Chapter 3
Mathematics Fluency—More than the Weekly Timed Test

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Pencils up, start…tick…tick…tick, stop, pencils down. If there is a shared cornerstone experience in education, the weekly timed test may be the winner. But why? Why across decades have teachers spoken those words and students furiously worked through sheets containing a range of problems from addition facts to division facts? This chapter attempts to provide answers to that fundamental question. We start with an exploration as to why fluency in mathematics is critical, examine interventions designed to increase fluency, and in the end provide an overview of the measures used to assess fluency and provide our thoughts to guide future work as the field gains a greater understanding of mathematics fluency.

As a nation, we compete in an international marketplace driven by technological innovation. Employment projections by the US Bureau of Labor Statistics indicate that the majority of the fastest growing occupations in the coming decade will require substantial preparation in mathematics or science (Lockard & Wolf, 2012). As policy-makers seek to address a dearth of workers prepared for science, technology, engineering, and mathematics (STEM) jobs in the USA, K-12 mathematics and science education is increasingly at the center of discussions about how to ensure international competitiveness. For instance, the current presidential administration has launched an "Educate to Innovate" campaign (The White House, 2012), designed to improve the coordination and facilitation of efforts to improve STEM education and prepare the students of today for the jobs of tomorrow.

In the STEM fields, mathematics and science education provide the foundation for advanced knowledge and professional skills that will prepare our nation's youth to compete for the surge of high-level jobs in engineering and technology (National

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Math Advisory Panel [NMAP], 2008). Arguably, students' understanding of mathematics, starting at an early age, is at the core of their ability to gain access to STEM jobs. Accordingly, proficiency in mathematics is receiving increasing attention, beginning in the early years of a student's education, because the early elementary years represent a critical first step in building a long-term foundation for success in mathematics. Emerging evidence suggests the long-term consequences of struggling early in mathematics exact the same or greater deleterious toll as early reading difficulties (Duncan et al., 2007; Morgan, Farkas, & Wu, 2009). For instance, students struggling to learn mathematics are ill-prepared for well-paying jobs in a modern, technological economy (National Academy of Sciences, 2007). Disparities in mathematical competency are evident between students from different racial and socioeconomic subgroups, impacting the life opportunities of a substantial portion of the population (Siegler et al., 2010). Moreover, mathematics difficulties are as persistent and difficult to remediate as reading difficulties (NMAP, 2008). In other words, just as early intervention in reading is critical, prevention of mathematics difficulties and effective early intervention should also be a primary focus of educational research and practice in mathematics.

Unfortunately, mathematics achievement in the USA is lagging. Results of the 2011 National Assessment for Educational Progress (NAEP) indicate that only 40% of fourth graders scored at or above *proficient* in mathematics, and nearly half of all fourth graders with a disability scored *below basic*. The percentage of students that demonstrate proficiency in mathematics also worsens over time (e.g., 35% of the eighth graders scored at or above *proficient* in mathematics in 2011). On international measures of achievement in fourth and eighth grades, the USA ranks ninth and twelfth, respectively, of approximately 50 countries participating in international benchmarking (Trends in International Mathematics and Science Study: TIMSS, 2011b). Although these rankings indicate students in the USA could be performing far worse, we are also failing to prepare students for the level of mathematics they may need, in order to acquire the 62% of American jobs that will require advanced math skills in the coming decade (Hanushek, Peterson, & Woessmann, 2010). Just 6% of the US students scored at the equivalent of the advanced level in mathematics on the Program for International Student Assessment (Organization for Economic Cooperation Development & Programme for International Student Assessment, 2007), while 30 other countries had a larger percentage of students scoring at this level out of 56 total countries that participated in the assessment (Hanushek et al., 2010). In sum, when it comes to ensuring the ability of our youth to successfully compete for jobs in an international marketplace that requires proficiency in mathematics for technological prowess, we are being outcompeted by a number of countries that do not share the same level of resources we possess in the USA (Hanushek et al., 2010; TIMSS, 2011b).

As competitors in an international marketplace, increasingly driven by technological innovation, it is imperative that US students acquire mathematical proficiency. Results from national and international assessments indicate that we, as a nation, have been inadequate in achieving this aim. The rest of this chapter emphasizes on the role of fluency in mathematical proficiency, discusses the types of interventions that are employed to promote mathematical fluency, and describes the assessment instruments used to measure mathematical fluency across grade levels.

Why Focus on Mathematics and Mathematical Fluency?

Despite a clear need to focus on mathematics education, the research based on the development of mathematical proficiency pales in comparison to extensive research that has been conducted in the area of reading (Clarke, Gersten, & Newman-Gonchar, 2010). But we can use what we know about the development of reading skills to inform our thinking about mathematics. For instance, there is broad consensus that foundational (e.g., phonological awareness) and higher order skills (e.g., vocabulary and reading comprehension) are critical areas of reading skill development that must be taught in concert. Congruently, mathematics experts agree that conceptual understanding (i.e., understanding mathematical ideas, the way they function, and the contexts where they apply) must be emphasized alongside efforts to teach procedural fluency, in an intertwined manner (NMAP, 2008; National Research Council [NRC], 2001).

There are also parallels between the types of skills that form the basis of understanding in reading in mathematics. We know, for example, that students learning to read must demonstrate phonemic awareness to have a solid understanding of the sounds that comprise language and become strong readers (National Reading Panel [NRP], 2000). In mathematics, to demonstrate proficiency, students must possess early numeracy skills (e.g., numeral identification, understanding one-to-one correspondence, and magnitude comparison) to understand relations between numbers and quantities (NRC, 2001). Although developmental trajectories in mathematics are often considered more linear (i.e., more advanced skills build directly upon basic skills over time) than the trajectories described for reading development (e.g., students apply similar reading skills in each grade to different types of texts that increase in difficulty as students make progress), the parallels between reading and mathematics in the types of skills and the need to simultaneously emphasize foundational and higher order thinking can inform efforts to improve mathematical proficiency.

Perhaps because research about the development of mathematical proficiency is relatively nascent, there is also substantially less evidence about effective practices for teaching mathematics when compared to our knowledge about effective practices for teaching reading (NMAP, 2008). However, we can learn from the research that has been conducted on reading instruction and intervention in several ways. First, as a result of No Child Left Behind (NCLB), there is increased emphasis on comprehensive systems of support to assist all students in meeting rigorous standards of achievement by 2014. Research in reading has informed the types of assessments (e.g., screening and progress monitoring) and scaffolded supports (e.g., Tier 2, Tier 3 interventions) that comprise these multitiered systems. Similarly, the Institute of Education Sciences (IES) Practice Guide, *Assisting Students Struggling* *with Mathematics: Response to Intervention for Elementary and Middle Schools* (Gersten et al., 2009), was written to provide guidance to schools and districts looking to establish Response to Intervention (RTI) systems of support in mathematics, using the best evidence available for interventions and assessments. The IES Practice Guide provides support that an RTI approach may be an effective mechanism for supporting the mathematical proficiency of all students.

As momentum shifts toward building service delivery systems of support and identifying the interventions that work to improve students' mathematical proficiency, increased attention has been given to the content that should comprise these interventions. Research on instruction and intervention in mathematics indicates there are key concepts (akin to the five "big ideas" in reading: Coyne, Zipolo, & Ruby, 2006) that should be targeted to support students' proficiency. These key concepts include a focus on whole number concepts in the elementary grades, and an emphasis on rational numbers beginning in fourth grade to support algebra readiness, and other critical foundations of algebra, including key topics in geometry and measurement (Gersten et al., 2009; NMAP, 2008). A number of states have sought to adopt the Common Core State Standards for Mathematics (CCSS-M, 2010). The CCSS-M are widely vetted standards that rest on the NCTM (2000) process standards (i.e., problem-solving, reasoning and proof, communication, connections, and representation) and the principles outlined by the National Research Council (2001) in their volume, *Adding It Up* (i.e., understanding, computing, applying, reasoning, and engaging). The CCSS-M is built on the consensus of experts that conceptual understanding and procedural fluency are critical constructs within mathematics topics, across grades. Recognizing the importance of fluency, one of the eight recommendations in the IES Practice Guide is to "devote about 10 min in each session to building fluent retrieval of basic arithmetic facts" in interventions at all grade levels (Gersten et al., 2009). That is, across grades, experts indicate a need for students to develop automaticity with whole and rational number operations.

Not surprisingly, the bulk of this chapter focuses on the importance of fluency in mathematics; however, we are not advocating that "fluency" is promoted at the cost of conceptual understanding, nor that fluency carries a narrow definition. In fact, we agree with the NCTM (2000) that "developing fluency requires a balance and connection between conceptual understanding and computational proficiency" (p. 35). In addition, we describe how mathematical fluency supports mathematical proficiency for students with learning disabilities (LD) and their typically developing peers, in terms of working memory demands and cognitive load theory.

What is Mathematical Fluency?

There is overwhelming support from cognitive scientists, researchers, and educators alike that fluency in mathematics supports mathematical proficiency, and should be a focus in Grades K–12 (e.g., NCTM, 2000; NMAP, 2008; NRC, 2001). Traditionally, fluency has been defined in terms of computational proficiency, or being able to quickly and accurately recall basic math facts and procedures. However, this narrow definition does not take into account the relation between conceptual understanding, procedural knowledge, and basic fact recall, and the notion that demonstrating mathematical fluency requires an awareness of these interconnections. Baroody (2011) defines fluency as the quick, accurate recall of facts and procedures, *and the ability to use them efficiently*. That is, as students develop procedural fluency, it is essential that mastery be tied to conceptual understanding to promote adaptive expertise. In other words, students need to know when they can use an algorithm and when they cannot, in order to demonstrate mathematical fluency. We contend, as do others (e.g., Fennell, 2011) that fluency is a broad construct, which refers to proficiency across mathematical domains (e.g., early numeracy, whole number concepts, rational number concepts, and algebra).

How Does Mathematical Fluency Support Mathematical Proficiency?

Mathematical fluency provides access to mathematical proficiency through several hypothesized mechanisms. As evidenced by the results of national assessments, students with LD tend to struggle in mathematics to a greater degree than their nondisabled peers (e.g., 2011 NAEP results). Research demonstrates that students with LD in mathematics typically struggle to attain fluency with basic number combinations and simultaneously demonstrate working memory deficits that may be contributing to these "developmental differences" in computational proficiency (Geary, 1996; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007). Students who struggle to automatically retrieve basic number combinations often work more slowly and tend to be more error prone when attempting more complex mathematical problems (Geary, 2004; Jordan & Montani, 1997). Furthermore, fluent basic number combination retrieval has been linked to successful word problem completion, presumably due to reduced working memory demands (Geary & Widaman, 1992; Geary, 2004). For the 5–8% of the students with LD in mathematics, it appears working memory deficits may be inhibiting mathematical fluency, and contributing to generalized difficulties in developing mathematical proficiency.

Working memory may also play a broader role in mathematical proficiency for a range of learners, where students who score lower on a range of working memory tasks demonstrate increased difficulty in mathematics (Raghubar, Barnes, & Hecht, 2010). Several studies have demonstrated that working memory skills predict mathematical fluency and problem-solving, even when controlling for cognitive variables, including attention, intelligence, and phonological processing (Fuchs et al., 2005; Swanson & Beebe-Frankenberger, 2004). In addition, research indicates a range of factors (e.g., age, language, and math representations) may interact with working memory to predict mathematical skill, including mathematical fluency (Raghubar et al., 2010).

There is also support that fluency in mathematics frees cognitive resources for higher order reasoning activities. In their seminal article on reading fluency, LaBerge and Samuels (1974) argued that human beings can only actively attend to only one thing at a time, thus learners can only do more than one thing at a time if one of the tasks can be performed automatically. Although initially applied to issues of reading performance, these conclusions are relevant to a discussion of the role of fluency in mathematics performance, as well. Advocates for mathematics interventions that train students to become more fluent at targeted mathematics tasks posit that being fluent with mathematics tasks reduces the learner's cognitive load and frees cognitive resources for more complex tasks (Geary, 2004; Geary & Widaman, 1992; Jordan & Montani, 1997).

Broadly, because foundational knowledge and skills unlock the door for understanding of higher order concepts, students who struggle to develop mathematical fluency will struggle to demonstrate mathematical proficiency across their schooling years, with the normative gap growing over time. Take, for instance, the student that is slow and methodical in performing math procedures and recalling number combinations. In elementary school, this student may simply require more time to complete instructional activities. However, when this student encounters a course in Algebra in middle or high school, she may have difficulty understanding daily lessons, because she cannot keep up with the pace of instruction (e.g., even though she understands the procedures, working memory deficits may be preventing access to new concepts). Alternatively, the student may struggle to learn new concepts because she is exhausting cognitive resources solving algorithms (e.g., the cognitive demand of both solving algorithms and learning new algebra concepts is overwhelming in combination). Regardless of the source of the deficit, it is clear that students who are unable to demonstrate fluency in mathematics will fall farther behind their peers who do not struggle with mathematical fluency. Mathematical fluency is, thus, a key ingredient for mathematical proficiency, achievement, and, ultimately, access to life opportunities.

Fluency-Based Interventions in Mathematics

Although mathematical fluency is a key skill for successful mathematics achievement, general mathematics interventions often do not focus primarily on fluency. Instead, general mathematics interventions tend to target concrete mathematical knowledge and skills such as number sense, algorithms, vocabulary, and proofs. However, as noted in the previous section, mathematics fluency interventions that train students to become more fluent at targeted mathematics tasks are important because possessing mathematical fluency frees students' cognitive resources for more complex tasks. If students struggle to automatically retrieve basic number combinations, they will work more slowly and make more errors when solving complex mathematics problems, whereas students who are fluent in basic number combination retrieval are able to complete word problems more accurately.

In school-based settings, versions or elements of fluency interventions are often implemented class wide in the elementary grades as students are expected to master all basic number combinations to 100. In Grades 6–12, fluency interventions are typically utilized in small group settings as part of specialized academic programs. When implementing targeted mathematics fluency interventions, interventionists aim to improve students' cognitive processes and resources that underlie fluent mathematical performance. However, it can be difficult to determine whether or not improved mathematical performance as measured by fluency assessments discussed in the following section is truly an indicator of cognitive development and not simply an artifact of rote memorization. Ultimately, all fluency interventions operate on the premise that improving mathematical fluency is fundamental to overall improvement in mathematics performance. Mathematics fluency interventions can be categorized into three general types: (1) those that utilize repeated trials with multiple forms to train students to become more fluent at a specific task; (2) interventions that target underlining academic and cognitive skills to teach students generalizable strategies that result in improved mathematics fluency; and (3) general mathematics interventions that include fluency skill-building components to impact both basic number combination proficiency and conceptual fluency.

In the following section, we describe each type of intervention, provide examples of interventions within each category that have been used in research and practice, and summarize research that has been conducted to evaluate the effectiveness of each type of intervention. We follow with a discussion of the challenges related to evaluating the generalizability of mathematics fluency interventions and conclude with summative recommendations to consider when selecting a mathematics fluency intervention.

Repeated Trials Fluency Training

For many years, researchers and educators have advocated for the use of repeated daily timed mathematics activities to build fact fluency with elementary students by implementing a variety of training components to build rate and accuracy (Miller & Heward, 1992). As repeated practice is a key feature of fluency interventions, most protocol-based fluency training programs rely on discrete learning trials with numerous practice opportunities of the same mathematical material to build speed and accuracy in responding. Some sample programs are detailed in Table 3.1.

Much of the research base for these interventions originates in special education literature and utilizes single-case designs to isolate specific learning gains. Fluency-based interventions tend to target repeated measures of basic number combinations with elementary-aged students as the focal population. As these interventions are tested with a small number of learners, computing effect sizes and making generalized claims about the research findings can be challenging. Single-subject researchers compute effect sizes using techniques that compare the distinct characteristics of student performance in each phase of the study (e.g., pre- and post-intervention). By comparing data points across phases, researchers generate either percentage of nonoverlapping data (PND), interpreted as a percentage with values

Table 3.1 Sample repeated trial fluency-training interventions

Intervention	Description
Cover, copy, and compare (Skinner, Turco, Beatty, & Rasavage, 1989)	Five step process: (1) look at a model of the math fact with the answer included, (2) cover the math fact with the answer, (3) write the fact with the answer, (4) uncover the original math fact with the answer, and (5) compare
Incremental rehearsal (Burns, 2005)	A flashcard-based drill procedure that combines unknown facts with known facts
Taped-problems (McCallum, Skinner, & Hutchins, 2004)	Using a list of problems on a sheet of paper, the learner is instructed to answer each problem before the answer is provided by an audiotape player using various time delay procedures to adjust the intervals between the problem and answer (adapted from Freeman & McLaughlin's (1984) taped-words intervention)
Detect, practice, and repair (Poncy, Skinner, & O'Mara, 2006)	Multicomponent intervention: (1) metronome-paced, group assessment administered to identify unknown facts, (2) cover, copy, and compare procedures used with unknown facts, (3) 1-min speed drill, and (4) learners graph their accuracy
Math to mastery (Doggett, Henington, & Johnson-Gros, 2006)	Multicomponent intervention: (1) preview problems, (2) repeated practice, (3) immediate corrective feedback, (4) summative and formative feedback, and (5) self-monitoring of progress
Great leaps math (Mercer, Mercer, & Campbell, 2002)	Multistage strategy: (1) greeting and set behavior expectations, (2) review previous facts and progress graph, (3) conduct instructional session with short-timed practice, error correction, and teaching, (4) administer a 1-min fluency probe, (5) graph accuracy

larger than 70% considered meaningful, or a metric called percentage of all nonoverlapping data (PAND) and convert this value to a Phi coefficient (φ) that serves a measure of effect size (Parker & Hagan-Burke, 2007; Parker, Hagan-Burke, & Vannest, 2007). Phi is intended to represent the effect of an intervention and can be interpreted with a rule of thumb where values ≤ 0.20 are considered small, 0.21 through 0.79 represent a medium effect, and ≥ 0.80 are considered large. It should be noted however, that because φ is directly tied to the number of data points collected in each phase of a single subject study, the potential values of φ are unbounded and it is not uncommon to generate extremely large values when studies have a large number of nonoverlapping data points.

In recent years, meta-analyses have been conducted to compare the treatment effects of various fluency interventions in mathematics and other academic areas (e.g., Codding, Burns, & Lukito, 2011; Joseph et al., 2012). By grouping interventions according to the fundamental strategy employed or by the general treatment component utilized (e.g., drill, practice with modeling, and self-management), researchers have been able to compare categories of interventions. Perhaps not surprisingly, meta-analytic findings suggest that interventions employing flashcard-based drill activities (e.g., incremental rehearsal) and practice sessions with a modeling component (e.g., math to mastery and great leaps) have proven most effective, with mean φ values of 92.00 (extremely high) and 0.71 (moderately strong), respectively. Self-management strategies that require learners to monitor their own understanding (e.g., cover, copy, and compare) demonstrated moderate effect sizes (mean $\varphi = 0.55$ and mean PND = 60.2–70.7) proving productive as well (Codding et al., 2011; Joseph et al., 2012). However, fluency interventions that prescribed learner practice without a modeling component had little to no impact on student performance (mean $\varphi = -0.003$).

When evaluating additional characteristics of fluency interventions, meta-analytic results suggested that fluency approaches including multiple components with combinations of rehearsal, correction, and practice strategies demonstrated better learner outcomes. Specifically, interventions with more than three components had a moderately strong effect size (mean $\varphi = 0.68$) and those with less than three components had a negligible mean φ value (Codding et al., 2011). Additionally, coupling mathematics fluency interventions based on self-management strategies with other instructional components was found to be effective across numerous studies, mean PND = 87.9–97.5 (Joseph et al., 2012).

In addition to conventionally delivered fluency interventions, technology-delivered fluency interventions have become increasingly popular and prolific. Traditionally, computer-aided interventions utilized drill-based procedures providing repeated practice of basic number combinations, but technological advances have allowed intervention developers to incorporate a variety of effective practice and self-management strategies into technology-delivered fluency programs. Programs that present sets of basic number combinations from a specified numerical range (e.g., *flash card program* and *Math Blaster)* are freely available for download and have been used in research programs to compare their utility to peer tutoring and other drill-based procedures (Cates, 2005; Mautone, DuPaul, & Jitendra, 2005). These studies have generated mixed results, with some students responding well to technology-based interventions and others performing better in traditional intervention conditions. Studies of both downloadable basic number combinations programs and researcher-developed mathematics drill programs such as *Math Facts in a Flash* (Renaissance Learning, 2003) have dedicated particular attention to at-risk students for mathematics difficulties. Results of this research suggest that computer-based interventions may result in not only improved mathematical fluency, but also increased on-task behavior (Mautone et al., 2005; Burns, Kanive, & DeGrande, 2012).

When comparing fluency interventions and evaluating their effectiveness for specific populations, it may be that distinct learner characteristics are predictive of the likelihood of responding well to a particular intervention. Research in this area has found that initial level of mathematics fluency can be a significant predictor of intervention effectiveness (Codding et al., 2007), and meta-analytic findings have suggested that baseline levels of fluency (instructional or frustration) may be associated with differential intervention effectiveness when comparing interventions that either (a) aim to support basic number combination acquisition (acquisition), or (b) intend to bolster learner fluency with known facts (rehearsal) (Burns, Codding, Boice, & Lukito, 2010). More specifically, the results of this study suggested that initial fluency performance was significantly linked to intervention outcomes such that acquisition interventions were more effective for learners with a frustration baseline fluency level (mean $\varphi = 0.84$) compared to learners with an instructional baseline fluency level (mean $\varphi = 0.49$). These findings provide support for the argument that effective mathematics fluency interventions should be implemented with careful consideration of initial learner performance, and also suggest that one should consider the phases of mathematical fluency (e.g., acquisition or rehearsal) when selecting a fluency intervention.

Targeting Generalizable Skills and Behaviors

Although initial level of fluency is a logical predictor of a learner's response to a repeated trial fluency-training intervention, research has shown that a variety of additional cognitive and behavioral factors are also predictive of both mathematics fluency and general mathematics achievement (Geary, Hoard, Nugent, & Bailey, 2013). Based on these correlational findings, mathematics fluency intervention developers have created and studied programs that target learners' underlying cognitive traits and behavioral tendencies. Rather than directly training learners with repeated trials and regular exposure to basic number combinations, these interventions use mathematics fluency probes primarily as outcome measures and attempt to strengthen the learners' foundational skills by teaching generalizable strategies.

Advocates for generalizable skill (e.g., self-management, goal setting, self-evaluation) interventions argue that teaching students to utilize their cognitive resources more efficiently and effectively will not only translate into improved fluency, but improved general mathematics achievement as well. Research on behavioral self-management interventions has suggested that these strategies can improve both mathematics fluency and academic engagement, and generalize to more complex mathematical tasks (McDougall & Brady, 1998; Farrell & McDougall, 2008). Performance feedback and goal setting have also been studied as mathematics fluency interventions. Results from these studies have indicated that there is an association between goal setting and feedback-based interventions and improved performance on mathematics fluency measures (Codding, 2003; Figarola et al., 2008). The challenge in evaluating these interventions is that it can be difficult to isolate the link between improved fluency and the underlying cognitive and behavioral factors. As these interventions rely on the repeated administration of fluency probes to monitor student progress, one could argue that mathematics fluency improvements could simply be due to the additional fluency practice and residual testing effects resulting from regular fluency probe administration.

Mathematics Interventions with Fluency Skill-Building Components

Rather than targeting underlying skills through cognitive training intended to support performance on both basic and complex mathematical tasks, others advocate for allocating intervention resources to boost general mathematical knowledge based on its relation with both accuracy in basic number combinations and general mathematics skill development. For example, because number sense performance in kindergarten can predict later calculation fluency above and beyond cognitive factors (Locuniak & Jordan, 2008), researchers claim that early academic interventions support the acquisition of foundational skills that are pre- or corequisites of mathematics fluency. In addition to boosting the development of foundational academic skills, many general mathematics interventions include fluency-training components to build speed and accuracy with targeted mathematical material. In fact, researchers recommend that mathematics interventions include fluency exercises (Fuchs et al., 2008a; Gersten, Jordan, & Flojo, 2005), and there is a high prevalence of fluency components in successful mathematics intervention curricula (Bryant et al., 2008; Fuchs, Fuchs, & Hollenbeck, 2007; Ketterlin-Geller, Chard, & Fien, 2008; Jitendra et al., 2013). Results from intervention research conducted by Fuchs et al. (2008) suggested that efforts to improve general mathematics skills and performance on complex mathematical tasks should be supported by mathematical fluency skill building. Although, improving fluency is not the primary objective of most general mathematics interventions, computational fluency is considered an essential aspect of mathematical performance and often explicitly addressed in intervention curricula aimed at at-risk students.

Challenges in Establishing Generalizable Interventions

Although some have argued that fluency is an interwoven component of applied problem-solving (Lin & Kubina, 2005) and improved fluency is associated with improved performance on more complex tasks (VanDerHeyden & Burns, 2009), others have found that fluency does not generalize across mathematics problems or skills (Poncy, Duhon, Lee, & Key, 2010). Poncy and colleagues suggest that fluency instruction targeting basic declarative skills (i.e., basic number combinations) needs to be supplemented with instruction that supports the fluent completion of procedural, multistep tasks for fluency to generalize to overall mathematics performance. In sum, general research evidence suggests that for the mathematics fluency interventions to be optimally effective, they should utilize a variety of strategies to train learners to be more fluent with basic number combinations and be integrated into the general mathematics instructional program to support skill transfer and generalization.

The variety of mathematics fluency intervention approaches speaks to the lingering debate about the generalizability of fluency and the role of automaticity with foundational material in facilitating advanced mathematical achievement. The debate about the role of fluency in mathematics parallels similar debates about the nature of the relation between fluency and comprehension in reading. Few years ago, Slocum, Street, and Gilbert (1995) found that interventions that proved effective at increasing reading rate had unreliable impacts on reading comprehension. They also noted challenges related to (a) identifying sensitive outcome measures of general reading performance and (b) the experimental design of the study when attempting to examine the mechanisms that link fluency and general reading achievement. Similar challenges abound in mathematics fluency research. Additional research is needed to investigate the mechanisms that link mathematical fluency and overall mathematics performance and determine how one can isolate intervention techniques that target rate of responding (considered true fluency) from repeated exposure or additional practice, two common features of fluency interventions that can increase overall mathematics performance on their own regardless of whether or not the interventions improve general fluency proficiency (Doughty, Chase, & O'Shields, 2004). The effect of mathematics fluency training interventions is evaluated by comparing pre- and posttests of student performance on basic number combination probes, but it can be difficult to isolate the source of those gains. Improved performance on fluency probes is often assumed to be evidence of improved rate of responding, but could also be the result of increased knowledge or the simple acquisition of basic number combinations alone. Effective assessments of mathematical fluency are critical to identifying factors of effective interventions and simply measuring student progress. In the next section, we will examine how mathematical fluency is measured and the role that fluency plays in mathematics assessment.

Fluency and Mathematics Assessment

The relation between fluency and mathematics assessment is complex. At first glance, the complexity of this relation is not readily apparent. In simple terms, a large number of commonly used mathematics assessments are timed, and a timed measure seems to imply that the measure functions as a fluency measure. However, a more in-depth examination of commonly used mathematics measures reveals a more dynamic relation between the construct of fluency and mathematics assessment.

To fully explore the role of fluency in mathematics assessment, we first examine the original development of widely used measures that are considered to be fluency-based mathematics assessments and their intended use in educational decision-making. We follow by providing an overview of measures currently in use and conclude with a discussion examining critical unanswered questions to which we feel the field should be attuned as we attempt to advance in both research and practice.

How Are We Measuring Fluency?

The construct of fluency in mathematics assessment is typically examined within the realm of a set of measures broadly classified as curriculum-based measures or CBM. Math CBM (M-CBM) measures have a long history and the general CBM category includes an expanding set of instruments used for a variety of purposes by schools such as screening, program evaluation, and monitoring student growth (Deno, 2003; Deno & Mirkin, 1977). Originally M-CBM measures focused on a student's understanding of computation objectives and application of conceptual understanding to problem-solving for the elementary school grades. But now the umbrella of M-CBM measures includes an array of measures designed to cover student development in mathematics from beginning number sense in the early elementary grades (Gersten et al., 2012) to a student's understanding of pre-algebra in middle school (Foegen, 2008). Across this spectrum, content-assessed ranges from students comparing the magnitude of two one-digit numbers to combining like integers. Yet across this vast range of mathematics content one central feature remains prevalent—a timing element. But why is a timing element a common universal feature of almost all M-CBM measures?

The design of M-CBM measures was governed by a multitude of considerations including the content assessed and technical characteristics (Deno, 2003). But serious consideration was also given toward the practical application of their use in schools. Because the original intent of CBM measures was to monitor the growth of at-risk students to gauge their response to instructional interventions and modifications (i.e., progress monitoring), the measures needed to have certain design characteristics that enabled them to be administered frequently and repeatedly over time (Deno, 1985). It was this consideration that played a major role in the inclusion of a timing element. Seminal articles detailing the use and design features of the measures were linked to their need to be used in a repeated fashion and the importance of efficient measures to meet that goal.

Typically, an M-CBM battery consists of two measures; a computation measure that covers major topics in the standards relating to computation, and a concepts and applications subtest that assesses all other topics including word problems, measurement, money and time, and geometry. While the computation and concepts and applications approach to M-CBM measures has long been utilized, a new theoretical framework has been advocated and initially researched that explores the possibility that math disabilities can occur in one of the two areas or both simultaneously (Fuchs, Fuchs, & Zumeta, 2008). M-CBM measures demonstrate acceptable test–retest, inter-rater and alternate-form reliability, and concurrent and predictive validity between .50 and .60 (Foegen, Jiban, & Deno, 2007). The timing of the measures varies by grade level with shorter durations (1 or 2 min) in the earlier grades, and up to 5 min for the later grades.

Although originally M-CBM measures were designed to align with actual curricula (i.e., the C in CBM stood for a specific curriculum) over time new iterations of M-CBM measures were designed to align to specific state standards (Gersten et al., 2012) and other similar but non-timed measures were aligned to foundational documents such as the National Council of Teachers of Mathematics Focal Points (2006; Clarke et al., 2011). This trend has specific implications for future measurement development as more contemporary standards, such as the Common Core, are adopted and implemented. Other advancements in the use of M-CBM have focused on extending the use of M-CBM-like measures to the early elementary and middle school grades. In the next section, we detail developments in those age and grade ranges.

Fluency-Based Measures Assessing Number Sense

At the early elementary grades (kindergarten and first grade) fluency-based measures are designed to tap into a student's beginning and developing number sense. Although the concept of number sense is widely accepted it has been elusive to operationalize. It has been postulated as a corollary to phonological awareness and described by Gersten and Chard (1999) as "a child's fluidity and flexibility with numbers, the sense of what numbers mean, and an ability to perform mental mathematics and look at the world and make comparisons" (p. 19). Other researchers have noted the complexity of attempting to define number sense but at the same time attempted to begin articulating exactly what is number sense (Berch, 2005).

> Possessing number sense ostensibly permits one to achieve everything from understanding the meaning of numbers to developing strategies for solving complex math problems; from making simple magnitude comparisons to inventing procedures for conducting numerical operations; and from recognizing gross numerical errors to using quantitative methods for communicating, processing, and interpreting information. (p. 334).

The complexity in defining number sense is often encapsulated by the wide range of specific number proficiencies put forth as indicating an underlying understanding of number. That is, although there is a general consensus on what number sense is, the specific proficiencies that capture number sense are varied. The National Research Council's (2009) *Mathematics Learning in Early Childhood* recognized the inherent difficultly in operationally defining number sense and noted that any attempt to measure number sense would likely focus on assessing key proficiencies (e.g., applying number properties or counting strategies to solving addition and subtraction problems and simple word problems). Thus, while measures developed to assess number sense would assess specific proficiencies, the larger goal was for the measure to tap into the underlying construct of number sense. Despite the complexity and difficulty in measuring number sense through examining specific skills, a number of assessments have been developed. Typically, these assessments focus on key constructs of beginning number sense.

In the next section, we detail and summarize that work focusing on three components of number sense judged to be critical by cognitive psychologists and education researchers: magnitude comparison (Booth & Siegler, 2006), strategic counting (Geary, 2004), and basic fact fluency (Jordan, Hanich, & Kaplan, 2003). The overview is not intended to suggest other aspects of mathematics development and number sense are not critical (e.g., solving word problems) or to suggest that other timed measures have not been developed to assess number sense or math readiness (e.g., numeral identification) but rather to focus on those measures tapping critical constructs and do so in a manner that is focused on a student's fluency with the construct. It should also be noted that although the measures and constructs reviewed focus on a specific skill, the original development of the measures mirrors that of early CBM development in that the goal is to provide a powerful indicator of a student's broader understanding of the domain. Thus, while a measure may have a student complete a specific number sense or mathematics task (e.g., noting the missing number in a sequence of numbers) the measures are intended to provide an indicator or overall level of understanding.

Magnitude Comparison

Magnitude comparison is made up of a number of specific skills but fundamentally it is based on the ability to draw comparisons about relative magnitude. Magnitude comparison can include the ability to determine which number is the greatest in a set and to be able to weigh relative differences in magnitude quickly and accurately. For example, initially children may know that 5 is bigger than 2 and then begin to understand that 7 is also bigger then 2 and that the difference between 7 and 2 is greater than the difference between 5 and 2. As children advance to developing a more nuanced understanding of number and quantity, they are able to make increasingly complex judgments about magnitude. In the earlier grades, the development of an understanding of magnitude is a critical underpinning of the ability to calculate.

It has been hypothesized that as children develop a greater understanding of magnitude, they map that understanding onto a mental number line and begin to use that mental number line to further understand magnitude and to solve initial calculation problems (Dehaene, 1997). For example, when a student is presented a problem to add 4 and 2, a student who can recognize 4 as the greater magnitude can then solve the problem by counting up 2 (this example also implies an understanding of the commutative property and the use of strategic counting) on a mental number line to derive a correct answer.

Typically, measures of magnitude comparison require a student to identify the greater number from a set of two numbers. A number of research teams have designed and tested similar measures of magnitude comparison for kindergarten and first grade with all measures including a timing element but varying the range of numbers used in the materials in response to potential concerns about floor or ceiling effects. For example, some measures uses number sets from 0 to 10 for kindergarteners (Lembke & Foegen, 2009; Seethaler & Fuchs, 2010), while others use 0–20 (Clarke et al., 2011).

A recent overview of screening measures in the early grades (Gersten et al., 2012), noted strong reliability coefficients across studies of examining magnitude comparison measures. Evaluations included interscorer, alternate-form, and test–retest, all of which reported coefficients consistently greater than .80, and concurrent and predictive validity data correlating with summative measures of mathematics falling mostly in the .50–.70 range.

Strategic Counting

Strategic counting is fundamental to developing mathematical understanding and proficiency and has been defined as the ability to understand how to count efficiently and to employ efficient counting strategies to solve an array of problems (Siegler & Robinson, 1982). Students who fail to develop strategic counting and to utilize counting principles efficiently to solve problems are more likely to be classified as having a mathematics learning disability (Geary, 1994). As children develop strategic counting strategies they are more able to efficiently solve addition and subtraction problems by applying this knowledge in combination with a growing understanding of number properties. For example, a child who understands counting up (e.g., 5+2 can be solved by counting up from 5) and the commutative property (i.e., a+b=b+a) can apply the min strategy (counting up from the larger addend) so if given a problem "what is 6 more than 3?" she will solve the problem by changing the problem to "what is 3 more than 6?" and simply count on from 6 to derive the answer.

The most common strategic counting measures require students to determine the missing number from a sequence of numbers. Similar to magnitude comparison measures, strategic counting measures include a timing element and vary the range of numbers used based on the grade level to avoid floor or ceiling effects. Some researchers have begun to experiment with measures that require skip counting (e.g., filling in the blank in a number series, 5, 10, __, 20) (Lembke & Foegen, 2009) An overview of strategic counting measures found moderate concurrent and predictive validities (range=.37–.72) and strong reliabilities (range from .59 to .98) (Gersten et al., 2012).

Retrieval of Basic Arithmetic Facts

An established finding in the research based on mathematics disabilities has been that students who are diagnosed as mathematics LD exhibit consistent and persistent deficits with the automatic retrieval of addition and subtraction number combinations (Goldman, Pellegrino, & Mertz, 1988; Hasselbring et al., 1987). Geary (2004) found that children with difficulties in mathematics typically fail to make the transformation from using simple strategies to solve problems (e.g., by counting on their fingers or with objects) to solving problems mentally without using these objects (also Jordan, Kaplan, Ramineni, & Locuniak, 2009).

Research trends seem to indicate that, although students with mathematics LD often make progress in their use of algorithms when provided with classroom instruction, significant deficits remain in their ability to retrieve basic number combinations (Geary, 2004; 2001; Jordan et al., 2003). A number of theories have been put forth to explain these difficulties. Geary (2004) hypothesized that the difficulty was related to issues with semantic memory (i.e., the ability to store and retrieve abstract information efficiently). Jordan et al. (2003) hypothesized that fact-retrieval difficulty was rooted in weak number sense, and that when students lack number sense and an understanding of the relations between and among numbers and operations they fail to develop automaticity with addition and subtraction number combinations. Whatever the root cause of difficulty with addition and subtraction number combinations, they remain a powerful predictor of later mathematics achievement (Jordan et al., 2009). Initial research on number combination or fact fluency measures shows promise in the early elementary grades (first and second grade; Bryant, Bryant, Gersten, Scammacca, & Chavez, 2008; Gersten, Clarke, Dimino, & Rolfhus, 2010).

Fluency-Based Measures in the Middle School Grades

As students advance to the middle school grades, new CBM-like measures have been designed to assess critical concepts of algebra (Foegen, 2008), problem-solving (Montague, Penfield, Enders, & Huang, 2010), and estimation (Foegen & Deno, 2001). Similar to M-CBM computation and concepts, and applications measures for the same grade range, these new CBM-like measures provide more time (e.g., 5 min) for students to work. The complexity of mathematics skills assessed by upper-grade measures brings into question how well we can assess mathematics using a timed measure. Consider one of the algebra measures developed by Foegen (2008) designed to assess, among other features, the following basic skills in algebra: applying the distributive property, working with integers, combining like terms, and simplifying equations. Whether or not a timed measure (and of what duration) is the best approach to assessing this content is a legitimate question along with considering how untimed or measures with a longer duration fit into different types of assessments (e.g., screening and progress monitoring).

In part, the issue of timing represents the larger issue of whether or not a timed measure is also a fluency measure. Given that the original purpose of developing CBM measures was to provide an initial gauge of student understanding in a topic and a long-term analysis of growth in that topic, one could argue that not all of the measures reviewed in this chapter are fluency measures. However, given that all the measures do assess how quickly and accurately a student applies specific skills (whether for 1 min or for 5), they do assess fluency. The answer likely lies between those two positions in that the measures provide useful information in both providing an indicator of overall student understanding and a student's fluency with greater overlap between the two in the earlier grades.

Conclusion

The concept of fluency and its importance is well established and accepted in the field of mathematics. Seminal documents on mathematics instruction readily acknowledge the role of fluency in the development of student proficiency in mathematics (NMAP, 2008; Gersten et al., 2009). Perhaps in a proactive attempt to avoid the "reading wars" that have plagued the field of reading instruction, the mathematics field has been more overt and proactive in advocating for viewing fluency in conjunction with the development of conceptual understanding (NMAP, 2008). That is, fluency and conceptual understanding are both of importance and that growth in one fuels increased growth in the other rather than one aspect of mathematics being developed at the cost of another (Wu, 2005).

Given the general acceptance of fluency's importance, continued evaluation of existing and development of new interventions specifically designed to impact fluency seems likely. We consider the development and research efforts reviewed in this chapter as a solid foundation for further work. We believe going forward two important considerations should guide the field. First, if researchers provide only a fluency intervention and evaluate the impact of that intervention with a measure that is closely aligned to the intervention, caution should be exercised when interpreting results. In particular if that measure is considered by the field to provide an overall index of understanding in the broader domain of mathematics. For example, an intervention may focus exclusively on building fluency in identifying the greater of a set of numbers and use a measure of magnitude comparison to examine impact. But because the intervention is specifically targeted on magnitude comparison, increased scores on a measure of the same content may not reflect a generalized improvement in the underlying domain of number sense. Second, given the high probability that low levels in fluency are accompanied by deficits in other areas of mathematics, fluency interventions should rarely be delivered in isolation. That is, students who struggle with fluency in mathematics need a comprehensive intervention that includes, but is not limited to, addressing fluency-related problems. This position is not to say that isolated intervention and research conducted to date lacks importance, it is rather to acknowledge that students with severe deficits in mathematics need an intervention of an intensity equal to their deficits and that likely involves a sustained effort to build conceptual understanding of critical mathematics concepts.

Lastly, developers of current and future measures of mathematics that include a timing element should be proactive in laying forth what constructs they are measuring and how they view the development and use of their measures. A cautionary tale from reading illustrates the point. When Reading First advanced the framework of five big ideas of beginning reading instruction, including accuracy and fluency with connected text (Baker, Fien, & Baker, 2010), states and districts viewed this framework as specifying a need to measure each big idea. The previous role of oral reading fluency as a measure of overall reading health was to some extent replaced with oral reading fluency serving only as a measure of accuracy and fluency with connected text despite the continued evidence that oral reading fluency continues to be validated as a strong measure of general reading achievement including comprehension (Fuchs et al., 2001). Thus, if developers and researchers design and view their mathematics measures as assessing student understanding in a broader domain but a timing element is also included, they should be proactive in discussing and demonstrating the link between their measure and greater understanding in mathematics.

We believe that efforts in all of these areas will help further our understanding of the role of fluency in developing mathematics proficiency. As we advance our understanding, we believe that the field will be better positioned to ensure that all children achieve success in mathematics.

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