Chapter 24
Multicriteria Decision Aid/Analysis in Finance

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Abstract Over the past decades the complexity of financial decisions has increased rapidly, thus highlighting the importance of developing and implementing sophisticated and efficient quantitative analysis techniques for supporting and aiding financial decision making. Multicriteria decision aid (MCDA), an advanced branch of operations research, provides financial decision makers and analysts with a wide range of methodologies well-suited for the complexity of modern financial decision making. The aim of this chapter is to provide an in-depth presentation of the contributions of MCDA in finance focusing on the methods used, applications, computation, and directions for future research.

Keywords Multicriteria decision aid • Finance • Portfolio theory • Multiple criteria optimization • Outranking relations • Preference disaggregation analysis

24.1 Introduction

Over the past decades, the globalization of financial markets, the intensification of competition among organizations, and the rapid social and technological changes that have taken place have only led to increasing uncertainty and instability in the business and financial environment. Within this more recent context, both the

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importance of financial decision making and the complexity of the process by which financial decision making is carried out have increased. This is clearly evident by the variety and volume of new financial products and services that have appeared on the scene.

In this new era of financial reality, researchers and practitioners acknowledge the requirement to address financial decision-making problems through integrated and realistic approaches utilizing sophisticated analytical techniques. In this way, the connections between financial theory, the tools of operations research, and mathematical modelling have become more entwined. Techniques from the fields of optimization, forecasting, decision support systems, MCDA, fuzzy logic, stochastic processes, simulation, etc. are now commonly considered valuable tools for financial decision making.

The use of mathematics and operations research in finance got its start in the 1950s with the introduction of Markowitz's portfolio theory [131, 133]. Since then, in addition to portfolio selection and management, operations research has contributed to financial decision making problems in other areas including venture capital investments, bankruptcy prediction, financial planning, corporate mergers and acquisitions, country risk assessment, etc. These contributions are not limited to academic research; they are now often found in daily practice.

Within the field of operations research, MCDA has evolved over the last three decades into one of its pillar disciplines. The development of MCDA is based upon the common finding that a sole objective, goal, criterion, or point of view is rarely used to make real-world decisions. In response, MCDA is devoted to the development of appropriate methodologies to support and aid decision makers across ranges of situations in which multiple conflicting decision factors (objectives, goals, criteria, etc.) are to be considered simultaneously.

The methodological framework of MCDA is well-suited to the growing complexities encountered in financial decision making. While there have been in finance MCDA stirrings going back 20–30 years, the topic of MCDA, as can be seen from the bulk of the references, really hasn't come into its own until recently (see [215] for a recent survey of the literature). As for early stirrings, we have, for example, Bhaskar [22] in which microeconomic theory was criticized for largely pursuing a single criterion approach arguing that things like profit maximization are too naive to meet the evolving decision-making demands in many financial areas. Also, in another paper [23], the unavoidable presence of multiple objectives in capital budgeting was noted and the necessity for developing ways to deal with the unique challenges posed by multiple criteria was stressed. It is upon what has taken place since these early roots, and on what are today promising directions in MCDA in finance, that this contribution is focused.

Such observations and findings have motivated researchers to explore the potentials of MCDA in addressing financial decision-making problems. The objective of this chapter is to provide a state-of-the-art comprehensive review of the research made up to date on this issue. Section 24.2 presents discussions to justify the presence of MCDA in financial decision making. Section 24.3, focuses on MCDA in resource allocation problems (continuous problems) as in the field of portfolio management. Section 24.4, presents the contribution of MCDA methodologies in supporting financial decisions that require the evaluation of a discrete set of alternatives (firms, countries, stocks, investment projects, etc.). Finally, Sect. 24.5 concludes the chapter and discusses possible future research directions on the implementation of multicriteria analysis in financial institutions and firms.

24.2 Financial Decision Making

Financial-economic decision problems come in great variety. Individuals are involved in decisions concerning their future pensions, the financing of their homes, and investments in mutual funds. Firms, financial institutions, and advisors are involved in cross-country mergers, complicated swap contracts, and mortgage-backed securities, to name just a few.

Despite the variety, such decisions have much in common. Maybe "money" comes first to mind, but there are typically other factors that suggest that financial-economic problems should most appropriately be treated as multiple criteria decision problems in general: multiple actors, multiple policy constraints, and multiple sources of risk (see e.g., Spronk and Hallerbach [177], and Hallerbach and Spronk [79, 80], Martel and Zopounidis [134], Zopounidis [202], and Steuer and Na [182]).

Two other common elements in financial decisions are that their outcomes are distributed over time and uncertainty, and thus involve risk. A further factor is that most decisions are made consciously, with a clear and constant drive to make "good", "better" or even "optimal" decisions. In this drive to improve on financial decisions, we stumble across an area of tension between decision making in practice on the one hand and the potential contributions of finance theory and decision tools on the other. Although the bulk of financial theory is of a descriptive nature, thus focusing on the "average" or "representative" decision maker, we observe a large willingness to apply financial theory in actual decision-making. At the same time, knowledge about decision tools that can be applied in a specific decision situation, is limited. Clearly, there is need of a framework that can provide guidance in applying financial theory, decision tools, and common sense to solving financial problems.

24.2.1 Issues, Concepts, and Principles

> Finance is a sub field of economics distinguished by both its focus and its methodology. The primary focus of finance is the workings of the capital markets and the supply and the pricing of capital assets. The methodology of finance is the use of close substitutes to price financial contracts and instruments. This methodology is applied to value instruments whose characteristics extend across time and whose payoffs depend upon the resolution of uncertainty. (Ross [158], p. 1)

The field of finance is concerned with decisions with respect to the efficient allocation of scarce capital resources over competing alternatives. The allocation is efficient when the alternative with the highest value is chosen. Current value is viewed as the (present) value of claims on future cash flows. Hence we can say that financial decisions involve the valuation of future, and hence uncertain or "risky," cash flow streams. Cash flow stream X is valued by comparing it with cash flow streams {A,. . . ,Z} that are traded on financial markets. When a traded cash flow stream Y has been identified that is a substitute for X, then their values must be the same. After all, when introducing X to the market, it cannot be distinguished value-wise from Y. Accepting the efficient market hypothesis (stipulating that all available information is fully and immediately incorporated in market prices), the market price of Y equals the value of Y, and hence the value of X. This explains the crucial role of financial markets.

The valuation of future cash flow streams is a key issue in finance. The process of valuation must be preceded by evaluation: without analyzing the characteristics of a cash flow stream, no potential substitute can be identified. Since it is uncertain what the future will bring, the analysis of the risk characteristics will be predominant. Moreover, as time passes, the current value must be protected against influences that may erode its value. This in turn implies the need for risk management. There are basically three areas of financial decisions:

1. **Capital budgeting**: to what portfolio of real investment projects should a firm commit its capital? The central issues here are how to evaluate investment opportunities, how to distinguish profitable from non-profitable projects and how to choose between competing projects.
2. **Corporate financing**: this encompasses the capital structure policy and dividend policy and addresses questions as: how should the firm finance its activities? What securities should the firm issue or what financial contracts should the firm engage in? What part of the firm's earnings should be paid as cash dividends and what part reinvested in the firm? How should the firm's solvency and liquidity be maintained?
3. **Financial investment**: this is the mirror image of the previous decision area and involves choosing a portfolio of financial securities with the objective to change the consumption pattern over time.

In each of these decision areas the financial key issues of valuation, risk analysis and risk management, and performance evaluation can be recognized, and from the above several financial concepts emerge: financial markets, efficient allocation and market value. In approaching the financial decision areas, some financial principles or maxims are formulated. The first is self-interested behavior: economic subjects are driven by *non-satiation* ("greed"). This ensures the goal of value maximization. Prices are based on financial markets, and under the efficient market hypothesis, prices of securities coincide with their value. Value has time and risk dimensions. With regard to the former, *time preference* is assumed (a dollar today is preferred to a dollar tomorrow). With respect to the latter, *risk aversion* is assumed (a safe dollar is preferred to a risky dollar). Overall risk may be reduced by *diversification*:

combining risky assets or cash flow streams may be beneficial. In one way or another, the trade-off between expected return and risk that is imposed by market participants on the evaluation of risky ventures will translate into a *risk-return trade-off* that is offered by investment opportunities in the market.

Since value has time and risk aspects, the question arises about what mechanisms can be invoked to incorporate these dimensions in the valuation process. There are basically two mechanisms. The first is the arbitrage mechanism. Value is derived from the presumption that there do not exist arbitrage opportunities. This no-arbitrage condition excludes sure profits at no cost and implies that perfect substitutes have the same value. This is the *law of one price*, one of the very few laws in financial economics. It is a strong mechanism, requiring very few assumptions on market subjects, only non-satiation. Examples of valuation models built on no-arbitrage are the Arbitrage Pricing Theory for primary financial assets and the Option Pricing Theory for derivative securities. The second is the equilibrium mechanism. In this case value is derived from the market clearing condition that demand equals supply. The latter mechanism is much weaker than the former: the exclusion of arbitrage opportunities is a necessary but by no means a sufficient condition for market equilibrium. In addition to non-satiation also assumptions must be made regarding the risk attitudes of all market participants. Examples of equilibrium-based models are the Capital Asset Pricing Model and its variants. Below we discuss the differences between the two valuation approaches in more detail. It suffices to remark that it is still a big step from the principles to solving actual decision problems.

24.2.2 Focus of Financial Research

An alternative, albeit almost circular, definition of finance is provided by Jarrow [101, p. 1].

> Finance theory (...) includes those models most often associated with financial economics. (...) [*A*] practical definition of financial economics is found in those topics that appear with some regularity in such publications as *Journal of Finance, Journal of Financial and Quantitative Analysis, Journal of Financial Economics*, and *Journal of Banking and Finance*.

Browsing through back volumes of these journals and comparing them to the more recent ones reveals a blatant development in nature and focus. In early days of finance, the papers were descriptive in a narrative way and in the main focused on financial instruments and institutions. Finance as a decision science emerged in the early 1950s, when Markowitz [130, 131] studied the portfolio selection decision and launched what now is known as "modern portfolio theory." In the 1960s and the early 1970s, many financial economic decision problems were approached by operational research techniques; see for example Ashford et al. [8] and McInnes and Carleton [138] for an overview. However, since then, this type of research has became more and more absorbed by the operations research community and in their journals.

But what direction did finance take? Over the last 25 years mathematical models have replaced the verbal models and finance has founded itself firmly in a neo-classical micro-economic tradition. Over this period we observe a shift to research that is descriptive in a sophisticated econometrical way and that focuses on the statistical characteristics of (mainly well-developed) financial markets where a host of financial instruments is traded. Bollerslev [27, p. 41], aptly describes this shift as follows.

> A cursory look at the traditional econometrics journals (. . .) severely underestimates the scope of the field [of financial econometrics], as many of the important econometric advances are now also published in the premier finance journals - the *Journal of Finance*, the *Journal of Financial Economics*, and the *Review of Financial Studies* – as well as a host of other empirically oriented finance journals.

The host of reported research addresses the behavior of financial market prices. The study of the pricing of primary securities is interesting for its own right, but it is also relevant for the pricing of derivative securities. Indeed, the description of the pricing of primary assets and the development of tools for pricing derivative assets mark the success story of modern finance.

The body of descriptive finance theory has grown enormously. According to modern definitions of the field of finance, the descriptive nature is even predominant.

> The core of finance theory is the study of the behavior of economic agents in allocating and deploying their resources, both spatially and across time, in an uncertain environment. (Merton [140], p. 7)

Compared to Ross' [158] definition cited earlier, the focus is purely positive. The question arises to what extent the insights gained from descriptive finance—how sophisticated they may be from a mathematical, statistical or econometric point of view—can serve as guidelines for financial decisions in practice. Almost 30 years ago, in the preface of their book *The Theory of Finance*, Eugene Fama and Merton Miller defended their omission of detailed examples, purporting to show how to apply the theory to real-world decision problems, as

> (. . .) a reflection of our belief that the potential contribution of the theory of finance to the decision-making process, although substantial, is still essentially indirect. The theory can often help expose the inconsistencies in existing procedures; it can help keep the really critical questions from getting lost in the inevitable maze of technical detail; and it can help prevent the too easy, unthinking acceptance of either the old clichs or new fads. But the theory of finance has not yet been brought, and perhaps never will be, to the cookbook stage. (Fama and Miller [65], p. viii)

Careful inspection of current finance texts reveals that in this respect not much has changed. However, pure finance theory and foolproof financial recipes are two extremes of a continuum. The latter cookbook stage will never be achieved, of course, and in all realism and wisdom this alchemic goal should not be sought for. But what we dearly miss is an extensive body of research that bridges the apparent gap between the extremes: research that shows how to solve real-world financial decision problems without violating insights offered by pure finance theory on the one hand and without neglecting the peculiarities of the specific decision problem on the other.

On another matter, the role of assumptions in modelling is to simplify the real world in order to make it tractable. In this respect the art of modelling is to make assumptions where they most contribute to the model's tractability and at the same time detract from the realism of the model as little as possible. The considerations in this trade-off are fundamentally different for positive (descriptive) models on the one hand and conditional-normative models on the other. In the next section we elaborate further on the distinctions between the two types of modelling as concerns the role of assumptions.

24.2.3 Descriptive vs. Conditional-Normative Modelling

In a positive or descriptive model simplified assumptions are made in order to obtain a testable implication of the model. The validity of the model is evaluated according to the inability to reject the model's implications at some level of significance. So validity is of an empirical nature, solely judged by the implications of the model. Consider the example of an equilibrium asset-pricing model. As a starting point, assumptions are made with respect to the preferences of an imaginary investor and the risk-return characteristics of the investment opportunities. These assumptions are sufficiently strong to allow solving the portfolio optimization problem. Next a homogeneity condition is imposed: all investors in the market possess the same information and share the same expectations. This allows focusing on "a representative investor". Finally the equilibrium market clearing condition is imposed: all available assets (supply) must be incorporated in the portfolio of the representative investor (demand). The first order conditions of portfolio optimality then stipulate the trade-off between risk and expected return that is required by the investor. Because of the market clearing, the assets offer the same trade-off. Hence a market-wide relationship between risk and return is established and this relationship is the object of empirical testing. As long as the pricing relationship is not falsified the model is accepted, irrespective of whether the necessary assumptions are realistic or not. When the model is falsified, deduction may help to amend the assumptions where after the same procedure is followed. This hypothetic-deductive cycle ends when the model is no longer falsified by the empirical data at hand.

In a conditional-normative model, simplifying assumptions are also made in order to obtain a tractable model. These assumptions relate to the preferences of the decision maker and to the representation of the set of choice alternatives. The object of the conditional-normative modelling is not to infer a testable implication but to obtain a decision rule. This derived decision rule is valid and can normatively be applied conditional on the fact that the decision maker satisfies the underlying assumptions; cf. Keynes [110].

In order to support decisions in finance, obviously both the preferences of the decision maker and the characteristics of the choice alternatives should be understood and related to each other. Unfortunately, the host of financial-economic modelling is of a positive nature and focuses on the "average" decision maker instead of addressing the particular (typically non-average) decision maker. The assumptions underlying financial theory at best describe "average individuals" and "average decision situations" and hence are not suited to describe specific individual decision problems. The assumptions made to simplify the decision situation often completely redefine the particular problem at hand. The real world is replaced by an over-simplified model-world. As a consequence, not the initial problem is solved but a synthesized and redefined problem that is not even recognized by the decision maker himself. The over-simplified model becomes a Procrustes bed for the financial decision maker who seeks advice.

For example, it is assumed that a decision maker has complete information and that this information can be molded into easily manipulated probability distributions. Even worse, positive knowledge and descriptive theories that by definition reflect the outcomes of decisions made by some representative decision maker are used to prescribe what actions to take in a specific decision situation. For example, equilibrium asset pricing theories predict the effects of decisions and actions of many individuals on the formation of prices in financial markets. Under the homogeneity condition the collection of investors is reduced to the representative investor. When the pricing implications of the model are simply used to guide actual investment behavior, then the decision maker is forced into the straitjacket of this representative investor.

Unfortunately we observe that conditional-normative financial modelling is only regarded as a starting point for descriptive modelling and is not pursued for its own sake. After almost 20 years, Hastie's [83] lament has not lost its poignancy.

> In American business today, particularly in the field of finance, what is needed are approximate answers to the precise problem rather than precise answers to the approximate problem.

Apart from the positive modelling of financial markets as described above, there is one other field in finance in which the achievements of applied modelling are apparent: option pricing theory, the set of models that enable the pricing of derivative securities and all kinds of contingent claims. Indeed, the option pricing formulas developed by Black and Scholes [25] and Merton [139] mark a huge success in the history of financial modelling. Contingent claims analysis made a flying start, and

> .... when judged by its ability to explain the empirical data, option pricing theory is the most successful theory not only in finance, but in all of economics. (Ross [158], p. 24)
> Given a theory that works so well, the best empirical work will be to use it as a tool rather than to test it. (Ross [158], p. 23)

Indeed, modern-day derivatives trading would be unthinkable without the decision support of an impressive coherent toolbox for analyzing the risk characteristics of derivatives and for pricing them in a consistent way. Compared to this framework, the models and theories developed and tested for primary assets look pale. What is the reason for the success of derivatives research?

For an explanation we turn to the principal tool used in option pricing theory: no-arbitrage valuation. By definition derivative securities derive their value from primary underlying assets. Under some mild assumptions, a dynamic trading strategy can be designed in which the derivative security is exactly replicated with a portfolio of the primary security and risk-free bonds. Under the no-arbitrage condition, the current value of the derivative security and the replicating portfolio should be identical. Looking from another perspective, a suitably chosen hedge combination of the derivative and the underlying security produces a risk-free position. On this position the risk-free rate must be earned, otherwise there exist arbitrage opportunities. Since the position is risk free, risk attitudes and risk aversion do not enter the story. Therefore a derivative security will have the same value in a market environment with risk neutral investors as in a market with risk averse investors. This in turn implies that a derivative can be priced under the assumption that investors are risk neutral. As a consequence, no assumptions are required on preferences (other than non-satiation), utility functions, the degree of risk aversion, and risk premia. Thus, option pricing theory can escape from the burden of modelling of preference structures. Instead, research attention shifts to analyzing price dynamics on financial markets. An additional reason for the success in derivatives research is that the analytical and mathematical techniques are similar to those used in the physical sciences (see for example Derman [44]).

Of course, even in derivatives modelling some assumptions are required. This introduces model risk. When the functional relationships stipulated in the model are wrong, or when relevant input parameters of the model are incorrectly estimated, the model produces the wrong value and the wrong risk profile of the derivative. To an increasing degree, financial institutions are aware that great losses can be incurred because of model risk. Especially in risk management and derivatives trading model risk is a hot item (see Derman [43]). This spurred Merton to ventilate this warning.

> At times, the mathematics of the models become too interesting and we lose sight of the models' ultimate purpose. The mathematics of the models is precise, but the models are not, being only approximations to the complex, real world. Their accuracy as a useful approximation to that world varies considerably across time and place. The practitioner should therefore apply the models only tentatively, assessing their limitations carefully in each application. (Merton [140], p. 14)

Ironically this quote was taken just after the very successful launch of Long Term Capital Management (LTCM), the hedge fund of which Merton and Myron Scholes were the founding partners. In 1998, LTCM collapsed and model risk played a very important role in this debacle.

Summarizing we draw the conclusion that successful applied financial modelling does exist, and blossoms in the field of derivatives. Here also the validity of the assumptions is crucial, this in contrast to positive modelling. However, in the field of derivatives with replicating strategies and arbitrage-based valuation, the concept of "absence of risk" is well defined and no preference assumptions are needed in the modelling process. For modelling decisions regarding the underlying primary assets, in contrast, assumptions on the decision maker's preferences and on the "risk" attached to the outcomes of the choice alternatives are indispensable. For these types of financial problems, the host of simplifying assumptions that are made in the descriptive modelling framework invalidate the use of the model in a specific decision situation. Thus we face the following challenge: how can we retain the conceptual foundation of the financial-economic framework and still provide sound advice that can be applied in multifarious practice? As a first step we will sketch the relationship between decision sciences and financial decision-making.

24.2.4 Decision Support for Financial Decisions

Over the last 50 years or so, the financial discipline has shown continuously rapid and profound changes, both in theory and in practice. Many disciplines have been affected by globalization, deregulation, privatization, computerization, and communication technologies. Hardly any field has been influenced as much as finance. After the mainly institutional and even somewhat *ad hoc* approaches before the 1950s, Markowitz [130, 131] has opened new avenues by formalizing and quantifying the concept of "risk". In the decades that followed, a lot of attention was paid to the functioning of financial markets and the pricing of financial assets including options. The year 1973 gave birth to the first official market in options (CBOE) and to crucial option pricing formulas that have become famous quite fast (Black-Scholes and Cox-Ross-Rubinstein, see Hull [90]), both in theory and practice. At that time, financial decision problems were structured by (a) listing a number of mutually exclusive decision alternatives, (b) describing them by their (estimated) future cash flows, including an estimation of their stochastic variation and later on including the effect of optional decisions, and (c) valuing them by using the market models describing financial markets.

In the 1970s, 1980s and 1990s, the financial world saw enormous growth in derivative products, both in terms of variety and in terms of market volumes. Financial institutions have learned to work with complex financial products. Academia has contributed by developing many pricing models, notably for derivatives. Also, one can say that financial theory has been rewritten in the light of contingent claims ("optional decisions") and will soon be further reshaped by giving more attention to game elements in financial decisions. The rapid development of the use of complex financial products has certainly not been without accidents. This has led regulators to demand more precise evaluations and the reporting of financial positions (cf. e.g., the emergence of the Value-at-Risk concept, see Jorion [107]).

In addition to the analysis of financial risk, the structured management of financial risk has come to the forefront. In their textbook, Bodie and Merton [26] describe the threefold tasks of the financial discipline as Valuation, Risk Management, and Optimization. We would like to amend the threefold tasks of financial management to Valuation, Risk Management, and Decision Making. The reason is that financial decision problems often have to be solved in dynamic environments where information is not always complete, different stakeholders with possibly conflicting goals and constraints play a role and clear-cut optimization problems cannot always be obtained (and solved).

At the same time, many efforts from the decision-making disciplines are misdirected. For instance, some approaches fail to give room for the inherent complexity of the decision procedure given the decision maker's specific context. Other approaches concentrate on the beauties of a particular decision method without doing full justice to the peculiarities of the decision context. Aside from being partial in this respect, useful principles and insights offered by financial-economic theory are often not integrated in the decision modelling. It is therefore no surprise that one can observe in practice unstructured *ad hoc* approaches as well as complex approaches that severely restrict the decision process.

24.2.5 Relevance of MCDA for Financial Decisions

The central issue in financial economics is the efficient allocation of scarce capital and resources over alternative uses. The allocation (and redistribution) of capital takes place on financial markets and is termed "efficient" when market value is maximized. Just as water will flow to the lowest point, capital will flow to uses that offer the highest return. Therefore it seems that the criterion for guiding financial decisions is one-dimensional: maximize market value or maximize future return.

From a financial-economic perspective, the goal of the firm, for example, is very much single objective. Management should maximize the firm's contribution to the financial wealth of its shareholders. Also the shareholders are considered to be myopic. Their only objective is to maximize their single-dimensional financial wealth. The link between the shareholders and the firm is footed in law. Shareholders are the owners of the firm. They possess the property rights of the firm and are thus entitled to decide what the firm should aim for, which according to homogeneity is supposed to be the same for all shareholders, i.e., maximize the firm's contribution to the financial wealth of the shareholders. The firm can accomplish this by engaging in investment projects with positive net present value. This is the neo-classical view on the role of the firm and on the relationship between the firm and its shareholders in a capitalist society. Figure 24.1 depicts a simplified graphical representation of this line of thought.

It is important to note that this position is embedded in a much larger framework of stylized thinking in among others economics (general equilibrium framework) and law (property rights theory and limited liability of shareholders). Until today, this view is seen as an ideal by many; see for example Jensen [102]. Presently, however, the societal impact of the firm and its governance structure is a growing topic of debate. Here we will show that also in finance there are many roads leading to Rome, or rather to the designation MCDA. Whether one belongs to the camp of Jensen or to the camp of those advocating socially responsible entrepreneurship, one has to deal with multiple criteria.

There is a series of situations in which the firm chooses (or has to take account of) a multiplicity of objectives and (policy) constraints. An overview of these situations is depicted in Fig. 24.2. One issue is who decides on the objective(s) of the firm. If there is a multiplicity of parties who may decide what the firm is aiming for, one generally encounters a multitude of goals, constraints and considerations that—more often than not—will be at least partially conflictive. A clear example is the

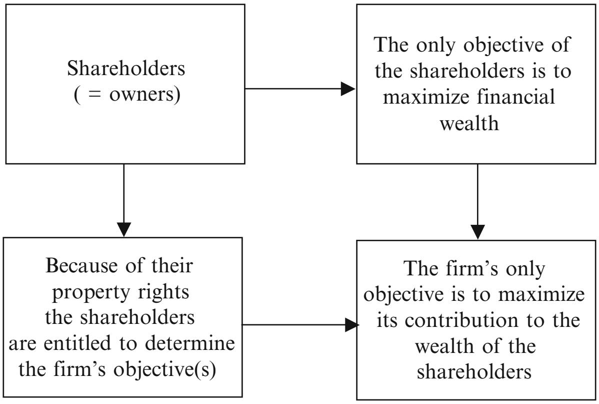

Fig. 24.1 The neo-classical view on the objective of the firm

conflicting objectives arising from agency problems (Jensen and Meckling [103]). This means that many decision problems include multiple criteria and multiple actors (viz. group decision making, negotiation theory, see Box 3 in Fig. 24.2). Sometimes, all those who decide on what the firm should aim for agree upon exactly the same objective(s). In fact, this is what neo-classical financial theory assumes when adopting shareholder value maximization (Box 1 in Fig. 24.2). In practice, there are many firms that explicitly strive for a multiplicity of goals, which naturally leads to decision problems with multiple criteria (Box 2 in Fig. 24.2).

However, although these firms do explicitly state to take account of multiple objectives, there are still very few of these firms that make use of tools provided by the MCDA literature. In most cases firms maximize one objective subject to (policy) constraints on the other objectives. As such there is nothing wrong with such a procedure as long as the location of these policy constraints is chosen correctly. In practice, however, one often observes that there is no discussion at all about the location of the policy constraints. Moreover, there is often no idea about the trade-offs between the location of the various constraints and the objective function that is maximized. In our opinion, multiple criteria decision methodologies may help decision makers to gain better insights in the trade-offs they are confronted with.

Now let us get back to the case in which the owner(s)/shareholders do have only one objective in mind: wealth maximization. Although this is by definition the most prominent candidate for single criteria decision-making, we will argue that even in this case there are many circumstances in which the formulation as a multiple criteria decision problem is opportune.

In order to contribute maximally to the wealth of its shareholders, an individual firm should maximize the value of its shares. The value of these shares is determined on the financial markets by the forces of demand and supply. Shares represent claims on the future residual cash flows of the firm (and also on a usually very limited right on corporate control). In the view of the financial markets, the value

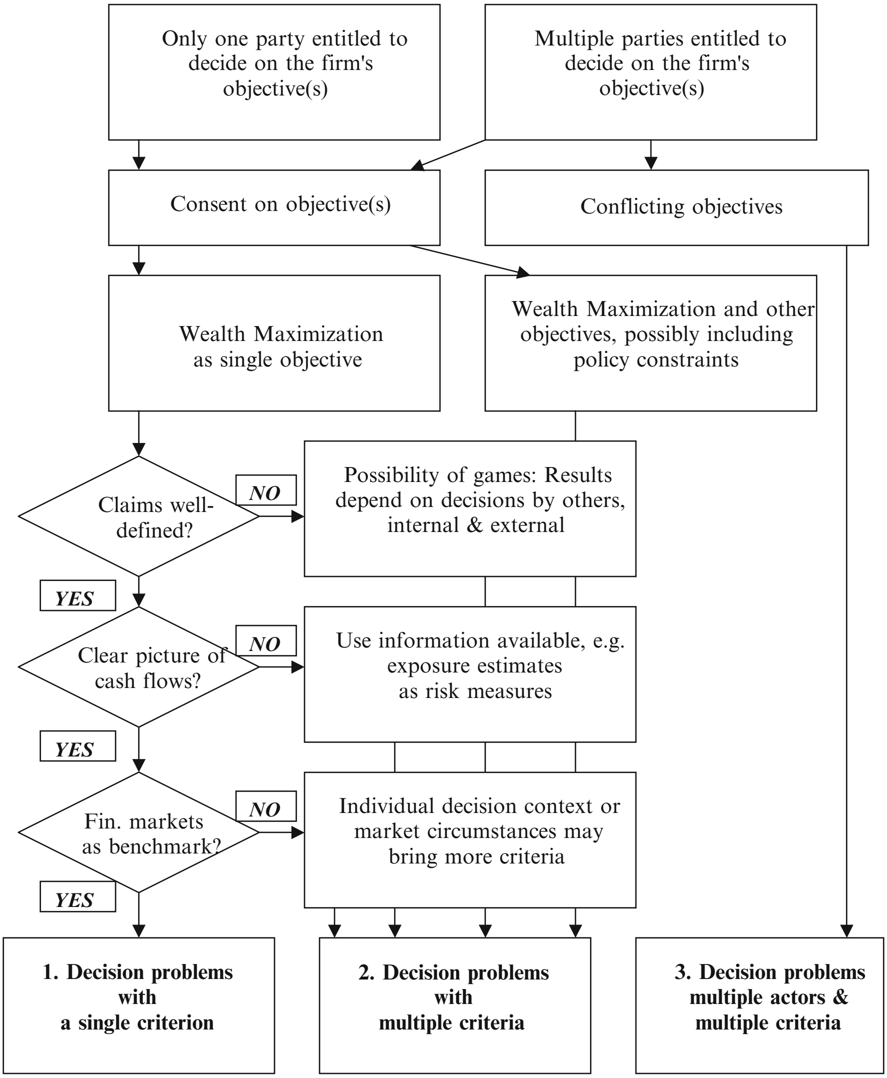

Fig. 24.2 Situations leading to MCDA in the firm

of such a claim is determined relative to the claims of other firms that are traded on these markets. The financial markets' perception of the quality of these cash flow claims is crucial for the valuation of the shares. Translated to the management of the individual firm, the aim is not only to maximize the quality of the future residual cash flows of the firm but also to properly communicate all news about these cash flows to the financial markets. Only by the disclosure of such information can informational asymmetries be resolved and the fair market value of a cash flow claim be determined. In evaluating the possible consequences of its decision alternatives, management should estimate the effects on the uncertain (future) cash flows followed by an estimation of the financial markets' valuation of these effects. Then (and only then) the decision rule of management is very simple: choose the decision alternative that generates the highest estimated market value.

The first problem that might arise while following the above prescription is that residual claims cannot always be defined because of "gaming effects" (see Fig. 24.2, Box 2). In other words, the future cash flows of the firm do not only depend on the present and future decisions of the firm's management, but also on the present and future decisions of other parties. An obvious example is the situation of oligopolistic markets in which the decisions of the competitors may strongly influence each other. Similar situations may arise with other external stakeholders such as powerful clients, powerful suppliers, and powerful financiers. Games may also arise within the firm, for instance between management and certain key production factors. The problem with game situations is that their effect on a firm's future cash flows caused by other parties involved cannot be treated in the form of simple constraints or as cost factors in cash flow calculations. MCDA may help to solve this problem by formulating multi-dimensional profiles of the consequences of the firm's decision alternatives. In these profiles, the effects on parties other than the firm are also included. These multi-dimensional profiles are the keys to open the complete MCDA toolbox.

A second problem in dealing with the single-objective wealth maximization problems is that the quality of information concerning the firm's future cash flows under different decision alternatives is far from complete. In addition, the available information may be biased or flawed. One way to approach the incomplete information problem is suggested by Spronk and Hallerbach [177]. In their multi-factorial approach, different sources of uncertainty should be identified after which the exposures of the cash flows to these risk sources are estimated. The estimated exposures can next be included in a multi-criteria decision method. In the case that the available information is not conclusive, different "views" on the future cash flows may develop. Next each of these views can be adopted as representing a different dimension of the decision problem. The resulting multi-dimensional decision problem can then be handled by using MCDA (see Fig. 24.2, Box 2).

A third potential problem in wealth maximization is that the financial markets do not always provide relevant pricing signals to evaluate the wealth effects of the firm's decisions, for example, because of market inefficiencies. This means that the firm may want to include attributes in addition to the market's signals in order to measure the riskiness and wealth effects of its decisions.

24.2.6 A Multicriteria Framework for Financial Decisions

In our view it, is the role of financial modelling to support financial decision making, as described in Hallerbach and Spronk [81], to build pointed models that take into account the peculiarities of the precise problem. The goal here is to bridge the gaps between decision-making disciplines, the discipline of financial economics, and the need for adequate decision support.

24.2.6.1 Principles

This framework is built on the principle that assumptions should be made where they help the modelling process the most and hurt the particular decision problem the least.[1] We call this the *Principle of Low Fat Modelling*. When addressing a decision situation, make use of all available information, but do not make unrealistic assumptions with respect to the availability of information. Do not make unrealistic assumptions that disqualify the decision context at hand. There should be ample room to incorporate idiosyncrasies of the decision context within the problem formulation, thus recognizing that the actual (non-average) decision maker is often very different from the "representative" decision maker. The preferences of the decision maker may not be explicitly available and may not even be known in detail by the decision maker himself. The uncertainty a decision maker faces with respect to the potential outcomes of his decisions may not be readily represented by means of a tractable statistical distribution. In many real-life cases, uncertainty can only be described in imprecise terms and available information is far from complete. And when the preferences of the decision maker are confronted with the characteristics of the decision alternatives, the conditional-normative nature of derived decision rules and advice should be accepted.

A second principle underlying our framework is the *Principle of Eclecticism*. One should borrow all the concepts and insights from modern financial theory that help to make better financial decisions. Financial theory can provide rich descriptions of uncertainty and risk. Examples are the multi-factor representation of risk in which the risk attached to the choice alternatives is conditioned on underlying factors such as the contingent claims approach in which the decision outcomes are conditioned on the opportunity to adjust or revise decisions in the future or game theory in which the outcomes are also conditioned on potential (conflicting) decisions made by other parties. But it is not the availability of theoretical insights that determines their application; it depends on the specific decision context at hand.

By restricting one thinking to a prechoosen set of problem characteristics, there is obviously more "to be seen" but at the same time it is possible to make observation errors, and maybe more worrisome, the problem and its context may be changing over time. This calls for the *Principle of Permanent Learning*, which stresses the process nature of decision making in which both the representation of the problem and the problem itself can change over time. Therefore, there is a permanent

[1]The underlying assumptions must be validated and the effectiveness and efficiency of the actions taken must be evaluated systematically. The latter calls for a sophisticated performance evaluation process that explicitly acknowledges the role of learning.

need to critically evaluate the problem formulation, the decisions made and their performance. Obviously, decision making and performance evaluation are two key elements in the decision-making process. As argued in Spronk and Vermeulen [178], performance evaluation of decisions should be structured such that the original idiosyncrasies of the problem (i.e., at the time the decision is made) are fully taken into account at the moment of evaluation, (i.e., *ex post*). By doing so, one increases the chance of learning from errors and misspecifications in the past.

24.2.6.2 Allocation Decisions

Financial decisions are allocation decisions, in which both time and uncertainty (and thus risk) play a crucial role. In order to support decisions in finance, both the preferences of the decision maker and the characteristics of the choice alternatives should be adequately understood and related to each other. A distinction can be made between "pure" financial decisions in which cash flows and market values steer the decision and "mixed" financial decisions in which other criteria are also considered. In financial theory, financial decisions are considered to be pure. In practice, most decisions are mixed. Hallerbach and Spronk [80] show that many financial decisions are mixed and thus should be treated as multiple criteria decision problems.

The solution of pure financial decisions requires the analysis, valuation, and management of risky cash flow streams and risky assets. The solution of mixed financial problems involves, in addition, the analysis of other effects. This implies that, in order to describe the effects of mixed decisions, multi-dimensional impact profiles should be used (cf. Spronk and Hallerbach [177]). The use of multi-dimensional impact profiles naturally opens the door to MCDA. Another distinction that can be made is between the financial decisions of individuals on one hand the financial decisions of companies and institutions on the other. The reason for the distinction results from the different ways in which decision makers steer the solutions. Individual decisions are guided by individual preferences (e.g., as described by utility functions), whereas the decisions of corporations and institutions are often guided by some aggregate objective (e.g., maximization of market value).

24.2.6.3 Uncertainty and Risk[2]

In each of the types of financial decisions just described, the effects are distributed over future time periods and are uncertain. In order to evaluate these possible effects, available information should be used to develop a "picture" of these effects and their likelihood. In some settings there is complete information but more often information is incomplete. In our framework, we use multi-dimensional risk profiles

[2]This section draws heavily on a part of Hallerbach and Spronk [79].

for modelling uncertainty and risk. This is another reason why multicriteria decision analysis is opportune when solving financial decision problems. *Two questions* play a crucial role:

1. Where does the uncertainty stem from or, in other words, what are the sources of risk?
2. When and how can this uncertainty be changed?

The answer to the first question leads to the decomposition of uncertainty. This involves attributing the inherent risk (potential variability in the outcomes) to the variability in several underlying state variables or factors. We can thus view the outcomes as being *generated* by the factors. Conversely, the stochastic outcomes are *conditioned* on these factors. The degree in which fluctuations in the factors propagate into fluctuations in the outcomes can be measured by response coefficients. These sensitivity coefficients can then be interpreted as exposures to the underlying risk factors and together they constitute the multi-dimensional risk profile of a decision alternative.

The answer to the second question leads to three prototypes of decision problems:

(1) The decision maker makes and implements a final decision and waits for its outcome. This outcome will depend on the evolution of external factors, beyond the decision maker's control.
(2) The decision maker makes and implements a decision and observes the evolution of external factors (which are still beyond the decision maker's control). However, depending on the value of these factors, the decision maker may make and implement additional decisions. For example, a decision maker may decide to produce some amount of a new and spectacular software package and then, depending on market reaction, he may decide to stop, decrease, or increase production.
(3) As in (2), but the decision maker is not the sole player and thus has to take account of the potential impact of decisions made by others sometime in the future (where the other(s) are of course confronted with a similar type of decision problem). The interaction between the various players in the field gives rise to dynamic game situations.

24.2.6.4 A Bird's-Eye View of the Framework

In Fig. 24.3, a bird's-eye view of the framework is presented. The framework integrates several elements in a process-oriented approach towards financial decisions. The left side of Fig. 24.3 represents the elements that lead to decisions, represented by the Resolution/Conclusion box at the lower left hand side. As mentioned above, performance evaluation (shown at the lower right hand side of the figure) is an integral part of the decision-making process. However, in this article we do not pay further attention to performance evaluation or to the feedback leading from performance evaluation to other elements of the decision-making process.

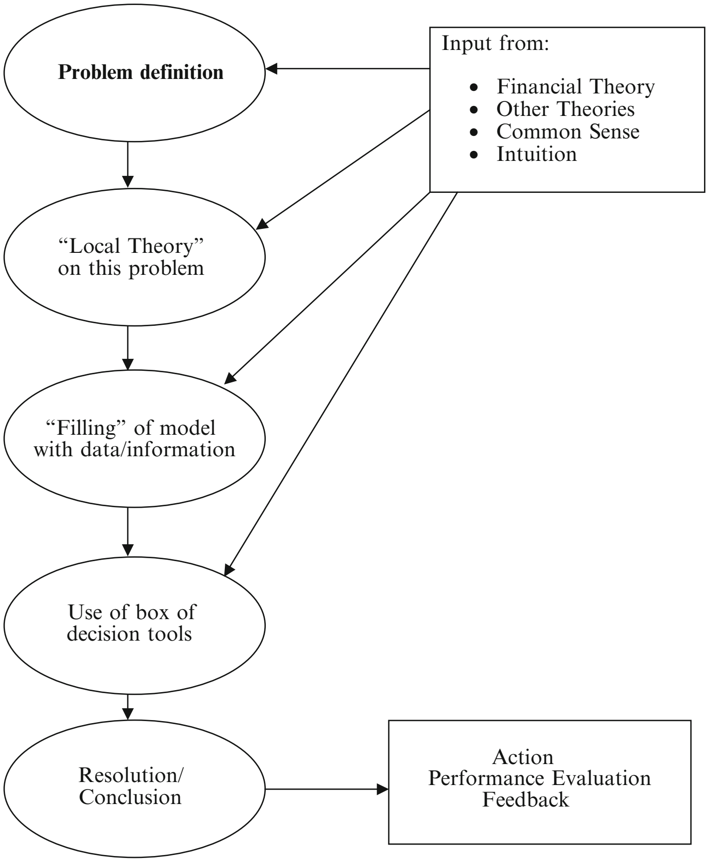

Fig. 24.3 A bird's-eye view of the framework

Financial decision problems will often be put as allocation problems. At this stage, it is important to determine whether the problem is a mixed or pure financial problem. Also, one should know who decides and which objectives are to be served by the decisions.

In the next step, the problem is defined more precisely. Many factors play a role here. For instance, the degree of upfront structure in the problem definition, the similarity with other problems, time and commitment from the decision makers, availability of time, similarity to problems known in theory and so on. In this stage, the insights from financial theory often have to be supplemented (or even amended) by insights from other disciplines and by the discipline of common sense. The problem formulation can thus be seen as a theoretical description (we use the label "local theory") of the problem.

After the problem formulation, data have to be collected, evaluated and sometimes transformed into estimates. These data are then used as inputs for the formalization of the problem description. The structure of the problem, together with the quality and availability of the data determines what tools can be used and in which way. As explained above, the use of multi-dimensional impact profiles almost naturally leads to the use of multicriteria decision analysis.

24.2.6.5 The Framework and Modern Financial Theory

In our framework we try to borrow all concepts and insights from modern financial theory that help to make better financial decisions. Financial theory provides rich and powerful tools for describing uncertainty and risk. Examples are the multi-factor representation of risk, which leads to multi-dimensional impact profiles that can be integrated within multicriteria decision analysis. A very important contribution of financial theory is the contingent claims approach in which the decision outcomes are conditioned on the opportunity to adjust or revise decisions in the future. This comes together with financial markets where contingent claims are being traded in volume. This brings us to the role of financial markets as instruments to trade risks, to redistribute risks, and even to decrease or eliminate risk. We believe and hope that contingent claims thinking will also be used in other domains than finance. In the first place because of what it adds when describing decision problems. Secondly, new markets may emerge in which also non-financial risks can be handled in a better way.

In addition to helping to better describe decision problems, financial theory provides a number of crucial insights. The most obvious (which is clearly not limited to financial economics) is probably the concept of "best alternative opportunity" thinking. Whenever making an evaluation of decision alternatives, one should take into account that the decision maker may have alternative opportunities (often but not exclusively provided by markets), the best of which sets a benchmark for the evaluation of the decision alternatives considered.

Other concepts are the efficient market hypothesis and the no-arbitrage condition. These point both to the fact that in competitive environments, it is not obvious that one can outsmart all the others. So if you find ways to make easy money, you should at least try to answer the question why you have been so lucky and how the environment will react.

24.3 MCDA in Portfolio Decision-Making Theory

We now turn our attention to the area of finance known as portfolio theory. In portfolio theory, we study the attributes of collections of securities called portfolios and how investors make judgements based upon these attributes. The problem that characterizes this area is the problem of portfolio selection.

Formulated as an optimization problem, this problem has been studied extensively. Thousands of papers have been written on it. A feel for many of these papers can be gained by scanning the references contained in Elton et al. [63]. As far as mainstream finance is concerned, the problem is only two-dimensional, able to address only tradeoffs between risk (typically measured by standard deviation) and return. To more realistically model the problem and be better prepared for a future which will only be more complicated, we now discuss issues involved in generalizing portfolio selection to include criteria beyond standard deviation and return, such as liquidity, dividend yield, sustainability, and so forth. See, for example, Lo et al. [123], Ehrgott et al. [62], Ben Abdelaziz et al. [1], Ballestero et al. [13], and Xidonas at al. [192]. In this way, MCDA in the form of multiple criteria optimization enters the picture. While the word "multiple" includes two, we will generally use it for more than two. We now explore the possibilities of multiple objectives in portfolio selection and discuss the effects of recognizing multiple criteria on the traditional assumptions and practice of portfolio selection in finance.

For this, we are organized as follows. In Sect. 24.3.1 we introduce the risk-return problem of portfolio selection, and in Sect. 24.3.2 we demonstrate the problem in a multiple criteria optimization framework. In Sect. 24.3.3 we discuss two variants of the portfolio selection model, and in Sect. 24.3.4 we discuss the bullet-shaped feasible regions that so often accompany portfolio selection problems. In the context of some key assumptions, in Sect. 24.3.5 we discuss the sensitivity of the nondominated set to changes in various factors, and in Sect. 24.3.6 we update the assumptions in accordance with the indicated presence of additional criteria. In Sect. 24.3.7 we talk about how to deal with resulting nondominated surfaces, and in Sect. 24.3.8 we report on the idea that the "modern portfolio analysis" of today can be viewed as the projection onto the risk-return plane of the real multiple criteria portfolio selection problem in higher dimensional space. In Sect. 24.3.9 we comment on future directions.

24.3.1 Portfolio Selection Problem

In finance, due to Markowitz [131–133], we have the canonical problem of portfolio selection as follows. Assume

(a) n securities
(b) a sum of money to be invested
(c) beginning of a holding period
(d) end of a holding period.

Let $\mathbf{x} = (x_1, \ldots, x_n)$ be a *portfolio* where the x_i are weights that specify the proportions of the sum to be invested in the different securities at the beginning of the holding period. For security i, let r_i be the random variable for the percent return realized on security i between the beginning of the holding period and the end of the holding period. Then for r_p, the random variable for the percent return realized on a portfolio between the beginning of the holding period and the end, we have

$$r_p = \sum_{i=1}^{n} r_i x_i$$

Unfortunately, it is not possible to know at the beginning of the holding period the value to be achieved by r_p at the end of the holding period. However, it is assumed that at the beginning of the holding period we have in our possession all expected values $E\{r_i\}$, variances σ_{ii}, and covariances σ_{ij} for the n securities.

Since r_p is not deterministic and an investor would presumably wish to protect against low values of r_p from turning out to be the case, the approach considered prudent in portfolio selection is to seek a portfolio solution that produces a high expected value of r_p and a low standard deviation of r_p. Using the $E\{r_i\}$, σ_{ii} and σ_{ij}, the expected value of r_p is given by

$$E\{r_p\} = \sum_{i=1}^{n} E\{r_i\} x_i \tag{24.1}$$

and the standard deviation of r_p is given by

$$\sigma\{r_p\} = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} x_i \sigma_{ij} x_j} \tag{24.2}$$

As for constraints, there is always the full investment constraint

$$\sum_{i=1}^{n} x_i = 1 \tag{24.3}$$

Depending on the version of the problem, there may be additional constraints such as

$$\ell_i \leq x_i \leq \mu_i \text{ for all } i \tag{24.4}$$

which are very common.

The way (24.1)–(24.4) is solved is as follows. First compute the set of all of the model's "nondominated" combinations of expected return and standard deviation. Then, after examining the set, which portrays as a non-negatively sloped concave curve, the investor selects the nondominated combination that he or she feels strikes the best balance between expected return and standard deviation.

With $E\{r_p\}$ to be maximized and $\sigma\{r_p\}$ to be minimized, (24.1)–(24.4) is a multiple objective program. Although the power of multiple criteria optimization is generally not necessary with two-objective programs (they can often be addressed with single criterion techniques), the theory of multiple criteria optimization, however, is necessary when wishing to generalize portfolio selection, as we do, to take into account additional criteria.

24.3.2 Background on Multicriteria Optimization

In multiple criteria optimization, to handle both maximization and minimization objectives, we have

$$\begin{aligned} &\max \textit{ or } \min \{f_1(\mathbf{x}) = z_1\} \\ &\quad\vdots \\ &\max \textit{ or } \min \{f_k(\mathbf{x}) = z_k\} \\ &\text{s.t.} \qquad \mathbf{x} \in S \end{aligned} \tag{MC}$$

where $\mathbf{x} \in \mathbb{R}^n$, k is the number of objectives, and z_i is a *criterion value*. In multiple criteria optimization, we have the feasible region in two different spaces. One is $S \subset \mathbb{R}^n$ in *decision space* and the other is $Z \subset \mathbb{R}^k$ in *criterion space*. Let $\mathbf{z} \in \mathbb{R}^k$. Then *criterion vector* $\mathbf{z} \in Z$ if and only if there exists an $\mathbf{x} \in S$ such that $\mathbf{z} = (f_1(\mathbf{x}), \ldots, f_k(\mathbf{x}))$. In this way, Z is the set of all *images* of the $\mathbf{x} \in S$.

Criterion vectors in Z are either *nondominated* or *dominated*, and points in S are either *efficient* or *inefficient*. Let $J^+ = \{ i \mid f_i(\mathbf{x})$ is to be maximized$\}$ and $J^- = \{j \mid f_j(\mathbf{x})$ is to be minimized$\}$. Then we have

Definition 1. Assume (MC). Then $\bar{\mathbf{z}} \in Z$ is a *nondominated* criterion vector if and only if there does not exist another $\mathbf{z} \in Z$ such that (i) $z_i \geq \bar{z}_i$ for all $i \in J^+$, and $z_j \leq \bar{z}_j$ for all $j \in J^-$, and (ii) $z_i > \bar{z}_i$ or $z_j < \bar{z}_j$ for at least one $i \in J^+$ or $j \in J^-$. Otherwise, $\bar{\mathbf{z}} \in Z$ is *dominated*.

The set of all nondominated criterion vectors is designated N and is called the *nondominated set*.

Definition 2. Let $\bar{\mathbf{x}} \in S$. Then $\bar{\mathbf{x}}$ is *efficient* in (MC) if and only if its criterion vector $\bar{\mathbf{z}} = (f_1(\bar{\mathbf{x}}), \ldots, f_k(\bar{\mathbf{x}}))$ is nondominated, that is, if and only if $\bar{\mathbf{z}} \in N$. Otherwise, $\bar{\mathbf{x}}$ is *inefficient*.

The set of all efficient points is designated E and is called the *efficient set*. Note the distinction with regard to terminology. While nondominance is a criterion space concept, in multiple criteria optimization, efficiency is only a decision space concept.

To define optimality in a multiple criteria optimization problem, let $U\colon \mathbb{R}^k \to \mathbb{R}$ be the decision maker's utility function. Then, any $\mathbf{z}^o \in Z$ that maximizes U over Z is an *optimal criterion vector*, and any $\mathbf{x}^o \in S$ such that $(f_1(\mathbf{x}^o), \ldots, f_k(\mathbf{x}^o)) = \mathbf{z}^o$ is an *optimal solution*. We are interested in the efficient and nondominated sets because if U is such that *more-is-better-than-less* for each z_i, $i \in J^+$, and *less-is-better-than-more* for each z_j, $j \in J^-$, then any $\mathbf{z}^o$ optimal criterion vector is such that $\mathbf{z}^o \in N$, and any feasible *inverse image* $\mathbf{x}^o$ is such that $\mathbf{x}^o \in E$. The significance of this is that to find an optimal criterion vector $\mathbf{z}^o$, it is only necessary to find a best point in N. After a $\mathbf{z}^o$ has been found, it is only necessary to obtain an $\mathbf{x}^o \in S$ inverse image to know what to implement to achieve the k simultaneous performances specified by the values in $\mathbf{z}^o$.

Although N in portfolio selection is a portion of the surface of $Z \in \mathbb{R}^k$, locating the best solution in N, when $k > 2$, is generally a non-trivial task because of the size of N. As a result, a large part of the field of multiple criteria optimization is concerned with procedures for computing or sampling N to locate an optimal or *near-optimal* solution, where a near-optimal solution is close enough to being optimal to terminate the decision process.

Within this framework, (24.1)–(24.4) can now be expressed in the form of a bi-objective multiple criteria optimization problem

$$\min\{\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} x_i \sigma_{ij} x_j} = z_1\} \tag{MC-O}$$

$$\max\{\sum_{i=1}^{n} E\{r_i\} x_i = z_2\}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} x_i = 1$$

$$\ell_i \leq x_i \leq \mu_i \text{ for all } i$$

24.3.3 Two Model Variants

Two model variants of (24.1)–(24.4) have evolved as classics. One is the *unrestricted* model

$$\min\{\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} x_i \sigma_{ij} x_j} = z_1\} \tag{MC-U}$$

$$\max\{\sum_{i=1}^{n} E\{r_i\} x_i = z_2\}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} x_i = 1$$

$$\text{all } x_i \text{ unrestricted}$$

meaning that there are no constraints beyond the full investment constraint in the model. The other is the *variable-restricted* model

$$\min\{\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} x_i\sigma_{ij}x_j} = z_1\} \tag{MC-B}$$

$$\max\{\sum_{i=1}^{n} E\{r_i\}x_i = z_2\}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} x_i = 1$$

$$\ell_i \leq x_i \leq \mu_i \text{ for all } i$$

in which lower and upper bounds exist on the x_i. In the unrestricted model there are no lower limits on the weights, meaning that *unlimited* short selling is permitted. To illustrate, let $x_3 = -0.2$. This says the following to an investor. Borrow a position in security 3 to the extent of 20 % of the initial sum to be invested and then sell. With the extra 20 % and the initial sum, invest it in accordance with the other x_i.

The unrestricted model is a favorite in teaching because of its elegant mathematical properties. For example, as long as the covariance matrix

$$\mathbf{\Sigma} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & & \\ \vdots & & & \vdots \\ \sigma_{n1} & & \cdots & \sigma_{nn} \end{bmatrix}$$

is nonsingular, every imaginable piece of information about the model appears to be analytically derivable in closed form (for instance see Roll [157]).

The variable-restricted model, despite requiring mathematical programming (typically some form of quadratic programming) because of the extra constraints, is the favorite in practice. For instance, in the US, short selling is prohibited by law in the $11 trillion mutual fund business. It is also prohibited in the management of pension assets. And even in hedge funds where short selling is almost standard, it is all but impossible to imagine any situation in which there wouldn't be limits. A question is, when trying to locate an optimal solution, how much difference might there be between the two models?

24.3.4 Bullet-Shaped Feasible Regions

When looking through the portfolio chapters of almost any university investments text, it is hard to miss seeing graphs of bullet-shaped regions, often with dots in them, with standard deviation on the horizontal axis and expected return on the

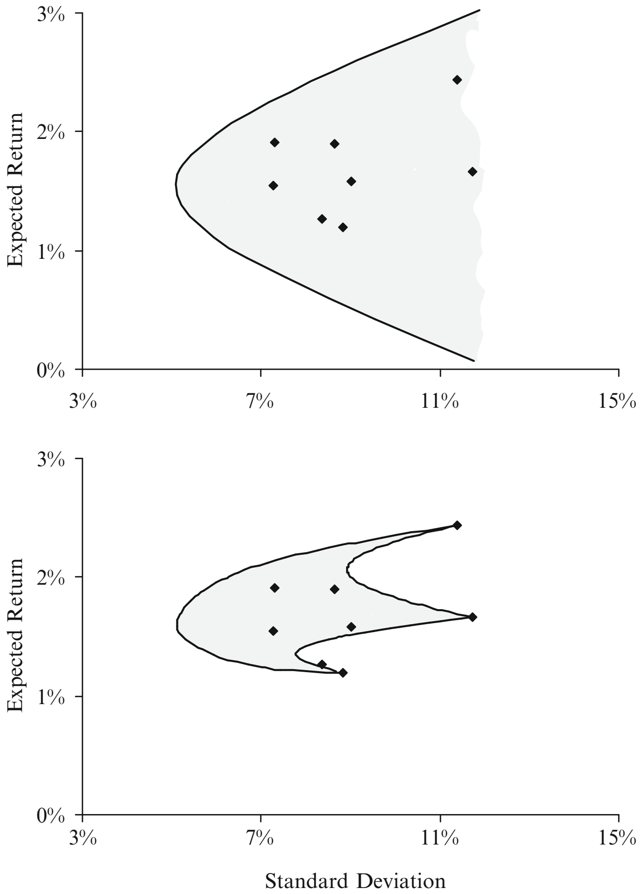

Fig. 24.4 Feasible regions Z of (MC-U) and (MC-B) for the same eight securities

vertical. When unbounded as in Fig. 24.4 (*top*), these are graphs of the feasible region Z of (MC-U) in criterion space. When bounded as in Fig. 24.4 (*bottom*), these are graphs of the feasible region Z of (MC-B) in criterion space. The dots are typically the criterion vectors $(\sigma\{r_i\}, E\{r_i\})$ of individual securities.

To see why a feasible region Z of (MC-U) is bullet-shaped and unbounded, consider securities A and B in Fig. 24.5. The unbounded line sweeping through A and B, which is a hyperbola, is the set of criterion vectors of all two-stock portfolios resulting from all linear combinations of A and B whose weights sum to one. In detail, all points on the hyperbola strictly between A and B correspond to weights $x_a > 0$ and $x_b > 0$; all points on the hyperbola above and to the right of A correspond to weights $x_a > 1$ and $x_b < 0$; and all points on the hyperbola below and to the right of B correspond to weights $x_a < 0$ and $x_b > 1$. The degree of "bow" toward the vertical axis of the hyperbola is a function of the correlation coefficient ρ_{ab} between

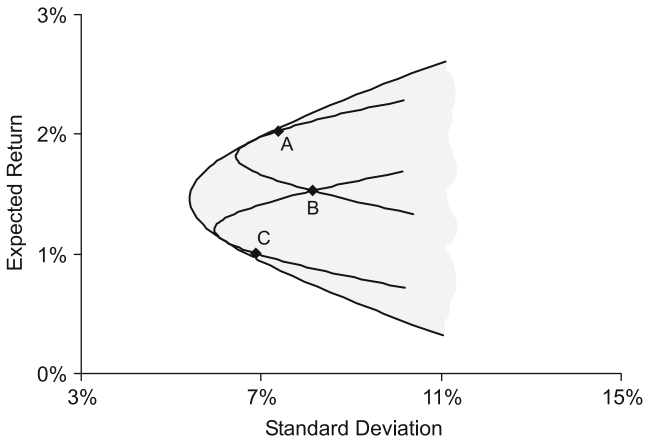

Fig. 24.5 Unbounded bullet-shaped feasible region Z created by securities A, B and C

A and B. This is seen by looking at the components of the $(\sigma\{r_{ab}\}, E\{r_{ab}\})$ criterion vector of any two-stock portfolio which are given by

$$\sigma\{r_{ab}\} = \sqrt{\sigma_{aa}x_a^2 + 2\rho_{ab}\sigma_a\sigma_b x_a x_b + \sigma_{bb}x_b^2}$$

and

$$E\{r_{ab}\} = E\{r_a\}x_a + E\{r_b\}x_b$$

in which $\sigma\{r_a\} = \sqrt{\sigma_{aa}}$ and $\sigma\{r_b\} = \sqrt{\sigma_{bb}}$.

Through B and C in Fig. 24.5 there is another hyperbola. Since through any point on the hyperbola through A and B and any point on the hyperbola through B and C there is yet another hyperbola, feasible region Z fills in and takes on its bullet shape whose leftmost boundary is, in the case of (MC-U), a single hyperbola.

With regard to the feasible region Z of (MC-B), the hyperbolic lines through the criterion vectors of any two financial products are not unbounded. In every case, they end at some point because of the bounds on the variables. While still filling in to create a bullet-shaped Z, the leftmost boundary, instead of being formed by a single hyperbola, is in general formed by segments from several hyperbolas. The rightmost boundary, instead of being unbounded, takes on a "scalloped" effect as in Fig. 24.4 (*bottom*).

Because standard deviation is to be minimized and expected return is to be maximized, we look to the "northwest" of Z for the nondominated set. This causes the nondominated set to be the upper portion of the leftmost boundary (the portion that is non-negatively sloped). In finance, it is called the "efficient frontier." Here, because of our interests in portfolio analysis with multiple criteria, we prefer to call it the "nondominated frontier."

24.3.5 Assumptions and Nondominated Sensitivities

The assumptions surrounding the use of (MC-U) and (MC-B) and theories based upon them in finance are largely as follows.

(a) There are many investors, each small, none of which can affect prices.
(b) There are no taxes.
(c) There are no transactions costs.
(d) Each investor's asset universe is all publicly traded securities.
(e) All investors are rational mean-variance optimizers.
(f) All investors have utility functions whose indifference curves are convex-to-the-origin.
(g) All investors share the same expected returns, variances, and covariances about the future. This is called *homogeneous expectations*.
(h) All investors have the same single holding period.
(i) Each security is infinitely divisible.

We now discuss the sensitivity of the nondominated frontier to factors that have implications about the appropriateness of this set of the assumptions. Sensitivity is measured by noting what happens to the nondominated frontier as the parameter associated with a given factor changes. We start by looking at the sensitivity of the nondominated frontier to changes in an upper bound common to all investment proportion weights. Then we discuss the likely sensitivities of the nondominated frontier to changes in other things such as dividend yield, a liquidity measure, a social responsibility attribute, and so forth. The computer work required for testing such sensitivities is outlined in the following procedure.

1. Start the construction of what is recognized in multiple criteria optimization as an ϵ-constraint program by converting the expected return objective in (MC-U) and (MC-B) to a $\geq$ constraint with right-hand side ϵ.
2. Set the factor parameter to its starting value.
3. Set ϵ to its starting value.
4. Solve the ϵ-constraint program and take the square root of the outputted variance to form the nondominated point $(\sigma\{r_p\}, E\{r_p\})$.
5. If ϵ has reached its ending value, go to Step 6. Otherwise, increment ϵ and go to Step 4.
6. Connect on a graph all of the nondominated points obtained from the current value of the factor parameter to achieve a portrayal of the nondominated frontier of this factor parameter value. If the factor parameter has reached its ending value, stop. Otherwise increment the factor parameter and go to Step 3.

For the procedure, the ϵ-constraint program is

$$\min\{\sum_{i=1}^{n}\sum_{j=1}^{n} x_i \sigma_{ij} x_j = \sigma^2\{r_p\}\} \tag{Eps-1}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} E\{r_i\} x_i \geq \epsilon$$

$$\sum_{i=1}^{n} x_i = 1$$

$$\ell_i \leq x_i \leq \mu \text{ for all } i$$

and to obtain results for the sensitivity of the nondominated frontier due to changes in the upper bound μ, let us consider a problem in which $n = 20$; $\ell = -0.05$ to permit mild short selling; and μ is set in turn to $1.00, 0.15, 0.10$ to generate three frontiers. Running 25 different ϵ values (experimenter's choice) for each μ-value, the three nondominated frontiers of Fig. 24.6 result. The topmost frontier is for $\mu = 1.00$, the middle frontier is for $\mu = 0.15$, and the bottommost frontier is for $\mu = 0.10$.

As seen in Fig. 24.6, the nondominated frontier undergoes major changes as we step through the three values of μ. Hence there is considerable sensitivity to the value of μ. Since, in the spirit of diversification, investors would presumably prefer smaller values of μ to larger values as long as portfolio performance is not seriously deteriorated in other respects, we can see that an examination of the tradeoffs among risk, return, and μ are involved before a final decision can be made. Since an investor would probably have no way of knowing in advance his or her optimal value of μ without reference to its effects on risk and return, μ conceivably could be a criterion to be optimized, too.

Using the same procedure, other experiments (results not shown) could be conducted. For example, if we wished to test the sensitivity of the nondominated frontier to changes in expected dividend yield, we would then work with the following ϵ-constraint program

$$\min\{\sum_{i=1}^{n}\sum_{j=1}^{n} x_i \sigma_{ij} x_j = \sigma^2\{r_p\}\} \tag{Eps-2}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} E\{r_i\} x_i \geq \epsilon$$

$$\sum_{i=1}^{n} E\{d_i\} x_i \geq \delta$$

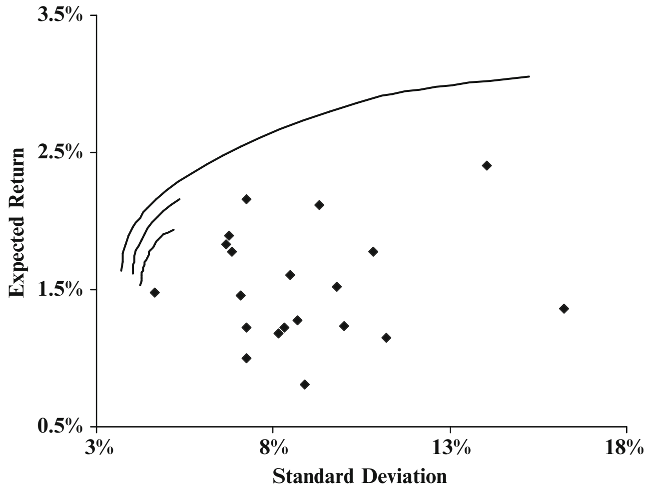

Fig. 24.6 Nondominated frontiers as a function of changes in the value of upper bound parameter μ

$$\sum_{i=1}^{n} x_i = 1$$

$$\ell_i \leq x_i \leq \mu_i \text{ for all } i$$

where d_i is the random variable for the dividend yield realized on security i between the beginning and end of the holding period and δ is the minimum dividend yield requirement parameter value to be changed in turn as μ was in (Eps-1) to test for different nondominated frontiers. A similar formulation could be set up for social responsibility.

For both dividend yield and social responsibility we can probably expect to see nondominated frontier sensitivities along the lines of that for μ. If this is indeed the case, this would signal that dividends and social responsibility could also be criteria. With μ, we now see how it is easy to have more criteria than two in investing. Whereas the assumptions at the beginning of this section assume a two-criterion world, we are led to see new things by virtue of these experiments. One is that the assumption about risk and return being the only criteria is certainly under seige. Another is that, in the company of μ, dividends, and social responsibility, the last of which can be highly subjective, *individualism* should be given more play. By individualism, no investor's criteria, opinions, or assessments need conform to those of another. In conflict with the assumption about homogeneous expectations, individualism allows an investor to have differing opinions about any security's expected return, risk profile, liquidity, dividend outlook, social responsibility quotient, and so forth. At the portfolio level, for example, individualism allows investors to possess different lists of criteria, have differing objective functions for even the same criteria, work from different asset universes, and enforce different attitudes about the nature of short selling. Therefore, with different lists of criteria, different objective functions, and different sets of constraints, all investors would not face the same feasible region with the same nondominated set. Each would have his or her own portfolio problem with its own optimal solution. The benefit of this enlarged outlook would be that portfolio theory would then not only have to focus on explaining equilibrium solutions, but on customized solutions as well.

24.3.6 Expanded Formulations and New Assumptions

Generally, in multiple criteria, we distinguish a constraint from an objective as follows. If when modelling we realize that we can not easily fix a right-hand side value without knowing how other output measures turn out, then we are probably looking at an objective. With this in mind, a list of possible extra objectives in portfolio selection could be

$$\max \{f_3(\mathbf{x}) = \text{dividend yield}\}$$
$$\min \{f_4(\mathbf{x}) = \text{maximum investment proportion weight}\}$$
$$\max \{f_5(\mathbf{x}) = \text{social responsibility}\}$$
$$\max \{f_6(\mathbf{x}) = \text{liquidity}\}$$
$$\max \{f_7(\mathbf{x}) = \text{momentum}\}$$
$$\max \{f_8(\mathbf{x}) = \text{investment in R\&D}\}$$

While one can imagine more exotic criteria, all of the above at least have the simplicity that they can be modelled linearly.

Updating to take a new look at portfolio selection, the following is proposed as a more appropriate set of assumptions with which to now approach the study of portfolio theory.

(a) There are many investors, each small, none of which can affect prices.
(b) There are no taxes.
(c) There are no transactions costs.
(d) An investor's asset universe can be any subset of all publicly traded securities.
(e) Investors may possess any mix of three or more objectives.
(f) All investors have utility functions whose indifference curves are convex-to-the-origin.

(g) Heterogeneity of expectations is the rule. That is, investors can have widely different forecasts and assessments about any security attribute including expected returns, variances, covariances, expected dividends, and so forth.
(h) Short selling is allowed but to only some limited extent.

The first three assumptions remain the same as they are nice to retain in that they establish benchmarks against which some of the world's imperfections can be measured. The assumption about convex-to-the-origin utility function contours is also retained as we see no compelling difficulty with it at the present time, but all the rest have either been modified or deleted.

24.3.7 Nondominated Surfaces

Let k be the number of criteria in a given portfolio selection model. Then the nondominated set of current-day finance that exists as a frontier in $\mathbb{R}^2$ is a *surface* in $\mathbb{R}^k$. The simplest case with a surface is with three criteria. The question is, how to solve? This is not a trivial question. Perhaps, to get a feel for the nondominated surface, the method that might first come to mind would be to solve repetitively the following ϵ-constraint program

$$\begin{aligned}
\min\{ \sum_{i=1}^{n}\sum_{j=1}^{n} x_i\sigma_{ij}x_j &= \sigma^2\{r_p\}\} \\
\text{s.t.} \qquad \sum_{i=1}^{n} E\{r_i\}x_i &\geq \epsilon_e \\
\sum_{i=1}^{n} E\{l_i\}x_i &\geq \epsilon_l \\
\sum_{i=1}^{n} x_i &= 1 \\
\ell_i \leq x_i \leq \mu_i &\text{ for all } i
\end{aligned}$$

where for sake of variety liquidity is the third criterion. We use ϵ_e and ϵ_l to distinguish between the ϵ's for expected return and liquidity. However, this approach involves many optimizations. If one might normally characterize a nondominated frontier with 50 points, up to a thousand points might be needed with a non-dominated surface to achieve about the same degree of representation density. Some references to help appreciate this might include Qi et al. [153], Şakar and Köksalan[164], and Mavrotas [137].

Instead of looking at the problem in ϵ-constraint terms, another approach is to look at it (since it contains three objectives) in tri-criterion form as follows

$$\min\{\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} x_i\sigma_{ij}x_j} = z_1\}$$

$$\max\{\sum_{i=1}^{n} E\{r_i\}x_i = z_2\}$$

$$\max\{\sum_{i=1}^{n} E\{l_i\}x_i = z_3\}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} x_i = 1$$

$$\ell_i \leq x_i \leq \mu_i \text{ for all } i$$

and then try to compute the whole nondominated surface exactly by multi-parametric quadratic programming. Whereas a nondominated frontier was shown earlier to be piecewise hyperbolic, a nondominated surface is platelet-wise or patch-wise hyperboloidic. One can think of the back of a turtle. This is new material and, as of this writing, the first paper on this is by Hirschberger et al. [84].

24.3.8 Idea of a Projection

In traditional risk-return finance there is the "market portfolio". By theory, the market portfolio contains every security in proportion to its market capitalization, is anticipated to be somewhere in the midst of the nondominated frontier, and is supposed to be everyone's optimal portfolio when not including the risk-free asset. Since the market portfolio is impractical, indices like the S&P 500 are used as surrogates. But empirically, the surrogates, which should be essentially as desirable as the market portfolio, have always been found to be quite below the nondominated frontier, in fact so below that this cannot be explained by chance variation. Whereas this is an anomaly in conventional risk-return finance, this is exactly what we would expect in multiple criteria finance.

To take a glimpse at the logic why, consider the following. In a risk-return portfolio problem, let us assume that the feasible region Z is the ellipse in Fig. 24.7. Here, the nondominated frontier is the portion of the boundary of the ellipse in the second quadrant emanating from the center of the ellipse. Similarly, in a k-criterion portfolio problem (with $k-2$ objectives beyond risk and return), let us assume that the feasible region is an ellipsoid in k-space. Here, the nondominated surface is the portion of the surface of the ellipsoid in a similar orthant emanating from the center of the ellipsoid.

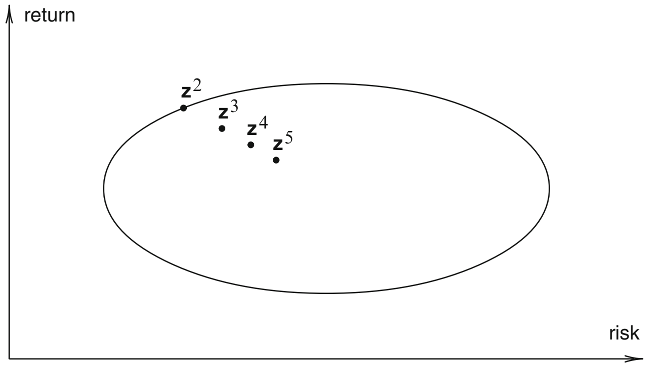

Fig. 24.7 An ellipsoidal feasible region projected onto two-dimensional risk-return space

Now assume that the market portfolio, which by theory is nondominated, is in the middle of the nondominated set. Then, when $k = 2$, the market portfolio would be at $\mathbf{z}^2$ on the ellipse. However, if (1) there is a third objective, (2) the feasible region is ellipsoidal in three-space, and (3) the market portfolio is in the middle of the nondominated surface in $\mathbb{R}^3$, then the market portfolio would project onto risk-return space at $\mathbf{z}^3$. If (1) there is a fourth objective, (2) the feasible region is ellipsoidal in four-space, and (3) the market portfolio is in the middle of the nondominated surface in $\mathbb{R}^4$, then the market portfolio would project onto risk-return space at $\mathbf{z}^4$. With five objectives under the same conditions, the market portfolio would project onto risk-return space at $\mathbf{z}^5$, and so forth, becoming deeper and deeper.

Consequently, it can be viewed that the "modern portfolio theory" of today is only a first-order approximation—a projection onto the risk-return plane of the real multiple criteria problem from higher dimensional criterion space.

24.3.9 Further Research in MCDA in Portfolio Analysis

So far we have only talked about extending the canonical model in the direction of multiple criteria. In addition to multiple criteria, we also find intriguing for future research the areas of special variable treatments and alternative risk measures. By special variable treatments, we mean conditions on the variables such as the following:

(a) No fewer than a given number of securities, and no more than a given number of securities, can be in a portfolio (either long or short).
(b) No more than a given number of securities can be sold short.
(c) If a stock is in a portfolio, then its weight must be in market cap proportion to the weights of all other stocks in the portfolio.
(d) No more than a given proportion of a portfolio can be involved in stocks sold short.

(e) Some or all of the x_i are semi-continuous. When an x_i is semi-continuous, x_i is either zero or in a given interval $[a, b], a > 0$.
(f) No more than a given number of stocks may have a given upper bound. For instance, at most one stock (but which one is not known beforehand) may constitute as much as 10 % of a portfolio, with all other stocks having an upper bound of 5 %.

While some of these can be modelled with auxiliary 0–1 variables as in Xidonas and Mavrotas [190], others may be best approached by evolutionary-style procedures as in Anagnostopoulos and Mamanis [4]. Having at one's disposal well-researched methods for dealing with special variable treatments would extend the power of our new look at portfolio analysis.

By alternative risk measures, we are thinking of measures like mean absolute deviation (MAD) as broached by Konno and Yamazaki [116] and conditional value at risk (CVaR) as integrated into a financial study such as by Şakar and Köksalan [164]. Finally, it may be that multiple criteria and behavioral finance (see Shefrin [168]) reinforce one another as both areas see more going on in investing than the traditional.

24.4 MCDA in Discrete Financial Decision-Making Problems

Several financial decision-making problems require the evaluation of a finite set of alternatives $A = \{a_1, a_2, \ldots, a_m\}$, which may involve firms, investment projects, stocks, credit applications, etc. These types of problems are referred to as "discrete" problems. The outcome of the evaluation process may have different forms, which are referred to as "problematics" [162]: (1) problematic α: Choosing one or more alternatives, (2) problematic β: Sorting the alternatives in pre-defined ordered categories, (3) problematic γ: Ranking the alternatives from the best to the worst ones, and (4) problematic δ: Describing the alternatives in terms of their performance on the criteria. The selection of an investment project is a typical example of a financial decision-making problem where problematic α (choice) is applicable. The prediction of business failure is an example of problematic β (classification of firms as healthy or failed), the comparative evaluation and ranking of stocks according to their financial and stock market performance is an example of problematic γ, whereas the description of the financial characteristics of a set of firms is a good example of problematic δ.

In all cases, the evaluation process involves the aggregation of all decision criteria $F = \{g_1, g_2, \ldots, g_n\}$. The aggregation process can be performed in many different ways depending on the form of the criteria aggregation model. Three main forms of aggregation models can be distinguished: (1) outranking relations (relational form), (2) utility functions (functional form), (3) decision rules (symbolic form). In order to make sure that the aggregation model is developed in accordance to the decision maker's judgment policy, some preferential information must be specified, such as the relative importance of the criteria. This information can be obtained either through direct procedures in which a decision analyst elicits it directly from the decision maker, or through indirect procedures in which the decision maker provides representative decision examples, which are used to infer the preferential parameters consistent with the decision maker's global evaluations. The latter approach is known in the MCDA field as "preference disaggregation analysis" [99, 100].

The subsequent subsections in this part of the chapter present several MCDA discrete evaluation approaches which are suitable for addressing financial decision-making problems. The presentation is organized in terms of the criteria aggregation model employed by each approach (outranking relations, utility functions, decision rules).

24.4.1 Outranking Relations

The foundations of the outranking relations theory have been set by Bernard Roy during the late 1960s through the development of the ELECTRE family of methods (**EL**imination **E**t **C**hoix **T**raduisant la **RE**alité; [160]). Since then, they have been widely used by MCDA researchers in several problem contexts.

An outranking relation is a binary relation that enables the decision maker to assess the strength of the outranking character of an alternative a_i over an alternative a_j. This strength increases if there are enough arguments (coalition of the criteria) to confirm that a_i is at least as good as a_j, while there is no strong evidence to refuse this statement.

Outranking relations techniques operate into two stages. The first stage involves the development of an outranking relation among the considered alternatives, while the second stage involves the exploitation of the developed outranking relation to choose the best alternatives (problematic α), to sort them into homogenous groups (problematic β), or to rank them from the most to the least preferred ones (problematic γ).

Some of the most widely known outranking relations methods include the family of the ELECTRE methods [161] and the family of the PROMETHEE methods [28]. These methods are briefly discussed below. A detailed presentation of all outranking methods can be found in the books of Roy and Bouyssou [163] and Vincke [186].

ELECTRE Methods The family of ELECTRE methods was initially introduced by Roy [160], through the development of the ELECTRE I method, the first method to employ the outranking relation concept. Since then, several extensions have been proposed, including ELECTRE II, III, IV, IS and TRI [161]. These methods address different types of problems, including choice (ELECTRE I, IS), ranking (ELECTRE II, III, IV) and sorting/classification (ELECTRE TRI).

Given a set of alternatives $A = \{a_1, a_2, \ldots a_m\}$ any of the above ELECTRE methods can be employed depending on the objective of the analysis (choice, ranking, sorting/classification). Despite their differences, all the ELECTRE methods are based on the identification of the strength of affirmations of the form Q ="alternative a_i is at least as good as alternative a_j". The specification of this strength requires the consideration of the arguments that support Q as well as the consideration of the arguments that are against it. The strength of the arguments that support Q is analyzed through the "concordance test". The measure used to assess this strength is the global concordance index $C(a_i, a_j) \in [0, 1]$. The closer is C to unity, the higher is the strength of the arguments that support the affirmation Q. The concordance index is estimated as the weighted average of partial concordance indices defined for each criterion:

$$C(a_i, a_j) = \sum_{k=1}^{n} w_k c_k(g_{ik} - g_{jk})$$

where w_k is the weight of criterion g_k ($\sum w_k = 1, w_k \geq 0$) and $c_k(g_{ik} - g_{jk})$ is the partial concordance index defined as a function of the difference $g_{ik} - g_{jk}$ between the performance of a_i and a_j on criterion g_k. The partial concordance index measures the strength of the affirmation Q_k ="a_i is at least as good as a_j on the basis of criterion g_k". The partial index is normalized in the interval [0, 1], with values close to 1 indicating that Q_k is true and values close to 0 indicating that Q_k is false.

Except for assessing the strength of the arguments that support the affirmation Q, the strength of the arguments against Q is also assessed. This is performed through the "discordance test", which leads to the calculation of the discordance index $D_k(g_{ik} - g_{jk})$ for each criterion g_k. The higher is the discordance index the more significant is the opposition of a criterion on the validity of Q.

The concordance C and the discordance indices D_k are combined to construct the final outranking relation. The way that this combination is performed, as well as the way that the results are employed to choose, rank, or sort the alternatives depends on the specific ELECTRE method that is used. Details on these issues can be found in the works of Roy [161, 162] as well as in the book of Roy and Bouyssou [163].

PROMETHEE Methods The development of the PROMETHEE family of methods (**P**reference **R**anking **O**rganization **METH**hod of **E**nrichment **E**valuations) began in the mid 1980s with the work of Brans and Vincke [28] on the PROMETHEE I and II methods.

The PROMETHEE method leads to the development of an outranking relation that can be used to choose the best alternatives (PROMETHEE I) or to rank the alternatives from the most preferred to the least preferred ones (PROMETHEE II). For a given set of alternatives A, the evaluation process in PROMETHEE involves pairwise comparisons (a_i, a_j) to determine the preference index $\pi(a_i, a_j)$ measuring the degree of preference for a_i over a_j, as follows:

$$\pi(a_i, a_j) = \sum_{k=1}^{n} w_k P_k(g_{ik} - g_{jk}) \in [0, 1]$$

The higher is the preference index (closer to unity) the higher is the strength of the preference for a_i over a_j. The calculation of the preference index depends on the specification of the criteria weights w_k ($\sum w_k = 1$, $w_k \geq 0$) and the criteria preference function $P_1, \ldots, P_n$. The criteria preference functions are increasing functions of the difference $g_{ik} - g_{jk}$ between the performances of a_i and a_j on criterion g_k. The preference functions are normalized between 0 and 1, with higher values indicating stronger preference for a_i over a_j in terms of criterion g_k. Brans and Vincke [28] proposed six specific types of criteria preference functions (generalized criteria) which seem sufficient in practice.

On the basis of all pairwise comparisons for m alternatives, two overall performance measures can be defined. The first is the leaving flow $\phi^+(a_i) = \frac{1}{m-1}\sum_j \pi(a_i, a_j)$ which indicates the strength of preference for a_i over all other alternatives in A. In a similar way, the entering flow $\phi^-(a_i) = \frac{1}{m-1}\sum_j \pi(a_j, a_i)$ is also defined to measure the weaknesses of a_i compared to all other alternatives.

On the basis of these measures the procedures of PROMETHEE I and II are employed to rank the alternatives [28]. PROMETHEE I builds a partial ranking (with incomparabilities) through the combination of the rankings defined from the leaving and entering flows. On the other hand, PROMETHEE II provides a complete ranking on the basis of the net flow index $\phi(a_i) = \phi^+(a_i) - \phi^-(a_i)$, which constitutes an overall index of the performance of the alternatives.

24.4.2 Utility Functions-Based Approaches

Multiattribute utility theory (MAUT; [109]) extends the traditional utility theory to the multi-dimensional case. The objective of MAUT is to model and represent the decision maker's preferential system into a utility/value function $U(a_i)$. The utility function is defined on the criteria space, such that:

$$U(a_i) > U(a_j) \Leftrightarrow a_i \succ a_j \quad (a_i \text{ is preferred to } a_j) \tag{24.5}$$

$$U(a_i) = U(a_j) \Leftrightarrow a_i \sim a_j \quad (a_i \text{ is indifferent to } a_j) \tag{24.6}$$

The most commonly used form of utility function is the additive one:

$$U(a_i) = w_1u_1(g_{i1}) + w_2u_2(g_{i2}) + \ldots + w_nu_n(g_{in}) \tag{24.7}$$

where, $u_1, u_2, \ldots, u_n$ are the marginal utility functions corresponding the evaluation criteria. Each marginal utility function $u_k(g_k)$ defines the utility/value of the alternatives for each individual criterion g_k. The constants $w_1, w_2, \ldots, w_n$ represent the criteria trade-offs that the decision maker is willing to take.

A detailed description of the methodological framework underlying MAUT and its applications is presented in the book of Keeney and Raiffa [109].

Generally, the process for developing an additive utility function is based on the cooperation between the decision analyst and the decision maker. This process involves the specification of the criteria trade-offs and the form of the marginal utility functions. The specification of these parameters is performed through interactive procedures, such as the midpoint value technique [109]. The realization of such interactive procedures is often facilitated by the use of multicriteria decision support systems, such as the MACBETH system [16].

However, the implementation of such interactive procedures in practice can be cumbersome, mainly because it is rather time consuming and it depends on the willingness of the decision maker to provide the required information and the ability of the decision analyst to elicit it efficiently. The preference disaggregation approach of MCDA (PDA; [99, 100]) provides a methodological framework for coping with this problem. PDA refers to the analysis (disaggregation) of the global preferences (judgement policy) of the decision maker in order to identify the criteria aggregation model that underlies the preference result (ranking or classification/sorting). In PDA, the parameters of the decision model are estimated through the analysis of the decision maker's overall preference on some reference alternatives A', which may include either examples of past decisions or a small subset of the alternatives under consideration. The decision maker is asked to provide some examples regarding the evaluation of the reference alternatives according to his decision policy (global preferences). Then, using regression-based techniques the global preference model is estimated so that the decision maker's global evaluation is reproduced as consistently as possible by the model. A comprehensive bibliography on preference disaggregation methods can be found in Jacquet-Lagrèze and Siskos [99, 100], whereas some recent trends are discussed in [170].

PDA methods are particularly useful in addressing financial decision-making problems [203]. The repetitive character of financial decisions and the requirement for real-time decision support are two features of financial decisions which are consistent with the PDA framework. Thus, several PDA methods have been extensively used in addressing financial decision problems, mainly in cases where a ranking or sorting/classification of the alternatives is required. The following subsections provide a brief description of some representative PDA methods which have been used in financial problems.

UTA Method The UTA method (**UT**ilités **A**dditives; [98]) is an ordinal regression method developed to address ranking problems. The objective of the method is to develop an additive utility function which is as consistent as possible with the decision maker's judgment policy. The input to the method involves a pre-order of a set of reference alternatives A' . The developed utility model is assumed to be consistent with the decision maker's judgment policy if it is able to reproduce the given pre-order of the reference alternatives as consistently as possible.

In developing the utility model to meet this requirement, there are two types of possible errors which may occur [171]): (1) the under-estimation error when the developed model assigns a reference alternative to a lower (better) rank than the one specified in the given pre-order (the alternative is under-estimated by the decision maker), and (2) the over-estimation error when the developed model assigns a reference alternative to a higher (worse) rank than the one specified in the given pre-order (the alternative is over-estimated by the decision maker). The objective of the model development process is to minimize the sum of these errors. This is performed through linear programming techniques [98].

UTADIS Method The UTADIS method (**UT**ilités **A**dditives **DIS**criminantes; [54, 97]) is a variant of the UTA method, developed for classification problems. Similarly to the UTA method, the decision maker is asked to provide a classification of a set of reference alternatives A' into ordered categories $C_1, C_2, \ldots, C_q$ defined such that $C_1 \succ C_2 \succ \cdots \succ C_q$ (i.e., group C_1 includes the most preferred alternatives, whereas group C_q includes the least preferred ones). Within this context, the developed additive utility model will be consistent with the decision maker's global judgment if $t_k < U(a_i) < t_{k-1}$ for any alternative a_i that belongs in category C_k, where $t_0 = 1 > t_1 > t_2 > \ldots > t_{q-1} > 0 = t_q$ are thresholds that discriminate the groups. Similarly, to the UTA method, the under-estimation and over-estimation errors are also used in the UTADIS method to measure the differences between the model's results and the predefined classification of the reference alternatives. In this case, the two types of errors are defined as follows: (1) the under-estimation error $\sigma_i^+ = \max\{0, t_\ell - U(a_i)\}$, $\forall\, a_i \in C_\ell,\ \ell = 1, 2, \ldots, q-1$, (2) the over-estimation error $\sigma_i^- = \max\{0, U(a_i) - t_{\ell-1}\}$, $\forall\, a_i \in C_\ell,\ \ell = 2, 3, \ldots, q$. The additive utility model is developed to minimize these errors using a linear programming formulation [54].

Several variants of the original UTADIS method have been proposed (UTADIS I, II, III) to consider different optimality criteria during the development of the additive utility classification model [54, 205]. Other recent extensions can be found in [50, 55, 73, 74, 115].

MHDIS Method The MHDIS method (**M**ulti-group **H**ie-rarchical **DIS**crimination [209]) extends the PDA framework of the UTADIS method in complex sorting/classification problems involving multiple groups. MHDIS addresses sorting problems through a hierarchical procedure, in which the groups are distinguished progressively, starting by discriminating group C_1 (most preferred alternatives) from all the other groups $\{C_2, C_3, \ldots, C_q\}$, and then proceeding to the discrimination between the alternatives belonging to the other groups. At each stage of this sequential/hierarchical process two additive utility functions are developed for the classification of the alternatives. Assuming that the classification of the alternatives should be made into q ordered classes $C_1 \succ C_2 \succ \cdots \succ C_q$, $2(q-1)$ additive utility functions U_ℓ and $U_{\sim\ell}$ are developed. The function U_ℓ measures the utility for the decision maker of a decision to assign an alternative into group C_ℓ, whereas the second function $U_{\sim\ell}$ corresponds to the classification into the set of groups

$C_{\sim\ell} = \{C_{\ell+1}, C_{\ell+2}, \ldots, C_q\}$. The rules used to perform the classification of the alternatives are the following:

$$\left.\begin{array}{l} \text{If } U_1(a_i) > U_{\sim 1}(a_i) \text{ then } a_i \in C_1 \\ \quad \text{Else if } U_2(a_i) > U_{\sim 2}(a_i) \text{ then } a_i \in C_2 \\ \quad\quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \quad\quad\quad \text{Else if } U_{q-1}(a_i) > U_{\sim(q-1)}(a_i) \text{ then } a_i \in C_{q-1} \\ \quad\quad\quad\quad \text{Else } a_i \in C_q \end{array}\right\} \tag{24.8}$$

The fitting of the decision model on the reference data is performed through a combination of linear and mix-integer programming formulation, which take into account the number of classification errors introduced by the model, as well as the robustness of the model's recommendations. A detailed description of the model optimization process in the MHDIS method can be found in Zopounidis and Doumpos [209].

24.4.3 Decision Rule Models: Rough Set Theory

Pawlak [151] introduced rough set theory as a tool to describe dependencies between attributes, to evaluate the significance of attributes and to deal with inconsistent data. The rough set approach assumes that every alternative is described by two types of attributes: condition and decision attributes. Condition attributes are those used to describe the characteristics of the alternatives (e.g., criteria), whereas the decision attributes define a one or multiple decision recommendations (usually expressed in a classification scheme). Alternatives that have the same description in terms of the condition attributes are considered to be indiscernible. The indiscernibility relation constitutes the main basis of the rough set theory. Any set of alternatives, which can be obtained through a union of some indiscernible alternatives is considered to be crisp otherwise it is a rough set. The existence of rough sets in a decision problem is due to imprecise, vague or inconsistent data. The rough set approach enables the identification of such cases, without requiring their elimination, which may actually lead to loss of useful information. Furthermore, it enables the discovery of important subsets of attributes as well as attributes that can be ignored without affecting the quality of the model's recommendations.

The rough set approach assumes a symbolic decision model expressed in the form of a set of "IF ... THEN ..." rules. Decision rules can be consistent if they include only one recommendation in their conclusion part, or approximate if their conclusion involves a disjunction of elementary decisions that describe rough sets.

This traditional framework of the rough set theory, has been extended towards the development of a new preference modelling framework within MCDA [71, 72]. The main novelty of the new rough set approach concerns the possibility of handling criteria, i.e., attributes with preference ordered domains, and preference ordered groups. Within this context the rough approximations are defined according to the dominance relation, instead of the indiscernibility relation used. The decision rules derived from these approximations constitute a preference model.

24.4.4 Applications in Financial Decisions

MCDA discrete evaluation methods are well suited for the study of several financial decision-making problems. The diversified nature of the factors (evaluation criteria) that affect financial decisions, the complexity of the financial, business and economic environments, the subjective nature of many financial decisions, are only some of the features of financial decisions which are in accordance with the MCDA modelling framework. This section reviews the up-to-date applications of MCDA discrete evaluation methods in some typical financial decision making contexts.

Bankruptcy and Credit Risk Assessment The assessment of bankruptcy and credit risk have been major research fields in finance for the last decades. The recent credit crisis that started from USA has highlighted once again the importance of these issues in a worldwide economic and business context. Bankruptcy risk is derived by the failure of a firm to meet its debt obligations to its creditors, thus leading the firm either to liquidation (discontinuity of the firm's operations) or to a reorganization program [204]. The concept of credit risk is similar to that of bankruptcy risk, in the sense that in both cases the likelihood that a debtor (firm, organization or individual) will not be able to meet its debt obligations to its creditors, is a key issue in the analysis. However, while bankruptcy is generally associated with legislative procedures, credit risk is a more general concept that takes into account any failure of a debtor to meet his/her debt obligations on the basis of a pre-specified payment schedule. In both bankruptcy and credit risk assessment, decision models are developed to classify firms or individuals into predefined groups (problematic β), e.g., classification of firms as bankrupt/non-bankrupt, or as high credit risk firms/low credit risk firms. Such models are widely used by financial institutions for credit granting decisions, loan pricing, credit portfolio risk analysis, and investment planning.

Statistical and econometric techniques (discriminant analysis, logit and probit analysis, etc.) have been widely used for developing bankruptcy prediction and credit risk models. Over the past couple of decades, however, new methodologies have attracted the interest of researchers and practitioners, including MCDA techniques [47, 146, 204].

Bankruptcy prediction and credit scoring models are fitted on historical default data. In that sense, the model construction process is mostly involved with the identification of powerful (statistical) patterns that explain past defaults and bankruptcies, which can also be used for handling future cases. However, there are a number of features that make MCDA methods particularly useful. First, every bankruptcy

Table 24.1 Applications of MCDA approaches in bankruptcy and credit risk assessment

Approaches	Methods	Studies
Multiattribute utility theory	AHP	[96, 179–181]
	MACBETH	[17]
Outranking relations	ELECTRE	[20, 46, 57, 88, 111]
	Other	[6, 89, 197]
Preference disaggregation	UTA	[198, 201]
	UTADIS	[51, 206–208]
	MHDIS	[52, 61, 148]
	Other	[31, 68, 76, 117, 121, 122, 180, 194]
Rough set theory		[32, 48, 70, 174, 176]

prediction and credit scoring model provides a risk rating, which is purely ordinal (e.g., the ratings of major rating agencies such as Moody's, Standard & Poor's, and Fitch). This is in accordance with the standard ordinal classification setting in MCDA. Furthermore, the attributes describing the performance and viability of corporate entities, organization, or individual clients (e.g., financial ratios) are not some arbitrary statistical predictor variables. Instead, their use in a prediction/decision model should be made in way that has clear economic and business relevance, not only in a general context, but also in the specific application setting of a particular country, region, business sector, or financial institution. Incorporating expert knowledge of senior credit risk analysts and policy makers into statistical models is not a straightforward process. On the other hand, MCDA methods provide this possibility, thus enhancing the model calibration process with information that is crucial for the successful use of the model in practice.

A representative list of the MCDA evaluation approaches applied in bankruptcy and credit risk assessment is presented in Table 24.1.

Portfolio Selection and Management Portfolio selection and management involves the construction of a portfolio of securities (stocks, bonds, treasury bills, mutual funds, etc.) that maximizes the investor's utility. This problem can be realized as a two stage process [92, 93, 192]: (1) the evaluation of the available assets to select the ones that best meet the investor's preferences, (2) specification of the amount of capital to be invested in each of the assets selected in the first stage. The implementation of these two stages in the traditional portfolio theory is based on the mean-variance approach introduced by Markowitz [131, 133].

Nevertheless, numerous studies have emphasized the multi-dimensional aspects of portfolio selection and management [29, 177, 183, 195]. Section 24.3 discussed this issue in a comprehensive manner in the context of portfolio optimization under multiple objectives. Except for the optimization phase, one could also consider the asset selection phase or even the process of selecting the most suitable capital allocation strategy among multiple Pareto efficient portfolios. The asset selection phase is most useful for large-scale portfolio problems with too many assets. In such

Table 24.2 Applications of MCDA approaches in portfolio selection and management

Approaches	Methods	Studies
Multiattribute utility theory	AHP	[77, 120, 165, 167]
	MACBETH	[14, 15, 124]
	Other	[3, 12, 36, 49, 64, 105, 155]
Outranking relations	ELECTRE	[91–93, 113, 135, 136, 184, 191]
	PROMETHEE	[2, 78, 112, 136]
	Other	[69, 85]
Preference disaggregation	UTA	[92, 93, 166, 200, 212]
	UTADIS	[10, 210, 214]
	MHDIS	[58]
Rough set theory		[106]

cases, investors often employ screening rules to select the assets that best suit their investment policy and have the best future growth prospects. Such rules are usually based on technical analysis and a careful examination of fundamental variables and factors. MCDA is well-suited in this context enabling the investor to combine multiple criteria related to the prospects of each investment option and its suitability to the investor's policy. The portfolio optimization process is then performed on a limited number of assets selected through a multicriteria evaluation and screening process. However, as demonstrated in Sect. 24.4, the optimization phase leads to a set of suitable portfolios (Pareto efficient portfolios), among which an investor must select the most suitable one. This can be achieved directly through the multiobjective optimization process, which may lead to a single efficient portfolio (the one that best meets the investor's policy), or through a multicriteria portfolio evaluation process implemented after a small number of representative efficient portfolios has been constructed. In the latter case, discrete MCDA methods can be employed to evaluate the performance of the selected efficient portfolios under multiple investment criteria.

Table 24.2 summarizes several studies involving the application of MCDA evaluations methods in portfolio selection and management, covering both the asset selection and the portfolio selection stages.

Corporate Performance Evaluation The evaluation of the performance of corporate entities and organizations is an important activity for their management and shareholders as well as for investors and policy makers. Such an evaluation provides the management and the shareholders with a tool to assess the strength and weakness of the firm as well as its competitive advantages over its competitors, thus providing guidance on the choice of the measures that need to be taken to overcome the existing problems. Investors (institutional and individual) are interested in the assessment of corporate performance for guidance to their investment decisions, while policy makers may use such an assessment to identify the existing problems in the business environment and take measures that will ensure a sustainable economic

Table 24.3 Applications of MCDA approaches in the assessment of corporate performance

Approaches	Methods	Studies
Multiattribute utility theory	AHP	[9, 11, 119, 142]
	Other methods	[45, 67, 193]
Outranking relations	ELECTRE	[24, 37, 66, 94]
	PROMETHEE	[11, 18, 37, 56, 108, 126–128]
		[129, 147, 196]
Preference disaggregation	UTA	[75, 173, 211, 213]
	UTADIS	[34, 66, 95, 141, 187]

growth and social stability. The performance of a firm or an organization is clearly multi-dimensional, since it is affected by a variety of factors of different nature, such as: (1) financial factors indicating the financial position of the firm/organization, (2) strategic factors of qualitative nature that define the internal operation of the firm and its relation to its customers and the market (organization, management, market trend, etc. [198], (3) economic factors that define the economic and business environment. The aggregation of all these factors into a global evaluation index is a subjective process that depends on the decision maker's values and judgment policy. This is in accordance with the MCDA paradigm, thus leading several operational researchers to the investigation of the capabilities that MCDA methods provide in supporting decision maker's in making decisions regarding the evaluation of corporate performance. An indicative list of studies on this topic is given in Table 24.3.

Investment Appraisal In most cases the choice of investment projects is an important strategic decision for every firm, public or private, large or small. Therefore, the process of an investment decision should be conveniently modelled. In general, the investment decision process consists of four main stages: perception, formulation, evaluation, and choice. The financial theory intervenes only in the stages of evaluation and choice based on traditional financial criteria such as the payback period, the accounting rate of return, the net present value, the internal rate of return, the discounted payback method, etc. [35]. This approach, however, entails some shortcomings such as the difficulty in aggregating the conflicting results of each criterion and the elimination of important qualitative variables from the analysis [202]. MCDA, on the other hand, contributes in a very original way to the investment decision process, supporting all stages of the investment process. Concerning the stages of perception and formulation, MCDA contributes to the identification of possible actions (investment opportunities) and to the definition of a set of potential actions (possible variants, each variant constituting an investment project in competition with others). Concerning the stages of evaluation and choice, MCDA supports the introduction in the analysis of both quantitative and qualitative criteria. Criteria such as the urgency of the project, the coherence of the objectives of the projects with those of the general policy of the firm, the social and environmental aspects should be taken into careful consideration. Therefore, MCDA contributes through the identification of the best investment projects according to the

Table 24.4 Applications of MCDA approaches in investment appraisal

Approaches	Methods	Studies
Multiattribute utility theory	AHP	[7, 114, 159]
	Other	[33, 40, 64, 82, 104, 152]
Outranking relations	ELECTRE	[30, 41]
	PROMETHEE	[118, 125, 154, 188]
	ORESTE	[41]
Preference disaggregation	UTA	[21, 169]
	UTADIS	[97]

Table 24.5 Applications of MCDA approaches in other financial decision-making problems

Topic	Methodology	Studies
Venture capital	Conjoint analysis	[143, 156]
	UTA	[172, 199]
	MAUT	[19, 87]
Country risk	MAUT	[86, 185]
	UTA	[5, 39]
	UTADIS	[5, 59, 205]
	MHDIS	[53, 59, 60]
	Other	[38, 42, 144, 145]
Mergers and acquisitions	UTADIS, MHDIS	[149, 150]
	Rough sets	[175]
	Other	[189]

problematic chosen, the satisfactory resolution of the conflicts between the criteria, the determination of the relative importance of the criteria in the decision-making process, and the revealing of the investors' preferences and system of values. These attractive features have been the main motivation for the use of MCDA methods in investment appraisal in several real-world cases. A representative list of studies in presented in Table 24.4.

Other Financial Decision Problems Except for the above financial decision-making problems, discrete MCDA evaluation methods are also applicable in several other fields of finance. Table 24.5 list some additional applications of MCDA methods in other financial problems, including venture capital, country risk assessment and the prediction of corporate mergers and acquisitions. In venture capital investment decisions, MCDA methods are used to evaluate firms that seek venture capital financing, and identify the factors that drive such financing decisions. In country risk assessment, MCDA methods are used to developed models that aggregate the appropriate economic, financial and socio-political factors, to support the evaluation of the creditworthiness and the future prospects of the countries. Finally, in corporate mergers and acquisitions MCDA methods are used to assess the likelihood that a firm will be merged or acquired on the basis of financial information (financial ratios) and strategic factors.

24.5 Conclusions and Future Perspectives

This chapter discussed the contribution of MCDA in financial decision-making problems, focusing on the justification of the multi-dimensional character of financial decisions and the use of different MCDA methodologies to support them.

Overall, the main advantages that the MCDA paradigm provides in financial decision making, could be summarized in the following aspects [202]: (1) the possibility of structuring complex evaluation problems, (2) the introduction of both quantitative and qualitative criteria in the evaluation process, (3) the transparency in the evaluation, allowing good argumentation in financial decisions, and (4) the introduction of sophisticated, flexible and realistic scientific methods in the financial decision-making process.

In conclusion, MCDA methods seem to have a promising future in the field of financial management, because they offer a highly methodological and realistic framework to decision problems. Nevertheless, their success in practice depends heavily on the development of computerized multicriteria decision support systems. Financial institutions as well as firms acknowledge the multi-dimensional nature of financial decision problems [23]. Nevertheless, they often use optimization or statistical approaches to address their financial problems, since optimization and statistical software packages are easily available in relatively low cost, even though many of these software packages are not specifically designed for financial decision-making problems. Consequently, the use of MCDA methods to support real time financial decision making, calls upon the development of integrated user-friendly multicriteria decision support systems that will be specifically designed to address financial problems. Examples of such systems are the CGX system [181], the BANKS system [128], the BANKADVISER system [126], the INVEX system [188], the FINEVA system [213], the FINCLAS system [206], the INVESTOR system [210], etc. The development and promotion of such systems is a key issue in the successful application of MCDA methods in finance.

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