Chapter 5
Implementation of the HELS-Based NAH

In this chapter we present the general guidelines for setting up measurement to get the desired accuracy and spatial resolution for the HELS-based NAH. There are several parameters that may influence the reconstruction results such as the number of measurement points, standoff distance, measurement aperture size versus source surface area, microphone spacing, and SNR. These parameters are generic for all NAH applications. The strategies for setting up the optimal measurement scheme are basically the same.

It is the hope of the present author that the proposed reconstruction guidelines and hybrid regularization strategy would help potential users to get an accurate reconstruction of the normal surface velocity for a non-spherical structure. This is because a straight application of the original HELS formulations can result in not-so-satisfactory reconstruction of the normal surface velocity. We believe that the HELS method possesses certain advantages for engineering applications that the Fourier acoustics- and BEM-based NAH do not. This is because the Fourier acoustics- and BEM-based NAH are based on the exact theories. They can provide an exact reconstruction of acoustic fields when the required conditions are satisfied, for example, separable source geometry for the Fourier acoustics and a source-free field for both Fourier acoustics and BEM-based NAH, but are invalid when these conditions are not met. Unfortunately, in engineering practice these conditions are seldom met. On the other hand, the restrictions on the HELS method are significantly relaxed because it only seeks approximate reconstruction and is suitable for patch reconstruction. Needless to say, the results obtained by using HELS are approximate and their accuracy depends on that of input data. So it is important to follow the proposed guidelines and strategies to obtain satisfactory reconstruction of all the acoustic quantities including the acoustic pressures, particle velocities, and acoustic intensities everywhere in three-dimensional space.

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5.1 Guidelines for Implementing the HELS Method

It is emphasized that the HELS method imposes no restrictions whatsoever on the use of the coordinate systems and the corresponding wave functions. The spherical coordinates and the spherical wave functions can yield good approximate solutions for a blunt radiator. The prolate, oblate, and elliptic coordinates and the corresponding spheroids can generate good approximate solutions for a slender, a flat, and an arbitrarily shaped radiator, respectively. Regardless of the coordinate systems selected, the expansion coefficients in HELS are specified by matching the assumed-form solution to the measured acoustic pressures, and the errors are minimized by the least-squares method and regularization. In practice, it is not easy to utilize the prolate, oblate, or elliptic spheroids because the corresponding analytic solutions do not exist. On the other hand, the spherical wave functions are readily available in any software library, making programming very straightforward and numerical computations very fast.

Although our ultimate goal is to extend HELS to reconstructing vibro-acoustic responses of an arbitrarily shaped structure, we begin investigation from a simple yet highly non-spherical surface, such as a baffled thin plate, and examine the reconstruction accuracy using the spherical wave functions that are readily available in most software tools (MATLAB, LabVIEW, etc.) [81, 82]. The choice of a baffled plate also allows for rigorous examinations on the HELS results because the corresponding analytic solutions are readily available.

Since the HELS-based NAH utilizes the expansion of certain basis functions, it is ideal if the geometry of a target source surface fits naturally with the basis functions. For example, if the spherical coordinates are used in the basis functions, then the HELS-based NAH will be naturally fit for reconstructing the acoustic quantities generated by a spherical source or blunt object, whose aspect ratio is close to 1:1:1. The accuracy in reconstruction will be quite high. If source geometry is different from a sphere or its aspect ratio is not 1:1:1, the HELS-based NAH is still applicable, but the accuracy in reconstruction may be compromised. The farther the source geometry is from the coordinates of the basis functions, the larger the errors will be in reconstruction. In practice, a vibrating object is usually of an arbitrary shape. In order to obtain satisfactory reconstruction using HELS, it will be a good idea to follow some tested guidelines.

In what follows, we consider a class of structures that are commonly used in practice, i.e., a plate, which is highly non-spherical and represents a serious challenge to the suitability of using HELS to reconstruct the resultant acoustic field in three-dimensional space. This is because there are additional factors that may affect reconstruction results, for example, location of the origin of the coordinate system. Since the thickness of a plate is usually negligible, there is no way of placing the origin at its geometric center. Thus it must be placed outside the planar surface, but where? How far should the origin of the coordinate system be? How far should the measurement aperture be? How large should the measurement aperture be?

It is emphasized that there are no analytic solutions to these questions because HELS is an approximation not an exact theory. Reference [74] gives a mathematical proof of the HELS method. For the exterior problems solutions are bounded in the three-dimensional domain Ω enclosed by the source surface Γ and a surface at infinity Γ_∞ (excluding the origin of the coordinate system), where the Helmholtz equation is satisfied. Moreover, HELS solutions converge logarithmically in Ω. There is no restriction on where the origin of the coordinate system should be placed. For a blunt object, it is natural to place the origin at the geometric center. For a thin plate, however, this is not possible. Our studies have illustrated that there is an optimum position for the origin of the coordinate system on the opposite side of the plate, which can produce satisfactory reconstruction results [83]. Experiments have validated the existence of such an optimal position. In addition, the number of the expansion terms and other parameters, namely, the number of measurement points, microphone spacing, standoff distances, measurement aperture sizes versus source surface areas, SNR, etc. are all important in implementing the HELS-based NAH.

Listed below are the guidelines for implementing the HELS-based NAH [84] to reconstruct the vibro-acoustic responses on the surface of a highly non-spherical surface (see Fig. 5.1).

1. Origin position d_{origin}: The inherent difficulty in HELS is to approximate the vibro-acoustic quantities on a highly non-spherical surface by using the spherical wave functions. If the origin of the coordinate system is placed too close to the surface, errors in reconstruction, though still bounded, can be quite large because the point of the origin is excluded in the region of validity for an exterior problem [85]. On the other hand, if the origin is placed too far from the surface, detailed features in the vibro-acoustic responses associated with the higher-order expansions in the HELS formulations may diminish, leading to discrepancies in reconstruction. Therefore, there exits an optimal position for placing the origin of the coordinate system. Since there are no analytic formulations for selecting the optimal origin of the coordinate system, numerical simulations are employed and results suggest that the optimal position d_{origin} falls within ± 10 % of the characteristic dimension of the plate D,

$$d_{\text{origin}} = (0.9 \sim 1.1)D, \tag{5.1}$$

where $D = 0.5\sqrt{L^2 + W^2}$, L and W are the length and width of the plate, respectively.
2. The critical spatial wavelength λ_{cr}: The spatial wavelengths of any vibrating structure are usually unknown, so λ_{cr} is a target value. Consider a rectangular plate of dimensions $L \times W$. Suppose that we aim at reconstructing up to the (n, m)th mode of this plate, where n is the modal index in the longitudinal direction with a dimension L and m is that in the transverse direction with a dimension W. Then $\lambda_{\text{cr}}/2$ is the smaller of L/n and W/m,

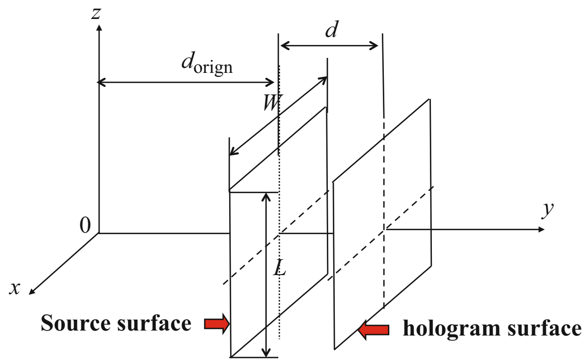

Fig. 5.1 Schematic of measurement setup for a plate type structure

$$\lambda_{cr}/2 = \min(L/n, W/m). \tag{5.2}$$

For example, consider a rectangular plate of dimensions 0.3 × 0.2 m^2 and the (4, 3)th mode of this plate is the highest mode to be reconstructed. The critical spatial wavelength is given by Eq. (5.2), where $n = 4$, $m = 3$, $L = 0.3$ m, and $W = 0.2$ m. Substituting these values into Eq. (5.2) yields $\lambda_{cr}/2 = 0.067$ m, or $\lambda_{cr} = 0.134$ m.

It is important to remember that the test setup in NAH is gauged with respect to the spatial wavelength or spatial frequency, not the acoustic wavelength or temporal frequency. If we can reconstruct structural waves up to the critical spatial wavelength λ_{cr}, the mechanical energies of all components of the structural waves up to the spatial wavelength λ_{cr} are captured. Since the vibration energy of a structural wave decays with spatial wavelength, we only miss a small portion of the total vibration energy that is the sum of structural waves whose wavelengths are shorter than λ_{cr}.

It is emphasized that the acoustic wave, regardless of its wavelength, is of no concern in NAH reconstruction. So long as the structural waves of wavelengths up to λ_{cr} are reconstructed, all acoustic waves can be reconstructed. For example, consider the plate as cited above. Suppose that structural waves of wavelengths up to $\lambda_{cr} = 0.134$ m are reconstructed. Then all acoustic waves whose wavelengths are shorter than 0.134 m or equivalently, the acoustic frequencies higher than $f > c/\lambda_{cr} = 340/0.134 = 2{,}537$ Hz can be reconstructed. The acoustic waves whose frequencies are lower than 2,537 Hz can always be reconstructed because the longest acoustic wavelength of the audible sound wave is always shorter than the longest spatial wavelength, i.e., the rigid body motion.

Note that if test setup is gauged with respect to the acoustic wavelength or temporal frequency, but not the spatial wavelength or spatial frequency, it will be acoustical holography, not NAH. Accordingly, one will only be able to reconstruct the radiated acoustic pressure and nothing else.

3. Number of measurement points M: This parameter is critical in practice. In theory, the more the measurement points are taken, the more information is

collected, and the more accurate the reconstruction becomes. However, an excessive number of measurement points may not be acceptable in practice. A compromise is to link the number of measurement points M to the required reconstruction surface area S through a target structural wavelength λ_{cr},

$$M > 4S/\lambda_{cr}^2 \quad \text{or} \quad M_{\min} = 44. \tag{5.3}$$

Note that we have imposed a minimal number of measurement points $M_{\min} = 44$ [84]. This is because for a small plate and a lower order of vibration mode, the value of M can be very small, leading to an omission of the critical information in the input data and large errors in a reconstruction. Once again, there is no analytic solution to determine $M_{\min}$. The value of $M_{\min} = 44$ is to ensure that the HELS expansion includes at least the fifth-order spherical Hankel function, namely, $n = 0$–5, in reconstruction, guaranteeing certain levels of details in the reconstructed vibro-acoustic responses.

4. Microphone spacing δ: Unlike the Fourier acoustics-based NAH, HELS does not require a uniform microphone spacing on the hologram surface. However, it is a good practice to set the microphone spacing to be less than one-half of the target structural wavelength λ_{cr} [86],

$$\delta < \lambda_{cr}/2. \tag{5.4}$$

5. Standoff distances d: The goal of NAH is to reconstruct vibro-acoustic quantities without the wavelength resolution limit in theory. This is possible when all the near-field effects are collected, which may be accomplished by placing microphones infinitely close to the target vibrating surface and infinitely close to each other. Such a scenario is unrealistic and unattainable in reality. Practical considerations such as the working condition, temperature, and accessory component attached to a structure require that microphones be placed at certain distances away from the structure. Thus, there is an upper limit in the spatial resolution in a reconstruction. To strike a balance between the theoretical goal and practical consideration, we recommend that the standoff distance d be less than one-eighth of the value of λ_{cr},

$$d < \lambda_{cr}/8. \tag{5.5}$$

Notice that there is an important distinction between the standoff distances for the Fourier acoustics-based NAH and those for the HELS-based NAH. The former utilizes the discrete spatial Fourier transform, and its accuracy in reconstruction is critically dependent on the spatial sampling frequency that is intimately related to the standoff distances. If the spatial sampling frequency is so low that the microphone spacing becomes larger than the standoff distance, "undersampling" may happen, causing spatial aliasing in a reconstruction. Hence the standoff distances in the Fourier acoustics-based NAH are kept at least one microphone spacing to avoid "undersampling" in data acquisition. The situations are quite different in the HELS-based NAH, where the acoustic quantities are reconstructed by superimposing the spherical wave functions. There is no direct correlation between the spatial resolution and spatial sampling frequency. In fact, the spatial resolution in HELS is directly related to the number of the spherical wave functions employed. In order for the high-order spherical wave functions to function the way they are supposed to, the standoff distances should be as close to the target surface as possible in order to collect enough near-field information. Experimental results have confirmed that the smaller the value d is, the more accurate the reconstruction is, regardless of the microphone spacing δ. This property of decoupling the measurement distance from microphone spacing is unique to the HELS method.

6. Target source surface S: Because the spherical wave functions are used in the HELS-based NAH to approximate the acoustic fields generated by non-spherical vibrating structures, it is a good idea to limit the overall size of a target source surface S so that reconstruction can be done all at once. Consider a plate of dimensions $S = L \times W$. We recommend that the length and width be no more than twice the target structural wavelength, namely,

$$L, W \leq 2\lambda_{\mathrm{cr}}. \tag{5.6}$$

This imposes some restriction on the overall dimensions of the structure that HELS may be attempted at once, but nevertheless leads to satisfactory reconstruction on a target surface. For surfaces whose overall lengths or widths are larger than $2\lambda_{\mathrm{cr}}$, patch reconstruction may be utilized. In performing patch reconstructions, the origin of the coordinate system should move with each patch, and the rest remains the same. The measurement aperture A_m must be at least one row and one column larger than a target reconstruction surface area S. Note that there is a difference between a patch measurement and patch reconstruction. The former refers reconstruction of the acoustic quantities on a portion of a large surface, whereas the latter indicates a specific measurement setup, which is often the case in practice for a finite number of microphones. For example, a specific reconstruction requires 100 measurement points, but only 20 microphones are available. Then reconstruction may be done by taking five patches of measurements sequentially.

7. Aspect ratio: For a planar surface, aspect ratio refers the ratio of its overall length to width. To reconstruct acoustic quantities on a planar surface of dimensions $S = L \times W$, its aspect ratio should be limited to the following range per reconstruction,

$$(1:1) \leq (L:W) \leq (2:1). \tag{5.7}$$

This is because the spherical wave functions and spherical harmonic functions are used in HELS to approximate the vibro-acoustic quantities on a planar surface. It may be difficult to ensure a satisfactory reconstruction over the entire surface area when the aspect ratio is larger than 2:1. For a planar surface with aspect ratio larger than 2:1, patch reconstruction should be used.

8. SNR: This parameter is universal to most measurement methods,

$$\mathrm{SNR} > 10(\mathrm{dB}) \tag{5.8}$$

Physically, this means that the energy or mean-squared acoustic pressure amplitude of the signal is at least ten times higher than that of background noise.

9. Number of reconstruction points N: There is no restriction on the number of reconstruction points, either on the source surface or in the field. However, for engineering applications it is recommended that vibro-acoustic quantities be reconstructed at four points per critical spatial wavelength λ_{cr} on the source surface to produce a smooth ODS. Excessive number of reconstruction points will not provide further information and should be avoided.
10. Number of the expansion functions J: Eq. (3.60) offers an effective way to estimate the optimal number of expansion functions $J_{\mathrm{op,MTR}}$ for reconstructing the acoustic pressure and normal velocity on the source surface. When $J_{\mathrm{op,MTR}} < M$, we have an overdetermined system. When $J_{\mathrm{op,MTR}} > M$, we have an under-determined system. Either way, the system of equations can be solved by SVD. However, an under-determined system tends to yield less satisfactory reconstruction results than an overdetermined system does. Therefore the maximal number of expansion terms is set to be equal to that of measurement points M.

Note that the above guidelines have accounted for the needs to simplify the measurement setup and data acquisition processes in engineering applications. In conducting research projects for which the accuracy of reconstruction is of primary concern, whereas time and effort are of no concern, some of the above guidelines may be tightened as needed.

For example, in our study the area of the square plate is $S = 0.22 \times 0.22 = 0.0484\ \mathrm{m}^2$, and the (4, 4)th natural mode of this plate is selected as the highest mode to be reconstructed. By using Eq. (5.2), we get $\lambda_{\mathrm{cr}}/2 = \min(0.22/4, 0.22/4) = 0.055$ m, so $\lambda_{\mathrm{cr}} = 0.11$ m. Next, we use Eq. (5.3) to determine the number of measurement points as $M > 0.0484/0.055^2 = 16$, which is smaller than 44, so we take $M_{\min} = 44$. By using Eqs. (5.4) and (5.5), we can set the microphone spacing at $\delta < \lambda_{\mathrm{cr}}/2 = 0.05$ m, which is less than 0.055 m as suggested by Eq. (5.4), and the standoff distance of $d = 0.0125$ m, which is shorter than 0.01375 m as suggested by Eq. (5.5).

These parameters would suffice to produce a quick reconstruction with a decent accuracy. However, we want to establish the accuracy in reconstructing the normal surface velocity by using the HELS-based NAH, and do not mind spending extra time and efforts in collecting input data. Therefore, we take three patches of measurements using a 12×4 microphone array, resulting in a total $M = 144$ measurement points over the measurement aperture A_m, which is one row and one column larger the surface area of the square plate S. The corresponding microphone spacing is set to be $\delta = 0.03$ m, which is less than the suggested microphone spacing ($\delta = 0.05$ m). Moreover, we set the standoff distance to be $d = 0.01$ m, which is less than the suggested standoff distance ($d = 0.0125$ m). This fine measurement grid and close measurement distance ensures the desired accuracy in reconstruction. The origin of the coordinate system is placed at $d_{\text{origin}} = 0.155$ m behind the plate as dictated by Eq. (5.1). The reconstruction results are exhibited in Figs. 3.6, 3.7, 3.8, 3.9, 3.10, and 3.11.

These results demonstrate that satisfactory reconstruction of the vibro-acoustic quantities on the surface of a highly non-spherical vibrating structure can be obtained by using the HELS method. In particular, the target (4, 4)th natural mode can be satisfactorily reconstructed.

5.2 Practical Considerations in Implementing the HELS Method

Noise and vibration abatement have always been one of the primary challenges facing the manufacturing industry, for example, the automobile, aircraft, and appliance manufacturers. The first step toward noise and vibration abatement is to identify their root causes, their interrelationships, and the key components that play the critical roles in generating undesirable noise and vibration.

The traditional technologies such as EMA [87] and operational modal analysis (OMA) [88] can provide an insight into the integrity of a vibrating structure by extracting its modal parameters that include the natural frequency, the natural mode, and the damping ratio. The knowledge acquired from EMA and OMA, however, may not be employed directly in noise abatement because these modal parameters are not related to sound radiation.

Traditional measurement devices such as microphones, intensity probes, and accelerometers enable one to measure the acoustic pressure, acoustic intensity, and normal surface velocity on specific locations that are very important to understanding the interrelationships between acoustic radiation and structural vibrations. However, the information captured in measurements is usually isolated and uncorrelated to each other. In other words, one can obtain a local and direct view of sound or vibration at a specific location, but not the global view of how sound is generated by a vibrating structure, and how it is correlated to structural vibrations.

Invention of the NAH technology has fundamentally changed the diagnostics and analyses of noise and vibration problems in that it enables one to visualize all acoustic quantities, including the acoustic pressure, particle velocity vector, acoustic intensity vector, and out-of-plane structural vibration distributions on the surface of a structure by taking acoustic pressure measurements at a very close distance. The insights acquired from NAH to the characteristics of vibrating structures and resultant acoustic radiation cannot be matched by traditional measurement methods.

Because NAH requires taking measurements at very close distances to a target structure to capture the near-field information, a conformal array of microphones is usually required. This conformal measurement will ensure that the standoff distance is uniform and the accuracy in the input data is consistent. The time and effort involved in setting up a conformal array can be quite intensive. Moreover, the measurement environment in engineering practice is usually not echo free, which means there are sound reflections and reverberation inside a test chamber. Accordingly, it is important to take these effects into consideration in order to get the desired reconstruction.

In addition to the general guidelines presented in Sect. 5.1, we offer further suggestions to optimize the measurement setups for various test configurations and test environments that are often encountered in engineering applications.

5.3 Test Configuration

Whenever possible, a conformal array should be utilized instead of a flat array. There is no doubt that a conformal microphone array will take time to make and set up. However, this is well worth the effort because the accuracy in reconstruction will be directly related to that in the input data. Accurate and consistent input data will lead to accurate and consistent reconstruction of all acoustic quantities in three-dimensional space, including the source surfaces. Figures 5.2 and 5.3 show schematics of conformal microphone arrays for reconstructing the acoustic fields generated by arbitrarily shaped vibrating structures in exterior and interior regions, respectively.

Oftentimes the source surfaces may be larger than the measurement aperture. Hence patch measurements are required. The number of measurement points as suggested in the guidelines is minimal for data acquisition. The number of reconstruction points, however, may be higher than that of measurement points. The suggested number of reconstruction points in HELS is up to but no more than four times that of measurement points.

Figures 5.4 and 5.5 illustrate, respectively, examples of using the HELS method to analyze the acoustic fields generated by an automobile transaxle in the exterior region, and by an aircraft inside its cabin while the aircraft was cruising at 0.8 Mach number 30,000 ft above the ground. The microphone array was mounted on a track so that it could travel along the longitudinal direction to measure the near-field acoustic pressure inside the cabin.

It must be pointed out that in engineering applications the demand for easy-of-use often overrides everything else. As a result, a planar array of microphones is used to collect the input data, even though a target source surface is nonplanar (see Fig. 5.6). As a result, the near-field information is completely lost because the measurement distances are varying and too large, and the reconstructed acoustic

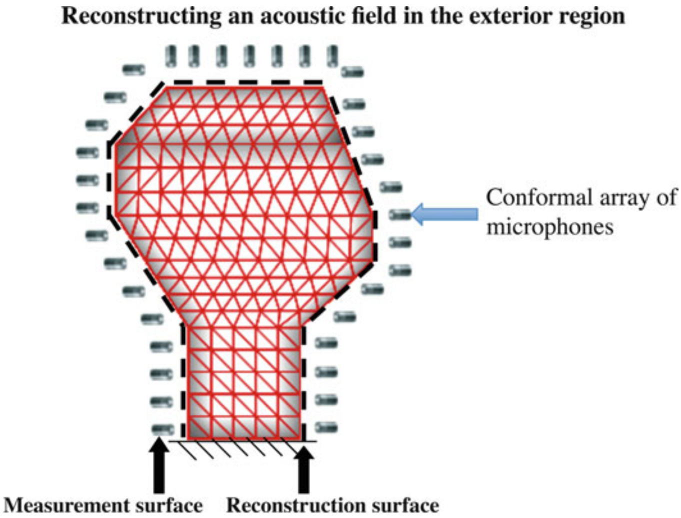

Fig. 5.2 Schematic of a conformal microphone array for reconstructing the acoustic field produced by an arbitrarily shaped source in the exterior region

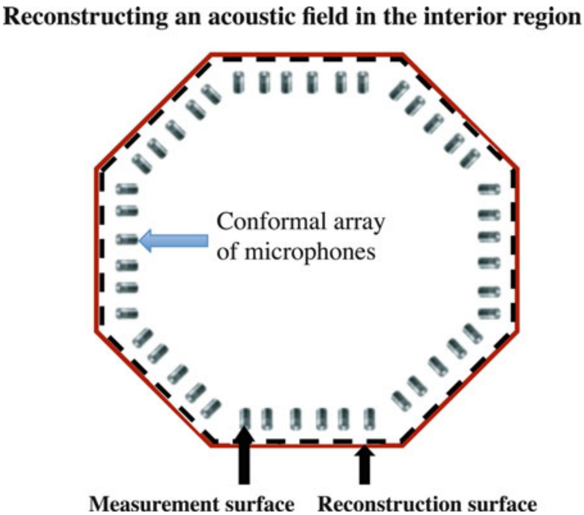

Fig. 5.3 Schematic of a conformal microphone array for reconstructing the acoustic field produced by an arbitrarily shaped source in the interior region

quantities are useless or even misleading. Therefore, even though this approach seems to save time and effort in data acquisition, it actually wastes all of them including those in post processing. The well-known statement in the field of computer science "Garbage in, garbage out" holds exactly true in this case.

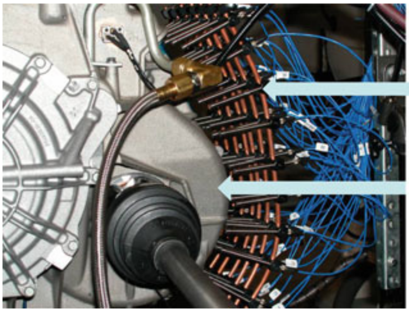

Fig. 5.4 A conformal array of measurement microphones around an automobile transaxle at very close distances to the target source surface

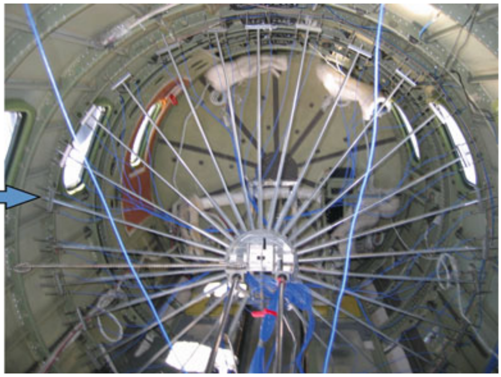

Fig. 5.5 A conformal array of measurement microphones around along the circumference of the interior space of an aircraft while it was cruising at 0.8 Mach number 30,000 ft above the ground. The microphone array was mounted on a track so that it could travel in the longitudinal direction to measure the nearfield acoustic pressure inside the cabin

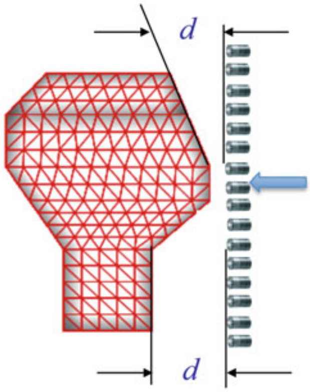

Fig. 5.6 Schematic of using a planar array of microphones to collect input data for an arbitrarily shaped vibrating structure. The near-field information is all lost in this case because measurement distances are varying and too large. As a result the reconstruction results are useless and even misleading

5.4 Test Environment

The HELS method is valid in both exterior and interior regions. In either case it is critical for the measurement surface to cover the entire source surface. If the source surface is relatively large, patch measurements should be taken. Covering a portion of the source surface may produce satisfactory reconstruction locally, but not acceptable globally.

Figure 5.7 displays a scenario of reconstructing the acoustic field using the HELS method in the exterior region. In this case the measurement surface A_m is on one side at close range. Hence, reconstruction may be acceptable on the covered source surface area, but not elsewhere.

Similarly, if a conformal array of microphones covers only a portion of the interior surface of a vibrating structure (see Fig. 5.8), $S = S_1 + S_2 + S_3$, the reconstructed acoustic quantities may be acceptable on the covered surfaces, but not on other surfaces, nor in the interior region.

Oftentimes we are dealing with a vibrating structure inside a large room (see Fig. 5.9), where the total acoustic pressure is the sum of the direct and reflected sound waves.

Under this condition, it will be critical to ensure that there is enough space between the source of interest and reflective walls, and SNR is at least 10 dB or higher in order to minimize the effects of reflected sound waves in data acquisition. If these conditions are met, the reconstructed results might be acceptable on the source surfaces covered by a conformal array of microphones. If these conditions cannot be met, reconstruction should not be carried out because the input data will be severely contaminated by the interfering sound signals.

This is because a reflecting surface behaves like an image source. Consider the case in which a source is situated on two infinitely large reflecting surfaces as shown in Fig. 5.10. Then the acoustic pressure measured in this confined space consists of the direct sound radiated from the source (ray 1) and those reflected from walls (rays 2–4). This is equivalent to the case where the source and its three images lie in free space, and the measured sound pressure will consist of the contributions

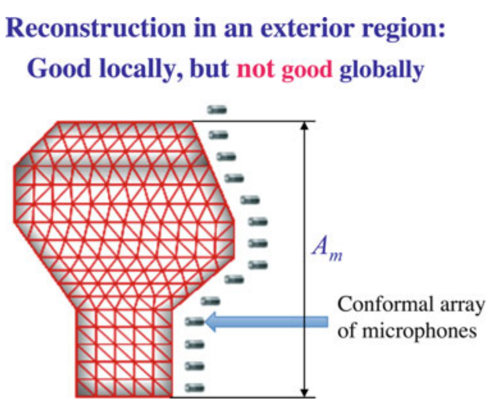

Fig. 5.7 Schematic of a conformal microphone array covering only a portion of the target surface *S* in the exterior region. Accordingly, the reconstructed acoustic quantities may be acceptable locally, but not globally

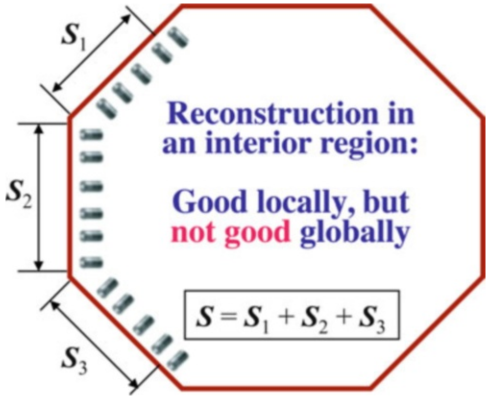

Fig. 5.8 Schematic of a conformal microphone array covering only a portion of the interior surface. The resultant reconstruction may be satisfactory on the cover surface area and immediately adjacent to it, but not satisfactory on other surfaces as well as in the interior region

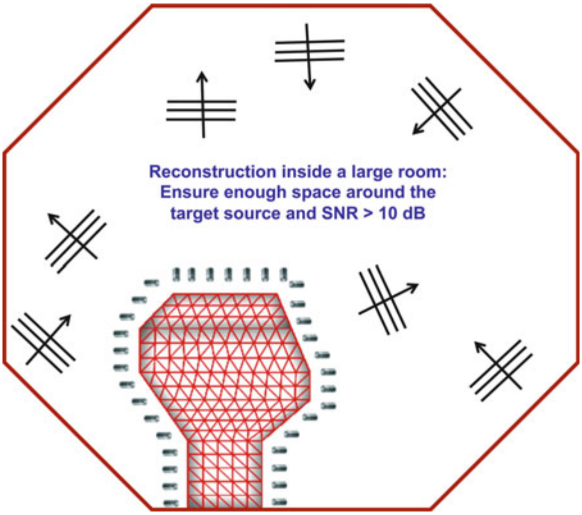

Fig. 5.9 Schematic of reconstructing the acoustic quantities generated by a vibrating structure inside a large room with reflecting surfaces. The total acoustic pressure is the sum of the direct and reflected sound waves

from all these sources. If a source is inside a room consisting four walls, one ceiling and one floor, then the measured acoustic pressure will consist of one direct sound wave emitted from the source and an infinite number of reflected sounds. Mathematically, this is expressible as

$$\hat{p}^{\,2}_{\text{rms}}(r,\theta,\phi;\omega) = \rho_0 c P(\omega)\left[\frac{Q_\theta}{4\pi r^2} + \frac{4}{R_{\text{rc}}(\omega)}\right], \tag{5.9}$$

where $\hat{p}^{\,2}_{\text{rms}}(r,\theta,\phi;\omega)$ indicates the measured mean-squared acoustic pressure inside a large room; $P(\omega)$ depicts the acoustic power, which is a function of frequency but independent of measurement location; Q_θ is the directivity factor given by

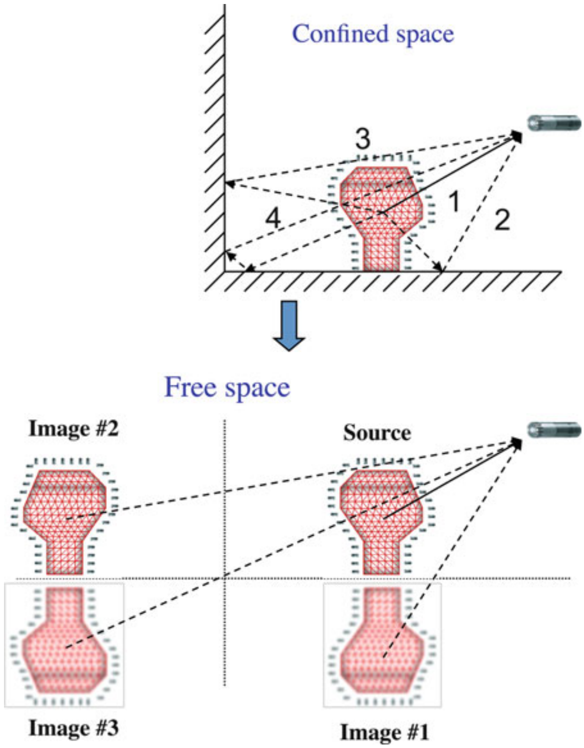

Fig. 5.10 Schematic of the effects of reflecting surfaces on the measured acoustic pressure

$$Q_\theta = \begin{cases} 1, & \text{in a free field;} \\ 2, & \text{on a large floor;} \\ 4, & \text{near any edge;} \\ 8, & \text{near any corner;} \end{cases} \tag{5.10}$$

The parameter $R_{rc}(\omega)$ in Eq. (5.9) represents the room constant defined as

$$R_{rc}(\omega) = \frac{S_{total}\overline{\alpha}(\omega)}{1 - \overline{\alpha}(\omega)}, \tag{5.11}$$

where S_{total} stands for the total reflecting surface area of the room and $\overline{\alpha}(\omega)$ represents the spatial averaged acoustic pressure absorption coefficient given by

$$\overline{\alpha}(\omega) = \frac{\sum_i S_i \alpha_i(\omega)}{S_{total}}. \tag{5.12}$$

The first term on the right side of Eq. (5.9) describes the direct sound wave emitted from the source, and the second term depicts the effect of reverberation of sounds inside the room. The smaller the room constant $R_{rc}(\omega)$ is, the higher the effect of reverberation of the room becomes and the worse the measurement condition is for reconstruction. Since in general the values of $\alpha_i(\omega)$ of all reflecting surfaces inside a room are unknown a priori, there is no way of determining the value of $R_{rc}(\omega)$. Hence, the measured mean-squared acoustic pressure does not reflect the true acoustic pressure emitted from the source, but includes all reverberation effects of the room. Accordingly, the input data to reconstruction will be severely contaminated.

There are several methods that can be used to determine the reverberation effect of a room, for example, reference source method and double-concentric-surface method [89]. All these methods require taking two sets of measurements and therefore are known as the indirect methods.

In the reference source method, a reference source with a known power spectrum is placed at the position of or close to a target source inside a room. Next the acoustic power radiated by this reference source is calculated based on the mean-squared acoustic pressure measured on a surface enclosing this reference source. Using Eq. (5.9), $R_{rc}(\omega)$ at any frequency can be written as

$$\frac{4}{R_{rc}(\omega)} = \frac{\hat{p}^2_{rms}(r_{ref}, \theta_{ref}, \phi_{ref}; \omega)}{\rho_0 c P_{ref}(\omega)} - \frac{Q_\theta}{4\pi r^2_{ref}}, \tag{5.13}$$

where $\hat{p}^2_{rms}(r_{ref}, \theta_{ref}, \phi_{ref}; \omega)$ represent the measured mean-squared acoustic pressures radiated by the reference source, and $P_{ref}(\omega)$ is the known acoustic power of this reference source. Once the value of room constant $R_{rc}(\omega)$ is specified, the true mean-squared acoustic pressure is given by

$$\hat{p}^2_{rms}(r, \theta, \phi; \omega) = \rho_0 c P(\omega) \left[\frac{\hat{p}^2_{rms}(r_{ref}, \theta_{ref}, \phi_{ref}; \omega)}{\rho_0 c P_{ref}(\omega)} + \frac{Q_\theta}{4\pi r^2} - \frac{Q_\theta}{4\pi r^2_{ref}} \right]. \tag{5.14}$$

In the double-concentric-surface method, the room constant $R_{rc}(\omega)$ is determined by taking the acoustic pressures over two concentric surfaces S_1 and S_2, respectively,

$$\hat{p}^2_{rms,1}(r_1, \theta_1, \phi_1; \omega) = \rho_0 c P(\omega) \left[\frac{Q_\theta}{4\pi r_1^2} + \frac{4}{R_{rc}(\omega)} \right], \tag{5.15}$$

$$\hat{p}^2_{rms,2}(r_2, \theta_2, \phi_2; \omega) = \rho_0 c P(\omega) \left[\frac{Q_\theta}{4\pi r_2^2} + \frac{4}{R_{rc}(\omega)} \right]. \tag{5.16}$$

Assume that $S_2 > S_1$, then $\hat{p}^2_{rms,2}(r_2, \theta_2, \phi_2; \omega) < \hat{p}^2_{rms,1}(r_1, \theta_1, \phi_1; \omega)$ because S_1 is closer to the source, and the amplitude of the acoustic pressure decays from S_1 to S_2, even though the decay rate is not known. Meanwhile, the acoustic power radiated from the target source $P(\omega)$ and directivity factor Q_θ remain the same. Therefore, combining Eqs. (5.15) and (5.16) we can express the room constant $R_{rc}(\omega)$ as

$$\frac{4}{R_{\mathrm{rc}}(\omega)}=\frac{\frac{Q_\theta}{4\pi r_1^2}\hat{p}^{\,2}_{\mathrm{rms},2}(r_2,\theta_2,\phi_2;\omega)-\frac{Q_\theta}{4\pi r_2^2}\hat{p}^{\,2}_{\mathrm{rms},1}(r_1,\theta_1,\phi_1;\omega)}{\hat{p}^{\,2}_{\mathrm{rms},1}(r_1,\theta_1,\phi_1;\omega)-\hat{p}^{\,2}_{\mathrm{rms},2}(r_2,\theta_2,\phi_2;\omega)}. \tag{5.17}$$

The acoustic power of the target source $P(\omega)$ can then be given by

$$\rho_0 cP(\omega)=\frac{\hat{p}^{\,2}_{\mathrm{rms},1}(r_1,\theta_1,\phi_1;\omega)-\hat{p}^{\,2}_{\mathrm{rms},2}(r_2,\theta_2,\phi_2;\omega)}{\frac{Q_\theta}{4\pi r_1^2}-\frac{Q_\theta}{4\pi r_2^2}}. \tag{5.18}$$

Once $R_{\mathrm{rc}}(\omega)$ and $P(\omega)$ are determined, the true mean-squared acoustic pressure emitted by the target source inside a large room is given by

$$\hat{p}^{\,2}_{\mathrm{rms}}(r,\theta,\phi;\omega)=\frac{\left[\hat{p}^{\,2}_{\mathrm{rms},1}(r_1,\theta_1,\phi_1;\omega)-\hat{p}^{\,2}_{\mathrm{rms},2}(r_2,\theta_2,\phi_2;\omega)\right]}{\left(\frac{Q_\theta}{4\pi r_1^2}-\frac{Q_\theta}{4\pi r_2^2}\right)}$$
$$\times\left[\frac{Q_\theta}{4\pi r^2}+\frac{\frac{Q_\theta}{4\pi r_1^2}\hat{p}^{\,2}_{\mathrm{rms},2}(r_2,\theta_2,\phi_2;\omega)-\frac{Q_\theta}{4\pi r_2^2}\hat{p}^{\,2}_{\mathrm{rms},1}(r_1,\theta_1,\phi_1;\omega)}{\hat{p}^{\,2}_{\mathrm{rms},1}(r_1,\theta_1,\phi_1;\omega)-\hat{p}^{\,2}_{\mathrm{rms},2}(r_2,\theta_2,\phi_2;\omega)}\right], \tag{5.19}$$

where $\hat{p}^{\,2}_{\mathrm{rms}}(r,\theta,\phi;\omega)$ depicts the true mean-squared acoustic pressure radiated from a target source inside a large room; $\hat{p}^{\,2}_{\mathrm{rms},1}(r_1,\theta_1,\phi_1;\omega)$ and $\hat{p}^{\,2}_{\mathrm{rms},2}(r_2,\theta_2,\phi_2;\omega)$ indicate the mean-squared acoustic pressures measured on two concentric surfaces. Note that this method works most effectively when the difference between two concentric surfaces is large enough so that the sound pressure level L_1 measured on the first surface S_1 is at least 3 dB higher than L_2 measured on the second surface S_2. In practice, hemispherical surfaces or rectangular parallelepiped surfaces are often selected for S_1 and S_2.

5.5 Clarifications

In implementing the Fourier acoustics-based NAH, measurement distance and microphone spacing should always be gauged with respect to the spatial frequency or spatial wavenumber, but not with respect to the acoustic frequency or acoustic wavenumber. That point was made clear in one of the original NAH papers [7], which stated "The minimum resolvable distance is on the order of $R=\pi/k_{\max}$, where $k_{\max}$ is the highest spatial frequency for a measurable Fourier component $\tilde{\psi}(k_x,k_y,z_H)$. In conventional optical and acoustical holography no evenescent waves are used in the field reconstructions so that $k_{\max}=k$ and $R=\lambda/2$, where λ is the acoustic wavelength."

Here the highest spatial frequency or the spatial wavenumber k_{max} is linked to the shortest spatial wavelength λ_{min} (=$2\pi/k_{max}$) that contains significant vibration energy [90]. Note that vibration energy of any structure decays with the spatial wavelength. The shorter the spatial wavelength is, the less the vibration energy it contains, whereas the longer the spatial wavelength is, the more the vibration energy it contains. Therefore, if measurement setup is gauged with respect to the highest spatial frequency k_{max} or the shortest spatial wavelength λ_{min}, structural vibrations that contain the vibration energy up to the shortest spatial wavelength can be reconstructed. Once this is done, the entire acoustic field, including the surface acoustic pressure, the normal surface acoustic intensity, and radiated acoustic power, can be reconstructed, and the correlation between structural vibration and acoustic radiation can be established. This is the advantage of NAH that cannot be matched by other methodologies. All that is lost during this process are the components of structural vibrations whose spatial wavelengths are shorter than λ_{min}, whose vibration energies are insignificant.

For example, in setting up the measurement microphone array, one can gauge the microphone spacing with respect to a target spatial resolution in reconstruction of structural vibration in, say, the z-axis direction. If the target spatial resolution is defined as R, then $R = \lambda_{cr}/2$, where λ_{cr} is the smallest axial wavelength corresponding to the maximum value of k_{max} [90]. This criterion agrees perfectly with Eq. (5.4). Note that the spatial sampling must be high enough to avoid spatial aliasing. In other words, the highest spatial wavenumber k_{max} "containing significant energy must be sampled at least at the rate of two samples per wavelength to prevent spatial aliasing which causes high wavenumbers to be converted to low wavenumbers [5]."

There is a huge difference between gauging microphone spacing with respect to the spatial wavelength and that with respect to the acoustic wavelength. This is because vibration of any structure can be expressed as a superposition of an infinite number of spatial waves, each of which has a specific spatial wavelength. This vibrating structure, however, can only produce a finite number of acoustic waves, each of which has a specific acoustic wavelength, which radiate into the surrounding fluid medium. The number of the acoustic waves generated by any vibrating elastic structure is always much less than that of the spatial waves. This is why we say, "Although sound can be generated by vibrations, not all vibrations can produce sound." In fact, only a small portion of structural vibrations can produce sound. The majority of the mechanical energy of an elastic structure stays close to the structure to maintain its vibration without emitting much sound into the surrounding fluid medium at all.

This phenomenon can be best illustrated by placing our ears next to a large window, where we can clearly sense the rumbly sound due to structural vibrations of the window, but nothing at all when we step back a little from the window. Another example is to put our ears near a railway track to find out if a train is coming. If a train is approaching, we will hear rumbling sound due to vibrations of the railway track excited by a train, even though we cannot see it. However, when we stand up, we hear nothing! This is because what we have sensed is the structural vibration whose amplitude decays exponentially with respect to the distance away from the structure. These exponentially decaying waves are known as the evanescent waves that are insignificant in acoustic radiation, but critically important for structural vibrations.

The primary objective of NAH is to reconstruct the evanescent waves so as to acquire a better understanding of structural vibrations, and how they are correlated to sound radiation. The insight into the interrelationships between structural vibrations and sound radiation will enable us to device the most cost-effective measures to tackle undesirable noise emission problems.

To illustrate this point, we consider an acoustic wave at 1,000 Hz or any frequency for that matter emitted by a vibrating panel of infinite dimensions in longitudinal and transverse directions. The acoustic wavelength for this 1,000 Hz sound wave is 0.343 m, given that the speed of sound is 343 m/s. If the microphone spacing is set with respect to the acoustic wavelength, we have $\delta = 0.171$ m $< \lambda/2$. Meanwhile, the measurement distance cannot be less than microphone spacing, so we can select $d = 0.172 > \lambda/2$, which is slightly more than one-half the acoustic wavelength.

To capture the critical evanescent component k_c that carries significant amount of energy, the dynamic range D (SNR of the measurement system) must satisfy the following condition:

$$10^{D/20} > e^{k_c(z_h - z_S)}, \tag{5.20}$$

where $(z_h - z_S) = d$ is the measurement distance.

Since the amplitudes of the evanescent waves decay exponentially as $e^{-k_c d}$, SNR will drop by 27.2 dB or 95.7 % at a measurement distance of $d = 0.172 \approx \lambda/2$! This will make it impossible for the evanescent components to be captured. Therefore, if we gauge the microphone array with respect to the acoustic frequency, we will not be able to reconstruct structural vibrations at all.

This example demonstrates that in order to capture the evanescent waves, the measurement setup must be gauged with respect to the target spatial, not the acoustic, wavelength. If the measurement spacing is gauged with respect to the acoustic frequency, we are in fact conducting acoustical holography, which will produce an image of the far-field component of the acoustic pressure radiated from a vibrating structure but nothing else! It cannot tell us anything about the acoustic pressure distribution on the surface of a vibrating structure, the normal surface velocity distribution of the structure, and the normal component of the time-averaged acoustic intensity or acoustic energy flow out of the structure. Moreover, the acoustic pressure reconstructed by using acoustical holography cannot be compared with respect to measured acoustic pressures because the measured data contain both near- and far-field components of the acoustic pressure. Further, the spatial resolution of acoustical holography is no better than one wavelength of the acoustic wave radiated from a target source. In contrast, NAH can produce, theoretically, an infinitely high spatial resolution if all near-field effects are captured, plus all the acoustic quantities anywhere in three-dimensional space.

Once the intent of NAH is understood, it becomes obvious that all documents, regardless what their titles claim, are in fact performing acoustic holography but not NAH, if the measurement setup is gauged with respect to the acoustic frequency or acoustic wavelength.

Problems

5.1. In implementing the HELS-based NAH or any NAH technologies, what is the single most important parameter that we should target in designing our measurement setup? Why?

5.2. Should we consider the frequency of the emitted sound wave in setting up our microphone array in conducting NAH measurements? Why?

5.3. Is the microphone spacing related to the standoff distance in the HELS-based NAH setup the same way as that in the Fourier transform-based NAH? Why?

5.4. How should input data be taken in general when reconstruction of the acoustic quantities on the surface of a vibrating structure is desired by using the HELS method in a nonideal environment such as inside a large room with unspecified reflecting objects and surfaces?

5.5. A prefixed and planar microphone array is easy to use and requires no setup time. Is it a good idea to use such a planar array to reconstruct the acoustic quantities generated by an arbitrarily shaped vibrating structure? Why?

5.6. Sometimes measurements can only be taken on one side of a vibrating structure in practice. In fact, this type of scenario happens almost all the time in engineering applications. What the impacts of this restriction may have on the resultant reconstruction? What should we expect under this condition?

5.7. When reconstruction must be conducted inside a large room in which the sound reflection and reverberation effects are not necessarily negligible, what should we do to minimize the impacts of sound reflection and reverberation?

5.8. What is wrong to target the measurement setup with respect to the frequency of the sound wave generated by a vibrating structure?