Chapter 6
PROMETHEE Methods

Jean-Pierre Brans and Yves De Smet

Abstract This paper gives an overview of the PROMETHEE-GAIA methodology for MCDA. It starts with general comments on multicriteria problems, stressing that a multicriteria problem cannot be treated without additional information related to the preferences and the priorities of the decision-makers. The information requested by PROMETHEE and GAIA is particularly clear and easy to define for both decision-makers and analysts. It consists in a preference function associated to each criterion as well as weights describing their relative importance. The PROMETHEE I, the PROMETHEE II ranking, as well as the GAIA visual interactive module are then presented. Additionally, comments about potential rank reversal occurrences are provided. The two next sections are devoted to the PROMETHEE VI sensitivity analysis procedure (human brain) and to the PROMETHEE V procedure for multiple selection of alternatives under constraints. A sorting method based on the PROMETHEE flow scores, called FlowSort, is described. An overview of the PROMETHEE GDSS procedure for group decision making is then given. Finally the D-Sight implementation of the PROMETHEE-GAIA methodology is presented.

Keywords MCDA • Outranking methods • PROMETHEE-GAIA • D-Sight

J.-P. Brans (✉)
Centrum voor Statistiek en Operationeel Onderzoek, Vrije Universiteit Brussel,
Pleinlaan 2, 1050 Brussels, Belgium
e-mail: brans.jp@skynet.be

Y. De Smet
Computer and Decision Engineering Department - CoDE-SMG, Université Libre de Bruxelles,
Boulevard du Triomphe CP 210-01, 1050 Brussels, Belgium
e-mail: yves.de.smet@ulb.ac.be

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6.1 Preamble

This chapter is an updated version of [15]. Since 2005, a number of works have been focused on the PROMETHEE and GAIA methods. We decided to include in this paper some of these contributions and more specifically those regarding the following papers:

- In 1996, W. De Keyser and P. Peeters [19] initially pointed out rank reversal occurrences in the PROMETHEE I ranking. Recently, several authors analyzed conditions under which these phenomena could potentially happen. Their main results will be presented in Sect. 6.6;
- In his Ph.D. thesis [36], P. Nemery de Bellevaux proposed a sorting method based on the PROMETHEE flow scores. This approach will be summarized in Sect. 6.10;
- A new PROMETHEE and GAIA based software, called D-Sight, is now available. Section 6.12 will be dedicated to its description.

Of course, we cannot address all the contributions that have been proposed since 2005 [more than 40 new articles have been published in scientific journals since 2005 with one of their keywords corresponding to PROMETHEE (source: Science Direct)]. Far from being exhaustive, we can cite applications to portfolio and stock selection problems [1, 32, 46], to environmental issues [24, 26, 39, 44, 49], to energy management [22, 31], to chemometrics [18, 38, 41, 50], to statistical distribution selection [27] ... Recent methodological extensions include the use of the Choquet integral to model interactions between criteria [20], an extension of the Promethee II method based on generalized fuzzy numbers [28], the use of PROMETHEE in new classification methods [25, 40] ... Finally, we would like to give prominence to the latest comprehensive literature review realized by Behzadian et al. [4]. The authors have listed more than 200 papers published in 100 different journals. The applications fields cover finance, health care, logistics and transportation, hydrology and water management, manufacturing and assembly ...

B. Mareschal decided, for personal reasons, not to be a co-author of this revised chapter. We respect his decision and thank him, once again, for his continuous involvement in the development of the PROMETHEE and GAIA methodology.

6.2 History

The PROMETHEE I (partial ranking) and PROMETHEE II (complete ranking) were developed by J.P. Brans and presented for the first time in 1982 at a conference organized by R. Nadeau and M. Landry at the Université Laval, Québec, Canada (L'Ingénierie de la Décision. Elaboration d'instruments d'Aide à la Décision). The same year several applications using this methodology were already treated by G. Davignon in the field of health care.

A few years later J.P. Brans and B. Mareschal developed PROMETHEE III (ranking based on intervals) and PROMETHEE IV (continuous case). The same authors proposed in 1988 the visual interactive module GAIA which is providing a marvellous graphical representation supporting the PROMETHEE methodology.

In 1992 and 1994, J.P. Brans and B. Mareschal further suggested two nice extensions: PROMETHEE V (MCDA including segmentation constraints) and PROMETHEE VI (representation of the human brain).

A considerable number of successful applications has been treated by the PROMETHEE methodology in various fields such as Banking, Industrial Location, Manpower planning, Water resources, Investments, Medicine, Chemistry, Health care, Tourism, Ethics in OR, Dynamic management, ... The success of the methodology is basically due to its mathematical properties and to its particular friendliness of use.

6.3 Multicriteria Problems

Let us consider the following multicriteria problem:

$$\max\{g_1(a), g_2(a), \ldots, g_j(a), \ldots, g_k(a) | a \in A\}, \tag{6.1}$$

where A is a finite set of possible alternatives $\{a_1, a_2, \ldots a_i, \ldots, a_n\}$ and $\{g_1(\cdot), g_2(\cdot), \ldots, g_j(\cdot), \ldots g_k(\cdot)\}$ a set of evaluation criteria. There is no objection to consider some criteria to be maximized and the others to be minimized. The expectation of the decision-maker is to identify an alternative optimizing all the criteria.

Usually this is an *ill-posed mathematical* problem as there exists no alternative optimizing all the criteria at the same time. However most (nearly all) human problems have a multicriteria nature. According to our various human aspirations, it makes no sense, and it is often not fair, to select a decision based on one evaluation criterion only. In most of cases at least technological, economical, environmental, social and educational criteria should always be taken into account. Multicriteria problems are therefore extremely important and request an appropriate treatment.

If A is finite, the basic data of a multicriteria problem (6.1) consist of an evaluation table (Table 6.1).

Let us consider as an example the problem of an individual purchasing a car. Of course the price is important and it should be minimized. However it is clear that in general individuals are not considering only the price. Not everybody is driving the cheapest car! Most people would like to drive a luxury or sports car at the price of an economy car. Indeed they consider many criteria such as price, reputation, comfort, speed, reliability, consumption, ... As there is no car optimizing all the criteria at the same time, a *compromise* solution should be selected. Most decision problems have such a multicriteria nature.

Table 6.1 Evaluation table

a	$g_1(\cdot)$	$g_2(\cdot)$	$\dots$	$g_j(\cdot)$	$\dots$	$g_k(\cdot)$
a_1	$g_1(a_1)$	$g_2(a_1)$	$\dots$	$g_j(a_1)$	$\dots$	$g_k(a_1)$
a_2	$g_1(a_2)$	$g_2(a_2)$	$\dots$	$g_j(a_2)$	$\dots$	$g_k(a_2)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
a_i	$g_1(a_i)$	$g_2(a_i)$	$\dots$	$g_j(a_i)$	$\dots$	$g_k(a_i)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
a_n	$g_1(a_n)$	$g_2(a_n)$	$\dots$	$g_j(a_n)$	$\dots$	$g_k(a_n)$

The solution of a multicriteria problem depends not only on the basic data included in the evaluation table but also on the decision-maker himself. All individuals do not purchase the same car. There is no absolute best solution! The best compromise solution also depends on the individual *preferences* of each decision-maker, on the *"brain"* of each decision-maker.

Consequently, *additional information* representing these preferences is required to provide the decision maker with useful decision aid.

The natural dominance relation associated to a multicriteria problem of type (6.1) is defined as follows:

For each $(a, b) \in A$:

$$\begin{aligned}
&\begin{cases} \forall j : g_j(a) \geq g_j(b) \\ \exists k : g_k(a) > g_k(b) \end{cases} &\Longleftrightarrow aPb, \\
&\forall j : g_j(a) = g_j(b) &\Longleftrightarrow aIb, \\
&\begin{cases} \exists s : g_s(a) > g_s(b) \\ \exists r : g_r(a) < g_r(b) \end{cases} &\Longleftrightarrow aRb,
\end{aligned} \tag{6.2}$$

where P, I, and R respectively stand for *preference*, *indifference* and *incomparability*. This definition is quite obvious. An alternative is better than another if it is at least as good as the other on all criteria. If an alternative is better on a criterion s and the other one better on criterion r, it is impossible to decide which is the best one without additional information. Both alternatives are therefore incomparable!

Alternatives which are not dominated by any other are called *efficient solutions*. Given an evaluation table for a particular multicriteria problem, most of the alternatives (often all of them) are usually efficient. The dominance relation is very poor on P and I. When an alternative is better on one criterion, the other is often better on another criterion. Consequently incomparability holds for most pairwise comparisons, so that it is impossible to decide without additional information. This information can for example include:

- Trade-offs between the criteria;
- A value function aggregating all the criteria in a single function (utility function) in order to obtain a single criterion problem for which an optimal solution exists;

- Weights giving the relative importance of the criteria;
- Preferences associated to each pairwise comparison within each criterion;
- Thresholds fixing preference limits;
- ...

Many multicriteria decision aid methods have been proposed. All these methods start from the same evaluation table, but they vary according to the additional information they request. The PROMETHEE methods require very clear additional information, that is easily obtained and understood by both decision-makers and analysts.

The purpose of all multicriteria methods is to enrich the dominance graph, i.e. to reduce the number of incomparabilities (R). When a utility function is built, the multicriteria problem is reduced to a single criterion problem for which an optimal solution exists. This seems exaggerated because it relies on quite strong assumptions (do we really make all our decisions based on a utility function defined somewhere in our brains?) and it completely transforms the structure of the decision problem. For this reason B. Roy proposed to build outranking relations including only realistic enrichments of the dominance relation (see [42, 43]). In that case, not all the incomparabilities are withdrawn but the information is reliable. The PROMETHEE methods belong to the class of outranking methods.

In order to build an appropriate multicriteria method some requisites could be considered:

Requisite 1: The amplitude of the deviations between the evaluations of the alternatives within each criterion should be taken into account:

$$d_j(a,b) = g_j(a) - g_j(b). \tag{6.3}$$

This information can easily be calculated, but is not considered in the efficiency theory. When these deviations are negligible the dominance relation can possibly be enriched.

Requisite 2: As the evaluations $g_j(a)$ of each criterion are expressed in their own units, *the scaling effects* should be completely eliminated. It is not acceptable to obtain conclusions depending on the scales in which the evaluations are expressed. Unfortunately not all multicriteria procedures are respecting this requisite!

Requisite 3: In the case of pairwise comparisons, an appropriate multicriteria method should provide the following information:

a is preferred to b;
a and b are indifferent;
a and b are incomparable.

The purpose is of course to reduce as much as possible the number of incomparabilities, but not when it is not realistic. Then the procedure may be considered as fair. When, for a particular procedure, all the incomparabilities are systematically withdrawn the provided information can be more disputable.

Requisite 4: Different multicriteria methods request different additional information and operate different calculation procedures so that the solutions they propose can be different. It is therefore important to develop methods being *understandable* by the decision-makers. "Black box" procedures should be avoided.

Requisite 5: An appropriate procedure should not include technical parameters having no significance for the decision-maker. Such parameters would again induce "Black box" effects.

Requisite 6: An appropriate method should provide information on the *conflicting nature* of the criteria.

Requisite 7: Most of the multicriteria methods are allocating weights of relative importance of the criteria. These weights reflects a major part of the *"brain"* of the decision-maker. It is not easy to fix them. Usually the decision-makers strongly hesitate. An appropriate method should offer *sensitivity tools* to test easily different sets of weights.

The PROMETHEE methods and the associated GAIA visual interactive module are taking all these requisites into account. On the other hand some mathematical properties that multicriteria problems possibly enjoy can also be considered. See for instance [47]. Such properties related to the PROMETHEE methods have been analyzed by [6] in a particularly interesting paper.

The next sections describe the PROMETHEE I and II rankings, the GAIA methods, as well as the PROMETHEE V and VI extensions of the methodology. The PROMETHEE III and IV extensions are not discussed here. Additional information can be found in [16]. Several actual applications of the PROMETHEE methodology are also mentioned in the list of references.

6.4 The PROMETHEE Preference Modelling Information

The PROMETHEE methods were designed to treat multicriteria problems of type (6.1) and their associated evaluation table.

The additional information requested to run PROMETHEE is particularly clear and understandable by both the analysts and the decision-makers. It consists of:

- Information between the criteria;
- Information within each criterion.

6.4.1 Information Between the Criteria

Table 6.2 should be completed, with the understanding that the set $\{w_j, j = 1, 2, \ldots, k\}$ represents weights of relative importance of the different criteria. These weights are non-negative numbers, independent from the measurement units of

Table 6.2 Weights of relative importance

$g_1(\cdot)$	$g_2(\cdot)$	...	$g_j(\cdot)$	...	$g_k(\cdot)$
w_1	w_2	...	w_j	...	w_k

the criteria. The higher the weight, the more important the criterion. There is no objection to consider normalized weights, so that:

$$\sum_{j=1}^{k} w_j = 1. \tag{6.4}$$

In the PROMETHEE software PROMCALC, DECISION LAB or D-Sight , the user is allowed to introduce arbitrary numbers for the weights, making it easier to express the relative importance of the criteria. These numbers are then divided by their sum so that the weights are normalized automatically.

Assessing weights to the criteria is not straightforward. It involves the priorities and perceptions of the decision-maker. The selection of the weights is his *space of freedom*. PROMCALC, DECISION LAB and D-Sight include several sensitivity tools to experience different set of weights in order to help to fix them.

6.4.2 Information Within the Criteria

PROMETHEE is not allocating an intrinsic absolute utility to each alternative, neither globally, nor on each criterion. We strongly believe that the decision-makers are not proceeding that way. The preference structure of PROMETHEE is based on *pairwise comparisons*. In this case the deviation between the evaluations of two alternatives on a particular criterion is considered. For small deviations, the decision-maker will allocate a small preference to the best alternative and even possibly no preference if he considers that this deviation is negligible. The larger the deviation, the larger the preference. There is no objection to consider that these preferences are real numbers varying between 0 and 1. This means that for each criterion the decision-maker has in mind a function

$$P_j(a,b) = F_j\left[d_j(a,b)\right] \quad \forall a,b \in A, \tag{6.5}$$

where:

$$d_j(a,b) = g_j(a) - g_j(b) \tag{6.6}$$

and for which:

$$0 \leq P_j(a,b) \leq 1. \tag{6.7}$$

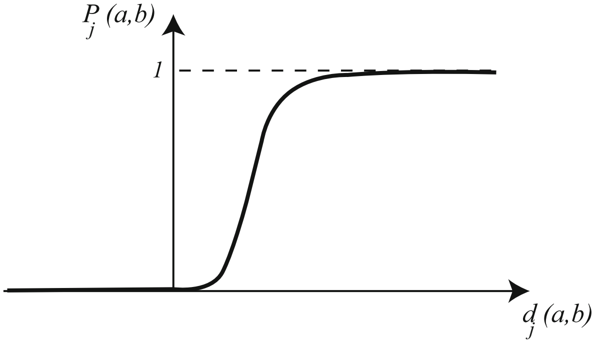


Fig. 6.1 Preference function

In case of a criterion to be maximized, this function is giving the preference of a over b for observed deviations between their evaluations on criterion $g_j(\cdot)$. It should have the following shape (see Fig. 6.1). The preferences equals 0 when the deviations are negative.

The following property holds:

$$P_j(a,b) > 0 \;\Rightarrow\; P_j(b,a) = 0. \tag{6.8}$$

For criteria to be minimized, the preference function should be reversed or alternatively given by:

$$P_j(a,b) = F_j\left[-d_j(a,b)\right]. \tag{6.9}$$

We have called the pair $\{g_j(\cdot), P_j(a,b)\}$ the *generalized criterion* associated to criterion $g_j(\cdot)$. Such a generalized criterion has to be defined for each criterion. In order to facilitate the identification six types of particular preference functions have been proposed (see Table 6.3). In each case 0, 1 or 2 parameters have to be defined, their significance is clear:

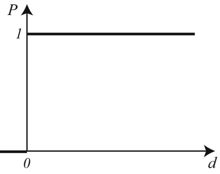
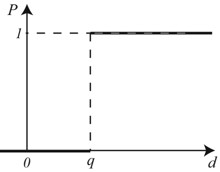
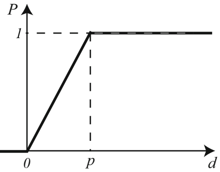
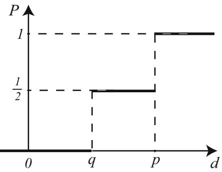
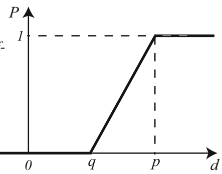
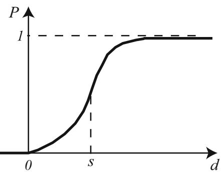
q is a threshold of indifference;
p is a threshold of strict preference ($P_j(a,b) = 1$);
s is an intermediate value between q and p.

The q indifference threshold is the largest deviation which is considered as negligible by the decision maker, while the p preference threshold is the smallest deviation which is considered as sufficient to generate a full preference.

The identification of a generalized criterion is then limited to the selection of the appropriate parameters. It is an easy task.

The PROMCALC, DECISION LAB and D-Sight software are proposing these six shapes only. As far as we know they have been satisfactory in most real-world applications. However there is no objection to consider additional generalized criteria.

Table 6.3 Types of generalized criteria ($P(d)$: preference function)

Generalized criterion	Definition	Parameters to fix
Type 1: Usual Criterion 	$P(d) = \begin{cases} 0 & d \leq 0 \\ 1 & d > 0 \end{cases}$	–
Type 2: U-shape Criterion 	$P(d) = \begin{cases} 0 & d \leq q \\ 1 & d > q \end{cases}$	q
Type 3: V-shape Criterion 	$P(d) = \begin{cases} 0 & d \leq 0 \\ \frac{d}{p} & 0 \leq d \leq p \\ 1 & d > p \end{cases}$	p
Type 4: Level Criterion 	$P(d) = \begin{cases} 0 & d \leq q \\ \frac{1}{2} & q < d \leq p \\ 1 & d > p \end{cases}$	p, q
Type 5: V-shape with indifference Criterion 	$P(d) = \begin{cases} 0 & d \leq q \\ \frac{d-q}{p-q} & q < d \leq p \\ 1 & d > p \end{cases}$	p, q
Type 6: Gaussian Criterion 	$P(d) = \begin{cases} 0 & d \leq 0 \\ 1 - e^{-\frac{d^2}{2s^2}} & d > 0 \end{cases}$	s

In case of type 5 a threshold of indifference q and a threshold of strict preference p have to be selected.

In case of a Gaussian criterion (type 6) the preference function remains increasing for all deviations and has no discontinuities, neither in its shape, nor in its derivatives. A parameter s has to be selected, it defines the inflection point of the preference function. We then recommend to determine first a q and a p and to fix s in between. If s is close to q the preferences will be reinforced for small deviations, while close to p they will be softened.

As soon as the evaluation table $\{g_j(\cdot)\}$ is given, and the weights w_j and the generalized criteria $\{g_j(\cdot), P_j(a,b)\}$ are defined for $i = 1, 2, \ldots, n; j = 1, 2, \ldots, k$, the PROMETHEE procedure can be applied.

6.5 The PROMETHEE I and II Rankings

The PROMETHEE procedure is based on pairwise comparisons (cf. [7–14, 17, 33, 34]). Let us first define aggregated preference indices and outranking flows.

6.5.1 Aggregated Preference Indices

Let $a, b \in A$, and let:

$$\begin{cases} \pi(a,b) = \sum_{j=1}^{k} P_j(a,b) w_j, \\ \pi(b,a) = \sum_{j=1}^{k} P_j(b,a) w_j. \end{cases} \tag{6.10}$$

$\pi(a,b)$ is expressing with which degree a is preferred to b over all the criteria and $\pi(b,a)$ how b is preferred to a. In most of the cases there are criteria for which a is better than b, and criteria for which b is better than a, consequently $\pi(a,b)$ and $\pi(b,a)$ are usually positive. The following properties hold for all $(a,b) \in A$.

$$\begin{cases} \pi(a,a) = 0, \\ 0 \leq \pi(a,b) \leq 1, \\ 0 \leq \pi(b,a) \leq 1, \\ 0 \leq \pi(a,b) + \pi(b,a) \leq 1. \end{cases} \tag{6.11}$$

It is clear that:

$$\begin{cases} \pi(a,b) \sim 0 \text{ implies a weak global preference of } a \text{ over } b, \\ \pi(a,b) \sim 1 \text{ implies a strong global preference of } a \text{ over } b. \end{cases} \tag{6.12}$$

In addition, it is obvious that $P_j(a,b), P_j(b,a), \pi(a,b)$ and $\pi(b,a)$ are real numbers (without units) completely independent of the scales of the criteria $g_j(.)$.

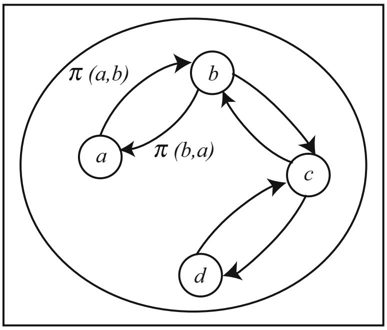


Fig. 6.2 Valued outranking graph

As soon as $\pi(a, b)$ and $\pi(b, a)$ are computed for each pair of alternatives of A, a complete valued outranking graph, including two arcs between each pair of nodes, is obtained (see Fig. 6.2).

6.5.2 Outranking Flows

Each alternative a is facing $(n - 1)$ other alternatives in A. Let us define the two following outranking flows:

- *the positive outranking flow:*

$$\phi^+(a) = \frac{1}{n-1} \sum_{x \in A} \pi(a, x), \tag{6.13}$$

- *the negative outranking flow:*

$$\phi^-(a) = \frac{1}{n-1} \sum_{x \in A} \pi(x, a). \tag{6.14}$$

The positive outranking flow expresses how an alternative a is *outranking* all the others. It is its *power, its outranking character*. The higher $\phi^+(a)$, the better the alternative (see Fig. 6.3a).

The negative outranking flow expresses how an alternative a is *outranked* by all the others. It is its *weakness, its outranked character*. The lower $\phi^-(a)$ the better the alternative (see Fig. 6.3b).

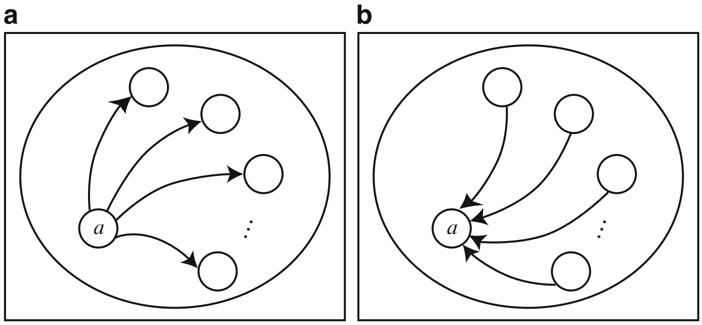


Fig. 6.3 The PROMETHEE outranking flows. (**a**) The $\phi^+(a)$ outranking flow. (**b**) The $\phi^-(a)$ outranking flow

6.5.3 The PROMETHEE I Partial Ranking

The PROMETHEE I partial ranking (P^I, I^I, R^I) is obtained from the positive and the negative outranking flows. Both flows do not usually induce the same rankings. PROMETHEE I is their intersection.

$$\begin{cases} aP^Ib & \text{iff} & \begin{cases} \phi^+(a) > \phi^+(b) \text{ and } \phi^-(a) < \phi^-(b), \text{ or} \\ \phi^+(a) = \phi^+(b) \text{ and } \phi^-(a) < \phi^-(b), \text{ or} \\ \phi^+(a) > \phi^+(b) \text{ and } \phi^-(a) = \phi^-(b); \end{cases} \\ aI^Ib & \text{iff} & \quad \phi^+(a) = \phi^+(b) \text{ and } \phi^-(a) = \phi^-(b); \\ aR^Ib & \text{iff} & \begin{cases} \phi^+(a) > \phi^+(b) \text{ and } \phi^-(a) > \phi^-(b), \text{ or} \\ \phi^+(a) < \phi^+(b) \text{ and } \phi^-(a) < \phi^-(b); \end{cases} \end{cases} \tag{6.15}$$

where P^I, I^I, R^I respectively stand for preference, indifference and incomparability.

When aP^Ib, a higher power of a is associated to a lower weakness of a with regard to b. The information of both outranking flows is consistent and may therefore be considered as sure.

When aI^Ib, both positive and negative flows are equal.

When aR^Ib, a higher power of one alternative is associated to a lower weakness of the other. This often happens when a is good on a set of criteria on which b is weak and reversely b is good on some other criteria on which a is weak. In such a case the information provided by both flows is not consistent. It seems then reasonable to be careful and to consider both alternatives as incomparable. The PROMETHEE I ranking is prudent: it will not decide which action is best in such cases. It is up to the decision-maker to take his responsibility.

6.5.4 The PROMETHEE II Complete Ranking

PROMETHEE II consists of the (P^{II}, I^{II}) complete ranking. It is often the case that the decision-maker requests a complete ranking. The *net outranking flow* can then be considered.

$$\phi(a) = \phi^{+}(a) - \phi^{-}(a). \tag{6.16}$$

It is the balance between the positive and the negative outranking flows. The higher the net flow, the better the alternative, so that:

$$\begin{cases} aP^{II}b & \text{iff} \;\; \phi(a) > \phi(b), \\ aI^{II}b & \text{iff} \;\; \phi(a) = \phi(b). \end{cases} \tag{6.17}$$

When PROMETHEE II is considered, all the alternatives are comparable. No incomparabilities remain, but the resulting information can be more disputable because more information gets lost by considering the difference (6.16).

The following properties hold:

$$\begin{cases} -1 \leq \phi(a) \leq 1, \\ \sum_{x \in A} \phi(a) = 0. \end{cases} \tag{6.18}$$

When $\phi(a) > 0$, a is more outranking all the alternatives on all the criteria, when $\phi(a) < 0$ it is more outranked.

In real-world applications, we recommend to both the analysts and the decision-makers to consider both PROMETHEE I and PROMETHEE II. The complete ranking is easy to use, but the analysis of the incomparabilities often helps to finalize a proper decision.

As the net flow $\phi(\cdot)$ provides a complete ranking, it may be compared with a utility function. One advantage of $\phi(\cdot)$ is that it is built on clear and simple preference information (weights and preferences functions) and that it does rely on comparative statements rather than absolute statements.

6.5.5 The Profiles of the Alternatives

According to the definition of the positive and the negative outranking flows (6.13) and (6.14) and of the aggregated indices (6.10), we have:

$$\phi(a) = \phi^{+}(a) - \phi^{-}(a) = \frac{1}{n-1} \sum_{j=1}^{k} \sum_{x \in A} \left[P_j(a,x) - P_j(x,a) \right] w_j. \tag{6.19}$$

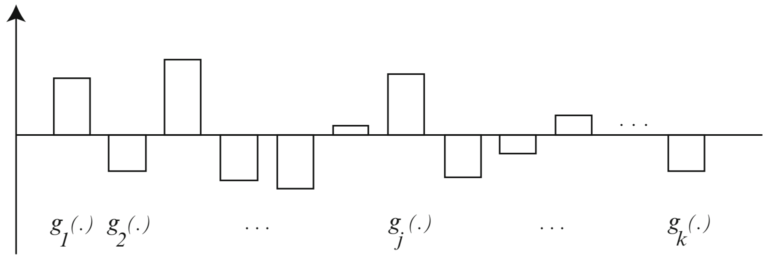


Fig. 6.4 Profile of an alternative

Consequently,

$$\phi(a) = \sum_{j=1}^{k} \phi_j(a) w_j \tag{6.20}$$

if

$$\phi_j(a) = \frac{1}{n-1} \sum_{x \in A} \left[P_j(a,x) - P_j(x,a) \right]. \tag{6.21}$$

$\phi_j(a)$ is the single criterion net flow obtained when only criterion $g_j(\cdot)$ is considered (100 % of the total weight is allocated to that criterion). It expresses how an alternative a is outranking ($\phi_j(a) > 0$) or outranked ($\phi_j(a) < 0$) by all the other alternatives on criterion $g_j(\cdot)$ only.

The profile of an alternative consists of the set of all the single criterion net flows: $\phi_j(a), j = 1, 2, \ldots, k$.

The profiles of the alternatives are particularly useful to appreciate their *"quality"* on the different criteria. It is extensively used by decision-makers to finalize their appreciation (Fig. 6.4).

According to (6.20), we observe that the global net flow of an alternative is the scalar product between the vector of the weights and the profile vector of this alternative. This property will be extensively used when building up the GAIA plane.

6.6 A Few Words About Rank Reversal

Pair-wise comparison methods, such as outranking methods, may suffer from the well-known rank reversal problem: the relative positions of two alternatives may be influenced by the presence of a third one. This phenomenon is not new and dates back from the beginning of social choice theory (see for instance the condition about irrelevant alternatives in the famous Arrow's theorem [3]).

A number of authors have already addressed this question in the context of multicriteria methods (see for instance [5] for the Analytic Hierarchy Process or [48] for ELECTRE methods). Let us stress that the debate is still very active and that a number of articles have been proposed to answer these issues. In the context of the PROMETHEE methods, W. De Keyser and P. Peeters [19] initially pointed out rank reversal occurrences in the context of the PROMETHEE I ranking. Following these observations, B. Mareschal et al. [35] and C. Verly et al. [45] have investigated conditions under which rank reversal could potentially occur in the PROMETHEE I and II rankings.

At first, it is important to stress that no unique definition of *rank reversal* exists. Some authors analyze if the positions of two alternatives can be affected by:

- the presence of a non-discriminating criterion;
- a copy of an alternative;
- a dominated alternative;
- any given alternative;
- ...

It is easy to prove that the PROMETHEE rankings will not be influenced by the presence or the elimination of a non discriminating criterion while it may be affected by copies of alternatives (see [45]). Furthermore, if a dominates b we will always have $\phi(a) \geq \phi(b)$ (whatever the other alternatives). No rank reversal could ever happen in such a situation.

If we investigate rank reversal occurrences induced by the deletion of a third alternative, we may come to the conclusion [35] that no rank reversal will occur in the PROMETHEE II ranking between a and b if

$$|\phi(a) - \phi(b)| > \frac{2}{n-1} \tag{6.22}$$

A direct corollary of this result is that rank reversal occurrences may only happen between alternatives which have close net flow scores. Additionally, C. Verly et al. [45] used computer simulations on artificial data sets to show that these rank reversal instances happened most of the time when the actual net flow differences were much lower than the $\frac{2}{n-1}$ threshold. This has led them to refine this bound. Finally, they extended the previous result in the context of the PROMETHEE I ranking and proved that no rank reversal will occur between a and b if the following conditions are satisfied:

$$|\phi^+(a) - \phi^+(b)| > \frac{1}{n-1} \tag{6.23}$$

$$|\phi^-(a) - \phi^-(b)| > \frac{1}{n-1} \tag{6.24}$$

6.7 The GAIA Visual Interactive Module

Let us first consider the matrix $M(n \times k)$ of the single criterion net flows of all the alternatives as defined in (6.21) (Table 6.4).

6.7.1 The GAIA Plane

The information included in matrix M is more extensive than the one in the evaluation Table 6.1, because the degrees of preference given by the generalized criteria are taken into account in M. Moreover the $g_j(a_i)$ are expressed on their own scale, while the $\phi_j(a_i)$ are dimensionless. In addition, let us observe, that M is not depending on the weights of the criteria. Consequently the set of the n alternatives can be represented as a cloud of n points in a k-dimensional space. According to (6.18) this cloud is centered at the origin. As the number of criteria is usually larger than two, it is impossible to obtain a clear view of the relative position of the points with regard to the criteria. We therefore project the information included in the k-dimensional space on a plane. Let us project not only the points representing the alternatives but also the unit vectors of the coordinate-axes representing the criteria.

The GAIA *plane* is the plane for which as much information as possible is preserved after projection. According to the *principal components analysis* technique it is defined by the two eigenvectors corresponding to the two largest eigenvalues of the covariance matrix $M'M$ of the single criterion net flows (Fig. 6.5).

Of course some information get lost after projection. The GAIA plane is a *meta model* (a model of a model). Let δ be the quantity of information preserved:

$$\delta = \frac{\lambda_1 + \lambda_2}{\sum_{j=1}^{k} \lambda_j} \tag{6.25}$$

Table 6.4 Single criterion net flows

	$\phi_1(\cdot)$	$\phi_2(\cdot)$	$\dots$	$\phi_j(\cdot)$	$\dots$	$\phi_k(\cdot)$
a_1	$\phi_1(a_1)$	$\phi_2(a_1)$	$\dots$	$\phi_j(a_1)$	$\dots$	$\phi_k(a_1)$
a_2	$\phi_1(a_2)$	$\phi_2(a_2)$	$\dots$	$\phi_j(a_2)$	$\dots$	$\phi_k(a_2)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
a_i	$\phi_1(a_i)$	$\phi_2(a_i)$	$\dots$	$\phi_j(a_i)$	$\dots$	$\phi_k(a_i)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
a_n	$\phi_1(a_n)$	$\phi_2(a_n)$	$\dots$	$\phi_j(a_n)$	$\dots$	$\phi_k(a_n)$

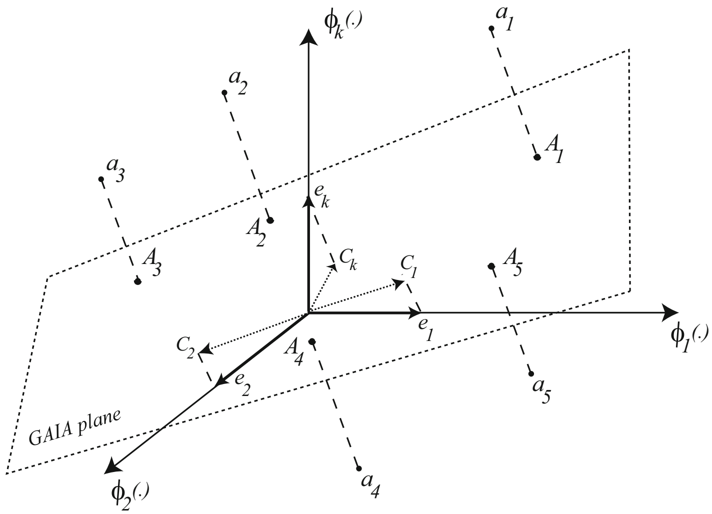


Fig. 6.5 Projection on the GAIA plane

where $\lambda_1, \lambda_2, \ldots, \lambda_j, \ldots, \lambda_k$ is the set of the k eigenvalues of $M'M$ ranked from the highest to the lowest one.

In most applications we have treated so far δ was larger than 60 % and in many cases larger than 80 %. This means that the information provided by the GAIA plane is rather reliable. This information is quite rich, it helps to understand the structure of a multicriteria problem. It is not often the case that δ is very small. When its value is too low (say $\delta < 0.5$) the GAIA plane becomes progressively useless.

6.7.2 Graphical Display of the Alternatives and of the Criteria

Let $(A_1, A_2, \ldots, A_i, \ldots, A_n)$ be the projections of the n points representing the alternatives and let $(C_1, C_2, \ldots, C_j, \ldots, C_k)$ be the projections of the k unit vectors of the coordinates axes of R^k representing the criteria. We then obtain a GAIA plane of the following type: Then the following properties hold (see [14, 33]) provided that δ is sufficiently high:

P 1: The longer a criterion axis in the GAIA plane, the more discriminating this criterion.

P 2: Criteria expressing similar preferences are represented by axes oriented in approximatively the same direction.

P 3: Criteria expressing conflicting preferences are oriented in opposite directions.

P 4: Criteria that are not related to each others in terms of preferences are represented by orthogonal axes.

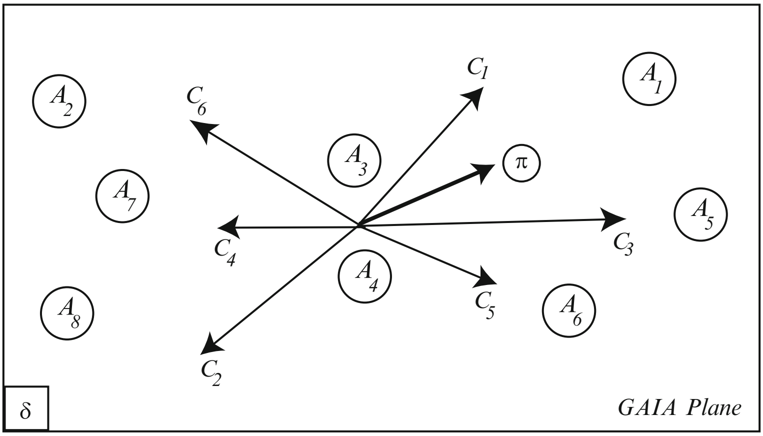


Fig. 6.6 Alternatives and criteria in the GAIA plane

P 5: Similar alternatives are represented by points located close to each other.
P 6: Alternatives being good on a particular criterion are represented by points located in the direction of the corresponding criterion axis.

On the example of Fig. 6.6, we observe:

- That the criteria $g_1(\cdot)$ and $g_3(\cdot)$ are expressing similar preferences and that the alternatives a_1 and a_5 are rather good on these criteria.
- That the criteria $g_6(\cdot)$ and $g_4(\cdot)$ are also expressing similar preferences and that the alternatives a_2, a_7, and a_8 are rather good on them.
- That the criteria $g_2(\cdot)$ and $g_5(\cdot)$ are rather independent
- That the criteria $g_1(\cdot)$ and $g_3(\cdot)$ are strongly conflicting with the criteria $g_4(\cdot)$ and $g_2(\cdot)$
- That the alternatives a_1, a_5 and a_6 are rather good on the criteria $g_1(\cdot)$, $g_3(\cdot)$ and $g_5(\cdot)$
- That the alternatives a_2, a_7 and a_8 are rather good on the criteria $g_6(\cdot)$, $g_4(\cdot)$ and $g_2(\cdot)$
- That the alternatives a_3 and a_4 are never good, never bad on all the criteria,
- ...

Although the GAIA plane includes only a percentage δ of the total information, it provides a powerful graphical visualisation tool for the analysis of a multicriteria problem. The discriminating power of the criteria, the conflicting aspects, as well as the "quality" of each alternative on the different criteria are becoming particularly clear.

6.7.3 *The PROMETHEE Decision Stick. The PROMETHEE Decision Axis*

Let us now introduce the impact of the weights in the GAIA plane. The vector of the weights is obviously also a vector of R^k. According to (6.20), the PROMETHEE net flow of an alternative a_i is the scalar product between the vector of its single criterion net flows and the vector of the weights:

$$\begin{aligned} a_i &: (\phi_1(a_i), \phi_2(a_i), \ldots, \phi_j(a_i), \ldots, \phi_k(a_i)), \\ w &: (w_1, w_2, \ldots, w_j, \ldots, w_k). \end{aligned} \tag{6.26}$$

This also means that the PROMETHEE net flow of a_i is the projection of the vector of its single criterion net flows on w. Consequently, the relative positions of the projections of all the alternatives on w provides the PROMETHEE II ranking. Clearly the vector w plays a crucial role. It can be represented in the GAIA plane by the projection of the unit vector of the weights. Let π be this projection, and let us call π the *PROMETHEE decision axis*.

On the example of Fig. 6.7, the PROMETHEE ranking is: $a_4 \succ a_3 \succ a_2 \succ a_1$. A realistic view of this ranking is given in the GAIA plane although some inconsistencies due to the projection can possibly occur.

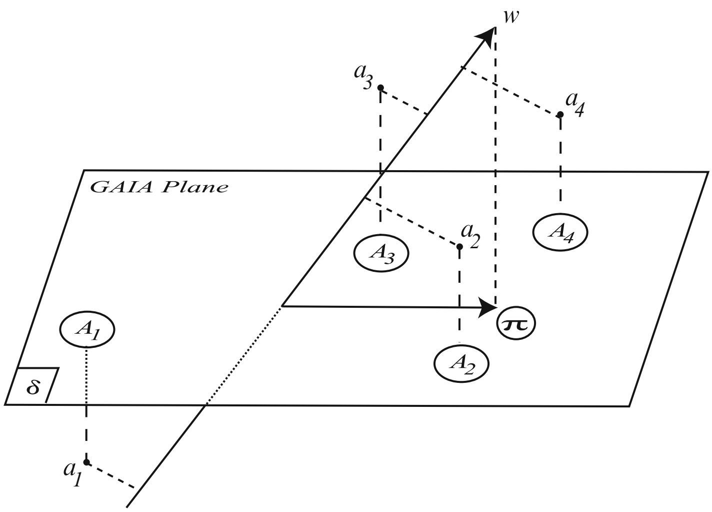


Fig. 6.7 PROMETHEE II ranking. PROMETHEE decision axis and stick

If all the weights are concentrated on one criterion, it is clear that the PROMETHEE decision axis will coincide with the axis of this criterion in the GAIA plane. Both axes are then the projection of a coordinate unit vector of $\mathbb{R}^k$. When the weights are distributed over all the criteria, the PROMETHEE decision axis appears as a weighted resultant of all the criterion axes $(C_1, C_2, \ldots, C_j, \ldots, C_k)$.

If π is long, the PROMETHEE decision axis has a strong decision power and the decision-maker is invited to select alternatives as far as possible in its direction.

If π is short, the PROMETHEE decision axis has no strong decision power. It means, according to the weights, that the criteria are strongly conflicting and that the selection of a good compromise is a hard problem.

When the weights are modified, the positions of the alternatives and of the criteria remain unchanged in the GAIA plane. The weight vector appears as a *decision stick* that the decision-maker can move according to his preferences in favour of particular criteria. When a sensitivity analysis is applied by modifying the weights, the PROMETHEE decision stick (w) and the PROMETHEE decision axis (π) are moving in such a way that the consequences for decision-making are easily observed in the GAIA plane (see Fig. 6.8).

Decision-making for multicriteria problems appears, thanks to this methodology, as a piloting problem. Piloting the decision stick over the GAIA plane. The PROMETHEE decision stick and the PROMETHEE decision axis provide a strong sensitivity analysis tool. Before finalising a decision we recommend to the decision-maker to simulate different weight distributions. In each case the situation can easily be appreciated in the GAIA plane, the recommended alternatives are located in the direction of the decision axis. As the alternatives and the criteria remain unchanged when the PROMETHEE decision stick is moving, the sensitivity analysis is particularly easy to manage. Piloting the decision stick is instantaneously operated by the PROMCALC, DECISION LAB and D-Sight software. The process is displayed graphically so that the results are easy to appreciate.

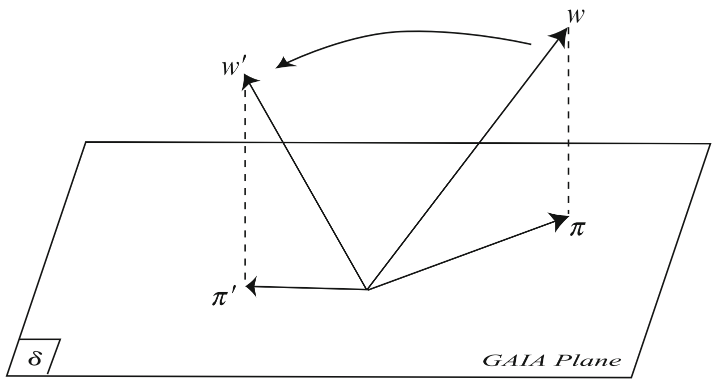


Fig. 6.8 Piloting the PROMETHEE decision stick

6.8 The PROMETHEE VI Sensitivity Tool (the "Human Brain")

The PROMETHEE VI module provides the decision-maker with additional information on his own personal view of his multicriteria problem. It allows to appreciate whether the problem is *hard or soft* according to his personal opinion.

It is obvious that the distribution of the weights plays an important role in all multicriteria problems. As soon as the weights are fixed, a final ranking is proposed by PROMETHEE II. In most of the cases the decision-maker is hesitating to allocate immediately precise values of the weights. His hesitation is due to several factors such as *indetermination, imprecision, uncertainty, lack of control,* …on the real-world situation.

However the decision-maker has usually in mind some order of magnitude on the weights, so that, despite his hesitations, he is able to give some intervals including their correct values. Let these intervals be:

$$w_j^- \leq w_j \leq w_j^+, \; j = 1, \ldots, k. \tag{6.27}$$

Let us then consider the set of all the extreme points of the unit vectors associated to all allowable weights. This set is limiting an area on the unit hypersphere in $\mathbb{R}^k$ (Fig. 6.9). Let us project this area on the GAIA plane and let us call (*HB*) (*"Human Brain"*) the obtained projection. Obviously (*HB*) is the area including all the extreme points of the PROMETHEE decision axis (π) for all allowable weights. Two particular situations can occur (Fig. 6.10):

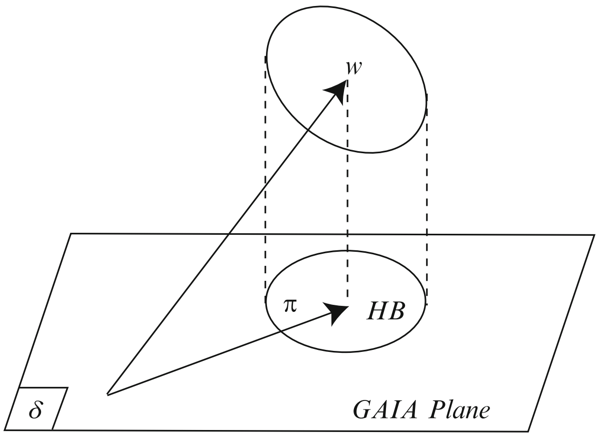


Fig. 6.9 "Human Brain"

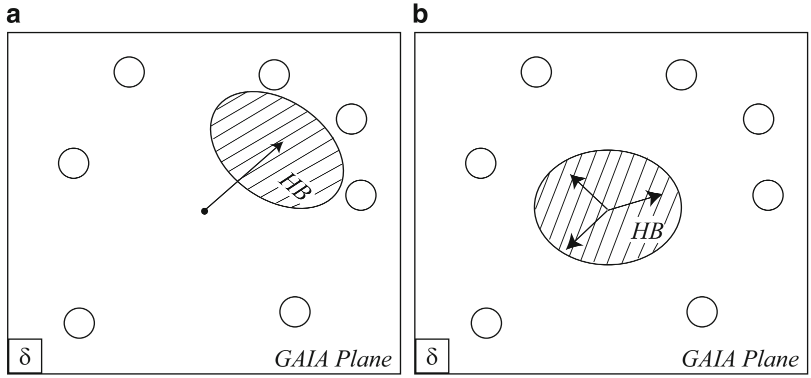


Fig. 6.10 Two types of decision problems. (**a**) Soft problem (S1). (**b**) Hard problem (S2)

S1: (*HB*) does not include the origin of the GAIA plane. In this case, when the weights are modified, the PROMETHEE decision axis (π) remains globally oriented in the same direction and all alternatives located in this direction are good. The multicriteria problem is rather easy to solve, it is a *soft* problem.

S2: Reversely if (*HB*) is including the origin, the PROMETHEE decision axis (π) can take any orientation. In this case compromise solutions can be possibly obtained in all directions. It is then actually difficult to make a final decision. According to his preferences and his hesitations, the decision-maker is facing a *hard* problem.

In most of the practical applications treated so far, the problems appeared to be rather soft and not too hard. This means that most multicriteria problems offer at the same time good compromises and bad solutions. PROMETHEE allows to select the good ones.

6.9 PROMETHEE V: MCDA Under Constraints

PROMETHEE I and II are appropriate to select one alternative. However in some applications a subset of alternatives must be identified, given a set of constraints. PROMETHEE V is extending the PROMETHEE methods to that particular case (see [11]).

Let $\{a_i, i = 1, 2, \ldots, n\}$ be the set of possible alternatives and let us associate the following boolean variables to them:

$$x_i = \begin{cases} 1 & \text{if } a_i \text{ is selected,} \\ 0 & \text{if not.} \end{cases} \tag{6.28}$$

The PROMETHEE V procedure consists of the two following steps:

Step 1: The multicriteria problem is first considered without constraints. The PROMETHEE II ranking is obtained for which the net flows $\{\phi(a_i), i = 1, 2, \ldots, n\}$ have been computed.

Step 2: The following $\{0, 1\}$ linear program is then considered in order to take into account the additional constraints (provided that they can be expressed linearly).

$$\max \left\{ \sum_{i=1}^{k} \phi(a_i) x_i \right\} \tag{6.29}$$

$$\sum_{i=1}^{n} \lambda_{p,i} x_i \sim \beta_p \quad p = 1, 2, \ldots, P \tag{6.30}$$

$$x_i \in \{0, 1\} \quad i = 1, 2, \ldots, n, \tag{6.31}$$

where $\sim$ holds for $=$, $\geq$ or $\leq$, and where the $\lambda_{p,i}$ are the coefficients of the constraints. The coefficients of the objective function (6.29) are the net outranking flows. The higher the net flow, the better the alternative. The purpose of the $\{0, 1\}$ linear program is to select alternatives collecting as much net flow as possible and taking the constraints into account.

The constraints (6.30) can include cardinality, budget, return, investment, marketing, … constraints. They can be related to all the alternatives or possibly to some clusters.

After having solved the $\{0, 1\}$ linear program, a subset of alternatives satisfying the constraints and providing as much net flow as possible is obtained. Classical 0–1 linear programming procedures may be used.

The PROMCALC software includes this PROMETHEE V procedure.

6.10 FlowSort

Recently, a number of researchers have proposed ways to extend the PROMETHEE methodology to sorting problems. Among them, we can cite PROMETHEE TRI [21] or PROMSORT [2]. In what follows, we describe a limited version of the FlowSort procedure developed by P. Nemery de Bellevaux in his Ph.D. thesis. From our point of view, this method constitutes the most natural extension of PROMETHEE to the sorting problematic.

The sorting problematic consists *in partitioning a set of alternatives into subsets with respect to pre-established norms* [47]. One way to interpret this definition is to assign a set of alternatives to predefined ordered groups (also called categories). For instance, one may think about the following applications:

- to assign a given patient to categories representing different disease grades according to a set of symptoms;
- to assign a company to categories representing different business failure risk levels according to financial criteria;
- ...

Let $Z_1, Z_2, \ldots, Z_V$ denote the V different categories. These are assumed to be ranked in order of preference: Z_1 is better than Z_2, Z_2 is better than Z_3, ... Consequently, Z_1 is considered to be best category while Z_V is the worst one. Let $\succ$ represent the preference order between the categories ($Z_1 \succ Z_2 \succ \ldots \succ Z_V$). We assume that each category Z_h is characterized by two limit profiles: the upper profile r_h and the lower profile r_{h+1} (let us note that the lower profile of Z_h corresponds to the upper profile of Z_{h+1}). Let $R = \{r_1, \ldots, r_{V+1}\}$ be the set of profiles. These are assumed to respect the following conditions:

Condition 1:

$$\forall a_i \in A : g_j(r_{V+1}) \leq g_j(a_i) \leq g_j(r_1) \ \forall j \in \{1, \ldots, q\} \tag{6.32}$$

Condition 2:

$$\forall r_h, r_l \in R | h < l : g_j(r_h) \geq g_j(r_l) \ \forall j \in \{1, \ldots, q\} \tag{6.33}$$

Condition 3:

$$\forall r_h, r_l \in R | h < l : \pi(r_h, r_l) > 0 \tag{6.34}$$

The first condition imposes that all the evaluations of the alternatives to be assigned are lying between r_{V+1} and r_1. As a natural consequence, no evaluation can be better than the one of the upper profile of the best category or worse than the lower profile of the worst category. Let us note that this condition is not restrictive since r_1 (respectively r_{V+1}) can always be defined as the ideal point of the problem (respectively the nadir point).

The two next conditions impose that some consistency should exist between the order of the categories and the preferences between the limit profiles:

- the evaluation of the upper limit profile of a better category should be at least as good as the evaluation of the upper profile of a worse category;
- the preference of the upper profile of a better category over the upper profile of a worse category should always be strictly positive.

Let us consider an alternative $a_i \in A$ to be sorted. The underlying idea of the FlowSort procedure is to compare a_i with respect to the elements of R by using the PROMETHEE I or PROMETHEE II ranking. Let us define $R_i = R \bigcup \{a_i\}$ (therefore $|R_i| = V + 2$). For all $x \in R_i$, the flow scores are computed as follows:

$$\phi^+_{R_i}(x) = \frac{1}{V+1} \sum_{y \in R_i} \pi(x, y) \tag{6.35}$$

$$\phi^-_{R_i}(x) = \frac{1}{V+1} \sum_{y \in R_i} \pi(y, x) \tag{6.36}$$

$$\phi_{R_i}(x) = \phi^+_{R_i}(x) - \phi^-_{R_i}(x) \tag{6.37}$$

The ranking based on the positive and negative flow scores can lead to two different situations:

$$Z_{\phi^+}(a_i) = Z_h \ \ if \ \ \phi^+_{R_i}(r_h) \geq \phi^+_{R_i}(a_i) > \phi^+_{R_i}(r_{h+1}) \tag{6.38}$$

$$Z_{\phi^-}(a_i) = Z_l \ \ if \ \ \phi^-_{R_i}(r_l) < \phi^-_{R_i}(a_i) \leq \phi^-_{R_i}(r_{l+1}) \tag{6.39}$$

where $Z_{\phi^+}(a_i)$ (respectively $Z_{\phi^-}(a_i)$) represents the assignment based on the positive (respectively negative) flow score only. Nevertheless, the assignment rule based on the PROMETHEE I ranking should integrate both of these aspects. As a consequence, let $b = \min\{h, l\}$ be the index of the category corresponding to the best assignment and let $w = \max\{h, l\}$ be the index of the category corresponding to the worst assignment. The first assignment rule will lead to conclude that a_i is assigned to the set of categories $[Z_b, \ldots, Z_w]$. Of course, if $w = b$ the assignment is unique.

Alternatively, the decision maker could force the assignment to a unique category by using a rule based on the net flow score:

$$Z_{\phi}(a_i) = Z_t \ if \ \phi_{R_i}(r_t) \geq \phi_{R_i}(a_i) > \phi_{R_i}(r_{t+1}) \tag{6.40}$$

As expected, the assignment procedures based on the PROMETHEE I and PROMETHEE II rankings are consistent. More formally [36]:

$$\forall a_i \in A : Z_b(a_i) \succeq Z_t(a_i) \succeq Z_w(a_i) \tag{6.41}$$

In other words, the assignment based on the net flow score will always lead to a category that is at least as good as ($\succeq$) the worst category and no better than the best category found by the first assignment rule.

These two assignment rules are the basics of FlowSort. Let us remind the reader that this section only constitutes a limited presentation of the method. We have to stress that a similar procedure exists when categories are represented by central profiles (instead of limit profiles) and that FlowSort is not limited to the PROMETHEE method [37] (even if the conditions imposed on the preference structure are close to it). Finally, it is worth noting that a number of theoretical properties have been analyzed to characterize the assignment rules. We refer the interested reader to [36] for a detailed analysis.

6.11 The PROMETHEE GDSS Procedure

The PROMETHEE Group Decision Support System has been developed to provide decision aid to a group of decision-makers $(DM_1), (DM_2), \ldots, (DM_r), \ldots (DM_R)$ (see [29]). It has been designed to be used in a GDSS room including a PC, a printer and a video projector for the facilitator, and R working stations for the DM's. Each working station includes room for a DM (and possibly a collaborator), a PC and Tel/Fax so that the DM's can possibly consult their business base. All the PC's are connected to the facilitator through a local network.

There is no objection to use the procedure in the framework of teleconference or video conference systems. It this case the DM's are not gathering in a GDSS room, they directly talk together through the computer network.

One iteration of the PROMETHEE GDSS procedure consists in 11 steps grouped in three phases:

- Phase I: Generation of alternatives and criteria
- Phase II: Individual evaluation by each *DM*
- Phase III: Global evaluation by the group

Feedback is possible after each iteration for conflict resolution until a final consensus is reached.

6.11.1 Phase I: Generation of Alternatives and Criteria

Step 1: First contact Facilitator—DM's The facilitator meets the DM's together or individually in order to enrich his knowledge of the problem. Usually this step takes place in the business base of each DM prior to the GDSS room session.

Step 2: Problem description in the GDSS room The facilitator describes the computer infrastructure, the PROMETHEE methodology, and introduces the problem.

Step 3: Generation of alternatives It is a computer step. Each DM implements possible alternatives including their extended description. For instance strategies, investments, locations, production schemes, marketing actions, …depending on the problem.

Step 4: Stable set of alternatives All the proposed alternatives are collected and displayed by the facilitator one by one on the video-screen, anonymously or not. An open discussion takes place, alternatives are canceled, new ones are proposed, combined ones are merged, until a stable set of n alternatives $(a_1, a_2, \ldots, a_i, \ldots, a_n)$ is reached. This brainstorming procedure is extremely useful, it often generates alternatives that were unforeseen at the beginning.

Step 5: Comments on the alternatives It is again a computer step. Each DM implements his comments on all the alternatives. All these comments are collected and displayed by the facilitator. Nothing gets lost. Complete minutes can be printed at any time.

Step 6: Stable set of evaluation criteria The same procedure as for the alternatives is applied to define a stable set of evaluation criteria $(g_1(\cdot), g_2(\cdot), \ldots, g_j(\cdot), \ldots g_k(\cdot))$. Computer and open discussion activities are alternating. At the end the frame of an evaluation table (Type Table 6.1) is obtained. This frame consists in a $(n \times k)$ matrix. This ends the first phase. Feedbacks are already possible to be sure a stable set of alternatives and criteria is reached.

6.11.2 Phase II: Individual Evaluation by Each DM

Let us suppose that each DM has a decision power given by a non-negative weight $(\omega_r, r = 1, 2, \ldots, R)$ so that:

$$\sum_{r=1}^{R} \omega_r = 1. \tag{6.42}$$

Step 7: Individual evaluation tables The evaluation table $(n \times k)$ has to be completed by each DM. Some evaluation values are introduced in advance by the facilitator if there is an objective agreement on them (prices, volumes, budgets, …). If not each DM is allowed to introduce his own values. All the DM's implement the same $(n \times k)$ matrix, if some of them are not interested in particular criteria, they can simply allocate a zero weight to these criteria.

Step 8: Additional PROMETHEE information Each DM develops his own PROMETHEE-GAIA analysis. Assistance is given by the facilitator to provide the PROMETHEE additional information on the weights and the generalized criteria.

Step 9: Individual PROMETHEE-GAIA analysis The PROMETHEE I and II rankings, the profiles of the alternatives and the GAIA plane as well as the net flow vector $\phi_r(\cdot)$ are instantaneously obtained, so that each DM gets his own clear view of the problem.

6.11.3 Phase III: Global Evaluation by the Group

Step 10: Display of the individual investigations The rankings and the GAIA plane of each DM are collected and displayed by the facilitator so that the group of all DM'S is informed of the potential conflicts.

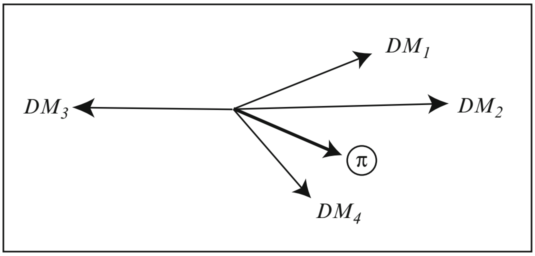


Fig. 6.11 Conflict between DM's

Step 11: Global evaluation The net flow vectors $\{\phi_r(\cdot), r = 1, \ldots, R\}$ of all the DM's are collected by the facilitator and put in a $(n \times R)$ matrix. It is a rather small matrix which is easy to analyzed. Each criterion of this matrix expresses the point of view of a particular DM.

Each of these criteria has a weight ω_r and an associated generalized criterion of Type 3 ($p = 2$) so that the preferences allocated to the deviations between the $\phi_i^r(\cdot)$ values will be proportional to these deviations.

A global PROMETHEE II ranking and the associated GAIA plane are then computed. As each criterion is representing a DM, the conflicts between them are clearly visualized in the GAIA plane. See for example Fig. 6.11 where DM_3 is strongly in conflict with DM_1, DM_2 and DM_4. The associated PROMETHEE decision axis (π) gives the direction in which to decide according to the weights allocated to the DM's. The alternatives (not represented on Fig. 6.11) to be considered are those in the direction of π.

If the conflicts are too sensitive the following feedbacks could be considered: Back to the weighting of the DM's. Back to the individual evaluations. Back to the set of criteria. Back to the set of alternatives. Back to the starting phase and to include an additional stakeholder ("DM") such as a social negotiator or a government mediator.

The whole procedure is summarized in the following scheme (Fig. 6.12):

6.12 The D-Sight Software

D-Sight [23] is the third generation of PROMETHEE based software; it has followed DECISION LAB 2000 and PROMCALC [12]. This application has been developed by Quantin Hayez at the CoDE-SMG laboratory. His work has been funded by the Walloon region under a First Spin-Off project supervised by Yves De Smet. Bertrand Mareschal initially acted as a scientific adviser. The software is available since February 2010 and despite the fact that it is quite new, many universities worldwide have already started to use it for educational and research purposes (http://www.d-sight.com/academic) [30]. Moreover, recent industrial projects testify its successful application in the fields of tenders evaluation, socio-economic assessment, infrastructure deployment . . . (http://www.d-sight.com/case-studies).

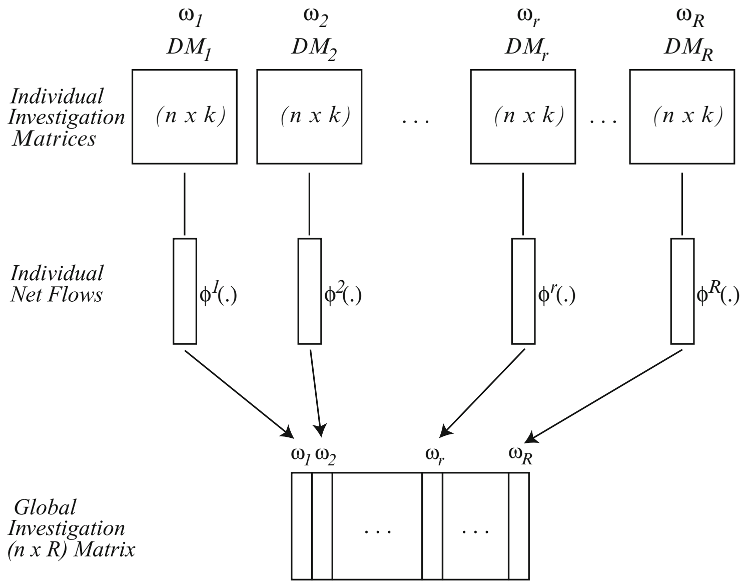


Fig. 6.12 Overview PROMETHEE GDSS procedure

D-Sight presents the same main functionalities as the preceding software (see Fig. 6.13). It is based on visual interactive tools that help the decision makers to better manage, understand and master their problems. The accustomed users of the PROMETHEE and GAIA methods will rediscover traditional tools such as an interactive GAIA plane, the PROMETHEE I and II rankings, the walking weights or weight stability intervals tools, in a new interface based on a flexible tabs system.

Additionally, D-Sight offers new features such as:

- the possibility to group criteria into a multiple layers hierarchy;
- an improved representation of the GAIA plane based on the explicit projections of the alternatives against the criteria or against the decision stick;
- a new representation of the PROMETHEE I ranking called the PROMETHEE Diamond (see Fig. 6.14);
- the PROMETHEE VI sensitivity tool (also called the “decision maker’s brain”) which was initially available in PROMCALC but not in Decision Lab 2000;
- the possibility to dynamically represent unicriterion net flow scores in a graph and, as a consequence, to better assess the impact of intra-criterion parameters;
- . . .

The software can easily be interfaced with other systems or databases and supports direct copy-paste with traditional applications. An automatic update procedure allows the users to always work with the latest release of the software. Finally, D-Sight offers a plugin system allowing the user to add features on the fly. These plugins are developed independently from the core system. They are available to

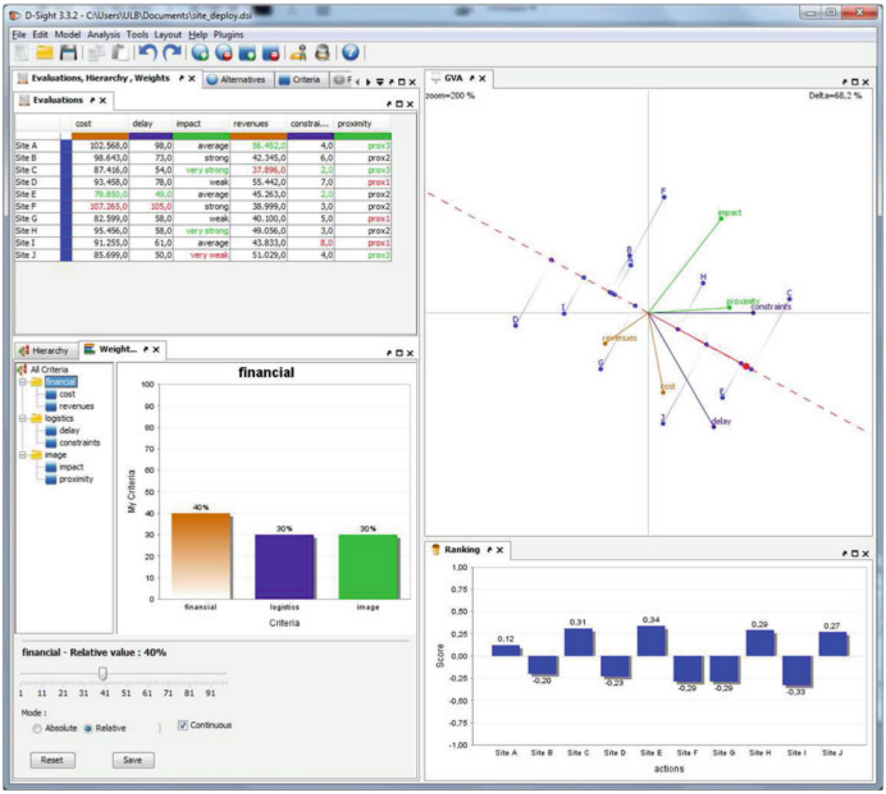


Fig. 6.13 Main functionalities of D-Sight

the user through an online plugin store accessible from D-Sight. With a single click, they are fully integrated in the software. Both D-Sight and the plugins are developed in Java. Some of the current available plugins are:

- a weights elicitation component based on an interactive tool;
- a module to geo-localize the alternatives in a complete interactive maps system directly connected to the mcda results (see Fig. 6.14);
- an optimization tool based on the PROMETHEE V procedure;
- a multi-actors plugin allowing decentralized decision making, while taking into account different stakeholders or scenarios;

Additional information about D-Sight can also be obtained on the website of the CoDE-SMG spin-off: http://www.d-sight.com.

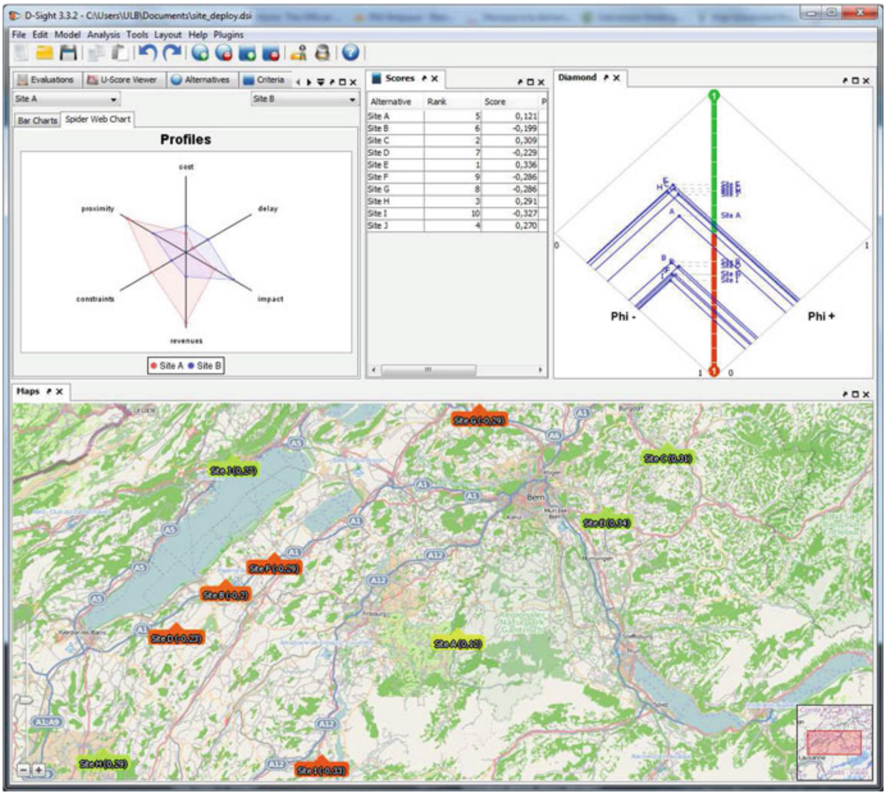


Fig. 6.14 D-Sight: geo-localization of the alternatives, PROMETHEE I diamond, comparisons of profiles

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