3

Doctoral Research

The account of Alfred Tarski's life begun in chapter 1 continues here.[1] This chapter describes the latter years of Alfred's studies at the University of Warsaw, and various other aspects of his life during 1921–1924. These include some personal details, his part-time employment as a teacher during his student years, his doctoral research, and his early participation in professional meetings. It provides a setting for both the detailed mathematics of Part Two of the book, and for Tarski's professional career during 1924–1939 as a researcher and secondary-school teacher and teacher trainer, which are the subject of Part Three.

Chapter 1, section 1.1, described Alfred's exciting first year of courses, 1919–1920; the Soviet invasion in summer 1920, the Polish victory that August—the Miracle of the Vistula—in the very suburbs of Warsaw; and Alfred's subsequent re-enrollment for autumn 1920, with courses from Stanisław Leśniewski, Jan Łukasiewicz, Stefan Mazurkiewicz, and Wacław Sierpiński. Three intellectual threads in Alfred's studies had taken form during those semesters: *logic*, *set theory*, and *measure theory*. They would extend far into his research career.

From autumn 1920 though autumn 1923, Alfred enrolled in many of the same classes as the future politician and logician Kazimierz Pasenkiewicz. Decades later, Pasenkiewicz described[2] the atmosphere that they shared:

> In the year 1920–1921, around ten thousand persons enrolled at the University of Warsaw. This number exceeded by many times the didactic possibilities of the university. In the lecture halls and laboratories it was very crowded; but not in all. Not many students enrolled in the theoretical directions, especially in mathematics, philosophy, and logic. ...
>
> This was a period of great liberalism and tolerance. The bureaucratic requirements were reduced to the most essential. ... The course of studies was very liberal. The only condition for receiving credit for a semester was to inscribe in a register a definite number of lecture-hours for subjects lectured in the department, and to obtain from the professor a confirmation of attendance. Very often these signatures were obtained through the janitors.
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> *[continued on the next page]*

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[1] For biographical information supporting this chapter, unless another source is cited, consult the biographies by Anita B. and Solomon Feferman (2004) and Jacek Juliusz Jadacki (2003a). Also note the descriptions of those works in chapter 17. The Fefermans emphasized personal-interview sources; Jadacki, published records. For historical and sociological information, consult the works by Norman Davies (1982, volume 2, chapters 18–19), Celia S. Heller (1994), and Richard M. Watt (1979).

[2] Pasenkiewicz 1984, 2–3. A portrait and biographical sketch of Pasenkiewicz are on page 32. The following quotation and those on pages 33 and 36 were translated by Jan Tarski, then lightly edited by the present editors to conform with the conventions of this book.

[quotation continued from the previous page]

There were no required lectures, nor examinations. One could, however, pass a discussion section for an attended lecture course. For a passed discussion section one received a confirmation with a grade.

After attending six semesters, one could take the final examination. Its successful result gave a diploma of completion of university studies and the right to lecture in secondary schools. The procedure for obtaining doctorates remained unchanged. It was necessary to obtain a positive evaluation of the doctoral dissertation and to pass through a traditional routine.

Kazimierz Pasenkiewicz
in 1920

Kazimierz Pasenkiewicz was born in 1897 in Kiev, and schooled there. He fought with the Russian army in World War I, then against it in the Polish–Soviet War of 1920, in which he lost a leg. Pasenkiewicz then entered the University of Warsaw to study mathematics and logic, and attended lectures and seminars alongside Alfred Tarski. Pasenkiewicz earned the doctorate there in 1933, with a dissertation supervised by Tadeusz Kotarbiński. He continued that study and research until World War II. During that war he worked with the Polish underground and with socialist organizations. Afterward he became very active in the Communist Party, continuing until its dissolution in 1991. During 1948–1968 Pasenkiewicz served as lecturer, professor, and dean in the philosophy faculty at the University of Cracow. He was an expert on the logical work of Leon Chwistek. Pasenkiewicz died in 1995 in Cracow.*

* Kutta 1997.

During the period from winter 1921 through autumn 1923, Alfred enrolled in courses following much the same pattern as that of his previous semesters:

- in nearly every term, two or three courses or seminars with Leśniewski;
- in nearly every term, courses from Mazurkiewicz—on analytic geometry, Jordan continua, topology of the plane, and entire functions;
- in most terms, seminars with Łukasiewicz and his courses on logic and philosophy;
- from winter 1921 through winter 1922, courses from Sierpiński on higher algebra and analysis;
- an autumn 1921 course on mechanics from Czesław Białobrzeski;
- in winter 1922 and autumn 1923, courses from Kazimierz Kuratowski on topology and set theory.[3]

Alfred was thus continuing to explore the *logic* and *set theory* threads that would extend into his research career. Moreover, with the courses on topology and continua he added a new strand that, with his previous study of measure theory, would become a thread of *applications of set theory to geometry*.

Pasenkiewicz enrolled in the lectures of Sierpiński and Mazurkiewicz that involved set theory, those of Leśniewski on logic, some lectures on physics, and lectures by Łukasiewicz on history of philosophy. Pasenkiewicz recalled,

> To the lectures on set theory there came then a fair number of students. These were the times of Émile Borel, Henri Lebesgue, Ernst Zermelo. The theory of sets was rather fashionable. … To the [very popular] lectures on theoretical physics of Białobrzeski from Kiev came students of mathematics and of philosophy; those were the times of Albert Einstein and the atom.
>
> In contrast, few came to the lectures of Leśniewski … regularly three persons: Jan Drewnowski, Aleksander Jabłoński, and I. … A few came irregularly, among them Alfred Tarski. He sat in the last row and read newspapers. After a lecture, or during intermission, he conferred with the professor; he did not enter into conversations with fellow students. During 1922–1923, Leśniewski acquired a few new students: there appeared Adolf Lindenbaum, Mordchaj Wajsberg, and a few others. Tarski livened up. At the seminars he had the possibility to evaluate those who spoke; he took interest in some of these.
>
> [Tarski] appeared from time to time also at the lectures of Sierpiński, primarily … when he had some matters to speak with him about. Tarski did not come to [the lectures on physics or] history of philosophy. He was consistent in his interests, did not distract himself. … In his surroundings Tarski did not notice those who did not show particular abilities or interests in logic. On the other hand, he valued people with whom he shared these interests. In discussions with them he did not take on attitudes or a position of authority.[4]

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[3] Tarski 1924f.

[4] Pasenkiewicz 1984, 2–3. For information about Lindenbaum, see a box in section 14.3; for the other students mentioned, see page 34. The present editors suggest the word *question* in place of *evaluate* in the second paragraph of this translation by Jan Tarski.

Aleksander Jabłoński
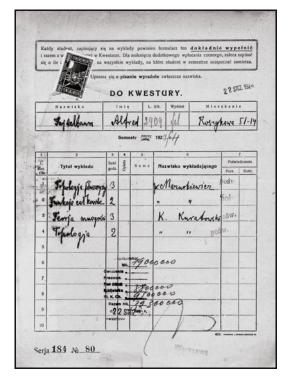*in 1935*

Mordchaj Wajsberg
around 1925

Jan Franciszek Drewnowski was born in Moscow in 1886. He attended lectures and seminars at the University of Warsaw during the early 1920s, alongside Alfred Tarski, and earned the doctorate in philosophy there in 1927, supervised by Tadeusz Kotarbiński. Drewnowski had studied finance and economics as well. He worked in industry and served as a government official both before and after World War II. Drewnowski was a member of the Cracow Circle of Catholic philosophers, and published in both economics and analytical philosophy. He died in Warsaw in 1978.*

Aleksander Jabłoński was born in 1898 in Woskresenówka, in eastern Poland, then in the Russian Empire. He served Poland as an engineer officer during the 1920 Polish–Soviet War. Afterward, he worked as a professional musician while studying at the University of Warsaw. He attended lectures alongside Alfred Tarski. Jabłoński earned the doctorate at Warsaw in physics in 1930, and received the *venia legendi* in 1934. As an engineer officer again in 1939, he was interned in the Soviet Union, left with the army of General Władysław Anders via Central Asia and the Middle East, then served as a lecturer in Scotland. After the Second World War, he helped build the Department of Physics at the University of Toruń. He was a world authority on the physics of light. Jabłoński died in 1980 in Skierniewice, in central Poland.†

Mordchaj Wajsberg was born in 1902 in Łomża, in central Poland, then in the Russian Empire. His schooling was interrupted by World War I, then by military service in the 1920 Polish–Soviet War. He finally completed secondary school in Łomża in 1923, and entered the University of Warsaw, to study mathematics and logic. He attended lectures and seminars with Alfred Tarski. Wajsberg soon began a stream of research results, concentrating on axiomatics of three-valued logic and modal logic. This area was intimately tied to the work of Tarski and of Jan Łukasiewicz, who supervised the research for Wajsberg's master's and doctoral degrees, awarded in 1928 and 1931. Wajsberg pioneered the use of algebraic and model-theoretic techniques to study those logics. In 1933 he began work as a secondary-school teacher, first in the eastern Polish town of Kowel (now Kovel, in Ukraine), then in Łomża. There he extended his techniques to include general multivalued and intuitionistic logic. Wajsberg perished in the Holocaust, probably during the Nazi massacre of Jews in Łomża in 1942.‡

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*Majdański and Lekka-Kowalik 2001. Note: there was   † Frąckowiak 1998
 another scholar of the same era with a similar name.   ‡ Surma 1977

The tuition costs entered in Alfred's enrollment record reveal a major ordeal for Polish citizens in the early 1920s: *hyperinflation*. The entries for autumn 1919 were pictured in section 1.1. Figures for subsequent autumn trimesters, shown in the table at right, illustrate exponentially increasing costs for 1919–1922, followed by a year of hyper-exponential growth. The bursar's certificate corresponding to that last line is pictured below. Poland's currency soon collapsed. In 1924, the government instituted reforms, and the new Bank Polski introduced the currency used today, the *złoty*.[5]

Autumn Trimester	*Tuition Costs (Polish Marks)*
1919	76
1920	600
1921	2,800
1922	14,500
1923	32,300,000

To the Bursar

Tajtelbaum, Alfred; Student 2909;
    Philosophical Faculty; Koszykowa 51-14
                Winter 1923–1924

Topology of the Plane	3 hr	Mr. Mazurkiewicz
Entire Functions	2 hr	" "
Theory of Sets	3 hr	K. Kuratowski
Topology	2 hr	" "

Payment:
                Mk. 19 000 000
⋮   ⋮   ⋮
Library   "   8 800 000
St. K. Ch.   "   4 500 000
Total   Mk. 32 300 000
        22 January 1924

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[5] Tarski 1924f, 8–11, and 1921–1926. It would be difficult to make precise year-to-year comparisons, because different aspects of tuition were billed in different ways. Moreover, Alfred evidently obtained partial tuition waivers for spring 1921 and spring 1922. But the exponential growth outweighs such details. For information about the Polish economy, see N. Davies 1982, volume 2, 307; Watt 1979, 202–207; and Wynot 1983, 46–47. *Złoty* is a Polish word for *gold*; the English spelling is *zloty*. The abbreviation *St. K. Ch.* in the caption means *Studencka Kasa Chorych* (Student Health Fund).

Alfred continued his development as a researcher in logic, guided by Łukasiewicz, Leśniewski, and Tadeusz Kotarbiński. All three had studied with Kazimierz Twardowski in Lwów. In 1920 yet another student of Twardowski had arrived in Warsaw to enrich its research climate: Kazimierz Ajdukiewicz.[6] At an April 1921 meeting of the Logical Section of the Warsaw Philosophical Institute, Alfred presented his first public scientific address, entitled *On the Notion of Proof* (*In Response to the Dissertation of K. Ajdukiewicz*). No copy of its text is known, but Alfred wrote later that it contained the first discussion of the deduction theorem, a logical tool that would play a fundamental role in his organization of metamathematics. In December 1921 Alfred presented another paper to the same group: *On the Problems of Extensionality in Logistic and in Ontology*. No copy of this text is known either, but the title suggests that it pertained to Leśniewski's developing system of foundations of mathematics, the subject of Alfred's doctoral research.[7]

At the age of twenty, Alfred was learning the etiquette of scientific discourse. Pasenkiewicz reported about Leśniewski's autumn 1920 seminar:

> The base of the seminar's activities was the book by Louis Couturat, *Algebra of Logic*. The criticism of this book was shattering. At one point Tarski stood up and asked whether it was at all worthwhile to busy oneself with this. Leśniewski felt a bit slighted, but asked with humor, "Do you think that my seminar is a waste of time?" Tarski sat down.

Pasenkiewicz attended the meetings at the Institute, too. Tarski's patience was improving, but it had limits:

> Tarski was unusually efficient intellectually. At the meetings…a few times papers were read…by Tarski. …I admired…[him for answering] the voices in the [subsequent] discussions without notes, but in the order in which they were presented. …[Unlike another speaker, who] answered to all the raised issues with equal attention and friendliness, Tarski, on the other hand, [responded] only to those which deserved an answer.[8]

Alfred still lived with his parents, and would do so until he married. According to his biographers Anita B. and Solomon Feferman, he felt a need to establish some financial independence. Around 1920, Alfred found a job teaching geometry at a girls' secondary school. After two years' service, according to his account many years later, he was fired for being Jewish. In 1921 Alfred obtained an appointment to teach logic at the National Pedagogical Institute. This organization had been founded in 1920 as an adjunct of the

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[6] For more information about Ajdukiewicz and Twardowski, see the box on page 37 and one in section 1.2.

[7] The two talks were listed in Warszawski Instytut Filozoficzny 1921–1922 as A. Tajtelbaum, *O pojęciu dowodu* (*z powodu rozprawy K. Ajdukiewicza*) and *O zagadnieniach ekstensjonalnych w logistyce i ontologii*. The dissertation mentioned in the parenthetical phrase was [1921] 1966, which Ajdukiewicz had submitted for the *venia legendi*. According to Coniglione and Betti 2001, and Betti 2008, 61, that dissertation introduced to Poland the structural definitions of *proof*, *theorem*, *consequence*, *logical theorem*, and *logical consequence* that Alfred would later organize precisely and forcefully. See Tarski [1930] 1983b, 32.

[8] Pasenkiewicz 1984, 5–6. The seminar date is from the enrollment record, Tarski 1924f. Couturat [1908] 1918 had recently been translated from French into Polish by Łukasiewicz and Bronisław Knaster, supported by a grant from the Mianowski Fund.

Kazimierz Ajdukiewicz
around 1920

Kazimierz Ajdukiewicz was born in 1890 in Tarnopol, then in the Austrian Empire (now Ternopil in Ukraine). His father was a government official. He was schooled in Cracow, then in Lwów, and entered the university there in 1908. He studied with Jan Łukasiewicz and Wacław Sierpiński, and earned the doctorate in 1912 with a thesis on Kant's philosophy of space, under supervision of Kazimierz Twardowski. Tadeusz Kotarbiński and Stanisław Leśniewski also earned doctorates from Twardowski that year. Ajdukiewicz qualified for a teaching credential, but spent the next year in Göttingen, where he could study the work of David Hilbert, Edmund Husserl, and Leonard Nelson. During World War I and its aftermath, he served in the Austrian and Polish armies, taught in a gimnazjum, and married Twardowski's daughter, a classical philologist. During this time Ajdukiewicz wrote his influential [1921] 1966 booklet on the methodology of deductive science, which won him the *venia legendi* and served as a basis of Alfred Tarski's later work in that area. During 1922–1928, Ajdukiewicz served in Lwów as gimnazjum teacher and university dozent, and in Warsaw as philosophy teacher at the same gimnazjum as Tarski and as professor at the university. He was appointed to the permanent faculty at the University of Lwów in 1928 and promoted to full professor in 1934. Ajdukiewicz was the originator of many concepts and techniques now familiar in the philosophy of science; he adapted and extended Tarski's approach to several areas of philosophical logic. During World War II, he worked as a teacher during the Soviet occupation, and as a clerk and clandestine teacher under the German oppression. Afterward, he became professor at the University of Poznań, where he served as rector during 1948–1952. From 1953 on he edited the journal *Studia Logica*. In 1955 Ajdukiewicz returned to Warsaw as head of the Division of Logic of the Polish Academy of Sciences and professor at the university. He retired from the latter position in 1961, and died suddenly in 1963.*

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* Giedymin 1978, Raabe 1926.

[continued from page 36]

University of Warsaw through the efforts of Łukasiewicz, then a university and government administrator. Its purpose was to train schoolteachers; its instructors were selected and supervised very seriously by university faculty.[9] Alfred continued with the institute for about four years. During that same period, he also taught at Zofja Kalecka's Gimnazjum, a girls' school that emphasized mathematics and physical science.[10]

During 1922–1924 Alfred was baptized a Roman Catholic, and he changed his surname legally to the Polish-sounding *Tarski*. At right are his signatures from this period. These acts may reflect a change in religious conviction, a desire to be more Polish, or an assessment of the disadvantages of being Jewish. Jewish scientists from Poland have painted a bleak picture of the situation facing Tarski there in academia. Alfred's decision has been discussed at some length in the literature; writers disagree about his motivation. In the early 1970s, he told the Israeli logician Menachem Magidor,

> I considered myself, at that point, to be Polish, culturally and nationally; and I didn't care about religion at all, so it had nothing to do with religious belief.

Alfred's former Berkeley colleague John Corcoran recently reported:

> Tarski and I talked about religion quite a bit. It was easy and natural.… Tarski never revealed a wavering from the atheistic humanism he came to as a young man … He never mentioned that he had ever converted to Catholicism; I think that "conversion" was never of any inner significance to him.[11]

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9 New educational regulations permitted schools to hire teachers after an oral examination, a sample lesson, and some university studies or completion of a program at the institute (Manteuffel 1936, 163). This permitted Alfred to teach at a secondary school, and created a demand for instruction at the institute.

10 The biographies by Steven R. Givant (1999, 50), Jacek Juliusz Jadacki (2003a, 142–143), and the Fefermans (2004, 54) disagree about the details of this paragraph. It follows Givant's most closely. Only Givant mentioned the first school, with no name. Jadacki corrected the others about the name of the institute: Państwowy Instytut Pedagogiczny. The Fefermans described the institute as a women's school whose headmistress fired Tarski. But its director, Paweł Sosnowski, described it in 1923 as coeducational; he listed "A. Tarski" as instructor for logic during 1921–1922 (Sosnowski 1923, 78, 81). Alfred must have notified him of the impending name change just before that publication. Tarski was still a staff member there in autumn 1923 (Zagórowski 1924–1926, volume 1, 106); Jadacki suggested that Alfred remained so until the institute closed in 1925. Only Jadacki, among the biographers, mentioned the Kalecka Gimnazjum; a Jewish school, it was not likely the one that fired Alfred. (That is confirmed in Zagórowski 1924–1926, volume 1, 151, and Shneiderman 1995.)

11 The signatures are from Tarski [1918] 2014, 1919–1920, and [1924] 2014a; the first change was discussed in section 1.1. See Feferman and Feferman 2004, 38–40, 269; this includes a reproduction of the application for change of name, Tarski [1924] 2007. See also Jadacki 2003a, 142–145, Woleński 1995a, note 7, and Woleński 2008. The autobiographies of physicist Leopold Infeld and probabilist Mark Kac record their impressions as students in Cracow and Lwów during 1920–1935 (Infeld 1980, chapters 1–2; Kac 1985, 28). Heller (1994, chapter 6) discusses assimilation and conversion in depth. Corcoran 2011a, 2011b.

Alfred was awarded the doctorate in March 1924, as recommended by Leśniewski.[12] His dissertation, *On the Primitive Term of Logistic*, was published in *Przegląd filozoficzny* in 1923, under the surname Tajtelbaum-Tarski. The term *logistic* referred to a system of logic whose features included material equivalence $\Leftrightarrow$ and perhaps other Boolean connectives, propositional variables $p, q, \ldots$, variables $f, g, \ldots$ for truth functions, and universal and existential quantifiers over these variables. A *truth* function f is one that maps truth values to truth values, such that if $p \Leftrightarrow q$ is valid, then so is $f(p) \Leftrightarrow f(q)$.[13] Alfred showed, roughly speaking, that negation and conjunction can be defined in terms of the equivalence connective $\Leftrightarrow$ and universal quantifier $\forall$ by regarding $\neg p$ and $(p \,\&\, q)$ as abbreviations for $(p \Leftrightarrow (\forall q)q)$ and $\forall f(p \Leftrightarrow (f(p) \Leftrightarrow f(q)))$. This can be justified intuitively,[14] but the steps that suggest themselves do not all have counterparts in Leśniewski's system. Alfred's achievement was to adapt and fit such definitions into the system; he had to use a more complicated one for conjunction. Alfred immediately published a two-part French translation of his dissertation in the journal *Fundamenta Mathematicae* during 1923 and 1924. He published the first part under the surname Tajtelbaum; the second, as Tajtelbaum-Tarski.[15] From that time on, he published all his work as Alfred Tarski.

Leśniewski worked on his logical system through the 1920s, and finally published its details at the end of the decade. In good humor, he emphasized his debt to Tarski:

> The system of the foundations of mathematics I have constructed owes a number of important improvements to Alfred Tarski … "my" Doctor in the year 1924. … I will endeavor to show them explicitly; but because of the nature of things I cannot show properly all of Tarski's occasional critical remarks, which undermined this or that link of my theoretical conceptions at the different stages in the building of my system, and all the subtle and sympathetic counsel and often impalpable suggestions, from which I had the opportunity to profit in numerous conversations with Tarski.[16]

Tarski regarded Leśniewski's system as equivalent to other better-known systems, but referred to its use in just one more publication, his 1927 report about geometry based on

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[12] The official certificates are displayed in Jadacki 2003, 168; the official critics are identified there on page 144 as Łukasiewicz and Sierpiński. For a detailed description of the presumably similar doctoral defense of mathematician Jerzy Neyman in Warsaw the same year, see Reid 1983, 53.

[13] Truth values are values of propositional variables. See Church 1956, §§06, 28, for more information on this sort of logical system.

[14] To see this, suppose first that p, q should both be valid; that would imply validity of $p \Leftrightarrow q$, $f(p) \Leftrightarrow f(q)$, and $p \Leftrightarrow (f(p) \Leftrightarrow f(q))$, no matter what f might be. Should p be valid but q not, then letting f denote the identity function, $f(p) \Leftrightarrow f(q)$ and hence $p \Leftrightarrow (f(p) \Leftrightarrow f(q))$ would be invalid. If p were invalid, then letting f be any constant truth-function, $f(p) \Leftrightarrow f(q)$ would be valid and thus $p \Leftrightarrow (f(p) \Leftrightarrow f(q))$ invalid, no matter what q might be. The definition of negation works because $(\forall q)q$ is invalid. The remaining connectives and the existential quantifier can then be defined as in elementary logic.

[15] See Tarski 1923a, 1923b, and 1924d.

[16] Leśniewski [1927–1930] 1992, 180. Leśniewski included a translation of the 1927 Polish version of this quotation in the German original version of Leśniewski [1929] 1992, 414. His Polish and German words for *impalpable* were *nieuchwytny* and *ungreifbar*: *elusive* and *ungraspable*.

the notion of solid body. Arianna Betti has analyzed Tarski's major scientific debt to Leśniewski, which would appear most clearly in Tarski's work of the 1930s:

> … among the things which Tarski was taught by Leśniewski were … how to analyse quotation marks, the language/metalanguage distinction, the idea that truth is language-relative, the notion of a closed language, and that natural language is such a language, namely that natural language is universal.[17]

During 1922–1923, Alfred had enrolled only in lectures and seminars on logic and philosophy, ostensibly concentrating on his doctoral research project in logic. Alongside those studies, however, he pursued intellectual threads that had evidently captivated him even earlier: set theory and its applications to geometry. As early as 1919–1920, he had studied geometry and calculus with Zygmunt Janiszewski and Stefan Mazurkiewicz, and set theory and measure theory with Wacław Sierpiński. Tarski had continued with the latter two, studying analysis and algebra, and during spring 1922 and autumn 1923, he also attended Kazimierz Kuratowski's topology courses. Sierpiński influenced Tarski to study in detail Felix Hausdorff's 1914a book *Grundzüge der Mengenlehre*.[18] Sierpiński had just published his major 1918 survey of the use in set theory and analysis of the axiom of choice, the basic principle according to which,

> for any family F of nonempty sets, there is a set that contains exactly one element from each member of F.

Moreover, for several years, Sierpiński and Mazurkiewicz had been studying seemingly paradoxical constructions in set theory. For example, in a 1914 paper, they described two disjoint unbounded subsets A and B of the plane, each of which is congruent to the union $A \cup B$. Their excitement about such results surely infected their lectures. Tarski immersed himself in that research area, too. His work would involve its most advanced aspects.

One filament of this set-theory thread involved delicate consideration of the axiom of choice. Tarski established striking relationships between this idea and that of the finiteness of a set, and discovered that certain other set-theoretic propositions, some quite familiar, were equivalent to the axiom. For example, in 1914a, Hausdorff included the theorem that for any infinite set A, there is a one-to-one correspondence between the elements of A and those of the set $A \times A$ of all ordered pairs of elements of A. The proof made essential use of the axiom of choice. Tarski showed that some use of that principle was inescapable, because that theorem actually implies the axiom.[19] Tarski published this work simultaneously with his dissertation, and continued to work on this thread in collaboration with Adolf Lindenbaum. Their joint papers would start appearing in 1926.

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[17] Lindenbaum and Tarski 1926, 299; Tarski [1927] 1983. Betti 2004, 278–279. Concerning the relationship between Leśniewski and Tarski, see also Betti 2008. For a brief account of Tarski's research in logic up to 1939, see section 8.1.

[18] See Feferman and Feferman 2004, 48, and Tarski 1924f. For biographical information about Sierpiński and Hausdorff, consult boxes in sections 4.1 and 4.4. Tarski always recommended Hausdorff 1914a to his own students, even decades later.

[19] Hausdorff 1914a, 127. Tarski 1924b and 1924e, 150. Lindenbaum and Tarski 1926. This theorem about $A \times A$ is due to Gerhard Hessenberg (1906, 108).

That research is not considered further here, because it is relatively accessible and lies beyond the scope of this book.

Soon after his graduation, Tarski applied to the dean of the faculty, the chemist Wiktor Lampe, for permission to pursue further study in mathematical sciences, particularly theoretical physics. His letter is translated in chapter 16, section 4. Tarski's signature included the title *Dr.*; it is reproduced on page 38. Tarski signed up for nine physics and astronomy courses in spring 1925 and five in spring 1926. There was evidently no tuition charge; he must have been going to sample those offerings.[20]

In 1925, probably on the basis of his 1924b research on the definition of finiteness, Tarski was awarded the *venia legendi* by the faculty: this allowed him to use the title *docent* and to give courses, receiving no salary but simply a fraction of his students' fees. For a year starting in February 1925, Tarski gave a course on deductive logic at the Free University of Poland, a private postsecondary institution in Warsaw; the course met for two hours each week.[21]

During 10–13 May 1923, Tarski had performed one of the duties of an advanced student: he had served as secretary of the logic section of the First Polish Philosophical Congress in Lwów. He did not present a paper, but participated in the discussions after talks on the principle of contradiction, type theory, axiomatization of physics, truth functions, and the logic of adjectives. At that meeting he met a new Lwów professor of mathematics, Stefan Banach, who was also fascinated by the intricacies of set theory and geometry.[22] They began a collaboration that led to their famous 1924 paper *On Decomposition of Point Sets into Respectively Congruent Parts*. Part Two of the present book is devoted to Tarski's research on decomposition of point sets, and chapter 6 contains a full translation of that paper.

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[20] Tarski [1924] 2014a; Tarski 1922–1926, documents 18 RP 2909 and 20 RP 2909. Jadacki 2003a, 144.

[21] Wolna Wszechnica Polska 1925, 8, 25. Many other University of Warsaw faculty taught part-time at the Free University; this was apparently Tarski's only involvement with it. For his service at the University of Warsaw, see section 9.3.

[22] Polski Zjazd Filozoficzny 1927, 266, 284–294, 361. Tarski's comments were not recorded. For biographical information about Banach, consult a box in section 4.2.