Chapter 4
Circuits

Abstract The observed quantity in eddy-current nondestructive evaluation is the electrical impedance of the probe coil. Proper interpretation of the impedance allows the inspector to infer material property information, and to detect and characterize defects. This chapter provides an introductory description of circuit theory that is relevant to eddy-current nondestructive evaluation, describing resistors, capacitors, and inductors and the impedance of circuits in which they are combined. The concept of an equivalent electrical circuit for an eddy-current probe is introduced and the equivalent circuit is given in its simplest form.

4.1 Introduction

The process of induction of eddy currents in a metal test-piece, due to a time-varying electric current flowing in an eddy-current coil, is most clearly understood in terms of the electric and magnetic fields introduced in Chap. 2. It is helpful, on the other hand, to describe certain characteristics of an eddy-current coil in terms of electrical circuit theory. In this chapter, the circuit quantities *resistance*, *capacitance*, and *inductance* are introduced and the quantity that is actually measured in an eddy- current inspection, the *impedance* of the coil, is defined. Some simple circuit configurations are analyzed in order to prepare the way for discussion of a method of correcting for "non-ideal" coil behavior, to be given in Sect. 6.3.6. A fairly brief overview is given here. More detail can be found in [1].

4.2 Electromotance and Potential Difference

Electromotance (commonly but misleadingly known as "electromotive force" [2]) must be applied to a conductor to compel the conduction electrons to move. A battery, for example, provides a DC source of electromotance. Another way of expressing this is that the battery terminals maintain a difference in *electrical potential*, V, which has the unit Volt (V). When connected to an electrical circuit, the *potential difference*

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provided by the battery compels the conduction electrons to move. The electrons move because an electric field exists between two points of different potential and a charge Q, with unit Coulomb (C), in an electric field experiences force $\mathbf{F}$ given by the product of the charge and the electric field:

$$\mathbf{F} = Q\mathbf{E}. \tag{4.1}$$

The potential energy W required to move the charge Q from point A to point B is

$$W = -\int_A^B \mathbf{F} \cdot d\mathbf{l} = -Q\int_A^B \mathbf{E} \cdot d\mathbf{l}. \tag{4.2}$$

The potential difference between these two points, V_{BA}, is the potential energy per unit charge:

$$V_{BA} = -\int_A^B \mathbf{E} \cdot d\mathbf{l}. \tag{4.3}$$

4.3 Resistance

As described in Sect. 1.5, a simple eddy-current coil is formed by winding multiple turns of wire on a nonconductive former. One intrinsic parameter of any coil is its DC resistance. Consider a conductor whose ends are maintained at a potential difference V, as shown in Fig. 4.1. The resistance R of the conductor is defined as the potential difference per unit current:

$$R = \frac{V}{I}. \tag{4.4}$$

From the point form of Ohm's Law, (2.15), the resistance of the conductor can be derived. The applied electric field of Fig. 4.1 is uniform, since the fields are not varying with time, and its magnitude is given by

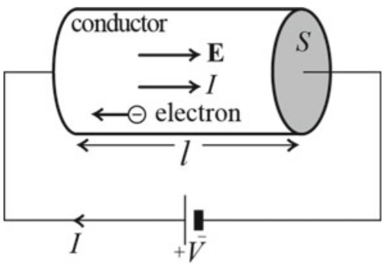

Fig. 4.1 A conductor with uniform cross section S under an applied electric field $\mathbf{E}$ due to the electromotance or potential difference V supplied to the circuit

$$E = \frac{V}{l}. \tag{4.5}$$

Since the conductor has uniform cross section and the current is DC, (2.1) holds. Combining (2.1), (2.15) and (4.5) gives

$$\frac{I}{S} = \sigma E = \frac{\sigma V}{l}.$$

From (4.4) it is now easy to see that (2.11) follows.

4.4 Capacitance

An eddy-current probe exhibits two forms of capacitance. One is *inter-winding capacitance* which arises from the fact that the windings of the coil are in close proximity to one another and separated by an insulating layer. The other is capacitance in the leads that connect the coil to the voltage source. Both of these sources of capacitance interfere with the operation of the probe, because their presence gives rise to an unwanted resonance in the probe circuit. This phenomenon will be described, and a method for correcting measured data to remove the effect of the probe resonance will be given, in Sect. 6.3.6.

A capacitor is formed by two conductors that carry equal and opposite charge, separated by an insulator (dielectric material). Broadly speaking, materials can be classified in terms of their conductivity σ and relative permittivity ϵ_r as either conductors (with $\sigma \gg 1$ and $\epsilon_r = 1$) or dielectrics (with $\sigma \ll 1$ and $\epsilon_r \gg 1$).

Consider a simple capacitor formed by two parallel conductive plates as shown in Fig. 4.2. The conductors are maintained at potential difference V given by

$$V = V_1 - V_2 = -\int_2^1 \mathbf{E} \cdot d\mathbf{l} \tag{4.6}$$

where $\mathbf{E}$ is the electric field between the capacitor plates. The capacitance, C, of the capacitor is defined as the ratio of the magnitude of the charge on one of the plates to the potential difference between them,

$$C = \frac{Q}{V}. \tag{4.7}$$

Capacitance can be regarded as a measure of how much electrical energy is stored by the capacitor. Equation (4.7) is useful for determining the capacitance of an ideal parallel-plate capacitor in which the plate separation d is much smaller than the dimensions of the plate. In this case, it is assumed that $\mathbf{E}$ is uniform in the gap between the capacitor plates, Fig. 4.2, and that the *fringing field* which leaks out at the edge of the capacitor plates is negligible. Then,

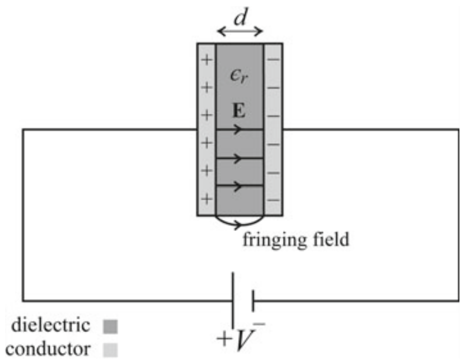

Fig. 4.2 A parallel-plate capacitor with plate area S, filled with a dielectric material with relative permittivity ϵ_r

$$C \approx \frac{\epsilon S}{d} \tag{4.8}$$

where ϵ is the permittivity of the dielectric filling the capacitor and S is the area of one of the identical plates. In fact, the ratio of the capacitance of an air-filled capacitor, C_0, compared with that of the same capacitor filled with dielectric, C, gives the relative permittivity of the dielectric:

$$\frac{C}{C_0} = \frac{\epsilon}{\epsilon_0} = \frac{\epsilon_0 \epsilon_r}{\epsilon_0} = \epsilon_r \tag{4.9}$$

where $\epsilon_0 = 8.854 \times 10^{-12}$ Farads per meter (F/m) is the permittivity of free space.

4.5 Discharge of a Capacitor Through a Resistor

Having defined resistance and capacitance, we are now in a position to consider a simple circuit formed by connecting a capacitor and resistor in series, as shown in Fig. 4.3. If the capacitor is given charge Q_0 at time $t = 0$, a potential difference $V = Q_0/C$ appears across the plates. In the absence of an electromotance, the capacitor discharges and current $I = dQ/dt$ flows through the circuit. At any time $t > 0$ therefore, the potential difference across the resistor is $R \times dQ/dt$. One of Kirchhoff's Laws of circuit theory (Kirchhoff's Voltage Law) states "The directed sum of the potential differences (voltages) around any closed loop is zero." Applying this law to the series RC circuit gives

$$R\frac{dQ(t)}{dt} + \frac{Q(t)}{C} = 0. \tag{4.10}$$

The solution of this equation shows that there is an exponential decay of charge with time,

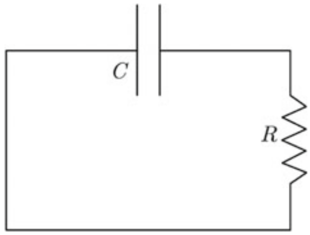

Fig. 4.3 Capacitor C and resistor R connected in a series circuit

$$Q(t) = Q_0 \exp\left(-\frac{t}{RC}\right) \tag{4.11}$$

where $\tau = RC$ is known as the relaxation time of the RC circuit.

4.6 Forced Oscillation of an RC Circuit by Alternating Electromotance

If an electromotance of the form $V_0 \cos(\omega t + \phi)$ is now introduced into the circuit, as shown in Fig. 4.4, applying Kirchhoff's Law (stated in the previous section) gives

$$R\frac{dQ(t)}{dt} + \frac{Q(t)}{C} = V_0 \cos(\omega t + \phi). \tag{4.12}$$

The solution of this equation, characteristic of alternating currents, is easily obtained by writing the equation in phasor form:

$$j\omega R\mathcal{Q} + \frac{\mathcal{Q}}{C} = \mathcal{V} \tag{4.13}$$

where $\mathcal{Q}$ represents phasor charge, related to Q by $Q = \mathrm{Re}\left\{\mathcal{Q}e^{j\omega t}\right\}$ as discussed in Sect. 2.3 and $\mathcal{V}$ represents phasor voltage, similarly. Then

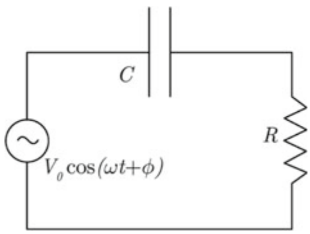

Fig. 4.4 Capacitor C and resistor R connected in a series circuit with time-harmonic electromotance $V_0 \cos(\omega t + \phi)$

$$\mathcal{Q} = \frac{\mathcal{V}}{j\omega R + 1/C} \quad (4.14)$$

and the phasor current $\mathcal{I} = j\omega\mathcal{Q}$ is given by

$$\mathcal{I} = \frac{\mathcal{V}}{R + 1/(j\omega C)}. \quad (4.15)$$

Next we will define another circuit component, inductance, and see the interesting effect of introducing an inductance into the circuit—a resonance is created in the circuit at a particular frequency.

4.7 Inductance

The inductance of an eddy- current coil is its most important circuit property because it is the property that allows the coil to detect changes in its local magnetic field that arise due to perturbations of the eddy currents induced in the test-piece due to the presence of defects or inhomogeneities. To understand inductance, it is necessary to define magnetic flux. By analogy with the relationship between electric current I and current density **J**, (2.2), magnetic flux Ψ is related to the field of magnetic induction **B** as

$$\Psi = \int_S \mathbf{B} \cdot d\mathbf{S}. \quad (4.16)$$

The unit of magnetic flux is the Weber (Wb) and, while the unit of **B** is the Tesla, **B** is also commonly quoted in Wb/m^2. One can conceive of a magnetic circuit in which Ψ is constrained to flow within a high-permeability material (e.g., a ferromagnet), by analogy with the way in which electrical current is confined within a high conductivity material (a conductor). Indeed, Faraday's transformer experiment relied upon this phenomenon to some extent (Sect. 1.2.1). The analogy has its limitations, however, because **B** easily "leaks" out of a material if μ_r is not especially large, unlike **J** which is confined strictly to the conductor.

In Chap. 2, the nature of the magnetic field produced by current flowing in a long, straight wire was discussed. If now a *closed* conducting path is considered, the current I produces a magnetic induction **B** that causes flux Ψ as defined in (4.16) to pass through the closed path. Further, if the circuit has N identical turns, the flux linkage λ can be defined as

$$\lambda = N\Psi. \quad (4.17)$$

If the relationship between I and **B** in the medium surrounding the circuit is linear, the flux linkage is proportional to the current producing it and $\lambda \propto I$. (The relationship between I and **B** is not linear in the case of a ferromagnetic material, as discussed in Chap. 3.) A constant of proportionality is introduced such that

$$\lambda = LI \tag{4.18}$$

where L is the *inductance* of the circuit. Inductance is measured in the unit Henry (H), after Joseph Henry (Sect. 1.2.2). From (4.17) and (4.18) the inductance of an inductor is defined as the ratio of the magnetic flux linkage to the current through the inductor:

$$L = \frac{\lambda}{I} = \frac{N\Psi}{I}. \tag{4.19}$$

Inductance can be regarded as a measure of how much magnetic energy is stored in an inductor.

Strictly, L is the *self*-inductance of an inductor since the flux linking the circuit is produced by the inductor itself. It is also possible to define mutual inductance, in which the flux linking the inductor is produced by a separate circuit. See, for example, [3].

An eddy-current coil with large self-inductance is desirable because the coil then responds more strongly to changes in the magnetic field in the vicinity of the coil. The probe is thus more sensitive to magnetic field variations caused by perturbations of the induced eddy-current density in a test-piece and, therefore, more sensitive to a defect or other feature that causes the perturbation. The self-inductance of an eddy-current coil is commonly increased in practice by winding the coil around a high-permeability ferrite core. This is discussed further in Sect. 6.3.5.

4.8 Forced Oscillation of an LRC Circuit by Alternating Electromotance

If an inductor is now introduced into the circuit, as shown in Fig. 4.5, a potential difference of $-L \times dI/dt$ appears across the inductor at any time $t > 0$. This can be shown by taking the derivative with respect to time of rearranged (4.19):

$$L\frac{dI}{dt} = \frac{d\Psi}{dt} = -V \tag{4.20}$$

where the final identity comes from Faraday's Law, given later in (5.1), Sect. 5.2. In this case, applying Kirchhoff's Law gives

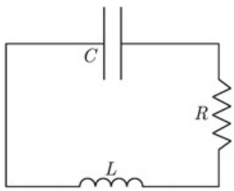

Fig. 4.5 Capacitor C, resistor R and inductor L connected in a series circuit

$$L\frac{d^2Q(t)}{dt^2} + R\frac{dQ(t)}{dt} + \frac{Q(t)}{C} = 0 \tag{4.21}$$

and, if an alternating electromotance $V(t) = V_0 \cos(\omega t + \phi)$ is applied to the circuit,

$$L\frac{d^2Q(t)}{dt^2} + R\frac{dQ(t)}{dt} + \frac{Q(t)}{C} = V_0 \cos(\omega t + \phi). \tag{4.22}$$

The solution of this equation, in phasor form, is

$$\mathcal{Q} = \frac{\mathcal{V}}{-\omega^2 L + j\omega R + 1/C} \tag{4.23}$$

and the corresponding current is

$$\mathcal{I} = \frac{\mathcal{V}}{R + j[\omega L - 1/(\omega C)]}. \tag{4.24}$$

4.9 Impedance

Now we define Z to be the complex impedance of the circuit, given by

$$Z = \frac{\mathcal{V}}{\mathcal{I}}. \tag{4.25}$$

The symbol "Z" was first introduced by Sir Oliver Heaviside (1850-1925, Fig. 4.6), an English physicist and electrical engineer who, despite being formally educated only to elementary level, made important advances in the application of mathematics to electrical circuits. His choice of mathematical notations and methods were often not celebrated by his peers, however, and for this reason, he was forced to publish his papers at his own expense [4]!

Returning to consideration of the series LRC circuit, the following expression for Z can now be obtained from (4.24) and (4.25);

$$Z = R + j\omega L + \frac{1}{j\omega C}. \tag{4.26}$$

In general, the real and imaginary parts of Z are given the symbols R and X and are known as the resistance and *reactance* of the circuit, respectively;

$$Z = R + jX. \tag{4.27}$$

In the specific example of the series LRC circuit, $X = \omega L - 1/(\omega C)$.

Fig. 4.6 Oliver Heaviside, English physicist and electrical engineer, 1850–1925 [5]

Impedance is a very important quantity in EC NDE! In most eddy- current inspections, the complex impedance of the probe is viewed by the inspector on an "impedance-plane plot" [1, 6], in which R is plotted on the horizontal (real) axis and X is plotted on the vertical (imaginary) axis. Variations in the impedance as a probe moves across a defective region in a test-piece, or from one material type to another, are manifested as movement in the impedance point on the complex plane.

Impedances connected in series or in parallel in a circuit can be manipulated in the same way as pure resistances;

$$Z = Z_1 + Z_2 + Z_3 + \dots, \quad \text{series,} \tag{4.28}$$

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \dots, \quad \text{parallel.} \tag{4.29}$$

In the case of parallel impedances, it is more convenient to work in terms of the admittance, $Y = 1/Z$. Then

$$Y = Y_1 + Y_2 + Y_3 + \dots, \quad \text{parallel.} \tag{4.30}$$

4.10 Frequency Response of an LRC Circuit

Consideration of (4.24) shows that the current amplitude varies as a function of frequency. If the alternating electromotance is maintained at constant amplitude for all frequencies, the current amplitude peaks when $\omega L = 1/(\omega C)$, for which

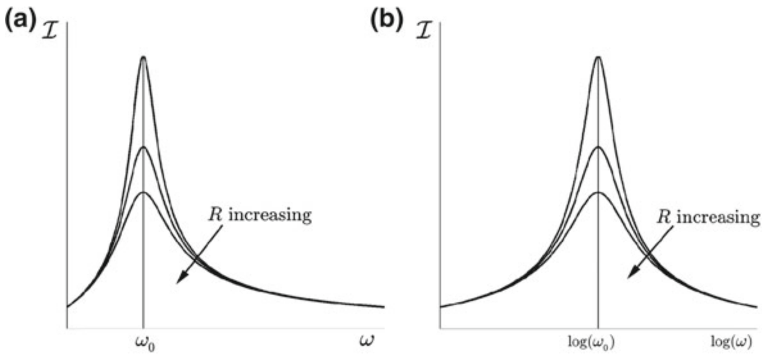

Fig. 4.7 Magnitude of current as a function of angular frequency, ω, for a series LRC circuit energized by an alternating electromotance whose amplitude is constant as a function of ω. The effects of varying R are shown on (**a**) linear and (**b**) logarithmic frequency scales

$$\omega = \omega_0 = \frac{1}{\sqrt{LC}} \tag{4.31}$$

and ω_0 is known as the resonance frequency of the circuit. If R increases, the curve becomes shallower and the peak height is reduced, as shown in Fig. 4.7.

The breadth of the peak is controlled by the quality, or Q-factor

$$\mathsf{Q} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R}. \tag{4.32}$$

Another type of resonant circuit is the subject of Exercise 3, at the end of this chapter.

4.11 Equivalent Electrical Circuit for an Eddy-Current Probe

An equivalent electrical circuit that accounts for the various contributions to the impedance of a real eddy-current probe is examined in Sect. 6.3.6 and is shown schematically in Fig. 6.15. It can be seen that the circuit representation is more complicated than any of those considered above. Nonetheless, under many circumstances it is reasonable to consider the impedance of an eddy-current probe to be described, to a first approximation, by the resistive and inductive contributions only. In other words, for a coil operating at a frequency well below its resonance frequency,

$$Z \approx R + j\omega L. \tag{4.33}$$

4.12 Summary

In this chapter, the three circuit quantities resistance, inductance, and capacitance have been introduced and their combination in various simple circuits has been developed to the point of defining impedance the quantity, that is measured in an EC inspection. The way in which the circuit components feature in the make-up of an EC probe has been outlined; resistance originates primarily in the resistivity of the wire of the coil windings, self-inductance arises from the coil's own time-varying magnetic flux changing in the vicinity of the coil's own windings, and capacitance arises from inter-winding effects. Acknowledging that in most cases an eddy-current coil impedance is given, to a good approximation, by the sum of resistive and inductive contributions only, (4.33), we turn in the next chapter to a discussion of the physical and mathematical framework that is needed to compute the probe impedance under various circumstances of practical significance. The framework that we need is provided by the field known as electromagnetism.

4.13 Exercises

1. If the ends of a cylindrical bar of carbon ($\sigma = 3 \times 10^4$ S/m) of radius 5 mm and length 8 cm are maintained at a potential difference of 9 V, find (a) the resistance of the bar, (b) the current through the bar.
2. The resistance per unit length of a long wire with a circular cross section and diameter 2 mm is 5.488 mΩ/m (milli-Ohm per meter). If a direct current of 40 mA flows through the wire, (a) find the conductivity of the wire; (b) identify the material of the wire: and (c) find the electric current density in the wire.
3. For the parallel resonance circuit shown in Fig. 4.8, in which L and R are in series with each other but in parallel with C, that is driven by alternating electromotance $V = V_0 \cos(\omega t + \phi)$, (a) express the impedance of the circuit, Z, in the form $R + jX$, (b) determine the resonance frequency ω_0 in terms of R, L and C, and (c) show that the current amplitude is *minimum* at resonance, rather than maximum as in the case of the series LRC circuit.

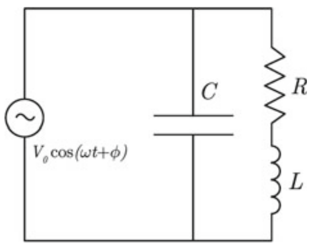

Fig. 4.8 Parallel resonance circuit in which L and R are connected in series with each other but in parallel with C, driven by alternating electromotance $V = V_0 \cos(\omega t + \phi)$

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