46], *Soundararajan* and *Vijayaraghavan* [15.47], *Kuralmani* and *Govindaraju* [15.48], and *Govindaraju* and *Subramani* [15.49] provided detailed tables and procedures for the design of MDS(r, b, m) plans for various requirements.

Vaerst [15.23, 46] modified the MDS(r, b, m) plan to make it on a par with the ChSP-1 plan. The operating procedure of the modified MDS(r, b, m) plan, called MDS-1(r, b, m), is given below:

1. For each lot, draw a sample of size n and observe the number of nonconforming units d.
2. If $d \le r$, accept the lot; if $d > r+b$, reject the lot. If $r+1 \le d \le r+b$, accept the lot if r or fewer nonconforming units are found in each of the consecutive m preceding (succeeding) lots.

When $r = 0$, $b = 1$, and $m = i$, MDS-1(r, b, m) becomes the ChSP-1 plan. The OC function of the MDS-1(r, b, m) plan is given by the recursive equation

$$P_{\text{a}}(p) = P_{\text{a},r}(p) + \left[P_{\text{a},r+b}(p) + P_{\text{a},r}(p)\right]\left[P_{\text{a},r}(p)\right]^m$$

Vaerst [15.46], *Soundararajan* and *Vijayaraghavan* [15.50], and *Govindaraju* and *Subramani* [15.51] provided detailed tables and procedures for the design of MDS-1(r, b, m) plans for various requirements.

The major and obvious shortcoming of the chain sampling plans is that, since they use sample information from past lots to dispose of the current lot, there is a tendency to reject the current lot of given good quality when the process quality is improving, or to accept the current lot of given bad quality when the process quality is deteriorating. Similar criticisms (in reverse) can be leveled against the deferred sentencing plans. As mentioned earlier, *Stephens* and *Dodge* [15.32] recognizedg this disadvantage of chain sampling and defined the ARL performance measures ARL_1 and ARL_2. Recall that ARL_2 is the average number of lots that will be accepted as a function of the true fraction nonconforming. ARL_1 is the average number of lots accepted after an upward shift in the true fraction nonconforming from the existing level. *Stephens* and *Dodge* [15.52] evaluated the performance of the two-stage chain sampling plans, comparing the ARLs with matching single- and double-sampling plans having approximately the same OC curve. It was noted that the slightly poorer ARL property due to chaining of lot results is well compensated by the gain in sampling economy. For deferred sentencing schemes, *Hill* et al. [15.40] investigated trends as well as sudden changes in quality. It was found that the deferred sentencing schemes will discriminate better between fairly constant quality at one level and fairly constant quality at another level than will a lot-by-lot plan scheme with the same sample size. However when quality varies considerably from lot to lot, the deferred sentencing scheme was found to operate less satisfactorily, and in certain circumstances the discrimination between good and bad batches may even be worse than for traditional unconditional plans with the same sample size. Furthermore, the deferred sentencing scheme may pose problems of flow, supp1y storage space, and uneven work loads (which is not a problem with chain sampling).

Cox [15.53] provided a more theoretical treatment and considered one-step forward and two-step backward schemes. He represented the lot-sentencing rules as a stochastic process, and applied Bayes's theorem for the sentencing rule. He did recognize the complexity of modeling a multistage procedure. When the submitted lot fraction nonconforming varies, say when a trend exists, both chain and deferred sentencing rules have disadvantages. But this disadvantage can be overcome by combining chain and deferred sentencing rules into a single scheme. This idea was first suggested by *Baker* [15.54] in his dependent deferred state (DDS) plan. *Osanaiye* [15.55] provided a complete methodology of combining chain and deferred sentencing rules, and developed the chain-deferred (ChDP) plan. The ChDP plan has two stages for lot disposition and its operating procedure is given below:

1. From lot number k, inspect n units and count the number of nonconforming units d_k. If $d_k \le c_1$, accept lot number k. If $d_k > c_2$, reject lot numbered k. If $c_1 < d_k \le c_2$, then combine the number of nonconforming units from the immediately succeeding and preceding samples, namely d_{k-1} and d_{k+1}. (Stage 1)
2. If $d_k \le c$, accept the k^{th} lot provided $d_k + d_{k-1} \le c_3$ (chain approach). If $d_k > c$, accept the k^{th} lot provided that $d_k + d_{k+1} \le c_3$ (deferred sentencing).

One possible choice of c is the average of c_1 and $c_3 + 1$. *Osanaiye* [15.55] also provided a comparison of ChDP with the traditional unconditional double-sampling plans as the OC curves of the two types of plans are the same (but the ChDP plan utilizes the neighboring lot results). *Shankar* and *Srivastava* [15.56] and *Shankar* and *Joseph* [15.57] provided a GERT analysis of ChDP plans, following the approach of *Ohta* [15.9]. *Shankar* and *Srivastava* [15.58] discussed the selection of ChDP plans using tables. *Osanaiye* [15.59] provided a multiple-sampling-plan extension of the ChDP plan (called the MChDP plan). MChDP plan uses several neighboring lot results to achieve sampling economy.

Osanaiye [15.60] provided a useful practical discussion on the choice of conditional sampling plans considering autoregressive processes, inert processes (constant process quality shift) and linear trends in quality. Based on a simulation study, it was recommended that the chain-deferred schemes are the cheapest if either the cost of 100% inspection or sampling inspection is high. He recommended the use of the traditional single or double sampling plans only if the opportunity cost of rejected items is very high. *Osanaiye* and *Alebiosu* [15.61] considered the effect of inspection errors on dependent and deferred double-sampling plans vis-a-vis ChDP plans. They observed that the chain-deferred plan in general has a greater tendency to reject nonconforming items than any other plans, irrespective of the magnitude of the inspection error.

Many of the conditional sampling plans, which follow either the approach of chaining or deferring or both, have the same OC curve as a double-sampling (or multiple-sampling) plan. Exploiting this equivalence, *Kuralmani* and *Govindaraju* [15.62] provided a general selection procedure for conditional sampling plans for given AQL and LQL. The plans considered include the conditional double-sampling plan of the ChSP-4A plans of *Frishman* [15.12], the conditional double-sampling plan of *Baker* and *Brobst* [15.13], the link-sampling plan of *Harishchandra* and *Srivenkataramana* [15.14], and the ChDP plan of *Osanaiye* [15.55]. A perusal of the operating ratio LQL/AQL of the tables by *Kuralmani* and *Govindaraju* [15.62] reveals that these conditional sampling plans apply in all type B situations, as a wide range of discrimination between good and bad qualities is provided. However the sample sizes, even though smaller than the traditional unconditional plans, will not be as small as the zero-acceptance-number single-sampling plans. This limits the application of the conditional sampling plans to this special-purpose situation, where the ChSP1 or MChSP-1 plans are most suitable.

Govindaraju [15.63] developed a conditional single-sampling (CSS) plan, which has desirable properties for general applications as well as for costly or destructive testing. The operating procedure of the CSS plan is as follows.

1. From lot numbered k, select a sample of size n and observe the number of nonconforming units d_k.
2. Cumulate the number of nonconforming units observed for the current lot and the related lots. The related lots will be either past lots, future lots or a combination, depending on whether one is using dependent sampling or deferred sentencing. The lot under consideration is accepted if the total number of nonconforming units in the current lot and the m related lots is less than or equal to the acceptance number, Ac. If d_k is the number of nonconforming units recorded for the k^{th} lot, the rule for the disposition of the k^{th} lot can be stated as:
   a) For dependent or chain single sampling, accept the lot if $d_{k-m}+\cdots+d_{k-1}+d_k \leq Ac$; otherwise, reject the lot.
   b) For deferred single sampling, accept the lot if $d_k+d_{k-1}+\cdots+d_{k+m} \leq Ac$; otherwise, reject the lot
   c) For dependent-deferred single sampling, where m is desired to be even, accept the lot if $d_{k-\frac{m}{2}}+\cdots+d_k+\cdots+d_{k+\frac{m}{2}} \leq Ac$; otherwise, reject the lot.

Thus the CSS plan has three parameters: the sample size n, the acceptance number Ac, and the number of related lot results used, m. As in the case of any dependent sampling procedure, dependent single sampling takes full effect only from the $(m+1)^{\text{st}}$ lot. To maintain equivalent OC protection for the first m lots, an additional sample of mn units can be taken from each lot and the lot be accepted if the total number of nonconforming units is less than or equal to Ac, or additional samples of size $(m+1-i)\,n$ can be taken for the i^{th} lot $(i=1,2,\ldots,m)$ and the same decision rule be applied. In either case, the results of the additional samples should not be used for lot disposition from lot $(m+1)$. *Govindaraju* [15.63] has shown that the CSS plans require much smaller sample sizes than all other conditional sampling plans. In case of trends in quality, the CSS plan can also be operated as a chain-deferred plan and this will ensure that the changes in lot qualities are somewhat averaged out.

15.6 Comparison of Chain Sampling with Switching Sampling Systems

Dodge [15.64] originally proposed quick-switching sampling (QSS) systems. *Romboski* [15.65] investigated the QSSs and introduced several modifications of the original quick-switching system, which basically consists of two intensities of inspection, say, normal (N) and tightened (T) plans. If a lot is rejected under normal inspection, a switch to tightened inspection will be made; otherwise normal inspection will continue. If a lot is accepted under the tightened inspection, then the normal inspection will be restored; otherwise tightened inspection will be continued. For a review of quick-switching systems, see *Taylor* [15.66] or *Soundararajan* and *Arumainayagam* [15.67].

Taylor [15.66] introduced a new switch number to the original QSS-1 system of *Romboski* [15.65] and compared it with the chain sampling plans. When the sample sizes of normal and tightened plans are equal, the quick-switching systems and the two-stage chain sampling plans were found to give nearly identical performance. Taylor's comparison is only valid for a general situation where acceptance numbers greater than zero are used. For costly or destructive testing, acceptance numbers are kept at zero to achieve minimum sample sizes. In such situations, the chain sampling plans ChSP-1 and ChSP-(0, 1) will fare poorly against other comparable schemes when the incoming quality is at AQL. This fact is explained in the following paragraph using an example.

For costly or destructive testing, a quick-switching system employing zero acceptance number was studied by *Govindaraju* [15.68], and *Soundararajan* and *Arumainayagam* [15.69]. Under this scheme, the normal inspection plan has a sample size of n_{N} units, while the tightened inspection plan has a higher sample size n_{T} $(>n_{\text{N}})$. The acceptance number is kept at zero for both normal and tightened inspection. The switching rule is that a rejection under the normal plan $(n_{\text{N}},0)$ will invoke the tightened plan $(n_{\text{T}},0)$. An acceptance under the $(n_{\text{T}},0)$ plan will revert back to normal inspection. This QSS system, designated as type QSS-$1(n_{\text{N}},n_{\text{T}};0)$, can be used in place of the ChSP-1 and ChSP(0,1) plans. Let AQL $=1\%$, $\alpha=5\%$, LQL $=15\%$, and $\beta=10\%$. The ChSP-1 plan for the prescribed AQL and LQL conditions is found to be $n=15$ and $i=2$ (Table 15.1). The matching QSS-1 system for the prescribed AQL and LQL conditions can be found to be

QSS-1($n_N = 5, n_T = 19$) from the tables given in *Govindaraju* [15.68] or *Kuralmani* and *Govindaraju* [15.70]. At good quality levels, the normal inspection plan will require sampling only five units. Only at poor quality levels, 19 units will be sampled under the QSS system. So, it is obvious that *Dodge*'s [15.1] chain sampling approach is not truly economical at good quality levels but fares well at poor quality levels. However, if the modified chain sampling plan MChSP-1 by *Govindaraju* and *Lai* [15.37] is used, then the sample size needed will only be three units (and i, the number of related lot results to be used, is fixed at seven or eight).

A more general two-plan system having zero acceptance number for the tightened and normal plans was studied by *Calvin* [15.71], *Soundararajan* and *Vijayaraghavan* [15.72], and *Subramani* and *Govindaraju* [15.73]. Calvin's TNT scheme uses zero acceptance numbers for normal and tightened inspection and employs the switching rules of MIL-STD-105 D [15.74], which is also roughly employed in ISO 2859-1:1989 [15.75]. The operating procedure of the TNT scheme, designated TNT (n_N, n_T; $Ac = 0$), is given below:

1. Start with the tightened inspection plan (n_T, 0). Switch to normal inspection (Step 2) when t lots in a row are accepted; otherwise continue with the tightened inspection plan.
2. Apply the normal inspection plan (n_N, 0). Switch to the tightened plan if a lot rejection is followed by another lot rejection within the next s lots.

Using the tables of *Soundararajan* and *Vijayaraghavan* [15.76], the zero-acceptance-number TNT(n_N, n_T; 0) plan for given AQL = 1%, $\alpha = 5\%$, LQL = 15%, and $\beta = 10\%$ is found to be TNT($n_N = 5, n_T = 16$; $Ac = 0$). We again find that the MChSP-1 plan calls for a smaller sample size when compared to Calvin's zero-acceptance-number TNT plan.

The skip-lot sampling plans of *Dodge* [15.77] and *Perry* [15.78] are based on skipping of sampling inspection of lots on the evidence of good quality history. For a detailed discussion of skip-lot sampling, *Stephens* [15.31] may be consulted. In the skip-lot sampling plan of type SkSP-2 by *Perry* [15.78], once m successive lots are accepted under the reference plan, the chosen reference sampling plan is applied only for a fraction f of the time. *Govindaraju* [15.79] studied the employment of the zero-acceptance-number plan as a reference plan (among several other reference sampling plans) in the skip-lot context. For given AQL = 1%, $\alpha = 5\%$, LQL = 15%, and $\beta = 10\%$, the SkSP-2 plan with a zero-acceptance-number reference plan is found to be $n = 15$ $m = 6$, and $f \simeq 1/5$. Hence the matching ChSP-1 plan $n = 15$ and $i = 2$ is not economical at good quality levels when compared to the SkSP-2 plan $n = 15$, $m = 6$, and $f \simeq 1/5$. This is because the SkSP-2 plan requires the zero-acceptance-number reference plan with a sample size of 15 to be applied only to one in every five lots submitted for inspection once six consecutive lots are accepted under the reference single-sampling plan ($n = 10$, $Ac = 0$). However, the modified MChSP-1 plan is more economical at poor quality levels when compared to the SkSP-2 plan. Both plans require about the same sampling effort at good quality levels.

15.7 Chain Sampling for Variables Inspection

Govindaraju and *Balamurali* [15.80] extended the idea of chain sampling to sampling inspection by variables. This approach is particularly useful when testing is costly or destructive provided the quality variable is measurable on a continuous scale. It is well known that variables plans do call for very low sample sizes when compared to the attribute plans. However not all variables plans possess a satisfactory OC curve, as shown by *Govindaraju* and *Kuralmani* [15.81]. Often, a variables plan is unsatisfactory if the acceptability constant is too large, particularly when the sample size is small. Only in such cases is it necessary to follow the chain sampling approach to improve upon the OC curve of the variables plan. Table 15.2 is useful for deciding whether a given variables sampling plan has a satisfactory OC curve or not. If the acceptability constant k_σ of a known sigma variables plan exceeds $k_{\sigma l}$ then the plan is deemed to have an unsatisfactory OC curve, like an $Ac = 0$ attributes plan.

The operating procedure of the chain sampling plan for variables inspection is as follows:

1. Take a random sample of size n_σ, say $(x_1, x_2, ..., x_{n_\sigma})$ and compute

$$v = \left(\frac{U - \bar{X}}{\sigma}\right), \text{ where } \quad \bar{X} = \frac{1}{n_\sigma}\sum_{i=1}^{n_\sigma} x_i \; .$$

Table 15.2 Limits for deciding unsatisfactory variables plans

n_σ	$k_{\sigma l}$	n_σ	$k_{\sigma l}$	n_σ	$k_{\sigma l}$	n_σ	$k_{\sigma l}$
1	0	16	2.3642	31	3.3970	46	4.1830
2	0.4458	17	2.4465	32	3.4549	47	4.2302
3	0.7280	18	2.5262	33	3.5119	48	4.2769
4	0.9457	19	2.6034	34	3.5680	49	4.3231
5	1.1278	20	2.6785	35	3.6232	50	4.3688
6	1.2869	21	2.7515	36	3.6776	51	4.4140
7	1.4297	22	2.8227	37	3.7312	52	4.4588
8	1.5603	23	2.8921	38	3.7841	53	4.5032
9	1.6812	24	2.9599	39	3.8362	54	4.5471
10	1.7943	25	3.0262	40	3.8876	55	4.5905
11	1.9009	26	3.0910	41	3.9384	56	4.6336
12	2.0020	27	3.1546	42	3.9885	57	4.6763
13	2.0983	28	3.2169	43	4.0380	58	4.7186
14	2.1904	29	3.2780	44	4.0869	59	4.7605
15	2.2789	30	3.3380	45	4.1352	60	4.8021

2. Accept the lot if $v \geq k_\sigma$ and reject if $v < k'_\sigma$. If $k'_\sigma \leq v < k_\sigma$, accept the lot provided the preceding i lots were accepted on the condition that $v \geq k_\sigma$.

Thus the variables chain sampling plan has four parameters: the sample size n_σ, the acceptability constants k_σ and k'_σ ($< k_\sigma$), and i, the number of preceding lots used for conditionally accepting the lot. The OC function of this plan is given by $P_a(p) = P_V + (P'_V - P_V)P^i_V$, where $P_V = \Pr(v \geq k_\sigma)$ is the probability of accepting the lot under the variables plan (n_σ, k_σ) and $P'_V = \Pr\left(v \geq k'_\sigma\right)$ is the probability of accepting the lot under the variables plan (n_σ, k'_σ). Even though the above operating procedure of the variables chain sampling plan is of general nature, it would be appropriate to fix $k'_\sigma = k_{\sigma l}$. For example, suppose that a variables plan with $n_\sigma = 5$ and $k_\sigma = 2.46$ is currently under use. From Table 15.2, the limit for the undesirable acceptability constant $k_{\sigma l}$ for $n_\sigma = 5$ is obtained as 1.1278. As the actual acceptability constant $k_\sigma (= 2.26)$ is greater than $k_{\sigma l} (= 1.1278)$, the variables plan can be declared to possess an unsatisfactory OC curve. Hence it is desirable to chain the results of neighboring lots to improve upon the shape of the OC curve of the variables plan $n_\sigma = 5$ and $k_\sigma = 2.46$. That is, the variables plan currently under use with $n_\sigma = 5$ and $k_\sigma = 2.46$ will be operated as a chain sampling plan fixing $i = 4$. A more detailed procedure on designing chain sampling for variables inspection, including the case when sigma is unknown, is available in *Govindaraju* and *Balamurali* [15.80]. The chain sampling for variables will be particularly useful when inspection costs are prohibitively high, and the quality characteristic is measurable on a continuous scale.

15.8 Chain Sampling and CUSUM

In this section, we will discuss some of the interesting relationships between the cumulative sum (CUSUM) approach of *Page* [15.82, 83] and the chain sampling approach of *Dodge* [15.1]. The CUSUM approach is largely popular in the area of statistical process control (SPC) but *Page* [15.82] intended it for use in acceptance sampling as well. *Page* [15.82] compares his CUSUM-based inspection scheme with the deferred sentencing schemes of *Anscombe* et al. [15.38], and the continuous sampling plan CSP-1 of *Dodge* [15.39] to evaluate their relative performance. In fact Dodge's CSP-1 plan forms the theoretical basis for his ChSP-1 chain sampling plan. A more formal acceptance sampling scheme based on the one-sided CUSUM for lot-by-lot inspection was proposed by *Beattie* [15.84]. Beattie's plan calls for drawing a random sample of size n from each lot and observing the number of nonconforming units d. For each lot, a CUSUM value is calculated for a given

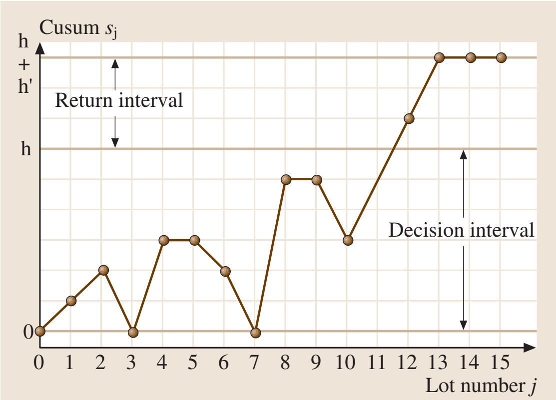

Fig. 15.6 Beattie's CUSUM acceptance sampling plan

slack parameter k. If the computed CUSUM is within the decision interval $(0, h)$, then the lot is accepted. If the CUSUM is within the return interval $(h, h+h')$, then the lot is rejected. If the CUSUM falls below zero, it is reset to zero. Similarly if the CUSUM exceeds $h+h'$, it is reset to $h+h'$. In other words, for the j-th lot, the plotted CUSUM can be succinctly defined as $S_j = \text{Min}\{h+h', \text{Max}\{(d_j - k) + S_{j-1}, 0\}\}$ with $S_0 = 0$. Beattie's plan is easily implemented using the typical number of nonconforming units CUSUM chart for lot-by-lot inspection Fig. 15.6. *Prairie* and *Zimmer* [15.85] provided detailed tables and nomographs for the selection of Beattie's CUSUM acceptance sampling plan. An application is also reported in [15.86].

Beattie [15.87] introduced a two-stage semi-continuous plan where the CUSUM approach is followed, and the product is accepted as long as the CUSUM, S_j, is within the decision interval $(0, h)$. For product falling in the return interval $(h, h+h')$, an acceptance sampling plan such as the single- or double-sampling plan is used to dispose of the lots. *Beattie* [15.87] compared the two-stage semi-continuous plan with the ChSP-4A plan of *Frishman* [15.12] and the deferred sentencing scheme of *Hill* et al. [15.40]. Beattie remarked that chain sampling plans (ChSP-4A type) call for a steady rate of sampling and are simple to administer. The two-stage semi-continuous sampling plan achieved some gain in the average sample number at good quality levels, but it is more difficult to administer. The two-stage semi-continuous plan also requires a larger sample size than the ChSP-4A plans when the true quality is poorer than acceptable levels.

We will now explore an interesting equivalence between the ChSP-1 plan, and a CUSUM scheme intended for high-yield or low-fraction-nonconforming production processes for which the traditional p or np control charts are not useful. *Lucas* [15.88] gave a signal rule for lack of statistical control if there are two or more counts within an interval of t samples. In the case of a process with a low fraction nonconforming, this means that, if two or more nonconforming units are observed in any t consecutive samples or less, a signal for an upward shift in the process fraction level is obtained. It should be noted that, if two or more nonconforming units are found even in the same sample, a signal for lack of statistical control will be obtained. *Govindaraju* and *Lai* [15.89] discuss the design of *Lucas*'s [15.88] scheme, and provided a method of obtaining the parameters n (the subgroup or sample size) and t (the maximum number of consecutive samples considered for a signal).

Lucas [15.88] has shown that his signal rule is equivalent to a CUSUM scheme having a reference value k of $1/t$ and decision interval $h = 1$ for detecting an increase in the process count level. It was also shown that a fast initial response (FIR) feature can be added to the CUSUM scheme (see *Lucas* and *Crosier* [15.90]) with an additional sub-rule that signals lack of statistical control if the first count occurs before the t-th sample. This FIR CUSUM scheme has a head start of $S_0 = 1 - k$ with $k = 1/t$ and $h = 1$. Consider the ChSP-1 plan of *Dodge* [15.1], which rejects a lot if two or more counts (of nonconformity or nonconforming units) occur but allows acceptance of the lot if no counts occur or a single count is preceded by t (the symbol i was used before) lots having samples with no counts. If the decision to reject a lot is translated as the decision of declaring the process to be not in statistical control, then it is seen that Lucas's scheme and the ChSP-1 plan are the same. This equivalence will be even clearer if one considers the operation of the two-stage chain sampling plan ChSP(0,1) of *Dodge* and *Stephens* [15.26] given in Sect. 15.3. When $k_2 = k_1 + 1$, the ChSP(0,1) plan is equivalent to the ChSP-1 plan with $t = k_1$. So it can also be noted that the sub-rule of not allowing any count for the first t samples suggested for the FIR CUSUM scheme of *Lucas* [15.88] is an inherent feature of the two-stage chain sampling scheme. This means that the ChSP-1 plan is equivalent to the FIR CUSUM scheme with the head start of $(1-k)$ with $k = 1/t$ and $h = 1$.

15.9 Other Interesting Extensions

Mixed sampling plans are two-phase sampling plans in which both variable quality characteristics and attribute quality measures are used in deciding the acceptance or rejection of the lot. *Baker* and *Thomas* [15.91] reported the application of chain sampling for acceptance testing for armor packages. Their procedure uses chain sampling for testing structural integrity (attributes inspection) while a variables sampling plan is used for testing penetration-depth quality characteristic. The authors also suggested the simultaneous use of control charts along with their proposed acceptance sampling procedures. *Suresh* and *Devaarul* [15.92] proposed a more formal mixed acceptance sampling plan where a chain sampling plan is used for the attribute phase. *Suresh* and *Devaarul* [15.92] also obtained the OC function for their mixed plan, and discussed various selection procedures. To control multidimensional characteristics, *Suresh* and *Devaarul* [15.93] developed multidimensional mixed sampling plans (MDMSP). These plans handles several quality characteristics during the variable phase of the plan, while the attribute sampling phase can be based on chain sampling or other attribute plans.

In some situations it is desirable to adopt three attribute classes, where items are classified into three categories: good, marginal and bad [15.94]. *Shankar* et al. [15.95] developed three-class chain sampling plans and derived various performance measures through the GERT approach and also discussed their design.

Suresh and *Deepa* [15.96] provided a discussion on formulating a chain sampling plan given a prior gamma or beta distribution for product quality. Tables for the selection of the plans and examples are also provided by *Suresh* and *Deepa* [15.96]. This approach will further improve the sampling efficiency of chain sampling plans.

15.10 Concluding Remarks

This chapter largely reviews the methodology of chain sampling for lot-by-lot inspection of quality. Various extensions of the original chain sampling plan ChSP-1 of *Dodge* [15.1] and modifications are briefly reviewed. The chain sampling approach is primarily useful for costly or destructive testing, where small sample sizes are preferred. As chain sampling plans achieve greater sampling economy, these are combined with the approach of deferred sentencing so that the combined plan can be used for any general situation. This chapter does not cover design of chain sampling plans in any great detail. One may consult textbooks such as *Schilling* [15.97] or *Stephens* [15.31, 98] for detailed tables. A large number of papers primarily dealing with the design of chain sampling plans are available only in journals, and some of them are listed as references. It is often remarked that designing sampling plans is more of an art than a science. There are statistical, engineering and other administrative aspects to be taken into account for successful implementation of any sampling inspection plan, including chain sampling plans. For example, for administrative and other reasons, the sample size may be fixed. Given this limitation, which sampling plan should be used requires careful consideration. Several candidate sampling plans, including chain sampling plans, must first be sought, and then the selection of a particular type of plan must be made based on performance measures such as the OC curve etc. The effectiveness of the chosen plan or sampling scheme must be monitored over time, and changes made if necessary.

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