6. Physical Acoustics

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An overview of the fundamental concepts needed for an understanding of physical acoustics is provided. Basic derivations of the acoustic wave equation are presented for both fluids and solids. Fundamental wave concepts are discussed with an emphasis on the acoustic case. Discussions of different experiments and apparatus provide examples of how physical acoustics can be applied and of its diversity. Nonlinear acoustics is also described.

Physical acoustics involves the use of acoustic techniques in the study of physical phenomena as well as the use of other experimental techniques (optical, electronic, etc.) to study acoustic phenomena (including the study of mechanical vibration and wave propagation in solids, liquids, and gasses). The subject is so broad that a single chapter cannot cover the entire subject. For example, recently the 25th volume of a series of books entitled *Physical Acoustics* was published [6.1]. *Mason* [6.2] began the series in 1964. The intermediate volumes are not repetitious, but deal with different aspects of physical acoustics. Even though all of physical acoustics cannot be covered in this chapter, some examples will illustrate the role played by physical acoustics in the development of physics.

Since much of physics involves the use and study of waves, it is useful to begin by mentioning some different types of waves and their properties. The most basic definition of a wave is a disturbance that propagates through a medium. A simple analogy can be made with a stack of dominoes that are lined up and knocked over. As the first domino falls into the second, it is knocked over into the third, which is knocked over into the next one, and so on. In this way, the disturbance travels down the entire chain of dominoes (which we may think of as particles in a medium) even though no particular domino has moved very far. Thus, we may consider the motion of an individual domino, or the motion of the disturbance which is traveling down the entire chain of dominoes. This suggests that we define two concepts, the average particle velocity of the individual dominoes and the wave velocity (of the disturbance) down the chain of dominoes. Acoustic waves behave in a similar manner. In physical acoustics it is necessary to distinguish between particle velocity and wave (or phase) velocity.

There are two basic types of waves: longitudinal waves, and transverse waves. These waves are defined according to the direction of the particle motion in the medium relative to the direction in which the wave travels. Longitudinal waves are waves in which the particle motion in the medium is in the same direction as the wave is traveling. Transverse waves are those in which the particle motion in the medium is at a right angle to the direction of wave propagation. Figure 6.1 is a depiction of longitudinal and transverse waves. Another less common type of wave, known as a torsional wave, can also propagate in a medium. Torsional waves are waves in which particles move in a circle in a plane perpendicular to the direction of the wave propagation. Figure 6.2 shows a commonly used apparatus for the demonstration of torsional waves.

There also are more-complicated types of waves that exist in acoustics. For example, surface waves (Rayleigh waves, Scholte–Stonley waves, etc.) can propagate along the boundary between two media. Another example of a more complicated type of wave propagation is that of Lamb waves, which can propagate along thin plates. A familiar example of Lamb waves are the waves that propagate along a flag blowing in a wind.

In acoustics, waves are generally described by the pressure variations that occur in the medium (solid or fluid) due to the wave. As an acoustic wave passes through the medium, it causes the pressure to vary as the acoustic energy causes the distance between the molecules or atoms of the fluid or solid to change periodically. The total pressure is given by

$$p_{\mathrm{T}}(x,t) = p_0(x,t) + p_1(x,t)\,. \tag{6.1}$$

Here p_0 represents the ambient pressure of the fluid and p_1 represents the pressure fluctuation caused by the acoustic field. Since pressure is defined as the force per unit area, it has units of newtons per square meter ($\mathrm{N/m^2}$). The official SI designation for pressure is the pascal ($1\,\mathrm{Pa} = 1\,\mathrm{N/m^2}$). Atmospheric pressure at sea level is 1 atmosphere (atm) $= 1.013 \times 10^5\,\mathrm{Pa}$. The types of sounds we encounter cause pressure fluctuations in the range from 10^{-3}–$10\,\mathrm{Pa}$.

One can also describe the strength of the sound wave in terms of the energy that it carries. Experimentally, one can measure the power in the acoustic wave, or the amount of energy carried by the wave per unit time. Rather than trying to measure the power at every point in space, it is usual to measure the power only at the location of the detector. So, a more convenient measurement is the power density, also referred to as the acoustic intensity I. In order to make a definition which does not depend on the geometry of the detector, one considers the power density only over an infinitesimal area of size $\mathrm{d}A$

$$I = \frac{\mathrm{d}P}{\mathrm{d}A}\,, \tag{6.2}$$

where $\mathrm{d}P$ is the portion of the acoustic power that interacts with the area $\mathrm{d}A$ *of the detector* oriented perpendicular to the direction of the oncoming acoustic wave. The units of acoustic intensity are watts per square meter ($\mathrm{W/m^2}$).

The human ear can generally perceive sound pressures over the range from about $20\,\mu\mathrm{Pa}$ up to about 200 Pa (a very large dynamic range). Because the range of typical acoustic pressures is so large, it is convenient to work with a relative measurement scale rather than an absolute measurement scale. These scales are expressed using logarithms to compress the dynamic range. In acoustics, the scale is defined so that every factor of ten increase in the amount of energy carried by the wave is represented as a change of 1 bel (named after Alexander Graham Bell). However, the bel is often too large to be useful. For this reason, one uses the decibel scale (1/10 of a bel). Therefore, one can write the sound intensity

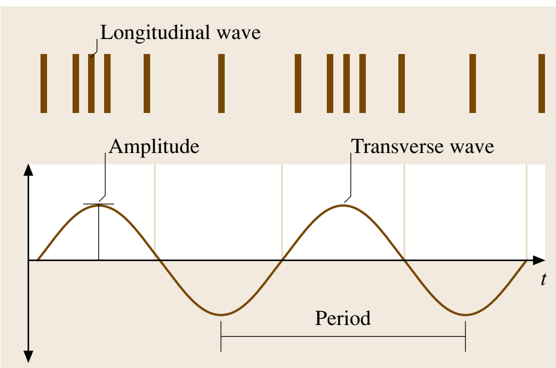


Fig. 6.1 Longitudinal and transverse waves

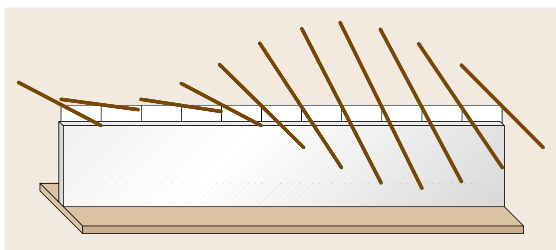


Fig. 6.2 Apparatus for the demonstration of torsional waves

level (SIL) as the logarithm of two intensities:

$$\mathrm{SIL(dB)} = 10\log\left(\frac{I}{I_{\mathrm{ref}}}\right), \quad (6.3)$$

where I is the intensity of the sound wave and I_{ref} is a reference intensity. (One should note that the bel, or decibel, is not a unit in the typical sense; rather, it is simply an indication of the relative sound level).

In order for scientists and engineers to communicate meaningfully, certain standard reference values have been defined. For the intensity of a sound wave in air, the reference intensity is defined to be $I_{\mathrm{ref}} = 10^{-12}\,\mathrm{W/m^2}$.

In addition to measuring sound intensity levels, it is also common to measure sound pressure levels (SPL). The sound pressure level is defined as

$$\mathrm{SPL(dB)} = 20\log\left(\frac{p}{p_{\mathrm{ref}}}\right), \quad (6.4)$$

where p is the acoustic pressure and p_{ref} is a reference pressure. (The factor of 20 comes from the fact that I is proportional to p^2.) For sound in air, the reference pressure is defined as 20 μPa (2×10^{-5} Pa).

For sound in water, the reference is 1 μPa (historically other reference pressures, for example, 20 μPa and 0.1 Pa, have been defined). It is important to note that sound pressure levels are meaningful only if the reference value is defined. It should also be noted that this logarithmic method of defining the sound pressure level makes it easy to compare two sound levels. It can be shown that $\mathrm{SPL}_2 - \mathrm{SPL}_1 = 20\log\left(\frac{p_2}{p_1}\right)$; hence, an SPL difference depends only on the two pressures and not on the choice of reference pressure used.

Since both optical and acoustic phenomena involve wave propagation, it is illustrative to contrast them. Optical waves propagate as transverse waves. Acoustic waves in a fluid are longitudinal; those in a solid can be transverse or longitudinal. Under some conditions, waves may propagate along interfaces between media; such waves are generally referred to as surface waves. Sometimes acoustic surface waves correspond with an optical analogue. However, since the acoustic wavelength is much larger than the optical wavelength, the phenomenon may be much more noticeable in physical acoustics experiments. Many physical processes produce acoustic disturbances directly. For this reason, the study of the acoustic disturbance often gives information about a physical process. The type of acoustic wave should be examined to determine whether an optical model is appropriate.

6.1 Theoretical Overview

6.1.1 Basic Wave Concepts

Although the domino analogy is useful for conveying the idea of how a disturbance can travel through a medium, real waves in physical systems are generally more complicated. Consider a spring or slinky that is stretched along its length. By rapidly compressing the end of the spring, one can send a pulse of energy down the length of the spring. This pulse would essentially be a longitudinal wave pulse traveling down the length of the spring. As the pulse traveled down the length, the material of the spring would compress or *bunch up* in the region of the pulse and *stretch out* on either side of it. The compressed regions are known as condensations and the stretched regions are known as rarefactions.

It is this compression that one could actually witness traveling down the length of the spring. No part of the spring itself would move very far (just as no domino moved very far), but the disturbance would travel down the entire length of the spring. One could also repeatedly drive the end of the spring back and forth (along its length). This would cause several pulses (each creating compressions with stretched regions around them) to propagate along the length of the spring, with the motion of the spring material being along the direction of the propagating disturbance. This would be a multipulse longitudinal wave.

Now, let us consider an example of a transverse wave, with particle motion perpendicular to the direction of propagation. Probably the simplest example is a string with one end fastened to a wall and the opposite end driven at right angles to the direction along which the string lies. This drive sends pulses down the length of the string. The motion of the particles in the string is at right angles to the motion of the disturbance, but the disturbance itself (whether one pulse or several) travels down the length of the string. Thus, one sees a transverse wave traveling down the length of the string.

Any such periodic pulsing of disturbances (whether longitudinal or transverse) can be represented mathematically as a combination of sine and/or cosine waves through a process known as Fourier decomposition. Thus, without loss of generality, one can illustrate ad-

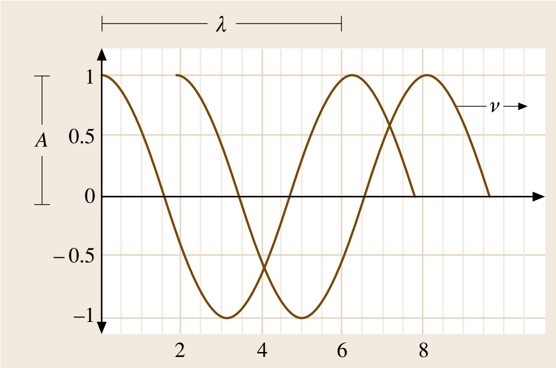


Fig. 6.3 One cycle of a sinusoidal wave traveling to the right

ditional wave concepts by considering a wave whose shape is described mathematically by a sine wave.

Figure 6.3 shows one full cycle of a sinusoidal wave which is moving to the right (as a sine-shaped wave would propagate down a string, for example). The initial wave at $t=0$ and beginning at $x=0$ can be described mathematically. Let the wave be traveling along the x-axis direction and let the particle displacement be occurring along the y-axis direction. In general, the profile or shape of the wave is described mathematically as

$$y(x) = A\sin(kx+\varphi)\,, \tag{6.5}$$

where A represents the maximum displacement of the string (i. e., the particle displacement) in the y-direction and k represents a scaling factor, the wave number. The argument of the sine function in (6.5) is known as the phase. For each value of x, the function $y(x)$ has a unique value, which leads to a specific y value (sometimes called a point of constant phase). The term φ is known as the phase shift because it causes a shifting of the wave profile along the x-axis (forward for a positive phase shift and backward for a negative phase shift). Such a sine function varies between $+A$ and $-A$, and one full cycle of the wave has a length of $\lambda = \frac{2\pi}{k}$. The length λ is known as the wavelength, and the maximum displacement A is known as the wave amplitude.

As this disturbance shape moves toward the right, its position moves some distance $\Delta x = x_f - x_o$ during some time interval Δt, which means the disturbance is traveling with some velocity $c = \frac{\Delta x}{\Delta t}$. Thus, the distance the wave has traveled shows this profile both before and after it has traveled the distance Δx (Fig. 6.3). The traveling wave can be expressed as a function of both position (which determines its profile) and time (which determines the distance it has traveled). The equation for a traveling wave, then, is given by

$$y(x,t) = A\sin[k(x-ct)+\varphi]\,, \tag{6.6}$$

where $t=0$ gives the shape of the wave at $t=0$ (which here is assumed constant), and $y(x,t)$ gives the shape and position of the wave disturbance as it travels. Again, A represents the amplitude, k represents the wave number, and φ represents the phase shift. Equation (6.6) is applicable for all types of waves (longitudinal, transverse, etc.) traveling in any type of medium (a spring, a string, a fluid, a solid, etc.).

Thus far, we have introduced several important basic wave concepts including wave profile, phase, phase shift, amplitude, wavelength, wave number, and wave velocity. There is one additional basic concept of great importance, the wave frequency. The frequency is defined as the rate at which (the number of times per second) a point of constant phase passes a point in space. The most obvious points of constant phase to consider are the maximum value (crest) or the minimum value (trough) of the wave. One can think of the concept of frequency less rigorously as the number of pulses generated per second by the source causing the wave. The velocity of the wave is the product of the wavelength and the frequency.

One can note that, rather than consisting of just one or a few pulses, (6.6) represents a continually varying wave propagating down some medium. Such waves are known as continuous waves. There is also a type of wave that is a bit between a single pulse and an infinitely continuous wave. A wave that consists of a finite number of cycles is known as a wave packet or a tone burst. When dealing with a tone burst, the concepts of phase velocity and group velocity are much more evident. Generally speaking, the center of the tone burst travels at the phase velocity – the ends travel close to the group velocity.

6.1.2 Properties of Waves

All waves can exhibit the following phenomena: reflection, refraction, interference and diffraction. (Transverse waves can also exhibit a phenomenon known as polarization, which allows oscillation in only one plane.)

Reflection

The easiest way to understand reflection is to consider the simple model of a transverse wave pulse traveling down a taut string that is affixed at the opposite end (as seen in Fig. 6.4. (A pulse is described here for purposes

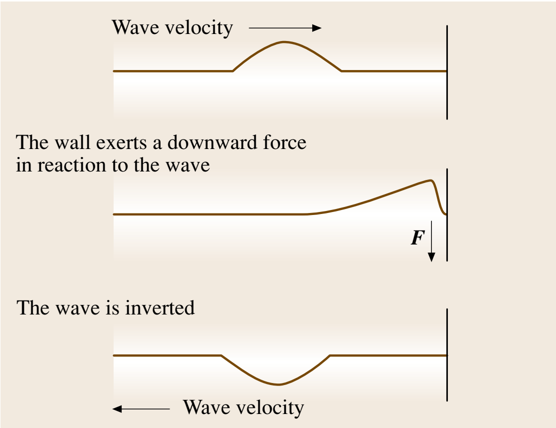


Fig. 6.4 Reflection of a wave from a fixed boundary

of clarity, but the results described apply to continuous waves as well.) As the pulse reaches the end of the string, the particles start to move upward, but they cannot because the string is fastened to the pole. (This is known as a fixed or rigid boundary.) The pole exerts a force on the particles in the string, which causes the pulse to rebound and travel in the opposite direction. Since the force of the pole on the string in the y-direction must be downward (to counteract the upward motion of the particles), there is a 180° phase shift in the wave. This is seen in the figure where the reflected pulse has flipped upside down relative to the incident pulse.

Figure 6.5 shows a different type of boundary from which a wave (or pulse) can reflect; in this case the end of the string is on a massless ring that can slide freely up and down the pole. (This situation is known as a free boundary.) As the wave reaches the ring, it drives

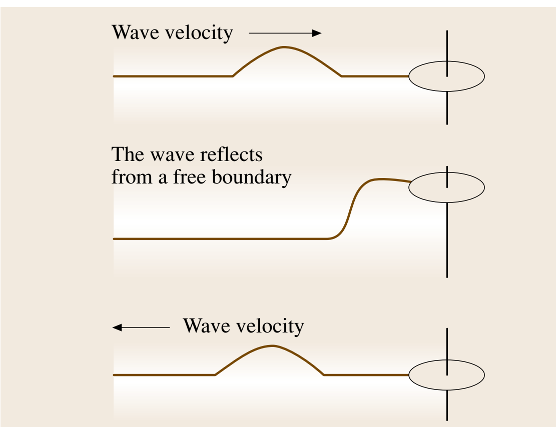


Fig. 6.5 Reflection of a wave from a free boundary

the ring upwards. As the ring moves back down, a reflected wave, which travels in a direction opposite to that of the incoming wave, is also generated. However, there is no 180° phase shift upon reflection from a free boundary.

Acoustic Impedance

Another important wave concept is that of wave impedance, which is usually denoted by the variable Z. When the reflection of the wave is not total, part of the energy in the wave can be reflected and part transmitted. For this reason, it is necessary to consider acoustic waves at an interface between two media and to be able to calculate how much of the energy is reflected and how much is transmitted. The definition of media impedance facilitates this. For acoustic waves, the impedance Z is defined as the ratio of sound pressure to particle velocity. The units for impedance are the Rayl, so named in honor of Lord Rayleigh. $1\,\text{Rayl} = 1\,\text{Pa}\,\text{s/m}$.

Often one speaks of the *characteristic impedance* of a medium (a fluid or solid); in this case one is referring to the medium in the *open space* condition where there are no obstructions to the wave which would cause the wave to reflect or scatter. The characteristic impedance of a material is usually denoted by Z_0, and it can be determined by the product of the mean density of the medium ρ with the speed of sound in the medium. In air, the characteristic impedance near room temperature is about, 410 Rayl.

The acoustic impedance concept is particularly useful. Consider a sound wave that passes from an initial medium with one impedance into a second medium with a different impedance. The efficiency of the energy transfer from one medium into the next is given by the ratio of the two impedances. If the impedances (ρc) are identical, their ratio will be 1; and all of the acoustic energy will pass from the first medium into the second across the interface between them. If the impedances of the two media are different, some of the energy will be reflected back into the initial medium when the sound field interacts with the interface between the two media. Thus the impedance enables one to characterize the acoustic transmission and reflection at the boundary of the two materials. The difference in Z, which leads to some of the energy being reflected back into the initial medium, is often referred to as the impedance mismatch. When the (usual) acoustic boundary conditions apply and require that the particle velocity and pressure be continuous across the interface between the two media, one can calculate the percentage of the energy that is reflected back into the medium. This is given in fractional form by the reflection coefficient given by

$$R = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1}\right)^2 , \quad (6.7)$$

where Z_1 and Z_2 are the impedances of the two media. Since both values of Z must be positive, R must be less than one. The fraction of the energy transmitted into the second medium is given by $T = 1 - R$ because 100% of the energy must be divided between T and R.

Refraction

Refraction is a change of the direction of wave propagation as the wave passes from one medium into another across an interface. Bending occurs when the wave speed is different in the two media. If there is an angle between the normal to the plane of the boundary and the incident wave, there is a brief time interval when part of the wave is in the original medium (traveling at one velocity) and part of the wave is in the second medium (traveling at a different velocity). This causes the bending of the waves as they pass from the first medium to the second. (There is no bending at normal incidence.)

Reflection and refraction can occur simultaneously when a wave impinges on a boundary between two media with different wave propagation speeds. Some of the energy of the wave is reflected back into the original medium, and some of the energy is transmitted and refracted into the second medium. This means that a wave incident on a boundary can generate two waves: a reflected wave and a transmitted wave whose direction of propagation is determined by Snell's law.

All waves obey Snell's law. For optical waves the proper form of Snell's law is:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 , \quad (6.8)$$

where n_1 and n_2 are the refractive indices and θ_1 and θ_2 are propagation directions. For acoustic waves the proper form of Snell's law is

$$\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2} , \quad (6.9)$$

where v_1 is the wave velocity in medium 1 and v_2 is the wave velocity in medium 2. These two forms are very similar since the refractive index is $n = c/C_m$, where c is the velocity of light in a vacuum and C_m is the velocity of light in the medium under consideration.

Interference

Spatial Interference. Interference is a phenomenon that occurs when two (or more) waves add together. Consider two identical transverse waves traveling to the right (one after the other) down a string towards a boundary at the end. When the first wave encounters the boundary, it reflects and travels in the leftward direction. When it encounters the second, rightward moving wave the two waves add together linearly (in accordance with the principle of superposition). The displacement amplitude at the point in space where two waves combine is either greater than or less than the displacement amplitude of each wave. If the resultant wave has an amplitude that is smaller than that of either of the original two waves, the two waves are said

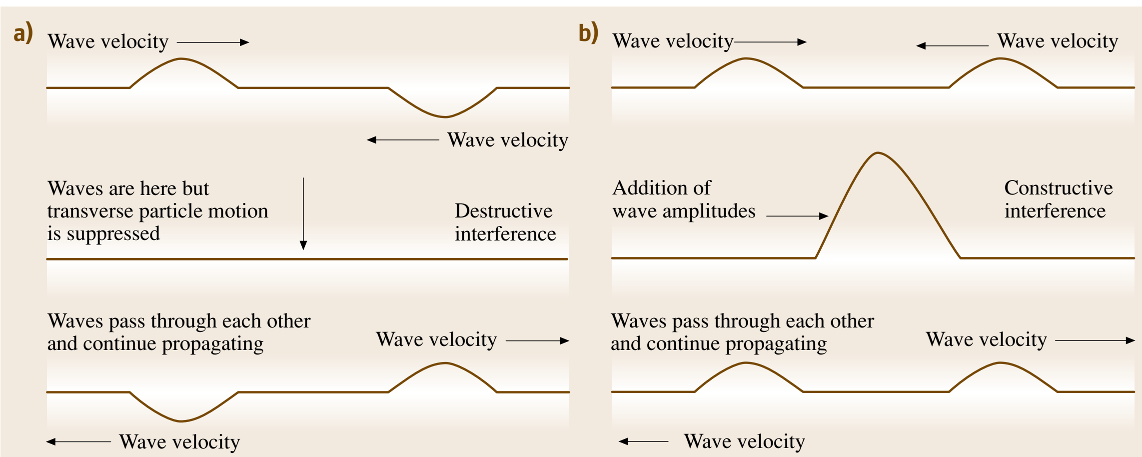


Fig. 6.6a,b Two waves passing through each other exhibiting **(a)** destructive and **(b)** constructive interference

to have destructively interfered with one another. If the combined wave has an amplitude that is greater than either of its two constituent waves, then the two waves are said to have constructively interfered with each other. The maximum possible displacement of the combination is the sum of the maximum possible displacements of the two waves (complete constructive interference); the minimum possible displacement is zero (complete destructive interference for waves of equal amplitude). It is important to note that the waves interfere only as they pass through one another. Figure 6.6 shows the two special cases of complete destructive and complete constructive interference (for clarity only a portion of the wave is drawn).

If a periodic wave is sent down the string and meets a returning periodic wave traveling in the opposite direction, the two waves interfere. This results in wave superposition, i. e., the resulting amplitude at any point and time is the sum of the amplitudes of the two waves. If the returning wave is inverted (due to a fixed boundary reflection) and if the length of the string is an integral multiple of the half-wavelength corresponding to the drive frequency, conditions for resonance are satisfied. This resonance produces a standing wave. An example of a standing wave is shown in Fig. 6.7. The points of maximum displacement are known as the standing-wave antinodes. The points of zero displacement are known as the standing-wave nodes.

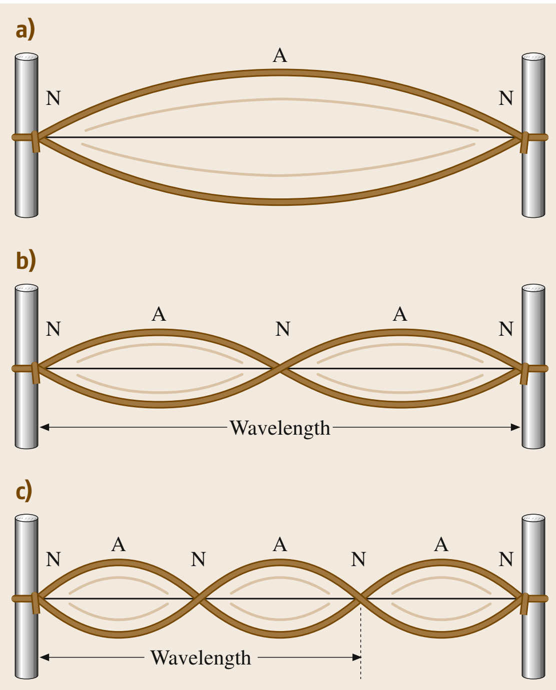


Fig. 6.7a–c Standing waves in a string with nodes and antinodes indicated. **(a)** The fundamental; **(b)** the second harmonic; **(c)** the third harmonic

Resonance Behavior. Every vibrating system has some characteristic frequency that allows the vibration amplitude to reach a maximum. This characteristic frequency is determined by the physical parameters (such as the geometry) of the system. The frequency that causes maximum amplitude of vibration is known as the resonant frequency, and a system driven at its resonant frequency is said to be in resonance. Standing waves are simply one type of resonance behavior.

Longitudinal acoustic waves can also exhibit resonance behavior. When the distance between a sound emitter and a reflector is an integer number of half wavelengths, the waves interfere and produce standing waves. This interference can be observed optically, acousticly or electronically. By observing a large number of standing waves one can obtain an accurate value of the wavelength, and hence an accurate value of the wave velocity.

One of the simplest techniques for observing acoustic resonances in water, or any other transparent liquid, is to illuminate the resonance chamber, then to focus a microscope on it. The microscope field of view images the nodal lines that are spaced half an acoustic wavelength apart. A screw that moves the microscope perpendicular to the lines allows one to make very accurate wavelength measurements, and hence accurate sound velocity measurements.

Temporal Interference. So far we have considered the interference of two waves that are combining in space (this is referred to as spatial interference). It is also possible for two waves to interfere because of a difference in frequency (which is referred to as temporal interference). One interesting example of this is the phenomenon of wave beating.

Consider two sinusoidal acoustic waves with slightly different frequencies that arrive at the same point in space. Without loss of generality, we can assume that these two waves have the same amplitude. The superposition principle informs us that the resultant pressure caused by the two waves is the sum of the pressure caused by each wave individually. Thus, we have for the total pressure

$$p_{\mathrm{T}}(t) = A\left[\cos(\omega_1 t) + \cos(\omega_2 t)\right] . \tag{6.10}$$

By making use of a standard trigonometric identity, this can be rewritten as

$$p_{\mathrm{T}}(t)=2A\cos\left(\frac{(\omega_1-\omega_2)}{2}t\right)\cos\left(\frac{(\omega_1+\omega_2)}{2}t\right). \tag{6.11}$$

Since the difference in frequencies is small, the two waves can be in phase, causing constructive interference and reinforcing one another. Over some period of time, the frequency difference causes the two waves to go out of phase, causing destructive interference (when $\omega_1 t$ eventually leads $\omega_2 t$ by 180°). Eventually, the waves are again in phase and, they constructively interfere again. The amplitude of the combination will rise and fall in a periodic fashion. This phenomenon is known as the beating of the two waves. This beating phenomenon can be described as a separate wave with an amplitude that is slowly varying according to

$$p(t)=A_0(t)\cos(\omega_{\mathrm{avg}}t) \tag{6.12}$$

where

$$A_0(t)=2A\cos\left(\frac{\omega_1-\omega_2}{2}t\right) \tag{6.13}$$

and

$$\omega_{\mathrm{avg}}=\frac{(\omega_1+\omega_2)}{2}. \tag{6.14}$$

The cosine in the expression for $A_0(t)$ varies between positive and negative 1, giving the largest amplitude in each case. The period of oscillation of this amplitude variation is given by

$$T_{\mathrm{b}}=\frac{2\pi}{\omega_1-\omega_2}=\frac{2\pi}{\omega_{\mathrm{b}}}=\frac{1}{f_{\mathrm{b}}}. \tag{6.15}$$

The frequency f_{b} of the amplitude variation is known as the beat frequency. Figure 6.8 shows the superposition of two waves that have two frequencies that are different but close together. The beat frequency corresponds to the difference between the two frequencies that are beating.

The phenomenon of beating is often exploited by musicians in tuning their instruments. By using a reference (such as a 440 Hz tuning fork that corresponds to the A above middle C) one can check the tuning of the instrument. If the A above middle C on the instrument is out of tune by 2 Hz, the sounds from the tuning fork and the note generate a 2 Hz beating sound when they are played together. The instrument is then tuned until the beating sound vanishes. Then the frequency of the instrument is the same as that of the tuning fork. Once the A is in tune, the other notes can be tuned relative to

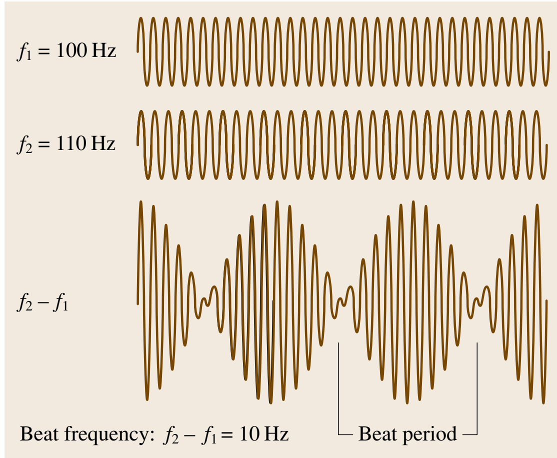


Fig. 6.8 The beating of two waves with slightly different frequencies

the A by counting the beats per unit time when different notes are played in various combinations.

Multi-frequency Sound

When sound consists of many frequencies (not necessarily all close together), one needs a means of characterizing the sound level. One may use a weighted average of the sound over all the frequencies present, or one may use information about how much energy is distributed over a particular range of frequencies. A selected range of frequencies is known as a frequency band. By means of filters (either acoustic or electrical, depending on the application) one can isolate the various frequency bands across the entire frequency spectrum. One can then talk of the acoustic pressure due to a particular frequency band. The *band pressure level* (PL) is given by

$$\mathrm{PL}_{\mathrm{band}}=20\log_{10}\left(\frac{p_{\mathrm{band}}}{p_{\mathrm{ref}}}\right) \tag{6.16}$$

where p_{band} is the room-mean-square (rms) average pressure of the sound in the frequency band range and p_{ref} is the standard reference for sound in air, 20 µPa. The average of the pressures of the frequency bands over the complete spectrum is the average acoustic signal. However, the presence of multiple frequencies complicates the situation: one does not simply add the frequency band pressures or the band pressure levels. Instead, it is the p^2 values which must be summed

$$p_{\mathrm{rms}}^2=\sum p_{\mathrm{band}}^2 \tag{6.17}$$

or

$$\mathrm{SPL} = 10\log_{10}\left(\frac{p_{\mathrm{rms}}^2}{p_{\mathrm{ref}}^2}\right) = 10\log_{10}\sum\left(\frac{p_{\mathrm{band}}}{p_{\mathrm{ref}}}\right)^2 \tag{6.18}$$

or, simplifying further

$$\mathrm{SPL} = 10\log_{10}\sum(10)^{(\mathrm{PL}_{\mathrm{band}}/10)} . \tag{6.19}$$

The octave is a common choice for the width of a frequency band. With a one-octave filter, only frequencies up to twice the lowest frequency of the filter are allowed to pass. One should also note that, when an octave band filter is labeled with its center frequency, this is determined by the geometric mean (not the arithmetic mean), i. e.,

$$f_{\mathrm{center}} = \sqrt{f_{\mathrm{low}} f_{\mathrm{high}}} = \sqrt{2 f_{\mathrm{low}}^2} = \sqrt{2} f_{\mathrm{low}} . \tag{6.20}$$

Coherent Signals

Two signals are coherent if there is a fixed relative phase relation between them. Two loudspeakers driven by the same source would be coherent. However, two loudspeakers being driven by two compact-disc (CD) players (even if each player was playing a copy of the same CD) would not be coherent because there is no connection causing a constant phase relationship. For two incoherent sources, the total pressure is

$$p_{\mathrm{tot}}^2 = \left(p_{1(\mathrm{rms})} + p_{2(\mathrm{rms})}\right)^2 = p_{1(\mathrm{rms})}^2 + p_{2(\mathrm{rms})}^2 \tag{6.21}$$

(where the $2p_{1(\mathrm{rms})}p_{2(\mathrm{rms})}$ term has averaged out to zero). For coherent sources, however, the fixed phase relationship allows for the possibility of destructive or constructive interference. Therefore, the signal can vary in amplitude between $(p_{1(\mathrm{rms})} + p_{2(\mathrm{rms})})^2$ and $(p_{1(\mathrm{rms})} - p_{2(\mathrm{rms})})^2$.

Diffraction

In optics it is usual to begin a discussion of diffraction by pointing out grating effects. In acoustics one seldom encounters grating effects. Instead, one encounters changes in wave direction of propagation resulting from diffraction. Thus, in acoustics it is necessary to begin on a more fundamental level.

The phenomenon of diffraction is the bending of a wave around an edge. The amount of bending that occurs depends on the relative size of the wavelength compared to the size of the edge (or aperture) with which it interacts. When considering refraction or reflection it is often convenient to model the waves by drawing a single ray in the direction of the wave's propagation. However, the ray approach does not provide a means to model the bending caused by diffraction. The bending of a wave by diffraction is the result of wave interference. The more accurate approach, then, is to determine the magnitude of each wave that is contributing to the diffraction and to determine how it is interfering with other waves to cause the diffraction effect observed.

It is useful to note that diffraction effects can depend on the shape of the wavefronts that encounter the edge around which the diffraction is occurring. Near the source the waves can have a very strong curvature relative to the wavelength. Far enough from the source the curvature diminishes significantly (creating essentially plane waves). Fresnel diffraction occurs when curved wavefronts interact. Fraunhofer diffraction occurs when planar wavefronts interact. In acoustics these two regions are known as the near field (the Fresnel zone) and the far field (the Fraunhofer zone), respectively. In optics these two zones are distinguished by regions in which two different integral approximations are valid.

6.1.3 Wave Propagation in Fluids

The propagation of an acoustic wave is described mathematically by the acoustic wave equation. One can use the approach of continuum mechanics to derive equations appropriate to physical acoustics [6.3]. In the continuum approach one postulates fields of density, stress, velocity, etc., all of which must satisfy basic conservation laws. In addition, there are constitutive relations which characterize the medium. For illustration, acoustic propagation through a compressible fluid medium is considered first. As the acoustic disturbance passes through a small area of the medium, the density of the medium at that location fluctuates. As the crest of the acoustic pressure wave passes through the region, the density in that region increases; this compression is known as the acoustic condensation. Conversely, when the trough of the acoustic wave passes through the region, the density of the medium at that location decreases; this expansion is known as the acoustic rarefaction.

In a gas, the constitutive relationship needed to characterize the pressure fluctuations is the ideal gas equation of state, $PV = nRT$, where P is the pressure of the gas, V is the volume of the gas, n is the number of moles of the gas, R is the universal gas constant ($R = 8.3145\,\mathrm{J/mol\,K}$), and T is the temperature of the gas. In a given, small volume of the gas, there is a variation of the density ρ from its equilibrium value ρ_0 caused by the change in pressure $\Delta P = P - P_0$ as the disturbance passes through that volume. (Here P is the pressure at any instant in time and P_0 is the equilibrium pressure.)

In many situations, there are further constraints on the system which simplify the constitutive relationship. A gas may act as a heat reservoir. If the processes occur on a time scale that allows heat to be exchanged, the gas is maintained at a constant temperature. In this case, the constitutive relationship can be simplified and expressed as $P_0 V_0 = PV =$ a constant. Since the number of gas molecules (and hence the mass) is constant, we can express this as the isothermal condition

$$\frac{P}{P_0} = \frac{\rho}{\rho_0} , \tag{6.22}$$

which relates the instantaneous pressure to the equilibrium pressure.

Most acoustic processes occur with no exchange of heat energy between adjacent volumes of the gas. Such processes are known as adiabatic or isentropic processes. Under such conditions, the constitutive relation is modified according to the adiabatic condition. For the adiabatic compression of an ideal gas, it has been found that the relationship

$$PV^\gamma = P_0 V_0^\gamma \tag{6.23}$$

holds, where γ is the ratio of the specific heat of the gas at constant pressure to the specific heat at constant volume. This leads to an adiabatic constraint for the acoustic process given by

$$\frac{P}{P_0} = \left(\frac{\rho}{\rho_0}\right)^\gamma . \tag{6.24}$$

When dealing with a real gas, one can make use of a Taylor expansion of the pressure variations caused by the fluctuations in density

$$\begin{aligned} P = P_0 &+ \left[\frac{\partial P}{\partial \rho}\right]_{\rho_0} (\Delta\rho) \\ &+ \frac{1}{2}\left[\frac{\partial^2 P}{\partial \rho^2}\right]_{\rho_0} (\Delta\rho)^2 + \ldots , \end{aligned} \tag{6.25}$$

where

$$\Delta\rho = (\rho - \rho_0) . \tag{6.26}$$

When the fluctuations in density are small, only the first-order terms in $\Delta\rho$ are nonnegligible. In this case, one can rearrange the above equation as

$$\Delta P = P - P_0 = \left[\frac{\partial p}{\partial \rho}\right]_{\rho_0} \Delta\rho = B\frac{\Delta\rho}{\rho_0} , \tag{6.27}$$

where $B = \rho_0 \left[\frac{\partial p}{\partial \rho}\right]_{\rho_0}$ is the adiabatic bulk modulus of the gas. This equation describes the relationship between the pressure of the gas and its density during an expansion or contraction. (When the fluctuations are not small one has finite-amplitude (or nonlinear) effects which are considered later.)

Let us now consider the physical motion of a fluid as the acoustic wave passes through it. We begin with an infinitesimal volume element $\mathrm{d}V$ that is fixed in space, and we consider the motion of the particles as they pass through this region. Since mass is conserved, the net flux of mass entering or leaving this fixed volume must correspond to a change in the density of the fluid contained within that volume. This is expressed through the continuity equation,

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \boldsymbol{u}) . \tag{6.28}$$

The rate of mass change in the region is

$$\frac{\partial \rho}{\partial t} \mathrm{d}V , \tag{6.29}$$

and the net influx of mass into this region is given by

$$-\nabla \cdot (\rho \boldsymbol{u})\, \mathrm{d}V . \tag{6.30}$$

Consider a volume element of fluid as it moves with the fluid. This $\mathrm{d}V$ of fluid contains some infinitesimal amount of mass, $\mathrm{d}m$. The net force acting on this small mass of fluid is given by Newton's second law, $\mathrm{d}\boldsymbol{F} = \boldsymbol{a}\,\mathrm{d}m$. It can be shown that Newton's second law leads to a relationship between the particle velocity and the acoustic pressure. This relationship, given by

$$\rho_0 \frac{\partial \boldsymbol{u}}{\partial t} = -\nabla p , \tag{6.31}$$

is known as the linear Euler equation.

By combining our adiabatic pressure condition with the continuity equation and the linear Euler equation, one can derive the acoustic wave equation. This equation takes the form

$$\nabla^2 p = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} , \tag{6.32}$$

where c is the speed of the sound wave which is given by

$$c = \sqrt{B/\rho_0} . \tag{6.33}$$

6.1.4 Wave Propagation in Solids

Similarly, one can consider the transmission of an acoustic wave through a solid medium. As an example, consider the one-dimensional case for propagation of an acoustic wave through a long bar of length L and cross-sectional area S. In place of the pressure, we consider the stress applied to the medium, which is given by the relationship

$$\sigma = F/S \tag{6.34}$$

where σ is the stress, F is the force applied along the length (L) of the bar, and S is the cross sectional area of the bar.

A stress applied to a material causes a resultant compression or expansion of the material. This response is the strain ζ. Strain is defined by the relationship,

$$\zeta = \Delta L / L_o \,, \tag{6.35}$$

where ΔL is the change in length of the bar, and L_o is the original length of the bar.

Let us consider the actual motion of particles as an acoustic wave passes through some small length of the bar $\mathrm{d}x$. This acoustic wave causes both a stress and a strain. The Hooke's law approximation, which can be used for most materials and vibration amplitudes, provides the constitutive relationship that relates the stress applied to the material with the resulting strain. Hooke's law states that stress is proportional to strain, or

$$\sigma = -Y\zeta \,. \tag{6.36}$$

If we consider the strain over our small length $\mathrm{d}x$, we can write this as

$$\frac{F}{S} = -Y\left(\frac{\mathrm{d}L}{\mathrm{d}x}\right) \tag{6.37}$$

or

$$F = -YS\left(\frac{\mathrm{d}L}{\mathrm{d}x}\right) . \tag{6.38}$$

The net force acting on our segment $\mathrm{d}x$ is given by

$$\mathrm{d}F = -\left(\frac{\partial F}{\partial x}\right)\mathrm{d}x = YS\left(\frac{\partial^2 L}{\partial x^2}\right)\mathrm{d}x \,. \tag{6.39}$$

Again, we can make use of Newton's second law, $\boldsymbol{F} = m\boldsymbol{a}$, and express this force in terms of the mass and acceleration of our segment. The mass of the segment of length $\mathrm{d}x$ is simply the density times the volume, or $\mathrm{d}m = \rho\,\mathrm{d}V = \rho S\,\mathrm{d}x$. Thus,

$$\mathrm{d}F = \left(\frac{\partial^2 L}{\mathrm{d}t^2}\right)\mathrm{d}m = \rho S\left(\frac{\partial^2 L}{\mathrm{d}t^2}\right)\mathrm{d}x \,, \tag{6.40}$$

where $\frac{\partial^2 L}{\mathrm{d}t^2}$ is the acceleration of the particles along the length $\mathrm{d}x$ as the acoustic wave stresses it. Equating our two expressions for the net force acting on our segment, $\mathrm{d}F$,

$$\frac{\partial^2 L}{\partial x^2} = \frac{1}{c^2}\left(\frac{\partial^2 L}{\partial t^2}\right) , \tag{6.41}$$

where

$$c = \sqrt{Y/\rho} \tag{6.42}$$

is the speed at which the acoustic wave is traveling through the bar. The form of the equation for the propagation of an acoustic wave through a solid medium is very similar to that for the propagation of an acoustic wave through a fluid.

Both of the wave equations developed so far have implicitly considered longitudinal compressions; for example, the derivation of the wave equation for an acoustic wave traveling down a thin bar assumed no transverse components to the motion. However, if we consider transverse motion, the resulting wave equation is of the same form as that for longitudinal waves. For longitudinal waves, the solution to the wave equation is given by

$$L(x,t) = A\cos(\omega t \pm kx + \phi) \text{ (longitudinal)} \,, \tag{6.43}$$

where $L(x,t)$ represents the amount of compression or rarefaction at some position x and time t. For transverse waves, the solution to the wave equation is given by

$$y(x,t) = A\cos(\omega t \pm kx + \phi) \text{ (transverse)} \,, \tag{6.44}$$

where $y(x,t)$ represents the vibration orthogonal to the direction of wave motion as a function of x and t. In both cases, A is the vibration amplitude, $k = 2\pi/\lambda$ is the wave number, and ϕ is the phase shift (which depends on the initial conditions of the system).

One can also consider an acoustic disturbance traveling in two dimensions; let us first consider a thin,

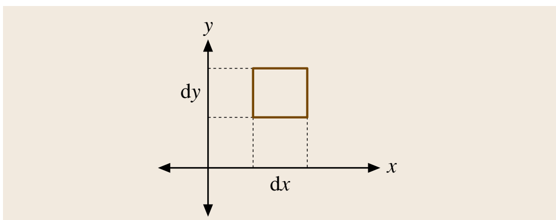


Fig. 6.9 Definitions necessary for the representation of a two-dimensional wave

stretched membrane as seen in Fig. 6.9. Let σ_s be the areal surface density (with units of kg/m^2), and let Γ be the tension per unit length. The transverse displacement of an infinitesimally small area of the membrane dS will now be a function of the spatial coordinates x and y along the two dimensions and the time t. Let us define the infinitesimal area $\mathrm{d}S = \mathrm{d}x\,\mathrm{d}y$ and the displacement of dS as the acoustic disturbance passes through it to be $u(x, y, t)$. Newton's second law can now be applied to our areal element dS

$$\begin{aligned}&\left[(\Gamma\partial u/\partial x)_{x+\mathrm{d}x,y}-(\Gamma\partial u/\partial y)_{x,y}\right]\mathrm{d}y\\&\quad+\left[(\Gamma\partial u/\partial y)_{x,y+\mathrm{d}y}-(\Gamma\partial u/\partial x)_{x,y}\right]\mathrm{d}x\\&=\Gamma\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)\mathrm{d}x\,\mathrm{d}y\end{aligned} \tag{6.45}$$

and

$$\Gamma\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)\mathrm{d}x\,\mathrm{d}y=\sigma_s\frac{\partial^2 u}{\partial t^2}\mathrm{d}x\,\mathrm{d}y \tag{6.46}$$

or

$$\nabla^2 u=\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)=\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}\,, \tag{6.47}$$

where $c=\sqrt{\Gamma/\sigma_s}$ is the speed of the acoustic wave in the membrane. Equation (6.47) now describes a two-dimensional wave propagating along a membrane.

The extension of this idea to three dimensions is fairly straightforward. As an acoustic wave passes through a three-dimensional medium, it can cause pressure fluctuations (leading to a volume change) in all three dimensions. The displacement u is now a function four variables $u = u(x, y, z, t)$. The volume change of a cube is given by

$$\Delta V=\Delta x\Delta y\Delta z\left(1+\frac{\partial u}{\partial x}\right)\left(1+\frac{\partial u}{\partial y}\right)\left(1+\frac{\partial u}{\partial z}\right), \tag{6.48}$$

$$\Delta V\approx\Delta x\Delta y\Delta z\left(1+\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}\right), \tag{6.49}$$

where the higher-order cross terms are negligibly small and have been dropped. This can be rewritten as

$$\Delta V=\Delta x\Delta y\Delta z\,(1+\nabla\cdot\boldsymbol{u})\,. \tag{6.50}$$

If the mass is held constant, and only the volume changes, the density change is given by

$$\rho_0+\rho_1=\frac{\rho_0}{1+\nabla\cdot\boldsymbol{u}}\,. \tag{6.51}$$

But since the denominator is very close to unity, we may rewrite this using the binomial expansion. Solving for ρ_1 we have

$$\rho_1\approx-\rho_0\nabla\cdot\boldsymbol{u}\,. \tag{6.52}$$

Again we can consider Newton's second law; without loss of generality, consider the force exerted along the x-direction from the pressure due to the acoustic wave, which is given by

$$F_x=\left[p(x,t)-p(x+\Delta x,t)\right]\Delta y\Delta z\,, \tag{6.53}$$

where $\Delta y \Delta z$ is the infinitesimal area upon which the pressure exerts a force. We can rewrite this as

$$F_x=-\Delta y\Delta z\left[\left(\frac{\partial p}{\partial x}\right)\Delta x\right], \tag{6.54}$$

where the results for F_y and F_z are similar. Combining these, we can express the total force vector as

$$\boldsymbol{F}=-\nabla p\Delta x\Delta y\Delta z\,. \tag{6.55}$$

Using the fact that the mass can be expressed in terms of the density, we can rewrite this as

$$-\nabla p=\rho_0\frac{\partial^2\boldsymbol{u}}{\partial t^2}\,. \tag{6.56}$$

As in the case with fluids, we need a constitutive relationship to finalize the expression. For most situations in solids, the adiabatic conditions apply and the pressure fluctuation is a function of the density only. Making use of a Taylor expansion, we have

$$p\approx p(\rho_0)+(\rho-\rho_0)\frac{\mathrm{d}p}{\mathrm{d}\rho} \tag{6.57}$$

but since $p = p_0 + p_1$, we can note that

$$p_1=\rho_1\frac{\partial p}{\partial\rho}\,. \tag{6.58}$$

Using (6.52), we can eliminate ρ_1 from (6.58) to yield

$$p_1=-\rho_0\left(\frac{\mathrm{d}p}{\mathrm{d}\rho}\right)\nabla\cdot\boldsymbol{u}\,. \tag{6.59}$$

We can eliminate the divergence of the displacement vector from this equation by taking the divergence of (6.56) and the second time derivative of (6.59). This gives two expressions that both equal $-\rho_0\partial^2\,(\nabla\cdot\boldsymbol{u})\,/\,\mathrm{d}t^2$ and thus are equal to each other. From this we can determine the full form of our wave equation

$$\frac{\partial^2 p_1}{\partial t^2}=c^2\nabla^2 p_1\,, \tag{6.60}$$

where c is again the wave speed and is given by

$$c^2 = \frac{\mathrm{d}p}{\mathrm{d}\rho} . \tag{6.61}$$

It should be noted that the above considerations were for an isotropic solid. In a crystalline medium, other complications can arise. These will be noted in a subsequent section.

6.1.5 Attenuation

In a real physical system, there are mechanisms by which energy is dissipated. In a gas, the dissipation comes from thermal and viscous effects. In a solid, dissipation comes from interactions with dislocations in the solid (holes, displaced atoms, interstitial atoms of other materials, etc.) and grain boundaries between adjacent parts of the solid, In practice, loss of energy over one acoustic cycle is negligible. However, as sound travels over a longer path, one expects these energy losses to cause a significant decrease in amplitude. In some situations, these dissipation effects must be accounted for in the solution of the wave equation.

The solution of the wave equation can be written in exponential form,

$$A = A_0\,\mathrm{e}^{\mathrm{i}(k'x-\omega t)} . \tag{6.62}$$

If one wishes to account for dissipation effects, one can assume that the wave number k' has an imaginary component, i. e.

$$k' = k + \mathrm{i}\alpha , \tag{6.63}$$

where k and α are both real and $\mathrm{i} = \sqrt{-1}$. Using this value of k' for the new wave number we have

$$A = A_0\,\mathrm{e}^{\mathrm{i}(k'x-\omega t)} = A_0\,\mathrm{e}^{\mathrm{i}\,[(k+\mathrm{i}\alpha)x-\omega t]} . \tag{6.64}$$

Simplifying, one has

$$\begin{aligned} A &= A_0\,\mathrm{e}^{\mathrm{i}(kx-\omega t)+\mathrm{i}^2\alpha x} = A_0\,\mathrm{e}^{\mathrm{i}(kx-\omega t)-\alpha x} \\ &= A_0\,\mathrm{e}^{-\alpha x}\,\mathrm{e}^{\mathrm{i}(kx-\omega t)} , \end{aligned} \tag{6.65}$$

where α is known as the absorption coefficient. The resulting equation is modulated by a decreasing exponential function; i. e., an undriven acoustic wave passing through a lossy medium is damped to zero amplitude as $x \to \infty$.

6.2 Applications of Physical Acoustics

There are several interesting phenomena associated with the application of physical acoustics. The first is the wave velocity itself. Table 6.1 shows the wave velocity of sound in various fluids (both gases and liquids). Table 6.2 shows the wave velocity of sound in various solids (both metals and nonmetals). The velocity increases as one goes from gases to liquids to solids. The velocity variation from gases to liquids comes from the fact that gas molecules must travel farther before striking another gas molecule. In a liquid, molecules are closer together, which means that sound travels faster. The change from liquids to solids is associated with increase of binding strength as one goes from liquid to solid. The rigidity of a solid leads to higher sound velocity than is found in liquids.

6.2.1 Crystalline Elastic Constants

Another application of physical acoustics involves the measurement of the crystalline elastic constants in a lattice. In Sect. 6.1.2, we considered the propagation of an acoustic field along a one-dimensional solid (in which

Table 6.1 Typical values of the sound velocity in fluids (25 °C)

Gas	Velocity (m/s)	Liquid	Velocity (m/s)
Air	331	Carbon tetrachloride (CCl_4)	929
Carbon dioxide (CO_2)	259	Ethanol (C_2H_6O)	1207
Hydrogen (H_2)	1284	Ethylene glycol ($C_2H_6O_2$)	1658
Methane (CH_4)	430	Glycerol ($C_3H_8O_3$)	1904
Oxygen (O_2)	316	Mercury (Hg)	1450
Sulfur dioxide (SO_2)	213	Water (distilled)	1498
Helium (H_2)	1016	Water (sea)	1531

Table 6.2 Typical values of the sound velocity in solids (25 °C)

Metals	Longitudinal velocity (m/s)	Shear (transverse) velocity (m/s)
Aluminum (rolled)	6420	3040
Beryllium	12 890	8880
Brass (0.70 Cu, 0.30 Zn)	4700	2110
Copper (rolled)	5010	2270
Iron	5960	3240
Tin (rolled)	3320	1670
Zinc (rolled)	4210	2440
Lead (rolled)	1960	610
Nonmetals	**Longitudinal velocity (m/s)**	**Shear (transverse) velocity (m/s)**
Fused silica	5968	3764
Glass (Pyrex)	5640	3280
Lucite	2680	1100
Rubber (gum)	1550	–
Nylon	2620	1070

the internal structure of the solid played no role in the propagation of the wave). Real solids exist in three dimensions, and the acoustic field propagation depends on the internal structure of the material. The nature of the forces between the atoms (or molecules) that make up the lattice cause the speed of sound to be different along different directions of propagation (since the elastic force constants are different along the different directions). The measurement of crystal elastic constants depends on the ability to make an accurate determination of the wave velocity in different directions in a crystalline lattice. This can be done by cutting (or lapping) crystals in such a manner that parallel faces are in the directions to be measured.

For an isotropic solid one can determine the compressional modulus and the shear modulus from a single sample since both compressional and shear waves can be excited. For cubic crystals at least two orientations are required since there are three elastic constants (and still only two waves to be excited). For other crystalline symmetries a greater number of measurements is required.

6.2.2 Resonant Ultrasound Spectroscopy (RUS)

Recently a new technique for measuring crystalline elastic constants, known as resonant ultrasound spectroscopy (RUS), has been developed [6.4]. Typically, one uses a very small sample with a shape that has known acoustic resonant modes (usually a small parallelepiped, though sometimes other geometries such as cylinders are used). The sample is placed so that the driving transducer makes a minimal contact with the surface of the sample (the boundaries of the sample must be pressure-free and (shearing) traction-free for the technique to work). Figure 6.10 shows a photograph of a small parallelepiped sample mounted in an RUS apparatus.

After the sample is mounted, the transducer is swept through a range of frequencies (usually from a few hertz to a few kilohertz) and the response of the material is

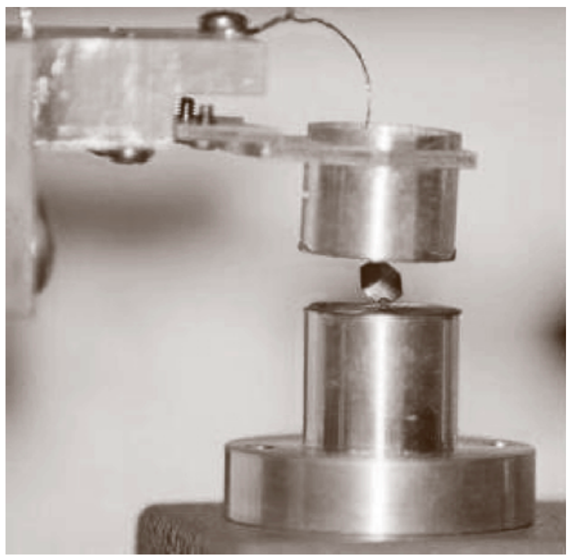


Fig. 6.10 Sample mounted for an RUS measurement. Sample dimensions are 2.0 mm × 2.5 mm × 3.0 mm

measured. Some resonances are caused by the geometry of the sample (just as a string of fixed length has certain resonant frequencies determined by the length of the string). In the RUS technique, some fairly sophisticated software eliminates the geometrical resonances; the remaining resonances are resonances of the internal lattice structure of the material. These resonances are determined by the elastic constants. The RUS technique, then, is used to evaluate all elastic constants from a single sample from the spectrum of resonant frequencies produced by the various internal resonances.

Measurement of Attenuation with the RUS Technique

The RUS technique is also useful for measuring the attenuation coefficients of solid materials. The resonance curves generated by the RUS experiment are plots of the response amplitude of the solid as a function of input frequency (for a constant input amplitude). Every resonance curve has a parameter known as the Q of the system. The Q value can be related to the maximum amplitude $1/\mathrm{e}$ value, which in turn can be related to the attenuation coefficient. Thus, the resonance curves generated by the RUS experiment can be used to determine the attenuation in the material at various frequencies.

6.2.3 Measurement Of Attenuation (Classical Approach)

Measurement of attenuation at audible frequencies and below is very difficult. Attenuation is usually measured at ultrasonic frequencies since the plane-wave approximation can be satisfied. The traditional means of measuring the acoustic attenuation requires the measurement of the echo train of an acoustic tone burst as it travels through the medium. Sound travels down the length of the sample, reflects from the opposite boundary and returns to its origin. During each round trip, it travels a distance twice the length of the sample. The transducer used to emit the sound now acts as a receiver and measures the amplitude as it strikes the initial surface. The sound then continues to reflect back and forth through the sample. On each subsequent round trip, the sound amplitude is diminished.

Measured amplitude values are then fit to an exponential curve, and the value of the absorption coefficient is determined from this fit. Actually, this experimental arrangement measures the insertion loss of the system, the losses associated with the transducer and the adhesive used to bond the transducer to the sample as well as the attenuation of sound in the sample. However, the values of the insertion loss of the system and the attenuation inside the sample are usually very close to each other. If one needs the true attenuation in the sam-

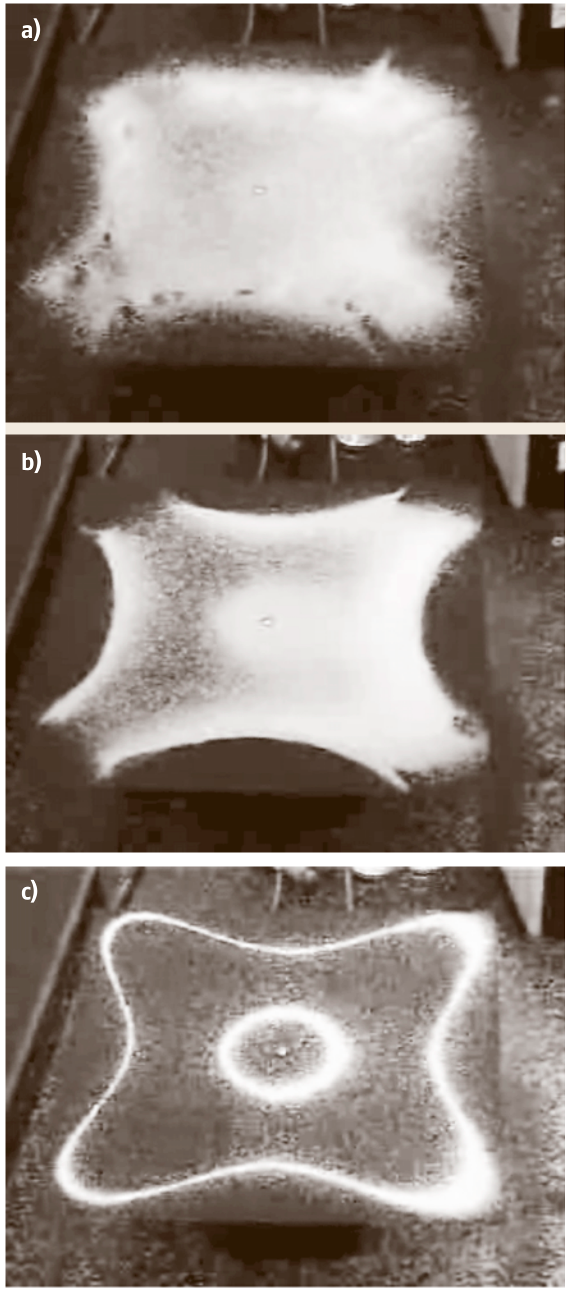


Fig. 6.11 **(a)** Fine rice on a plate; **(b)** as the plate is excited acoustically the rice begins to migrate to the nodes; **(c)** the Chladni pattern has formed

ple, one can use various combinations of transducers to account for the other losses in the system [6.5].

Losses in a sample can come from a number of sources: viscosity, thermal conductivity, and molecular relaxation. Viscosity and thermal conductivity are usually referred to as classical losses. They can be calculated readily. Both are linearly dependent on frequency. Relaxation is a frequency-dependent phenomenon. The maximum value occurs when the sound frequency is the same as the relaxation frequency, which is determined by the characteristics of the medium. Because of this complication, determination of the attenuation to be expected can be difficult.

6.2.4 Acoustic Levitation

Acoustic levitation involves the use of acoustic vibrations to move objects from one place to the other, or to keep them fixed in space. Chladni produced an early form of acoustic levitation to locate the nodal planes in a vibrating plate. Chladni discovered that small particles on a plate were moved to the nodal planes by plate vibrations. An example of Chladni figures is shown in Fig. 6.11. Plate vibrations have caused powder to migrate toward nodal planes, making them much more obvious.

The use of radiation pressure to counterbalance gravity (or buoyancy) has recently led to a number of situations in which levitation is in evidence.

1. The force exerted on a small object by radiation pressure can be used to counterbalance the pull of gravity [6.6].
2. The radiation force exerted on a bubble in water by a stationary ultrasonic wave has been used to counteract the hydrostatic or buoyancy force on the bubble. This balance of forces makes it possible for the bubble to remain at the same point indefinitely. Single-bubble sonoluminescence studies are now possible [6.7].
3. Latex particles having a diameter of 270 μm or clusters of frog eggs can be trapped in a potential well generated by oppositely directed focused ultrasonic beams. This makes it possible to move the trapped objects at will. Such a system has been called *acoustic tweezers* by *Wu* [6.8].

6.2.5 Sonoluminescence

Sonoluminescence is the conversion of high-intensity acoustic energy into light. Sonoluminescence was first discovered in water in the early 1930s [6.9, 10]. However, interest in the phenomenon languished for several decades.

In the late seventies, a new type of sonoluminescence was found to occur in solids [6.11, 12]. This can occur when high-intensity Lamb waves are generated along a thin plate of ferroelectric material which is driven at the frequency of mechanical resonance in a partial vacuum (the phenomenon occurs at about 0.1 atm). The acoustic fields interact with dislocations and defects in the solid which leads to the generation of visible light in ferroelectric materials. Figure 6.12 shows a diagram of the experimental setup and a photograph of the light emitted during the excitation of solid-state sonoluminescence.

In the early 1990s, sonoluminescence emissions from the oscillations of a single bubble in water were discovered [6.7]. With single-bubble sonoluminescence, a single bubble is placed in a container of degassed water (often injected by a syringe). Sound is used to push the bubble to the center of the container and to set the bubble into high-amplitude oscillation. The dynamic range

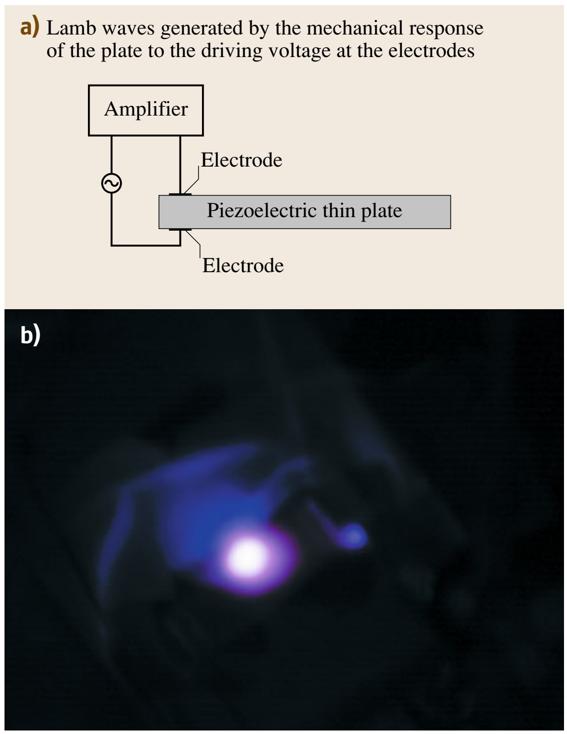


Fig. 6.12 (a) Block diagram of apparatus for solid-state sonoluminescence and **(b)** photograph of light emitted from a small, thin plate of $LiNbO_3$

for the bubble radius through an oscillation can be as great as 50 μm to 5 μm through one oscillation [6.13–15]. The light is emitted as the bubble goes through its minimum radius. Typically, one requires high-amplitude sound corresponding to a sound pressure level in excess of 110 dB. The frequency of sound needed to drive the bubble into sonoluminescence is in excess of 30 kHz, which is just beyond the range of human hearing.

Another peculiar feature of the sonoluminescence phenomenon is the regularity of the light emissions. Rather than shining continuously, the light is emitted as a series of extremely regular periodic flashes. This was not realized in the initial sonoluminescence experiments, because the rate at which the flashes appear requires a frequency resolution available only in the best detectors. The duration of the pulses is less than 50 ps. The interval between pulses is roughly 35 μs. The time between flashes varies by less than 40 ps.

The discovery of single-bubble sonoluminescence has caused a resurgence of interest in the phenomenon. The light emitted appears to be centered in the near-ultraviolet and is apparently black body in nature (unfortunately water absorbs much of the higher-frequency light, so a complete characterization of the spectrum is difficult to achieve). Adiabatic compression of the bubble through its oscillation would suggest temperatures of about 10 000 K (with pressures of about 10 000 atm). The temperatures corresponding to the observed spectra are in excess of 70 000 K [6.13]. In fact, they may even be much higher.

Since the measured spectrum suggests that the actual temperatures and pressures within the bubble may be quite high, simple compression does not seem to be an adequate model for the phenomenon. It is possible that the collapsing bubble induces a spherically symmetric shockwave that is driven inward towards the center of the bubble. These shocks could possibly drive the temperatures and pressures in the interior of the bubble high enough to generate the light. (Indeed, some physicists have suggested that sonoluminescence might enable the ignition of fusion reactions, though as of this writing that remains speculation).

6.2.6 Thermoacoustic Engines (Refrigerators and Prime Movers)

Another interesting application of physical acoustics is that of thermoacoustics. Thermoacoustics involves the conversion of acoustic energy into thermal energy or the reverse process of converting thermal energy into sound [6.16]. Figure 6.13 shows a photograph of a thermoacoustic engine. To understand the processes involved in thermoacoustics, let us consider a small packet of gas in a tube which has a sound wave traveling through it (thermoacoustic effects can occur with either standing or progressive waves). As the compression of the wave passes through the region containing the packet of gas, three effects occur:

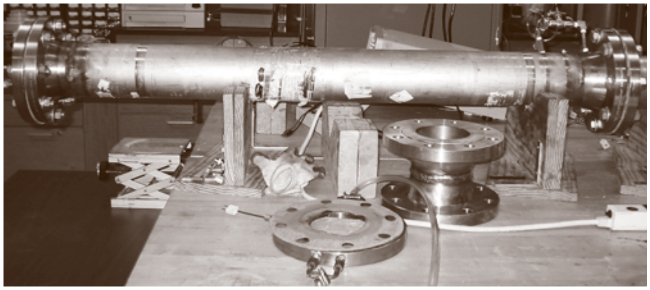


Fig. 6.13 A thermoacoustic engine

1. The gas compresses adiabatically, its temperature increases in accordance with Boyle's law (due to the compression), and the packet is displaced some distance down the tube.
2. As the rarefaction phase of the wave passes through the gas, this process reverses.
3. The wall of the tube acts as a heat reservoir. As the packet of gas goes through the acoustic process, it deposits heat at the wall during the compression phase (the wall literally conducts the heat away from the compressed packet of gas).

This process is happening down the entire length of the tube; thus a temperature gradient is established down the tube.

To create a useful thermoacoustic device, one must increase the surface area of wall that the gas is in contact with so that more heat is deposited down the tube. This is accomplished by inserting a *stack* into the tube. In the inside of the tube there are several equally spaced plates which are also in contact with the exterior walls of the tube. Each plate provides additional surface area for the deposition of thermal energy and increases the overall thermoacoustic effect. (The stack must not impede the wave traveling down the tube, or the thermoacoustic effect is minimized.) Modern stacks use much more complicated geometries to improve the efficiency of the thermoacoustic device (indeed the term stack now is a misnomer, but the principle remains the same). Figure 6.14 shows a photograph of a stack used in a modern thermoacoustic engine. The stack shown in Fig. 6.14 is made of a ceramic material.

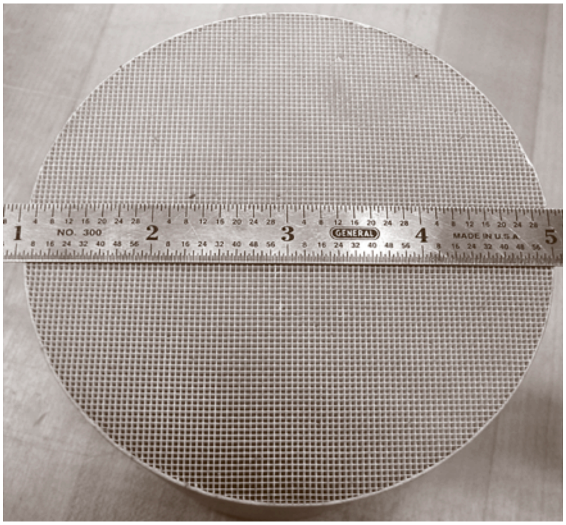


Fig. 6.14 A stack used in a thermoacoustic engine. Pores allow the gas in the tube to move under the influence of the acoustic wave while increasing the surface area for the gas to deposit heat

A thermoacoustic device used in this way is a thermoacoustic refrigerator. The sound generated down the tube (by a speaker or some other device) is literally pumping heat energy from one end of the tube to the other. Thus, one end of the tube gets hot and the other cools down (the cool end is used for refrigeration).

The reverse of this thermoacoustic refrigeration process can also occur. In this case, the tube ends are fitted with heat exchangers (one exchanger is hot, one is cold). The heat delivered to the tube by the heat exchangers does work on the system and generates sound in the tube. The frequency and amplitudes of the generated waves depend on the geometry of the tube and the stacks. When a thermoacoustic engine is driven in this way (converting heat into sound), it is known as a prime mover.

Though much research to improve the efficiency of thermoacoustic engines is ongoing, the currently obtainable efficiencies are quite low compared to standard refrigeration systems. Thermoacoustic engines, however, offer several advantages: they have no moving parts to wear out, they are inexpensive to manufacture, and they are highly reliable, which is useful if refrigeration is needed in inaccessible places.

One practical application for thermoacoustic engines is often cited: the liquefaction of natural gas for transport. This is accomplished by having two thermoacoustic engines, one working as a prime mover and the other working as a refrigerator. A small portion of the gas is ignited and burned to produce heat. The heat is applied to the prime mover to generate sound. The sound is directed into a second thermoacoustic engine that acts as a refrigerator. The sound from the prime mover pumps enough heat down the refrigerator to cool the gas enough to liquefy it for storage. The topic of thermoacoustics is discussed in greater detail in Chap. 7.

6.2.7 Acoustic Detection of Land Mines

Often, during wars and other armed conflicts, mine fields are set up and later abandoned. Even today it is not uncommon for mines originally planted during World War II to be discovered still buried and active. According to the humanitarian organization CARE, 70 people are killed each day by land mines, with the vast majority being civilians. Since most antipersonnel mines manufactured today contain no metal parts, electromagnetic-field-based metal detectors cannot locate them for removal. Acoustic detection of land mines offers a potential solution for this problem.

The approach used in acoustic land-mine detection is conceptually simple but technically challenging. Among the first efforts made at acoustic land-mine detection were those of *House* and *Pape* [6.17]. They sent sounds into the ground and examined the reflections from buried objects. *Don* and *Rogers* [6.18] and *Caulfield* [6.19] improved the technique by including a reference beam that provided information for comparison with the reflected signals. Unfortunately, this technique yielded too many false positives to be practical because one could not distinguish between a mine and some other buried object such as a rock or the root of a tree.

Newer techniques involving the coupling of an acoustic signal with a seismic vibration have been developed with much more success [6.20–24]. They make use of remote sensing and analysis by computer. Remote measurement of the acoustic field is done with a laser Doppler vibrometer (LDV), which is a an optical device used to measure velocities and displacements of vibrating bodies without physical contact. With this technique, the ground is excited acousticly with lower frequencies (usually on the order of a few hundred Hz). The LDV is used to measure the vibration of the soil as it responds to this driving force. If an object is buried under the soil in the region of excitation, it alters the resonance characteristics of the soil and introduces nonlinear effects. With appropriate digital signal processing and analysis, one can develop a system capable of recognizing different types of structures buried beneath the soil. This reduces the number of false positives and makes the system more efficient.

6.2.8 Medical Ultrasonography

For the general public, one of the most familiar applications of physical acoustics is that of medical ultrasonography. Medical ultrasonography is a medical diagnostic technique which can use sound information to construct images for the visualization of the size, structure and lesions of internal organs and other bodily tissues. These images can be used for both diagnostic and treatment purposes (for example enabling a surgeon to visualize an area with a tumor during a biopsy). The most familiar application of this technique is obstetric ultrasonography, which uses the technique to image and monitor the fetus during a pregnancy. An ultrasonograph of a fetus is shown in Fig. 6.15.

This technique relies on the fact that in different materials the speed of sound and acoustic impedance are different. A collimated beam of high-frequency sound is projected into the body of the person being examined. The frequencies chosen will depend on the application. For example, if the tissue is deeper within the body, the sound must travel over a longer path and attenuation affects can present difficulties. Using a lower ultrasonic frequency reduces attenuation. Alternatively, if a higher resolution is needed, a higher frequency is used. In each position where the density of the tissue changes, there is an acoustic impedance mismatch. Therefore, at each interface between various types of tissues some of the sound is reflected. By measuring the time between echoes, one can determine the spatial position of the various tissues.

If a single, stationary transducer is used; one gets spatial information that lies along a straight line. Typically, the probe contains a phased array of transducers that are used to generate image information from different directions around the area of interest. The different transducers in the probe send out acoustic pulses that are reflected from the various tissues. As the acoustic signals return, the transducers receive them and convert the information into a digital, pictorial representation. One must also match the impedances between the surface of the probe and the body. The head of the probe is usually soft rubber, and the contact between the probe and the body is impedance-matched by a water-based gel.

Fig. 6.15 Ultrasonograph of a fetus

To construct the image, each transducer (which themselves are separated spatially somewhat) measures the strength of the echo. This measurement indicates how much sound has been lost due to attenuation (different tissues have different attenuation values). Additionally, each transducer measures the delay time between echoes, which indicates the distance the sound has traveled before encountering the interface causing the echo (actually since the sound has made a round trip during the echo the actual displacement between the tissues is $\frac{1}{2}$ of the total distance traveled). With this information a two-dimensional image can be created. In some versions of the technique, computers can be used to generate a three-dimensional image from the information as well.

A more esoteric form of ultrasonograpy, Doppler ultrasonography, is also used. This technique requires separate arrays of transducers, one for broadcasting a continuous-wave acoustic signal and another for receiving it. By measuring a frequency shift caused by the Doppler effect, the probe can detect structures moving towards or away from the probe. For example, as a given volume of blood passes through the heart or some other organ, its velocity and direction can be determined and visualized. More-recent versions of this technique make use of pulses rather than continuous waves and can therefore use a single probe for both broadcast and reception of the signal. This version of the technique requires more-advanced analysis to determine the frequency shift. This technique presents advantages because the timing of the pulses and their echoes can be measured to provide distance information as well.

6.3 Apparatus

Given the diverse nature of physical acoustics, any two laboratories conducting physical acoustics experiments might have considerably different types of equipment and apparatus (for example, experiments dealing with acoustic phenomena in water usually require tanks to hold the water). As is the case in any physics laboratory, it is highly useful to have access to a functioning machine shop and electronics shop for the design, construction, and repair of equipment needed for the physical acoustics experiment that is being conducted. Additionally, a wide range of commercial equipment is available for purchase and use in the lab setting.

6.3.1 Examples of Apparatus

Some typical equipment in an acoustic physics lab might include some of the following:

- Loudspeakers;
- Transducers (microphones/hydrophones);
- Acoustic absorbers;
- Function generators, for generating a variety of acoustic signals including single-frequency sinusoids, swept-frequency sinusoids, pulses, tone bursts, white or pink noise, etc.;
- Electronics equipment such as multimeters, impedance-matching networks, signal-gating equipment, etc.;
- Amplifiers (acoustic, broadband, Intermediate Frequency (IF), lock-in);
- Oscilloscopes. Today's digital oscilloscopes (including virtual oscilloscopes implemented on personal computers) can perform many functions that previously required several different pieces of equipment; for example, fast Fourier transform (FFT) analysis previously required a separate spectrum analyzer; waveform averaging previously required a separate boxcar integrator, etc.);
- Computers (both for control of apparatus and analysis of data).

For audible acoustic waves in air the frequency range is typically 20–20 000 Hz. This corresponds to a wavelength range of 16.5 m–16.5 mm. Since these wavelengths often present difficulties in the laboratory, and since the physical principles apply at all frequencies, the laboratory apparatus is often adapted to a higher frequency range. The propagating medium is often water since a convenient ultrasonic range of 1–100 MHz gives a convenient wavelength range of 0.014–1.4 mm. The experimental arrangements to be described below are for some specialized applications and cover this wavelength range, or somewhat lower. The results, however, are useful in the audio range as well.

6.3.2 Piezoelectricity and Transduction

In physical acoustics transducers play an important role. A transducer is a device that can convert a mechanical vibration into a current or vice versa. Transducers can be used to generate sound or to detect sound. Audible frequencies can be produced by loudspeakers and received by microphones, which often are driven electromagnetically. (For example, a magnet interacting with a current-carrying coil experiences a magnetic force that can accelerate it, or if the magnet is moved it can induce a corresponding current in the coil.) The magnet could be used to drive the cone of a loudspeaker to convert a current into an audible tone (or speech, or music, etc.). Similarly, one can use the fact that the capacitance between two parallel-plate capacitors varies as a function of the separation distance between two plates. A capacitor with one plate fixed and the other allowed to vibrate in response to some form of mechanical forcing (say the force caused by the pressure of an acoustic wave) is a transducer. With a voltage across the two plates, an electrical current is generated as the plates move with respect to one another under the force of the vibration of an acoustic wave impinging upon it. These types of transducers are described in Chap. 24.

The most common type of transducers used in the laboratory for higher-frequency work are piezoelectric-element-based transducers. In order to understand how these transducers work (and are used) we must first examine the phenomenon of piezoelectricity.

Piezoelectricity is characterized by the both direct piezoelectric effect, in which a mechanical stress applied to the material causes a potential difference across the surface of the faces to which the stress is applied, and the secondary piezoelectric effect, in which a potential difference applied across the faces of the material causes a deformation (expansion or contraction). The deformation caused by the direct piezoelectric effect is on the order of nanometers, but leads to many uses in acoustics such as the production and detection of sound, microbalance applications (where very small masses are measured by determining the change in the resonance frequency of a piezoelectric crystal when it is loaded with the small mass), and frequency generation/control for oscillators.

Piezoelectricity arises in a crystal when the crystal's unit cell lacks a center of symmetry. In a piezoelectric crystal, the positive and negative charges are separated by distance. This causes the formation of electric dipoles, even though the crystal is electrically neutral overall. The dipoles near one another tend to orient themselves along the same direction. These small regions of aligned dipoles are known as domains because of the similarity to the magnetic analog. Usually, these domains are oriented randomly, but can be aligned by the application of a strong electric field in a process known as poling (typically the sample is poled at a high temperature and cooled while the electric field is maintained). Of the 32 different crystal classifications, 20 exhibit piezoelectric properties and 10 of those are polar (i. e. spontaneously polarized). If the dipole can be reversed by an applied external electric field, the material is additionally known as a ferroelectric (in analogy to ferromagnetism).

When a piezoelectric material undergoes a deformation induced by an external mechanical stress, the symmetry of the charge distribution in the domains is disturbed. This gives rise to a potential difference across the surfaces of the crystal. A 2 kN force ($\approx$ 500 lbs) applied across a 1 cm cube of quartz can generate up to 12 500 V of potential difference.

Several crystals are known to exhibit piezoelectric properties, including tourmaline, topaz, rochelle salt, and quartz (which is most commonly used in acoustic applications). In addition, some ceramic materials with perovskite or tungsten bronze structures, including barium titanate ($BaTiO_3$), lithium niobate ($LiNbO_3$), PZT [$Pb(ZrTi)O_3$], $Ba_2NaNb_5O_5$, and $Pb_2KNb_5O_{15}$, also exhibit piezoelectric properties. Historically, quartz was the first piezoelectric material widely used for acoustics applications. Quartz has a very sharp frequency response, and some unfavorable electrical properties such as a very high electrical impedance which requires impedance matching for the acoustic experiment. Many of the ceramic materials such as PZT or lithium niobate have a much broader frequency response and a relatively low impedance which usually does not require impedance matching. For these reasons, the use of quartz has largely been supplanted in the acoustics lab by transducers fashioned from these other materials. Although in some situations (if a very sharp frequency response is desired), quartz is still the best choice.

In addition to these materials, some polymer materials behave as electrets (materials possessing a quasi-permanent electric dipole polarization). In most piezoelectric crystals, the orientation of the polarization is limited by the symmetry of the crystal. However, in an electret this is not the case. The electret material polyvinylidene fluoride (PVDF) exhibits piezoelectricity several times that of quartz. It can also be fashioned more easily into larger shapes.

Since these materials can be used to convert a sinusoidal electrical current into a corresponding sinusoidal mechanical vibration as well as convert a mechanical vibration into a corresponding electrical current, they provide a connection between the electrical system and the acoustic system. In physical acoustics their ability to produce a single frequency is especially important. The transducers to be described here are those which are usually used to study sound propaga-

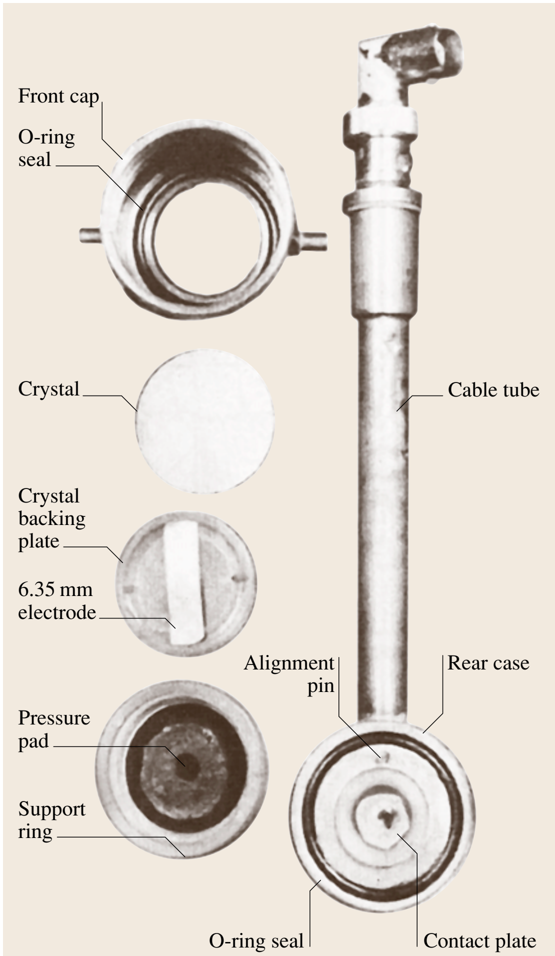


Fig. 6.16 Transducer housing

tion in liquids or in solids. The frequencies used are ultrasonic.

The first transducers were made from single crystals. Single crystals of x-cut quartz were preferred for longitudinal waves, and y-cut for transverse waves, their thickness determined by the frequency desired. Single crystals are still used for certain applications; however, the high-impedance problems were solved by introduction of polarized ceramics containing barium titanate or strontium titanate. These transducers have an electrical impedance which matches the $50\,\Omega$ impedance found on most electrical apparatus. Low-impedance transducers are currently commercially available.

Such transducers can be used to generate or receive sound. When used as a receiver in liquids such transducers are called hydrophones. For the generation of ultrasonic waves in liquids, one surface of the transducer material should be in contact with the liquid and the other surface in contact with air (or other gas). With this arrangement, most of the acoustic energy enters the liquid. In the laboratory it is preferable to have one surface at the ground potential and electrically drive the other surface, which is insulated. Commercial transducers which accomplish these objectives are available. They are designed for operation at a specific frequency. A transducer housing is shown in Fig. 6.16. The transducer crystal and the (insulated) support ring can be changed to accommodate the frequency desired. In the figure a strip back electrode is shown. For generating acoustic vibration over the entire surface, the back of the transducer can be coated with an electrode. The width which produces a Gaussian output in one dimension is described in [6.25]. The use of a circular electrode that can produce a Gaussian amplitude distribution in two dimensions is described in [6.26].

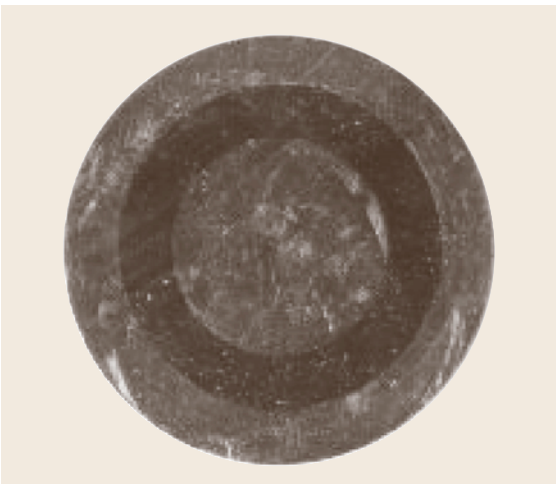


Fig. 6.17 Coaxial transducer

For experiments with solids the transducer is often bonded directly to the solid without the need for an external housing. For single-transducer pulse-echo operation the opposite end of the sample must be flat because it must reflect the sound. For two-transducer operation one transducer is the acoustic transmitter, the other the receiver.

With solids it is convenient to use coaxial transducers because both transducer surfaces must be electrically accessible. The grounded surface in a coaxial transducer can be made to wrap around the edges so it can be accessed from the top as well. An example is shown in Fig. 6.17. The center conductor is the high-voltage terminal. The outer conductor is grounded; it is in electrical contact with the conductor that covers the other side of the transducer.

6.3.3 Schlieren Imaging

Often it is useful to be able to see the existence of an acoustic field. The Schlieren arrangement shown in Fig. 6.18 facilitates the visualization of sound fields in water. Light from a laser is brought to focus on a circular aperture by lens 1. The circular aperture is located at a focus of lens 2, so that the light emerges parallel. The water tank is located in this parallel light. Lens 3 forms a real image of the contents of the water tank on the screen, which is a photographic negative for photographs. By using a wire or an ink spot on an optical flat at the focus of lens 3, one produces conditions for dark-field illumination. The image on the screen, then, is an image of the ultrasonic wave propagating through the water. The fact that the ultrasonic wave is propagating through the water means that the individual wavefronts are not seen. To see the individual wavefronts, one must add a stroboscopic light source synchronized with the ultrasonic wave. In this way the light can be on at the frequency of the sound and produce a photograph which shows the individual wavefronts.

On some occasions it may be useful to obtain color images of the sound field; for example, one can show an incident beam in one color and its reflection from an interface in a second color for clarity. The resultant photographs can be beautiful; however, to a certain extent their beauty is controlled by the operator. The merit of color Schlieren photography may be more from its aesthetic or pedagogical value, rather than its practical application. This is the reason that color Schlieren pho-

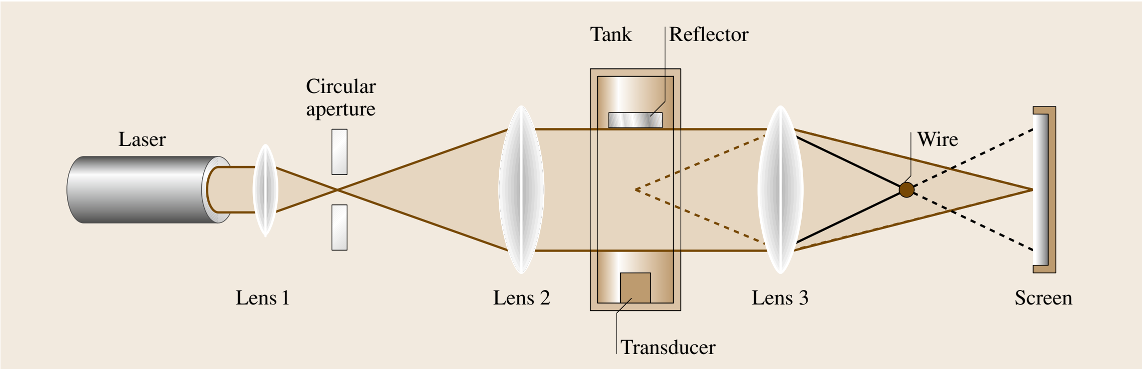


Fig. 6.18 Diagram of a Schlieren system

tographs are seldom encountered in a physical acoustic laboratory. For completeness, however, it may be worthwhile to describe the process.

The apparatus used is analogous to that given in Fig. 6.18. The difference is that a white-light source

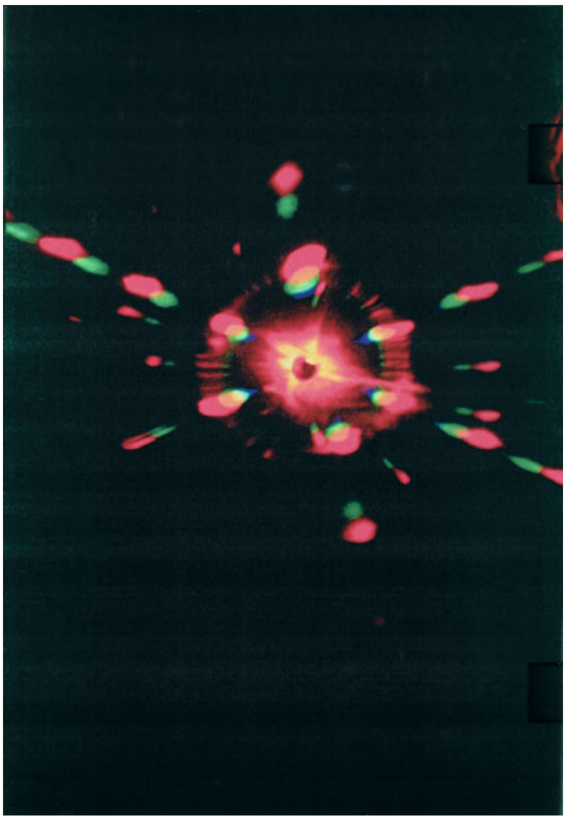


Fig. 6.19 Spectra formed by the diffraction of light through a sound field

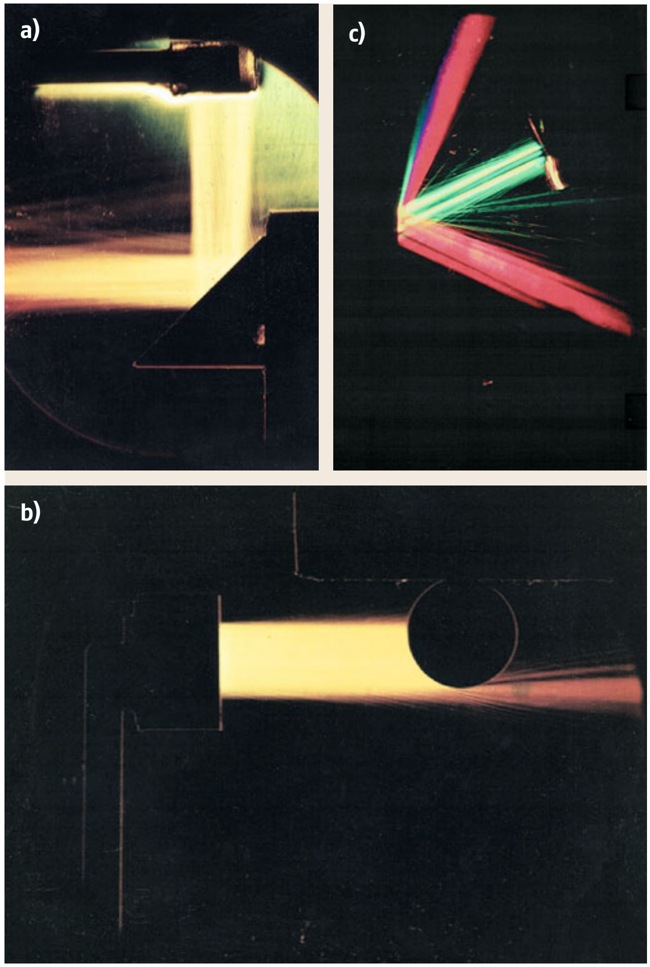
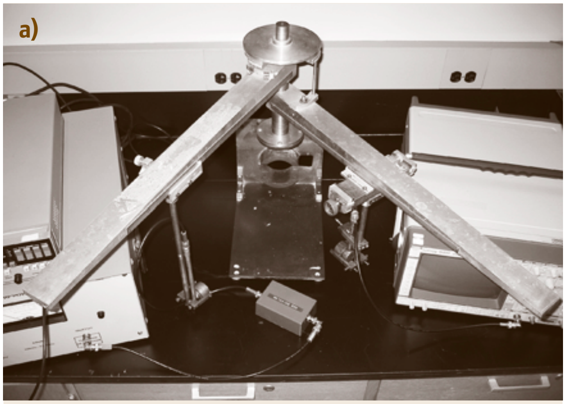


Fig. 6.20a–c Schlieren photographs showing reflection and diffraction of ultrasonic waves by a solid immersed in water



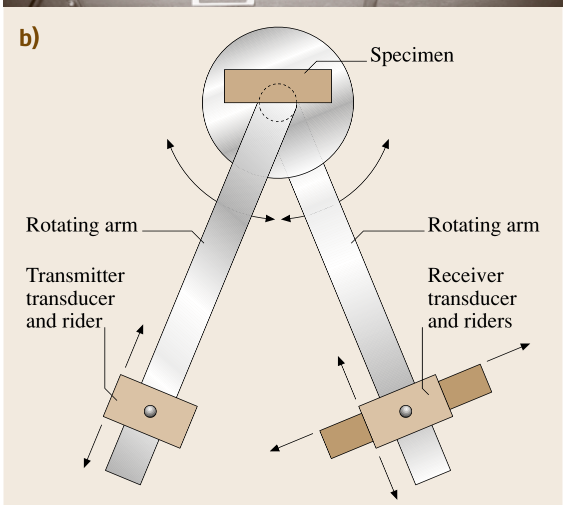


Fig. 6.21 **(a)** Photograph of a goniometer system; **(b)** diagram of a goniometer system ◀

is used in the place of the monochromatic light from the laser. As the light diffracts, the diffraction pattern formed at the focus of lens 3 is made up of complete spectra (with each color of the spectra containing a complete image of the acoustic field). A photograph showing the spectra produced is shown in Fig. 6.19. If the spectra are allowed to diverge beyond this point of focus, they will combine into a white-light image of the acoustic field. However, one can use a slit (rather than using a wire or an ink spot at this focus to produce dark-field illumination) to pass only the desired colors. The position of the slit selects the color of the light producing the image of the sound field. If one selects the blue-colored incident beam from one of the spectra, and the red-colored reflected beam from another spectrum (blocking out all the other colors), a dual-colored image results. Since each diffraction order contains enough information to produce a complete sound field image, the operator has control over its color. Spatial filtering, then, becomes a means of controlling the color of the various parts of the image. Three image examples are given in Fig. 6.20. Figure 6.20a is reflection of a sound beam from a surface. Figure 6.20b shows diffraction of sound around a cylinder. Figure 6.20c shows backward displacement at a periodic interface. The incident beam is shown in a different color from that of the reflected beam and the diffraction order.

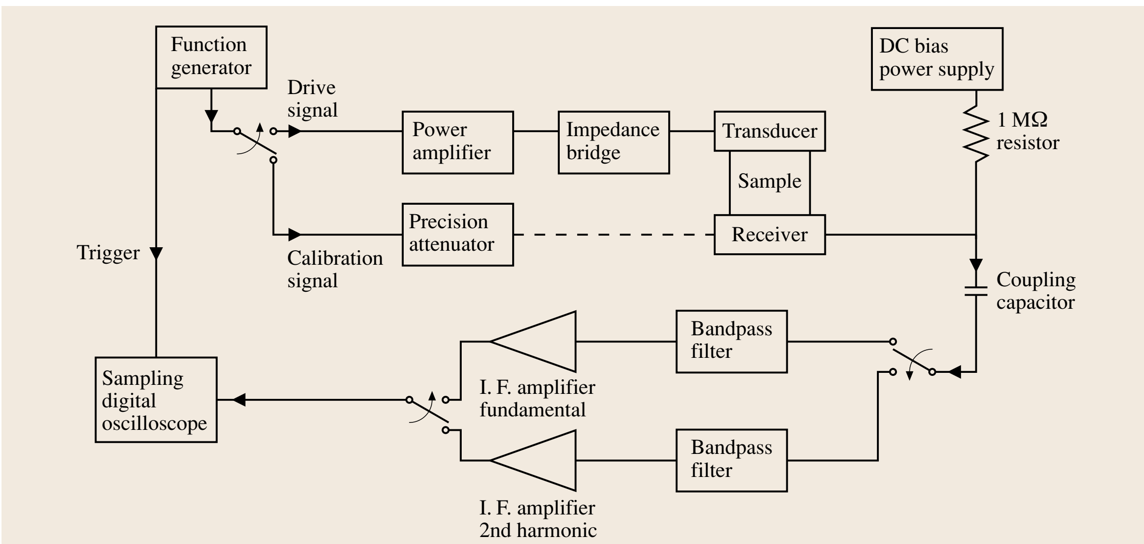


Fig. 6.22 Block Diagram of apparatus for absolute amplitude measurements

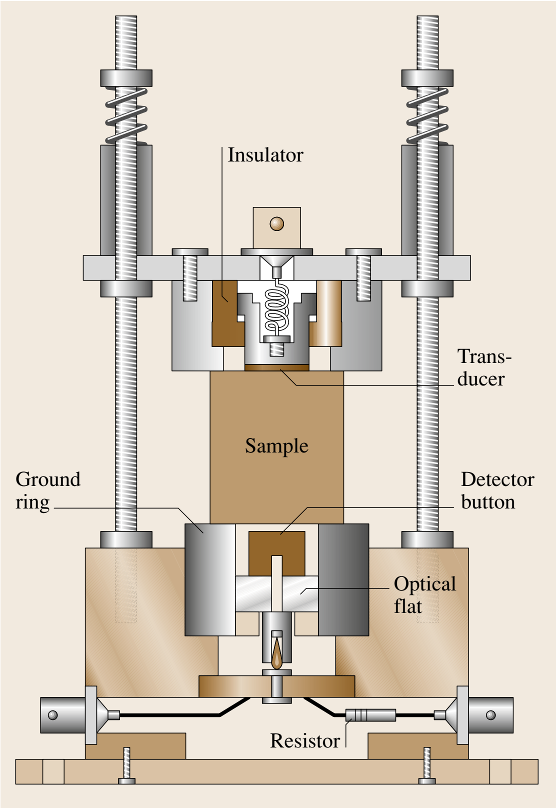


Fig. 6.23 Mounting system for room-temperature measurement of absolute amplitudes of ultrasonic waves in solids

6.3.4 Goniometer System

For studies of the propagation of an ultrasonic pulse in water one can mount the ultrasonic transducers on a goniometer such as that shown in Fig. 6.21. This is a modification of the pulse-echo system. The advantage of this system is that the arms of the goniometer allow for the adjustments indicated in Fig. 6.21b. By use of this goniometer it is possible to make detailed studies of the reflection of ultrasonic waves from a variety of water–solid interfaces. By immersing the goniometer in other liquids, the type of liquid can also be changed.

6.3.5 Capacitive Receiver

In many experiments, one measures acoustic amplitude relative to some reference amplitude, which is usually determined by the parameters of the experiment. However, in some studies it is necessary to make a measurement of the absolute amplitude of acoustic vibration. This is especially true of the measurement of the nonlinearity of solids. For the measurement of the absolute acoustic amplitude in a solid, a capacitive system can be used. If the end of the sample is coated with a conductive material, it can act as one face of a parallel-plate capacitor. A bias voltage is put across that capacitance, which enables it to work as a capacitive microphone. As the acoustic wave causes the end of the sample to vibrate, the capacitor produces an electrical signal. One can relate the measured electrical amplitude to the acoustic amplitude because all quantities relating to them can be determined.

The parallel-plate approximation, which is very well satisfied for plate diameters much larger than the plate separation, is the only approximation necessary. The electrical apparatus necessary for absolute amplitude measurements in solids is shown in the block diagram of Fig. 6.22. A calibration signal is used in such a manner that the same oscilloscope can be used for the calibration and the measurements. The mounting system for room-temperature measurements of a sample is shown in Fig. 6.23. Since stray capacitance affects the impedance of the resistor, this impedance must be measured at the frequencies used. The voltage drop in the resistor can be measured with either the calibration signal or the signal from the capacitive receiver. A comparison of the two completes the calibration. With this system acoustic amplitudes as small as 10^{-14} m (which is approximately the limit set by thermal noise) have been measured in copper single crystals [6.27].

6.4 Surface Acoustic Waves

It has been discovered that surface acoustic waves are useful in industrial situations because they are relatively slow compared with bulk waves or electromagnetic waves. Many surface acoustic wave devices are made by coating a solid with an interdigitated conducting layer. In this case, the surface acoustic wave produces the desired delay time and depends for its generation on a fringing field (or the substrate may be piezoelec-

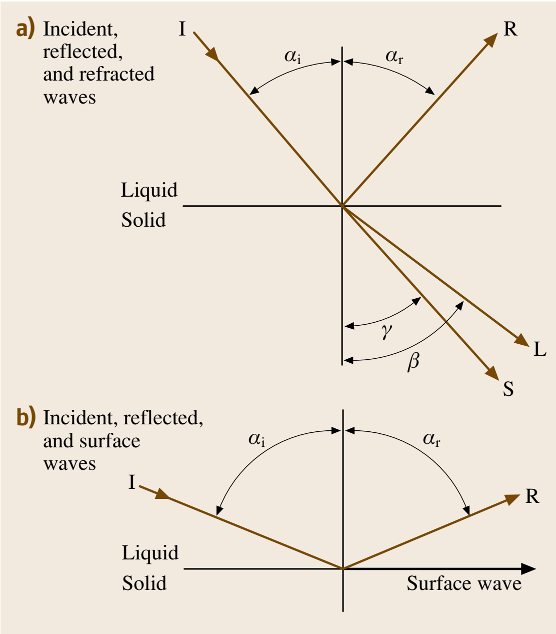


Fig. 6.24a,b Sound waves at a liquid–solid interface

tric). The inverse process can be used to receive surface acoustic waves and convert them into an electrical signal, which can then be amplified.

Another type of surface wave is possibly more illustrative of the connection between surface acoustic waves and physical acoustics. This is the surface acoustic wave generated when the trace velocity of an incident wave is equal to the velocity of the surface acoustic wave. This occurs when a longitudinal wave is incident on a solid from a liquid. This is analogous to the optical case of total internal reflection [6.28], but new information comes from the acoustic investigation.

The interface between a liquid and a solid is shown in Fig. 6.24, in which the various waves and their angles are indicated. The directions in which the various waves propagate at a liquid–solid interface can be calculated from Snell's law, which for this situation can be written

$$\frac{\sin\theta_i}{v} = \frac{\sin\theta_r}{v} = \frac{\sin\theta_L}{v_L} = \frac{\sin\theta_S}{v_S}\,, \tag{6.66}$$

where the velocity of the longitudinal wave in the liquid, that in the solid, and the velocity of the shear wave in the solid are, respectively, v, v_L, and v_S. The propagation directions of the various waves are indicated in Fig. 6.24.

Since much of the theory has been developed in connection with geology, the theoretical development of *Ergin* [6.29] can be used directly. Ergin has shown that the energy reflected at an interface is proportional to the square of the amplitude reflection coefficient, which can be calculated directly [6.29]. The energy reflection coefficient is given by

$$R_E = \left(\frac{\cos\beta - A\cos\alpha\,(1-B)}{\cos\beta + A\cos\alpha\,(1-B)}\right)^2 , \tag{6.67}$$

where

$$A = \frac{\rho_1 V_L}{\rho V} \quad \text{and}$$

$$B = 2\sin\gamma\sin 2\gamma\left(\cos\gamma - \frac{v_S}{v_L}\cos\beta\right). \tag{6.68}$$

The relationship among the angles α, β and γ can be determined from Snell's law as given in (6.66). The book by *Brekhovskikh* [6.30] is also a good source of information on this subject.

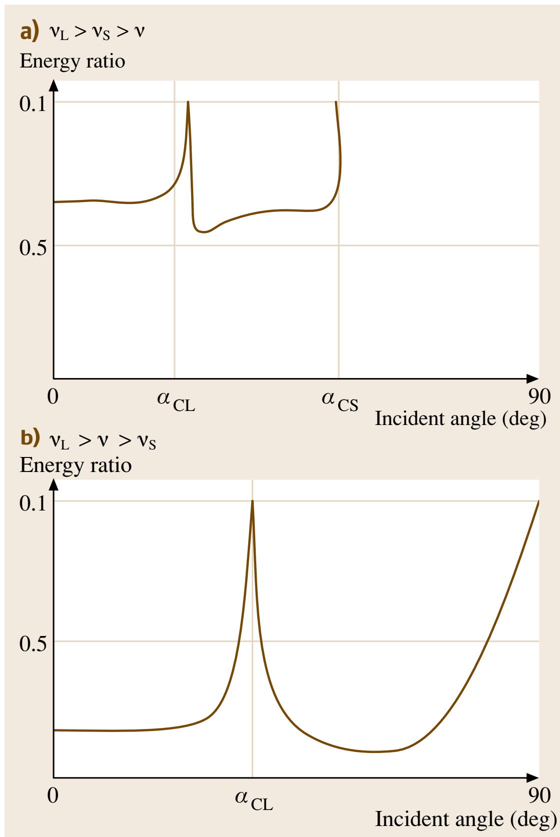


Fig. 6.25a,b Behavior of energy reflected at a liquid–solid interface

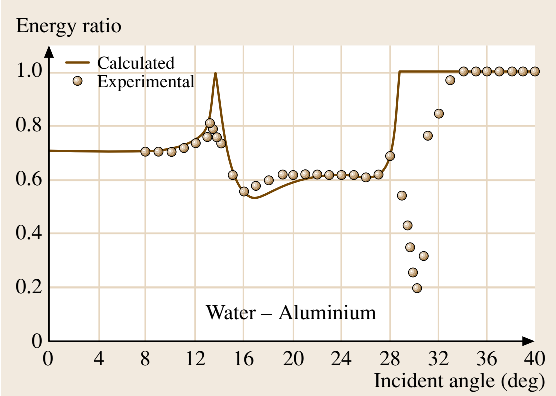


Fig. 6.26 A 2 MHz ultrasonic wave reflected at a liquid–solid interface

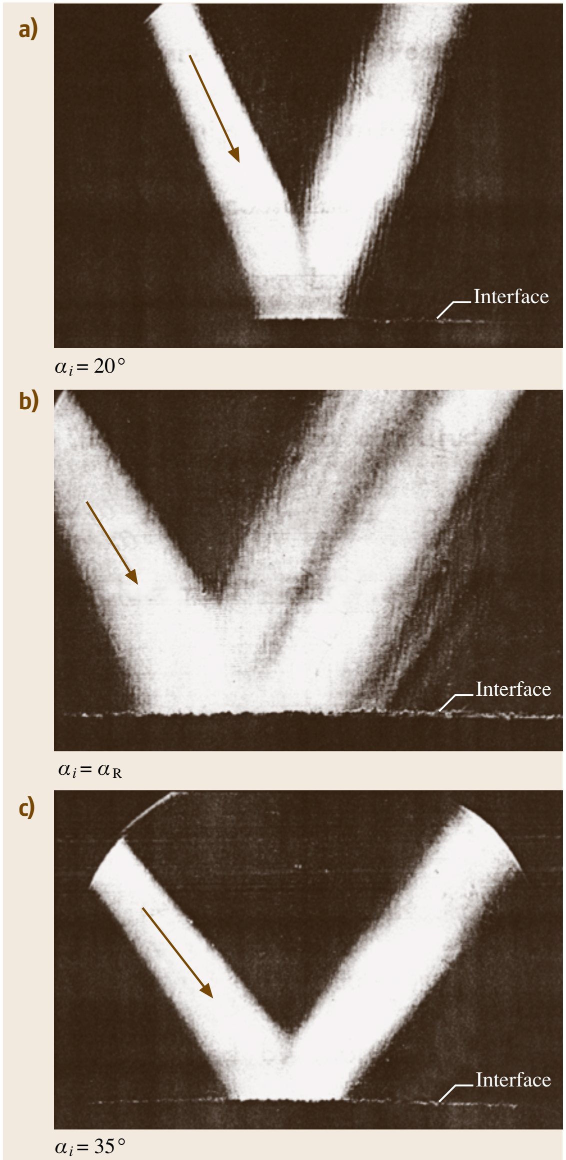


Fig. 6.27a–c Schlieren photographs showing the behavior of a 4 MHz ultrasonic beam reflected at a water–aluminium interface

Typical plots of the energy reflection coefficient as a function of incident angle are given in Fig. 6.25 in which the critical angles are indicated. Usually, $v_L > v_S > v$, so the curve in Fig. 6.25a is observed. It will be noticed immediately that there is a critical angle for both the longitudinal and transverse waves in the solid. In optics there is no longitudinal wave; therefore the curve has only one critical angle.

If one uses a pulse-echo system to verify the behavior of an ultrasonic pulse at an interface between a liquid and a solid, one gets results that can be graphed as shown in Fig. 6.26. At an angle somewhat greater than the critical angle for a transverse wave in the solid, one finds a dip in the data. This dip is associated with the generation of a surface wave. The surface wave is excited when the projection of the wavelength of the incident wave onto the interface matches the wavelength of the surface wave. The effect of the surface wave can be seen in the Schlieren photographs in Fig. 6.27.

Figure 6.27 shows the reflection at a water–aluminum interface at an angle less than that for excitation of a surface wave (a Rayleigh surface wave), at the angle at which a surface wave is excited, and an angle greater. When a surface wave is excited the reflected beam contains two (or more) components: the specular beam (reflected in a normal manner) and a beam displaced down the interface. Since most of the energy is contained in the displaced beam, the minimum in the data shown in Fig. 6.24 is caused by the excitation of the displaced beam by the surface wave. This has been shown to be the case by displacing the receiver to follow the displaced beam with a goniometer system, as shown in Fig. 6.21. This minimizes the dip in data shown in Fig. 6.24. *Neubauer* has shown that the ultrasonic beam excited by the surface wave is 180° out of phase with the specularly reflected beam [6.31]. Destructive interference resulting from phase cancelation causes these beams to be separated by a null strip. Although a water–aluminum interface has been used in these examples, the phenomenon occurs at all liquid–solid interfaces. It is less noticeable

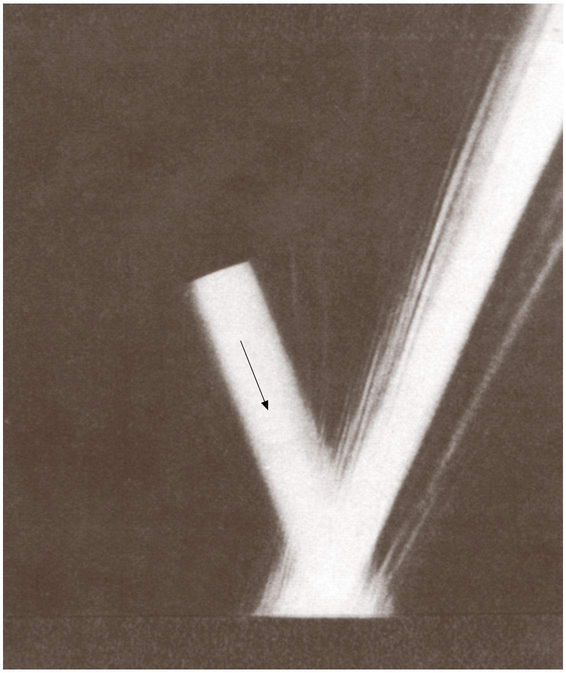


Fig. 6.28 Schlieren photograph showing backward displacement of a 6 MHz ultrasonic beam at a corrugated water–brass interface

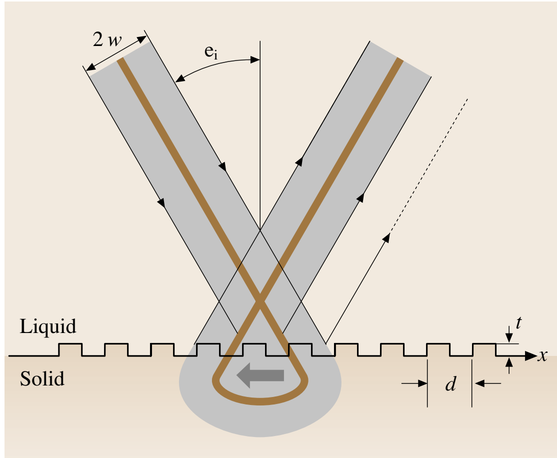


Fig. 6.29 Diagram of incident beam coupling to a backward-directed leaky wave to produce backward displacement of the reflected beam

at higher ultrasonic frequencies since the wavelength is smaller.

At a corrugated interface it is possible that the incident beam couples to a negatively directed surface wave so that the reflected beam is displaced in the negative direction. This phenomenon was predicted for optical waves by *Tamir* and *Bertoni* [6.32]. They determined that the optimum angle for this to occur is given by

$$\sin\theta_\mathrm{i} = V_\mathrm{liq}\left(\frac{1}{fd} - \frac{1}{V_\mathrm{R}}\right) \tag{6.69}$$

where d is the period, f is the frequency, V_liq is the wave propagation velocity in the liquid, and V_R is the propagation velocity of the leaky surface wave. Figure 6.28 is a Schlieren photograph which shows what happens in this case [6.33]. Figure 6.29 is a diagram of the phenomenon [6.33]. The incident beam couples to a backward-directed surface wave to produce backward displacement of the reflected beam.

6.5 Nonlinear Acoustics

There are several sources of nonlinearity whether the propagating medium be a gas, liquid, or solid. They are described in more detail in Chap. 8. Even in an ideal medium in which one considers only interatomic forces, there is still a source of nonlinear behavior since compression requires a slightly different force from dilatation. With Hooke's law (strain is proportional to stress) one assumes that they are equal. This is seldom true. The subject of nonlinear acoustics has been developed to the point that it is now possible to go beyond the linear approximation with many substances.

6.5.1 Nonlinearity of Fluids

If one assumes an ideal gas and keeps the first set of nonlinear terms, *Beyer* has shown that the equation of motion in one dimension becomes [6.34]

$$\frac{\partial^2\xi}{\partial t^2} = \frac{c_0^2}{\left(1+\frac{\partial\xi}{\partial a}\right)^{\gamma+1}}\frac{\partial^2\xi}{\partial a^2}\,. \tag{6.70}$$

This form of the nondissipative wave equation in one dimension in Lagrangian coordinates includes nonlinear terms. In this equation γ is the ratio of specific heats and

$$c_0^2 = \frac{\gamma p_0}{\rho_0} \frac{1}{\left(1+\frac{\partial \xi}{\partial a}\right)^{\gamma-1}} . \qquad (6.71)$$

One can also generalize this equation in a form which applies to all fluids. By expanding the equation of state $p = p_0 \left(\frac{\rho}{\rho_0}\right)^{\gamma}$ in powers of the condensation $s = \frac{\rho - \rho_0}{\rho_0}$, one obtains

$$p = p_0 + As + \frac{B}{2!}s^2 + \ldots \quad \text{and}$$
$$c^2 = c_0^2 \left[1 + \left(\frac{B}{A}\right) s + \ldots\right] . \qquad (6.72)$$

This makes it possible to obtain the nonlinear wave equation in the form [6.24]

$$\frac{\partial^2 \xi}{\partial t^2} = \frac{c_0^2}{\left(1+\frac{\partial \xi}{\partial a}\right)^{2+\frac{B}{A}}} \frac{\partial^2 \xi}{\partial a^2} . \qquad (6.73)$$

In this form one can recognize that the quantity $2 + B/A$ for fluids plays the same role as $\gamma + 1$ for ideal gases. Values of B/A for fluids given in Table 6.3 indicate that nonlinearity of fluids, even in the absence of bubbles of air, cannot always be ignored. The nonlinearity of fluids is discussed in greater detail in Chap. 8.

6.5.2 Nonlinearity of Solids

The propagation of a wave in a nonlinear solid is described by first introducing third-order elastic constants. When extending the stress–strain relationship (which essentially is a force-based approach) it becomes difficult to keep a consistent approximation among the various nonlinear terms. However, If one instead uses an energy approach, a consistent approximation is automatically maintained for all the terms of higher order.

Beginning with the elastic potential energy, one can define both the second-order constants (those determining the wave velocity in the linear approximation) and the third-order elastic constants simultaneously. The elastic potential energy is

$$\phi(\eta) = \frac{1}{2!} \sum_{ijkl} C_{ijkl} \eta_{ij} \eta_{kl} + \frac{1}{3!} \sum_{ijklmn} C_{ijklmn} \eta_{ij} \eta_{kl} \eta_{mn} + \ldots , \qquad (6.74)$$

Table 6.3 Values of B/A

Substance	T (°C)	B/A
Distilled water	0	4.2
	20	5.0
	40	5.4
	60	5.7
	80	6.1
	100	6.1
Sea water (3.5%)	20	5.25
Methanol	20	9.6
Ethanol	0	10.4
	20	10.5
	40	10.6
N-propanol	20	10.7
N-butanol	20	10.7
Acetone	20	9.2
Beneze	20	9/0
Chlorobenzene	30	9.3
Liquid nitrogen	b.p.	6.6
Benzyl alcohol	30	10.2
Diethylamine	30	10.3
Ethylene glycol	30	9.7
Ethyl formate	30	9.8
Heptane	30	10.0
Hexane	30	9.9
Methyl acetate	30	9.7
Mercury	30	7.8
Sodium	110	2.7
Potassium	100	2.9
Tin	240	4.4
Monatomic gas	20	0.67
Diatomic gas	20	0.40

Table 6.4 K_2 and K_3 for the principal directions in a cubic crystal

Direction	K_2	K_3
[100]	C_{11}	C_{111}
[110]	$\frac{C_{11}+C_{12}+2C_{44}}{2}$	$\frac{C_{111}+3C_{112}+12C_{166}}{4}$
[111]	$\frac{C_{11}+2C_{12}+4C_{44}}{3}$	$\frac{1}{9}(C_{111}+6C_{112}+12C_{144}+24C_{166}+2C_{123}+16C_{456})$

Table 6.5 Comparison of room-temperature values of the ultrasonic nonlinearity parameters of solids. BCC = body-centered cubic; FCC = face-centered cubic

Material or structure	Bonding	β_{avg}
Zincblende	Covalent	2.2
Flourite	Ionic	3.8
FCC	Metallic	5.6
FCC (inert gas)	van der Waals	6.4
BCC	Metallic	8.2
NaCl	Ionic	14.6
Fused silica	Isotropic	−3.4
$YBa_2Cu_3O_{7-\delta}$ (ceramic)	Isotropic	14.3

where the η are the strains, C_{ijkl} are the second-order elastic constants, and C_{ijklmn} are the third-order elastic constants. For cubic crystals there are only three second-order elastic constants: C_{11}, C_{12} and C_{44}, and only six third-order elastic constants: C_{111}, C_{112}, C_{144}, C_{166}, C_{123} and C_{456}. This makes the investigation of cubic crystals relatively straightforward [6.35]. By using the appropriate form of Lagrange's equations, specializing to a specific orientation of the coordinates with respect to the ultrasonic wave propagation direction, and neglecting attenuation and higher-order terms, one can write the nonlinear wave equation for propagation in the directions that allow the propagation of purely longitudinal waves (with no excitation of transverse waves). In a cubic crystalline lattice there are three of these *pure mode* directions for longitudinal waves and the nonlinear wave equation has the form [6.35]

$$\rho_0 \frac{\partial^2 u}{\partial t^2} = K_2 \frac{\partial^2 u}{\partial a^2} + (3K_2 + K_3) \frac{\partial u}{\partial a} \frac{\partial^2 u}{\partial a^2} + \dots , \tag{6.75}$$

where both K_2 and K_3 depend on the orientation considered. The quantity K_2 determines the wave velocity: $K_2 = c_0^2 \rho_o$. The quantity K_3 contains only third-order elastic constants. The quantities K_2 and K_3 are given for the three pure-mode directions in a cubic lattice in Table 6.4. The ratio of the coefficient of the nonlinear term to that of the linear term has a special significance. It is often called the *nonlinearity parameter* β and its magnitude is $\beta = 3 + \frac{K_3}{K_2}$. Since K_3 is an inherently negative quantity and is usually greater in magnitude than $3K_2$, a minus sign is often included in the definition

$$\beta = -\left(3 + \frac{K_3}{K_2}\right) . \tag{6.76}$$

Table 6.6 Parameters entering into the description of finite-amplitude waves in gases, liquids and solids

Parameter	Ideal gas	Liquid	Solid
c_0^2	$\frac{\gamma P_0}{\rho_0}$	$\frac{A}{\rho_0}$	$\frac{K_2}{\rho_0}$
Nonlinearity parameter β	$\gamma + 1$	$\frac{B}{A} + 2$	$-\left(\frac{K_3}{K_2} + 3\right)$

The nonlinearity parameters of many cubic solids have been measured. As might be expected, there is a difference between the quantities measured in the three pure-mode directions (usually labeled as the [100], [110] and [111] directions). These differences, however, are not great. If one averages them, one gets the results shown in Table 6.5. The nonlinearity parameters cover the range 2–15. This means that for cubic crystals the coefficient of the nonlinear term in the nonlinear wave equation is 2–15 times as large as the coefficient of the linear term. This gives an impression of the approximation involved when one ignores nonlinear acoustics.

There is also a source of nonlinearity of solids that appears to come from the presence of domains in lithium niobate; this has been called *acoustic memory* [6.36].

It is possible to measure all six third-order elastic constants of cubic crystals. To do so, however, it is necessary to make additional measurements. The procedure that minimizes errors in the evaluation of third-order elastic constants from combination of nonlinearity parameters with the results of hydrostatic-pressure measurements has been considered by *Breazeale* et al. [6.37] and applied to the evaluation of the third-order elastic constants of two perovskite crystals.

6.5.3 Comparison of Fluids and Solids

To facilitate comparison between fluids and solids, it is necessary to use a binomial expansion of the denominator of (6.73)

$$\left(1 + \frac{\partial \xi}{\partial a}\right)^{-\left(\frac{B}{A}+2\right)} = 1 + \left(\frac{B}{A} + 2\right) \frac{\partial \xi}{\partial a} + \dots \tag{6.77}$$

Using this expression, (6.73) becomes

$$\frac{\partial^2 \xi}{\partial t^2} = c_0^2 \frac{\partial^2 \xi}{\partial a^2} + c_0^2 \left(\frac{B}{A} + 2\right) \frac{\partial \xi}{\partial a} \frac{\partial^2 \xi}{\partial a^2} + \dots \tag{6.78}$$

This form of the equation can be compared directly with (6.74) for solids. The ratio of the coefficient of the nonlinear term to that of the linear term can be evaluated directly. The nonlinearity parameters of the various substances are listed in Table 6.6. Use of Table 6.6 allows one to make a comparison between the nonlinearity of fluids as listed in Table 6.3 and the nonlinearity parameters of solids listed in Table 6.5. Nominally, they are of the same order of magnitude. This means that solids exhibit intrinsic nonlinearity that is comparable to that exhibited by fluids. Thus, the approximation made by assuming that Hooke's law (strain is proportional to stress) is valid for solids is comparable to the approximation made in the derivation of the linear wave equation for fluids.

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