

# Chapter 2

## Friction Between Ski and Snow

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### 2.1 Introduction

The first skis were developed to improve locomotion across the natural, wind packed snow surface in the European northern countries. The skis were made of flat planks with shovels at the tips. Under load, the tips and ends of the skis bended up causing resistance against forward movement. An improvement of the gliding of skis was the invention of the bow-shaped cambered ski, arched up towards his center. Under load, the ski lies flat on the snow surface and the load is more evenly distributed along the ski. With the appearance of downhill skiing, the turning properties of skis became more important. In 1928, Lettner (AT) invented steel edges to give the skis more grip. During the first half of the twentieth century, the technique was developed to produce laminated skis composed of a wooden core with different bottom and upper layers. In 1955, Kofler (AT) introduced the first ski with a polyethylene base, which remarkably improved the gliding properties. In addition, the repair of minor scratches was easily possible. In the recent past, the gliding properties of skis were further developed by special grinding techniques for the ski base and by the development of special waxes.

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Besides practical ski development, a lot of knowledge about the friction processes was gained. The initial considerations of friction on snow regarded dry friction. Soon the reason for the low friction on snow and ice was presumed to lie in a thin water film. This water film was first supposed to be generated by pressure melting. Ice melts at lower temperature when additional pressure is applied [e.g., 1]. Energy considerations led to another theory. At snow temperatures relevant for skiing, the heat generated by friction is sufficient to melt snow [2]. This process causes lubricated friction, which is now considered as the dominant friction regime of skis on snow. In the last decade, an additional theory came up. Quasi-liquid films were observed due to premelting even below the melting point of crystals [3] which supposedly have an effect on snow friction.

## 2.2 Theory of Snow Friction

### 2.2.1 Dry Friction

For friction between a solid and its supporting surface the friction force,  $F_F$  is given by  $F_F = \mu F_N$ .  $F_N$  is the normal load and  $\mu$  is the coefficient of friction. One has to distinguish between static and kinetic friction, depending on whether the object is resting or moving. Static coefficients usually are larger than kinetic ones.

Leonardo da Vinci (1452–1519) already stated the two basic laws of friction: (1) the area of contact has no effect on friction and (2) if the load of an object is doubled, its friction will be doubled. Two centuries later Amontons (1663–1705) investigated friction. According to Amontons's first law the friction force is proportional to the applied load, and due to Amontons's second law the constant of proportionality is independent of the contact area. Coulomb (1736–1806) continued Amontons's studies and found that the coefficient of kinetic friction is independent of speed. After another century, [4, 5] stated that the friction between solid bodies arises from the contact of the asperities on the surface of a solid body. As contact pressure increases, more asperities come into contact, and so frictional resistance increases.

In skiing dry friction is usually modeled as solid-to-solid interaction. The frictional resistance originates from shearing off the tips of the snow asperities and/or wax or running base asperities. The coefficient of friction is given according to [6] as

$$\mu = \frac{\tau}{\sigma} \quad (2.1)$$

(which is equivalent to  $F_F = \mu F_N$ ).  $\tau$  represents the shear stress and  $\sigma$  the normal stress acting between ski base and snow. For the shear stress,  $\tau$  one has to use the shear strength of the snow asperities or the ski base asperities including wax. The smaller value is the appropriate one. At a temperature near 0 °C, the snow asperities are softer than the ski base but harder as most ski waxes [7, Fig. 10]. For snow/ice

temperatures from 0 down to  $-32\text{ }^{\circ}\text{C}$  the shear strength of the ice asperities increases about 15 % and the compression strength roughly about 400 %. Consequently, the coefficient of dry friction drops for decreasing temperatures [8, Table 1].

### 2.2.2 Wet Friction

A lubricating water film between ski and snow causes reduced friction compared to dry friction. Bowden and Hughes [2] related the formation of the water film to frictional heating. Friction between ski and snow generates heat, which dissipates into snow and ski. If the melting temperature of snow is reached, water acts as lubricant between ski and snow. When a complete water film exists between snow asperities and ski base, the friction force for a laminar flow is given by

$$F_{F,\text{wet}} = \frac{\eta_w A_C v}{h}. \quad (2.2)$$

$A_C$  denotes the real contact area between ski and snow asperities,  $h$  the thickness of the water film, and  $v$  the speed of the ski. The dynamic viscosity of water is given by  $\eta_w = 1.79 - 0.054\vartheta_S$  mPa s with  $\vartheta_S$  the snow temperature in  $^{\circ}\text{C}$ .

Depending on the properties of the snow and the texture of the ski base, the real contact area between snow and ski may be quite small. The snow is composed of ice crystals with various shapes and sizes, which are packed and sintered together. The structure of the ski's running surface is given by its texture. The whole load applied by the ski is supported by a small fraction of the snow surface. For small contact pressures, the ski will run on some spikes. For larger contact pressures, these spikes collapse and so the real contact area gets larger. Let  $A$  be the area of the ski base and  $A_C = aA$  be the real contact area. According to [8], the fraction of the real contact area,  $a$  ranges between 0.001 and 0.015 and to [9] between 0.01 and 0.1.

The critical Reynolds number gives the transition from laminar to turbulent flow. For a rectangular cross section the flow remains laminar as long as

$$R = \frac{\rho_w h v}{\eta_w} < R_{\text{crit}} = 1500 \quad \text{or if} \quad h v < 2 \cdot 10^{-3} \text{ m}^2 \text{ s}^{-1}. \quad (2.3)$$

In skiing speeds are below  $40 \text{ m s}^{-1}$  and the water film thickness is clearly below  $50 \mu\text{m}$  [10]. Therefore, the flow in the water film is laminar.

### 2.2.3 Mixed Friction

Dry friction occurs when meltwater lubrication is absent and wet friction when the ski-snow contacts are completely covered by a layer of water. In many cases, snow asperities and ski base may not be fully separated by a water film and so

solid-to-solid contacts as well as lubricated contacts occur, i.e. mixed lubrication conditions prevail. Due to frictional heating the water film thickens and the number of lubricated contacts increases. Squeezing the water away from the snow asperities reduces the water film thickness and increases the number of solid-to-solid contacts.

Along the ski there may exist solid-to-solid contacts at the front of the ski and towards the tail of the ski, due to frictional heating, mixed conditions, and finally solely wet friction. Therefore, the friction coefficient varies along the ski too. To calculate the overall coefficient of friction for a ski, one may partition the ski into several segments. Then the coefficient of friction for the whole ski is given by

$$\mu = \frac{1}{F_N} \sum_{i=1}^n F_{F,i}. \quad (2.4)$$

The friction force ( $F_{F,i}$ ,  $i = 1, \dots, n$ ) has to be calculated for every segment separately by using the appropriate friction model (dry, mixed, or wet friction) with the appropriate parameters (water film thickness, normal load, contact area, etc.).

In an advanced approach, one has additionally to consider contributions due to ploughing and compression [8] specifically in the front region of a ski, and for turns, skidding and carving processes.

## 2.2.4 Quasi-Liquid Layer

At snow temperatures below  $0^\circ\text{C}$ , no melt water is produced. Yet, there is a boundary layer, called quasi-liquid layer, with the thickness of some few water molecules [3, 11]. It is in an intermediate state between solid and liquid. Above  $-80^\circ\text{C}$  the crystalline structure of the ice begins to alter and for temperatures between  $-30$  and  $0^\circ\text{C}$  the ice surface is covered with a 1–10 nm thick quasi-liquid layer of quasi-liquid water [3, Fig. 9]. This tiny layer rounds the edges of the ice asperities and reduces the solid-to-solid interactions between the ice crystals. It is presumed that this makes the ice asperities slippery and contributes to the small coefficients of friction at moderate snow temperatures below  $0^\circ\text{C}$ .

## 2.3 Heat Considerations

### 2.3.1 Basic Considerations

The solid state of water exists in nature as snow and ice. The amount of heat per unit mass necessary to raise the temperature of a certain material by 1 K is called specific heat capacity  $c_p$ . Table values for ice, snow, and water are 2.110, 2.009, and 4.186  $\text{kJ kg}^{-1} \text{K}^{-1}$ , respectively. The heat to raise the temperature of the snow mass  $m_S$

with volume  $V$  and density  $\rho_S$  by  $\Delta T$  degrees of Kelvin is given by  $Q = c_{p,S}m_S\Delta T$ . When the snow temperature reaches the melting point of snow any further supplied heat is used to melt snow. When the whole snow is molten, the water temperature rises according to the specific heat capacity of water. If water is cooled, the water temperature drops until  $0^\circ\text{C}$  and freezing starts. The water-snow mixture keeps the temperature of  $0^\circ\text{C}$  until all water is frozen. Once the water is fully frozen, the temperature of the snow continues to fall. The specific melting heat or enthalpy of fusion is the heat per unit mass added during the phase transition from snow to water and vice versa. Its value is  $H_{\text{fus},W} = 335 \text{ kJ kg}^{-1}$ . The heat required to melt snow is given by  $Q = H_{\text{fus},W}m_S$ . This amount of energy is needed to release the water molecules from the lattice structure of the snow crystals. The same amount of energy is released when the water freezes again.

Because of frictional heating, the temperature of the tips of the snow asperities below the ski is increased. When the temperature reaches  $0^\circ\text{C}$ , snow asperities melt causing a layer of water. We consider a particular location on the snow surface. The passage of a 1.75 m long and 0.06 wide ski loaded with 400 N produces an amount of 35 J of heat ( $Q = \mu F_N L$ ) when presuming a friction coefficient of 0.05. The heat flows partly into snow and ski and when the snow temperature is high enough, snow melts. The energy needed to heat snow is much lower than the energy needed to melt snow. In the following two extreme cases are examined. Firstly, it is assumed that all 35 J of heat is used to heat snow. The snow mass of 1.74 g can be heated from  $-10$  to  $0^\circ\text{C}$  ( $Q = c_{p,S}m_S\Delta T$ ). This mass of snow has a volume of  $3480 \text{ mm}^3$  if one assumes a snow density of  $500 \text{ kg m}^{-3}$ . This corresponds to a  $0.033 \text{ mm}$  thick layer of snow between ski and snow ( $A = 1.75 \times 0.06 = 0.105 \text{ m}^2$ ). In reality, the snow is not heated at once and one has to solve the heat equation to study the propagation of heat. Nevertheless, one can conclude that during the passage of a ski the heat transfer is restricted to a quite thin layer of snow, whose thickness is approximately the magnitude of the snow asperities. Secondly, it is assumed that all 35 J of heat is used to melt snow. The snow mass of 0.11 g is converted to water ( $Q = H_{\text{fus},W}m_W$ ). If the amount of melt water is uniformly distributed along the complete running surface of the ski ( $A = 0.105 \text{ m}^2$ ), the thickness of the water film is  $1.0 \mu\text{m}$ . The real contact area is not known, since the melt water is concentrated on the snow asperities with contact to the ski. Published results differ by two orders of magnitude (see Sect. 2.2). If only a tenth of the running surface of the ski is covered by water, the water film thickness is  $10.0 \mu\text{m}$ . Experimentally, the water film thickness was measured between  $5$  and  $20 \mu\text{m}$  using a dielectric device [10]. Because their measurement device had a hydrophilic coating, the measured values are likely too large. Strausky et al. [12] used fluorescence spectroscopy to detect the water film. The lower limit of their measurement range was  $0.05 \mu\text{m}$ . They did not detect any water in their measurement range. This may be reasonable for the measured speed below  $0.1 \text{ m s}^{-1}$ . Bäurle et al. [9] presented simulations of the heat flow with water film thicknesses considerably lower than  $1.0 \mu\text{m}$ . Their results suggest that a large part of the produced heat flows into ski and snow and only a small fraction is used to melt snow.

### 2.3.2 Modeling of Heat Flow Between Ski and Snow

In this section, the location along the ski is determined, where the transition from dry to mixed/wet friction occurs. For this, it is not necessary to know the exact amount of the produced melt water. For simplicity, we focus on the heat flow into the snow and neglect the flow into the ski. Thus, the melting temperature at the snow surface is reached faster, since the complete produced heat is transmitted into the snow.

At a particular location on the snow surface a Cartesian coordinate system is introduced with the  $x$ -coordinate along the ski's longitudinal direction, the  $y$ -coordinate transverse to the ski, and the  $z$ -coordinate normal to the snow surface. The snow surface is given by the plane  $z = 0$  and the positive  $z$ -coordinate points towards the snow. The heat flow in  $z$ -direction is computed by solving the one-dimensional heat equation. During the passage of the ski, heat is produced with the rate  $\frac{\Delta Q}{\Delta t} = F_F v$ . Here  $F_F$  is the friction force and  $v$  the speed of the ski. This causes a constant heat flux

$$J = \frac{1}{A} \frac{\Delta Q}{\Delta t} = \frac{F_F v}{A} \quad (2.5)$$

into the snow.  $A$  denotes the contact area of the whole ski with the snow. The friction force along the ski is assumed to be constant.  $T(z, t)$  is the snow temperature at the time  $t$  and at the penetration depth  $z$ . The start time ( $t = 0$ ) is given with the first contact and the end time ( $t = L/v$ ) with the last contact of the ski. A complete set of equations to calculate the snow temperature,  $T$  is given by the heat equation for the snow, the initial temperature of the snow,  $T_0$ , and the heat flux into the snow surface,  $J$ :

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial z^2}, \quad T(z, 0) = T_0, \quad -\lambda \frac{\partial T}{\partial z}(0, t) = J. \quad (2.6)$$

Because the ski is only supported by snow asperities, for the thermal diffusivity,  $\alpha$  and the thermal conductivity,  $\lambda$  table values for ice are used:  $\alpha_I = 0.93 \text{ mm}^2 \text{ s}^{-1}$  and  $\lambda_I = 1.8 \text{ W m}^{-1} \text{ K}^{-1}$ . The solution for the heat equation is [13]:

$$T(z, t) = T_0 + \frac{2J}{\lambda} \sqrt{\frac{\alpha t}{\pi}} f\left(\frac{z}{\sqrt{4\alpha t}}\right) \quad \text{with} \quad f(q) = e^{-q^2} - \sqrt{\pi} q \operatorname{erfc}(q). \quad (2.7)$$

Here,  $\operatorname{erfc}$  denotes the complementary error function. From this equation the temperature rise on the snow surface ( $z = 0$ ) can be computed. We get

$$\Delta T = \frac{2J}{\lambda} \sqrt{\frac{\alpha t}{\pi}}. \quad (2.8)$$

The time with dry friction is defined by  $t_{\text{dry}} = L_{\text{dry}}/v$  with  $L_{\text{dry}}$  the length of the section in the frontal part of the ski with dry friction. In this time, the temperature of

the tips of the snow asperities raises from  $T_S$  to  $0^\circ\text{C}$ . We set  $\Delta T = \vartheta_I$ , with  $\vartheta_I$  the temperature of the snow asperities in degree Celsius. Putting all together, one gets

$$L_{\text{dry}} = \frac{\pi}{\alpha_I v} \left( \frac{\lambda_I A_C \vartheta_I}{2\mu_{\text{dry}} F_N} \right)^2. \quad (2.9)$$

As numerical example we consider a cross-country ski of the size  $A = 1.75 \times 0.06 = 0.105 \text{ m}^2$ , with a real contact area of a tenth ( $a = 0.1$ ), loaded with  $F_N = 400 \text{ N}$ , and moving at a speed of  $v = 7 \text{ m s}^{-1}$ . For dry friction  $\mu = 0.25$  and  $0.22$  are used for snow temperatures of  $\vartheta_I = -5$  and  $-10^\circ\text{C}$  [8, Table 2]. In this case, the sections with dry friction have a length  $L_{\text{dry}} = 0.11$  and  $0.55 \text{ m}$ , respectively. If the real contact area is only a twentieth ( $a = 0.05$ ), or if the speed is  $v = 28 \text{ m s}^{-1}$  (e.g. downhill skiing), the corresponding numbers are  $L_{\text{dry}} = 0.03$  and  $0.14 \text{ m}$ . The calculated  $L_{\text{dry}}$  is a lower bound for the real length with dry friction, since some heat also flows into the ski.

Note, the sections with dry friction are longer for lower temperatures due to the lower temperature of the snow as well as due to the smaller coefficient of dry friction. The smaller coefficient of friction causes lower frictional heating. This contributes to the well-known fact that the total friction for the whole ski increases with decreasing temperature.

The presented modeling approach has two major limitations. The heat production rate along the ski is not constant and heat is not only transmitted into the snow but also into the ski. To obtain a more realistic approximation of the amount of produced melt water, the heat equation for the heat flow into snow and ski has to be solved. The major difficulties are the rate of heat production for the different kinds of friction (dry, mixed, wet), the heat consumption for melting of snow, and the thinning of the water film by squeezing. Such models were presented for ice [14, 15] and for snow [9]. All these approaches needed the numeric solution of a complicated set of equations.

## 2.4 Experiments

Friction between skies and snow is affected by numerous factors, e.g. speed, load, snow properties (temperature, density, liquid-water content, hardness, texture, etc.), and ski properties (stiffness, thermal conductivity, base material, base roughness, etc.). Most of the knowledge about ski-snow friction is from laboratory measurements. The results of the laboratory studies are important for explaining the basic processes of snow friction; however, the results are restricted to the conditions in which the experiments were conducted. Load, speed, slider characteristics, and snow properties used in the laboratory do usually not correspond to ski gliding, where long, highly loaded skies move at high speed under a diversity of man-made and natural snow conditions. Furthermore, friction varies along a sliding ski, which cannot be mirrored with short probes.

Tests under ski-specific conditions have to be carried out in the field, e.g. on long gliding tracks with skiers. The results of these field tests are regrettably impaired by changing environmental conditions and variation of the skier's motion. To determine snow friction, air and gliding resistance have to be separated which constitutes a major problem, especially when measuring at high speeds where drag is overwhelming. In addition, the conduction of outdoor tests is time-consuming and cumbersome. Hence, only few measurements have been published. They are presented in the next chapter.

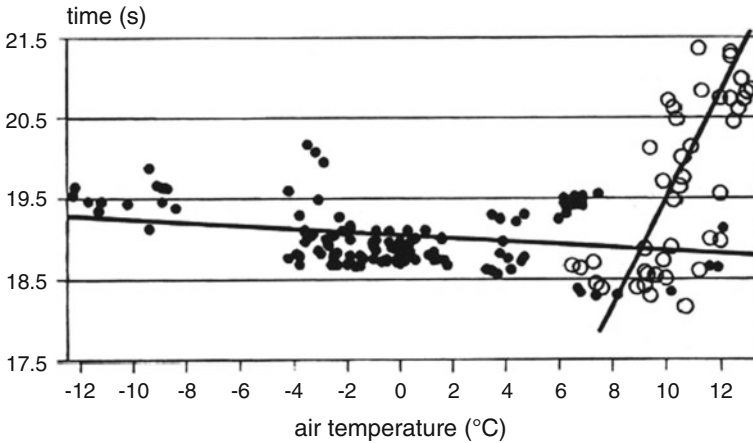
To investigate the factors influencing snow friction one by one in a controlled environment, various experimental devices were developed. Among these, pin-on-disk tribometers are the most common types. In these rotational tribometers, a stamp carrying the probe is pressed against a snow-filled rotating disk and the frictional force acting on the probe is measured. Tribometers with rotational disks have serious drawbacks. One is the repeated contact of the probe with the snow or ice on each rotation. Temperature and water film can subsequently build up and the snow surface is polished with each passage of the probe. A second drawback is the centrifugal force at high speeds. Melt water might be moved radially outwards altering the measured friction force. Consequently, at the Centre of Technology of Ski and Alpine Sports at the University of Innsbruck a linear tribometer was developed on which snow friction can be measured for a whole ski at speeds up to  $30 \text{ m s}^{-1}$ . A description of this tribometer follows in a separate chapter below.

### **2.4.1 Field Tests**

In competitive alpine and cross-country skiing, every year numerous gliding tests are conducted by ski manufacturers and national ski federations to select the fastest skies for their athletes. Results of these tests are usually not published. For alpine skiing, experienced ski testers glide straight down the test courses in the tucked downhill position. The test courses are about 350 m long; it is intended to reach maximal speeds above  $30 \text{ m s}^{-1}$ . Besides split and end times using photo cells some environmental conditions like air and snow temperature, air humidity, and snow moisture are taken. Running times and corresponding speeds are used to assess the sliding friction of different base material, base texture, and waxes.

On a permanent 342 m long test course located in Seefeld (AT) an experienced ski tester performed 175 test runs with the same equipment during a winter season. Air and snow conditions were measured for each run [16]. As an example running time vs air temperature for snow with and without liquid water content is presented in Fig. 2.1. At about  $6^\circ\text{C}$  liquid water was partly measured in the snow. One can see different tendencies: Decreasing friction with increasing air temperature on dry snow and increasing friction with increasing temperature on wet snow. It may be speculated that the decreasing friction on dry snow is due to increased lubrication by increased water film thickness and the increasing friction on wet snow is caused by an increase of capillary attachments between ski and snow.





**Fig. 2.1** Run time versus air temperature in a straight-run gliding test. The *filled circles* correspond to snow without and the *open circles* to snow with liquid water content [16]. Reprinted, with permission, from ASTM STP1266 Skiing Trauma and Safety: Tenth Volume, copyright ASTM International, 100 Barr Harbor Drive, West Conshohocken, PA 19428

In Table 2.1 the published field friction measurements for straight gliding, traversing, and turning are summarized. In the following the field experiments conducted by our group are briefly outlined. On the permanent test course in Seefeld (AT) the coefficient of friction and the drag area of skiers during straight gliding were determined [17]. Nine photocells to measure split times were installed. By geodetic surveying slope inclination and the locations of the photocells were determined. The motion of the skier was formulated as differential-algebraic equation and an optimization method was used to separate drag and friction force. The results revealed a drag area of  $0.22 \text{ m}^2$  and a coefficient of friction of 0.0085 (Table 2.1). The value for the drag area compares to wind tunnel experiments, whereas the value for the coefficient of friction is lower than the value found in other studies. A reason might be that the coefficient of friction may depend on a term proportional to  $v^2$ , too, but such a term cannot be separated from drag.

Coefficients of friction and drag areas for traversing and turning are published by Kaps and Nachbauer [18] and Kaps et al. [17, 19]. Position-time of the center of mass of the skiers was determined by two- or three-dimensional video analysis. The skier's equation of motion was formulated as differential-algebraic and by dynamic optimization friction coefficients and drag area were determined. In these cases, snow friction is significantly affected by ploughing, compression, and skidding and carving actions of the ski. As one can see from the coefficients of friction (Table 2.1), the frictional interaction at the ski-snow interface plays a minor role especially for turning.

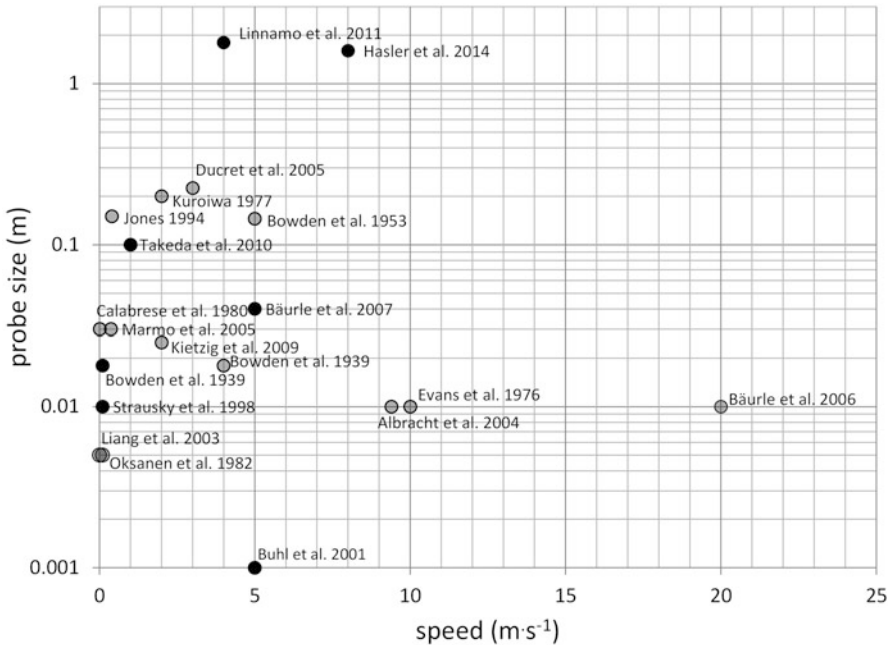
**Table 2.1** Overview of friction measurements during straight gliding, traversing, and turning

$\mu$	Speed (m s <sup>-1</sup> )	Method	Source
Straight running			
0.03–0.04	2.2	Towing method	[20]
0.1	8–27	Run-out, light cells	[20]
0.02–0.1	8–22	Light cells	[21]
0.02 (dry snow)	0.5–10	Run-out, light cells	[22]
0.03–0.1 (wet snow)	0.5–10	Run-out, light cells	[22]
0.02–0.05	5.5–10.5	Run-out, light cells	[23]
0.0085	3–29	Light cells	[17]
0.041	0–4	Towing method	[24]
Traversing			
0.064	1–11	Video	[17]
0.128	11–13	Video	[17]
0.153	15–17	Video	[17]
Turning			
0.2–0.3	Carving	Video	[18]
0.45	Downhill racing turn	Video	[19]
< 0.3	Stem and parallel turn, Wedeln, and snow plow	Serial images	[25]

## 2.4.2 Laboratory Tests

Bowden and Hughes [2] first investigated in the laboratory the frictional resistance when gliding on snow or ice. They developed a friction apparatus to analyze friction on snow for different materials, loads, and speeds. Based on their observations, they presumed the generation of a water film by frictional heating between slider and snow. At present, this is a generally accepted theory. In the following, many different laboratory tribometers were developed. Figure 2.2 gives an overview of the publications in the field. Studies on snow were carried out to a maximum speed of 5 m s<sup>-1</sup> and a sample size of 0.1 m. Even on ice, a much easier manageable surface than snow, probe size never was in regions of interest for snow sports.

The pin-on-disk tribometer is the most common device type used in the research on friction on snow and ice. It consists of a rotating disk covered with ice or snow and a probe pushed with given normal force to the surface. The force against the rotating direction parallel to the disk is measured. It has been built in a wide range of sizes from a diameter of several mm [26] to 1.8 m [27]. Evans et al. [28] chose a design with a pendulum hanging around a revolving drum of ice to imitate the movement of an ice skating blade on the ice. Jones et al. [29] determined the friction coefficient using a conveyor belt and measuring the friction force on a fixed ice sample. Several studies have been carried out using ring-on-disk or disk-on-disk tribometers, where instead of a pin a ring or a disk is pressed against the snow or



**Fig. 2.2** Overview of laboratory experiments on snow (black circles) and ice (grey circles) [34–41]

ice surface [e.g., 21, 30, 31]. Temperatures ranged from  $-150^{\circ}\text{C}$  [26] to the melting point of  $0^{\circ}\text{C}$ .

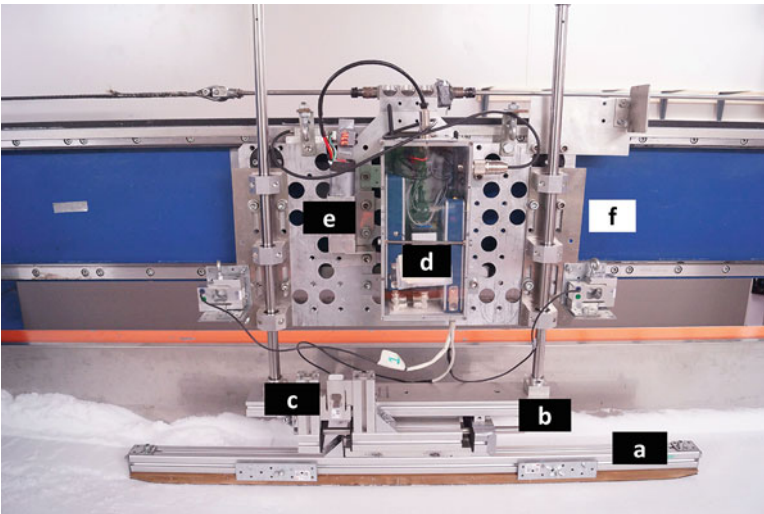
The first laboratory measurements were carried out on ice and on snow [2]. Later, mainly ice friction measurements were done because of the easier setup of the testing surface and the better reproducibility of the experiments. Furthermore, on ice mechanisms subject to friction are easier to model since the contact area is flat and hence well defined. Especially with rotating measurement devices snow is difficult to handle due to its graininess and the change of the contact area by the abrasion that occurs with each rotation.

### 2.4.3 Friction Measurements with the Linear Tribometer

At the Centre of Technology of Ski and Alpine Sports of the University of Innsbruck a linear tribometer (Fig. 2.3) was developed for investigating friction between snow and skies. The device consists of a linear beam, a trough for the snow, and carriage for the probe or ski. The trough can be moved laterally in order to have up to ten fresh tracks of the same snow for the measurements available. The carriage is pulled by an 80 kW high torque electric-motor. At the end of a run, the carriage is decelerated by the electric motor and an eddy current brake. Each run consists



**Fig. 2.3** Linear tribometer



**Fig. 2.4** Carriage of the linear tribometer. (a) Ski base probe, (b) lower carriage, (c) load cell horizontal force, (d) data acquisition and transmission system, (e) battery, (f) linear beam

of three phases: acceleration, measurement, and deceleration. In the measurement section, the carriage runs with almost constant speed. Available speeds range between  $0.1$  and  $30 \text{ m s}^{-1}$  resulting in a measurement length of  $20\text{--}5 \text{ m}$ . The skies or probes are attached to the carriage (Fig. 2.4). With vertical springs, normal loads

are applied to the ski between 10 N and 700 N. Normal and horizontal forces acting on the ski are measured with load cells. The signals of the load cells are transmitted wireless to a computer. The speed of the carriage is determined by an inductive length measurement system. The whole tribometer is located in a cooling chamber in which air can be cooled down to approximately  $-20\text{ }^{\circ}\text{C}$ .

Snow is produced in a separate cooling chamber using a snow lance. The snow is placed in the trough, compacted and leveled using a stainless steel blade. Cooling coils positioned on the bottom of the trough are used to control snow temperature. Snow properties (temperature, density, hardness, water content, grain form, and grain size), air temperature, and air humidity are determined before each test.

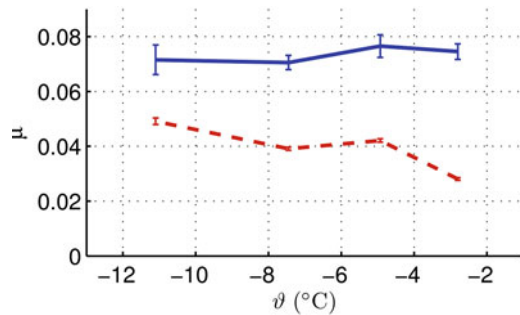
In the following, the effects of snow temperature, speed, and normal load on snow friction are presented. The probes with a size of  $1.03 \times 0.05\text{ m}$  were mounted below a stiff aluminum profile to get a uniform load distribution. Snow temperature was set between  $-11$  and  $-3\text{ }^{\circ}\text{C}$ , speed to 5, 10, 15, and  $20\text{ m s}^{-1}$ , and normal load between 10 and 600 N. Each measurement consisted of ten consecutive runs on the same track. For runs four to ten the means of the measured horizontal and vertical force were calculated and from these the mean coefficient of friction was determined. The horizontal force was adjusted to account for air drag. The measurements are highly reliable with an accuracy, which decreases with increasing speed to approximately 1.3 % at  $10\text{ m s}^{-1}$ . The mean coefficients of friction with the 95 % confidence intervals are presented in Figs. 2.5 and 2.6.

### 2.4.3.1 Effect of Snow Temperature

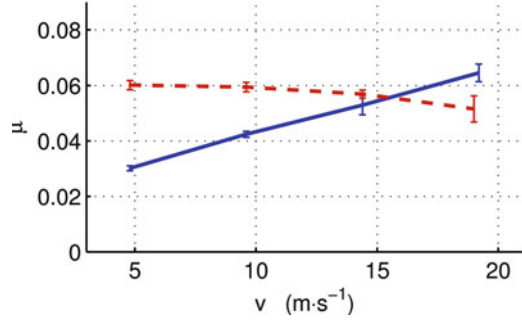
At the low speed of  $5\text{ m s}^{-1}$  the friction coefficient decreased with increasing snow temperature (Fig. 2.5). This behavior is in accordance with experimental results of Buhl et al. [32] on snow and with a manifold of studies on ice [e.g., 27, 31]. Buhl and colleagues found the minimal friction between  $-5$  and  $0\text{ }^{\circ}\text{C}$ . It is assumed that the decrease of the friction coefficient with increasing temperature is due to enhanced lubrication at higher snow temperatures.

In contrast to the measurements at low speed, there was no clear temperature effect observed on the coefficient of friction at a high speed of  $20\text{ m s}^{-1}$  (Fig. 2.5).

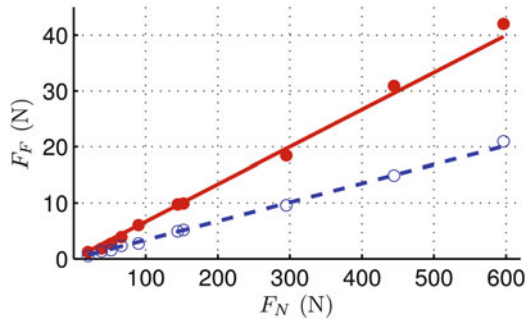
**Fig. 2.5** Coefficient of friction,  $\mu$  with 95 % confidence intervals versus snow temperature,  $\vartheta$  at a speed of  $5\text{ m s}^{-1}$  (dashed line) and  $20\text{ m s}^{-1}$  (solid line). Normal load  $F_N = 146\text{ N}$ , snow density  $\rho = 400\text{ kg m}^{-3}$



**Fig. 2.6** Coefficient of friction,  $\mu$  with 95 % confidence intervals versus speed  $v$  at snow temperature of  $-3^\circ\text{C}$  (solid line) and  $-11^\circ\text{C}$  (dashed line). Normal load  $F_N = 146\text{ N}$ , snow density  $\rho = 400\text{ kg m}^{-3}$



**Fig. 2.7** Friction force,  $F_F$  versus normal load,  $F_N$  at speed of  $5\text{ m s}^{-1}$  (open circles) and  $15\text{ m s}^{-1}$  (filled circles). Snow temperature  $\vartheta = -4^\circ\text{C}$ , snow density  $\rho = 390\text{ kg m}^{-3}$



### 2.4.3.2 Effect of Speed

Contrary to the measurement at the snow temperature of  $-11^\circ\text{C}$ , at  $-3^\circ\text{C}$  the friction coefficient increased from 0.03 to 0.07 with increasing speed (Fig. 2.6). This increase at high temperatures was also found by Kuroiwa [21]. The increase of friction for the higher snow temperature may be explained by the increase of speed assuming wet friction  $F_{F,\text{wet}} = \frac{\eta_w A_C v}{h}$ .

### 2.4.3.3 Effect of Normal Load

Friction force was directly proportional to the normal force for the measured speeds of  $5$  and  $15\text{ m s}^{-1}$  (Fig. 2.7). Thus, the friction coefficient was independent of the normal force as it is stated for Coulomb type friction. For snow, load-independent static [33] and dynamic [32] friction coefficients were measured, which is in agreement with our findings. For ice, it was repeatedly documented that the friction coefficient decreases at a certain load [e.g., 27, 28]. This is explained by the higher friction force with increasing loads, which causes more meltwater due to higher frictional heating and a reduction of the coefficient of friction by improved lubrication. The friction coefficient on snow might not be affected by the load because additional meltwater after a certain level is squeezed out or absorbed by the snow.

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