

# Chapter 12

## Smoke Stratification

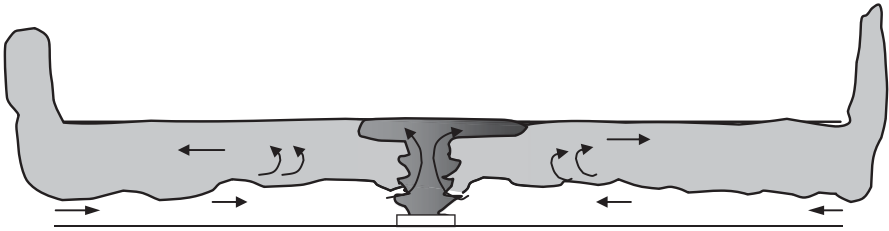
**Abstract** The phenomena and formation mechanisms of smoke stratification and engineering solutions to estimate smoke stratification in tunnel fires are described. Smoke stratification is an important issue for evacuation and fire fighting in tunnel fires. Smoke released from a fire contains some hazardous combustion products. If the smoke stratification in a tunnel section dissolves, tunnel users in this region could be in great danger. In no ventilation or very low ventilation conditions, smoke exists on both sides of the fire, and good stratification could exist at the early stages but generally not after the fire becomes larger. For a ventilation velocity slightly lower than the critical velocity, smoke backlayering and good stratification exist upstream of the fire, however, the smoke stratification downstream becomes worse. Under high ventilation rates, all the smoke flows towards the downstream side and stratification is difficult to maintain even at a short distance downstream. The theory of the smoke movement along the tunnel is introduced. An empirical model of smoke stratification in tunnels with longitudinal ventilation is also presented.

**Keywords** Stratification · Ventilation velocity · Entrainment · Smoke layer height · Smoke backlayering · Simple model · Froude number

### 12.1 Introduction

Smoke stratification is dependent on the longitudinal ventilation velocity and the buoyancy forces created by the fire. Due to the buoyancy forces, the hot smoke flows upward and occupies the upper region of a tunnel cross section. Therefore, a clear stratification may exist in some cases.

Smoke released from a fire is a combination of combustion products and air. The four factors affecting tenable conditions are asphyxiant fire gases, irritant fire gases, heat, and visual obscuration. More detailed information on these parameters is given in Chaps. 14 and 15. All of these factors are intimately related to smoke, with the exception of flame radiation in the vicinity of a fire source. In the early stages of a fire the heat could be a minor problem, but high concentrations of some combustion products such as carbon monoxide and other toxic gases could easily cause deaths. In any case, after the smoke descends to the head height of the tunnel



**Fig. 12.1** A sketch showing the smoke stratification in a tunnel with a very low air velocity, typically 0–0.5 m/s for group 1

users, people inside the region will be in great danger. In engineering applications in building fires, maintaining the smoke layer height above head height is a key task. In tunnel fires, this issue is even more important due to the limited possibilities for evacuation and often unfamiliar surroundings for the users.

The phenomena and formation mechanisms of smoke stratification are described, and engineering solutions to estimate the smoke stratification in tunnel fires are presented in the following sections.

## 12.2 Phenomenon of Smoke Stratification

In tunnel fires, the characteristics of the smoke spread are highly dependent on the air velocity inside the tunnel, especially in the vicinity of the fire site. In order to illustrate this, we can identify three typical air velocity ranges (groups):

- low or no forced air velocity (0–1 m/s),
- moderate forced air velocity (1–3 m/s), and
- high forced air velocity (> 3 m/s).

Note that the values of 1 and 3 m/s inside the parentheses are only approximate values. Specifically, 3 m/s is an estimate of the critical velocity under which no backlayering exists. The critical velocity in tunnel fires is defined as the minimum longitudinal ventilation velocity required to prevent any backlayering of smoke. The concept of critical velocity is discussed thoroughly in Chap. 13. The critical velocity in a tunnel fire could in reality be greater than 3 m/s for a large fire or less than this value for a small fire.

In the *first group* (low air velocity) the stratification of the smoke is usually high in the vicinity of the fire source. Tunnels with natural ventilation normally fall into this group. The backlayering length of the smoke is relatively long and in some cases the smoke travels nearly uniformly in both directions, see Fig. 12.1. When the velocity increases and is close to about 1 m/s the smoke upstream of the fire is inhibited by the ventilation and prevented from further spreading, and the length of this backlayering smoke layer from the fire site could be of the order of 25 times the tunnel height, see Fig. 12.2. The estimation of the backlayering length can be found in Chap. 13 on tunnel fire ventilation, and is therefore not discussed further here.

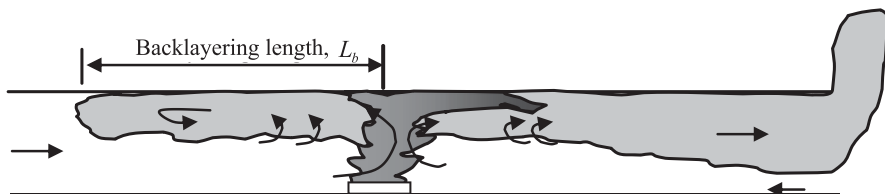


Fig. 12.2 A sketch showing the smoke stratification with a flow velocity of 1 m/s

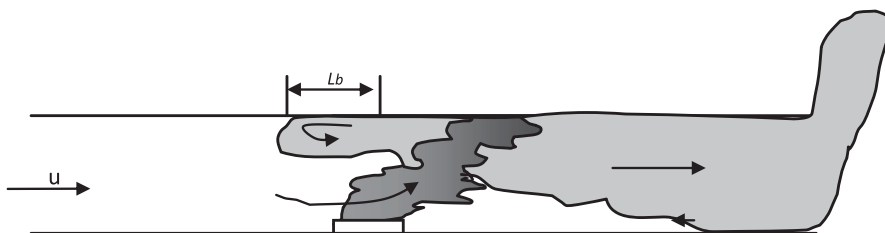


Fig. 12.3 A sketch showing a typical smoke stratification for group 2

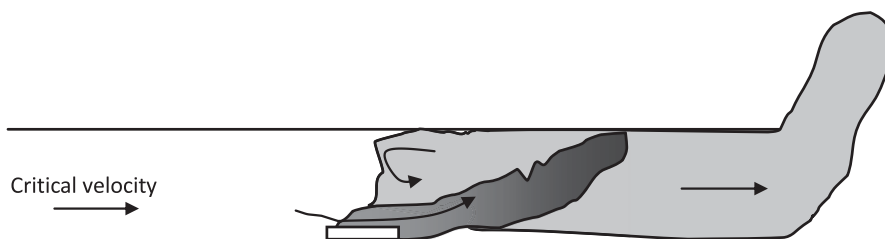


Fig. 12.4 A sketch showing a typical smoke stratification for group 3, i.e. a flow velocity larger than the critical flow velocity,  $u_c$

In the *second group* (moderate flow) the stratification in the vicinity of the fire is strongly affected by the air velocity, especially at the higher velocities. Tunnels with natural ventilation or forced ventilation can reside in this group. The backlayering length could vary from 0 up to around 25 times the tunnel height, see Fig. 12.3.

In the *third group* (high flow velocity) the stratification of the smoke downstream usually disappears and no backlayering exists upstream of the fire, see Fig. 12.4. This group generally corresponds to tunnels with forced ventilation [13].

The stratification downstream from the fire is a result of the mixing process between the cold air stream and the hot plume created by the fire. The phenomenon is three-dimensional in the region close to the fire plume. The principal pathways of the two flows are indicated in Fig. 12.5. The gravitational forces tend to suppress turbulent mixing between the two flows having different densities.

This explains why it is possible for cold unreacted air to bypass the fire plume without mixing, even though the flow is turbulent. The longitudinal dimension of

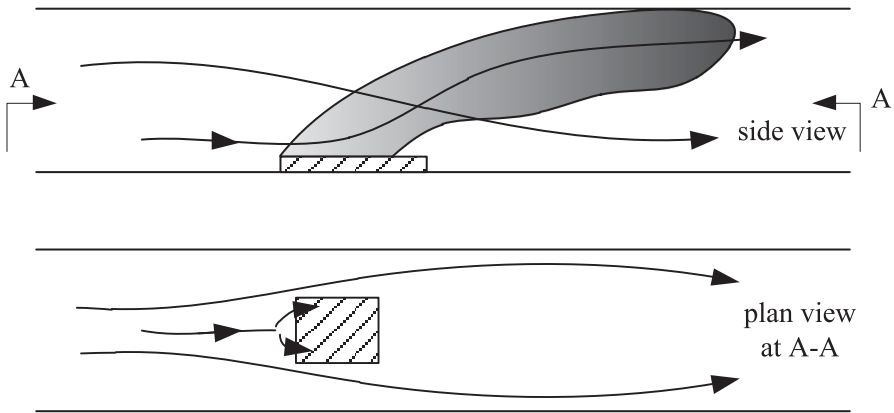


Fig. 12.5 The principal flow pathways in the vicinity of a fire plume [13]

the fuel involved in the fire may therefore, play an important role for the mixing process between the longitudinal flow and the fuel vapours generated by the fire.

### 12.3 Mechanism of Smoke Stratification

Stratification is a common phenomenon, for example, floating oil on water. Hot smoke flows have lower density which produces a pressure difference between the hot gas and ambient air, that is, thermal pressure. Due to the thermal pressure or buoyancy force, the hot smoke flows upward, impinges on the tunnel ceiling, and then flows along the tunnel ceiling longitudinally. As the smoke travels along the ceiling, the smoke temperature decreases rapidly with distance mainly due to heat loss to the tunnel structure. This indicates that the thermal pressure also decreases with distance. Therefore, smoke stratification becomes more and more difficult to maintain as the distance from the fire increases.

Inertia forces also play a role in formation of stratification. Consider a cold air jet beneath the tunnel ceiling at ambient temperature. There will be no heat transfer between the air jet and the homogenous environment. However, as the distance from the injection outlet increases, the air jet continually entrains air from the layer beneath and the thickness of the layer increases gradually. This suggests that the inertia forces tend to destroy the stratification of the layer.

This means that the thermal pressure tends to maintain the smoke stratification but the inertia force tends to destroy the smoke stratification. A nondimensional parameter to describe the balance between these two forces is the global Richardson number,  $Ri$ , defined as:

$$Ri = \frac{\Delta\rho gh}{\rho\Delta u^2} \quad (12.1)$$

where  $\rho$  is the hot gas density ( $\text{kg/m}^3$ ),  $\Delta\rho$  is the density difference ( $\text{kg/m}^3$ ),  $g$  is the gravitational acceleration ( $\text{m/s}^2$ ),  $h$  is the layer thickness (m), and  $\Delta u$  is the gas velocity difference between the layers (m/s). The characteristic depth,  $h$ , in fact should be the depth of the mixing layer, however, for smoke flow beneath the ceiling it is reasonable to use the layer thickness instead. It can be expected that as the Richardson number increases, the smoke stratification becomes more stable.

### 12.3.1 Entrainment

An important phenomenon observed in density stratified flows is the entrainment between the layers. For upper layer smoke flows in tunnel fires, it can be expected that the entrainment of air flows into the upper smoke layer is mainly due to turbulent mixing between the two layers, that is, the entrained air is mainly carried by the large vortices produced by turbulence.

The entrainment coefficient based on the mixing theory is introduced in the following set of equations. By applying mixing length turbulent theory [1] to the mixing layer between the hot smoke layer and the lower layer, it is known that the turbulent mixing velocity,  $u_t$  (m/s), can be expressed as:

$$u_t = Cl_m \frac{\partial u}{\partial z} \quad (12.2)$$

where  $C$  is a proportionality constant,  $l_m$  is the mixing length (m) and  $z$  is the vertical distance (m). The entrainment velocity,  $v_e$ , can be expected to approximate the turbulent mixing velocity. Further, given that the mixing length is proportional to the thickness of mixing layer, that is,  $l_m = 0.07\delta$  [1], the entrainment velocity,  $u_e$  (m/s), can be expressed as follows:

$$u_e = 0.07C\delta \frac{\Delta u}{\delta} = 0.07C\Delta u \quad (12.3)$$

The entrainment coefficient,  $\beta$ , can therefore be expressed as:

$$\beta = \frac{u_e}{\Delta u} = 0.07C \quad (12.4)$$

The entrainment coefficient has been extensively investigated in different scenarios in the last several decades. The entrainment coefficient for isothermal jet flows is close to constant, however, for density stratified flows, buoyancy affects the entrainment. The entrainment coefficient for the stratified layer has been found to be a function of the Richardson number,  $Ri$ .

There is a large amount of research on the entrainment of stratified layers in the field of fluid dynamics.

Ellison and Turner [2] carried out a classic study on mixing between a turbulent fluid and a stationary fluid and correlated the entrainment coefficient  $\beta$  with the Richardson number in the following relationship:

$$\beta = \alpha_1 \exp(-\alpha_2 Ri) \quad (12.5)$$

where  $\alpha_1$  could be considered as the entrainment coefficient for vertical fire plumes and  $\alpha_2$  is a correction coefficient. Alpert [3] found that the above equation with  $\alpha_2 = 3.9$  is a good fit to Ellison and Turner's [2] data, that is, the expression can be [4, 5]:

$$\beta = 0.12 \exp(-3.9 \text{ Ri}) \quad (12.6)$$

Based on a numerical study and a comparison with test data, Alpert [3] proposed alternative equations for the entrainment coefficient:

$$\beta = 0.075 \exp(-5 \text{ Ri}) \quad (12.7)$$

and in the form of distance:

$$\beta = 0.12[1 - \exp(-0.6H/x)] \quad (12.8)$$

You and Faeth [4] carried out a numerical study of ceiling fire plumes and compared numerical results to their test data. The equation for entrainment has the same form as Ellison and Turner's [2] but with different coefficients. They found that the following equation best test results best for weak plume [4]:

$$\beta = 0.14 \exp(-1.5 \text{ Ri}) \quad (12.9)$$

Ding and Quintiere [6] pointed out that the density ratio also needed to be accounted for and corrected Alpert's equation [3] to:

$$\beta = 0.075 \rho / \rho_o \exp(-5 \text{ Ri}) \quad (12.10)$$

where  $\rho_o$  is the density of the gas in the lower layer and  $\rho$  is the smoke density.

The equations presented above are basically of the same form as proposed by Ellison and Turner [2]. However, the expression proposed by Ellison and Turner [2] is only valid for  $\text{Ri} < 0.8$  [7]. Further, nearly all the equations presented above date back to Ellison and Turner's [2] liquid experiments. For entrainment into hot smoke flows, especially into flame flows, the validity of these equations must be confirmed. According to Fernando [7], for intermediate Richardson numbers ranging from 0.1 to 10, the entrainment law was proposed to be  $\beta \sim \text{Ri}^{-1}$  and within this range the Kelvin–Helmholtz instabilities are active.

Wilkinson and Wood [2] investigated the possible density jump in density stratified flows and related it to a critical Froude number below which entrainment will not occur, based on which Delichatsios [8] analyzed smoke flow under a beamed ceiling. However, according to Kunsch's study on smoke movement in tunnel fires [9], no density jump occurred in the tunnel fire cases that were studied, although further study was recommended. Fernando [7] conducted a systematic review of turbulent mixing in stratified flows. The analysis showed that for a Richardson number ranging from 0 to 100, the entrainment coefficient ranged from  $10^{-1}$  to  $10^{-5}$ . This appears to be contrary to the definition of density jump. It could be interpreted that, as the Richardson number increases to a certain number the entrainment will decrease to a very low level which could be ignored.

For smoke flows in tunnel fires, the smoke flow could be very different from the ceiling flow in the open or in a short corridor due to the confinement of the tunnel geometry and ventilation. The entrainment in these scenarios needs to be further investigated.

### 12.3.2 Smoke Layer Height

Although this chapter discusses smoke stratification in tunnel fires, the variance of smoke layer height along the tunnel is more meaningful. In tunnel fires, the smoke released from a fire impinges on the ceiling and then flows along the ceiling, and the depth of smoke flow generally increases with distance from the fire. In reality, the descent of the smoke layer is the outcome of a combination of mass entrainment, momentum loss, and heat transfer.

The profile of gas velocity and density for smoke flows is complicated and could differ from one location to another. For simplicity, top hat profiles are assumed for these parameters. For smoke flow in a tunnel with natural ventilation, the time-resolved differential controlling equations for mass, momentum, and energy can approximately be expressed as:

Mass:

$$\frac{\partial}{\partial t}(\rho A) + \frac{\partial}{\partial x}(\rho u A) = \rho_o u_e W \quad (12.11)$$

Momentum:

$$\frac{\partial}{\partial t}(\rho u A) + \frac{\partial}{\partial x}(\rho u^2 A) + \frac{\partial}{\partial x}\left(\frac{1}{2} \Delta \rho g h A\right) = -\frac{1}{2} C_f \rho u^2 w_p \quad (12.12)$$

Energy:

$$\frac{\partial}{\partial t}(\rho A c_p T) + \frac{\partial}{\partial x}(\rho u A c_p T) = \rho_o u_e W c_p T_o - h_t w_p (T - T_w) \quad (12.13)$$

where  $A$  is the cross sectional area of smoke flow ( $\text{m}^2$ ),  $t$  is the time (s),  $x$  is a distance along the tunnel length axis (m),  $h$  is the smoke layer depth (m),  $h_t$  is the total heat transfer coefficient ( $\text{kW}/(\text{m}^2 \text{K})$ ),  $w_p$  is the wet perimeter of the smoke layer (m),  $C_f$  is the skin friction coefficient,  $T_w$  is the wall temperature (K).

It is evident from the momentum equation that the definition of the Richardson number in fact is the ratio of buoyancy to momentum in a steady state. The controlling equations imply that besides the initial conditions, the three factors that affect the smoke layer height are heat transfer, wall friction, and entrainment. Note that the influence of heat transfer and momentum on stratification has been discussed previously and integrated into a global Richardson number.

For a steady smoke flow, the mass flow rate along the tunnel,  $\dot{m}$  (kg/s), can be approximated using:

$$\dot{m}(x) = \dot{m}_o + \beta W \int |u - u_o| dx \quad (12.14)$$

where  $u_o$  is the air velocity at lower layer (m/s) and  $\beta$  is the entrainment coefficient between the layers.

As a rough approximation, the entrainment coefficient could be assumed to be around 0.01 based on the equations presented previously and the resulting mass flow should be conservative.

The tunnel length is an influencing factor, especially after the smoke front reaches the exit.

The smoke layer depth at a position  $x$  m from the fire before the smoke descends to the floor level can be estimated using:

$$h(x) = \frac{\dot{m}(x)(T_o + \Delta T(x))}{\rho_o T_o u(x) W} \quad (12.15)$$

where  $\Delta T$  is smoke excess gas temperature (K).

The smoke layer height can be expressed as:

$$H_{smoke}(x) = H - h(x) \quad (12.16)$$

Note that the tunnel height is the upper limit for the smoke layer depth.

The initial conditions of the ceiling jets must be known, however, there is a lack of this knowledge for tunnel fires, especially for large fires in longitudinal ventilated tunnels. Further, in a tunnel fire with longitudinal ventilation, the controlling equations become more complicated. Moreover, as mentioned in the previous section, the validity of the entrainment equations needs to be confirmed in such an environment before any practical use.

Therefore, the purpose of this section is mainly to improve understanding of the mechanisms of smoke stratification and smoke layer height. Further research on this topic is needed.

## 12.4 Simple Model of Smoke Stratification in Tunnels

Although the theoretical model and the related key parameters have been discussed in the previous section, the solution is complex and some of the key parameters required for the solution cannot be accurately estimated.

As an alternative, a simple model is presented in this section for estimation of smoke stratification in tunnels according to Newman [10] and Nyman and Ingason's work [11].

Newman [10] has shown, for duct fires, that there is a correlation between the local temperature stratification and the local mass concentration of chemical compounds. Further, Ingason and Persson [12] have shown that there is a correlation between local smoke optical density (or visibility) and the local density (or temperature) and the oxygen concentration in tunnels. Therefore, it is reasonable to assume that there is a correlation between the local temperature stratification as given by Newman and the gaseous composition (CO, CO<sub>2</sub>, O<sub>2</sub> etc) and smoke stratification in tunnels. The temperature stratification is not only related to the air velocity but also to the heat release rate (HRR) and the tunnel height. These parameters can be related through the local Froude number ( $Fr$ ) or Richardson number ( $Ri$ ).



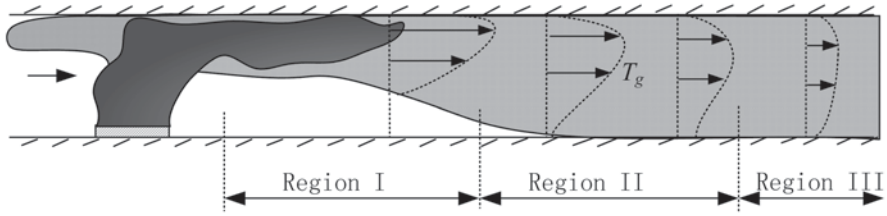


Fig. 12.6 A sketch of three stratification regions after Ingason [13]

Newman [10] presented a very simple method to identify three distinct regions of smoke stratification in terms of a specific Froude number,  $Fr$ , which is defined as follows:

$$Fr = \frac{u_{avg}^2}{\sqrt{gH\Delta T_{cf}/T_{avg}}} \quad (12.17)$$

where  $H$  is the tunnel height (m),  $T_{avg}$ , the average gas temperature over the entire cross-section at a given position (K),  $\Delta T_{cf} = T_c - T_f$ , gas temperature difference between ceiling and floor (K) and  $u_{avg} = uT_{avg}/T_a$  (m/s). The average temperature at a specific location can be estimated using the one dimensional equations proposed in Chap. 8, Sect. 8.6, on gas temperatures. The definition of the Froude number has a similar physical meaning to the Richardson number defined previously: both of them correlate buoyancy force with inertia force somehow, but in an inverse way to each other.

A sketch of the temperature stratification regions is shown in Fig. 12.6. The vertical temperature profile varies significantly from Region I to Region III. Newman's classification for smoke stratification [10] is described in the following. In the first region (Region I), when  $Fr \leq 0.9$ , there is severe stratification in which hot combustion products travel along the ceiling. For Region I, the gas temperature near the floor is essentially ambient. This region consists of buoyancy dominated temperature stratification. The second region (Region II)  $0.9 \leq Fr \leq 10$  is dominated by strong interaction between imposed horizontal flow and buoyancy forces. Although not severely stratified or layered, it has vertical temperature gradients and is mixing controlled. In other words, there is significant interaction between the ventilation velocity and the fire-induced buoyancy. The third region (Region III),  $Fr > 10$ , has insignificant vertical temperature gradients and consequently insignificant stratification.

Newman [10] also established correlations for the excess temperatures near the ceiling at  $0.88 \times H$  (ceiling) and temperatures near the floor at  $0.12 \times H$  (floor) for the different regions of stratification, which are based on a weighted average of the gas temperature ( $T_{avg}$ ) and the gas velocity ( $u_{avg}$ ) over the entire cross section. He postulated that these correlations could be used for various applications, such as in assessing flame spread and detector response. These correlations have not been tested for tunnel fires and are therefore not presented here. The majority of the tests used to identify these stratification regions were performed in a large scale (rectan-

gular) duct measuring 2.4 m wide ( $B$ ), 2.4 m high ( $H$ ), and 47.6 m long ( $L$ ) with  $D_h/L = 19.8$  m, where  $D_h$  is the hydraulic diameter. The hydraulic diameter is defined as  $\sqrt{4A/P}$  where  $A$  is the cross sectional area ( $\text{m}^2$ ) and  $P$  is the perimeter (m).

To estimate the smoke stratification according to the Froude number defined above, the temperature difference between the ceiling and floor must be known. Nyman and Ingason [11] investigated Newman's temperature correlations based on a large amount of data from small- and large-scale tunnel fire tests. It was concluded that Newman's equation for ceiling temperatures did not fit the large-scale data, and a new equation in Region II was proposed for estimation of the temperature difference between the ceiling and floor,  $\Delta T_{cf}$  (K), which can be expressed as:

$$\Delta T_{cf} = 0.225 \frac{gH\Delta T_{avg}^2}{T_{avg}u_{avg}^2} \quad (12.18)$$

The average temperature and average gags velocity in the above equation can be estimated using Eqs. (8.45) and (8.48). Afterward, the temperature difference between the ceiling and floor can be estimated using Eq. (12.18). The calculated values can thereafter be used to calculate the Froude number by Eq. (12.17) and then the smoke stratification can be estimated for any specific location downstream of a fire.

It should be kept in mind that there is no clear distinction between Region II and Region III. A Froude number of 10 and ratio of  $\Delta T_{cf}/\Delta T_{avg}$  of 0.1 is used by Newman [10] as an approximation. However, if the work done by Nyman and Ingason [11] is applied here and the same criteria of  $\Delta T_{cf}/\Delta T_{avg} = 0.1$  is used, the value of the Froude number at the Region II–III interface can then be determined as 3.2, compared to a value of 10 proposed by Newman. In general, the Froude number of 3.2 should be a more reasonable value for the interface between Region II and Region III in tunnel applications.

Therefore, the first region (Region I) corresponds to  $Fr \leq 0.9$ , results in severe stratification. Region II corresponds to  $0.9 \leq Fr \leq 3.2$  where strong interaction between imposed horizontal flow and buoyancy forces exists. Region III corresponds to  $Fr > 3.2$  where smoke stratification is insignificant. For practical use, the calculated Froude number should be less than 0.9 to ensure severe stratification, that is,

$$Fr \leq 0.9 \quad (12.19)$$

Further, it should be kept in mind that the gas temperature equations proposed in this section are purely empirical without any physical meaning, which, therefore, cannot be used in estimation of ceiling gas temperatures and instead should be used only for estimation of the Froude number.

### Example 12.1

What is the stratification region after 10 min (600 s) at  $x = 150$  m from the fire location? The longitudinal ventilation is 2 m/s, ambient temperature  $T_o = 20^\circ\text{C}$ , the tunnel geometry is  $H = 6$  m and  $W = 9$  and the fire grows linearly and reaches a peak HRR of 120 MW after 10 min.

*Solution:* First the simple method proposed in Sect. 8.6 to correct the time is applied here, that is,  $\tau = t - x/u_o = 600 - 150/2 = 525$  s, which means  $\dot{Q}(\tau) = 105$  MW. Thus,

By using Eq. (8.46), the average temperature at  $x=0$  becomes:  $T_{\text{avg}}(x=0, \tau) = 560^\circ\text{C}$  where  $\dot{m}_a = 1.2 \times 2 \times 6 \times 9 = 130 \text{ kg/s}$ . The reader must note that the corresponding ceiling temperature may be much higher than  $560^\circ\text{C}$  ( $\sim 1000^\circ\text{C}$ ), but since this is an average bulk temperature used as input for determine the Fr number it is acceptable. The average temperature 150 m from the fire at time  $t=600 \text{ s}$  can be calculated using Eq. (8.45):  $T_{\text{avg}}(x=150, t=600 \text{ s}) = 247^\circ\text{C}$  where  $h = 0.025 \text{ kW}/(\text{m}^2 \text{ K})$ ,  $P = 30 \text{ m}$ ,  $x = 150 \text{ m}$ ,  $\dot{m}_a = 130 \text{ kg/s}$ ,  $T_o = 20^\circ\text{C}$  and  $c_p = 1 \text{ kJ}/(\text{kg K})$ . The temperature difference is  $\Delta T_{\text{cf}} = 123^\circ\text{C}$  by Eq. (12.18). Thus, the Froude number can be determined using Eq. (12.17), that is,  $Fr = 3.4 > 3.2$ . This corresponds to Region III, that is, smoke stratification is insignificant. It could be expected that at this moment the whole tunnel is full of smoke and evacuation here is very difficult.

## 12.5 Summary

Smoke stratification is an important phenomenon for evacuation and fire fighting in tunnel fires. Smoke released from a fire contains some deadly combustion products. If smoke stratification disappears, people in the region could be in great danger.

The phenomenon of smoke stratification in a tunnel fire is illustrated. Under no ventilation or very low ventilation, smoke exists on both sides of the fire, and good stratification could exist at the early stages but generally not after the fire becomes larger. For a ventilation velocity slightly lower than the critical velocity, smoke backlayering and good stratification exist upstream of the fire. However, the smoke stratification downstream becomes worse. Under high ventilation, all smoke flows toward downstream and stratification is difficult to maintain even at a short distance downstream of the fire.

The mechanism of the smoke stratification is closely related to the global Richardson number, which indicates the stability of the smoke layer. Entrainment is a key mechanism that causes the descent of the smoke layer. The entrainment velocity is proportional to the velocity difference between layers.

The time-resolved controlling equation for smoke movement is proposed. From the controlling equations, it is known that besides the initial conditions, the three factors that affect the smoke layer height are heat transfer, wall friction, and entrainment. At present, there is a lack of knowledge on initial conditions for ceiling jets and the applicability of the entrainment equations, especially for large fires in longitudinal ventilated tunnels. Further, in a tunnel fire with longitudinal ventilation, the controlling equations become more complicated. Moreover, as mentioned in the previous section, the validity of the entrainment equations must be confirmed in such an environment before practical use.

A simple model of smoke stratification in tunnels is described in Sect. 12.4 which could be used for estimation of smoke stratification in tunnels with longitudinal ventilation.

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