Chapter 3
Dynamics

Contents

This chapter introduces the reader to a non-classical understanding of the concept of dynamics. While the classical concept relates to changes in the *same* entity in classical space and time, here different approaches, suitable for the novel conceptual categories previously dealt with in Chap. 2, are considered. For instance, dynamics can be related to changes in the structural properties of the entities studied where entities are considered as collective (beings) and properties are related to networks, regimes of validity, levels and intra-levels and coherences.

This approach will enable us to understand and model this form of structural dynamics. Such a new view of dynamics is of paramount importance for post-GOFS.

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3.1 A Short Introduction to the Classical Concept

The word 'dynamics' has several disciplinary meanings. However, they all have in common the property of being related to *change* (Minati, Abram, & Pessa, 2012).

A partial, introductory disciplinary list may be:

(a) In classical physics, it describes, for instance, changes in metrical, structural and topological properties of bodies over time (see, e.g. Meriam & Kraige, 2012), the behaviour of gases, fluid dynamics (Ruban & Gajjar, 2014), Brownian motion (Schilling, Partzsch, & Bottcher, 2012) and the relationships between heat and mechanical energy in thermodynamics both at the microscopic and macroscopic level (see, e.g. De Pablo & Schieber, 2014). It is also possible to consider the dynamics of states adopted, for instance, by electronic (Mladenov & Ivanov, 2014) and chemical (Kuramoto, 2003) systems.
(b) In biology, it describes motion at the molecular level as well as changes at the macroscopic level (DiStefano, 2013).
(c) In computer science and information technology, it is used to describe the dynamics of information, its flows, and its processing (Vogiatzis, Walteros, & Pardalos, 2014).
(d) In cognitive science and psychology, it refers to the dynamical changes occurring in cognitive systems and cognitive models, produced, for instance, by learning (Gros, 2013).
(e) In economics and sociology, it is used when dealing with social, economic and cultural changes (see, e.g. Skyrms, 2014).
(f) The *general theory of relativity* introduced a very different understanding of the gravitational motion responsible for dynamics (Skinner, 2014). As is well known, the *special theory of relativity* consists of a reformulation of classical mechanics where the mathematical relationship between the measurements of space and time performed by two *inertial*[1] observers is given by a Lorentz transformation rather than a Galilean transformation. The *general theory of relativity* generalises special relativity and Newton's law of universal gravitation, by introducing a representation of gravity as a geometric property of space and time. The two theories introduced a new representation and understanding of interactions and dynamics (currently being exploited by modern gauge theories).

At this point, we stress that dynamics is considered as being related to:

- Possible dynamical parameters describing properties *possessed* by *entities* such as physical, chemical, informational, cognitive and cultural over time.
- Interactions mediated, for instance, by some exchange of matter/energy and dependent on eventual environmental and space-time properties.

[1]That is two observers, each one of which is at rest with respect to a specific inertial reference frame. The latter expression denotes a reference frame in which the first principle of classical dynamics holds. In special relativity these frames cannot undergo rotations or accelerations.

In the following we will understand the dynamics as being given by various *kinds* of changes such as *changes* in (1) constraints or degrees of freedom; (2) properties; (3) *ways of interacting;* (4) *structure,*[2] due to relational and interactional changes such as parametrical ones; (5) *states*; (7) coherences; (8) phases like in *sequences* of phase transitions; and (9) attractors. We will consider also sequences of structural changes as for complex systems – examining specifically the case of the cytoskeleton – intended as *sequences* of phase transitions where the *properties of such sequences* should be understood as constituting a structural dynamics, sometimes *coherent* within the context of particular complex systems.

3.2 Dynamical Coherence in Processes of Self-Organization and Emergence

Before entering into the topics of this and the following sections, several other aspects related to the concept of *coherence* already considered above in Sect. 2.1 must be introduced. For instance, a better understanding of coherence may be related to processes of *synchronisation*. Let us consider, for example, populations of oscillators, such as clocks, organized in dynamic clusters where synchronization is the *source* of their coherence (see, for instance, Mikhailov & Calenbuhr, 2002). Things become more interesting when oscillators interact and the internal cyclic dynamics of a population of N coupled oscillators, each characterized by a time-variable phase and a natural frequency can be given, for example, by (Acebrón, Bonilla, Vicente, Ritort, & Spigler, 2005; Kuramoto, 2003):

$$\dot{\theta}_i = \omega_i + \sum_{j=1}^{N} K_{ij} \sin\left(\theta_j - \theta_i\right)$$

where:

- $i = 1, \ldots, N$.
- $\dot{\theta}_i$ is the time derivative of the phase of the i-th oscillator.
- ω_i is the natural frequency of the i-th oscillator.
- K_{ij} denotes a coupling matrix.

Here the natural frequencies of the different oscillators are randomly distributed with a given probability density $g(\omega)$.

This model is known as *Kuramoto model*. It has been the subject of intensive studies, as its different implementations display a large variety of synchronization patterns. Here we will limit ourselves to mention the simplest case in which $K_{ij} = K/$

[2]While *organization* deals with networks of relationships with undefined parameters, *structure deals with* networks of relationships having well-defined parameters. Relationships may consist of rules of interaction, see Sects. 2.3 and 3.2.2.

$N > 0$, where K is a suitable constant. Usually one refers to this case as *mean-field coupling*. It is possible to show that, when $K \rightarrow 0$, the synchronization disappears and each oscillator rotates with an angular frequency given by its own natural frequency. Instead, when $K \rightarrow \infty$, all oscillators become synchronized to their average phase (global synchronisation). Finally, if $K_C < K < \infty$, where K_C denotes a suitable *critical value* of K, we have the appearance of a *partial synchronization* state, in which a part of oscillators have the same (constant) phase, while other oscillators rotate out of synchrony. The value of K_C depends on the form of the function $g(\omega)$, and we will avoid any discussion about the details of its computation. In any case, the main contribution of Kuramoto and other similar models of interacting oscillators consists in the evidence of a number of different kinds of synchronisation, a circumstance which opens the way to the search for models describing the occurrence of multiple synchronisations within the same system (see, for examples of application of this model to Neuroscience Breakspear, Heitmann, & Daffertshofer, 2010; Schmidt, LaFleur, de Reus, van den Berg, & van den Heuvel, 2015).

As a matter of fact, such phenomena have been observed in a number of models, together with the occurrence, in some cases, of different synchronisations over time *when such multiplicity is in its turn synchronized*, possible in more complicated contexts such as the human nervous system. When such *upper synchronization* of multiple local instantaneous synchronisations is maintained, it can be considered as a form of *coherence* (see, for instance, Boccaletti, 2008). This applies also to the case of populations of chaotic systems (see, for instance, Ciszak, Euzzor, Geltrude, Arecchi, & Meucci, 2013; Boccaletti, Kurths, Osipov, Valladares, & Zhouc, 2002; Manrubia & Mikhailov, 2004).

A first popular example of these phenomena is given by the ensembles of globally coupled chaotic maps, first introduced by Kaneko (see, e.g. Kaneko, 1990; see also Mikhailov & Calenbuhr, 2002, p. 155). In the simplest case, their dynamics is described by laws of the form:

$$x_i(n+1) = (1-\varepsilon)f(x_i(n)) + \frac{\varepsilon}{N}\sum_{j=1}^{N} f\left(x_j(n)\right)$$

where:

- N is the number of chaotic maps.
- $i = 1, \ldots, N$ is a space index.
- $x_i(n)$ denotes the value of the i-th map in correspondence to the discrete time $n = 0, 1, \ldots.$
- The function $f(x)$ is given by $f(x) = a\,x\,(1-x)$ (logistic map).
- a denotes the nonlinearity parameter of the logistic map.
- ε denotes the coupling parameter.

The numerical simulations of the dynamics of such a system evidence that, when the coupling parameter overcomes a critical value ε_c (for instance, $\varepsilon_c \approx 0.355$ when $a = 3.8$), a state of *full synchronization* occurs, in which all maps, at any instant, have the same value, so that the whole system behaves like a single chaotic map. When $\varepsilon < \varepsilon_c$ the full synchronization disappears, and we observe the occurrence of a number of different clusters, each one containing a number of mutually synchronized units (for a detailed study of this phenomenon, see, e.g. Popovych, Maistrenko, & Mosekilde, 2001). If, now, we consider a system of globally couple maps in which the coupling parameter ε is allowed to grow, starting from a very small value, far lesser than ε_c, up to the situation of full synchronisation, we obtain a dynamics characterized by an ordered sequence of different synchronisations, ending in a situation of global coherence, similar to the one described above and quoted, for instance, in Boccaletti, 2008.

As expected, a far more complex phenomenology occurs when we consider more complicated systems, such, for instance, the ones in which the couplings are local instead than global. A typical case is the one of chains of coupled limit-cycle oscillators (see, e.g. Osipov & Kurths, 2001), generalizing the Kuramoto model previously quoted and described by equations having a generic form of the kind:

$$\dot{\varphi}_n = \omega_n + F(\varphi_n) + d\big(\sin\left(\varphi_{n+1} - \varphi_n\right) + \sin\left(\varphi_{n-1} - \varphi_n\right)\big)$$

where φ_n denotes the phase of the n-th oscillator, ω_n its natural frequency, d a suitable parameter and $F(\varphi_n)$a non-linear function responsible for the non-uniformity of rotations of the oscillator taken into consideration.

In these systems, besides the occurrence of clusters of synchronized elements, it is possible to observe the occurrence of *defects* which are present in the zones separating different and adjacent clusters. In many models these defects follow a specific kind of dynamics, which can imply even their appearance and disappearance. More complex patterns of synchronization phenomena can appear in spatially extended systems of non-linear oscillators (see, among the others, Hong, Park, & Choi, 2005).

The detection of the different forms of synchronization phenomena is more generally based on the use of various kinds of *correlation measures* such as those resorting to linear approaches like the ones underlying Bravais-Pearson coefficient. As well known, in statistics correlation refers to classes of statistical relationships involving dependence among random variables (Drouetm & Kotz, 2001). There is a large number of different correlation measures, most of which is introduced within the context of the study of brain signals. They can be subdivided into two classes: the *linear* and the *nonlinear* measures (see, for a review, Kreuz, 2011). Among the linear measures, which generalize the traditional Bravais-Pearson quoted before, the most popular is given by the *cross-correlation* function, applied to two time series having the same length N, whose values are denoted, respectively, by x_n and y_n (these values have been previously normalized so as to have a zero mean and a unitary variance). This function depends on the time lag τ, running within the interval from $-(N-1)$ to $N-1$, according to the following rule:

$$C_{XY}(\tau) = \begin{cases} \frac{1}{N-\tau}\sum_{n=1}^{N-\tau} x_{n+\tau}y_n & \textit{if} \quad \tau \geq 0 \\ C_{XY}(-\tau) & \textit{if} \quad \tau < 0 \end{cases}$$

The cross-correlation values can run from 1 (maximum synchronization) to −1 (loss of correlation). When the focus is on the frequency, rather than on time, the cross-correlation can be replaced by the so-called *cross spectrum*, defined by:

$$C_{XY}(\omega) = E\left[F_X(\omega)\, F_Y^*(\omega)\right]$$

where ω denotes the frequency, E the estimation function, F_X the Fourier transform of x and the star the complex conjugation. From the cross spectrum, it is possible to compute the *coherence* function through the relationship:

$$\Gamma_{XY}(\omega) = \frac{|C_{XY}(\omega)|^2}{|C_{XX}(\omega)|\,|C_{YY}(\omega)|}$$

As regards the nonlinear correlation measures, the domain is far more complicated than in the linear case, and many different choices are available. Without entering into further details (within the wide literature on this subject, we can quote only few references, such as Kantz & Schreiber, 1997; Pereda, Quiroga, & Bhattacharya, 2005; Dauwels, Vialatte, Musha, & Cichocki, 2010), we limit ourselves to mention only the names of the main kinds of measures, including mutual information, transfer entropy, Granger causality, nonlinear interdependence and phase synchronization.

As it can be seen both from the quoted literature and the previous considerations, synchronization (Pikovsky, Rosenblum, & Kurths, 2001), for example, between pairs of data, signals or waves, is the most often used among the possible measures of their *similarity* as a function of a suitable time-lag. While neglecting a further discussion about the possible measures of synchronization, we mention only a very simple and easily computable synchronization index, also called *coherence parameter*, used when dealing with the dynamical evolution of networks of interacting units. In order to introduce it (see, for instance, Van Wreeswijk & Hansel, 2001), we can supposedly deal with a network of N interconnected units (like neurons), each one of which is described by its momentary state of activation $V_i(t), (i = 1, \ldots, N)$. This knowledge allows to compute the momentary average network activation through:

$$A_N(t) = \frac{1}{N}\sum_i V_i(t)$$

The fluctuations of the latter have a variance given by:

$$\Delta_N = \left\langle A_N(t)^2 \right\rangle_t - \langle A_N(t) \rangle_t^2$$

As customary, the symbol $\langle \ldots \rangle_t$ denotes an averaging with respect to t. An analogous variance can be computed with respect to $V_i(t)$ through the formula:

$$\Delta = \frac{1}{N} \sum\nolimits_i \left(\left\langle V_i(t)^2 \right\rangle_t - \langle V_i(t) \rangle_t^2 \right)$$

Then, the coherence parameter is given by:

$$\Sigma_N = \frac{\Delta_N}{\Delta}$$

Higher values of Σ_N (close to 1) denote high synchronization between the network units, while very low values are associated to a diffuse asynchrony.

Another kind of correlation function has been introduced when studying the fluctuations in velocity within flocks of birds (Cavagna et al., 2010). Namely, in that case, one must take into account two different kinds of variables: the direction of the individual motion and the modulus of its velocity. In other applications of statistics (for instance, to data coming from psychology or sociology), the *coefficient of multiple correlation* can be used as a measure of how values adopted by a specific variable are given by a linear function of a set of one or more other variables (Huber & Ronchetti, 2009), provided, however, we exclude nonlinearity from our hypothesis, a circumstance still common in those domains.

Another example of a source of coherence is the occurrence of ergodicity in collective behaviours (see, for instance, Minati & Pessa, 2006, pp. 291–313) where the *same* system can be *both* ergodic and non-ergodic depending upon the time scale of the observer, as in polymers, or even temporarily ergodic. Moreover, it is possible to introduce degrees or indices of ergodicity. See, in this regard, the Sect. 4.5.1.

After these considerations on the concepts of synchronization and correlation, we now remark that the Post-GOFS approach requires the introduction of new possible variations of the concepts of *classical dynamics*. These latter could be applied, for instance, to networks or meta-structures in order to describe the nine structural changes mentioned in Sect. 3.1. We anticipate here a concept – the one of meta-structure – which is introduced in a more detailed way in the Sect. 3.8. *In short, a meta-structure is intended here as a dynamical set of simultaneous, superimposed and possibly* ***interfering***[3] *structures of interactions between elements, acting as rules (examples are shown in the* Table 3.1 *in the Sect. 3.8.2). Such different structures may of course be characterized by different starting times or durations.*

[3] *As we will see two or more interactions are considered here to interfere when one is function of the other ones* in conceptual correspondence with the original formalization of system introduce by Bertalanffy as reminded at the Sect. 2.3. See Sect. 3.8.2.

Table 3.1 Example of populations of interactions for flock-like collective behaviours

Multiple structural interactions within a flock-like collective behaviour			
Agents	*Interacts by varying their*	*Depending on the*	*Rules of interaction* $Rint_{J:1\text{-}13}$
e_k	Speed	Speed of the closest agent or the average speed of the closest agents	$Rint_1$
e_k	Speed	Speed of agent(s) having its same direction	$Rint_2$
e_k	Speed	Speed of agent(s) having its same altitude	$Rint_3$
e_k	Speed	Speed of agent(s) having symmetrical, topological position	$Rint_4$
e_k	Direction	Direction of the closest agent or the average direction of the closest	$Rint_5$
e_k	Direction	Direction of agent(s) having its same speed	$Rint_6$
e_k	Direction	Direction of agent(s) having its same altitude	$Rint_7$
e_k	Direction	Direction of agent(s) having symmetrical topological position	$Rint_8$
e_k	Altitude by varying direction	Altitude of the closest agent or the average altitude of closest agent(s)	$Rint_9$
e_k	Altitude by varying direction	Altitude of agent(s) having its same direction	$Rint_{10}$
e_k	Altitude by varying direction	Number of agents having its same altitude	$Rint_{11}$
e_k	Altitude by varying direction	Altitude of the agent(s) having symmetrical topological position	$Rint_{12}$
e_k	Speed	Speed of the closest agent or the average speed of the closest agent	$Rint_{13}$

Among the new conceptual generalizations of classical dynamics, here we will limit ourselves to mention the ones related to the four different ways of understanding the *dynamical constraints* listed below. This list, for instance, is contained in two papers of Hooker (Hooker, 2011, pp. 3–90; Hooker, 2013) and includes the following conceptions of constraints, together with their possible generalizations:

1. Constraints intended as *variable* rather than *fixed* degrees of freedom and which can also vary with respect to single or multiple *coherences*, such as sequential or parallel ones (Raynor, 1977). Moreover, we should also take into account the cases in which system's dynamics *generates* new constraints during its behaviour like, for instance, '...a river altering its own banks, an accumulative process where the current constraints (banks) are a function of the history of past flows (currents), intra-cellular biochemical reaction processes where molecular structures constraining some processes are the products of other processes and vice versa; ...' (Hooker, 2011, p. 217). The mathematical description of such situations has mostly been obtained by resorting to a particular section of the theory of differential equations, dealing with *moving* or *free boundary problems* (among the textbooks on this subject we can quote Crank, 1984; Alexiades & Solomon, 1993;

Figueiredo, Rodrigues, & Santos, 2007). The typical moving boundary problems arise from the attempts to describe phase change phenomena. The most celebrated example is given by the so-called *Stefan problem* (see, e.g. Meirmanov, 1992). In its simplest formulation, the problem takes into consideration a semi-infinite one-dimensional block of a substance in a solid phase (for instance, ice) whose global boundaries go from 0 to $+\infty$. The initial temperature of the substance is the critical one corresponding to the melting of the solid phase (0 in our example). The introduction of a heat flux at the left boundary of the block produces a melting leaving the left part of the block occupied by the liquid phase (in our example water). Let us now denote by $u(x,t)$ the value of the temperature in correspondence to the position x at time t and by $s(t)$ the position of the point of separation between liquid and solid phase (i.e. between water and ice). Moreover, let us denote by $f(t)$ the function describing the time dependence of the heat influx. It is immediate to see that within the liquid region defined by $0 \leq x < s(t)$, the system must obey the heat equation which, in terms of suitable rescaled variables, can be written as:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

Of course, in order to grant for the solvability of this equation, we need to add an initial condition for $u(x,t)$, that is:

$$u(x,0) = 0$$

Besides, the presence of a heat influx requires the introduction of a boundary condition holding at the left extremity of our system and given by:

$$-\frac{\partial u}{\partial t}(0,t) = f(t)$$

As regards the solid region, lying within the spatial interval $s(t) < x$ < $+\infty$, in this simple version of the model, we can only assert that within it the temperature is kept constant, that is:

$$u(x,t) = 0$$

Unfortunately, it is easy to understand that the previous equation and the enclosed conditions are not enough for finding a solution to the problem of finding the form of $u(x,t)$. Namely, they are unable to help us to find the form of the function $s(t)$, specifying the dynamics of the *moving boundary* between the two phases. In this regard, Stefan added a further equation (expressing a principle of energy conservation) ruling the behaviour of $s(t)$ and given by:

$$\frac{ds}{dt} = -\frac{\partial u}{\partial x}(s(t), t)$$

The presence of this new equation, complemented by the conditions:

$$s(0) = 0, \quad u(s(t), t) = 0$$

allowed Stefan to solve the problem of finding the functions $u(x, t)$ and $s(t)$. Such a circumstance justifies the name of *Stefan problem* attributed to the problem itself.

The model introduced within the context of Stefan problem, despite its linear nature and its apparent simplicity, stimulated an extended search for more general and complex models of phase change. Among these models the most popular one is described by the Cahn-Hilliard equation (see, for review papers, Novick-Cohen, 2008; Lee et al., 2014). Originally the latter has been introduced to describe a process of phase separation occurring within a binary fluid, when the two components separate and give rise to two spatial domains, each containing a single pure component. This description is based on a function $c(x, t)$ specifying how the fluid composition depends on spatial position and time. Usually the values of this function are restricted within the closed interval from -1 to 1, each extremely corresponding to the presence of only a specific pure component. Thus, the function itself can be interpreted also as a measure of concentration. The basic form of Cahn-Hilliard equation is:

$$\frac{\partial c}{\partial t} = D\,\nabla^2\left(c^3 - c - \gamma\,\nabla^2 c\right)$$

Here D is diffusion coefficient, while γ is a parameter related to the width of the transition layer between the two regions containing the single pure phases. Namely, an equilibrium solution of this equation is given by $c(x) = \tanh\left(\frac{x}{\sqrt{2\gamma}}\right)$, a function of a sigmoidal form describing the transition from a left region in which $c = -1$ to another region on the right in which $c = 1$. Moreover, the symbol ∇^2 denotes the n-dimensional Laplace operator, that is:

$$\nabla^2 = \frac{\partial^2}{\partial x_1^2} + \ldots + \frac{\partial^2}{\partial x_n^2}$$

There are many relationships between the different models of phase change, often dealt with as moving boundary problems. Among these relationships we will limit ourselves to mention the one evidenced by Pego some years ago (see Pego, 1989). He showed that the asymptotic behaviour of solutions of Cahn-Hilliard equation (which is a non-linear equation) can be described by the solution of a (non-linear) Stefan problem.

Without entering into further details on this subject, we are content with remarking that many moving boundary problems can be transformed into *free boundary problems* (in which even the overall boundary of the problem is not fixed and changes with time). This circumstance occurs, for instance, when the changed phase is immediately removed from the system. Problems of this kind are usually called *ablation problems* (see, e.g. Akbari & Hsieh, 1994; Betterton, 2001).

2. Constraints characterized by a *holonomic* nature. The latter is endowed with a considerable importance mostly if we deal with systems hopefully described by suitable conservation principles and a Lagrangian or Hamiltonian dynamics. In this regard we shortly remind that for a classical system described by N position variables $x_1, x_2, \ldots, x_N$and a time variable t, an holonomic constraint must be expressed under the form:

$$f(x_1, x_2, \ldots, x_N, t) = 0$$

where f is a suitable function (see, e.g. the classical textbook Goldstein, Safko, & Poole, 2014, Chap. 1). In short, constraints are holonomic when they can be expressed in a purely geometrical way, independently from the behaviour of the system. 'While smooth (frictionless) sliding under gravity on a sloping plane is a case of holonomic constraint, a spherical bead rolling smoothly on the outside of a cylinder is not because the constraint alters its basic character when the bead falls off' (Hooker, 2011, p. 216). Unfortunately, most constraints used in dynamical system theory are *nonholonomic* (or, as some people uses to say, *anholonomic*). In the more general case, the existence of constraints of the latter type entails that the final state of the dynamical evolution of a given system with nonholonomic constraints depends on the intermediate values of its trajectory along the phase space. This circumstance, in turn, prevents from the existence of a conservative potential function. The impossibility of resorting to traditional methods of mathematical physics when dealing with systems of this kind stimulated a large number of researches trying to obviate to this inconvenient, at least in special cases (see for reviews Koon & Marsden, 1997; Bloch, Baillieul, Crouch, & Marsden, 2003; Flannery, 2005). However, despite the remarkable obtained results, the presence of nonholonomic constraints often induces to abandon the traditional methods of system dynamics for shifting towards new approaches.
3. Constraints of *different natures* may simultaneously act upon the system with *additive* (assumption of linearity) or non-linear effects. In their turn, such constraints may be dependent or independent of one another. Examples are given by mechanical or chemical constraints. However, in more recent times, the need for a theory of these multiple constraints arose within the domain of multi-objective optimization problems (see, for instance, Barichard, Ehrgott, Gandibleux, & T'Kindt, 2009). A typical application is given by mobile ad hoc networks, which are autonomous systems of mobile nodes connected by wireless links but devoid of any static infrastructure (Kumar Sarkar, Basavaraju,

& Puttamadappa, 2013; Loo, Mauri, & Ortiz, 2012). They can be used in many different contexts, such as military applications, emergency search and rescue operation, and require an autonomous self-programming system able to cope with the dynamical change of network topology. Instead of resorting to traditional optimization techniques, researchers directed their attention to methods based on multiple genetic algorithms, which allowed to achieve encouraging results (see, for an example, Sun et al., 2008).

4. Constraints of *passive* or *active* nature. Both these attributes are borrowed from a number of different disciplines, in which they assume different meanings. The latter, however, can be easily applied to characterize the constraints if we refer to the distinction between the system under consideration and the external environment. In order to simplify our considerations, we will assume that both can be distinguishable one from another even if, in most realistic situations, this is not always the case. Our conceptual distinctions are inspired by a clear analysis of the relationships between a biological cell and its extracellular environment, described in Ricca, Venugopalan, and Fletcher (2013). Let us, now, assume, as a reference system, the system itself under consideration and, as environment, just its environment. We are thus assessing every system-environment process by using, as a vantage point, the considered system. Then we use the attribute *active* for the actions produced by the system which are able to give rise to deep modifications of the environment, while we use the attribute *passive* for the system actions which give rise only to environment modifications compatible with the intrinsic properties of this latter. To make an example taken from biology, if the system consists of a cell and the environment of the surrounding substrate, an active action produced by the cell could, for instance, be the one changing the activation state of the chemical regulators of actin assembly present in the substrate, thus changing its nature and operation. On the contrary, a passive action produced by the system could be the one exerted by a mechanical pressure of the cell on its surround, resulting only in a viscoelastic deformation of the latter, ruled by the same laws of viscoelasticity which are used for inanimate bodies. If, now, we change our vantage point, going from the system to its environment, it is easy to understand that the same attributes can be used to characterize the actions of the environment itself. Thus, an action exerted by the environment on the system can be considered as *active* if it produces a deep change of the nature itself of the system, while is *passive* if the action produces only modifications of the system compatible with its intrinsic nature. Thus, for instance, a surround injecting a chemical substance inside the cell produces an active action, while a mechanical pressure exerted by the surround able to produce only a shift of the cell is a passive action. At this point we can apply the previous considerations to our main concern, that is, the role of constraints. Namely, we can see the constraints as special cases of the environment. Therefore we can qualify a constraint as *passive* if its occurrence does not change the intrinsic nature of the system, while it is *active* (a better attribute would be *reactive*) if its occurrence change the nature itself of the laws ruling the system. In most cases studied in system science, people takes into consideration only

passive constraints. They are related, for instance, to limiting resources such as geometrical spaces for the movement of bodies. This is the conventional *dis*-abling understanding of the term. However, constraints and their *dis*abling effects may also *en*able the system to adopt new states and properties which are not available to the *unconstrained* system. At a first sight, this seems also the case in which the system *changes* its structure. Examples of structureless systems acquiring properties when structured include metal atoms in the vapour phase acquiring electrical conductivity when structured into their solid state lattice. In biology, a skeleton, although limiting the movements of limbs, also provides a frame for muscular attachments allowing articulated motions unavailable to the *unconstrained* system. However, a deeper analysis shows that all these cases occurred owing to the presence, even if difficult to detect, of active constraints. As it easy to understand, the study of models including active constraints (typically called *reactive media*) is very difficult. Many years ago it was appanage of a very small number of mathematicians. However the technological development following the introduction of quantum electronics and of the associated devices (mainly lasers) allowed the domain of reactive media to gain popularity, as witnessed by the appearance of books like Aris, Aronson, and Swinney (1991). As a matter of fact, this domain found practical applications in a number of interesting fields, such as the study of combustion (see, e.g. Yarin & Hetsroni, 2004), the understanding of phase transitions and the transport phenomena in geological media (Dentz, Le Borgne, Englert, & Bijeljic, 2011a). In more recent times, the study of reactive media became a part of a more general domain of study, of foremost importance for biologists, named *soft active matter* (among the main contributions, we can quote Marchett et al., 2013; Hemingway et al., 2015). It is, however, to be remarked that the research activities related to reactive media still require a very high mathematical competence. As a somewhat shocking example, we limit ourselves to show the explicit form of the reactive transport equation describing the space-time evolution of the concentration of a mobile solute liquid in presence of solidification, chemical reaction, diffusion and porosity (a case of interest in geology). The equation in question has the form: (see Dentz, Gouze, & Carrera, 2011b)

$$\varphi_m \frac{\partial c_m(x,t)}{\partial t} + \frac{\partial}{\partial t} \int_0^t dt' \, \varphi_r(t-t')\, c_m(x,t') \\ + \nabla \left[q(x)\, c_m\big(x,t\big) - D_m \nabla c_m\big(x,t\big) \right] = - \int_0^t dt' \, k(t-t') \left[c_m(x,t') - c^{eq} \right]$$

Here the symbol $c_m(x,t)$ denotes the solute concentration, $q(x)$ is the liquid flow, φ_m the porosity of the medium, and D_m, c^{eq} are suitable constants. What creates serious mathematical problems are the two functions $\varphi_r(t-t')$ and $k(t-t')$. Namely, they describe *memory effects*, due to the fact that the local value of solute concentration depends on the local value of the solid concentration, in turn depending on the past history of the system. In other words, they act as nonholonomic constraints in the sense specified before. This obviously entails that we cannot resort to the usual methods of mathematical physics in order to study the previous equation. Moreover, this situation is general when we deal with reactive media and requires new advances of systems science allowing to cope with the problems raised by these interesting systems.

Another phenomenon which can be included in this category is that of *allostasis* (Levy, Levy, Barto, & Meyer, 2013; Nibuya, Tanaka, Satoh, & Nomura, 2012), that is, the process through which a biological organism achieves stability through changes following deviation of the regulatory system from its normal homeostatic level. Allostasis is a mechanism which maintains stability through continuous, adaptive, constraint changes. In a number of cases, the allostasis is related to changes which could be dangerous for the organism (as occurs for substance dependence). In other cases, however, as the ones related to *psychological resilience*, the allostasis could give rise to positive outcomes (see, e.g. Ong, Bergeman, Bisconti, & Wallace, 2006; Reich, Zautra, & Hall, 2010, Sarkar & Fletcher, 2014).

New concepts and assumptions about dynamics will be considered here to study and model *collective phenomena* such as the establishment, sustaining and varying of generic *collective systems*, i.e. established by multiple interacting entities, and their properties such as collective motion. The latter subject, as it is well known, has been widely reported in the literature (see the review by Tamás & Zafeiris, 2012 with reference to collective motion).

The ***nature*** *of collective phenomena can vary and may be metrical, topological, networked, temporal, acoustic, relating to information or signals, economic or biological. The significance of the adjective* ***collective*** *relates, for instance, to the* ***nature*** *of a) relations and networks, b) interactions among* ***entities*** *establishing the phenomena, c) correlations among multiple systems and partial properties or d) the dependence of the acquired properties upon preserving the collective behaviour.*

3.2.1 Entities, Relationships and Interactions

Entities, relationships and interactions belong to the fundamental concepts used when dealing with dynamics. As regards *entities* they may be of different nature: words, physical bodies, agents, signals, processes, systems, networks establishing dynamics as on the Internet and *anything* considered in relation to and/or in interaction with, even with themselves at different times or on different scales. Similar considerations can be applied to relationships and interactions themselves, networks, nodes and agents. Generally entities, relationships and interactions are detected through the usage of suitable levels of representation and by cognitive systems applying different kinds of cognitive models as occurring in *constructivist approaches*. As is well known, *constructivism* (see Sects. 5.1.4 and 5.1.5) was introduced by authors such as H. von Foerster, E. von Glasersfeld, H. Maturana, F. Varela and P. Watzlawick (Butts & Brown, 1989; Von Foerster, 1979; Von

Glasersfeld, 1996; Maturana & Varela, 1980, 1992; Varela, Thompson, & Rosch, 1991; Watzlawick, 1983). This understanding and approach can be briefly represented by using the strategy of thinking based on *how it is more convenient to think that something is* rather than *trying to find out how something really is*. However, both questions should be considered, even if to differing extents, adopting empirical and not ideological viewpoints. In some contexts, like within DYSAM-like approaches (see Appendix 1, point 6), one question may be more helpful than the other. It may be more effective, for example, to account for a phenomenon in electromagnetic terms rather than in thermodynamical ones or vice versa.

Actually, sometimes it may be more *effective* to think that something *really* exists: in fact this approach may be considered as a particular case of the supremacy attributed to the first question (Minati & Pessa, 2006, pp. 50–54). However, suitable approaches must be adopted for establishing which are the entities to be taken into consideration as well as for dealing with and modelling the phenomena of interest. Some approaches may simply consist in assuming that entities have absolute validity, i.e. independently from the observer or the problem under study. Various levels of description are possible, however, when considering different variables and scaling transformations. Within this conceptual framework, some problems may arise, such as the need for *detecting communities* of elements as in social network analysis (Missaoui & Sarr, 2015) or in generic graphs (see, e.g. Fortunato, 2010) as well as in multilayer networks (Boccaletti et al., 2014). Moreover, among the methods useful to detecting the presence of suitable entities, we should include those studied by the approach based on the *renormalization group* (Creswick, Farach, & Poole, 2015).

The renormalization group allows systematic mathematical investigation of changes in a system on various distance scales. While self-similarity is related to scale invariance when the properties under consideration are independent from the scales and the most important information contained in the *flow* of renormalization is given by its *fixed points*, we should also focus on scale changes where, for instance, different laws and symmetries occur, energy-momentum and resolution distance scale in conformity with the uncertainty principle, thus making a *leap* from discrete to continuous as in quantum field theory (see Chap. 6). In such cases *non-equivalent representations of the same system are possible*.

> The attractiveness of the latter stems from the fact that within QFT, and only within it, there is the possibility of having different, *non-equivalent*, representations of the same physical system (*cfr*. Haag, 1961; Hepp, 1972; a more recent discussion on the consequences arising from this result, often denoted as 'Haag Theorem', can be found in Bain, 2000; Arageorgis, Earman, & Ruetsche, 2002; Ruetsche, 2002). As each representation is associated with a particular class of macroscopic states of the system (via quantum statistical mechanics) and this class, in turn, can be identified with a particular thermodynamical *phase* of the system (for a proof of the correctness of such an identification, see Sewell, 1986), we are forced to conclude that *only* QFT allows for the existence of different phases of the system itself. (Pessa, 2009, pp. 606–607)

This framework could be generalized by allowing the entities to be considered as being superimposed and entangled as in quantum models. In this case the nature of an entity should be intended as a ***role*** *rather than a state or property***.** These considerations acquire a foremost importance mainly when we deal with specific entities denoted in the literature as *agents*, which can behave, interact and possibly have cognitive abilities such as memory and learning (Taylor, 2014).

Taking now into consideration the relationships, we remark that a *relationship* is intended as a correspondence of any kind, such as quantitative, topological, logical, functional, phenomenological, philosophical, linguistic or any *combination* of these, between entities suitable for identifying that (those) *corresponding* to the other(s). Relationships apply to entities in a variety of possible ways such as causal and non-causal, simultaneous or not, homogeneous-inhomogeneous, constant or variable and context or non-context sensitive. An *interaction* is classically intended as occurring between entities when *properties (behaviour) of one affect the properties of another (behaviour)* and when collective entities such as collective systems affect in different possible ways other collective entities.

An interaction may be intended, for instance, as a process of mutual exchange of matter/energy, goods or money in the economy or information between entities affecting their mutual properties. In this view interactions are assumed to occur because of the properties possessed/acquired by the entities involved. Interactions may not only affect entity properties but also occur through possible structural modifications, such as adapting or learning.

However, beyond this classical understanding of the process of interaction, one should take into account also:

- The case of *active* entities, i.e. possessing autonomous behaviour or embedded into an environment structured in such a way that entity behaviour is induced to become interactive. This case, for instance, can occur when reducing degrees of freedom and increasing environmental density.
- The case of a hosting and *unavoidable* environment, occurring when the latter is a source, for instance, of energy and fluctuations. In these cases the entities may be considered as passive, interacting only in a suitable environment such as happens for many ecosystems.
- The presence of *fields* changing entities or making them to acquire properties.
- The case of dynamical geometrical properties of space such as deformations or relativistic effects. Other interesting cases occur when entities are dynamically networked and the structure of the network establishes the way of interacting between entities themselves (nodes).
- The case where two processes may be considered to interact when they simultaneously happen to the same entities. In this case there are *resulting* effects.
- The case where the interactions themselves may be allowed to interact through *interference*. This phenomenon is considered in physics when there is, for instance, a superposition of two or more waves, disturbances and distortions. The interference can change the interactions themselves *when parts of processes of interacting are inserted into one another* (see Sect. 3.8.2). This latter case

includes the situations in which several entities, of the same or different kinds, in a stable or varying quantity, performing single or multiple interactions, may establish *collective entities*, possessing and acquiring properties different from those possessed by the single interacting entities.

The occurrence of multiple interactions, as considered below when introducing the concept of Multiple System (Sect. 4.5), is related to a) the ability of a generic agent both to interact with other agents by using dynamical, context-sensitive ***combinations*** *of specific rules of interaction and b) the* ***contextual*** *multiple roles or multiple significances of the results produced by specific interactions.*

We stress that this may apply to populations of interactions themselves interfering with each other. In this case, the interactions between entities will occur through resulting interactions *as discussed in Sect. 3.8.2*.

This may be of help from a phenomenological and interpretative point of view. Often models and simulations of collective behaviours are, however, based on different approaches such as stating constraints rather than combinations of rules of interaction. This is the case of the classical model (Reynolds, 1987) in which the agents acquire a flock-like behaviour by collectively moving while respecting behavioural constraints.

Furthermore, as will be seen below, many collective entities are considered to acquire coherence(s) between sequences of acquired properties. This regards the well-known processes of self-organization and emergence (Sect. 3.2.3) where suitable models are based on networks and meta-structures.

3.2.2 Organization, Structure and Abstract Structure

We need to specify, at this point, how we will use the concepts of *organization* and *structure*. Regarding the two concepts, a huge variety of disciplinary, and even non-equivalent meanings, is available in the literature.

According to Ashby, as proposed in his fundamental article (Ashby, 1947), the organization of a system consists of the functional dependence of its future state on its present state and its external inputs, if any. This suggests that it is possible to conceive *organization* as a set of relationships and *kinds* of interactions among entities of any nature (Maturana & Varela, 1973).

While organization relates to properties of sets of relationships and interactions, such as sequential, hierarchical, networked, exclusive, combined, based on levels, stable or dynamical or dealing with undefined parameters, *structure* is a *specification* of organization dealing with well-defined parameters (see Sect. 2.3 for a more specific discussion on the concept of structure). When dealing with *organization*, reference is made even to multiple and variable *networks* of relationships with *undefined* parameters, whereas in the case of *structure*, reference is made to networks having well-defined parameters.

An example of the difference between organization and structure is given by the existence of two different ways of describing an artificial neural network: either as a system, for instance, with *n inputs*, *m hidden layers* and *s outputs*, or as a network with precise values of connection weights and well-defined transfer functions associated with the individual neurons. Some authors speak of the former organizational description as a specification of network *architecture*.

When dealing with systems, *organization* is intended as relating to their *general architecture*, i.e. subsystems, active kinds of interactions, relationships, network and input-output processes. *Structure* relates to specified, parametrized organization when considering particular interactions, relationships and networks, with their current parameters. For instance, the organization of an electronic device is given by a general organizational scheme between types of components. The structure of an electronic device is given by well-specified interconnections between its individual components.

However, we remark that in mathematics, we can consider *abstract structures* over a set, such as algebraic structures (e.g. groups, rings and fields), equivalences of relationships, measures and metric structures (i.e. geometries), orders and topologies (see, for instance, Satake, 2014; Tonti, 2013). More generally, an *abstract structure* is then a *formal object* defined by a set of composition rules, properties and relationships. Such a formal object is defined by a set of coherent laws, rules, properties and relationships like occurs in games and juridical codes. In this case, organization and abstract structure may be considered as being generally *equivalent*.

3.2.3 Dynamics of Self-Organization and Emergence

When speaking of self-organization, one refers to sequences of structures, each associated with a different organization, and to their coherence, as discussed below.

In order to discuss a first distinction between the processes of self-organization and the ones of emergence, about which the literature reports a number of definitions (see, for instance, De Wolf & Holvoet, 2005; Fernandez, Maldonado, & Gershenson, 2014), it is useful to introduce the concept of dynamical coherence to allow generalization and adaptation to different conceptual frameworks. Such a distinction will enable effective approaches for acting upon such processes in order to have prospective suitable conceptual methodologies and tools to induce, maintain, modify, combine and eventually avoid or *deactivate* self-organization and emergence.

Before discussing such differentiation, one should recall that both processes of self-organization and emergence (particularly radical emergence) are characterized by radical structural changes as originally studied in the case of phase transitions. The reference is to physical phenomena associated with macroscopic changes in structure. In this regard one must resort to classical macroscopic thermodynamics, which constitutes the best starting point for a more precise analysis of these phenomena. It is virtually impossible to list here the plethora of textbooks on classical thermodynamics: traditional and comprehensive treatises include Callen, 1960; Rumer & Rivkyn, 1980; and Sears, 1955, and in the case of quantum phenomena, Gitterman, 2014 and Mahler, 2015.

At the end of this section, we ask why neither self-organization nor emergence can be considered as *coincident* with the traditional definition of a phase transition. We use here the attribute 'traditional' (or 'classical') to characterize the theories in which the phase transitions (PT) are studied in presence of volumes tending to infinity and in absence of external fluctuations. Most theories of this kind are based on classical thermodynamics. Quantum aspects, such as the ones related to quantum phase transitions (QPT), will be discussed in Chap. 6.

Within classical theories the processes of *phase transitions* are intended as the acquisition of, or change in, structure (Minati & Pessa, 2006, pp. 201–229; Pessa, 2008). This is the case for first-order phase transitions, e.g. water-ice-vapour allowing the coexistence of structures such as water and vapour or water and ice. In contrast, second-order phase transitions consist of an internal rearrangement of the entire system structure, occurring simultaneously at all points within the system. Each transition occurs because the conditions necessary for the stable existence of the structure corresponding to the initial phase *cease to be valid* being replaced by a new one. Standard examples are given by transitions from paramagnetic to ferromagnetic states or the occurrence of superconductivity or superfluidity. Theories which partly differ from the classical ones have been applied to study the very complicated transient dynamics between phases taking place when classical and quantum aspects mix (Gauger, Rieper, Morton, Benjamin, & Vedral, 2011; Sewell, 1986; Vattay, Kauffman, & Niiranen, 2014).

Furthermore it is possible to consider like phase transitions phenomena occurring in different domains as for cognitive processes with the occurrence, on suitable short temporal scales, of abilities and behaviours not predictable or explained on the basis of previous knowledge of the state or the abilities possessed by the agent considered. The inclusion of these phenomena within the category of phase transitions is often based on analogies rather than on rigorous thermodynamic criteria (which often are not fulfilled). In any case they are useful to suggest the need for a generalization of traditional PT theory. Other examples occur a) in language learning and usage through the extension of vocabulary and the frequency of using plurals (Robinson & Mervis, 1998), b) in cognitive science through the transition from the wrong hypothesis to the right one during the process of the discovery of a rule (Terai, Miwa, & Koga, 2003), c) in evolutionary psychology when a child gains the ability to grasp an object (Wimmers, Savelsbergh, Beek, & Hopkins, 1998) and d) in cognitive science when we have a transition from non-analogical to analogical reasoning (Hosenfeld, van der Maas, & van den Boom, 1997).

In order to understand the difference between the classical theory of PT and the theories of *self-organization*, we now shift our interest towards the latter concept. In this regard we remind that it was introduced by Ashby (Ashby, 1947) who understood a system to be self-organising when the system is changing by itself its own organization rather than being changed by an external action.

We start our considerations by remarking the difference between the processes of self-organization and the ones of *self-structuring* which have different disciplinary meaning like in ecology for spatial self-structuring (Lion & van Baalen, 2008), in the study of networked systems (Gang Chen & Song, 2014; Kermarrec, Mostéfaoui, Raynal, Trédan, & Viana, 2009) and in psychology, communication and education. The distinction between self-organization and self-structuring emphasises that processes of self-organization consists in the adoption of different possible organizations, each of them allowing different *possible* compatible structures.

Processes of *self-organization* are considered here as corresponding to continuous but *predictable*, for instance, periodic or quasi-periodic (Hemmingsson & Peng, 1994), variability in the acquisition of new structures. Examples are given by Rayleigh-Bénard rolls, structures formed in the Belousov-Zhabotinsky reaction, dissipative structures such as whirlpools in the absence of any internal or external fluctuations, and swarms having repetitive behaviour. In particular, in the Rayleigh-Bénard (Ching, 2013) case, there is metastability. In the experiments, the acquired direction of the rotation of the cells, or rolls, is stable and alternates from clockwise to counterclockwise horizontally. Their properties are very sensitive to initial conditions and show a distinct inability to predict long-term conditions typical of chaotic systems. When the temperature of the bottom plane is further increased, cells tend to approximate regular hexagonal prisms like the hexagonal cells of beehives (Getling, 1998).

Processes of self-organization may be understood as ***regular*** *sequences* ***of phase transitions when their changing or transition*** *over time is regular, e.g. cyclic and quasi-periodic when adopting a* ***single*** *coherence.*

Let us now take in consideration the processes of *emergence* (Minati & Pessa, 2006, pp. 145–279). They are considered here as corresponding to the continuous but *irregular* and *unpredictable* (a typical case is given by some kinds of symmetry breaking processes) *coherent* acquisition of new multiple sequences of different structures. Due to coherence, such sequences display to the observer the *same* emergent, acquired property. Examples include the properties of collective behaviours adopted by bacterial colonies, cells, flocks, industrial districts, markets, mobile phone networks, morphological properties of cities, nano-swimmers, nematic fluids, networks such as the Internet, protein chains and their folding, queues and traffic signals, rods on vibrating surfaces, shaken metallic rods (interaction involves reacting), swarms and systems of boats (Minati & Licata, 2012, p. 9; Vicsek & Zafeiris, 2012). In the literature, the difference between *strong* and *weak* emergence has been considered, which can be related, for instance, respectively, to *non-deducibility* and *unexpectedness* from low levels of treatment (see, for instance, Bar-Yam, 2004; Bedau, 2008; Chalmers, 2006; Hovda, 2008).

Processes of emergence may be understood as the occurrence of possibly multiple simultaneous sequences of processes of self-organization when the corresponding acquired dynamic structures are ***coherent****, i.e. display the same* *property in spite of adopting multiple coherences (an example is given by the theory of 'dual evolution' for adaptive systems, introduced by Paperin, Green, & Sadedin,* 2011*).*

Let us now deal with the fundamental question if the PT can be considered as examples of processes of emergence. If we resort to traditional PT theory, the answer is obviously negative. However, if we adopt more complex theoretical models, it is very difficult to prove the validity of this answer. The interest for this question arose when studying the symmetry breaking PT within the context of quantum field theory (see Minati & Pessa, 2006, Chap. 5.4; Liu & Emch, 2005; Batterman, 2011; Landsman, 2013). Without entering in too hard technical details (a very good reference is given by Brauner, 2010), we limit ourselves to remind that a spontaneous symmetry breaking occurs when the dynamical equations ruling a given system continue to keep an invariance with respect to a specific symmetry group, while its ground state loses it. In other words, the system changes its previous ground state (invariant with respect to the same symmetry group) for assuming a new ground state (no more invariant). The phenomenon is spontaneous when it is generated by the change of value of a parameter, without any external force. The two ground states (before and after the symmetry breaking) are different and non-equivalent with respect to unitary transformations acting on system states. In short, they describe two different kinds of physics (just like what happens in traditional PT). In most models of interest for physics, we have a plurality (or even infinity) of possible ground states available after the symmetry breaking, and the specific choice of the new ground state is unpredictable by traditional PT theories. This circumstance is suggested to identify the symmetry breaking transformations with cases of radical emergence.

But is this picture correct? A number of deeper analyses (see Brauner, 2010; Landsman, 2013) showed that it is incomplete. First of all, already in the sixties, first Nambu (Nambu, 1960) and then Goldstone (Goldstone, 1961) showed that the occurrence of a symmetry breaking transition is associated with the presence of bosonic long-range excitations of zero mass, the so-called *Nambu-Goldstone* (NG) bosons (these results have been generalized to quantum field theoretical models by Goldstone, Salam, & Weinberg, 1962). This circumstance holds under the hypotheses of continuity of the symmetry to be broken and of Lorentz invariance of the dynamical equations ruling the theory under consideration. However, it has been shown (see, Brauner, 2010; Watanabe & Maruyama, 2012) that a similar situation occurs also in the case of spontaneous breaking of Lorentz invariance (or of other space-time symmetries) or of rotational or translational invariance. The only change consists of the fact that NG bosons are replaced by suitable quasi-particles.

In the second place, it has been shown that the choice of the new ground state after the symmetry breaking is, in the realistic contexts, not casual and unpredictable but dictated by the influence of external environment upon the system under study. A simple example is given by the second-order PT from the paramagnetic to ferromagnetic state. Here the rotational symmetry is broken (namely, we are in presence of a preferred magnetization direction) and the corresponding NG boson is replaced by a quasiparticle called *magnon*, consisting in a spin wave produced by a collective oscillation of the magnetization direction. But who is the actor specifying the preferred magnetization direction – a random, unpredictable choice made by the system itself during the transition, in absence of any external influence? We understand that this answer would be absurd, just because the divergence of magnetic susceptibility is close to the transition critical point. A factory producing magnets would go bankrupt if expecting the inner system random fluctuations for designing its products! Namely, what really happens is that the preferred magnetization is one of the *external magnetic field* acting on the system in the moment of transition. This implies that a theory of PT which not includes the role of the environment is useless.

The combination of the two aforementioned circumstances gives rise to a somewhat paradoxical situation. On the one hand, a PT is an emergent phenomenon, owing to the presence of NG bosons which help to 'keep' the choice of the new ground state after the symmetry breaking (a fact denoted as 'generalized rigidity' by Anderson in some celebrated papers; see Anderson, 1981; Anderson & Stein, 1985). So they act as 'coherence keepers', a role characterising one of most important aspects of emergence. On the other hand, this emergence is far from being unpredictable, being determined by a specific choice made by external environment. And, as a matter of fact, the NG bosons (or magnons in the case of ferromagnetism) undergo amplitude oscillations around the preferred direction.

For a number of years, the solution of the paradox has been based on the choice of making all volumes tending to infinity. Namely, in this way the role of the local choice of preferred direction made by the environment loses its primary importance. At the same time, we can deal with an exact theory of PT instead of obtaining only approximate results. However, even this hypothesis leaves unsolved an important question: what can make NG bosons? What is their dynamics? In this regard we remark that all previous results do not give any information about the amplitudes of the NG modes which, in principle, could have a whatsoever value. Moreover, the few studies performed on this subject evidenced the existence of different kinds of NG bosons, some of which characterized by different forms of dispersion relations, that is of relationships between ω and κ or, which is the same, between energy and momentum.

This situation suggest the need for adopting a point of view based on the primary role for which NG bosons have been introduced: the one *reacting* to inner and external perturbations in such a way as to act as coherence keepers. It is easy to understand, in this regard, that both kinds of perturbations are, in principle, unpredictable. And, because they must be counteracted by NG bosons which they are free to act in different ways, we must conclude that the whole story of perturbations and corresponding reactions, allowing to keep the coherence of the chosen ground state, is not only endless but consists of a series of acts, each one of which is unpredictable. We can thus assert that a PT associated with a symmetry breaking must be followed by an infinite series or different and unpredictable emergences, each one granting for the keeping of the global coherence corresponding to the new ground state. This story, could, in principle, be experimentally detected by resorting to microscopic observations. As regards the magnetic materials, it is possible to observe some partial effects of this story by looking at the structure of magnetic domains. In short, the previous paradox can be solved, and PT can be considered as cases of radical emergence, provided we take into account realistic contexts of interaction between the system and the environment, taking into account random fluctuations and finite volumes.

We may summarise by saying that PTs relate to order-disorder transitions and can be viewed as cases of radical emergence only if we take into account fluctuations and finite volumes. The self-organization allows to acquire coherence, and emergence allows to acquire ***possibly*** *multiple coherent coherences (**coherent collective self-organization**[4]) when distinguishing, for instance, from* ***multiple synchronizations*** (see Chap. 7 and Pikovsky et al., 2001). Synchronization also relates to multiple maintaining of the same distances of any nature, e.g. spatial, electrical, acoustical, etc., between phenomena. Coherence is considered here, see Sects. 2.1, 3.2.4, and 7.2.1, as maintaining the same emergent property(ies) notwithstanding a continuous structural change.

With reference to scale-free correlations in collective behaviours (Cavagna et al., 2010; Hemelrijk & Hildenbrandt, 2015), we consider self-organization as corresponding to the establishment of a single correlated domain, and emergence as corresponding to the correlation of multiple correlated domains where different, but constant, correlation lengths occur, such as, for instance, when changes in size occur.

Different understandings about the difference between processes of self-organization and emergence (De Wolf & Holvoet, 2005), as well as the *self-organization of processes of emergence* are available in the literature (De Wolf, Holvoet, & Samaey, 2006; De Wolf, Samaey, & Holvoet, 2005a; De Wolf, Samaey, Holvoet, & Roose, 2005b; Samaey, Holvoet, & De Wolf, 2008). Processes of emergence, for instance, of coexisting states, multi-stability and attractors within different disciplinary contexts should also be considered (Feudel, 2008).

An example of multiplicity for processes of self-organization and emergence is given by considering the *hopping* itinerancy of neural activities between attractors (Marro, Torres, & Cortés, 2007) and in sequences of *quasi-attractors*, local regions of convergent/divergent flows. The quoted paper by Marro et al. can be considered as representative of the modelling works in the domain of biologically inspired neural networks. Typically in this context, the multiplicity is produced by resorting to probabilistic processes ruled by stochastic equations. In the paper cited above, the authors introduce networks of N binary neurons whose individual activities

[4]We consider cases where a specific phenomenon of self-organization *differentiates* into different coherent self-organized possibly subsequent, superimposed phenomena such as swarms or flocks having repetitive regular behaviour following perturbation or when subjected to internal fluctuations due to predator attack. This corresponds to the concept of Multiple Systems, Collective Beings (see Sect. 4.5), or *quasi-synchronization* consisting of multiple superimposed synchronisations (Pikovsky et al., 2001), and is at the base of the concept of meta-structures, see below and Sect. 3.8.

s_i $(i = 1,\ldots,N)$ can have only the values 1 or -1. These totally connected neurons communicate through synapses whose intensities are given by a general law having the form:

$$w_{ij} = w_{ij}^L x_j$$

where w_{ij}^L is an average weight value, while x_j is a random value. The model is designed to act as an associative memory, loaded from the beginning by a set of M random binary patterns, stored according to the traditional Hebbian learning rule:

$$w_{ij}^L = M^{-1} \sum_{\mu=1}^{M} \xi_i^\mu \xi_j^\mu$$

If we denote by $m^\mu = N^{-1} \sum_{i=1}^{N} \xi_i^\mu s_i$ the *overlap* between the μ-th memory pattern and the activities of network neurons, it is possible, once introduced a probability distribution for the values of x_j, to compute the local activity fields deriving from the interactions between the neurons through the formula:

$$h_i = \left[1 - \gamma \sum_{\mu=1}^{M} (m^\mu)^2\right] \cdot \sum_{\nu=1}^{M} \xi_i^\nu m^\nu$$

Here the symbol γ is given by the expression:

$$\gamma = (1 + \Phi) \cdot (1 + \alpha)^{-1}$$

in which $\alpha = M/N$. The constant denoted by Φ appears because one of the goals of the model is to describe the neurobiological phenomenon of *synaptic depression* and consisting in the fact that the synaptic weight of a neural connection decreases under repeated presynaptic activation. The value of Φ is just a measure of the amount of this decrease and, as such, appears within the law describing the probability distribution for the values of x_j and, therefore, into the formula for computing h_i.

The final part of model description regards its time evolution which, obviously, has a stochastic nature. This means that, for each network unit, the probability $P(s_i \to s_i')$ that its state s_i at time t be updated to the state s_i' at time $t + 1$ is given by a law having the form:

$$P(s_i \to s_i') = \Psi[\beta_i(s_i' - s_i)] \cdot [1 + \Psi(2\beta_i s_i')]^{-1}$$

where $\beta_i = h_i/T$ and T is a parameter controlling the degree of stochasticity (the so-called *temperature*), while the function $\Psi(u)$ is arbitrary, except for the fact that it must fulfil the conditions:

$$\Psi(u) = \Psi(-u)\exp(u)\ ,\ \Psi(0) = 1\ ,\ \Psi(\infty) = 0$$

A practical example of a function fulfilling these conditions is given by:

$$\Psi(u) = \exp[-(1/2)(u - u_0)]$$

where u_0 is a generic constant.

Needless to say, the behaviour of the model must be studied not only by resorting to analytical considerations but mostly performing numerical computer simulations. The latter evidence, both a chaotic evolutionary trend as well as attractor hopping phenomena, however occurs when the number M of the stored pattern is large. The previous model has been worked out with some details in order to show in an explicit way the mathematical techniques most often used to describe emergence in complex systems endowed with attractors. As well known, attractors and quasi-attractors are associated with memories, perceptions and thoughts, the chaos between them occurring with searches, sequences and itineraries in processes of recalling, thinking, speaking and writing (Kanamaru, Fujii, & Aihara, 2013). Chapter 7 shows that it is possible to consider, for instance, *layers of emergence* and *top-down emergence*, whereas the same *self-organization* is rarer.

Another aspect of the dynamics of self-organization and emergence considers quasi-emergence, quasi-self-organization and their dynamics of changing as in Sect. 4.7.

3.2.4 Dynamical Coherence

When dealing with collective systems, their dynamics is here identified with the changes in the way through which their elements interact, contrarily to classical dynamics which is given by parametrical changes in the fixed form of evolutionary laws.

In the former case, the *structure* of the system is considered as being given by the ways in which each element interacts with the others. It is thus possible to take into consideration temporal sequences of different rules and temporal sequences of different combinations of rules (Sect. 3.8.2), with different coherent networks governing the system.

Different *kinds* of change are possible, such as changes in the way of interacting mentioned above, subsequent *structural* changes as for the cytoskeleton and for complex systems intended as *sequences* of phase transitions where the properties of such sequences should be understood as a structural dynamics, coherent in complex systems (Minati & Licata, 2013). Different possible cases may occur separately or together in any combination:

1. *Change* in structure, i.e. from one structure to another.
2. *Acquisition* of a structure, i.e. change from a non-structured configuration to a structured one.

3. *Loss* of structure, i.e. change from a structured configuration to a non-structured one.
4. *Combinations* of structures.

These may occur both for PTs and networks.

We may also consider *structural regimes*, where for structural regime we intend the current validity, given appropriate thresholds and distributions, of some sequences and combinations of rules of interaction or networks (Sect. 3.8.5). These include *single structural regimes of rules*, *multiple and overlapping* fixed *structural regimes of rules* and *multiple and overlapping* variable *structural regimes* (see Tables 3 and 4).

The dynamical coherence of collective systems has a phenomenological nature, given by the *preservation* of acquired properties, such as behaviour and shape, in spite of the underlying structural dynamics. This is known only a posteriori, and the idea *to zip* the essential characteristics of change and particularly its coherence by using a set of ideal equations is often unsuitable. This occurs because the coherence we have in mind is related to multiple continuous changes which can be represented by sequences of analytical models suitable for representing coherence when used one at a time.

Actually, this conceptual framework has been dealt with by using statistical approaches, whereas here we are considering new post-GOFS approaches, such as networks, meta-structures, preservation of scale-invariance and power laws (see Sect. 3.7). Moreover, it is to be taken into account that more recent advances in the theory of modelling and simulations (see, for instance, Zeigler, Praehofer, & Kim, 2000; Zeigler & Sarjoughian, 2013) make available a number of tools helping the modeller to increase its storage of usable models. Among these tools we can quote the *systems of agents* and the *molecular dynamics* (see, for overviews, Schweitzer, 2003; Helbing, 2010). They allow, mainly in presence of a suitable amount of phenomenological data, to detect a number of useful regularities, in turn suggesting specific local (or global) models, endowed with a suitable, even if temporary, validity (an example of application within a social domain is contained in Budka, Juszczyszyn, Musial, & Musial, 2013).

The concept of coherence, when suitably modelled using ideal approaches (here the attribute 'ideal' is used by making reference to the distinction between ideal and non-ideal models made in Sect. 5.6), can be applied to collective systems working under *stable* environmental conditions, i.e. considered conceptually as a phenomenon occurring *within* closed systems without an active environment with which to interact. Examples include synchronized oscillators, non-perturbed swarms established by suitable initial conditions, populations of fireflies (Buck & Buck, 1966) and traffic jams with hovering data clouds (Fekete, Schmidt, Wegener, & Fischer, 2006) reaching stationary states in a non-perturbed environment.

In contrast, processes of dynamical coherence, i.e. coherence which is changing or the development of multiple coherences which may together show coherence, which often cannot be suitably modelled using ideal approaches, occur, for instance, when a system must also *process* environmental perturbations.

Finally, there is the case in which the system must process *internal changes*, due to reasons such as the occurrence of *intrinsic fluctuations* (of various natures: non-linearity, stochastic noise, chaotic behaviour or quantum-like phenomena) or *decisions* made by autonomous entities. *It should be stressed that the concepts considered above also apply when dynamics relates to changes occurring within* populations of properties *and* configurations *to be intended as entities, as for the dynamics of networks* (Nolte, 2014).

A more comprehensive discussion is given in Sect. 7.2.2 and Appendix 1 when dealing with levels of emergence and with networks.

3.3 The Case of the Dynamics of the Cytoskeleton

One example of complex structural dynamics is given by the dynamics of the ***cytoskeleton*** *(Fletcher & Mullins,* 2010*). Within the cell cytoplasm, the cytoskeleton consists of a network of protein fibres and is characterized by its structural dynamics since its parts are continuously destroyed, renewed or newly created.*

In recent years there has been an increased interest in the dynamics of the cytoskeleton, fomented by the theories of Penrose and Hameroff on the role that quantum processes regarding the microtubules might have in explaining the phenomena associated with cognitive activity and, more generally, consciousness (see, for example, Hameroff, 1994; Hameroff & Penrose, 1996; Penrose, 1994; more recent formulations and proofs are contained in Hameroff & Penrose, 2014a, 2014b). Given the difficulty of carrying out experiments to confirm or deny the validity of these theoretical proposals, it is necessary to build models of the dynamics of the cytoskeleton which allow the prediction of effects which can be experimentally verified.

Currently such model-building is very difficult, given, on the one hand, the complexity of the structure of the cytoskeleton and, secondly, the existence of major limitations linked to the simulation of quantum processes. In all the modelling approaches proposed so far, the cytoskeleton has been considered as a network of biopolymers comprising three main types of filaments (for a review see Pullarkat, Fernández, & Ott, 2007): those of actin, the microtubules and the intermediate filaments. Usually these are disregarded, given that they seem to play only a passive role of reinforcement. Almost all models are based on descriptions of a classical type, focused on the macroscopic hydrodynamics of the cell, and mainly on the rheology of the cytoskeleton, related to the role of the cytoskeleton in determining the mechanical properties of the cell (for reviews, see Jülicher, Kruse, Prost, & Joanny, 2007; Levine & MacKintosh, 2009). Some of these models are inspired by a general theory concerning biological matter, known as the theory of tensegrity, proposed by Ingber (Ingber, Heidemann, Lamoureux, & Buxbaum, 2000). This theory postulates that all biological structures, on any scale, guarantee the stability of their shape, as well as the ability to perform movements in a coordinated manner through the combined action of forces of tension and compression exercised locally. In particular, in the cytoskeleton, tensions would be sustained by filaments of actin, while the microtubules would be responsible for compression (for an example of a model of the cytoskeleton based on tensegrity, see Cañadas, Laurent, Oddou, Isabey, & Wendling, 2002).

Computer simulations of microtubule models (see, for instance, Deymier, Yang, & Hoying, 2005; Baulin, Marques, & Thalmann, 2007; Glade, 2012; Zelirski & Kierfeld, 2013; Gao, Blackwell, Glaser, Betterton, & Shelley, 2015; Muratov & Baulin, 2015), often conducted on systems comprising hundreds of microtubules, revealed two critical aspects: (1) the rheological properties of the cytoskeleton observed so far can only be obtained with a very careful choice of the values of the parameters of the model, suggesting that these properties do not have generality and (2) there is no evidence of any particularly significant influence of the quantum character of microtubule dynamics, except the case where interactions between microtubules and the intracellular fluid are particularly intense. These circumstances suggest, on the one hand, the need to reflect upon the theories proposed relating to the role of the cytoskeleton and, on the other hand, the opportunity of extending the models to avoid too rough approximations of a very complex biological reality. In any case, the simulations performed and the critical examination of their results are a necessary step towards the construction of a general theory of the dynamics of the cytoskeleton.

3.4 Ontological Dynamics of Systems

Ontology (see also Sect. 9.4) is the philosophical study of the nature of existence, of *being* (Brenner, 2008; Effingham, 2013). It is considered a part of the branch of philosophy known as *metaphysics*. Ontology deals with questions concerning the *existence* of entities, their categorization, grouping within hierarchies or according to similarities or differences related to different kinds of applications (Casellas, 2011).

Ontology is intended in philosophy as the science of what is *currently existent*, of the kinds, structures and properties of objects, events, processes and their relationships in every area of reality (van Inwagen, 2014).

*However, the term 'ontology' is associated with different meanings in different disciplines, the bridge between them being given by making reference to **cognitive existence**.*

Ontology, then, is a matter of inquiry, research, development and application in disciplines related to computation, information and knowledge like, e.g. artificial intelligence, knowledge representation and information science, dealing with *categorising and structuring concepts and entities of interest* (see Sect. 9.4). Examples of disciplines applying ontological principles include information science, communication, geography, linguistics, mathematics, medicine and sociology. In all cases each discipline establishes some specific ontological domain in order to consider structures of concepts and meanings pertaining to that discipline.

Let us consider *represented knowledge* (Jakus, Milutinovic, Omerovic, & Tomazic, 2013; Mazzieri & Dragoni, 2012). Formally, it is based on the conceptualization as being a formal, symbolic representation of entities, such as objects and concepts, assumed to be *existent*. *Ontology is then intended as an explicit specification of such conceptualization*. In *computer science*, for instance, the term is used to denote a file containing the formal definition of terms and relationships.

An ontology should be built by analysing the domain to be represented and by conceptualizing it *explicitly*, i.e. symbolically. That is, to allow a Turing machine *to understand* the conceptualization, being endowed with a complete deductive system to logically *infer* all consequences of the available domain knowledge. The *intelligence* of the machine is intended as its ability to find *implicit consequences* of the explicitly represented knowledge.

Such ontologies are studied and used in many fields such as web semantics and databases (Kishore, Sharman, & Ramesh, 2004) for classifications, search engines and web languages (Glimm, Horrocks, Motik, Shearer, & Stoilosm, 2012) and as *computational models* enabling certain kinds of *automated reasoning* (Steward, 1997).

Structured knowledge representations, i.e. ontologies and terminologies, are widely used in *biomedicine* (see, for instance, Gruber, 1993 and the World Health Organization (WHO, 2013).

Another related disciplinary field is the *Gene Ontology project* (see the Gene Ontology Consortium in the References) whose goal is to standardize the representation of gene and gene product attributes across species and databases. As a byproduct, vocabularies of terms for describing *gene product characteristic* and *gene product annotation* are available in the literature (see in the References the entry geneontology).

Let us consider now processes implying *changes of ontologies, which* appear, from the point of view of mathematical logic, as a matter of syntactical change through either the addition or removal of an axiom in the formal system under study. These processes introduce problems of consistency since the ontology might acquire sets of axioms which are mutually incompatible (Haase, van Harmelen, Huaang, Stuckenschmidt, & Sure, 2005).

The changes of ontologies are taken here into consideration as they could be relevant for representing structural changes and changes in properties, i.e. *acquisition or loss, of a system and its levels of coherence(s) during processes of emergence*.

The subject is not new and has been explored by several researchers with reference to the presence and evolution of levels within systems (see, for instance, Baas, 1994; Heard, 2006; Silberstein & McGeever, 1999; Wimsatt, 1994). It is, however, to be taken into account that in this context, it is virtually impossible to establish simple and understandable links between the ontology changes and the processes of emergence occurring within systems. Namely, if we deal with systems made by entities endowed with some sort of cognitive system, as it is the case when we study social systems, we are faced with two fundamental difficulties: (1) there is no commonly shared definition of ontology and (2) we still lack a sound theory explaining how an ontology (which is a mental entity) can have a relation with actions of the members of a social system (which are physical processes). The solution of the latter problem, if any, would be equivalent to the solution of the 'hard problem of consciousness' (using the terminology introduced by David Chalmers; see Chalmers, 1995, 1996). It consists in understanding how the private and subjective personal experience (of mental nature) can be connected with our action-perception system operating in the physical environment.

In this situation, all we can practically do requires the introduction of a specific research context in which all concepts can acquire well-defined meanings. Among the available contexts, so far the most convenient is the one of artificial intelligence. Namely, within it the ontologies are important elements for the design of software tools having specific concrete applications. This allowed the introduction of formalized definitions of ontologies, which overcome the problems related to the older definitions, based on natural language and directly derived from the philosophical tradition. A very popular formalized definition of ontology is, for instance, the one introduced by Kalfoglou and Schorlemmer (2003), according to which an ontology is a pair $\langle S, A \rangle$, where S is the *vocabulary* (often called *signature*), that is, mathematical structure whose elements are the terms used in the ontology, and A the set of *ontological axioms* specifying the interpretation of the vocabulary within a given domain. Such an approach allowed the formalized logico-mathematical study of most processes concerning ontologies, such as ontology changes (see, for instance, Flouris, Manakanatas, Kondylakis, Plexousakis, & Antoniou, 2008; Khattak, Batool, Pervez, Khan, & Lee, 2013; Mahfoudh, Forestier, Thiry, & Hassenforder, 2015).

In turn, the results obtained in these studies allowed practical implementations within specific kinds of models, designed to perform quantitative computer simulations. Among these models we quote the *agent models*, already mentioned in this chapter, and the ones based on the so-called *memetic algorithms* (an introductory paper is the one of Ong, Lim, & Chen, 2010; reviews are contained in Le, Ong, Jin, & Sendhoff, 2009; Chen, Ong, Lim, & Tan, 2011; textbooks are the ones of Goh, Ong, & Tan, 2009; Neri, Cotta, & Moscato, 2012). As it is well known, the term *meme* has been introduced many years ago by the biologist Richard Dawkins to denote a unit of cultural evolution which can undergo biological-like processes such as evolution, propagation and refinement (see Dawkins, 1976). With the years, the original (but imprecise) ideas of Dawkins have been transformed to denote a class of models and algorithms, more often designed to solve optimization problems, but having in common the characteristic of working under a suitable combination of global evolutionary algorithms (like, for instance, genetic algorithms) with local (that is, acting on single individuals) search techniques (like, for instance, the ones based on learning procedures). When these tools are used to simulate the behaviour of agents, whose cognitive systems include ontologies based on memes, it is immediate to understand that models of this kind are suited to describe many evolutionary processes occurring in social systems.

Without entering into technical details, we shortly illustrate a general scheme concerning the application of a memetic algorithm within the context of problem solving through artificial neural networks. This scheme is adapted from a paper by

Chandra (2014). The latter deals with the solution of grammatical inference problems through recurrent neural networks with Elman architecture. In practice these networks consist of three layers of units: the *input* layer, the *hidden* layer and the *output* layer. These layers are connected through standard feedforward links, like usual perceptrons. However, they differ from the latter because the hidden layer has also a feedback link which sends the activations of its units to another layer, parallel to the input layer and called *context* layer. This circumstance allows the units of the hidden layer to receive at the same time t two kinds of inputs: the ones coming from the input layer and the others coming from the context layer (containing the activations of the hidden layer at time $t - 1$. Therefore the activation values of the hidden layer units are given by a law of the form:

$$y_i(t) = f\left[\sum_{k=1}^{K} v_{ik} y_k(t-1) + \sum_{j=1}^{J} w_{ij} x_j(t-1)\right]$$

In this formula K and J denote, respectively, the numbers of units belonging to the hidden and input layers, while v_{ik} and w_{ij} are the weights associated with the related links. The symbol f denotes a traditional sigmoid activation function.

In order to implement the memetic algorithm, the first step consists in decomposing the set of problems to be solved in such a way that each network can be subdivided into subcomponents, each one of which is deputed to solve a specific subset of problems. Without entering into details about the subdivision procedure, here we will limit ourselves to remark that each subcomponent (coded through the connection weights that define it) can be interpreted as a representation of a specific *meme*. Now the next step implies that, once introduced a particular set of memes (that is, subcomponents), we must compute the *fitness* of each meme in solving the subset of problems associated with the considered subcomponent. Obviously, the method used to perform this computation depends on the chosen fitness measure and, therefore, on the nature of the problems to be solved. For this reason we will not insist on the details of this procedure. Let us now introduce the further step of this processing scheme, which is based, for each subcomponent, on a global evolution of the population of memes according to standard rules, for instance, used when applying a *genetic algorithm*. This evolution will give rise, after a suitable number of generations, to a new population of memes, including the ones characterized by the highest fitness. At this point we can introduce a local search procedure, acting on the latter memes, designed to further improve their fitness. While neglecting the details of this procedure (for instance, it could be based on hill-climbing methods), we must remark that it is applied to specific selected memes rather than to their whole population. At the end of this procedure, we can re-assemble the obtained best memes in such a way as to reconstruct the whole network, which, then, is the best suited one for solving the problems belonging to the original set.

While the scheme previously sketched can appear as complex and resource-consuming with respect to traditional learning methods, the experience showed that it is far more effective, also because it helps to understand the deep nature of the problems to be dealt with. This effectiveness, then, becomes evident when we are interested in simulating the behaviour of social systems rather than solving optimization problems.

We recall also an important aspect of ontology, consisting in the fact that often it is used to individuate entities which exist independently from an observer such as a human subject, that is, without any subject having thought of them or otherwise related itself to the entity. They then exist not only epistemologically but also ontologically, i.e. having independent, objective and materialistic existence: reality. This area of research aims to explain emergence by considering the ontology of levels (Emmeche, Koppe, & Stjernfelt, 1997).

This line of thought and research is being considered here not for any interest in classical objectivism, but because the independence from an observer can be viewed as equivalent to considering the observed and observer represented as one in terms of the other, (the case where conceptually the system *contains* the generator of meaning) and also because *different coherences,* as introduced above, might be considered as levels of emergence. In this case we may speak of *super-coherence*, i.e. coherences between coherences, as an ontology of levels.

This involves transformations and transitions. This is the case even within GOFS for the transformation of structured sets into systems where composing elements interact in suitable ways. It also includes phase transitions where the change relates to the structure of the system moving from one phase to another. *Radical emergence* is yet another case.

As made already evident in artificial intelligence, the ontological aspect of transitions is shown through the acquisition of new properties from entities, requiring new ***names*** *and new specifications of relationships among them. The references quoted before when speaking of the formalized theories of ontologies illustrate the achievements already obtained in the study of changing ontologies.*

The subject is considered here in order to explore the problems of a) the ***identity*** *of emergent systems and b) equivalences. Identity (*see Sect. 3.5) *is considered as being related to the robustness of coherence(s) and their possible* ***super coherence***[5] *as in the case of networks* (see, for instance, Cohen & Havlin, 2010; Peixoto & Bornholdt, 2012; Zhou, Gao, Liu, & Cui, 2012) where the coherence of multiple emergent properties is maintained.

In this regard it is important to mention the fact that for a long time, the notion of multiple coherences has been introduced mainly in the study of stochastic systems described by suitable time series of experimental data (a very old contribution on this subject is the one of Goodman, 1963; among more recent contributions, we can quote the ones of Brillinger, 1975, Potter, 1977; Kay, 1999; Box, Jenkins, Reinsel, & Ljung, 2015). However, despite the sound mathematical origin of this notion, it has been generalized to account for multiple local coherences in conceptual changes related to learning process in school students (see, for instance, Rosenberg, Hammer, & Phelan,

[5]We recall that the concept of super coherence originates and is specific to quantum physics when dealing with coherence among dominions of coherences considered in the case of water (Del Giudice & Tedeschi, 2009).

2006; Scherr & Hammer, 2009). In any case, the concept of multiple coherences has acquired a paramount importance mainly within quantum physics. Namely, in this context a state can be formed through the coherent linear superposition of a whatever number of elementary states, a circumstance that allows the superposed state to be characterized by a number of different frequencies, each one corresponding to a particular kind of coherence. Then, a suitable detecting apparatus can work in such a way as to extract from the same superposed state, one at time, different frequencies. Such a property widens the possibilities of spectral analysis of complex system behaviours and is at the basis, for instance, of techniques such as nuclear magnetic resonance (see, e.g. Ernst, Bodenhausen, & Wokaun, 1987; Mathew et al., 2009).

We conclude this section by focussing upon correspondences between aspects of super coherence, identity, ontological dynamics and structural dynamics all of which can be considered as ontological when *the* system goes through levels of emergence or mutations (see Sect. 7.2.2). This is valid when considering the possible *persistence* of properties following the disappearance of original constituents from which structures having such properties emerged (see Klaers, Schmitt, Vewinger, & Weitz, 2010 for a case where photons can *autonomously* persist in Bose-Einstein condensation).

Ontological dynamics of systems relates to the applicability of the same, different or equivalent models and their coherence to be used as within DYSAM-like approaches (see Chap. 5 and Appendix 1), and non-equivalent unitarily quantum representations (Blasone, Jizba, & Vitiello, 2011).

Furthermore, structural system dynamics can be considered as transformation, redefinition or equivalence between ontological identities and the transient as well as the ***dynamics of meanings and their coherence****.*

3.5 Systems Identity

Possession of clear demarcation, stability and permanence, no fuzziness, and structural invariance, all denoting *systemic closure*, are examples of requirements classically considered to deal with *identity*.

Since the opposite, such as openness as non-closure, may be achieved in a variety of dynamical cases, it may be more difficult to define identity rather than through related properties such as coherence, stability or regular dynamics. The subject of identity in philosophy is also called *sameness*, making an entity definable, recognizable and entities distinguishable (see, for instance, Wiggins, 2001).

Here ***identity*** *is considered as being given by the* ***permanence*** *of* ***emergent*** *properties or the* ***permanence of properties of the way in which change can occur at any level*** *such as coherence(s), super coherence and ontological dynamics*. One typical example is life itself.

Such an understanding of identity may be considered within various representations and scales such as in the cases of networks or mesoscopic scale, intermediate between microscopic and macroscopic ones, when dealing with the *middle way* (Laughlin, Pines, Schmalian, Stojkovic, & Wolynes, 2000).

The crucial point is that some representations, such as network or mesoscopic ones, have in common the adoption and validity of specific criteria and thresholds decided upon by the *theoretically active observer*, no longer a noise-generator or source of relativism, but a generator of *cognitive reality* as in constructivism (see Sect. 5.1 and Licata & Minati, 2010). On the other hand, representations could be introduced by considering nodes and links for networks, or clusterisations and introduction of thresholds for mesoscopic representations (Haken, 2005), or based on other criteria such as *optimisations* of the number of variables represented, or even by adopting mixed approaches (Giuliani, 2014).

The subject of 'systems identity' can be understood as being articulated into various issues having possible multiple philosophical ontological interests and scientific aspects. For instance:

1. *Relationships among identities*. The issue arises in various cases, such as when (a) identities are given by stable systemic properties; (b) the same entities establish different systems due to different interactions, e.g. multiple systems where the same elements have multiple roles and synchronization is the source of their coherence; and (c) identity is given by coherence(s) or the properties of the dynamics of their sequences. Mesoscopic identities are explored as meta-structural in Sect. 3.8 and Chap. 4. We should consider *multiple identities* as well the *nature* of this multiplicity. Identity may be given, for instance, by the properties of networks, indices of ergodicity or correlations. The ontological aspects relate to the possibility of acting upon a semantic classificatory network and considering its properties in order to detect properties such as absences, irregularities, or defects as clues of other possible *cognitive realities*. An example is given by the *missing* elements in Mendeleev's table where coherence is intended as *phenomenological*.
2. *Acquisition of identity and the acquisition of properties*. The subject becomes more interesting when identity relates to the *ability to acquire properties* rather than to the acquisition of a specific property. It is a kind of *system* currently *without systemic properties*, in a systemic situation of 'metastability' and *readiness* to acquire systemic properties. This readiness and metastability should be considered as a *pre-identity* of the system available to *adopt*, for instance, its collapse, to degenerate or to acquire a real property. Although a structured system such as an electronic device acquires systemic properties as functionalities and degenerates into structured sets when no longer powered on or when *broken*, we can refer to populations of configurations of interacting elements as being ready to *collapse* into one of a variety of possible *equivalent* (see Sect. 3.6) systems, due, for instance, to noise, fluctuations or symmetry breaking. This relates to processes of the acquisition of coherence(s) and requires a minimum level of complexity.
3. *Maintaining properties*. This subject is more interesting when identity relates to the *ability to keep properties* and their relationships, e.g. sequential, simultaneous or in any other way, rather than to keeping a specific property. It is a kind of *transversal* general property. It may be considered as a *virtual property* ready

to be applied within specific contexts and for configurations having a suitable level of complexity. It is *potential*. It is typical of systems having the property of maintaining acquired properties such as the ones of systemic nature, their sequences, coherence(s), networks or meta-structural ones. In this case a very special *stability* ensues, i.e. maintaining those properties or the ways of acquiring them, whatever they be.

4. *Maintaining equivalence*. This case relates to the ability of a system to maintain as *equivalent* any version of itself over time, e.g. *without* going through structural changes for any reason. The case is trivial when considering the *same* system without acquiring any new properties. It, however, may be interesting when considering multiple systems or sequences of systems. Equivalence in this case may refer to equivalent structural dynamics or coherences. It is also possible to consider equivalence within systems going through evolutionary phases, such as, for instance, growing or aging. This issue relates to topics such as the possibility to *transfer cognitive systems*, then operating as *joint* cognitive systems (Thraen, Bair, Mullin, & Weir, 2012; Woods & Hollnagel, 2006); in linguistics the equivalence among formal languages or among non-formal languages (Dreyer & Marcu, 2012; Jumarie, 1981, 1982; Kapetanios & Sugumaran, 2008); or in knowledge transfer (Holyoak & Morrison, 2013).
5. *Maintaining transience*. This relates to the *same way* of changing of a system when, for instance, it is acquiring or losing or changing its properties, coherence or structures. The same transience can occur in different situations. Trivial cases relate to modalities such as linear, exponential or periodic. Non-trivial cases occur where *uniqueness is repeated*, that is, when evolutionary systems acquire unique configurations or properties in different possible ways. The issues considered in the preceding point, related to cognitive systems, languages and knowledge, equally concern us here, considering, for instance, *processes of generation of singularities*, through fluctuations or noise. These are categories of logical and physical processes able to generate uniqueness. A typical example is given by chaotic systems. Can this transience be considered autonomously and various versions of it be applied to systems in general? Transience should become an object of study as in physics when considering classical and non-classical aspects of transitions since it is the *place* where uniqueness is generated as, for example, in the dynamics between quantum and classical stages (see, for a review, Kapral, 2006).

The above comments about *system identity* are related to the original classical approach considering a *theory of the general system* (singular) introduced by von Bertalanffy (Von Bertalanffy, 1968, 1975) and as also presented by Boulding (Boulding, 1985; Mesarovic, 1972; Rapoport, 1968).

With regard to the term *general*, the subject has been previously discussed (Minati & Pessa, 2006, p. 4):

> 'A collection of his essays was published in 1975, three years after his death. This collection (Von Bertalanffy, 1975) included forewords written by Maria Bertalanffy (his wife) and Ervin Laszlo. The latter added the following considerations about the term

General Systems Theory: 'The original concept that is usually assumed to be expressed in the English term *General Systems Theory* was Allgemeine Systemtheorie (or Lehre). Now – Theorie- or Lehre, just as Wissenschaft, has a much broader meaning in German than the closest English words *theory* and *science*.'

The word *Wissenschaft* refers to any organized body of knowledge. The German word *Theorie* applies to any systematically presented set of concepts. They may be philosophical, empirical, axiomatic, etc. Von Bertalanffy's reference to *Allgemeine Systemtheorie* should be interpreted by understanding a new perspective, a new way of *doing science* more than a proposal of a *General Systems Theory* in the dominion of science, i.e. a *Theory of General Systems*'.

We may consider that von Bertalanffy and the early system scientists had in mind a kind of idealistic, ontological view concerning the properties of *existence* of systems in general. Von Bertalanffy wrote:

'... we postulate a new discipline called *General System Theory*. Its subject matter is the formulation and derivation of those principles which are valid for 'systems' in general'. (Von Bertalanffy, 1968, p. 32).

It is a line of research looking for such general principles, such as that relating to identity listed above, that is still acceptable.

This understanding is not reducible to approaches such as considering the general validity of the *same* models by changing the meanings of variables or by them having the *same* model properties. This is the *local* view of interdisciplinarity dealing with families of problems and approaches mutually translatable and reformulated one into the other.

The ontological approach may be intended as the search for *fundamental systems*, if not *the* system, to be then considered in different non-equivalent *actualisations* into *real* systems. Is such an approach still viable? *Can we look for **the** general network?*

Such an approach may be considered appropriate for collective systems with structural dynamics and where coherence(s) and related properties are the *invariants*.

3.6 Equivalence/Non-equivalence

The problem of *equivalence* can be considered from different points of view (within the domain of mathematics see, for instance, Olver, 2009). It consists, generally speaking, in finding the criteria enabling to consider as *equivalent*, for instance, actions, approaches, configurations, drugs, inputs, levels of descriptions, models, processes, outputs, properties, states and systems.

A trivial case occurs when it is possible to *substitute* one *issue* with another, equivalent because they have the *same* property, such as effect, meaning or role. They are assumed to be *interchangeable*, because one can substitute, replace, the other. Various kinds or degrees of substitutability are possible: total, partial or temporary. The degrees determine the difference between equivalence and equality.

Another case occurs when considering processes. A viable approach may consist of considering them as being equivalent when they provide outputs possessing the *same* properties. Furthermore, processes may be considered as equivalent when the processing of a specific input produces an output equivalent to various possible degrees: total, partial or temporary equality.

Another case occurs when dealing with *equifinal* systems . The topic related to finality has been discussed over a long period in philosophy and science. The subject has been considered by the fathers of systemics, such as Kenneth Boulding who stated (Boulding, 1956, p. 204]):

> The fifth level might be called the genetic-societal level; it is typified by the *plant*, and it dominates the empirical world of the botanist. The outstanding characteristics of these systems are first, a division of labour among cells to form a cell-society with differentiated and mutually dependent parts (roots, leaves, seeds, etc.), and second, a sharp differentiation between the genotype and the phenotype, associated with the phenomenon of equifinal or "blueprinted" growth. At this level there are no highly specialized sense organs and information receptors are diffuse and incapable of much throughput of information – it is doubtful whether a tree can distinguish much more than light from dark, long days from short days, cold from hot.

The subject was also present in von Bertalanffy's founding book (Von Bertalanffy, 1968). von Bertalanffy wrote (von Bertalanffy, 1950, p. 25):

> A profound difference between most inanimate and living systems can be expressed by the concept of *equifinality*. In most physical systems, the final state is determined by the initial conditions. Take, for instance, the motion in a planetary system where the positions at a time t are determined by those of a time t_0, or a chemical equilibrium where the final concentrations depend on the initial ones. If there is a change in either the initial conditions or the process, the final state is changed. Vital phenomena show a different behaviour. Here, to a wide extent, the final state may be reached from different initial conditions and in different ways. Such behaviour we call equifinal.

von Bertalanffy discussed three kinds of finalities, respectively associated with the following situations:

- The dynamical evolution of a system reaches asymptotically over time a stationary state.
- The dynamical evolution never reaches this state.
- The dynamical evolution is characterized by periodic oscillations.

In the first case, the variations in the values of the state variables may be expressed as a function of their distance from the stationary state. System changes may be described as if they were to depend upon a future final state. Such a circumstance could be related to a teleological view expressed, for instance, by *minimum or maximum principles* (of a local or global nature). von Bertalanffy noticed how this form of description is nothing but a *different expression of causality*: the final state corresponds simply to a condition of extreme in the differential equations ruling the dynamical evolution. We could, however, view such a condition also as describing a particular kind of finality, that is, the so-called *equifinality*. The latter characterizes those dynamical systems which are able to reach the same final state independently from their initial conditions or input .

On the contrary, there are situations where the system displays very high sensitivity to initial conditions, as for *chaotic systems*.

An interesting situation occurs when the behaviour of systems occurs in situations where the next state to be adopted is one of several different ones all *equivalent* for the given system. For instance, the direction of rotation of Bènard rolls. The *decision* is 'made' by noise and fluctuations. Let us now consider the case for models. In order to assess their equivalence/non-equivalence, there are different criteria:

- The level of description adopted and possible correspondences.
- The *transformability* of one model into another.
- The possible *transformability* of representations modelled, one into the other.

Examples of general incompatibility, i.e. non-equivalence, are given when considering quantum and non-quantum models, Turing-machines and quantum computing devices, thermodynamic and electromagnetic models.

The practice of DYSAM (Minati & Pessa, 2006, pp. 64–75 and Appendix 1) can be used by considering both equivalent and non-equivalent models since the focus is on the changing of models and the properties of their sequences, such as coherence.

The DYSAM approach considers systems, in real time or not, in parallel, synchronously or sequentially, depending on the kind of process to be dealt with, the dynamic identification of levels of representation of the case to be modelled which allow *multi-model*-based processing. This is typical for processes of emergence where the complex system acquires coherent sequences of new properties and the observer must use *n* different levels of description corresponding to *n* different models.

From an ontological viewpoint, equivalence/non-equivalence could be considered as the ***ontological essence*** *of the relationships among identities.* ***We recall the relationship between*** *equivalence* ***and*** *non-completeness,* ***where the latter is the space for multiple equivalences*** .

We conclude this section by mentioning the interest in studying the possible equivalence/non-equivalence between coherences modelled, for instance, using network models or meta-structures as introduced below.

3.7 Acting on the Dynamics of Emergence

The subject of this section concerns examples of prospective conceptual representations, models and approaches, methodologies and tools, to induce, maintain, modify, combine and eventually *deactivate* the dynamics of processes of emergence.

Examples of suitable interventions are given by acting *macroscopically* on the resources available such as energy, by setting obstacles and *distortions* in the interactions among agents and by changing general environmental conditions.

This is the subject of the current science of complexity. Among the various possible research approaches related to the *observability* of complex systems (Yang-Yu Liua, Slotine, & Barabási, 2013), below there are some examples of research topics for tools suitable for acting, for instance, upon:

1. *Acquisition, change and the use of constraints or degrees of freedom*. The concept of degree of freedom in mathematics relates to the number of independent quantities necessary to express the values of all the variables describing a system. For instance, a point moving without constraints in *3D* space has three degrees of freedom because three coordinates are necessary to specify its position. Eventual constraints *reduce* the number of degrees of freedom, for instance, when considering a *simple pendulum* having only one degree of freedom since its angle of inclination is specified by a single number. *In this book we consider the concept of degree of freedom in a more generic way as used in daily language,* i.e. *intended as a constraint on values adopted by single independent variables, such as geometrical or physical*. Following the discussion in Sect. 3.2, we may also consider values of max and min and the usage of the *between*. For instance, we may consider that the value of a variable adopted to respect such constraints may *use* a well-defined percentage of the degree of freedom, i.e. *[Dmax - Dmin]* allowing the researcher to detect that such usage has properties such as always being close to the max or min, or is periodic, random or given by distributions having suitable properties. Moreover, the degrees of freedom may be variable, multiple and quantitatively related.
2. *Environmental properties*. As we stated above, the *separation* of a system from its environment is a matter of simplification, whereas research focuses upon open, non-complete representations, layers (Sect. 2.7), environment (Sect. 2.3) and the *between* (Sect. 1.3.8 and 7.1), where systems and environment may be represented one as a function of the other, as for an observer and observed.
3. *Ways of interacting*. Ways of interacting are covered in Sect. 3.8.2. They may be *fixed*, based on the exchange of matter-energy, *context sensitive*, depending on environmental properties, or *evolutionary,* based on learning for autonomous systems provided with sufficiently complex cognitive systems. They may be multiple and apply in different ways.
4. *Available states*. The system may have available a predefined set of possible states to occupy. Interest may focus, for instance, on two different *modalities*. In the case of *multistability*, we consider both states and attractors when stability is given by the restoring or changing of stability following perturbation of the system. The other states are the *metastable equilibrium states* discussed in Chap. 2 and Box 3.3. The states available tell us something about the degrees of freedom of the system, but without saying anything about the *modalities* for reaching them, moving among them, their possible combinations, or temporal constraints.

5. *Coherences*. More emphasis is placed on coherence rather than, for instance, on equilibrium. Dissipative systems, for example, can maintain stationary states *far from thermodynamic equilibrium* through the transfer of entropy to the environment through the dissipation of matter, as do whirlpools (the same kinds of structure exist in atmospheric phenomena such as hurricanes) and living structures dissipating material flows such as air, water, food and, in certain cases, light to remain far from thermodynamic equilibrium, i.e. thermodynamic death. The process of dissipation allows emergence and the preservation of ordered structures and properties. However, there are processes of emergence which do not require dissipation to establish coherence(s), as is the case for collective behaviours in general. The focus is on the search for *coherence* (Sects. 2.1, 4.7, and 7.2.1), rather than equilibrium, and coherence(s) among eventual multiple dynamic equilibriums, and levels of coherence(s) as for super-coherence discussed in Sect. 3.4. Interventions are then made on processes of dissipation and the establishment of coherence(s) by acting, for instance, on networks, scale invariance, power laws or meta-structural properties introduced later.
6. *Emergent properties*. In the following chapters, particularly Chap. 5, we present new theoretical frameworks to be adopted when studying emergence and representations of its dynamics using strategies without *explicit prescribability*, no- or low-intensive invasiveness, and low energy in order to *induce* processes of emergence without *regulation* since *explicit*, *intensive* interventions are *incompatible*, non-processable by complex emergent systems, as discussed in Sects. 1.3, 4.2.7, and 5.6. Examples include *weak* (with reference to original values) changes in prices, taxations and exchange rates in economy and biochemical equilibria in living systems. Examples of radical invasive interventions are given by possible *necessary substitutions* then continuing with processes such as transplants or social rejection. Then the approach based on using Perturbative Collective Behaviour (PCB) to influence collective behaviour (see Sect. 3.8.4.5) will be considered.

3.8 Methods and Approaches to Model and Act upon the Dynamics of Emergence: Research on Meta-Structures

As we have previously showed, there are different possible methods and approaches to act upon the dynamics of emergence. Their list includes:

- The science of networks (see, for instance, Barabási, 2002; Baker, 2013; Lewis, 2009; Valente, 2012), discussed in Chap. 8.
- The quantum theories (see, for instance, Carati & Galgani, 2001; Clifton & Halvorson, 2001; Del Giudice, Doglia, Milani, & Vitiello, 1985; Pessa, 1998; Sewell, 1986), discussed in Chap. 6.

- The study of meta-structures (Minati & Licata, 2012; Minati, Licata, & Pessa, 2013; Pessa, 2012), presented immediately below.

The meta-structures are to be intended as structures whose elements are in turn structures (Pessa, 2012). In biology, for instance, a meta-structure may be an organism, consisting of structured arrays of cells, each of which is a complex structure composed of a large number of macromolecules. Another example is socio-economical and cognitive phenomena where the hierarchical networks of complex relationships offer examples of meta-structures, often even more complex than biological ones. In physics, meta-structures involve interactions between different structured and coherent domains, as in liquids or magnetic materials.

From the point of view of the relationships between components of meta-structures, it is possible to consider different *types* of meta-structures, for instance:

1. Those in which individual components can simultaneously belong to different structures, which are not related through their hierarchical relationships (*horizontal* meta-structures).
2. Those in which individual components can simultaneously belong to different structures which do have hierarchical relationships between them (*vertical* meta-structures).

Examples of horizontal meta-structures include individuals who have relationships both with their colleagues and with those who share the same hobby.

Examples of vertical meta-structures include individuals who have relationships with both colleagues and executives of the company in which they work, supermolecules and multiple networks (Nicosia, Bianconi, Latora, & Barthelemy, 2013).

Vertical meta-structures are very common in the world of physics and biology, and therefore their study is important.

The interest for a theory of meta-structures arose after the birth of so-called *mesoscopic physics* (for introductory reviews see Imry, 1986; Altshuler, Lee, & Webb, 1991; Katsoulakis, Plecháč, & Tsagkarogiannis, 2005).

As introduced above in Sect. 2.4, mesoscopic physics deals with the domain of length scales in between the microscopic and macroscopic, where unexpected phenomena can occur.

A number of different descriptions of meta-structures and their dynamics have been introduced in many different domains, such as *metalattices* (Han & Crespi, 2001), *multilevel neural networks* (Breakspear & Stam, 2005) and *agent systems* (Johnson & Iravani, 2007). However, we are still lacking models of emergence of meta-structures from situations in which they were initially absent. When introducing the approach considered below, the concept of structure will be taken as the *structure of interaction* between entities.

Multiple Systems (Minati & Pessa, 2006) are considered to be based upon the occurrence of multiple interactions, having possibly different durations and starting time, involving the same entities which may belong (simultaneously or successively) to different systems (see Sect. 4.1 and Fig. 3.1 corresponding to specific

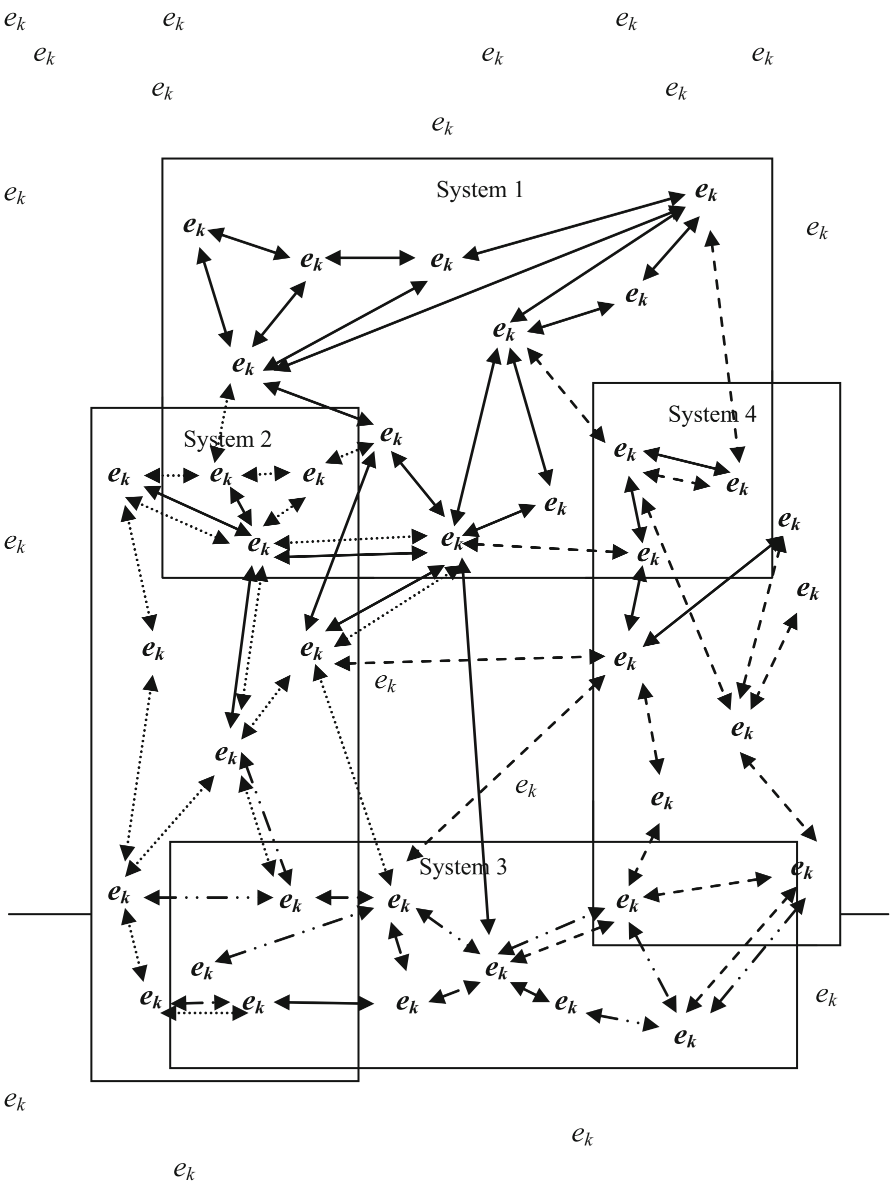

Fig. 3.1 Multiple interactions

interaction). An example of multiple interactions is given, for instance, by the rules listed in Table 3.1.

In this case it is possible to consider a meta-structure as being given by a *set of structures* of the different systems, i.e. multiple interactions, establishing a given

Multiple System together with possible relationships between the component systems. Multiple structures may also relate to multiple networks and sequences of adjacent units in lattices.

It is possible to consider a simplified case by taking a Multiple System established by two systems with binary and classifiable relationships (Pessa, 2012, p. 115) as:

- *Fully hierarchical*, in which the elements of one of the two systems are a proper subset of the set of elements of the other system and, moreover, the larger system influences the average dynamical behaviour of the smaller one (at this stage a detailed description of this influence is not necessary).
- *Partially hierarchical*, where, while almost all the above conditions are satisfied, the sets of elements of the two systems have only a partial overlap.
- *Non-hierarchical*, where the sets of elements of the two systems are totally disjointed.

However, three main characteristics seem to be indispensable to give rise to hierarchical structures (Pessa, 2006, 2012, p. 120), which are:

- *Locally causal* interactions between system elements.
- *Long-range* correlations between those elements.
- *Local inhomogeneities* in the activities of those elements.

3.8.1 The Meta-Structure Research Project

Collective behaviour can be distinguished from collective interaction, such as Brownian notion (Nelson, 1967), since the former adopts emergent properties due to coherence(s) as correlation(s). This may work as a criterion, as may other approaches which can be distinguished by considering the presence or absence of properties such as scale invariance or power laws.

In the meta-structure research project (Minati, 2016a, 2016b; Minati et al., 2013; Minati & Licata, 2012, 2013, 2015; Pessa, 2012), a meta-structure consists of sets of multiple structures of interaction, i.e. more than one, and their properties which may simultaneously be *combined*, for instance, linearly or non-linearly, or involve their *interference* as in Sect. 3.8.2.

Thus, the research considers as meta-structural *real interactions*, i.e. combined single rules of interaction or through interference among them, occurring within populations of entities establishing collective behaviours. A simplified case is given by *bipolar* meta-structures, i.e. when real interactions, occurring for specific couples of entities per instant, involves the same elements belonging to other couples interacting in turn in different ways with different entities as represented in Fig. 3.1.

It should be stressed that this understanding is conceptually different from approaches based on considering *effects* of interactions to which, for instance, statistical methods or macroscopic approaches such as looking for indices (see, for instance, Stephen et al., 2011) are applied. Macroscopic properties may also be considered as having a meta-structural nature since they *summarise* as *global indices*, e.g. temperature and pressure, the effects of multiple structures of interactions. However, they are of limited interest here since they miss all microscopic references and, because of that, *they allow very limited actions on the process itself*.

In the project, there is the assumption that a well-defined and stable, however, contextually parameterized, *library* of structures of interaction is available to the entities involved during the process.

The typical process to study is collective behaviour, natural or simulated, established by a number of interacting *agents*, from here onwards referred to with the more general term *entity*, where all microscopic information is available and to which non-macroscopic approaches are applied. Microscopic data are considered to be available from suitable processes, such as (1) ad hoc simulations (Minati, 2016a, 2016b) where the software simulates a flock-like collective behaviour based on the classic Reynolds approach (Reynolds, 1987); (2) *stereometric digital photogrammetry* data related to real flocks (Cavagna et al., 2010) where authors detected scale-invariance (3) ad hoc electronic devices of coupled oscillators generating emergence (Minati, 2014; Minati, 2015); and processes with available phenomenological data such as social, economical and financial from so-called *big data*, very large data sets where analysers apply techniques of data mining to find, for instance, regularities, cross-correlations, frequency, performance and statistical evaluations (Davenport, 2014; Franks, 2012).

The approach considered here was inspired by von Bertalanffy with the concepts of *dynamic morphology* (Von Bertalanffy, 1975, p. 47) and by considering that 'Life is a dynamic equilibrium in a polyphasic system' (Von Bertalanffy, 1968, p. 123).

Meta-structures are an attempt to model structural dynamics and its eventual coherence as introduced above.

Moreover, the coherence of emergent collective behaviours cannot be suitably modelled by considering *only* rules of interactions. This latter approach conceptually corresponds to considering *networked sequences of stimulus-reaction* when dealing with agents.

The point missed regards the *usages of rules of interactions*. In the case of living agents, it is important to consider their cognitive systems which are responsible for using the rules of interactions and for processing information which is not reducible to networked sequences of stimulus-reaction being, for instance, context-dependent.

However, evidence that biological agents establishing emergent collective behaviours do so by using the *same cognitive* system is given by the fact that they are all of the *same* type, i.e. of the same species or same genus.

The sharing of the same cognitive system using the *same* cognitive model may be assumed as possibly being a *necessary* but not *sufficient* condition for establishing collective behaviours among agents.

Different usages of rules of interactions may be assumed to occur for non-living agents, i.e. without natural cognitive systems. Analytical intractability combines with generic equivalences (considered hereafter as interchangeability) and mesoscopic approaches continuously trading between the microscopic and macroscopic.

Several possibly necessary conditions may be considered, such as assuming the conceptual interchangeability of agents playing the same roles at different times and allowing *ergodicity,* in this case *responsible for coherence* (see for a discussion Minati & Pessa, 2006, pp. 104–110).

Such conditions, i.e. possession of ergodic interchangeability or meta-structural properties, also apply to collective behaviours established by *living systems provided with no cognitive systems* such as amoeba, bacterial colonies, cells and macromolecules and by *non-living systems* such as electrical systems, mobile phones or Internet networks, morphological properties of cities and traffic signalling systems.

In the latter cases, possible ergodic-like interchangeability or meta-structural usage of rules is not due to *decisions* taken by cognitive systems through cognitive models but is rather a way to *model the coherence of collective behaviours*.

Another possibly necessary condition considered here is the coherent usage of rules of interaction represented, for instance, by meta-structural properties as introduced later (see Sect. 3.8.4) and meta-structural regimes introduced in Sects. 3.2.4 and 3.8.3. Discussed below is the approach based on considering the properties of mesoscopic variables, as in Sect. 3.2.4, in order to represent, at a suitable level, multiple interactions, as in Sect. 3.8.3.

However, examples of other approaches where *meta-structural properties are not mesoscopically represented* consider, for instance, scale invariance (Cavagna et al., 2010; Hemelrijk & Hildenbrandt, 2015), topological distance (Balle Ballarini, et al., 2008), maximum entropy (Cavagna et al., 2013), network properties (see Chap. 8 and Barabási, 2002; Lewis, 2009), the global consistency of an adjacency matrix in lattices (Tasdighian et al., 2014), topological constraints and scale-free graphs for Self-Organizing Networks (Licata & Lella, 2007).

As will be seen below, *meta-structural properties are all properties of multiple structural dynamics as for Multiple Systems. This understanding is based on switching*

- *From* ***a priori*** *approaches based on adopting known fixed general analytical rules of interaction.*
- *To* ***a posteriori*** *approaches, different from* ***statistical ones at the microscopic level*** *while looking, for instance, for collective mesoscopic properties,* i.e. *meta-structural properties, assumed to represent analytically* ***incognizable*** *rules of interaction.*

Multiple Systems are always metastable *too* (Kelso & Tognoli, 2006) presenting criticalities and invariance of scale (Chialvo, 2010). Multi-structural dynamics, a possible conceptual example of which is shown in Table 3.1, is analytically, explicitly *intractable*. Classical approaches are of a statistical nature. However, our interest is in finding possible alternative representations, such as networks in order to consider coherence, levels of coherence, quasi-coherences and multiple coherences possibly superimposed. In cases such as those considered in Table 3.1 related to flock-like collective behaviours, possibly generalizable when rules are analytically represented, structural dynamics occur, for instance, as variations in altitude, direction, distance or velocity.

Properties of their dynamical parametrical combinations and interferences should be considered as ***clues*** *and* ***representations*** *of aspects of coherence. Such properties are intended in the following as being represented by meta-structural properties of suitable mesoscopic variables and clusterisations* ***transversally*** *intercepting such structural dynamics* as in Sects. 3.8.2, 3.8.3 and 3.8.4.

Meta-structural representations and understanding of complex behaviours are introduced to allow strategies of intervention in order to modify complex behaviours and their properties, such as for systems of cells, traffic, markets and crowds.

The general purposes of considering meta-structural properties is to contribute towards a post-GOFS developing approaches and models which can act upon complex systems by participating *in their change rather than regulating, prescribing or deciding it.*

3.8.2 Interactions

Consider a hypothetical library of *rules of interactions such as* $Rint_{j:1-13}$ as in Table 3.1. This table shows an example of multiple rules of interactions for flock-like collective behaviours where we consider a population of $k > 3$ interacting agents, with k fixed as a simplified case for the entire observational time T. This example considers the simplistic case where interactions may be explicitly represented by symbolic rules, considered to *completely* represent the phenomenon.

More realistically, ***resulting interactions*** *will be due to any combinations, interference or timing since the time scalarity might not simply coincide with the beginning or end of* ***any*** *interaction.*

Resulting interactions $Res\text{-}int_j$[6] applied to agents e_k per instant will be due to possible *partial* (because of different durations) linear or non-linear combinations of $Rint_j$ as well from interferences among $Rint_j$, i.e. as a function of various $Rint_j$ as will f_i introduced below.

Example of linear combination is given by adding the effects of rules.

Example of non-linear combination is given by computing the resultant effects of rules such as $(\textit{effect of } Rint_1 + \textit{effect of } Rint_j)^2$.

Example of a generic interference f is given by

$$Res - \text{int} = f(Rint_{1,}, Rint_2, Rint_6),$$

[6]Resulting interaction $Res\text{-}int_i$ may be a variable number where i is the number of resulting interactions per instant.

Table 3.2 Original rules of interaction from Table 3.1 considered for the following example

$Rint_1$	Consists of varying	Speed	Depending on	Speed or average speed of closest agents
$Rint_2$	Consists of varying	Speed	Depending on	Speed of agent(s) having same direction
$Rint_6$	Consists of varying	Direction	Depending on	Direction of agent(s) having same speed

Table 3.3 Resulting rule of interaction *Res-int* following interference *f* among $Rint_1$,$Rint_2$, $Rint_6$

Res-int	Consists of varying	Speed and	Depending on	Speed of closest agent having same direction [1]and
		Direction		Direction of closest agent having same speed [2]

[1] Added to $Rint_2$ the required *closest agent. Average speed of closest agents* in $Rint_1$ is not considered
[2] Added to $Rint_6$ the required *closest agent*

where from original rules of interaction (Table 3.2) following interference *f* (see notes 1 and 2 in Table 3.3) one obtains the resulting rule of interaction *Res-int* (Table 3.3).

Examples of more complex cases occur when, for instance:

$$\begin{cases} Res - int_1 = f_1(Rint_2,\ Rint_6) \\ \dots \\ Res - int_5 = f_5(Res - int_1,\ Rint_5,\ Rint_{10}). \end{cases}$$

The e_k agents may interact, for instance, in pairs by using any linear or non-linear combination of the interactions $Rint_j$ and/or given by any f_i of $Rint_{J:1-13}$:

$$Res - \mathrm{int}_i = f_i(Rint_1, Rint_2, \dots, Rint_{13}).$$

In the simplest cases, f_i will act on parameters. Resulting interactions $Res\text{-}int_j$ will, of course, be time dependent in correspondence with $f_i(t)$. Furthermore agents e_k may interact in any possible combinations per instant.

Furthermore, interactions could be represented by non-symbolic rules and in a non-comprehensive manner, such as probabilistically or fuzzy.

Position, speed, direction and altitude of a specific agent e_k at time t_{i+1} is *considered calculated* by the model, using one or more combinations of, or interference with, the *13* rules and using the values possessed by the agent (s) considered at time t_i.

Computation of the new state at the time t_{i+1} by applying the rules above gives specific, positive or negative, incremental changes regarding the state, as for speed and/or altitude and/or direction.

The elementary cases listed in Table 3.1 should be considered as *parameterized* by considering, for instance, context-sensitive parameterisations.

Furthermore, incremental changes should be computed by considering the need to respect ranges allowing *continuity* given, for instance, by maximum discontinuities, levels and degrees of inhomogeneity within the collective behaviour and *compactness* allowing consistency.

Computation of the new state, depending on interaction rules, is also carried out by *choosing* from among several possible equivalent incremental changes. For instance, in the classical Reynolds model (Reynolds, 1987), the choice is made in such a way as to ensure:

- Alignment: agents must compute the interaction by pointing toward the average direction of the local or adjacent agents,
- Cohesion: agents must compute the interaction by pointing toward the average position of the local or adjacent agents, *being able to appropriately vary speed, direction and altitude*.

Different and more complex options are, of course, possible for rules of interactions, computing and selection from among equivalent possible incremental changes.

An example of interactions occurring through multiple rules of interaction is considered in Table 3.1 and graphically represented in Fig. 3.1 (Minati & Licata, 2012, p. 292).

Interactions may occur between *properties* of behaviours of agents e_k such as topological ones, properties of systems of rules of interactions, multiple ones, or those having different dynamics possibly represented by systems of macroscopic indices, such as volume.

With reference to the ***temporal granularity*** *for both simulations and detection of real collective phenomena, it is important to cope with the fact that interactions are assumed to occur with* ***dynamically changing different starting times and durations****, being values of mesoscopic variables representing those phenomena.*

3.8.3 Mesoscopic Variables

The *microscopic* level of description is that corresponding to descriptions of properties of entities considered as *ultimate*, i.e. when they can no longer be suitably further decomposed. Examples are descriptions in terms of molecular variables, such as position or speed of pollen grains or water molecules.

The *macroscopic* level of description corresponds to descriptions of properties of entities whose composition is not of interest. For instance, this level could be adopted for describing the motion of a ball or of a fluid, by considering only the *resultant* effects of properties of a large number of microscopic variables.

The *mesoscopic* level is between these two. At this level reduced variables are considered *as* at the macroscopic level, but without completely ignoring the degrees of freedom present at the microscopic level, i.e. when dealing with the *middle way* (Laughlin et al., 2000). See Sect. 2.4.

For instance, by considering the system established by road traffic circulation, a mesoscopic variable is given by considering cars that *cannot accelerate*. With this selection, both cars can be considered as stationary, in line with constant speed or decelerating when, for instance, approaching an obstacle. Another example considers the quantity of people on the stairs of a building. Here, people are considered as walking up or down or standing on the stairs.

Meaningful variables at the mesoscopic level are known, in the science of complexity, as *order parameters* introduced with synergetics (Haken, 1987, 1988). When complex systems undergo phase transitions, a special type of ordering occurs at the microscopic level. Instead of addressing *each* of a very large number of atoms of a complex system, Haken showed, mathematically, that it is possible to address their fundamental *modes* by means of *order parameters*. The very important mathematical result obtained using this approach consists of drastically lowering the number of degrees of freedom to only a few parameters. Haken also showed how *order parameters* guide complex processes in self-organizing systems.

When an *order parameter* guides a process, it is said to *slave* the other parameters, and this slaving principle is the key to understanding self-organizing systems. Complex systems organize and generate themselves under far-from-equilibrium conditions:

> *In general just a few collective modes become unstable and serve as 'order parameters' which describe the macroscopic pattern. At the same time the macroscopic variables, i.e. the order parameters, govern the behavior of the microscopic parts by the 'slaving principle'. In this way, the occurrence of order parameters and their ability to enslave allows the system to find its own structure.* (Graham & Haken, 1969, p. 13)

'*In general, the behavior of the total system is governed by only a few order parameters that prescribe the newly evolving order of the system*' (Haken, 1987, p. 425). Mesoscopic order parameters in the science of complexity have the purpose of extending to systems far from thermal equilibrium concepts used for systems in equilibrium. It is possible to obtain an effective mesoscopic description by considering a very limited number of order parameters: only a few may manifest instability and be taken as significant in transitions. Others may be ignored either because of their very fast dynamics or because of their essentially stability.

A subsequent step is then taken using the so-called *collective variables* widely used in theoretical physics, as mesoscopic ones '...where it allows a shift from a representation of a system based, for example, upon a set of isolated atoms, mutually interacting in a very complicated way, to a new collective representation (physically equivalent to the previous one) based on isolated atoms interacting in a simple way only with suitable collective excitations (so-called *quasi-particles*)'. (Minati & Pessa, 2006, pp. 236–237).

As introduced above, mesoscopic variables are essentially suitable clusterisations (Minati, 2016a, 2016b).

We are interested in considering mesoscopic variables ***representing structural dynamics*** *occurring through combinations, interference and various temporal durations as shown in the examples in* Table 3.1 *and Fig.* 3.1 *where the collective interactions are coherent. Coherence of collective interactions – meta-structures – is studied here as represented by properties of mesoscopic variables.*

This approach uses mesoscopic variables whose values *indirectly* represent the *effects* on entities of multiple interactions in *3D* as listed above and which are suitable for simulations.

Examples of mesoscopic variables, clusterisations, suitable for representing multiple simultaneous, different processes of structural dynamics occurring where each agent *may select*, for any reason such as perturbations, energetic reasons, boundary conditions or possibly cognitive reasons when provided with a cognitive system, to use any combinations of the available rules (see Sects. 3.2.4 and 3.8.1) are presented below. Consider a situation, typically a simulation, where the number k of interacting agents e_k (such as oscillators or logistic maps) is finite and fixed for the entire finite observational time T. This approach is a conceptual extension of the simpler case when dealing with populations of interacting oscillators which consider variations in phases or frequency. In the following, we consider the case of flock-like collective behaviours as introduced above in Sect. 3.8.2.

3.8.3.1 Correlation and Synchronization *of Single Agents*

Mesoscopic variables are considered here as synchronized, multiply synchronized or correlated clusters of *agents*. Processes of synchronization and correlations were considered in Sects. 3.2 and 3.2.3.

A simplified view consists of considering an ***optimized temporal granularity*** *where all synchronisations and correlations start and end within the* ***same*** *temporal interval*.

We recall the non-transitivity of the *property of being positively correlated* as demonstrated by Langford (Langford, Schwertman, & Owens, 2001).

Another form of correlation occurs when such explicit data may be represented as *networked* (Lewis, 2009).

Mesoscopic variables are given in this case by clusters of networked synchronized or correlated agents, corresponding parametrical values such as phases, correlation values, ergodic parameters or, for instance, by numbers of agents, their spatial distributions, data on their possible multiple belonging or density when considering the space identified by the cluster.

3.8.3.2 Communities and Clusters

Several approaches are presented below for considering aggregations *among agents* as mesoscopic variables when considering their general *similarity in behaviour*. The problem may be approached in different ways such as looking for community detection in complex networks (Kaneko, 1990; Ovelgönne & Geyer-Schulz, 2013; Shalizi, Camperi, & Klinkner, 2006; Sobolevsky, Campari, Belyi, & Ratti, 2014), functional clustering (Filisetti, Villani, Roli, Fiorucci, & Serra, 2015; Tononi, McIntosh, Russel, & Edelman, 1998) or large aggregates of data by adopting approaches such as data clustering (Aggarwal & Reddy, 2013; Gan, 2011), data matching (Christen, 2014) and data mining (Gorunescu, 2011).

There are also the usual well-known statistical approaches (Shevlyakov & Oja, 2016):

- Multivariate Data Analysis (MDA) and Cluster Analysis, to identify classes (Everitt & Landau, 2011; Hair & Black, 2013).
- Pearson Product Moment Correlation Coefficient (PPMCC), to measure possible linear dependence between two or more attributes (Rupp & Walk, 2010).
- Principal Component Analysis (PCA) to identify non-explicit rhythms and deterministic structures (Jolliffe, 2002).
- Principal Components (PCs) to generate low-dimensional descriptions (Vidal, Ma, & Sastry, 2016).
- Recurrence Plot Analysis (RPA), see (Webber, Ioana, & Marwan, 2016).
- Recurrence Quantification Analysis (RQA) to quantify the number and duration of recurrences as trajectories in phase space (Webber & Marwan, 2016).
- Time-Series Analysis (Box et al., 2015).

Mesoscopic variables are given in this case by clusters of agents and, for instance, their number of agents, spatial distributions, possible multiple belonging and density when considering the space identified by the cluster.

3.8.3.3 Sameness

Similarities are considered as suitably represented by clusters of agents grouped by closely similar values of a specific variable considered *as if* respecting *virtual* thresholds computed ex-post, i.e. after clusterization.

It is possible to consider clusters of agents at a given instant having the same or different *thresholds* per type of cluster allowing to assume two values adopted by a variable be considered as equal when less than the threshold value:

1. The *maximum* distance(s).
2. The *minimum* distance(s).
3. The *same* distance(s) from the nearest neighbour.
4. The *same* speed(s).
5. The *same* direction(s).
6. The *same* altitude(s).
7. The *same* topological position, such as at a *boundary*. Generic agents e_k are considered to be at a boundary at instant t_i by considering properties of their position (x_k, y_k, z_k). Agents are at the boundary when their geometrical coordinates respect at least one of the following conditions *max or min*(x_k), *max or min* (y_k), *max or min*(z_k) or any of their possible combinations.

Thresholds can be statistically *derived* when considering the ordered sets of values adopted by specific variables per instant in order to identify the more significant ones. By using suitable statistical methods, it is possible to identify *statistical extremes*, i.e. *aggregates* of agents possessing the four properties considered above (distance, speed, direction and position), allowing computation of the resulting corresponding thresholds to be considered for subsequent modelling purposes.

Examples of techniques used include top-down and bottom-up clustering, the so-called *Self-Organizing Maps* (SOM) and in particular processes of clustering techniques, *K-Means*, *K-median*, and *K-medoids* (Everitt, Landau, Leese, & Stahl, 2011; Mirkin, 2012).

In cases 1, 2 and 7 listed above, we have a single corresponding set of values per instant.

In cases 3–6 we may have more than one set of values at any instant when ordered elements are clusterized in classes such as:

- $n\text{-}dis_1$ number of agents e_k at same distance $dist_1$, $n\text{-}dis_2$ number of agents e_k at same distance $dist_2$, etc.
- $n\text{-}spe_1$ number of agents e_k at same speed $speed_1$, $n\text{-}spe_2$ number of agents e_k at same speed $speed_2$, etc.
- $n\text{-}dir_1$ number of agents e_k having same direction dir_1, $n\text{-}dir\ _2$ number of agents e_k having same direction dir_2, etc.
- $n\text{-}alt_1$ number of agents e_k at same altitude alt_1, n_2 number of agents e_k at same altitude $d\text{-}alt_2$, etc.

It is thus possible to consider vectors consisting of a) values of the property considered, b) the number of agents belonging to the cluster and c) the values of the thresholds computed *ex-post* as minimum and maximum values.

For instance, in the case of distance when n_1 agents e_k are at distance d_1, n_2 are at distance d_2, etc. It is then possible to consider a vector $Vd(t_i)$ given by triple scalar values $Vd(t_i) = [(d, q, t)_1, (d, q, t)_2, \ldots, (d, q, t)_v]$where

- d is the distance considered.
- q is the number of elements e_k at the *same* distance d.
- t is the threshold value computed.

The same applies to the other variables.

Mesoscopic variables are given in this case by the values adopted by vectors $Vd(t_i)$, and consider eventual spatial distributions of agents, their possible multiple belonging and density when considering the space identified by the cluster.

3.8.3.4 Differences among Agents per Instant

Consider, for instance, operating with the sets of *all differences* between values of positions or speeds or directions or altitudes possessed *per instant* by *all* $[k!/(k-2)!]/2$ *couples* of agents such as $[e_m(t_i), e_j(t_i)] \equiv [e_j(t_i), e_m(t_i)]$where $m \neq j$, $m > 0, j > 0$ *and* $m \leq k, j \leq k$.

It is thus possible to consider, at given point in time, significant clusterisations of differences, e.g. possessing minimum differences among them.

Mesoscopic variables are given in this case by clusters of differences having the minimum differences between them.

3.8.3.5 Variations of Single Agents over Time

Consider, for instance, operating with the displacement (as a particular case of a variation) vector $Vspace(t_i)$ of size k whose elements correspond to the individual agents $e_k(t_i)$ and which contains their respective spatial positions x_k, y_k *and* z_k at a given instant.

For a generic agent $e_k(t_i)$, it will be possible to consider, for example, its spatial positions in (t_{i-1}) and (t_i) which allow one to calculate the displacement vector Vs $[e_k(t_i), e_k(t_{i-1})] = [Vspace_k(t_{i-1})\text{-}\ Vspace_k(t_i)]$.

One can thus construct a vector of size k $Vspost(t_i)$ whose elements correspond to the individual agents e_k and contain the spatial displacement x, y, z of each $e_k(t_i)$ at a given instant relative to the previous position and $e_k(t_{i-1})$.

One can then consider the historical sequences related to variations in position, speed, direction and altitude for each agent $e_k(t_i)$ and study *homogeneous* correlations, i.e. between historical sequences of changes in speed *or* position *or* direction *or* altitude, or *non-homogeneous* correlations, i.e. between historical sequences of changes in all variables.

It is thus possible to consider clusterizations having the *same* or *correlated* variations as displacement, per instant, and at the same or at different computed thresholds per type of cluster. It is possible to cluster on the basis of the *same* variation as displacement of homogeneous variables.

Related *mesoscopic variables* are given, for instance, by the number of clusterized variations, their possible correlations and properties of related agents possibly belonging to other possible different clusterisations.

3.8.3.6 Classes

We consider here clusters as introduced in Sect. 3.8.3.3. The maximum and minimum values assumed by a variable establishing a cluster, considered ex-post as given by suitable threshold, can be intended to identify classes. Clustered values of variables may by aggregated in classes *h:1-C* as in the table below.

Class *h*	*1*	*2*	...	*C*
Distances	$M_1 < dist(e_i,e_r) < M_2$	$M_3 < dist\ (e_i,e_r) < M_4$	...	$M_n < dist\ (e_i,e_r) < M_s$
Speeds	$S_1 < speed\ (e_i) < S_2$	$S_3 < speed\ (e_r) < S_4$	...	$S_p < speed\ (e_s) < S_q$
...	...	...	...	...

At any given point in time any e_k may:

1. Belong only to a single cluster.

2. Belong simultaneously to j ($j < C \wedge j > 0$) different clusters. Here, one must consider that a specific distance may a) have different extremes, i.e. distances between different agents e_k, or b) share one *extreme*, i.e. a *same* agent e_k. In the second case, a same element e_k can belong simultaneously more times to the same distance class and to different distance classes. At any given point in time t, each distance class will be characterized by the number of elements e_k falling within it.

This applies to the classes such as differences in altitudes, directions and velocities for each agent for all temporal periods t_i and t_{i+1} where $i{:}1{,}T$.

Classes and their relative number of agents per instant are considered to constitute *mesoscopic variables*.

This section is also preparatory to Sect. 3.8.4.2 on Ergodicity.

3.8.3.7 Degrees of Freedom

In this case, mesoscopic variables are considered as being given by statistical clusters of percentages per agent of their usage of degrees of freedom.

Consider the *absolute* maximum and minimum values, for instance, reached ex-post, at the end of the observational or simulation time, among all speeds, directions, altitudes and distances.

At each instant values of speed, direction and altitude of each agent may be computed as specific percentages of the maximum or minimum values as detected above a posteriori.

Consider sets of all the percentages of maximums or minimums per agent and per variable detected a posteriori.

At this point it is possible to consider clusterisations of percentages: clusterisations given by aggregations of agents whose values of corresponding variables respect such percentages.

Mesoscopic variables will then be given by clusterization of percentages per instant and per corresponding agents when considering, for instance, their number.

3.8.4 Meta-Structural Properties

We apply here the principles outlined in Chap. 2, such as the need to be non-complete; non-precise, to assume lightness; and non-explicitness as properties to capture complexity when meta-structural, i.e. *multiple, multiphase, and superimposed, interactions and interference is the **place** of partial or dynamic equivalences, **trading** between possibilities contending to become **effective becoming**, the emergence of new coherences.*

We consider possible multiple, simultaneous, properties of clusters and communities established not through commonalities of microscopic properties, e.g. *speeds, but by clusters and communities of clusters having* ***properties of multiple relational properties***[7] *and* ***properties of their dynamical intersections*** *as introduced below.*

Several approaches are possible to formulate meta-structural properties. With reference to mesoscopic variables, as mentioned in Sect. 2.4 and 3.8.3, one can consider their values and the properties of the sets of their values. The values of mesoscopic variables are considered to *intercept* and represent the structural dynamics as the application of multiple rules.

Generic examples of meta-structural properties are given by:

(a) Properties of the values acquired by mesoscopic variables, single or crossed, such as any regularities including periodicity, quasi-periodicity and chaotic regularities possibly with attractors which characterize specific collective behaviours.
(b) Properties, e.g. geometrical, topological, of distribution, or statistical, of sets of generic agents constituting mesoscopic variables and their change over time.
(c) Properties related to the usage of degrees of freedom as introduced above.
(d) Relationships between properties of sets of clustered generic agents and macroscopic properties such as density, distribution, scale-freeness or numerical properties such as percentages.
(e) Properties of the thresholds adopted for specifying the mesoscopic general vector.
(f) Possible topological properties of network representations, power laws and scale-invariance.
(g) Possible levels of ergodicity.

However, examples of some specific meta-structural properties are presented here below.

3.8.4.1 Correlation and Synchronization *of Mesoscopic Variabl*es

In this case synchronized, correlated values of mesoscopic variables are considered rather than microscopic values related to properties of agents such as speed, weight, age, etc. as in Sect. 3.8.3.1.

Here, a meta-structural property is given by synchronization and correlation parameters and their possible dynamics among the values taken by mesoscopic variables, such as their number of elements. In the latter case, the meta-structural property also consists of considering the properties and parameters of such dynamics.

[7] *Multiple relational properties* represented by mesoscopic clusterisations. *Multiple relational properties* and *properties of their dynamical intersections* represented by meta-structural properties.

3.8.4.2 Ergodic Passage from one Class to another and *Mesoscopic Ergodicity*

With reference to classes introduced in Sect. 3.8.3.6, for any value of t, there is a distribution of agents within the different classes. Let π_{ht} denote the total number of agents belonging to class h at time t. Then the vector $\pi_t = (\pi_{1t}, \pi_{2t}, ..., \pi_{ct})$ defines the state of this distribution at time t.

This allows the introduction of the probability P of the transition of an agent from a class i at time t_{-1} to a class j at time t, denoted as p_{ij}.

The first order Markov assumption (which turns out to be a very good approximation in most real cases) implies that the status of the world π_t depends only on $\pi_{t\text{-}1}$ through Markov's transition matrix $[P_{ij}]$.

This implies that $\pi'_t = \pi'_{t1}P$.

A distribution is ergodic if $\pi' = \pi' P$.

In this case, classes allow detection of ergodicity.

In such cases meta-structural properties are given by ergodic properties.

At this point we can introduce the concept of *mesoscopic ergodicity*. As considered at the Sect. 4.5.1, it is well known that over a given observational time and considering a system composed by finite, constant over time number of elements, if:

- $Y_\varphi\%$ is the average percentage of time spent by a single element in state S.
- $X_\varphi\%$ *is* the average percentage of elements lying in the same state, the degree of ergodicity is given by:

$$E_\varphi = 1/[1 + (X_\varphi\% - Y_\varphi\%)^2].$$

We have ergodicity when $X_\varphi\% = Y_\varphi\%$ and the degree E_φ then adopts its maximum value of *1*.

However, in a correspondent way, we may consider as state S, called here *mesoscopic state*, the belonging of elements to a specific cluster.

Consider n interacting entities e_k.

The simpler single instantaneous mesoscopic state is given when considering a single instantaneous cluster related to values of a single variable. For instance, a mesoscopic state is given by the clustered elements

$$e_j, ..., e_h$$

having all similar value of a variable, for instance, aggregated in clusters where elements e_k have the *same* distances $dist_1, dist_2, ..., dist_n$ between each other.

Clusterisations per instant will occur by considering different clustering distances $dist_1(t), dist_2(t), ..., dist_n(t)$.

The mesoscopic variable related to distances $[n_{dist1}, n_{dist2}, ..., n_{distn}]$ considers the *number* of elements e_k having per instant the same distance, $dist_1(t), dist_2(t), ..., dist_n(t)$ between each other. We know the number of elements $n_{dist1}, n_{dist2}, ..., n_{distn}$, but we do not know *which* elements, being them mesoscopically equivalent, i.e. one can play the role of the other, that is to increase the number of elements belonging to the cluster.

The same values of a mesoscopic correspond to a variety of different microscopic configurations of elements.

In this way clusters of the same mesoscopic variable are established by different *equivalent* configurations of same elements, then considerable in equivalent different ordered sets. In this way are considered as equivalent elements having possible important differences given however by properties related to other variables such as their altitude, speed and direction. Crossing evaluations will occur when consider crossing correlations.

Furthermore an element e_k belonging to the cluster $dist_1(t_n)$ can belong to a different cluster $dist_1(t_m)$ or do not belong to any cluster at a different time with $m \neq n$).

We may summarise by considering:

- $Y_\varphi\%$ as the average percentage of time spent by *equivalent* elements belonging to a specific cluster.
- $X_\varphi\%$ *as* the average percentage of *equivalent* elements belonging to this specific cluster. The degree of mesosocopic ergodicity is then given by:

$$E_\varphi = 1/[1 + (X_\varphi\% - Y_\varphi\%)^2].$$

Also in this case, we have mesoscopic ergodicity when $X_\varphi\% = Y_\varphi\%$ and the degree E_φ adopts its maximum value of 1.

Notes:

- The number of clusters per mesoscopic variable is fixed for the entire process (for instance, when clustering by using K-means).
- The number of elements belonging to the same cluster is different along time.
- The mesoscopic variable is then composed of the same number of clusters having different numbers of belonging elements along time.
- We consider the total time spent by each element to belong to a specific cluster along time and how many elements belong to this specific cluster per instant.
- Correspondingly we may consider the average of all percentages of time spent by each element to belong to a specific cluster along time and the average of all percentages of the number of elements belonging to this specific cluster per instant.
- In this case ergodicity relates to single specific clusters. It is possible to consider the ergodicity of each cluster along time and different ergodicities are possible for the different clusters constituting the mesoscopic variable.
- Furthermore we may consider mesoscopic ergodicity when averaging among all the clusters constituting the mesoscopic variable.

We then consider the ergodicity among mesoscopic states, given by taking in count percentage of equivalent elements belonging to a mesoscopic state mesoscopic, i.e. to a cluster, in an instant t_i, versus percentage of time spent by those equivalent elements to belong to that mesoscopic state, by ways in which E_φ oscillates around 1 in time. Other related meta-structural properties are given by correlations among ergodicities for different variables.

However we stress that the same degree of mesoscopic ergodicity can be given by different microscopic configurations due to possible multiple roles played by *interchangeable* elements along time. This is the case for Multiple Systems and Collective Beings considered in the Sect. 4.5.1. *A specific mesoscopic state identifies a set of instantaneous* ***equivalent*** *microscopic states. For example, the set of elements establishing clusters where microscopic states of elements are considered equivalent,* e.g. *having similar values of the same variable and considered* ***interchangeable*** *when do not* ***altering*** *the global coherence,* e.g. *of a collective behaviour, or when* ***inducing*** *assumption of equivalent coherences,* i.e. *different equivalent collective configurations.*

Mesoscopic ergodicity does not *prescribe microscopic properties but equivalences allowing theoretical incompleteness* (Minati, 2016a, 2016b)*, reason of unpredictability.*

Suitable levels of degrees of mesoscopic ergodicity can be considered as meta-structural properties since corresponding to levels of coherence (see Sect. 3.4*).* The suitability is given by the possibility to *represent* or *prescribe* not only local temporal or spatial coherence, but generalized coherence typical of collective behaviours. *In this case it is matter of coherence having* ergodic nature.

3.8.4.3 Mesoscopic *Slaving*

This section considers an approach corresponding, conceptually, to the identification of order parameters (variables in this case) representing a kind of *mesoscopic slaving* as considered in synergetics.

It is important to find dynamical *summarizing variables representing the collective behaviour and considered suitable for modifying it, by using non-explicit approaches.*

Consider a matrix $K \bullet M(t_i)$ where K is the number of agents e_k, and M is the number of mesoscopic properties considered. Element $KM_{k,m}(t_i)$is equal to *0* if the generic agent e_k does not possess the mesoscopic property m at time t_i or to *1* if the generic agent e_k does possess that property m at time t_i:

$$\begin{array}{lll} KM_{11} & KM_{12}\ldots & KM_{1m} \\ KM_{21} & KM_{22} & KM_{2m} \\ \ldots\ldots & & \\ KM_{k1} & KM_{k2} & KM_{1m} \end{array}$$

It is possible to consider at time T, i.e. at the end of the simulation or of the real phenomenon under study, for instance, the sequences of previous matrices.

Properties of such sequences are considered as meta-structural properties.

Examples of properties are given when considering trends, periodicities, correlations and statistical properties of sets of values, such as:

(a) Number of agents and which agents possess at least one mesoscopic property and the total number of properties and which properties are possessed by agents after the global observational computational time. The trends of acquisition of properties should be detected.

(b) Number and *which* agents have the same or more or several or no mesoscopic properties over time. This labelling allows to identify *zones* of agents possessing mesoscopic properties, their topology and dynamics.

(c) The repetitiveness or quasi-repetitiveness (unless one, two, ..., *n* cases being level of repetitiveness) of same matrixes and their temporal distributions.

(d) Number of agents and *which* agents possess a specific topological position. Agents may:

- Be *topological centre* of the flock, i.e. all topological distances between the agent under study and all the agents belonging to the geometrical surface are equal. This agent may be *virtual* and be considered as a *topological attractor* for the flock. Its trajectory may *represent* the trajectory of the flock.
- Belong to the geometrical surface or to a specific zone of interest.
- Have a specific topological distance from one of the agents such as temporary leaders and agents belonging to the geometrical surface or a specific area of interest.

These are examples of meta-structural properties both *representing* the collective behaviour under study and the meta-structural variables to be used to influence the possible further evolution of the collective system after time *T*.

However, from the data above, it is possible to compute a posteriori, i.e. at the end of the collective behaviour, the sequences and the sum of all the previous matrices per instant:

$$\sum_{\substack{k\,:\,1,K \\ m\,:\,1,M \\ t\,:\,1,T}} KM_{km}(t_i)$$

It is thus possible to identify the ***maximum intersections****,* i.e. *not only the agents which possessed the maximum number of mesoscopic properties, but those which possessed the maximum number of* ***specific*** *mesoscopic properties, with special reference to the case where* ***this possession occurred at the same time*** *or with particular sequences and correlations in time*. In the latter case, it is of great interest to identify the sequences of agents possessing multiple mesoscopic properties per instant and their *persistence* over time.

Such sequences, their properties and their possible correlations are intended as meta-structural properties.

When properties of sequences and of their intersections are significant, they are intended to meta-structurally represent the collective behaviour under study.

The significance of such sequences allows representation and possible modifying actions upon them leading to generalized effects on the global collective behaviour, for instance, by introducing suitable environmental perturbations having the purpose to facilitate or avoid specific properties of sequences. Examples of perturbations are given by introduction of obstacles and changing environmental properties *to which agents are sensitive,* e.g. *temperature, lighting, air currents and acoustic*. We will consider at the Sect. 3.8.4.5 the insertion of suitable *Perturbative Collective Behaviour(s).*

3.8.4.4 Networks

This is the case where there are no microscopic data to be networked but clusters, as above, and so networks of clusters have to be considered. It is question of networked mesoscopic variables. Properties of such networks, see Sect. 8, are intended here as meta-structural properties.

3.8.4.5 Perturbed Meta-Structures

It is possible to consider the *introduction* of suitable Perturbative Collective Behaviour (PCB) allowing *combinations* of meta-structural properties of the perturbed collective behaviour and of the PCB. As introduced previously (Minati et al., 2013), it is possible to consider various approaches such as when elements of the original collective behaviour to be modified are *invisible* to the component elements of the PCB, appearing as *dynamic obstacles*.

This approach is inspired by the *order parameter* used in synergetics or in the *doping* of materials such as silicon, processes of delocalization and restructuring within damaged brains and networks and meta-materials.

A PCB, having meta-structural properties different from those of the collective behaviour to be modified, may consist of external elements or even of some original *mutated* elements, i.e. when artificially adopting different meta-structural rules. Therefore the insertion of a suitable PCB may occur, for instance, in at least two ways:

- By allowing the original collective behaviour to interact with another one, inserted in a suitable way and acting as mobile coherent *obstacles*, i.e. nothing to do, for instance, with prey-predator interactions. Components of the collective behaviour must adapt their behaviour, whereas the PCB acts independently.
- Some elements of the collective behaviour *mutate* their behaviour, i.e. interact differently from before. Such mutation may be stable, temporal, following some temporal regularities, have different possible levels of homogeneity or coherence and possibly following rules of another type of collective behaviour. The distribution of such mutated agents may be of any type such as following topological or metrical criteria.

The number of components of the PCB can vary. In order to model or to adopt approaches to modify the original collective behaviour, it is possible to consider, for instance, the dynamic percentage of mutated or external agents, their distribution, lifespan and topology.

This also relates to complex systems where aspects such as multiple meta-structural properties are simultaneously active each with their own distributions over time, having scale invariance or topological properties as for networks.

3.8.4.6 Further Considerations

Macroscopic variables such as measures of *Vol*(t_i), volume of the collective entity over time (used to compute density) and *Sur*(t_i), measure of the *surface* of the collective entity over time, can be used to complement the models even when correlated with other meta-structural properties. The volume and surface of a collective entity should be modelled by using suitable approaches such as considering lattices.

We stress that the examples considered here relate to spatial properties in ***3D*** *although similar approaches can be used for non-spatial contexts such as for economics.*

It is also possible, as mentioned above, to consider properties of *physical* clusters of *corresponding* agents, i.e. represented by mesoscopic variables. For instance, when considering the mesoscopic variable given by the clusters of elements having the *same* distance from the nearest neighbour at a given point in time or above the average, *instead* of taking into account the *number* of elements one can consider other properties of each cluster, such as:

- The measure of the volume and surface of the cluster.
- Its density; the distribution of belonging agents within the cluster.
- Geometrical and topological properties of the configuration of the belonging agents.

On the basis of such properties one can consider, for instance:

1. *Structure* of individual clusters, such as topology, distribution and properties of the connections, i.e. networks, between components.
2. *Topological position* and distribution of the clusters in the collective system overall.
3. *Connections and compactness*. Consider the *space occupied* by a cluster whose volume and surface is measured, and its inside where there are possibly components *extraneous* to the cluster (i.e. they do not 'belong' to the mesoscopic variable). One can then consider the extraneous entities, such as agents belonging to other clusters or not belonging to any cluster, as contextually fixed, e.g. obstacles, or moving entities, such as preys. This allows an evaluation of the properties of physical structures *where* clusters of agents are. For instance, by considering the inside of the space occupied by a specific cluster of agents, it is possible to evaluate how *diluted* it is, percentages of agents and extraneous entities, separation of agents by extraneous entities and superpositions of configurations.

4. *Persistence,* or even partial iteration, over time of properties for the same or different clusters is also possible. The properties of their sequences and relationships can be studied.
5. *Sequences of clusters*, corresponding to the same mesoscopic variable by considering their possible homological or co-homological relationships.

3.8.4.7 Mesoscopic Dynamics

Consider the collective behaviours of agents e_k as above and, in particular, the cases considered in Sect. 3.8.4.3, for which one can study the values adopted by the *mesoscopic general vector*, i.e. lines of the previous matrix:

$$V_{k,m} = [e_{k1}(t_i), e_{k2}(t_i), \ldots e_{km}(t_i)].$$

This mesoscopic general vector represents the diffusion over time of the mesoscopic properties possessed by single e_k agents per instant. The evolution of this vector represents the *mesoscopic history* of single agents of the collective behaviours under study.

Reversely we may consider as mesoscopic general vector the columns of the previous matrix:

$$V_{km}(t_i) = [e_{1,m}(t_i), e_{2,m}(t_i), \ldots e_{k,m}(t_i)].$$

The mesoscopic general column vector represents how specific mesoscopic properties are diffused, i.e. possessed by single agents per instant. The evolution of this vector represents the *mesoscopic history* of single mesoscopic properties of the collective behaviours under study.

Thus one can consider the general *mesoscopic dynamics* of the matrices or of specific mesoscopic general vectors whose eventual coherence represented by properties such as synchronisation, periodicity, statistical or, more generally, correlations represents collective behaviour (see Table 3.4) as specified below (De Wolf et al., 2005b; Minati et al., 2013). There are at least four exemplary cases, as shown in Table 3.2.

1. *All agents that simultaneously possess all the same mesoscopic properties and values of associated mesoscopic and parametric variables, such as thresholds, are **constant** over time*. Agents all simultaneously respect the degrees of freedom *and* the parametrical values defining mesoscopic variables that are *constant*, i.e. changes are *insignificant* within the adopted threshold.

For any agent e_k and for $\forall$ mesoscopic property $m(t_i)$, $V_{km}(t_i) = [1, 1, \ldots, 1]$, where $m(t_i) = m(t_{i+1})$ and parameters are constant *over time*.

2. *All agents simultaneously possess all the same mesoscopic properties and values of associated mesoscopic and parametric variables, such as thresholds, are **constant** per instant, but **variable** over time*. Agents all simultaneously respect the degrees of freedom and the parametrical values defining mesoscopic variables that are constant per instant, but variable over time, i.e. changes are

Table 3.4 Mesoscopic dynamics

	Mesoscopic dynamics		
Structural properties / Properties of the collective behaviours	Structure of interaction	Mesoscopic properties	Meta-structural properties
Case 4 Collective behaviours structurally at high variability, e.g., flock under attack	Multiple and superimposed variations in the structures of interaction	Agents possess different mesoscopic properties per instant and over time. However their parametrical values, such as thresholds, are *constant* per instant, but *variable* over time.	Non-trivial meta-structural properties
Case 3 Collective behaviours structurally variable, e.g. perturbed flock	Multiple and superimposed variations in the same structures of interaction	Agents possess different mesoscopic properties per instant and over time. However, their parametrical values, such as thresholds, are *constant* over time.	Non-trivial meta-structural properties
Case 2 Collective behaviours structurally at low variability, e.g., flock dealing with fixed obstacles	Changes in the *same* structure of interaction	All the agents simultaneously possess all the same mesoscopic properties, and values of associated mesoscopic and parametric variables, such as thresholds, are *constant* per instant, but *variable* over time.	Trivial meta-structural properties
Case 1 Collective behaviours structurally 'fixed', e.g., flock with repetitive behaviour	Structure of interaction fixed	All the agents simultaneously possess all the same mesoscopic properties and values of associated mesoscopic and parametric variables, such as thresholds, are *constant* over time.	Trivial meta-structural properties

insignificant within the threshold adopted per instant, whereas they can change significantly over time.

For any agent e_k and for $\forall$ mesoscopic property $m(t_i)$, $V_{km}(t_i) = [1, 1, \ldots, 1]$, where $m(t_i) \neq m(t_{i+1})$ and parameters are constant *per instant*, i.e. for all instantaneous different situations.

3. *Agents possess different mesoscopic properties per instant and over time. However, parametrical values, such as thresholds, are* ***constant*** *over time*. Agents simultaneously respect the degrees of freedom and the parametrical values

defining mesoscopic variables are constant, i.e. changes are insignificant within the adopted threshold. For any agent e_k and for ∀ mesoscopic property $m(t_i)$, per instant, there will be different configurational varieties of the vector $V_{k,m} = [e_{k1}(t_i), e_{k2}(t_i), \ldots e_{km}(t_i)]$ such as:

$$\begin{aligned} V_{1,m}(t_i) &= [1, 0, 0, \ldots, 0] \\ V_{2,m}(t_i) &= [0, 1, 0, \ldots, 1] \\ V_{3,m}(t_i) &= [0, 1, 1, \ldots, 0] \\ V_{4,m}(t_i) &= [1, 0, 1, \ldots, 1] \\ V_{5,m}(t_i) &= [0, 0, 0, \ldots, 1] \\ &\ldots \\ V_{k,m}(t_i) &= [0, 1, 0, \ldots, 1] \end{aligned}$$

Reversely the same situation is represented by the column vector $V_{km}(t_i) = [e_{1,m}(t_i), e_{2,m}(t_i), \ldots e_{k,m}(t_i)]$.

Parameters are constant *over time*.

4. *Agents possess different mesoscopic properties per instant and over time. However, parametrical values, including thresholds, are* ***constant*** *per instant, but* ***variable*** *over time*. Agents simultaneously respect the degrees of freedom, and parametrical values defining mesoscopic variables are constant per instant, but variable over time i.e. changes are insignificant within the threshold adopted per instant, whereas they can change significantly over time.

For any agent e_k and for ∀ mesoscopic property $m(t_i) \neq m(t_{i+1})$, per instant, there will be different configurational varieties of the vector $V_{k,m} = [e_{k1}(t_i), e_{k2}(t_i), \ldots e_{km}(t_i)]$ such as:

$$\begin{aligned} V_{1,m}(t_i) &= [1, 1, 0, \ldots, 0] \\ V_{2,m}(t_i) &= [0, 1, 1, \ldots, 0] \\ V_{3,m}(t_i) &= [1, 0, 1, \ldots, 1] \\ V_{4,m}(t_i) &= [1, 0, 1, \ldots, 0] \\ V_{5,m}(t_i) &= [0, 0, 0, \ldots, 1] \\ &\ldots \\ V_{k,m}(t_i) &= [0, 1, 0, \ldots, 1] \end{aligned}$$

Reversely the same situation is represented by the column vector $V_{km}(t_i) = [e_{1,m}(t_i), e_{2,m}(t_i), \ldots e_{k,m}(t_i)]$.

Parameters are constant *per instant*, i.e. for all instantaneous different situations.

An interesting research issue could consider the four classes of *mesoscopic dynamics* as possibly conceptually related to the four classes of cellular automata introduced by Wolfram (Wolfram, 2002) as in Table 3.5.

Table 3.5 Four classes of cellular automata

Classes	Kinds of evolution
Class 4	Emergence of local and surviving dynamic structures
Class 3	Chaotic evolution. Spread randomness
Class 2	Evolution into stable or oscillating structures. Local randomness
Class 1	Evolution into stable, homogeneous structures

3.8.5 Structural Regimes of Validity

A number of possible *regimes of structural validity* should be considered for the behaviour of agents interacting by respecting degrees of freedom, whether single, multiple, fixed or variable. As considered in Sect. 3.8.2 and discussed above, there are various possibilities, at least the four listed in Table 3.4.

Elementary examples of *extreme* structural regimes of validity are given by:

1. Usage of the *same* rule of interaction by *all* interacting agents.
2. Usage of the *same* rule of interaction by subsets of interacting agents.

   In this case various options are possible, such as:

   - Single *fixed* subsets or clusters of agents using the *same* rules of interaction over time, the rules being different from subset to subset.
   - Single *fixed* subsets or clusters of agents using the *same* rules of interaction per instant, the rules being different from subset to subset.
   - *Variable* single subsets or clusters of agents using the *same* rules of interaction over time, the rules being different from subset to subset and varying per instant.

   It should be noted that subsets or clusters can have any intersection or diffusion, while the same agents may even belong to more than one subset.
3. The usage of different rules of interaction may be variable and multiple. In this case, fixed or variable subsets or clusters of agents use the rules of interaction by following *specific,* whether *fixed or variable, modalities*, such as:

   - *Regular* repetition of different rules per single agent while the rules used may be single or multiple.
   - *Regular* repetition of different rules per fixed, or possibly variable, subsets or clusters of agents.
   - Probabilistic assumption of different rules per fixed or possibly variable subsets or clusters of agents.

In this view, the *minimum degree of freedom* for structures or, better, for a structural regime of validity, is given by case 1.

The *maximum degree of freedom* is given by the *random* adoption of different rules for any subsets or clusters of agents.

Properties of structural regimes (see Table 3.6) are significant when related to the area *between* such extremes and when having some regularities such as

Table 3.6 Elementary structural regimes

Single structural regime	At each step all the agents will interact according to one of the 13 rules valid for all.
Multiple structural regime	At each step each agent can choose which of the 13 rules should be used to interact.
Multiple, fixed and superimposed structural regimes	At each step each agent can choose to interact with $m > 1$ of the *13* rules. The number *m* is constant for all agents per instant.
Multiple, variable and superimposed structural regimes	At each step each agent can choose to interact with *any* $s > 1$ of the 13 rules. The number *s* is variable per agent.

periodicities or distributions *between* such extremes. Given the 13 rules of interaction listed in Table 3.1, we can summarise as shown in Table 3.4.

Such structural regimes may be valid with various combinations and timings in an inhomogeneous way. At this point, we note that rules of interactions and adoptions of structural regimes of validity do not ensure the *uniqueness* of the global configuration identified at time t_{i+1} nor *coherence(s)* among sequences of configurations.

The coherence between configurations is considered here as being given and represented by the validity of suitable properties, possibly meta-structural properties as in Sect. 3.8.4. This should be intended as a *degree of freedom* in selecting the structural regimes and their possible combinations. Several configurations may respect suitable current properties or meta-structural properties and are thus *authorized* to occur. This can happen while respecting different structural degrees of freedom.

There are thus $q(t_i)$*-equivalent* configurations for which there must be a strategy of choice.

Sect. 3.8, dedicated to ***Methods and approaches to model and act upon the dynamics of emergence: research on meta-structures****, summarises the research on meta-structures and its modelling. Its purpose is to provide approaches for detecting the establishment of emergence of collective phenomena, their dynamics and possible interventions for modifying them.*

3.9 The Transient

As mentioned above, structural dynamics can be understood as changes between, for instance, phases, ontologies, levels of emergence and properties.

Here, we consider aspects related to the *between* as given by modalities, *properties* of potentialities and boundary conditions, as already mentioned in Sects. 2.4, 2.6 and 2.7.

Focus is on modalities and properties of transience, such as continuity, discretisation, convergence or irregularity.

Networks and meta-structural properties are intended here as a way of representing and prescribing structural properties, modalities and properties of their dynamics.

Networks and meta-structural representations of processes and phenomena are intended to obtain and represent their *structural* invariants, modalities, and their *non-explicit properties*, i.e. non-analytically, that cannot be *zipped*, non-exhaustible in analytical formulas. That is represented by their implicit imprinting intended as *implicit* because it is not represented *symbolically*, but by using networks and meta-structural properties.

On the other hand, approaches suitable to *prescribe* networks and meta-structural properties can be applied to processes having a *significant 'between'* amidst their phases such as processes of emergence. Prescriptions of networks and meta-structural properties are expected to be able to induce and orient complex behaviours by allowing *varieties of equivalences* as for the structural regimes considered above. Prescription of networks and meta-structural properties may be intended as a way to *prescribe a general future*, by prescribing modalities able to ensure the acquisition of *kinds* of properties through the processing of almost any environmental or internal inputs or fluctuations. This may apply, for instance, to complex systems in general whereas it may be not suitable for systems having very tight degrees of freedom as in closed, deterministic systems or devices.

Thus, the focus is on the study and prescriptions of equivalences, by setting meta-structural and network levels where alternatives may become equivalent. Here, two phenomena should be mentioned:

- *Meta-structural transience* where the transience relates to the acquisition, change or loss of a specific meta-structural property.
- Transience between meta-structural regimes of validity where meta-structural properties are still maintained, but in different ways, i.e. through different parameters in given structural regimes.

These are important lines of trans-disciplinary research dealing with *general systemic properties*, i.e. *properties of properties*, impossible to deal with in the context of GOFS.

Examples of non-explicit prescription consist of varying meta-structural properties as presented in Sect. 3.8.4. For instance, by using *mesoscopic slaving* as introduced in Sect. 3.8.4.3; properties of Networks of mesoscopic variables as mentioned in Sect. 3.8.4.4 (see Chap. 8); by *inserting* a Perturbative Collective Behaviour *within* the collective behaviour to be influenced as in Sect. 3.8.4.5; or by acting upon properties such as parameters of synchronisation, correlation or usage of degrees of freedom, and environmental as in Sects. 3.8.3.1 and 3.8.3.7.

3.10 Further Remarks

This concluding section focuses upon general aspects considered in this chapter and considers possible future research.

Identity and meaning is considered from a dynamical point of view, i.e. through the properties of dynamics such as coherence or meta-structural properties.

The ontological meaning of existence should here be considered as the properties of change. By adopting the sayings of *Heraclitus*, we can consider change as *coming first* as the quantum vacuum precedes matter, not being simply the lack of matter. Levels and states should be intended as *simplifications* at certain levels of descriptions.

Emergence could be intended as *normality* represented using simplified levels assumed to be states, with their changes and dynamics considered as the dynamics among those states.

Emergence could be intended as coming first, as a property of *pre-matter*, of the vacuum. The quantum void could thus be intended as a kind of field of potentialities ready to collapse but always pervasive as are the probabilistic features of Quantum mechanics (QM).

The identity of matter should then be given by the properties of levels where one can consider ontological being and non-being.

What are the advantages of considering such approaches and assumptions? The idea is that in the new, post-GOFS the standard is not given by the statics, its states and their properties but by a continuous flux of change and the properties of its dynamics, the static option being a mere simplification. The reconstruction of the dynamics from given states will be very complicated for complex systems whereas it could be simplified by choosing the reverse. The same is true when considering openness starting from closed systems, intending *openness as non-closeness, rather than the reverse*.

Such comments should be considered as a preview of the need for new *tools* to describe dynamics in mathematics other than the classical approach.

We conceptually refer to approaches where dynamics, openness and environment ***come first*** *and then a state, closure and bodies can be defined* ***through*** *them.*

Examples are given by representations of change not by states but, rather, through the properties of the change as for networks, meta-structural properties and structural regimes. In the dynamics of such change, several microscopic configurations are *equivalent* and possible.

Such properties are also able to prescribe microscopic behaviour, e.g. topological distance and number of links, other than that given by the classical fixed *degrees of freedom* as for macroscopic properties.

Box 3.1: Mermin-Wagner theorem

States that, in QM and QFT, in dimensions ≤ 2 *within systems with sufficiently short-range interactions, continuous symmetries cannot be spontaneously broken at finite temperature*[1]*,* i.e. *long-range fluctuations can be created with little energy cost and they are favoured since increase the entropy*. This allows an understanding of why it is impossible to have phase transitions in a one-dimensional system, and it is nearly impossible in a two-dimensional system.

In general, reduction in the number of degrees of freedom increases stability. For instance, by constraining a spiral motion to lie only on a two-dimensional plane, escape along the third dimension (and whence the loss of stability) would be precluded. It can thus be considered that, in general, *3D* CollectiveBehaviours is an entity which, in principle, is more stable than its local constituent parts, and this stability is, in turn, granted only by the constraints defining it.

This explains, using a further example, why some collective behaviours, such as those of two-dimensional flocks, seem to violate this theorem (Mermin & Wagner, 1966). This occurs because a flock exists and survives as a consequence of suitable constraints between the motions of individual birds belonging to it and the presence of these constraints lowers the dimensionality of the available phase space, in turn increasing the stability of the whole system and rendering untenable the thermodynamic arguments upon which the Mermin-Wagner theorem itself is based.

[1]In order to allow that heat exchange takes place between two bodies, a finite difference of temperature between them is required, even if ideally this difference temperature may be infinitesimal. In the later case for exchanging a finite amount of heat are necessary a surface infinitely extended or infinite time. The concept applies in thermal quantum field theory or finite temperature field theory.

Box 3.2: Theorem of Smale

It was shown that, on increasing the number of variables and parameters, it became impossible to group the patterns of change into a small number of categories (e.g. Arnold, Afrajmovich, Ilyashenko, & Shilnikov, 1999). This circumstance, already present in previous and celebrated theorems such as that of Smale on *structural stability* (see, e.g. Arnold, 1988; Palis & de Melo, 1982; Smale, 1966), practically dominates the world of chaotic phenomena and of partial differential equations.

In short: *given a system of dynamic equations that describe the evolution in time of the values of at least three dependent variables, the probability that it has chaotic solutions is infinitely close to 1.*

Box 3.3: Metastability
Once identified a global equilibrium state (in principle we could have more different states of this kind), in some contexts called a *ground state*, all other equilibrium states are called *metastable equilibrium states*. This term denotes the fact that all these states, in presence of fluctuations, have a finite lifetime, as there will be a nonzero probability of having a fluctuation of such amplitude that ~~it~~ will push the system outside the basin of attraction of the metastable equilibrium state, letting it to fall into the global equilibrium state. If the latter is unique, its lifetime in presence of fluctuations will be instead infinite, as every fluctuation, even if putting temporarily the system into the basin of attraction of a metastable equilibrium state, will be first or later counterbalanced by another fluctuation letting the system abandon the metastable situation and fall again in the global equilibrium state. For this reason the metastable equilibrium states are also called *far from equilibrium stationary states*.

A typical example of stationary state far from equilibrium is given by the case of Bénard cells. When the considered system, in order to manifest Bénard instability, gradually moves away from equilibrium (equilibrium in this case is when there is uniform temperature in the whole liquid), it reaches a critical instability point where the so-called Bénard cells, ordered hexagonal cells, honeycomb-like, emerge.

The most celebrated examples of systems lying in far from equilibrium states is given by the *dissipative structures* introduced by Prigogine and his school.

More generally when while at short time scales the system appears to be in a quasi-equilibrium, i.e. metastable state, at longer time scales rapid transitions, induced by random fluctuations, between meta-stable states occur (Antman, Ericksen, & Kinderlehrer, 2011; Kelso, 2012; Tognoli & Kelso, 2014).

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