Chapter 2
Fields

Abstract In this chapter, the concept of the electromagnetic field is introduced in the context of current-carrying, possibly ferromagnetic, conductors. The electric current density, itself a vector field, is defined and related to electric current. Phasors are introduced and their usefulness for treating electrical systems whose current is alternating sinusoidally is described. The related material parameters, conductivity, and resistivity, are defined and discussed. The relationships between electric current, electric field, and magnetic field are introduced in relation to a line current. The relation of electromagnetic fields to power transfer is introduced. Magnetization field, the field of magnetic induction, and the permeability of a material are defined. The definition of the electromagnetic skin depth is provided, highlighting the material parameters that influence it. The consequences of the skin effect for EC NDE are discussed, utilizing the example of an air-cored eddy-current coil adjacent to an unflawed metal test-piece. For completeness, the electric displacement and polarization fields are also defined.

2.1 Introduction

What do we mean by an electric or magnetic "field"? A field is a way of referring to a spatial distribution of a certain quantity. Knowledge of field distributions is useful for predicting the behavior of physical systems. Faraday was the first to propose thinking of the universe as consisting of fields of various kinds. For example, knowledge of the form of the earth's gravitational field enables us to predict that an object will fall when dropped, and indeed what its velocity will be on impact with the ground. Similarly, knowledge of the forms of electric and magnetic fields allow us to predict the behavior of electric charges and currents when they interact with those fields.

In the previous chapter, it was mentioned that a time-varying current, flowing in a loop of wire, produces an associated magnetic field that is spatially similar to the field in the vicinity of a small bar magnet. A significant difference between them, however, is that the magnetic field associated with the time-varying current flow also fluctuates in time, whereas that associated with a bar magnet does not. On the other hand, a constant magnetic field similar to that produced by a bar magnet is produced

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by direct current flowing in a circular loop. In this chapter the relationship between the electric current flowing in the coil of an eddy-current probe, the magnetic field it produces, and the eddy currents induced in the conductive test-piece will be explored.

One characteristic of metals is the existence of conduction electrons which are free to move when an electro-motive force is present. The electro-motive force can be provided by a battery or AC power supply, for example. The conduction electrons are the carriers of electric current although, by historical accident, the conventional current I flows in the opposite direction to the flow of the negatively charged electrons. The unit of electric current is the Ampère (A), named after the French mathematician and physicist André Ampère (1775–1836). Within one week of the report of Oersted's experiment (Sect. 1.2.1), Ampère had formulated the right-hand rule (Sect. 2.6), in which the *direction* of deflection of a small bar magnet (e.g., a compass needle) in the vicinity of a current-carrying wire is specified [1]. In fact, it was in the formulation of the right-hand rule that it was incorrectly assumed that current flows from positive to negative poles, that is, that the charge carriers were positively charged.

2.2 Current Density

For the purpose of describing eddy currents in a conductor, it is more convenient to use the current density, $\mathbf{J}$, a vector field whose unit is Ampères per meter squared (Am^{-2}), rather than the current I. This is because eddy currents in a test-piece follow a path which offers the least electrical resistance to their flow. Their direction changes to accommodate the presence of resistive obstacles in the conductor, in the same way that smoothly flowing water separates to flow around a rock in a stream. This means that the direction as well as the magnitude of the eddy currents needs to be described, and this is best accomplished by the vector field $\mathbf{J}$. Examples of resistive obstacles in a conductor are cracks, pores, and regions of corrosion.

To begin to understand the relationship between I and $\mathbf{J}$, consider current flowing in a wire with cross-sectional area S, Fig. 2.1. In the case of direct current (DC), which does not vary with time, the current density in the wire is spatially uniform and the total current flowing in the wire is given by the product of the current density and the cross-sectional area of the wire:

$$I = JS. \tag{2.1}$$

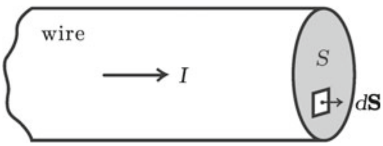

Fig. 2.1 Current I flowing in a wire with cross-sectional area S

In the case of time-varying current, however, the current density in the wire is generally not spatially uniform, and the current flowing in the wire is obtained by means of a surface integral.

$$I = \int_S \mathbf{J} \cdot d\mathbf{S}, \tag{2.2}$$

where $d\mathbf{S}$ is an element of surface area. The dot product ensures that the component of $\mathbf{J}$ that is normal to the surface S is the one that contributes to I. For refreshment on vector analysis, see Appendix 10.3.

Example: Uniform current density in a wire Determine the total current in a wire of radius 1 mm, placed along the z-axis, if $\mathbf{J} = (500/\pi)\hat{z}$ Am^{-2}.

Solution From (2.1), $I = JS = J\pi\alpha^2$ where α is the wire radius. Hence,

$$I = \frac{500}{\pi}\pi(10^{-3})^2 = 500 \times 10^{-6} = 0.5 \text{ mA}.$$

Example: Nonuniform current density in a wire The current density in a cylindrical conductor of radius α placed along the z-axis is $\mathbf{J} = 10\exp[-(1-\rho/\alpha)]\hat{z}$ Am^{-2}. Evaluate the current through the cross section of the conductor.

Solution From (2.2),

$$\begin{aligned} I &= \int \mathbf{J} \cdot d\mathbf{S} \\ &= \int_0^{2\pi} \int_0^{\alpha} 10\, e^{-(1-\rho/\alpha)} \hat{z} \cdot (\rho d\rho d\phi \hat{z}) \\ &= 10 \int_0^{2\pi} d\phi \int_0^{\alpha} \rho\, e^{-(1-\rho/\alpha)} d\rho \\ &= \frac{20\pi}{e} \int_0^{\alpha} \rho\, e^{\rho/\alpha} d\rho \end{aligned}$$

Use of the standard integral [2, relation 4.2.55]

$$\int x e^{\beta x} dx = \frac{e^{\beta x}}{\beta^2}(\beta x - 1)$$

then gives $I = (20\pi/e)\alpha^2 = 23.11\alpha^2$ A.

2.3 Alternating Current and Phasor Representation

Direct current, produced by a battery, for example, flows steadily. The magnitude of the current, I, is constant as a function of time, as shown in Fig. 2.2. Alternating current is one form of time-varying current and often has a simple-harmonic waveform, as shown in Fig. 2.3.The current at any instant in time is given by

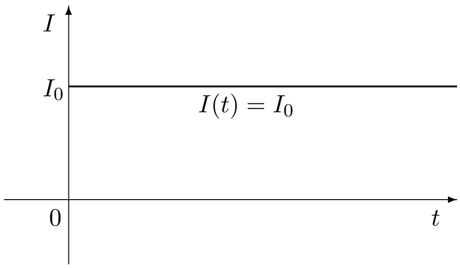

Fig. 2.2 Direct current $I(t) = I_0$

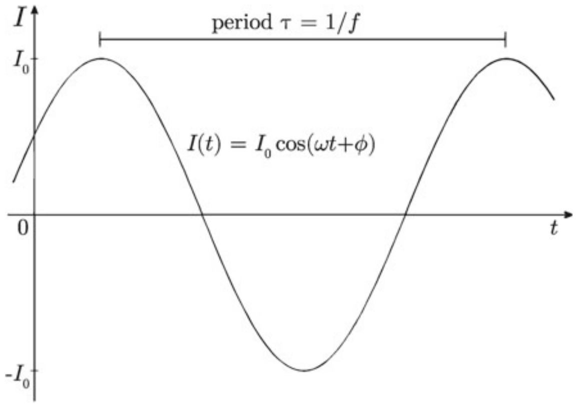

Fig. 2.3 Sinusoidal alternating current $I(t) = I_0 \cos(\omega t + \phi)$

$$I(t) = I_0 \cos(\omega t + \phi) \tag{2.3}$$

where I_0 is the amplitude of the current, $\omega = 2\pi f$ is the angular frequency, t is time elapsed and ϕ is the phase of $I(t)$. It is popular to work in terms of the angular frequency ω (unit radian^{-1} or rad^{-1}) rather than the frequency f (unit Hertz, written Hz, or second^{-1}, written s^{-1}) because the period of the cosine function is 2π. In other words

$$\cos \alpha = \cos(\alpha + 2n\pi) \tag{2.4}$$

where n is an integer ... 2, 1, 0, 1, 2, ... Hence,

$$\cos(2\pi f t) = \cos[2\pi(f t + n)] \tag{2.5}$$

and the length of one cycle is defined by $f t = 1$ which means that the period of the oscillation τ (measured in seconds, s) is simply

$$\tau = \frac{1}{f}. \tag{2.6}$$

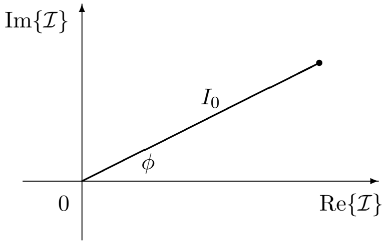

Fig. 2.4 Complex-plane representation of phasor $\mathcal{I} = I_0 \exp(j\phi)$

Note that sinusoidal alternating current, given in terms of the cosine function in (2.3), may equivalently be expressed in terms of the sine function $I(t) = I_0 \sin(\omega t + \phi + \pi/2)$. In this text, another representation will be used, in which

$$
\begin{aligned}
I(t) &= \mathrm{Re}\{\mathcal{I} \exp(j\omega t)\} \\
&= \mathrm{Re}\{I_0 \exp[j(\omega t + \phi)]\} \\
&= I_0 \mathrm{Re}[\cos(\omega t + \phi) + j \sin(\omega t + \phi)] \\
&= I_0 \cos(\omega t + \phi).
\end{aligned}
\tag{2.7}
$$

In these equations, $j = \sqrt{-1}$, $\mathrm{Re}\{Z\}$ denotes the real part of Z,

$$\exp(j\alpha) = \cos\alpha + j \sin\alpha$$

is Euler's relationship and

$$\mathcal{I} = I_0 \exp(j\phi) \tag{2.8}$$

is the *phasor* representation of $I(t)$. $\mathcal{I}$ is a complex number representing a time-harmonic physical quantity. Note that, whereas, $I(t)$ is a real quantity that is a function of t, $\mathcal{I}$ is a complex quantity that does not depend on t. Phasor $\mathcal{I}$ is plotted in the complex-plane in Fig. 2.4. In this text, the complex time dependence $\exp(j\omega t)$ is used throughout rather than $\exp(-i\omega t)$, $i = \sqrt{-1}$, which is another convention. For revision of the complex-plane representation of a complex number, see Appendix 10.1.

The phasor notation of (2.7) is especially convenient because when the derivative with respect to time is taken, the factor $j\omega$ is brought down but the exponential term itself is unchanged and can often be subsequently canceled, simplifying the analysis. In other words, the conversion between the time derivative of $I(t)$ and its phasor form $\mathcal{I}$ is

$$\frac{\partial}{\partial t} I(t) \Leftrightarrow j\omega\mathcal{I}. \tag{2.9}$$

Example: Phasor addition Prove that the addition of two time-harmonic functions with the same frequency, $I(t) = I_0 \cos(\omega t + \phi)$ and $K(t) = K_0 \cos(\omega t + \psi)$ can be represented in phasor form by the sum $\mathcal{I} + \mathcal{K}$.

Solution

$$I(t) = \text{Re}\left\{\mathcal{I}e^{j\omega t}\right\}$$
$$K(t) = \text{Re}\left\{\mathcal{K}e^{j\omega t}\right\}$$

and

$$I(t) + K(t) = \text{Re}\left\{(\mathcal{I} + \mathcal{K})e^{j\omega t}\right\}.$$

Hence,

$$I(t) + K(t) \Leftrightarrow \mathcal{I} + \mathcal{K}.$$

Note that two phasors that represent time-harmonic functions with different frequencies *cannot* be summed in this way.

Example: Phasor multiplication Show that the product of two time-harmonic functions with the same frequency, $I(t) = I_0 \cos(\omega t + \phi)$ and $K(t) = K_0 \cos(\omega t + \psi)$ *cannot* be represented in phasor form by the product $\mathcal{I}\mathcal{K}$.

Solution

$$\begin{aligned} I(t)K(t) &= I_0 K_0 \cos(\omega t + \phi)\cos(\omega t + \psi) \\ &= \frac{1}{2} I_0 K_0 \left[\cos(2\omega t + \phi + \psi) + \cos(\phi - \psi)\right] \end{aligned}$$

from identity (10.13). This is the sum of two oscillations with different frequencies (2ω and 0), which means that the product $I(t)K(t)$ cannot be represented by a single phasor of any form.

Example: Phasor differentiation Prove relation (2.9).

Solution

$$\begin{aligned} \frac{\partial}{\partial t} I(t) &= \frac{\partial}{\partial t}\left[I_0 \cos(\omega t + \phi)\right] \\ &= -\omega I_0 \sin(\omega t + \phi) \\ &= -\omega I_0 \cos(\omega t + \phi - \pi/2) \\ &= \text{Re}\left\{-\omega I_0 e^{j\omega t} e^{j\phi} e^{-j\pi/2}\right\} \\ &= \text{Re}\left\{j\omega I_0 e^{j\omega t} e^{j\phi}\right\} \\ &= \text{Re}\left\{j\omega \mathcal{I} e^{j\omega t}\right\} \end{aligned}$$

2.4 Conductivity and Resistivity

Different metals vary in their current-carrying ability. A measure of the ability of a material to convey electric current is its electrical *conductivity*, a parameter which is intrinsic to each material and arises in a metal from interactions between the conduction electrons and the crystal lattice. Also commonly used is the electrical *resistivity* of the material, which is its ability to impede (or resist) the passage of electric current. The resistivity is simply the reciprocal of the conductivity. In terms of current flowing in a wire, the DC resistance of the wire, R measured in Ohms (Ω), is proportional to the length of the wire, l, and inversely proportional to its cross-sectional area S, so that

$$R = \rho \frac{l}{S}. \tag{2.10}$$

The constant of proportionality, ρ, is the resistivity of the material out of which the wire is made. The units of ρ are Ohm meter (Ωm). Equivalently (2.10) can be written in terms of the material conductivity σ, measured in Siemens per meter (Sm^{-1}).

$$R = \frac{l}{\sigma S}, \tag{2.11}$$

where, evidently,

$$\sigma = \frac{1}{\rho}. \tag{2.12}$$

A higher value of conductivity is associated with better conductors than with poorer conductors. The conductivity of common metals varies by around two orders of magnitude. Copper is highly conductive and lends its name to the International Annealed Copper Standard (IACS), a measure of conductivity used to compare electrical conductors to a traditional copper-wire standard, in which 100% IACS represents conductivity 58 MSm^{-1}. Titanium, for example, has conductivity 1% IACS. The conductivities of selected metals are given in Table 2.1.

Electrical conductivity σ is essentially constant as a function of frequency for the electromagnetic inspection techniques discussed in this book, that operate up to frequencies of a few MHz. Conductivity is very sensitive, on the other hand, to variations in the temperature of a conductor [9]. Increasing the temperature of a conductor reduces its conductivity due to increased vibrations of the crystal lattice that impede the motion of the conduction electrons. Extensive tables of resistivity values at various temperatures are given in [6, 10]. Since conductivity values are commonly stated at 20 °C (degrees Celsius), measurements made at other temperatures must be corrected in order to properly analyze and sort metals, for example. The following formula for correcting for the effect of small temperature changes on the conductivity is given in [9, 11]:

$$\sigma(T_1) = \frac{\sigma(T_2)}{[1 + \alpha(T_1 - T_2)]}. \tag{2.13}$$

In (2.13), $\sigma(T_i)$ is conductivity in MS/m at temperature T_i in °C and α is the temperature coefficient of the material in °C^{-1}. The temperature coefficient for selected metallic elements is listed in Table 2.2 [11]. A detailed discussion on how to improve the accuracy of conductivity measurements made using EC NDE is provided in [9]. Two complementary methods of conductivity measurement, by EC NDE and by the

Table 2.1 Electrical conductivity of selected metals at 20 °C

Metal	Alloy	Conductivity, σ		Reference
		(MSm^{-1})	(% IACS)	
Aluminum, pure		35.38	61.00	[3]
Aluminum	2024	17.6	30.3	[4]
Brass	C26000	16.42	28.31	[5]
Bronze, commercial annealed		25.52	44.00	[3]
Chromium		5.10	8.80	[3]
Cobalt		16.01	27.60	[3]
Copper		58.00	100.00	[3]
Gold		40.60	70.00	[3]
Iron, pure		10.44	18.00	[3]
Nickel		14.62	25.20	[3]
Silver		68.03	117.3	[6]
Steel, Carbon	1018	5.18	8.93	[7]
Steel, Stainless	316	1.379	2.378	[5]
Steel, Spring	C1074/75	5.50	9.48	[5]
Titanium	Ti-6Al-4V	0.58	1.00	[8]
Tungsten		18.21	31.40	[3]
Zinc, commercial rolled		16.24	28.00	[3]

Table 2.2 Temperature coefficient for selected metallic elements at 20 °C [11]

Metal	Temperature coefficient ($\times 10^{-3}$ °C^{-1})
Aluminum	4.3
Copper	4.0
Gold	3.7
Iron	6.0
Nickel	5.9
Silver	3.8
Tungsten	4.4
Zinc	3.8

four-point potential drop method, are compared in [7]. The latter method is particularly useful for measuring the conductivity of ferromagnetic metals, where the eddy-current method commonly fails due to its inability to separate the effects of conductivity and permeability on the probe impedance except at frequencies typically lower than the operating range of most probes. Permeability is the parameter that describes the way in which a ferromagnetic material responds to an applied magnetic field and is described in Sect. 2.8.

Example: Temperature dependence of conductivity The aluminum alloy whose temperature coefficient is listed in Table 2.2 has conductivity 63.6% IACS at 25 °C. Evaluate the conductivity of this alloy at 20 °C.
Solution

$$\begin{aligned}\sigma(20\ ^\circ\mathrm{C}) &= \frac{\sigma(25\ ^\circ\mathrm{C})}{[1+\alpha\times(-5)]} \\ &= \frac{63.6}{[1-(0.0043\times 5)]} \\ &= 65.0\ \%\mathrm{IACS}.\end{aligned}$$

2.5 Electric Field

The fundamental source of an electrostatic field, or stationary electric field **E**, is a stationary electric charge. **E** has unit Volt per meter (V/m). Electric charges may be either positive, as in the case of a proton which resides in the atomic nucleus for example, or negative, as in the case of an electron. Static fields are not the concern of this text since EC NDE is founded on inherently dynamic processes, i.e., fields and their sources that vary with time.

When an electric current flows in a wire, or eddy currents are induced in a metal test-piece, there is an electric field associated with the moving electrons. The current I and current density **J** both exist only in the conductor because there are no conduction electrons in the region beyond the metal test-piece or coil. By contrast, the electric field is not spatially restricted to the region of the conductor alone, but exists everywhere. For example, the electric field in air due to an infinitesimally thin, long, straight wire (mathematically, the wire radius $\to 0$) carrying phasor current $\mathcal{I}$ in the z-direction is given by

$$\mathbf{E} = \hat{z}\,\frac{\mathcal{I}}{2\pi} j\omega\mu_0 \ln\rho, \qquad \rho > 0, \tag{2.14}$$

where the wire coincides with the z-directed axis of a cylindrical coordinate system of which ρ is the radial coordinate and μ_0 is the permeability of free space (Sect. 2.8). To discover how (2.14) is obtained, see Exercise 2 at the end of this chapter. This is just one example of the electric field external to a current-carrying body. In fact, it is also an example of the working of Faraday's Law of electromagnetic induction. The field expressed in (2.14) does not exist unless the current in the wire is time-varying. As you can see in (2.14), $\mathbf{E} \to 0$ if $\omega \to 0$. This is a consequence of the fact that the current flowing in the wire is the source of a magnetic field external to the wire, and it is actually the time variation in the magnetic field that in turn induces the electric field expressed in (2.14).

Inside the conductor, things are much simpler. There is a linear relationship between **J** and **E**, known as a constitutive relation. This particular constitutive relation is the "point form" of Ohm's Law, which will be discussed in more detail in Chap. 4. In an *isotropic* conductor, the conductivity is scalar which means that the electric field and the current density have the same direction at every point in the conductor. Ohm's Law is then stated as follows, with the magnitudes of **J** and **E** being proportional to one another and the constant of proportionality being σ:

$$\mathbf{J} = \sigma \mathbf{E}. \tag{2.15}$$

In the case of anisotropic conductivity such as exists in a carbon–fiber composite material it is necessary to express Ohms' Law as follows, wherein the conductivity $\overline{\overline{\sigma}}$ is a second-rank tensor:

$$\mathbf{J} = \overline{\overline{\sigma}} \cdot \mathbf{E} \qquad \text{and} \qquad \overline{\overline{\sigma}} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix}. \tag{2.16}$$

In this text, it is generally assumed that material properties are isotropic rather than changing with the direction of the applied field, which is true for most metals routinely inspected by EC NDE.

2.6 Magnetic Field

The fundamental source of a magnetic field **H** is *charge in motion*. A current flowing in a wire always has a magnetic field associated with it, regardless of whether the current is time-varying or flowing steadily. **H** has unit Ampère per meter (A/m). Continuing the discussion surrounding (2.14), the magnetic field in air due to an infinitesimally thin, long, straight wire carrying current $\mathcal{I}$ in the z-direction is given by

$$\mathbf{H} = \hat{\phi} \frac{\mathcal{I}}{2\pi\rho}, \qquad \rho > 0, \tag{2.17}$$

where ϕ is the azimuthal coordinate of the cylindrical system. (This is a demonstration of the right-hand rule of Ampère: If the right hand is wrapped around a conductor such that the thumb indicates the direction of current flow, here $\hat{z}$, then the direction in which the curling fingers point is the direction of the associated magnetic field, here $\hat{\phi}$.) To discover how expression (2.17) is obtained, see Exercise 2 at the end of this chapter. Unlike in the case of the electric field associated with this same wire, given in (2.14), note that there is no explicit frequency dependence (ω) in (2.17). In fact, a similar expression holds for direct current $I = I_0$ flowing in the same wire:

$$\mathbf{H} = \hat{\phi} \frac{I_0}{2\pi\rho}, \qquad \rho > 0 \tag{2.18}$$

whereas, as explained above, **E** expressed in (2.14) tends to zero when direct current flows in the wire. This is not to say, however, that there is no electric field external to a long straight wire carrying direct current I_0. In fact, the component of the electric field tangential to the wire surface must be continuous (see Sect. 5.6), which means that external to a DC current-carrying wire with axis parallel to the $\hat{z}$-direction, $\mathbf{E} = \hat{z}E_0$ is also $\hat{z}$-directed and is *constant* everywhere in space. Unlike the form expressed in (2.14), this constant term has no connection with the magnetic field.

The source of permanent magnetization in a ferromagnetic material lies in the orbital- and spin angular momentum of atomic electrons as described by quantum mechanics. So one can say, loosely, that the magnetic field due to a permanent magnet also originates with charge in motion. Ferromagnetism cannot, however, be fully explained in terms of classical physics. This type of magnetism is discussed in Sect. 2.8 and also in Chap. 3.

The existence of electric and magnetic fields external to a conductor is at the very heart of EC NDE. These are the fields that account for "action-at-a-distance" phenomena such as the attraction between two electric charges of opposite sign or the attraction between opposite poles of two permanent magnets. The fields store electromagnetic energy which is converted to kinetic energy (energy of motion) when such oppositely charged objects move toward each other. In the context of EC NDE, an EC drive coil is the source of an external electromagnetic field, which then couples into the metal test-piece and induces eddy currents there, whose current density can be determined from the electric field by means of (2.15).

2.7 Poynting Vector

It will be useful later on, Chap. 9, to have introduced the Poynting vector. The instantaneous power density, or Poynting's vector $\mathcal{P}$, has unit Watt-per-meter-squared (Wm^{-2}) and is given by

$$\mathcal{P} = \mathbf{E} \times \mathbf{H}. \tag{2.19}$$

When the fields vary sinusoidally, the average power per unit area $\overline{\mathcal{P}}$ can be computed as

$$\overline{\mathcal{P}} = \frac{1}{2}\mathrm{Re}\left(\mathbf{E} \times \mathbf{H}^*\right) \tag{2.20}$$

where **E** and **H** are now phasor representations of the time-harmonic fields in terms of their amplitudes and the superscript "$*$" indicates the complex conjugate. The average power $\overline{P}$ through a surface S is then

$$\overline{P} = \int_S \overline{\mathcal{P}} \cdot d\mathbf{S} = \frac{1}{2}\mathrm{Re}\int_S \left(\mathbf{E} \times \mathbf{H}^*\right) \cdot d\mathbf{S} \tag{2.21}$$

where the direction of the elemental surface $d\mathbf{S}$ gives the direction of power flow.

2.8 Permeability and Magnetic Induction

Ferromagnetic materials such as iron or iron-based alloys respond to the application of a magnetic field, such as that produced by alternating electric current flowing in an eddy-current coil, by becoming strongly magnetized. Such materials, especially steels, are commonly used in various structures and are routinely inspected using NDE methods. The ferromagnetism exhibited by these materials means that they interact with eddy currents in a way that is somewhat different from the interaction with non-ferromagnetic materials such as copper or aluminum. For this reason, ferromagnetic materials deserve special attention and will be dealt with in more detail in Chap. 3. Here, the concept of permeability is briefly introduced since it is needed in the discussion of the electromagnetic skin effect which follows.

When an applied magnetic field **H** interacts with a ferromagnetic material, **H** is augmented by the magnetization of the material, **M**, which also has units A/m in SI. In fact, **M** is related to **H** via the susceptibility χ, a material-dependent dimensionless parameter which embodies the strength of magnetization of the ferromagnet in response to the applied magnetic field,

$$\mathbf{M} = \chi \mathbf{H}. \tag{2.22}$$

Hence, the total magnetic field in the presence of a ferromagnet can be written

$$\mathbf{H} + \chi \mathbf{H} = (1 + \chi)\mathbf{H} = \mu_r \mathbf{H}$$

where $\mu_r = 1 + \chi$ is a material-dependent dimensionless parameter known as the relative permeability. Typically, χ is a function of **H** which means that μ_r is not constant for any particular ferromagnet but changes as the value of **H** changes. The relationship between **M** and **H** is inherently *nonlinear*.

When working with ferromagnetic materials, it is convenient to work in terms of the field of magnetic induction, **B**, whose unit is the Tesla (T), since **B** represents the combined effect of **H** and **M**. In the absence of magnetic material, there is a simple linear relationship between **B** and **H**;

$$\mathbf{B} = \mu_0 \mathbf{H}, \tag{2.23}$$

in which the parameter μ_0 is the permeability of free space with unit Henry per meter (H/m) and value $\mu_0 = 4\pi \times 10^{-7}$ H/m. In the presence of a ferromagnet,

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}). \tag{2.24}$$

By manipulating the above definitions, the following relationship can be obtained:

$$\mathbf{B} = \mu \mathbf{H} \tag{2.25}$$

where μ is the permeability of the material, unit H/m, and $\mu = \mu_0\mu_r$. For the reasons described in the discussion following (2.22), the relationship between **B** and **H** is also inherently nonlinear although, if the magnitude of the applied **H** field is not very large, **B** is approximately proportional to **H**. Relation (2.25) holds in the case that the material permeability is isotropic, as discussed in relation to conductivity, (2.15). In the case of a material that exhibits anisotropic permeability, a tensor form for permeability is required.

2.9 Electromagnetic Skin Effect

As mentioned in the discussion around (2.1), the current density inside a wire carrying DC current is spatially uniform. As the frequency increases, however, a phenomenon known as the electromagnetic skin effect comes into play. This phenomenon has the effect of confining the current to a thin skin near the surface of the conductor. The effect is observed in every conductor carrying AC current, whether it be the wires in the windings of the eddy-current coil, or the metal test-piece in which the eddy currents are induced. As the frequency of the alternating current flowing in the probe coil increases, the eddy-current density induced in the test-piece is confined to an increasingly thin layer (or "skin") near its surface. In other words, the depth of penetration of the eddy currents into the test-piece can be controlled by adjusting the frequency of the inspection. The fact that the depth of penetration can be varied in this way provides a tool for optimizing an electromagnetic inspection to a particular depth in the test-piece. For surface-breaking defects, it is best to work at higher frequencies for which the induced eddy currents are concentrated near the surface of the specimen. Inspection sensitivity is increased by concentrating the eddy currents in the vicinity of the flaw. For deep-lying flaws, lower frequencies are needed so that the eddy currents penetrate sufficiently far into the specimen to interact with the flaw.

An approximate guide to the depth of penetration of electric current flowing in a conductor is known as the electromagnetic skin depth, δ (m), given by the following formula:

$$\delta = \sqrt{\frac{2}{\omega\sigma\mu}} \tag{2.26}$$

which can also be expressed

$$\delta = \frac{1}{\sqrt{\pi f \sigma\mu}}. \tag{2.27}$$

In the definition of (2.26) and (2.27), δ is inversely proportional to the square root of the frequency of the alternating current exciting the eddy-current coil f, the electrical conductivity of the test-piece σ, and its magnetic permeability μ. This definition emerges from the analysis of a two-dimensional system in which eddy currents are excited in a half-space conductor (an infinitely deep conductor with a flat surface)

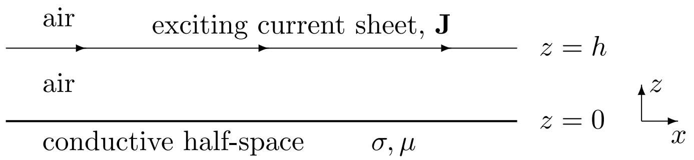

Fig. 2.5 Conductive half-space excited by a current sheet $\mathbf{J} = J(z)\hat{x}$

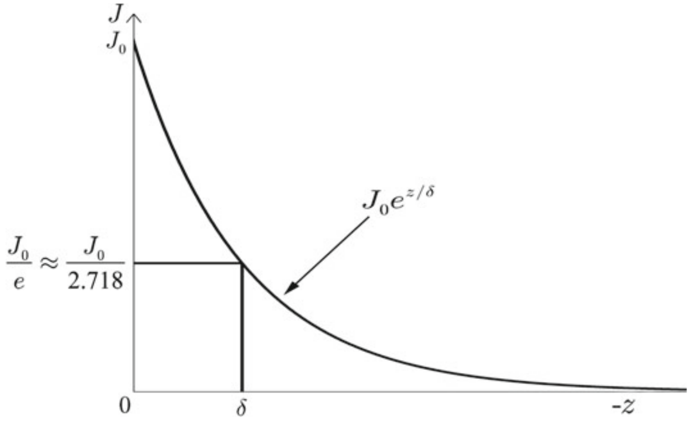

Fig. 2.6 Exponential decay of induced current density as a function of depth in a half-space conductor

by currents flowing in a thin sheet above the surface of the conductor, and parallel to it, as shown in Fig. 2.5. In this case, the magnitude of the eddy-current density in the test-piece falls off exponentially with depth from the surface:

$$J(z) = J_0 \exp(z/\delta), \qquad z < 0, \tag{2.28}$$

where $J_0 = J(0)$, as shown in Fig. 2.6. The value of electromagnetic skin depth versus frequency is plotted in Fig. 2.7 for various metals [12]. The derivation of (2.26)–(2.28) is presented in Sect. 6.3.1.

The value of δ is easy to compute for any given metal and serves as a useful guide to the depth of penetration of the electromagnetic field. In a real eddy-current measurement, however, the cylindrical geometry of a typical coil usually leads to lesser field strength, for a given depth in the test-piece, than a uniform field excitation, especially for a small probe. In Fig. 2.8 [3], eddy-current contours produced by a surface coil of the type shown in Figs. 1.8 and 1.9 are compared at three distinct frequencies. It is obvious that the depth of penetration of the eddy-current density decreases as the frequency increases.

Example: Electromagnetic skin effect What thickness of copper sheet, conductivity 58 MS/m, is needed to block 99% of incoming cellphone signal at 800 MHz?

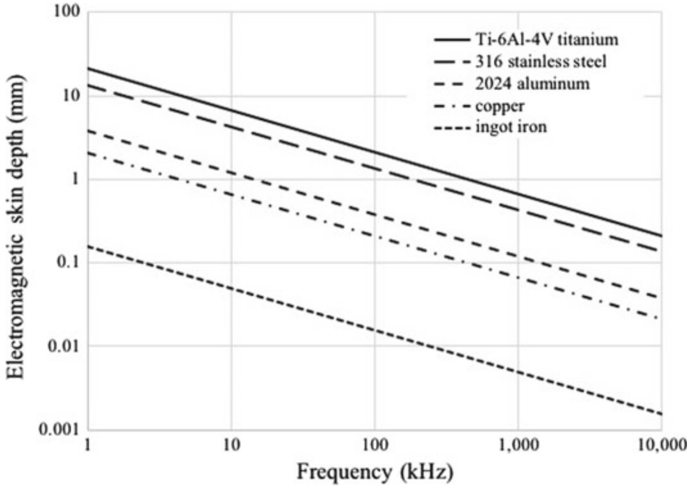

Fig. 2.7 Electromagnetic skin depth δ versus frequency for various metals at 20 °C, whose conductivities are given in Table 2.1. Relative permeability of 316 stainless steel is here taken to be $\mu_r = 1.02$ [5] and that of ingot iron to be a representative value $\mu_r = 1{,}000$, Table 3.2

Solution The electromagnetic wave carrying the cellphone signal also obeys a law of exponential decay in a metal similar to (2.28),

$$A(z) = A_0 e^{-|z/\delta|}.$$

When 99% of the signal is blocked,

$$\frac{A(z)}{A_0} = 0.01 = e^{-|z/\delta|}$$

and

$$-\frac{z}{\delta} = \ln(0.01) = -4.61$$

or $z = 4.61\delta$. Now, in copper at 800 MHz,

$$\delta = \frac{1}{\sqrt{\pi \times 800 \times 10^6 \times 58 \times 10^6 \times 4\pi \times 10^{-7}}} = 2.34\ \mu\text{m},$$

using (2.27). Finally, the depth of copper that blocks 99% of incoming cellphone signal at 800 MHz is

$$z = 4.61 \times 2.34 = 10.8\ \mu\text{m}.$$

The copper does not need to be very thick at all!

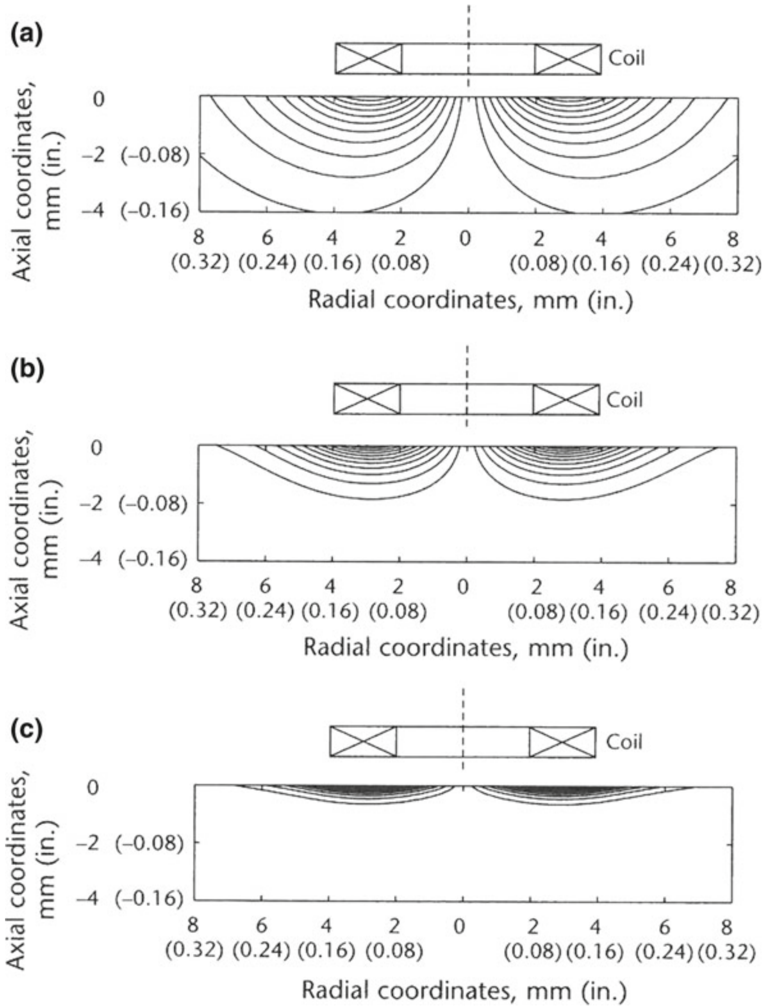

Fig. 2.8 Contours of eddy-current density $|J_\phi|$ induced by a surface coil at frequencies 1 kHz (**a**), 10 kHz (**b**) and 100 kHz (**c**). The probe parameters are $r_\mathrm{i} = 2$ mm, $r_\mathrm{o} = 4$ mm, $l = 1$ mm, $N = 800$ and $s = 1$ mm. The test-piece is a half-space ($T \to \infty$) with $\sigma = 35.4$ MS/m and $\mu_\mathrm{r} = 1$. The electromagnetic skin depth takes values $\delta = 2.7$, 0.85 and 0.27 mm in **a**, **b** and **c**, respectively. Reprinted with permission from the NDT Handbook: Electromagnetic Testing. Copyright © 2004, ASNT, Columbus, Ohio [3]

2.10 Polarization and Electric Displacement

We have now met three of the four electromagnetic fields, namely the electric field, **E**, the magnetic field, **H**, and the field of magnetic induction, **B**. The fourth is the *electric displacement*, **D**, measured in Coulombs per meter squared (C/m^2). **D** is also known as *electric flux density* because electric flux Ψ_e can be defined in terms of **D** as follows:

$$\Psi_e = \int_S \mathbf{D} \cdot d\mathbf{S} \tag{2.29}$$

(compare with (4.16) that relates magnetic flux Ψ with magnetic induction **B**). For a dielectric material, **D** is related to **E** in a way that parallels the relationship between **B** and **H** for a magnetic material.

In EC NDE, we are dealing with metals, for which conductivity $\sigma \gg 1$ and relative permittivity $\epsilon_r = 1$, rather than dielectrics (insulators) for which $\sigma \ll 1$ and $\epsilon_r > 1$. This means that we do not rely heavily on the use of **D** in EC NDE. Nonetheless, occasionally we shall need to be aware of the meaning of **D** and for this reason the relations between **D**, **E** and the polarization **P** of a dielectric are summarized briefly here.

As we already know, there are free electrons available in a conducting material to conduct electricity when a force is applied. A dielectric material has no such free charges. Instead, at an atomic level in a dielectric, the negatively charged electron cloud is balanced by the positively charged atomic nuclei. On application of an external force, the charge clouds are not free to move in a macroscopic sense, but they do exhibit some degree of displacement from one another, so that an electric dipole is created. The displaced charges give rise to local dipoles whose moment is expressed

$$\mathbf{p} = Q\mathbf{d} \tag{2.30}$$

where **d** is the distance vector from $-Q$ to $+Q$ of the dipole.

On a macroscopic scale, over a collection of many atoms, it is useful to introduce the polarization vector field of a material, **P** measured in C/m^2, where **P** is the dipole moment per unit volume of the dielectric and is a measure of the intensity of polarization in the material. The electric displacement inside a material of polarization **P** is then

$$\mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P}. \tag{2.31}$$

For some dielectrics, **P** is proportional to the applied electric field and

$$\mathbf{P} = \chi_e\epsilon_0\mathbf{E} \tag{2.32}$$

where χ_e is the (dimensionless) electric susceptibility of the material. If (2.32) is combined with (2.31), the constitutive relation

$$\mathbf{D} = \epsilon\mathbf{E} \tag{2.33}$$

is obtained, in which $\epsilon = \epsilon_0(1 + \chi_e) = \epsilon_0\epsilon_r$. Relation (2.33) implies that the material permittivity is isotropic, as discussed in relation to conductivity, (2.15). In the case of a material that exhibits anisotropic permittivity, a tensor form for permittivity is required.

2.11 Summary

In this chapter, the electromagnetic fields and the constitutive relations by which they are related to material properties have been introduced. The relative importance of these fields and material properties to the EC NDE method have been described. The conductivity of a material is of primary importance in EC NDE. The permeability of a ferromagnetic test-piece also strongly influences the signals measured in an EC inspection, as will be seen in Chap. 6. In the next chapter, a description of ferromagnetic phenomena is given that attempts to explain the origins of ferromagnetism and the mechanisms that underly it, which gives insight into the fact that the magnetic history of a test-piece can strongly affect the measured EC signals obtained from it.

2.12 Exercises

1. Obtain the phasor notation of the following time-harmonic functions, if possible.
   (i) $A(t) = 2\sin(\omega t) - 3\cos(\omega t)$,
   (ii) $C(t) = 4\cos(80\pi t - \pi/4)$,
   (iii) $D(t) = 2 - \cos(2\omega t)$,
   (iv) $I(t) = -7\sin(\omega t)$,
   (v) $U(t) = \sin(\omega t + \pi/4)\sin(\omega t + \pi/8)$,
   (vi) $V(t) = 5\cos(\omega t + \pi/3)$.
2. Obtain $C(t)$ in terms of ω from the following phasors.
   (i) $\mathcal{C} = 1 + 3j$,
   (ii) $\mathcal{C} = 3\,e^{j0.9}$,
   (iii) $\mathcal{C} = 2\,e^{j\pi/2} + 3\,e^{j0.7}$.
3. A coil is made of 150 turns of silver wire wound on a circular cylindrical core and carries current 0.1 A. If the mean radius of the turns is 6.5 mm and the diameter of the wire is 0.4 mm, (i) calculate the DC resistance of the coil (you may make the approximation that all the turns on the coil (windings) have the same radius as the mean radius) and (ii) calculate the current density in the wire.
4. Determine the total current in a wire of radius 1.6 mm placed along the z-axis if

$$\mathbf{J} = \frac{500}{\rho}\hat{z} \qquad \mathrm{Am}^{-2}.$$

5. Given that the electrical conductivity of nickel is 14.62 MS/m at 20 °C, evaluate the electrical conductivity of nickel at 25 °C.
6. According to (2.13), sketch the dependence of conductivity on temperature. Rewrite (2.13) in terms of resistivity ρ and now sketch the dependence of ρ on temperature. Explain the features of your two sketches.
7. Using the data shown in Fig. 2.7, estimate the conductivities of graphite, titanium, and copper. Give your answers in terms of (a) MS/m and (b) % IACS. What additional piece of information do you need to determine the conductivity of ingot iron from this data? Conduct a little research to find this information and use it to (c) estimate the conductivity of ingot iron from the data shown in Fig. 2.7.

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