Instrumentation, Electronics, and Signal Analysis

16

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Abbreviations

τ	Tau
AC	Alternating current
ACNS	American clinical neurophysiology society
ADC	Analog-to-digital conversion
C	Capacitor
CMRR	Common mode rejection ratio
dB	Decibels
DC	Direct current
DR	Dynamic range
EEG	Electroencephalography
ELI	Voltage–Inductor–Current
HFF	High-frequency filter
HPF	High-pass filter
ICE	Current–Capacitance–Voltage
L	Inductor
LFF	Low-frequency filter
LPF	Low-pass filter
R	Resistor
RC	Tau
X_C	Capacitive reactance
X_L	Inductive reactance
Z	Impedance

Introduction

In order to understand the significance of polysomnographic (PSG) and other clinical neurophysiologic tests to practice first-rate sleep medicine, it is important to have a basic knowledge about the principles of physics and electronics underlying the techniques for recording multiple physiological characteristics. Biological, physical, and chemical environment of the body tissues (e.g., brain, heart, lungs, and others) continually generates electromagnetic signals movements of which tell us about internal physiological changes in the body from which we can differentiate normal from abnormal phenomena. The physiological signals are minute in magnitude and hence must be amplified to recognize them visually. Amplification of essential signals and filtering of unwanted signals are the two most fundamental processes in understanding PSG and other neurophysiologic recording techniques. It should be noted that electrical signals in human body manifested as waveforms are generated by flow of charged ions (e.g., Na, K, Cl) opposed by the resistance and capacitance of the tissues. Using analog and now digital electronic devices, we can measure and analyze current flow and potential differences between different areas of the body and scalp to assess normal functions and alterations by disease. This chapter briefly outlines the basic electronics for sleep specialists and clinical neurophysiologists.

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Definitions and Circuit Analysis

Charge

In a copper wire at room temperature with no externally applied force, the outermost shell of a copper atom loses an electron as a result of surrounding thermal energy. When a copper atom loses a free electron, it becomes a positive ion because electrons carry a negative charge. With no applied force on the wire, electrons continue to move about in a random manner without a net flow. As the electron moves about randomly the positive ion it leaves behind does little more than oscillate in place; the electron thus acts as a charge carrier. Coulombs law states that like charges repel and opposite charges attract and is the reason why free negatively charged electrons are attracted to positive ions. The symbol for charge is "Q." One coulomb of electrons is equal to 6.242×10^{18}. If one coulomb of electrons flows at uniform velocity through a circular cross-sectional area of conductor in one second, the flow of charge is said to be one ampere. Charge or Q may also be expressed as shown in Eq. 16.1 where I equals current and t is time in seconds [1–3]. For current to flow and perform work in a circuit, such as to light an incandescent bulb, electrons must move in the same direction through the load. Recording an electrographic event requires the movement of ions in large populations of neurons. An applied force causes the electrons or ions to move directionally, and in a basic electric circuit, this applied force is the source voltage also referred to as an electromotive force. In the case of a neuron, the applied force is the stimulus (spontaneous or applied) with the electrons replaced by ions. The flow of charge regardless of form is accomplished through the application of an applied force.

$$Q = I * t \tag{16.1}$$

Power Sources

The two types of power sources are alternating current or AC and direct current or DC. The most commonly encountered DC source is the battery. A battery is the conversion of chemical or solar energy using positive and negative electrodes and electrolytes to provide direct current. Direct current moves in one direction and remains constant for the life of the source. Generators on the other hand produce AC power through the use of any number of energy sources to turn a rotor housed in a set of windings called a stator, inducing a voltage in the wires of the stator. One end product is the common 120-V household outlet. In North America, AC power is delivered at 60 Hz, whereas in Europe 50 Hz is the predominant frequency. Both AC and DC provide the electromotive force to move electrons directionally supplying current to power the recording equipment.

Fig. 16.1 Schematic symbol of a resistor

Resistors and Resistance

Resistance is the ability to inhibit the flow of current or charge. Resistance is one of the things that slows down electron movement described earlier. While copper wire has resistive properties, it has better conductive properties which make it ideal for movement of charge. Resistance occurs naturally, and resistors are specifically manufactured for use in circuit design. Resistors behave the same regardless of the type of power applied. The current through a resistor is in phase with the applied voltage. This means both current and voltage follow the same path at the same time. Resistors do not store energy, and they dissipate energy through heat. Figure 16.1 illustrates the schematic symbol of the resistor. The unit of resistance for a resistor is ohms and is indicated by the capital Greek symbol omega "Ω." Resistors can be connected in parallel or series.

The manner in which resistors are connected determines the total resistance of the circuit. Figure 16.2a, b illustrates simple series and parallel resistive circuits, respectively, arrows indicating direction of voltage and current. Notice that in the series circuit, the voltage has only one path to travel; it must pass through each resistor in an orderly manner. In the parallel circuit, the full voltage is applied to each resistor at the same time. When voltage is applied, it pushes the electrons in a uniform direction; in Fig. 16.2a, b, it is through the resistors of the circuit. When electrons are pushed, they develop a charge or Q which is current over time as shown in Eq. 16.1.

As voltage passes through each resistor in the series circuit, some of the supply voltage is lost because the resistor dissipates the supplied energy in the form of heat and this is called voltage drop. The amount of voltage dropped or consumed by each resistor is determined by the size of the resistor and the charge or current through it. In Fig. 16.2a, resistor R_1 receives the full voltage and is the first to consume some of voltage dissipating the energy as heat. Resistor R_2 receives the voltage left over following the voltage drop from R_1, and it too consumes voltage passing on to R_3 the remaining voltage leftover from the source.

In the parallel circuit, the full voltage is impressed upon each resistor immediately. Each resistor drops voltage, and how much voltage is consumed by each resistor is dependent

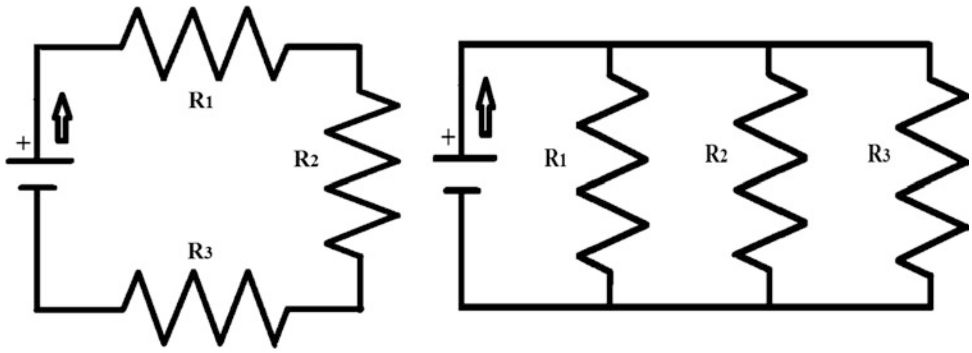


Fig. 16.2 Resistors in series in (**a**) and in parallel in (**b**); *arrow* indicates direction of voltage and current flow

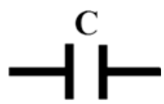


Fig. 16.3 Schematic symbol of a capacitor

upon the resistive value and current through each branch. The relationship between current, voltage, and resistance is explained and easily calculated by Ohm's law discussed further on.

Calculation of the total amount of resistance in a given circuit is different for series or parallel configurations. In a series circuit, the resistance value of each resistor is added to obtain the total resistance of the circuit. The total resistance in a series circuit is higher than the highest value of any single circuit resistor. In a parallel circuit, total resistance is calculated by adding the reciprocal of each individual resistor and then taking the reciprocal of that sum. The total resistance in a parallel circuit is lower than the lowest single resistive value in the circuit. Equations 16.2 and 16.3 illustrate the formulas used to calculate the total resistance in series and parallel resistive circuits, respectively [1–4]. Whether in parallel or series circuit, the current and voltage in a purely resistive circuit are in phase with one another. This means they both follow the same path at the same time.

$$R_T = R_1 + R_2 + R_3 + \cdots + R_n \quad \textbf{\textit{Series Circuit}} \tag{16.2}$$

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}} \quad \textbf{\textit{Parallel Circuit}} \tag{16.3}$$

Capacitors and Capacitance

Capacitors are formed by two metal plates or any two conducting surfaces that are separated by a dielectric. A dielectric is an electrical insulator, e.g., air is a dielectric. Two parallel ribbons of wire on a printed circuit board create a capacitor. Power lines running parallel between poles create capacitors by virtue of the definition of a capacitor. An electrode on the surface of the body creates a capacitor. Capacitors are no more than two conductive surfaces separated by a dielectric. There are additional criteria to further define the properties of capacitance; however, this discussion is limited to series and parallel configurations of capacitors and the resistive value they pose in a circuit based on applied frequency. Figure 16.3 shows the schematic symbol of a capacitor.

A capacitor is rated in farads, "*F*" described below, its symbol is "*C*," and the resistance it poses to a circuit is called capacitive reactance symbolized by X_C (X of $_C$) and is calculated as shown in Eq. 16.4 [1, 4].

$$X_C = \frac{1}{2\pi fC} \tag{16.4}$$

Similar to a resistor, the capacitive reactance or X_C is indicated in ohms "Ω," and unlike a resistor, the capacitive reactance of a capacitor changes with the applied frequency. In Eq. 16.4, the only variable that is not a constant is the frequency or "*f*," the remaining terms are fixed with 2π equivalent to 6.28, and C is the value of the capacitor in farads. In the case of a DC source, "*f*" is equal to zero. If the denominator becomes zero, the resistance is infinite. If that is true what happens when a capacitor is connected across a DC source such as a battery? Electrons move between the battery terminals and the metal surfaces of the capacitor until the capacitor is fully charged with one of the metal conductive plates (positive or negative). You now have an additional power source. How long does this take? The answer depends on time constants. It is important that you understand the behavior of capacitors. Capacitors are effectively an open circuit with a DC source after charging up to their final capacitive value [1, 4].

We still have to account for the energy applied to a capacitor because the law of conservation of energy dictates that energy cannot be lost. Unlike resistors which dissipate energy in heat and drop voltage, capacitors store charge. The ability of a capacitor to store charge is termed capacitance, and its units are farads (*F*), the higher the rating (*F*), the more charge it can store. The symbol of the charge stored on

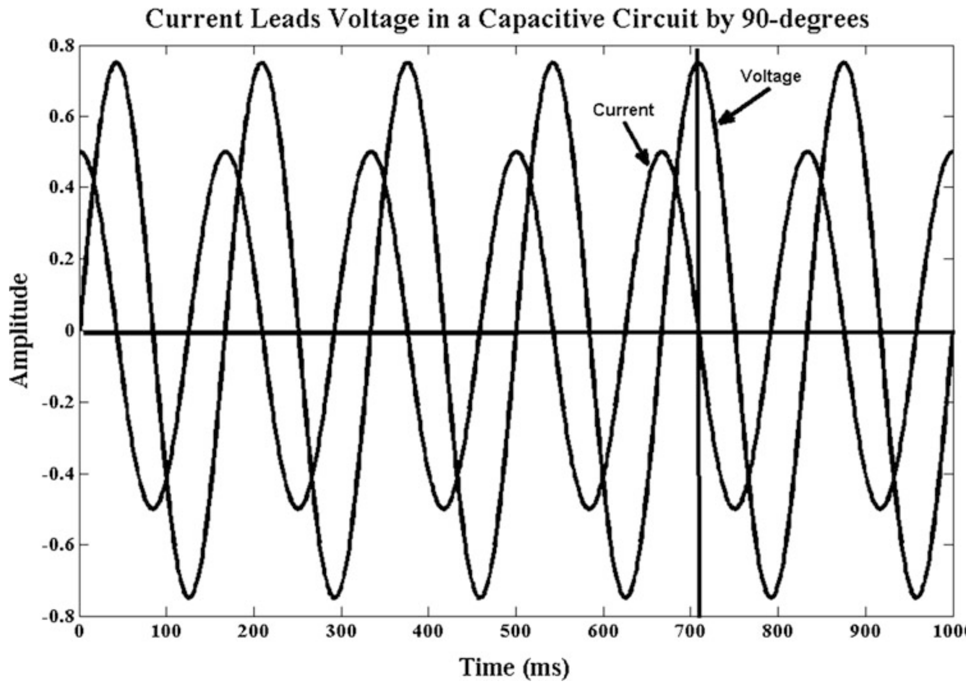


Fig. 16.4 The relationship between current and voltage in a purely capacitive circuit. Voltage lags current by 90°. When the voltage waveform (higher amplitude) is at its peak, the current waveform (lower amplitude) is crossing zero, and this equates to 90°. This is illustrated by the *vertical line* just beyond the 700 ms point. Current flow began 90° prior to any voltage change on the capacitor. Code and output created using MatLab R2012a

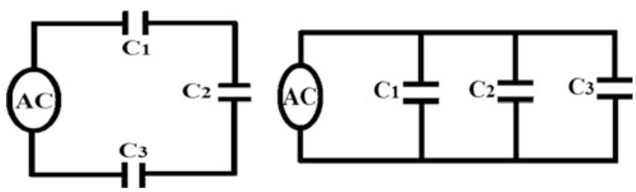


Fig. 16.5 Circuit configurations of capacitors in series (**a**) and in parallel (**b**)

a capacitor is termed "Q," because it is electrons that move charging the capacitor. It is the nature of capacitors to resist a change in voltage across them; therefore, the applied voltage lags the current in a purely capacitive circuit by 90°. Figure 16.4 illustrates the phase difference between voltage and current in a purely capacitive circuit. Equation 16.5 states the law of capacitors, where C is the capacitor in farads, Q is the charge, and V is the applied voltage [1, 4].

$$C = \frac{Q}{V} \tag{16.5}$$

Figure 16.5 illustrates capacitors in series (a) and parallel (b) left to right, respectively. In a series, circuit capacitors add like resistors in parallel that is the total capacitance in the circuit is lower than the lowest capacitive value in the entire series circuit. Capacitors in parallel, as you may have guessed, add like resistors in series that is the total capacitance in the circuit is higher than the highest capacitive value in the entire parallel circuit. Formulas for total capacitance in parallel and series circuits are shown in Eqs. 16.6 and 16.7, respectively [1, 3, 4].

$$C_T = C_1 + C_2 + C_3 + \cdots + C_n \quad \textbf{\textit{Parallel Circuit}} \tag{16.6}$$

$$C_T = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_n}} \quad \textbf{\textit{Series Circuit}} \tag{16.7}$$

Table 16.1 illustrates the changes in resistance of a 1 μF (10^{-6}) capacitor as the applied frequency increases. Equation 16.4 was used to calculate the change in resistance. It is easily seen that there is a dramatic change in the resistance offered to the circuit by the capacitor as the frequency increases even slightly. This is due to the alternating positive and negative phases of an AC power source. As the alternating phases of the input signal change, more and more rapidly the capacitor ceases to become a resistive factor in the circuit.

Inductors and Inductance

Inductors are coils of wire. The coils of wire can be hollow or the coil can be wrapped around a magnetic or non-magnetic core. Inductors are rated in henries and are indicated by a capital H and are identified in a circuit by capital L. Figure 16.6 illustrates the circuit schematic symbol for an inductor.

Inductors also store energy, but in a magnetic field, they cannot, however, store energy in the absence of a power source. Once the power source is removed from an inductor, it releases its stored energy. Unlike a capacitor, the resistance of an inductor is directly proportional to the applied

Table 16.1 Capacitive reactance as a result of applied frequency

Capacitor in farads (μF)	Applied frequency	Capacitive reactance (X_C)
1	0.5 Hz	318.3 kΩ
1	1 Hz	159.2 kΩ
1	5 Hz	31.8 kΩ
1	15 Hz	10.6 kΩ
1	20 Hz	8 kΩ
1	25 Hz	6.4 kΩ
1	50 Hz	3.2 kΩ
1	100 Hz	1.59 kΩ
1	1 kHz	159 Ω
1	10 kHz	15.9 Ω
1	100 kHz	1.59 Ω

Fig. 16.6 Schematic symbol of an inductor

frequency of the source. The nature of an inductor is to resist a change in current through it; thus, current lags voltage in a purely inductive circuit by 90°. Like a capacitor, the resistance of an inductor, called inductive reactance which is rated in ohms, is termed X_L (X of $_L$) with symbol Ω and is calculated as indicated in Eq. 16.8 [1].

$$X_L = 2\pi f L \tag{16.8}$$

There is only one variable in Eq. 16.8 that is not a constant, and this is "f" or the applied frequency, 2π is equivalent to 6.28, and L is the measure of the inductor in henries. The phase relationship between an inductor and capacitor is 180°.

Representative circuits for inductors are not shown. The circuit configuration for inductors in series and parallel is identical to the circuit configurations previously shown for resistors and capacitors with the simple change of circuit components to inductors. Figure 16.7 illustrates the output of an inductive circuit with voltage leading current by 90°; this is identical to Fig. 16.4, but voltage and current have changed places. This phase relationship exists because inductors resist a change in current through them so current lags voltage.

It should be remembered that the physical properties of a capacitor cause it to resist an instantaneous change in voltage across it and that an inductor resists an instantaneous change in current through it. This is why each of these properties lags in their respective circuit as illustrated in Fig. 16.7 for inductors and Fig. 16.4 for capacitors.

Circuit calculations for the total inductance of inductors in series or parallel configurations are identical to those of resistors in the same configuration and are shown in Eqs. 16.9 and 16.10, respectively [1].

$$L_T = L_1 + L_2 + L_3 + \cdots + L_n \tag{16.9}$$
Series Circuit

$$L_T = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_n}} \tag{16.10}$$
Parallel Circuit

Cartesian Plane

A Cartesian plane is a coordinate system with four quadrants that are labeled with Roman numerals I–IV counterclockwise beginning in the upper right quadrant. The "X" or real axis is the horizontal plane, and the "Y" or imaginary axis is the vertical plane. The zero point in the Cartesian coordinate system is where the X and Y axes cross. All X-coordinate points to the left of zero are negative; all X-coordinate points to the right of zero are positive. All Y-coordinate points below zero are negative; all Y-coordinate points above zero are positive. This coordinate system allows for the unique identification of points in space with two specific coordinates, an X value and a Y value. These points are always shown in that order and usually in parentheses as (X, Y). For instance, a point on the Cartesian coordinate system of (3, −3) indicates that it is located three positive units along the X axis horizontally and three negative units along the Y axis vertically, so the point is located in the lower right quadrant or quadrant IV. Figure 16.8 shows an illustration of a Cartesian coordinate system or Cartesian plane.

Here, the Cartesian system is used to create an impedance diagram by plotting resistance, capacitive reactance, and inductive reactance using vectors. Vectors are lines with arrowheads that indicate direction and magnitude. The longer the vector the greater the magnitude of the representative quantity; the arrowhead is an indication of direction. Discussion of phase is not addressed other than what

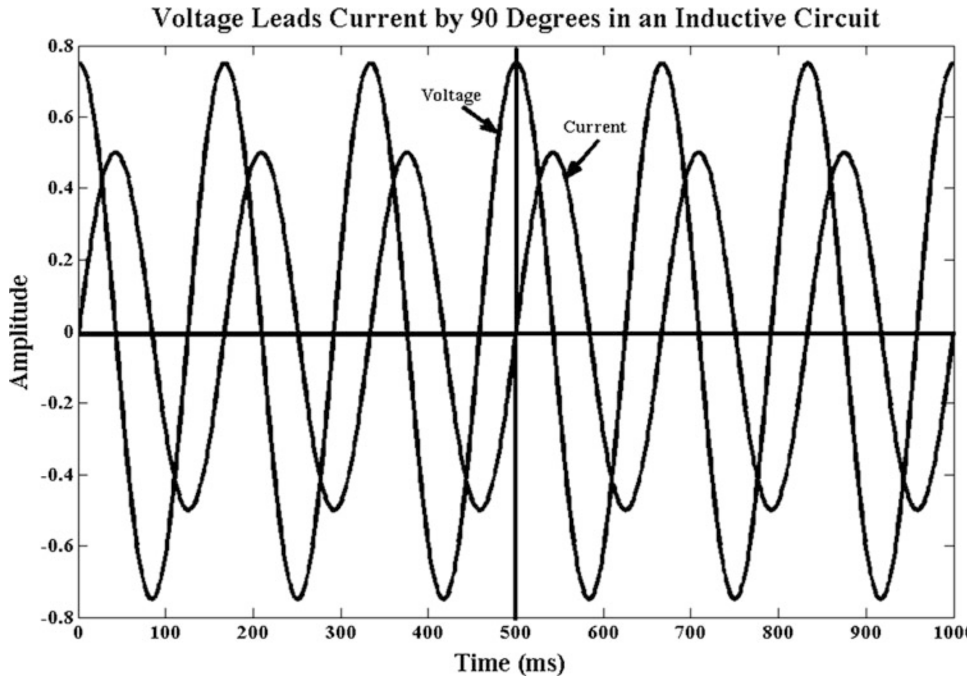


Fig. 16.7 The relationship between current and voltage in a purely inductive circuit. Voltage leads current by 90°. When the voltage waveform (higher amplitude) is at its peak, the current waveform (lower amplitude) is crossing zero, and this equates to 90°. The *vertical line* through the 500 ms point illustrates this point. The voltage through the inductor began 90° prior to current flow. Code and output created using MatLab R2012a

has previously been described in resistive and purely capacitive or inductive circuits. Resistance R is always plotted on the positive X axis, capacitive reactance X_C on the negative Y axis, and inductive reactance X_L on the positive Y axis all by vectors. The phase relationship between inductive and capacitive reactance is 180° which is why X_L points up and X_C points down. The length of the vector indicates the magnitude of the resistance in each case. Because X_C and X_L are 180° out of phase, the smaller value is subtracted from the larger value to determine their combined reactance. The direction, pointing up or down in the impedance diagram, is dependent upon the larger value. The resultant vector will point up if X_L is larger than X_C or will point down if X_C is larger than X_L and should be scaled to indicate the relative reactance value. In Fig. 16.8, X_C is a longer vector than X_L, so capacitive reactance has a higher value than inductive reactance leaving a resultant X_C vector on the Y axis pointing downward. Drawing a straight line from the arrowhead of the resultant vector, here it would be X_C to the vector representing R forms a right triangle. This new line is the hypotenuse of a right triangle, and Pythagoreans theorem shown in Eq. 16.11 is used to calculate this value which is the total impedance of the circuit indicated by the capital letter "Z" [1, 4, 5].

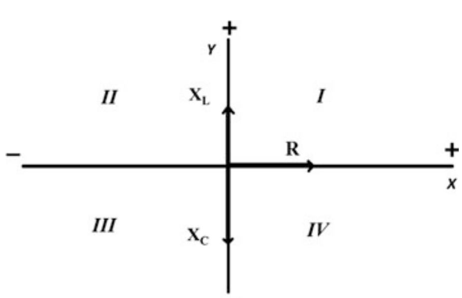


Fig. 16.8 Cartesian coordinate system with labeled quadrants and vectors indicating resistance "R," inductive reactance "X_L," and capacitive reactance "X_C"

In a circuit containing R, X_L, and X_C, the phase angle between voltage and current is less than 90° and depends on the relative size of the resistor compared to the resultant reactance.

$$Z = \sqrt{(R^2 + (X_{\text{Larger}} - X_{\text{Smaller}})^2)} \qquad (16.11)$$

Impedance Z is a complex value; in simple AC circuits such as those with a single voltage or current source, Eq. 16.11 will suffice, but for complex circuitry, more sophisticated circuit analysis is required.

Ohm's Law

Ohm's law is the equation that defines the relationship between resistance, voltage, and current. Equation 16.12 illustrates the Ohm's law voltage equation where "R" is resistance, "E" is voltage, and "I" is current, voltage may be shown as "V," and this does not change the formula [1–4].

$$E = I * R \qquad (16.12)$$

From Eq. 16.12, it is clear that voltage (E) is directly proportional to resistance and current. Manipulation of this basic formula reveals that both current (I) and resistance (R) are proportional to the applied voltage and inversely proportional to one another. These relationships are illustrated in Eqs. 16.13 (a) and (b) for current and resistance, respectively [1, 3, 4].

$$\underset{(\mathbf{a})}{I = \frac{E}{R}} \quad \underset{(\mathbf{b})}{R = \frac{E}{I}} \tag{16.13}$$

In Eq. 16.13 (a), for a fixed resistance as the applied voltage increases so does the current. Conversely, if the applied voltage remains the same and the resistance is increased, the current decreases. Using Eq. 16.13 (b), the applied voltage and current resistance are easily calculated.

Power

Power in an electrical circuit is a measure of the rate of performing work and is measured in watts, indicated by a capital "W," and is calculated by the formula in Eq. 16.14 [1]. A light bulb rated 60 W utilizes 60 W of energy, and a 120-W bulb provides more light and consumes more energy.

$$P = E * I \tag{16.14}$$

Through direct substitution of Ohm's law, Eq. 16.13 (a) for current, and Eq. 16.12 for voltage, power can be expressed as indicated in Eqs. 16.15 (a) and (b), respectively.

$$\underset{(\mathbf{a})}{P = \frac{E^2}{R}} \quad \underset{(\mathbf{b})}{P = I^2 * R} \tag{16.15}$$

Kirchhoff's Voltage and Current Laws

Kirchhoff's laws will be defined simply without consideration or explanation of power supply orientation or polarity of any circuit devices. Kirchhoff's voltage law states that the sum of the voltage drops in a closed loop must equal the applied source. Kirchhoff's current law states that the current into a junction, node, or system must equal the current exiting that same junction, node, or system [1–6]. Energy must always be accounted because it cannot be lost. Kirchhoff's voltage and current laws are mathematically stated in Eqs. 16.16 (a) and (b), respectively, and illustrated graphically in Fig. 16.9.

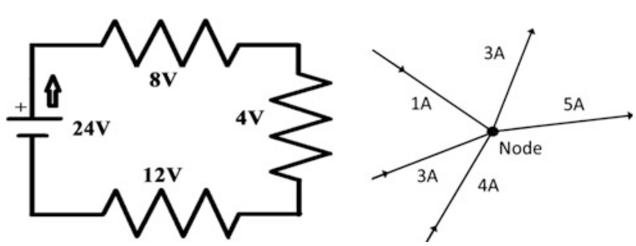


Fig. 16.9 Kirchhoff's voltage law in **a** the sum of the voltage drops 8 V + 12 V + 4 V = 24 V, which equals the source voltage. In **b**, three current values are entering one central node (*arrows* pointing into the node) and two exiting (*arrows* pointing away from the node) current pathways. The sum of the input current is 8 A which equals the sum of the output current meeting the requirements of Kirchhoff's current law

$$\begin{matrix} \Sigma_{\text{Current In}} = \Sigma_{\text{Current Out}} & (\mathbf{a}) \\ \Sigma_{\text{Source}} = \Sigma_{\text{Voltage Drops}} & (\mathbf{b}) \end{matrix} \tag{16.16}$$

Frequency and Period

Frequency is a rate quantity measured in cycles per second. Frequency is an indication of how often something happens in one second time. The symbol for frequency is lower case "f," and its units are Hertz, abbreviated Hz. Period is a time quantity and is a measure of the time it takes for one cycle of a periodic waveform to occur. The symbol for period is capital "T" and is expressed as a unit of time. Frequency and period are inversely related, as one increases the other decreases. The period of a signal with a frequency of 60 Hz is 1/60th of a second. This means there are 60 cycles per second in a 60 Hz periodic signal, and it takes 1/60th of a second for one of those cycles to occur [1]. If you know one quantity you know the other. Equation 16.17 indicates the inverse relationship between period and frequency.

$$f = \frac{1}{T} \quad T = \frac{1}{f} \tag{16.17}$$

When identifying frequency of any waveform, verify the time line and determine the frequency in one second. Figure 16.10 shows a 3 Hz signal with one period indicated.

Decibels, Logarithms, Gain, and Bode Plots

These topics are discussed briefly because filter frequency response curves are shown in logarithmic scale in "Bode" plots, filters are designed with a cutoff stated in decibels, and gain is usually indicated in decibels. Gain is the output of an amplifier; in neurodiagnostic technology, this is termed a differential amplifier which is a sophisticated device made up of resistors, capacitors, and transistors. When transistors are used in filter design, they are called active filters, whereas passive filters only use resistors and capacitors in their design.

The voltage gain of a circuit denoted "A_V" is a ratio of the output voltage to the input voltage as shown in Eq. 16.18 (a). Gain is a measure of how much larger the output is

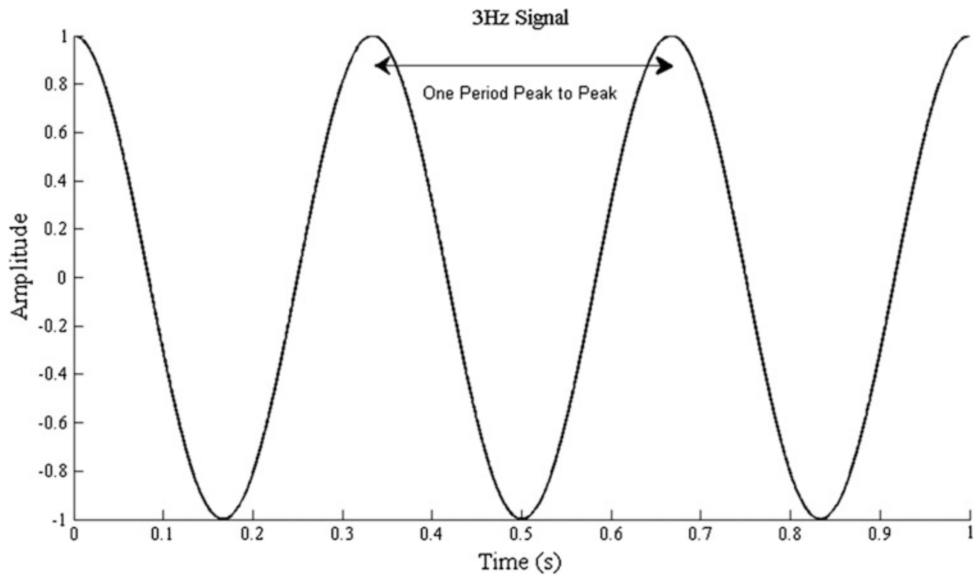


Fig. 16.10 3 Hz signal, its period from one peak to the next, is indicated by the *double arrow*, and this equals one cycle whose period is 1/3 s the inverse of the signal frequency. Code and output created using MatLab R2012a

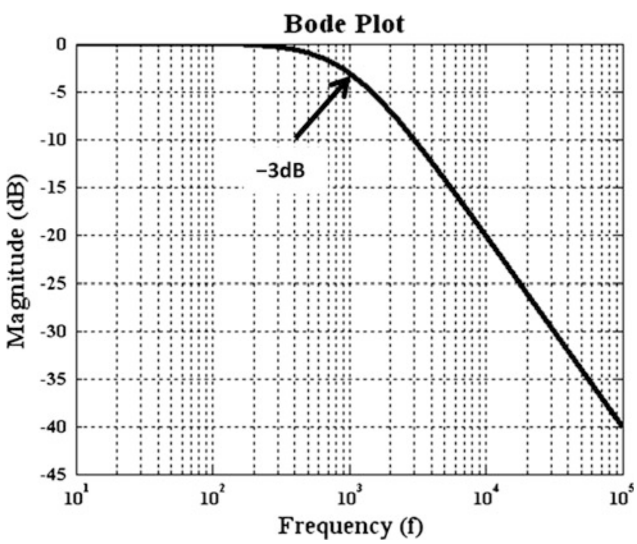


Fig. 16.11 The "*y*" axis in the Bode plot indicates magnitude in dB, and a logarithmic "*x*" axis indicates frequency. Maximum signal magnitude begins at "0" and slopes negative as the frequency increases. The −3 dB point or filter cutoff is indicated by the *arrow*, and this is a high-frequency filter (HFF) with cutoff of 1 kHz. Code and output created using MatLab R2012a

compared to the input and is a term used in reference to amplifiers. The voltage gain of an amplifier expressed in decibels (dB) is found by using logarithms as illustrated in Eq. 16.18 (b) [1, 7].

$$\underset{(\mathbf{a})}{A_V = \frac{V_{\text{Out}}}{V_{\text{In}}}} \quad \underset{(\mathbf{b})}{A_{V\text{dB}} = 20\log\frac{V_{\text{Out}}}{V_{\text{In}}}} \tag{16.18}$$

The technical specifications of a machine can indicate the gain in decibels (dB) or as a whole number, i.e., 100,000 that can be obtained by using Eqs. 16.18 (a) and (b). For example, given 1 μV input and an output of 0.1 V, the gain would be 100,000 or 100 dB.

Bode plots represent the frequency response of a filter and are plotted on a log frequency axis. An example of a Bode plot is shown in Fig. 16.11; in this example, the cutoff frequency of the filter is 1 kHz and is indicated on the plot as the −3 dB point.

There are several names for the cutoff of a filter. We already are familiar with −3 dB; there is also f_C for frequency cutoff, and half power point, and there are others. The term half power point means that the effective value of the voltage is 0.707 V, and the signal is now equivalent to half its original power. Rather than introducing additional equations, I will illustrate this property using the power formula of Eq. 16.15(a). In reiteration, I have just stated that −3 dB is equal to the half power point of the signal, and the output effective value has dropped to 0.707 V of its full value [1, 6, 7]. By inserting 0.707 into Eq. 16.15(a) as shown in Eq. 16.19, the output at −3 dB is half the power of the input.

$$P = \frac{E_{\text{Max}}^2}{R} = \frac{(0.707 * E_{\text{Max}})^2}{R} = \frac{0.707^2 * E_{\text{Max}}^2}{R}$$

$$= 0.5 * \frac{E_{\text{Max}}^2}{R} = 0.5 * P \tag{16.19}$$

The Bode plot of Fig. 16.11 is initially flat at zero and does not begin to drop off until the frequency of the signal approaches the filter cutoff. This flat part of the Bode plot is maximum power passing through the filter; any signal in this frequency range is passing through the filter at full amplitude or full power. As frequency increases and nears the −3 dB point, or the filter cutoff, the amplitude of the input voltage drops to 70.7 % of its maximum value. For the Bode plot of Fig. 16.11, the filter cutoff is 1 kHz; if a 2 V 1 kHz signal was passed through the filter, the output would be 1.414 V or 70.7 % of the input amplitude [1]. This is illustrated in Eq. 16.20 and is called **attenuation**. This is how filters work; they are not perfect, and they incrementally remove the power from the signal as the frequency increases by dropping the signal amplitude.

$$2V * 0.707 = 1.414V \tag{16.20}$$

Filters vary in design and can be cascaded to improve attenuation at the filters cutoff point. In this chapter, we discuss simple single-stage filter circuits with a −3 dB cutoff.

Filters

The three types of filters used in neurodiagnostic equipment are high pass, low pass, and notch [2–6]. The low-pass filter (LPF), in clinical neurophysiology, is commonly referred to as a high-frequency filter (HFF) because it eliminates high frequencies allowing low frequencies to pass up to the design cutoff of the filter. The high-pass filter (HPF) is commonly called a low-frequency filter (LFF) because it excludes low frequencies and passes high frequencies. The notch filter is designed to eliminate a specific frequency band. In North America, power is generated at a frequency of 60 Hz, whereas in Europe it is generated at 50 Hz. In equipment design, the notch filter would be set to match the generated frequency, and thus, in North America, the notch would eliminate 60 Hz, and in Europe, it would eliminate 50 Hz.

What separates a LFF from a HFF is the location of the capacitor in the circuit which dictates filter behavior. If a capacitor is the first component the signal encounters, the capacitor will resist a change in voltage across it and have a very high resistance at low frequencies, and so this must be a low-frequency filter. The high resistance at low frequencies causes these signals to be severely attenuated. As the capacitive reactance of the capacitor decreases with increasing frequency, the filter will incrementally pass the higher-frequency components. When the incoming signal frequency reaches the filter cutoff, 70.7 % of the signal amplitude passes (this assumes a—3 dB design). As frequency increases beyond the −3 dB point, increasing amounts of the signals are passed until the full signal strength is passed through the filter.

If a capacitor is second in line in a filter circuit, the design is that of a high-frequency filter. A high-frequency filter will allow low frequencies to pass until the frequency of the incoming signal approaches the design cutoff of the filter. At the filter cutoff, the signal is attenuated to 70.7 % of its amplitude. Amplitude continues to decrease and is attenuated as frequency increases. The frequencies that filters pass are called the pass band of the filter, and those that are attenuated are considered in the stop band of the filter. In any filter, attenuation is indicated in a Bode plot by the slope of the line, the steeper the slope, the higher the percentage of attenuation with increasing frequency. Figure 16.12a, b illustrates the circuit design, frequency behavior, and the stop and pass bands of high- and low-frequency filters, respectively.

The output of a HFF as illustrated in Fig. 16.12a is taken across the capacitor; for an LFF, the output is taken across the resistor as in Fig. 16.12b. The filter behavior at the respective output can be demonstrated through the use of Eqs. 16.21 (a) and (b) as shown in Table 16.2.

$$\begin{gathered} V_{\text{Out}} = V_{\text{In}} * \frac{X_C}{\sqrt{R^2 + X_C^2}} \\ \textbf{(a) HFF} \\ V_{\text{Out}} = V_{\text{In}} * \frac{R}{\sqrt{R^2 + X_C^2}} \\ \textbf{(b) LFF} \end{gathered} \tag{16.21}$$

The cutoff frequency of a filter is determined by the combination of the resistive and capacitive values of the

Fig. 16.12 High-frequency filter circuit (**a**), low-frequency filter circuit (**b**). Frequency behavior, −3 dB and the stop and pass bands are identified for each circuit

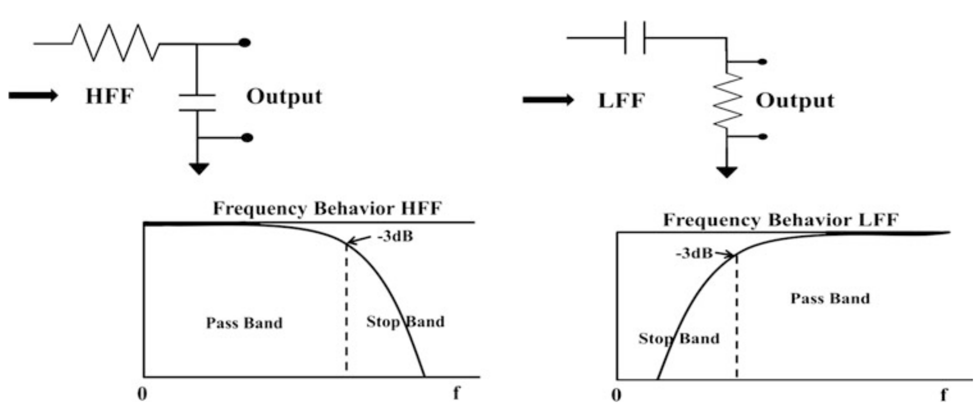


Table 16.2 Filter output at varying input frequencies with V_{In} 1 V, resistance 5 kΩ, and capacitance 50 nF. Equation 16.4 was used to calculate X_C. Equations 16.21 (a) and (b) were used to calculate V_{Out} for the respective filter

Frequency (f)	Capacitive reactance X_C	V_{Out} HFF	V_{Out} LFF
2 Hz	1.59 MΩ	1 V	0 V
200 Hz	15.9 kΩ	0.95 V	0.30 V
2 kHz	1.59 kΩ	0.30 V	0.95 V
20 kHz	159 Ω	0.03 V	1 V
200 kHz	15.9 Ω	0 V	1 V

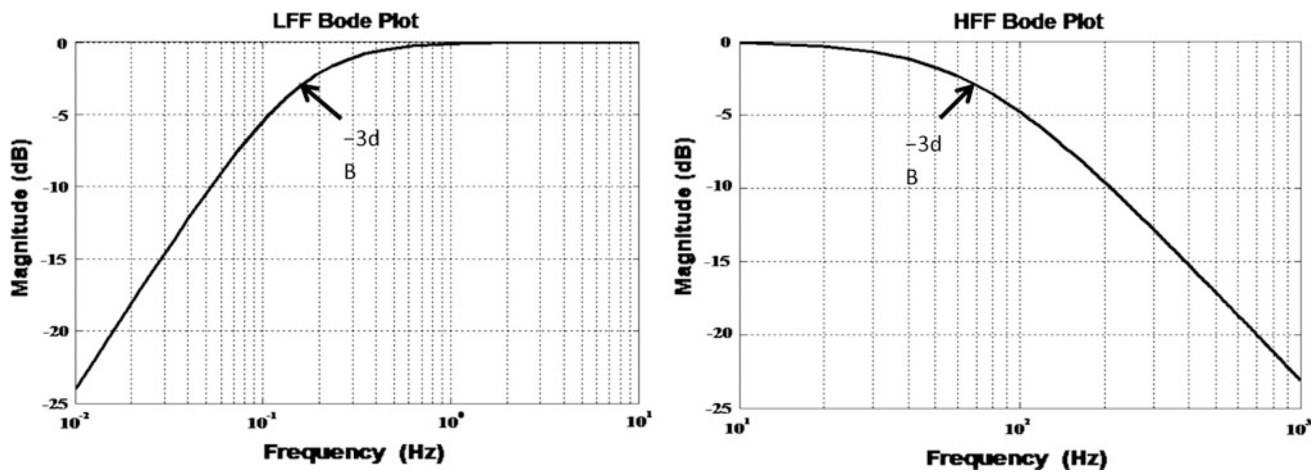


Fig. 16.13 Frequency response plots of the filters in Fig. 16.12a, b, respectively. In **a**, the filter attenuates high frequencies, and in **b**, the filter attenuates low frequencies. An *arrow* indicates the −3 dB point; in (**a**), f_C is 70 Hz, and in (**b**), f_C is 0.16 Hz. Code and output created using MatLab R2012a

filter circuit. This is shown in Eq. 16.22 where 2π is equivalent to 6.28, R is the resistor value in ohms, and C is the capacitor value in farads. The product $R * C$ is called tau, is indicated by the lower case Greek letter τ, represents a time constant, and can be substituted for RC as shown in Eq. 16.22 [1–5]. Time constants are discussed in the next section; however, it is important to understand that filters may be discussed in their cutoff point in Hz or by their time constant tau.

$$f_C = \frac{1}{2\pi RC} = f_C = \frac{1}{2\pi\tau} \quad (16.22)$$

Bode plots of HFF and LFF are shown in Fig. 16.13a, b, respectively. In the Bode plot of Fig. 16.13a, the HFF pass band extends from the vertical axis to the −3 dB point, and beyond the filter cutoff is the filter stop band. This is also true for the LFF Bode plot in Fig. 16.13b; however, the stop band and pass bands are reversed relative to the vertical axis.

Time Constants

A time constant is the product of the resistor and capacitor values in a filter circuit (RC) and is measured in seconds [1–5]. How does the product of a resistor in ohms and a capacitor in farads result in a time quantity? Using formulas already discussed for resistance (R), capacitance (C), and current (I), this can be answered and is shown in Eq. 16.23.

$$RC = \frac{E}{I} * \frac{Q}{E} = \frac{E}{\frac{Q}{t}} * \frac{Q}{E} = t \quad (16.23)$$

Filters may be described in terms of their time constant or frequency cutoff. It is important to be conversant between the two and to understand the behavior of the charge and discharge cycles for each filter. The voltage charge and discharge cycles of a capacitive transient are illustrated in Fig. 16.14a, b, respectively. A typical HFF setting in clinical neurophysiology is 70 Hz, with tau of 2.2 ms, and a 1-Hz LFF is synonymous with tau of 0.159 s. The term tau was introduced in the section on Filters and illustrated in Eq. 16.22, the formula for frequency cutoff of a filter. Equation 16.22 solved for tau is shown in Eq. 16.24.

$$\tau = \frac{1}{2\pi f_C} \quad (16.24)$$

Time constants can quite effectively be explained through the use of calculus; however, I will eliminate derivations and illustrate the equations along with a table. Equation 16.25

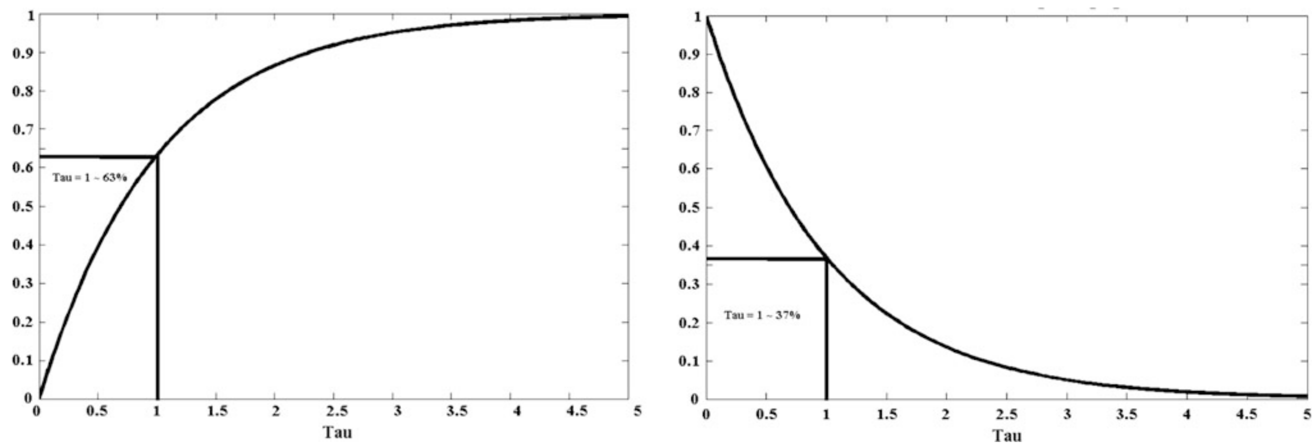


Fig. 16.14 The capacitive transient characteristics of Eqs. 16.25 (a) and (b) are shown in (**a**) and (**b**), respectively. In 1τ, the output has risen to 63 % in (**a**) and declined to 37 % in (**b**). In 5τ, in a DC circuit the components are either fully charged or discharged. Code and output created using MatLab R2012a

Table 16.3 Behavior of rising and decaying exponential time constants 0τ is initial conditions

Tau (τ)	Rise $V(t) = 1 * (1 - e^{\frac{-t}{\tau}})$		Decay $V(t) = 1 * e^{\frac{-t}{\tau}}$	
τ = tau	V	% of rise	V	% of decay
0	0	0	1	0
1	0.63	63	0.37	63
2	0.86	86	0.14	86
3	0.95	95	0.05	95
4	0.98	98	0.02	98
5	0.99	99	0.01	99

(a) and (b) are the rise and decay of voltage in a capacitor, respectively.

$$\underset{(\mathbf{a})}{V(t) = V(1 - e^{\frac{-t}{\tau}})} \quad \underset{(\mathbf{b})}{V(t) = Ve^{\frac{-t}{\tau}}} \tag{16.25}$$

The results of Eqs. 16.25 (a) and (b) are shown graphically in Fig. 16.14a, b respectively and numerically in Table 16.3.

It is clear from Fig. 16.14a, b and the values in Table 16.3 that in five time constants (5τ), the capacitor resistor combination has either completely charged or discharged. Another useful observation is this: Following one time constant, the circuit has completed 63 % of its course regardless of direction, charging, or discharging. With a DC source after 5τ, the capacitor has fully charged, and current has stopped flowing. With a DC source applied after 5tau, the capacitor has fully charged and current has stopped flowing. With an AC source applied charging and discharging of the capacitor alternate with the alternating AC source. The higher the applied frequency of the AC source the faster the alternating state of the capacitor. The higher the frequency the lower the capacitive reactance as illustrated in Eq. 16.4. The higher the frequency, the faster the alternating states. The time constant remains the product of the resistance and capacitance (RC), and this is the design of the circuit.

In terms of Bode plots if the time constant is a small value, the −3 dB point would be further from zero on the frequency scale, and the filter cutoff would be a larger value. If the time constant is a relatively large value, the −3 dB point would be closer to zero on the frequency scale, and the frequency cutoff would be a smaller value. This can be illustrated in several ways: One is using Eqs. 16.25 (a) and (b) to form Table 16.3. Another method is to use Eq. 16.24 to derive tau with various values of f_C as illustrated in Table 16.4. A third way is to view the concept graphically as in Fig. 16.15.

Differential Amplifiers and Polarity Convention

This is a general discussion of the operation of differential amplifiers in clinical neurophysiology. There is no in-depth electrical or electronic explanation, and the internal operation and electrical requirements of the operational amplifier are assumed satisfied.

Differential amplifiers do exactly what their name implies, and they take the difference of two inputs, amplify, and output the result. The schematic diagram of a differential amplifier is indicated in Fig. 16.16.

Table 16.4 The inverse relationship between tau and f_C. Values are rounded up to the highest integer. Note the change in units from seconds (s) to milliseconds (ms) at 3 Hz

Cutoff −3 dB f_C (Hz)	$\tau = \frac{1}{2\pi f_C}$
0.25	0.64 s
0.5	0.32 s
1	0.16 s
1.5	0.11 s
3	53 ms
5	32 ms
10	16 ms
15	10.6 ms
35	4.5 ms
70	2.3 ms
100	1.6 ms

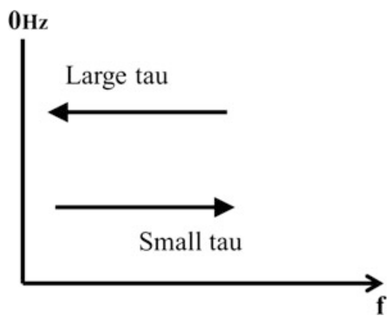
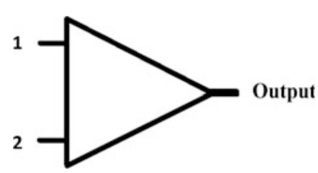


Fig. 16.15 Graphic illustration of the inverse relationship between the filter frequency cutoff and the value of its time constant. Horizontal axis begins at 0 Hz with increasing frequency moving to the right. With larger values of tau, the frequency cutoff moves toward 0 Hz indicated by a lower-frequency value cutoff. With smaller values of tau, the frequency cutoff moves away from 0 Hz taking on a larger-frequency cutoff value

Fig. 16.16 Schematic symbol of a differential amplifier



As can be seen in Fig. 16.16, the amplifier has two inputs, input 1 and input 2, and a single output representing the amplified difference of the two inputs. Input 1 is referred to as the active input and input 2 the reference input [2–6]. Each amplifier represents one channel of digital recording. In clinical neurophysiology, electrodes are connected to the differential amplifier with one on each input. The differential amplifier then subtracts input 2 from input 1, amplifies, and outputs the difference as one channel of recording.

The deflection of the channel output up or down is a result of the polarity convention [4, 5]. All electroencephalography (EEG) machines are designed the same way allowing for a standard definition of polarity. If input 1 is relatively more negative than input 2, the deflection of the output is upward, and if input 2 is more negative than input 1, the deflection of the output is downward. In short, the deflection of the output follows the negativity [4].

Common mode rejection ratio (CMRR) is a design parameter of a differential amplifier. It is a measure of the amplifiers ability to reject common signals [2–7]. Common mode rejection is a ratio of two measured outputs of the amplifier. One output, called A_{Diff}, is measured with a different and controlled voltage level applied to each amplifier input. This may also be accomplished by applying a voltage to one input and grounding the other. The other output termed A_{Com} is measured with an identical, i.e., common and controlled voltage level applied to both amplifier inputs [1, 7]. In this way, it is possible to determine the integrity of the amplifier. Does the amplifier properly take the difference of the inputs, amplify, and output the result and is the output zero when identical voltage levels are applied to both inputs? There are inherent imperfections in electronic design, so the output is rarely if ever zero when identical and controlled voltages are applied to the inputs; however, the output is extremely low. CMRR is indicated in decibels (dB) although it may be shown as a whole number. The equation for CMRR and its conversion to decibels are shown in Eqs. 16.26 (a) and (b) [2, 3, 7].

$$\underset{\textbf{(a)}}{A_{\text{CMRR}} = \frac{A_{\text{Diff}}}{A_{\text{Com}}}} \qquad \underset{\textbf{(b)}}{\text{dB}_{\text{CMRR}} = 20\log(A_{\text{CMRR}})} \tag{16.26}$$

This is an important design criterion of the amplifier, the higher the CMRR, the better the component design. Common mode voltage would be noise, such as 60 Hz induced on the electrode wires from a variety of sources. If it is common to both inputs, it should be rejected, and so it is not amplified as part of the signal of interest which could potentially lead to misinterpretation. One way to assist an amplifier to perform up to its design potential is to bundle all connected electrode wires together. Bundling the wires causes induced noise to be common to all the wires allowing the differential amplifier to remove common mode noise.

Analog-to-Digital Conversion

Neurodiagnostic recording captures analog signals through the application of externally applied electrodes. Analog signals are continuous in time versus a digital signal which is composed of discrete points in time. In older pen and paper machines, it was the analog signal that was captured and faithfully recorded in full. Digital equipment has many advantages over older pen-driven machines; however, computers cannot capture and store signals that are continuous in time; they must be converted to digital form for storage and display. All neurodiagnostic equipments begin with an analog signal, and all convert it to a digital signal through a circuit called an analog-to-digital converter or ADC. Digital signals are discrete points in time represented by 1's and 0's and stored in memory on a computer.

All biological signals are analog, but all stored signals are digitally obtained by sampling and analog-to-digital conversion. The signal being recorded is filtered, sampled at equidistant points in time, and stored transforming it into a digital signal. An ADC has several design parameters, which come in various speeds and have differing degrees of accuracy and resolution, and an input voltage range, but all are rated in the number of bits of information they can store [4, 6, 7]. A bit is a single unit of information represented by a 1 or a 0 in binary notation; calculation of bits is shown in Eq. 16.27.

$$2^n = \# \text{ of bits} \quad 2^4 = 16 \text{ bits} \tag{16.27}$$

The design criterion of the ADC dictates the price, and hence the more bits, the higher is the cost. A typical ADC size of 2^{16} is 65,536 bits, and if the ADC had a resolution of 0.06 μV, its dynamic range would be ±1.97 mV. Calculation of dynamic range requires knowledge about the size of the ADC and its resolution. Calculation of dynamic range (DR) indicated in our example is shown in Eq. 16.28.

$$\begin{aligned} \text{DR} &= 2^n * \frac{\text{resolution}}{\text{per bit}} = 65536 \text{ bits} * \frac{0.06\ \mu\text{V}}{\text{bit}} \\ &= \pm 0.001966 \text{ V} \end{aligned} \tag{16.28}$$

This means that the ADC can assign values to the samples with a minimum or maximum value of 1.97 mV. Without consideration for sign bits, the number of bins available to assign values to these samples is 65,536, with half (32,768) dedicated to the positive values and the other 32,768 is dedicated to the negative values. Each bin in this ADC represents a 60 nV increment, that is 10^{-9}, lending to very accurate placement of the digital samples to match their true analog values. The current American Clinical Neurophysiology Society (ACNS) guideline #8 calls for a minimum of an 11 bit ADC with 12 or higher preferred, to resolve the EEG to 0.5 μV or better and record up to plus or minus several millivolts without clipping [8]. Clipping may occur when the sample falls out of the range of the ADC as it has nowhere left to place a value larger than its dynamic range (positive or negative). Anything beyond these values is assigned to the maximum available value and appears clipped off in the positive or negative range.

Quantization (or signal processing is the process of mapping a large set of input values to a smaller set) is an indication of how much rounding, up or down, the ADC will have to do with the samples taken to make the value "fit" into its available steps [6, 7]. The higher the number of bits, the less rounding the ADC has to do, and thus, the sample will be assigned a value closer to its actual value. Our example of ±32,768 available steps at a resolution of 0.06 μV decreases the quantization error, which is defined as the round-off error introduced by quantization, for example, a decimal number of 12.65 representing as 13 in which there is an inherent error. This is what a larger number of bits does; it provides more alternatives for the ADC to more accurately represent the value of the samples taken as they are converted to digital form.

Nyquist Theorem

We need to sample and convert analog to digital signal, but we do not know yet how often to sample. This is where the Nyquist sampling theorem will help. Nyquist theorem states that a band-limited signal can be faithfully reproduced, providing the sampling rate is twice the maximum frequency of the signal being sampled [4–6]. For example, if the highest frequency content of a signal of interest is 70 Hz, then per Nyquist theorem it must be sampled at a **minimum** of 140 samples per second or 140 Hz for perfect reconstruction. What happens to the information between each sample? It is discarded, and the computer does not store any information other than the value of the analog signal at the time a sample is taken. One must understand the frequency content of a signal and ensure Nyquist criterion is met to obtain perfect reconstruction of an analog signal in digital form. If Nyquist theorem is not followed, "**aliasing**" of the signal occurs. Aliasing describes a situation that generates false (i.e., alias) frequency signals with jagged distortions making it difficult to recognize the original signal. Aliasing results in misinterpretation of the recorded signal because the original signal can no longer be perfectly reconstructed from its digital samples as there is not enough information from which to properly reconstruct the signal [4–6].

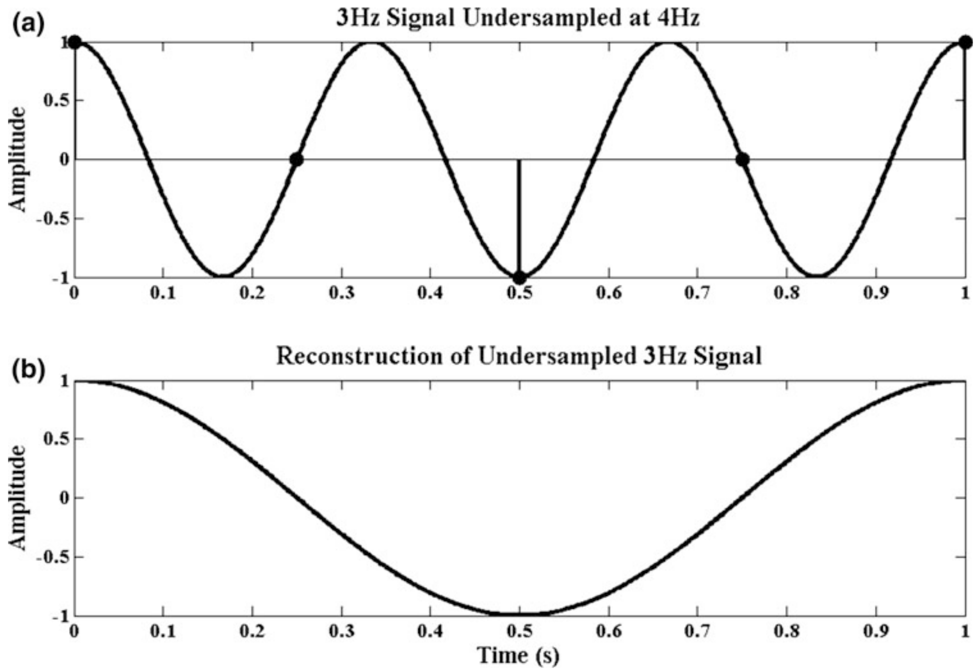


Fig. 16.17 The consequences of violating Nyquist sampling criteria. In **a**, 3 Hz signal is sampled at 4 Hz. In **b**, it has been reconstructed from its samples and appears to have been a 0.5-Hz signal, and this is aliasing, improper reconstruction due to insufficient sampling. Code and output created using MatLab R2012a

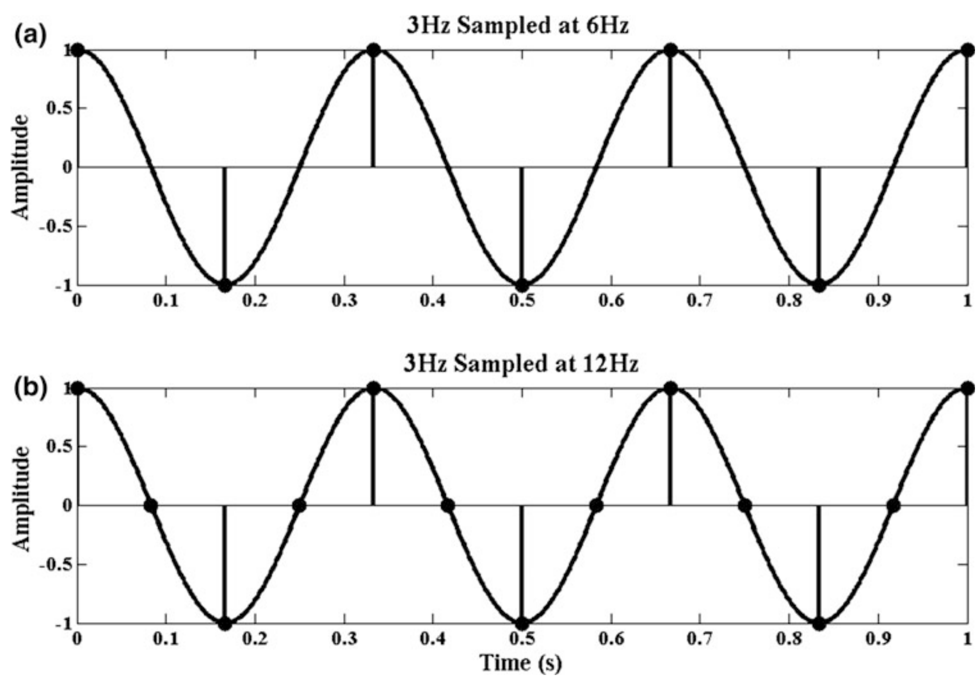


Fig. 16.18 In **a**, 3-Hz signal is sampled at 6 Hz meeting Nyquist criteria, and in **b**, the signal is over sampled at four times the signal frequency or 12 Hz. Perfect reconstruction is virtually ensured in both (**a**) and (**b**); however, (**b**) provides additional information for reconstruction with preserved morphology. Code and output created using MatLab R2012a

To faithfully follow the Nyquist theorem, the signal is filtered before sampling so the maximum frequency is known and Nyquist sampling theorem can be applied. In practice, sampling is performed at a higher rate than Nyquist theorem. The current American Clinical Neurophysiology Society (ACNS) guideline #8 requires a minimum sampling rate of three times the highest frequency content of the signal [8]. Higher sampling rate or over sampling is preferable and not generally an issue as the cost of storage is fairly inexpensive. More storage capacity is needed with a higher sampling rate because the faster a signal is sampled the more discrete points in time are taken that must be stored by the analog-to-digital converter. Taking constant snapshots of an event at evenly spaced intervals, stacking up all the snapshots sequentially, and flipping through them will give a fairly accurate and animated picture of the event. If sufficient snapshots are not available, there would be missing information and voids without properly depicting the event as it occurred. During sampling, the values are briefly held up in a sample, and the circuit reassembled time locked and then displayed as "*X*" number of channels.

Figure 16.17a illustrates a 3 Hz signal that has been under sampled at 4 Hz violating Nyquist criteria. Note that the samples would never faithfully reproduce a 3 Hz signal, and on reconstruction, it would appear to have been a 0.5 Hz signal as shown in Fig. 16.17b; this is **aliasing**. Remember information between samples is lost, and the only values available to reconstruct the signal are the stored sampled values.

In Fig. 16.18a, the 3 Hz signal has been sampled at 6 Hz, meeting Nyquist criteria; notice, however, that only the peaks have been captured during sampling. While this would be sufficient to reconstruct the original signal, its morphology could be compromised upon reconstruction. It may appear to have had sharper peaks than the original signal. It is best to oversample as in Fig. 16.18b to capture additional information and ensure reconstruction.

Conclusion

The recording of a biological signal requires the application of electrodes that interface with sophisticated electronic equipment in order to capture and record minute electrical activity generated by the movement of ions in biological tissue. Neurodiagnostic machines have a variety of settings that must be adjusted during the course of the recording to enhance data acquisition providing meticulous recording.

In order to appropriately adjust variables, it is vital to possess an understanding of the controls available, the appropriateness of making variable adjustments, and a complete understanding of the effect on the integrity of the recording.

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