Chapter 8
Evaluating Two-Stage Network Structures: Bargaining Game Approach

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Abstract This chapter presents a Nash bargaining game model to measure the performance of two-stage decision making units (DMUs) in data envelopment analysis (DEA). The two stages are viewed as players to bargain for a better payoff, which is represented by DEA ratio efficiency score. The efficiency model is developed as a cooperative game model. It is shown that when only one intermediate measure exists between the two stages, the newly-developed bargaining approach yields the same results as applying the standard DEA approach to each stage separately.

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8.1 Introduction

In order to address the potential conflict caused by the dual role of intermediate measures, quite a number of scholars propose their own versions of solutions. For example, Kao and Hwang (2008) combine the efficiency scores of the two stages in a multiplicative (geometric) manner, while Chen et al. (2009) use a weighted additive model. Liang et al. (2008) develop a number of DEA models using game theory concept. Specifically, they develop a leader-follower model borrowed from the notion of Stackelberg games, and a centralized or cooperative game model where the combined stage is of interest.

This chapter presents the study of Du et al. (2011) which applies directly the Nash bargaining game theory to the efficiency of DMUs that have the aforementioned two-stage processes. The two stages are regarded as two individuals bargaining with each other for a better payoff, which is characterized by the DEA ratio efficiency of each individual stage. In general, the resulting bargaining game model is highly non-linear, given the nature of ratio forms of DEA efficiency. This chapter shows that this non-linear bargaining model can be converted equivalently into a parametric linear programming problem with one parameter, whose lower and upper bounds can be determined. As a result, a global optimal solution can be found using a heuristic search on the single parameter.

In the bargaining model, the breakdown or status quo point is determined via the standard DEA model. The bargaining efficiency scores of the two stages may depend on the selection of the breakdown point. Thus in applications, a sensitivity analysis is carried out to study the stability of the bargaining DEA efficiency scores with respect to different status quo points. Also, it is shown that when only one intermediate measure exists between the two stages, the Nash bargaining game model in this study yields the same results as applying the standard DEA model to each stage separately.

8.2 Background

Consider a two-stage process shown in Fig. 8.1. Suppose there are n DMUs and each DMU_j ($j = 1, 2, \ldots, n$) has m inputs to the first stage, denoted by x_{ij} ($i = 1, 2, \ldots, m$), and D outputs from this stage, denoted by z_{dj} ($d = 1, 2, \ldots, D$). These D outputs then become the inputs to the second stage, which are referred to as intermediate measures. The s outputs from the second stage are denoted by y_{rj} ($r = 1, 2, \ldots, s$).

Based upon the constant returns to scale (CRS) model (Charnes et al. 1978), the (CRS) efficiency scores for each DMU_j ($j = 1, 2, \ldots, n$) in the first and second stages can be defined by e_j^1 and e_j^2, respectively,

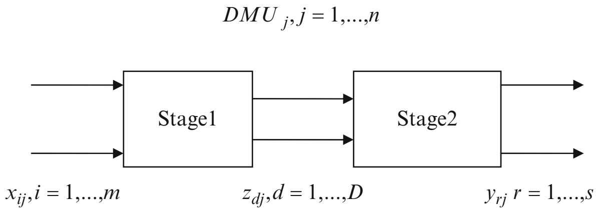

Fig. 8.1 A two-stage process

$$e_j^1 = \frac{\sum_{d=1}^{D} w_d^1 z_{dj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1 \quad \text{and} \quad e_j^2 = \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{d=1}^{D} w_d^2 z_{dj}} \le 1 \tag{8.1}$$

where v_i, w_d^1, w_d^2 and u_r are unknown non-negative weights. These ratios are then optimized in a linear fractional programming problem which can be converted into a linear CRS DEA model (Charnes et al. 1978).

As noted both in Kao and Hwang (2008) and in Liang et al. (2008), it is reasonable to set w_d^1 equal to w_d^2, since the "worth" or value assigned to the intermediate measures should be the same regardless of whether they are viewed as outputs from the first stage or inputs to the second stage. Then in this case, given the individual efficiency scores e_j^1 and e_j^2, it is reasonable to define the overall efficiency of the entire two-stage process for DMU_j ($j = 1, \ldots, n$) as $e_j = e_j^1 \cdot e_j^2$ since

$$e_j = \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} = \frac{\sum_{d=1}^{D} w_d z_{dj}}{\sum_{i=1}^{m} v_i x_{ij}} \cdot \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{d=1}^{D} w_d z_{dj}} = e_j^1 \cdot e_j^2 \tag{8.2}$$

The above overall efficiency definition ensures that $e_j \le 1$ from $e_j^1 \le 1$ and $e_j^2 \le 1$, and the overall process is efficient if and only if $e_j^1 = e_j^2 = 1$.

Clearly, separate DEA analysis can be applied to each individual stage as in Seiford and Zhu (1999). However, as pointed out by Liang et al. (2008), such an approach could cause inherent conflict between these two separate analyses.

The efficiency-evaluation problem can be approached from two game theory perspectives. One is to view the two-stage process as a non-cooperative game model, in which one stage is assumed to be a leader and solved for its CRS efficiency first, and the other stage a follower, whose efficiency is computed without changing the leader's efficiency score. The other approach is to regard the process as a centralized model, where the overall efficiency given in (8.2) is maximized, and a decomposition of the overall efficiency is obtained by finding a set of multipliers producing the largest first (or second) stage efficiency score while maintaining the overall efficiency score.

Note that in fact, the two stages can be regarded as two players in Nash bargaining game. Therefore, the efficiency evaluation of two-stage processes can be approached by using Nash bargaining game theory directly. But before that, we first briefly introduce the Nash bargaining game approach.

Denote the set of two individuals participating in the bargaining by $N = \{1, 2\}$, and a payoff vector is an element of the payoff space R^2, which is the 2-dimensional Euclidean space. A feasible set S is a subset of the payoff space, and a breakdown or status quo point $\vec{b}$ is an element of the payoff space. A bargaining problem can then be specified as the triple $\left(N, S, \vec{b}\right)$ consisting of participating individuals, feasible set, and breakdown point. Nash (1950) requires that the feasible set be compact, convex, and contain some payoff vector such that each individual's payoff is at least as large as the individual's breakdown payoff. The solution is a function F that is associated with each bargaining problem $\left(N, S, \vec{b}\right)$, expressed as $F\left(N, S, \vec{b}\right)$. Nash (1950, 1953) argue that a reasonable solution should satisfy the four properties: (i) Pareto efficiency (*PE*), (ii) invariance with respect to affine transformation (*IAT*), (iii) independence of irrelevant alternatives (*IIA*), and (iv) symmetry (*SYM*). Due to extensive discussion about these properties in the literature, no detailed explanation will be provided here. For the traditional bargaining problem, Nash (1950, 1953) has shown that there exists a unique solution, called the Nash solution, which satisfies the above-mentioned four properties, and can be obtained by solving the following maximization problem

$$\underset{\vec{u}\in S,\, \vec{u}\geq \vec{b}}{Max} \prod_{i=1}^{2} \left(u_i - b_i\right) \tag{8.3}$$

where $\vec{u}$ is the payment vector for the individuals, and u_i, b_i is the ith element of vector $\vec{u}$, $\vec{b}$, respectively.

8.3 Nash Bargaining Game Model for Two-Stage Structures

In the current case, we view the two individual stages as two players in the bargaining procedure, the efficiency ratios as the payoffs, and weights chosen for efficiency scores as strategies. To proceed, one needs to find a breakdown point for stages 1 and 2 which is the starting point for bargaining. Note that the breakdown point or status quo represents possible payoff pairs obtained if one decides not to bargain with the other player. As mentioned in Binmore et al. (1986), the choice of the breakdown point is a matter of modeling judgment. A number of elements in the underlying situation can be natural candidates for this role. For example, Lundberg and Pollak (1993) use a non-cooperative equilibrium as the breakdown point in their bargaining model. In application section, we will use different breakdown points, including the ones based upon the leader-follower (non-cooperative) model of Liang et al. (2008), to perform sensitivity analysis to study the stability of our bargaining DEA efficiency scores with respect to different breakdown points.

We here first construct the least ideal DMU and use its DEA efficiency scores as the breakdown point. By doing that, we assume that if the two stages do not negotiate, their efficiency scores will be the worst. Note that such a DMU may not exist, however, its inputs and outputs are observed. Let $x_i^{\max} = \max_j \{x_{ij}\}$, $y_r^{\min} = \min_j \{y_{rj}\}$, $z_d^{\min} = \min_j \{z_{dj}\}$ and $z_d^{\max} = \max_j \{z_{dj}\}$. Then $(x_i^{\max}, z_d^{\min})$ $(i = 1, \ldots, m,\ \ d = 1, \ldots, D)$ represents the least ideal DMU in the first stage, which consumes the maximum amount of input values, while producing the least amount of intermediate measures. Similarly, we denote $(z_d^{\max},\ y_r^{\min})$ $(d = 1, \ldots, D,$ $r = 1, \ldots, s)$ the least ideal DMU in the second stage, which consumes the maximum amount of intermediate measures while producing the least amount of output values.

The CRS efficiency for the above two least ideal DMUs is the worst among the existing DMUs. We denote the (CRS) efficiency scores of the two least ideal DMUs in the first and second stage as $\theta^1_{\min}$ and $\theta^2_{\min}$, respectively, and use $\theta^1_{\min}$ and $\theta^2_{\min}$ as the breakdown point. The (input-oriented) DEA bargaining model for a specific DMU_o with respect to (8.3) can be expressed as

$$
\begin{aligned}
& Max \left(\frac{\sum_{d=1}^{D} w_d z_{do}}{\sum_{i=1}^{m} v_i x_{io}} - \theta^1_{\min} \right) \cdot \left(\frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{d=1}^{D} w_d z_{do}} - \theta^2_{\min} \right) \\
& s.t. \ \frac{\sum_{d=1}^{D} w_d z_{do}}{\sum_{i=1}^{m} v_i x_{io}} \geq \theta^1_{\min} \\
& \quad \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{d=1}^{D} w_d z_{do}} \geq \theta^2_{\min} \\
& \quad \frac{\sum_{d=1}^{D} w_d z_{dj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, \quad j = 1, \ldots, n \\
& \quad \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{d=1}^{D} w_d z_{dj}} \leq 1, \quad j = 1, \ldots, n \\
& \quad v_i,\ u_r,\ w_d > 0, \quad i = 1, \ldots, m,\ r = 1, \ldots, s,\ d = 1, \ldots, D
\end{aligned}
\tag{8.4}
$$

Denote all the constraints in model (8.4) by S, which represents the feasible set for this bargaining problem. Then the bargaining problem here can be specified as the triple $(\{1, 2\}, S, \{\theta^1_{\min}, \theta^2_{\min}\})$. Next we will prove that the feasible set S is both compact and convex.

Lemma 1 *The feasible set* S *is compact and convex.*

Proof Since the feasible set S is bounded and closed in Euclidean space, then S is compact. Next we will prove that S is also convex.

Suppose $(v'_1, \ldots, v'_m, u'_1, \ldots, u'_s, w'_1, \ldots, w'_D) \in S$ and $(v''_1, \ldots, v''_m, u''_1, \ldots, u''_s, w''_1, \ldots, w''_D) \in S$. For any $\lambda \in [0, 1]$ we have $\lambda v'_i + (1-\lambda) v''_i > 0, i = 1, \ldots, m$, $\lambda u'_r + (1-\lambda) u''_r > 0$, $r = 1, \ldots, s$ and $\lambda w'_d + (1-\lambda) w''_d > 0$, $d = 1, \ldots, D$. Since $\sum_{i=1}^{m} v_i x_{ij} > 0$ and $\sum_{d=1}^{D} w_d z_{dj} > 0$ for all $j = 1, \ldots, n$, the constraints in S, $\frac{\sum_{d=1}^{D} w_d z_{dj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1$ and $\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{d=1}^{D} w_d z_{dj}} \le 1$ are equivalent to $\sum_{d=1}^{D} w_d z_{dj} \le \sum_{i=1}^{m} v_i x_{ij}$ and $\sum_{r=1}^{s} u_r y_{rj} \le \sum_{d=1}^{D} w_d z_{dj}$, respectively, for all $j = 1, \ldots, n$, and the constraints $\frac{\sum_{d=1}^{D} w_d z_{do}}{\sum_{i=1}^{m} v_i x_{io}} \ge \theta^1_{\min}$ and $\frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{d=1}^{D} w_d z_{do}} \ge \theta^2_{\min}$ are equivalent to $\sum_{d=1}^{D} w_d z_{do} \ge \theta^1_{\min} \sum_{i=1}^{m} v_i x_{ij}$ and $\sum_{r=1}^{s} u_r y_{rj} \ge \theta^2_{\min} \sum_{d=1}^{D} w_d z_{do}$, respectively. Then we have

$$\begin{aligned}
\sum_{d=1}^{D} \left[\lambda w'_d + (1-\lambda) w''_d\right] z_{dj} &= \lambda \sum_{d=1}^{D} w'_d z_{dj} + (1-\lambda) \sum_{d=1}^{D} w''_d z_{dj} \\
&\le \lambda \sum_{i=1}^{m} v'_i x_{ij} + (1-\lambda) \sum_{i=1}^{m} v''_i x_{ij} \\
&= \sum_{i=1}^{m} \left[\lambda v'_i + (1-\lambda) v''_i\right] x_{ij}
\end{aligned}$$

and

$$\begin{aligned}\sum_{r=1}^{s}\left[\lambda\, u'_r+(1-\lambda)\, u''_r\right] y_{rj} &= \lambda\sum_{r=1}^{s}u'_r y_{rj}+(1-\lambda)\sum_{r=1}^{s}u''_r y_{rj}\\ &\le \lambda\sum_{d=1}^{D}w'_d z_{dj}+(1-\lambda)\sum_{d=1}^{D}w''_d z_{dj}\\ &= \sum_{d=1}^{D}\left[\lambda\, w'_d+(1-\lambda)\, w''_d\right] z_{dj}\end{aligned}$$

Similarly, we have $\sum_{d=1}^{D}\left[\lambda w'_d+(1-\lambda)\, w''_d\right] z_{do}\ge\theta^1_{\min}\sum_{i=1}^{m}\left[\lambda\, v'_i+(1-\lambda)\, v''_i\right]$ x_{ij} and $\sum_{r=1}^{s}\left[\lambda\, u'_r+(1-\lambda)\, u''_r\right] y_{rj}\ge\theta^2_{\min}\sum_{d=1}^{D}\left[\lambda\, w'_d+(1-\lambda)\, w''_d\right] z_{do}$.

Therefore $(\lambda\, v'_i + (1-\lambda)\, v''_i,\ \lambda\, u'_r + (1-\lambda)\, u''_r,\ \lambda\, w'_d + (1-\lambda)\, w''_d) \in S$, where $i = 1, \ldots, m$, $r = 1, \ldots, s$, $d = 1, \ldots, D$, or equivalently, $\lambda\,(v'_1, \ldots, v'_m, u'_1, \ldots, u'_s, w'_1, \ldots, w'_D) + (1-\lambda)(v''_1, \ldots, v''_m, u''_1, \ldots, u''_s, \ldots, w''_1, \ldots, w''_D) \in S$. Consequently S is a convex set.

Q.E.D.

Let $t_1=\left(\sum_{i=1}^{m}v_i x_{io}\right)^{-1}$, $t_2=\left(\sum_{d=1}^{D}w_d z_{do}\right)^{-1}$, $\gamma_i=t_1\ v_i$, $\omega_d=t_1\ w_d$, $\mu_{r1}=t_1\ u_r$, $\mu_{r2}=t_2\ u_r$. Note that $\mu_{r1}=t_1\ u_r$ and $\mu_{r2}=t_2\ u_r$ imply a linear relationship of $\mu_{r1}=\frac{t_1}{t_2}\ \mu_{r2}$ between μ_{r1} and μ_{r2}. Therefore, we denote $\frac{t_1}{t_2}$ by α (>0) and have $\mu_{r1}=\alpha\mu_{r2}$ for all $r=1,\ldots,s$. Then model (8.4) is converted into model (8.5).

$$\begin{aligned}
Max\ & \sum_{r=1}^{s}\mu_{r1}y_{ro}-\theta^1_{\min}\sum_{r=1}^{s}\mu_{r2}y_{ro}-\theta^2_{\min}\sum_{d=1}^{D}\omega_d z_{do}+\theta^1_{\min}\cdot\ \theta^2_{\min}\\
s.t.\ & \sum_{d=1}^{D}\omega_d z_{do}\ge\theta^1_{\min}\\
& \sum_{r=1}^{s}\mu_{r2}y_{ro}\ge\theta^2_{\min}\\
& \sum_{i=1}^{m}\gamma_i x_{io}=1\\
& \sum_{d=1}^{D}\omega_d z_{do}=\alpha\\
& \sum_{d=1}^{D}\omega_d z_{dj}-\sum_{i=1}^{m}\gamma_i x_{ij}\le 0,\quad j=1,\ldots,n\\
& \sum_{r=1}^{s}\mu_{r1}y_{rj}-\sum_{d=1}^{D}\omega_d z_{dj}\le 0,\quad j=1,\ldots,n\\
& \mu_{r1}=\alpha\mu_{r2},\quad r=1,\ldots s\\
& \alpha>0,\ \gamma_i,\ \omega_d,\ \mu_{r1},\ \mu_{r2}>0,\quad i=1,\ldots,m,\ r=1,\ldots,s,\ d=1,\ldots,D
\end{aligned}\tag{8.5}$$

Model (8.5) is equivalent to the following nonlinear model (8.6).

$$
\begin{aligned}
& Max\ \alpha \times \sum_{r=1}^{s} \mu_{r2} y_{ro} - \theta_{\min}^{1} \sum_{r=1}^{s} \mu_{r2} y_{ro} - \theta_{\min}^{2} \sum_{d=1}^{D} \omega_d z_{do} + \theta_{\min}^{1} \cdot \theta_{\min}^{2} \\
& s.t.\ \sum_{d=1}^{D} \omega_d z_{do} \geq \theta_{\min}^{1} \\
& \quad \sum_{r=1}^{s} \mu_{r2} y_{ro} \geq \theta_{\min}^{2} \\
& \quad \sum_{i=1}^{m} \gamma_i x_{io} = 1 \\
& \quad \sum_{d=1}^{D} \omega_d z_{do} = \alpha \\
& \quad \sum_{d=1}^{D} \omega_d z_{dj} - \sum_{i=1}^{m} \gamma_i x_{ij} \leq 0, \quad j = \quad 1, \ldots, n \\
& \quad \alpha \times \sum_{r=1}^{s} \mu_{r2} y_{rj} - \sum_{d=1}^{D} \omega_d z_{dj} \leq 0, \quad j = 1, \ldots, n \\
& \quad \alpha > 0,\ \gamma_i,\ \omega_d,\ \mu_{r2} > 0, \quad i = 1, \ldots, m,\ r = 1, \ldots, s,\ d = 1, \ldots, D
\end{aligned}
\tag{8.6}
$$

Note the constraints in model (8.6) that $\sum_{i=1}^{m} \gamma_i x_{io} = 1$, $\sum_{d=1}^{D} \omega_d z_{do} \geq \theta_{\min}^{1}$, $\sum_{d=1}^{D} \omega_d z_{do} = \alpha$, and for any $j = 1, \ldots, n$, $\sum_{d=1}^{D} \omega_d z_{dj} - \sum_{i=1}^{m} \gamma_i x_{ij} \leq 0$. Then we have $\theta_{\min}^{1} \leq \alpha = \sum_{d=1}^{D} \omega_d z_{do} \leq \sum_{i=1}^{m} \gamma_i x_{io} = 1$, which provides both upper and lower bounds on α, and indicates that the optimal value of α represents the first-stage efficiency score for each DMU.

Thus α can be treated as a parameter within $[\theta_{\min}^{1}, 1]$. As a result, model (8.6) can be solved as a parametric linear program via searching over the possible α values within $[\theta_{\min}^{1}, 1]$. In computation, we set the initial value for α as the upper bound one, and solve the corresponding linear program. Then we begin to decrease α by a very small positive number ε (=0.0001 for example) for each step t, namely, $\alpha_t = 1 - \varepsilon \times t$, $t = 1, 2, \ldots$, until the lower bound $\theta_{\min}^{1}$ is reached, and solve each linear program of model (8.6) corresponding to α_t and denote the corresponding optimal objective value by Ω_t. Note that not all values taken by α within $[\theta_{\min}^{1}, 1]$ lead to feasible solutions to program (8.6). Let $\Omega^* = \max_t \Omega_t$ and denote the specific α_t associated with Ω^* as α^*. Note that it is likely that Ω^* is associated with several α^* values.

Then Ω^* associated with α^* is solution to model (8.6). Denote $e_o^{1*} = \alpha^*\left(= \sum_{d=1}^{D} \omega_d^* z_{do}\right)$, $e_o^{2*} = \sum_{r=1}^{s} \mu_{r2}^* y_{ro}$ and $e_o^* = e_o^{1*} \cdot e_o^{2*}$ as DMU_o's bargaining efficiency scores for the first and second stages and the overall process, respectively.

With respect to the four properties associated with a bargaining solution, we have (i) Pareto efficiency (*PE*) indicates that for the bargaining efficiency scores, there is no possibility to improve one stage's individual efficiency score without decreasing the other individual efficiency score; (ii) invariance with respect to affine transformation (*IAT*) reveals that if both the feasible region of bargaining model (8.6) and the breakdown point are subjected to an affine transformation on the payoff space R^2, then the bargaining efficiency scores satisfy the same affine transformation; (iii) independence of irrelevant alternatives (*IIA*) shows that the bargaining efficiency scores will not change when the feasible region of bargaining model (8.6) is decreased but still includes the bargaining solution; and (iv) symmetry (*SYM*) demonstrates that if $\left(S, \vec{b}\right)$ is symmetric, where S is the feasible region of bargaining model (8.6) and $\vec{b}$ is breakdown point, then the bargaining efficiency scores of both individual stages are equal to each other.

8.4 Mathematical Relationship

We finally look at the relationship between the bargaining efficiency scores obtained from model (8.6) and the standard CRS efficiency scores. Let θ_o^1 and θ_o^2 represent the standard (CRS) efficiency scores for the first and second stages, respectively. It will be shown that when there is only one intermediate measure linking the two stages, $e_o^{1*} = \theta_o^1$ and $e_o^{2*} = \theta_o^2$.

Theorem 1 *For any specific* $\mathrm{DMU_o}$, $\Omega_\mathrm{o} \leq (\theta_o^1 - \theta_{\min}^1) \cdot (\theta_o^2 - \theta_{\min}^2)$, *where* Ω_o *is the (maximum) optimal value to model (8.6) (or model (8.4)).*

Proof θ_o^1 and θ_o^2 can be obtained by solving the following two regular DEA models (8.7) and (8.8), respectively.

$$\begin{aligned} \theta_o^1 &= Max \ \frac{\sum_{d=1}^{D} \hat{w}_d^1 z_{do}}{\sum_{i=1}^{m} \hat{v}_i x_{io}} \\ s.t. &\quad \frac{\sum_{d=1}^{D} \hat{w}_d^1 z_{dj}}{\sum_{i=1}^{m} \hat{v}_i x_{ij}} \leq 1, \ j = 1, \ldots, n \\ &\quad \hat{v}_i, \hat{w}_d^1 > 0, \ i = 1, \ldots, m, d = 1, \ldots, D \end{aligned} \tag{8.7}$$

$$
\begin{aligned}
\theta_o^2 &= Max \ \frac{\sum_{r=1}^{s} \hat{u}_r y_{ro}}{\sum_{d=1}^{D} \hat{w}_d^2 z_{do}} \\
s.t. \ & \frac{\sum_{r=1}^{s} \hat{u}_r y_{rj}}{\sum_{d=1}^{D} \hat{w}_d^2 z_{dj}} \le 1, \ j = 1, \ldots, n \\
& \hat{u}_r, \hat{w}_d^2 > 0, \ r = 1, \ldots, s, d = 1, \ldots, D
\end{aligned}
\tag{8.8}
$$

Let v_i^*, w_d^* and u_r^* be an optimal solution to the bargaining model (8.4). By comparing the constraints in models (8.4), (8.7) and (8.8), we note that the feasible regions of model (8.7) and (8.8) both contain the feasible region of model (8.4). Thus, v_i^* and w_d^* are a feasible solution to model (8.7), and w_d^* and u_r^* are a feasible solution to model (8.8). Therefore, we have $\frac{\sum_{d=1}^{D} w_d^* z_{do}}{\sum_{i=1}^{m} v_i^* x_{io}} \le \theta_o^1$ and $\frac{\sum_{r=1}^{s} u_r^* y_{ro}}{\sum_{d=1}^{D} w_d^* z_{do}} \le \theta_o^2$, and furthermore $\Omega_o \le \ (\theta_o^1 - \theta_{\min}^1) \ \cdot (\theta_o^2 - \theta_{\min}^2)$.

Q.E.D.

Based upon Theorem 1, under the special case of one intermediate measure, we have

Theorem 2 *If there is only one intermediate measure, then* $\Omega_o = (\theta_o^1 - \theta_{\min}^1) \cdot (\theta_o^2 - \theta_{\min}^2)$.

Proof Under the situation of one intermediate measure where $D = 1$, let $\hat{v}_i^*$ and $\hat{w}_1^{1*}$ be an optimal solution to model (8.7), and $\hat{u}_r^*$ and $\hat{w}_1^{2*}$ be an optimal solution to model (8.8), then we have $\theta_o^1 = \frac{\hat{w}_1^{1*} z_{1o}}{\sum_{i=1}^{m} \hat{v}_i^* x_{io}}$, $\theta_o^2 = \frac{\sum_{r=1}^{s} \hat{u}_r^* y_{ro}}{\hat{w}_1^{2*} z_{1o}}$, and $\left(\frac{\hat{w}_1^{1*}}{\hat{w}_1^{2*}}\right) \hat{u}_r^*$ and $\hat{w}_1^{1*}$ can be another optimal solution to model (8.8). By the definition of $\theta_{\min}^1$ and $\theta_{\min}^2$, we know that $\theta_o^1 \ge \theta_{\min}^1$, and $\theta_o^2 \ge \theta_{\min}^2$. Therefore $\hat{v}_i^*, \hat{w}_1^{1*}$ and $\left(\frac{\hat{w}_1^{1*}}{\hat{w}_1^{2*}}\right) \hat{u}_r^*$ satisfy all the constraints in our bargaining game model (8.4), indicating that $\hat{v}_i^*$, $\hat{w}_1^{1*}$ and $\left(\frac{\hat{w}_1^{1*}}{\hat{w}_1^{2*}}\right) \hat{u}_r^*$ are a feasible solution to model (8.4). Thus, we have $\Omega_o \ge \ (\ \theta_o^1 - \theta_{\min}^1) \ \cdot (\theta_o^2 - \theta_{\min}^2)$.

From Theorem 1, we have $\Omega_o \le \ (\theta_o^1 - \theta_{\min}^1) \ \cdot (\theta_o^2 - \theta_{\min}^2)$. Therefore, $\Omega_o = (\theta_o^1 - \theta_{\min}^1) \ \cdot (\theta_o^2 - \theta_{\min}^2)$.

Q.E.D.

Since the regular (CRS) efficiency scores for the first stage θ_o^1 and for the second stage θ_o^2 are the maximum achievable efficiency scores for individual stages, based upon Theorems 1 and 2, we have

Theorem 3 *If there is only one intermediate measure, then* $\mathrm{e}_o^{1*} = \theta_o^1$ *and* $\mathrm{e}_o^{2*} = \theta_o^2$*, where* e_o^{1*} *and* e_o^{2*} *represent the bargaining efficiency scores to the first and second stage of any specific* $\mathrm{DMU_o}$ *obtained via model (8.6), respectively.*

Proof In the case of one intermediate measure where $D = 1$, let v_i^*, w_1^* and u_r^* be an optimal solution to the bargaining model (8.4), and then $e_o^{1*} = \frac{w_1^* z_{1o}}{\sum_{i=1}^{m} v_i^* x_{io}} \geq \theta_{\min}^1$ and $e_o^{2*} = \frac{\sum_{r=1}^{s} u_r^* y_{ro}}{w_1^* z_{1o}} \geq \theta_{\min}^2$. From the proof of Theorem 1, we have $e_o^{1*} \leq \theta_o^1$ and $e_o^{2*} \leq \theta_o^2$, and based on Theorem 2, we have $(e_o^{1*} - \theta_{\min}^1) \cdot (e_o^{2*} - \theta_{\min}^2) = \Omega_o$ $=(\theta_o^1 - \theta_{\min}^1) \cdot (\theta_o^2 - \theta_{\min}^2)$. Therefore $e_o^{1*} = \theta_o^1$ and $e_o^{2*} = \theta_o^2$ must be true.

Q.E.D.

Theorem 3 also indicates that a unique pair of (bargaining) efficiency scores for both stages are obtained for each DMU, which is $(\theta_o^1,\ \theta_o^2)$, regardless the choice of breakdown point. i.e., with one intermediate measure, model (8.4) is independent of the breakdown point. However, such independence can no longer hold when multiple intermediate measures are considered, which will be discussed later. Liang et al. (2008) prove the same conclusion with respect to their leader-follower and centralized models. We note, however, that Liang et al. (2008) models are fundamentally different from this bargaining model. To further explain, we present the centralized model in Liang et al. (2008) as follows.

$$
\begin{aligned}
& e_o^{centralized} = Max \sum_{r=1}^{s} \mu_r y_{ro} \\
& s.t. \quad \sum_{r=1}^{s} \mu_r y_{rj} - \sum_{d=1}^{D} \omega_d z_{dj} \leq 0, \quad j = 1, \ldots, n \\
& \qquad \sum_{d=1}^{D} \omega_d z_{dj} - \sum_{i=1}^{m} \gamma_i x_{ij} \leq 0, \quad j = 1, \ldots, n \\
& \qquad \sum_{i=1}^{m} \gamma_i x_{io} = 1 \\
& \qquad \mu_r, \gamma_i, \omega_d \geq \varepsilon, \quad r = 1, \ldots, s, \quad i = 1, \ldots, m, \quad d = 1, \ldots, D
\end{aligned}
\tag{8.9}
$$

It can be seen that the Nash bargaining game model reduces to the centralized model of Liang et al. (2008) when the breakdown point is set equal to (0, 0). Or, when their centralized efficiency scores are used as breakdown points, the Nash bargaining game model cannot improve the breakdown point, namely, the centralized model of Liang et al. (2008) provides a set of "best" overall bargaining efficiency scores. However, this does not necessary imply that the results from the centralized model should be used. The centralized model solution may not be acceptable to the two stages, or ideal with respect to improving the two stages' operations. The bargaining model is not about finding the best overall efficiency score, or the best solution, but rather is about finding the best achievable efficiency through negotiation. A breakdown point (0, 0) only leads to the best overall efficiency score, but not necessarily the best achievable efficiency for Stage 1 or 2. A breakdown point of (0, 0) simply implies that the two stages will get an efficiency score of zero if they do not negotiate. This may further imply that (0, 0) is not a good candidate for a breakdown point in bargaining model.

8.5 Output-Oriented Bargaining Model

The above DEA bargaining model (8.4) is input-oriented. If an output-orientation is taken into account, the bargaining model becomes

$$
\begin{aligned}
& Max \left(\frac{\sum_{i=1}^{m} v_i x_{io}}{\sum_{d=1}^{D} w_d z_{do}} - h_{\min}^1 \right) \cdot \left(\frac{\sum_{d=1}^{D} w_d z_{do}}{\sum_{r=1}^{s} u_r y_{ro}} - h_{\min}^2 \right) \\
& s.t. \quad \frac{\sum_{i=1}^{m} v_i x_{io}}{\sum_{d=1}^{D} w_d z_{do}} \le h_{\min}^1 \\
& \qquad \frac{\sum_{d=1}^{D} w_d z_{do}}{\sum_{r=1}^{s} u_r y_{ro}} \le h_{\min}^2 \\
& \qquad \frac{\sum_{i=1}^{m} v_i x_{ij}}{\sum_{d=1}^{D} w_d z_{dj}} \ge 1, \quad j = 1, \ldots, n \\
& \qquad \frac{\sum_{d=1}^{D} w_d z_{dj}}{\sum_{r=1}^{s} u_r y_{rj}} \ge 1, \quad j = 1, \ldots, n \\
& \qquad v_i,\ u_r,\ w_d > 0, \quad i = 1, \ldots, m, \quad r = 1, \ldots, s, \quad d = 1, \ldots, D
\end{aligned} \tag{8.10}
$$

where $h^1_{\min}$ and $h^2_{\min}$ represent the output-oriented CRS efficiency scores of the two least ideal DMUs for the first and second stage, respectively. Since $h^1_{\min} = \frac{1}{\theta^1_{\min}}$ and $h^2_{\min} = \frac{1}{\theta^2_{\min}}$, model (8.4) and (8.10) are structurally the same except for the different objective functions.

Let $t_1 = \left(\sum_{d=1}^{D} w_d z_{do}\right)^{-1}$, $t_2 = \left(\sum_{r=1}^{s} u_r y_{ro}\right)^{-1}$, $\gamma_{i1} = t_1 \ v_i$, $\gamma_{i2} = t_2 \ v_i$, $\omega_d = t_2 \ w_d$, $\mu_r = t_2 \ u_r$. Note that $\gamma_{i1} = t_1 \ v_i$ and $\gamma_{i2} = t_2 \ v_i$ imply a linear relationship of $\gamma_{i2} = \frac{t_2}{t_1} \gamma_{i1}$ between γ_{i1} and γ_{i2}. Therefore, we denote $\frac{t_2}{t_1}$ by β (> 0) and have $\gamma_{i2} = \beta \gamma_{i1}$ for all $i = 1, \ldots, m$. Then model (8.10) can be equivalently converted into model (8.11) with parameter β.

$$
\begin{aligned}
& Max \ \beta \sum_{i=1}^{m} \gamma_{i1} x_{io} - h^2_{\min} \sum_{i=1}^{m} \gamma_{i1} x_{io} - h^1_{\min} \sum_{d=1}^{D} \omega_d z_{do} + h^1_{\min} \cdot \ h^2_{\min} \\
& s.t. \ \sum_{d=1}^{D} \omega_d z_{do} \le h^2_{\min} \\
& \qquad \sum_{i=1}^{m} \gamma_{i1} x_{io} \le h^1_{\min} \\
& \qquad \sum_{r=1}^{s} \mu_r y_{ro} = 1 \\
& \qquad \sum_{d=1}^{D} \omega_d z_{do} = \beta \\
& \qquad \sum_{d=1}^{D} \omega_d z_{dj} - \sum_{r=1}^{s} \mu_r y_{rj} \ge 0, \quad j = 1, \ldots, n \\
& \qquad \beta \sum_{i=1}^{m} \gamma_{i1} x_{ij} - \sum_{d=1}^{D} \omega_d z_{dj} \ge 0, \quad j = 1, \ldots, n \\
& \qquad \beta > 0, \ \gamma_{i1}, \ \gamma_{i2}, \ \omega_d, \ \mu_r > 0, \quad i = 1, \ldots, m, \quad r = 1, \ldots, s, \quad d = 1, \ldots, D
\end{aligned}
\tag{8.11}
$$

We have $h^2_{\min} \ge \beta = \sum_{d=1}^{D} \omega_d z_{do} \ge \sum_{r=1}^{s} \mu_r y_{ro} = 1$, which provides both upper and lower bounds on β, and indicates that the optimal value of β represents the second-stage efficiency score for each DMU. Therefore model (8.11) can be solved as a parametric linear program via searching over the possible β values within $[1, h^2_{\min}]$.

We should point out that the DEA bargaining model presented in this chapter is not suitable for situations when one stage is input-oriented and the other is output-oriented. It is because the resulting bargaining model cannot be transformed into a parametric linear program like model (8.6) or (8.11).

8.6 Applications

The Nash bargaining game approach is applied to two real world data sets. The first one consists of 30 top US commercial banks with two intermediate measures, which was used in Seiford and Zhu (1999) first, and then in Liang et al. (2008). The second data set, which was previously studied both in Kao and Hwang (2008) and in Chen et al. (2009), also has two intermediate measures and consists of 24 Taiwanese non-life insurance companies.

8.6.1 Top US Commercial Banks

The data set consisting of 30 top US commercial banks is presented in Table 8.1. The inputs to the first stage are number of employees, assets ($ million) and equity ($ million). The intermediate measures connecting two stages are revenue ($ million) and profit ($ million). The outputs from the second stage are market value ($ million), earning per share ($) and returns to the investors (%). See Seiford and Zhu (1999) for discussion on the above measures.

The CRS efficiency scores for the least ideal DMUs in the first and second stages are calculated as $\theta^1_{\min} = 0.0775$ and $\theta^2_{\min} = 0.0515$, respectively. We next begin with the initial value for α in model (8.6) as one, then decrease α by a small positive number $\varepsilon = 0.0001$ for each step t, namely, $\alpha_t = 1 - 0.0001 \times t,\ t = 1,2,$ …, until the lower bound $\theta^1_{\min} = 0.0775$ is reached. Solving the linear program of model (8.6) for each step t corresponding to α_t, we obtain a best heuristic search solution to the bargaining efficiency scores of both individual stages and the overall process, which are reported in columns 2 through 4 in Table 8.2. Column 5 shows the corresponding value of the parameter α when the best heuristic search solution is obtained. In this case, the value for α associated with the optimal solution is unique for each DMU, indicating we have a unique pair of efficiency scores for both individual stages.

For comparison, columns 6 through 8 display the corresponding results from Liang et al. (2008) via the centralized model, which, as indicated above, could be viewed as a special case of Nash bargaining model with breakdown point (0, 0). Note that the efficiency scores of both individual stages and the overall process, obtained through the bargaining game approach, are almost the same with those obtained from Liang et al. (2008)'s centralized model, except for DMU 10. This indicates that bargaining results are very similar to those obtained from the centralized model for this particular data set.

The centralized scores obtained from Liang et al. (2008) represent efficiency pairs under the cooperative game structure that lead to the best overall efficiency scores. Thus, if the centralized efficiency scores are used as breakdown point, model (8.6) cannot further improve the bargaining efficiency scores for the two stages and model (8.6) must yield scores identical to the centralized scores.

Table 8.1 US commercial bank data

Bank	Employees	Assets	Equity	Revenue	Profit	Market value	Earning	Returns
1. Citicorp	85,300	256,853	19,581	31,690	3,464	33,221.7	7.21	66.1
2. BankAmerica Corp.	95,288	232,446	20,222	20,386	2,664	27,148.6	6.49	69.4
3. NationsBank Corp.	58,322	187,298	12,801	16,298	1,950	20,295.9	7.13	59.7
4. Chemical Banking Corp.	39,078	182,926	11,912	14,884	1,805	16,971.3	6.73	70.5
5. J.P. Morgan & Co.	15,600	184,879	10,451	13,838	1,296	15,003.5	6.42	49.4
6. Chase Manhattan Corp.	33,365	121,173	9,134	11,336	1,165	12,616.4	5.76	82.4
7. First Chicago NBD	35,328	122,002	8,450	10,681	1,150	12,351.1	3.45	50
8. First Union Corp.	44,536	131,879.9	9,043.1	10,582.9	1,430.2	16,815	5.04	39.9
9. Banc One Corp.	46,900	90,454	8,197.5	8,970.9	1,277.9	14,807.4	2.91	54.9
10. Bankers Trust New York	14,000	104,000	5,000	8,600	215	5,252.4	2.03	28.3
11. Fleet Financial	30,800	84,432.2	6,364.8	7,919.4	610	10,428.7	1.57	31.8
12. Norwest Corp.	45,404	72,134.4	5,312.1	7,582.3	956	12,268.6	2.76	45.5
13. PNC Bank Corp.	26,757	73,404	5,768	6,389.5	408.1	9,938.2	1.19	61.4
14. KeyCorp	28,905	66,339.1	5,152.5	6,054	825	8,671.2	3.45	51.6
15. Bank of Boston	17,881	47,397	3,751	5,410.6	541	5,310.1	4.55	84.7
16. Wells Fargo & Co.	19,700	50,316	4,055	5,409	1,032	11,342.5	20.37	52.8
17. Bank of New York	15,850	53,685	5,223	5,327	914	10,101.5	4.57	69.9
18. First Interstate Bancorp	27,200	58,071	4,154	4,827.5	885.1	12,138	11.02	108.5
19. Mellon Bank	24,300	40,129	4,106	4,514	691	7,476.7	4.5	83.8
20. Wachovia Corp.	15,996	44,981.3	3,773.8	3,755.4	602.5	7,623.6	3.5	46.9
21. SunTrust Banks	19,415	46,471.5	4,269.6	3,740.3	565.5	7,922.5	4.94	46.9
22. Barnett Banks	20,175	41,553.5	3,272.2	3,680	533.3	5,774.9	5.3	59
23. National City	20,767	36,199	2,921	3,449.9	465.1	4,912.2	3.03	33.9
24. First Bank System	13,231	33,874	2,725	3,328.3	568.1	8,304	4.19	54.3
25. Comerica	13,500	35,469.9	2,607.7	3,112.6	413.4	4,537	3.54	71.7
26. Boatmen's Bancshares	17,023	33,703.8	2,928.1	2,996.1	418.8	4,997	3.25	57.3
27. U.S. Bancorp	14,081	31,794.3	2,617	2,897.3	329	4,865.1	2.09	66.8
28. CoreStates Financial	13,598	29,620.6	2,379.4	2,868	452.2	5,788	3.22	52
29. Republic New York	4,900	43,881.6	3,007.8	2,859.6	288.6	3,218	4.66	41.1
30. MBNA	11,171	13,228.9	1,265.1	2,565.4	353.1	6,543.3	1.54	60.7

Table 8.2 Results for US commercial banks with breakdown point $\{\theta^1_{\min},\ \theta^2_{\min}\}$

	Bargaining efficiency scores				Centralized		
Bank	e_o^{1*}	e_o^{2*}	$e_o^{1*} \cdot e_o^{2*}$	α	$e_o^{1,Centralized}$	$e_o^{2,Centralized}$	$e_o^{Centralized}$
1	1.0000	0.4487	0.4487	1.0000	1.0000	0.4487	0.4487
2	0.6821	0.5327	0.3634	0.6821	0.6821	0.5327	0.3634
3	0.7946	0.5305	0.4215	0.7946	0.7946	0.5305	0.4216
4	0.8463	0.5050	0.4274	0.8463	0.8463	0.5050	0.4274
5	1.0000	0.6061	0.6061	1.0000	1.0000	0.6061	0.6061
6	0.8179	0.5111	0.4180	0.8179	0.8180	0.5110	0.4180
7	0.7816	0.5042	0.3941	0.7816	0.7816	0.5042	0.3940
8	0.7451	0.6371	0.4747	0.7451	0.7451	0.6371	0.4747
9	0.7021	0.6389	0.4486	0.7021	0.7021	0.6389	0.4486
10	0.5868	0.5735	0.3365	0.5868	0.4884	0.6946	0.3393
11	0.6619	0.6281	0.4157	0.6619	0.6619	0.6282	0.4158
12	0.6906	0.6576	0.4541	0.6906	0.6906	0.6576	0.4541
13	0.5843	0.7640	0.4464	0.5843	0.5843	0.7641	0.4464
14	0.7131	0.5852	0.4173	0.7131	0.7131	0.5852	0.4173
15	0.8469	0.7582	0.6421	0.8469	0.8469	0.7582	0.6421
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
17	1.0000	0.7144	0.7144	1.0000	1.0000	0.7144	0.7144
18	0.7974	1.0000	0.7974	0.7974	0.7974	1.0000	0.7974
19	0.7477	0.7811	0.5840	0.7477	0.7478	0.7811	0.5841
20	0.7541	0.7844	0.5915	0.7541	0.7542	0.7844	0.5916
21	0.6550	0.8660	0.5672	0.6550	0.6550	0.8661	0.5673
22	0.6491	0.8005	0.5196	0.6491	0.6491	0.8005	0.5196
23	0.6280	0.6330	0.3975	0.6280	0.6280	0.6330	0.3975
24	0.8711	0.9478	0.8256	0.8711	0.8711	0.9478	0.8257
25	0.7403	1.0000	0.7403	0.7403	0.7403	1.0000	0.7403
26	0.6344	0.8363	0.5305	0.6344	0.6345	0.8363	0.5306
27	0.6549	1.0000	0.6549	0.6549	0.6549	1.0000	0.6549
28	0.7735	0.8012	0.6197	0.7735	0.7736	0.8011	0.6198
29	0.8092	1.0000	0.8092	0.8092	0.8093	1.0000	0.8093
30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Note: The optimal value of parameter α represents the first-stage bargaining efficiency score for the corresponding DMU

In this case, we assume that such a payoff pair or breakdown point is acceptable to the two stages if they do not bargain. Also, any breakdown point other than the centralized efficiency scores will yield a smaller efficiency score for the overall process.

The choice of the breakdown point cannot be arbitrary. For example, it is likely that model (8.4) is infeasible if we use the minimum CRS efficiency score for each stage as the breakdown point. Also, if both breakdown points are greater than the corresponding centralized efficiency scores, model (8.4) will become infeasible. This infeasibility is mainly caused by the fact that some breakdown points will violate the constraints for individual efficiency scores in model (8.4).

Recall that Liang et al. (2008) also develop a non-cooperative leader-follower model where one of the two stages is treated as the leader and is given pre-emptive priority to maximize its efficiency. That is, for example, when the first stage is treated as the leader, the efficiency score for the first stage is calculated CRS score, θ_o^1, because this θ_o^1 is the best efficiency score DMU_o can achieve. Then the efficiency score for the second stage, e_o^2, is maximized given that the first stage's efficiency is fixed at θ_o^1.

Then the breakdown point of ($\min_j \left\{ \theta_j^1 \right\} = 0.6345$, $\min_j \left\{ e_j^2 \right\} = 0.3094$) based upon the leader-follower model of Liang et al. (2008) will ensure that model (8.4) is feasible. Here, 0.6345 is the smallest (CRS) efficiency score for the first stage, and 0.3094 is the smallest leader-follower score for the second stage.

Similarly, based upon the case when the second stage is treated as the leader, another breakdown point of ($\min_j \left\{ e_j^1 \right\} = 0.3056$, $\min_j \left\{ \theta_j^2 \right\} = 0.4859$) can be obtained.

Table 8.3 reports in columns 2 through 4 the bargaining efficiency scores for both individual stages and the overall process corresponding to breakdown point ($\min_j \left\{ \theta_j^1 \right\} = 0.6345$, $\min_j \left\{ e_j^2 \right\} = 0.3094$); columns 5 through 7 report the results corresponding to ($\min_j \left\{ e_j^1 \right\} = 0.3056$, $\min_j \left\{ \theta_j^2 \right\} = 0.4859$).

We note that for DMUs 5, 8, 9, 16, 17, 18, 20, 21, 24, 25, 26, 29, 30, their bargaining efficiency scores remain the same under the three different breakdown points, which also are the centralized efficiency scores. Also bargaining efficiency scores for DMU 26 under the breakdown point ($\min_j \left\{ \theta_j^1 \right\} = 0.6345$, $\min_j \left\{ e_j^2 \right\} = 0.3094$), and scores for DMU 1 under the breakdown point ($\min_j \left\{ e_j^1 \right\} = 0.3056$, $\min_j \left\{ \theta_j^2 \right\} = 0.4859$) are equal to their respective leader-follower (noncooperative) efficiency results. This indicates that under the bargaining model, DMU26 achieves its CRS efficiency score for the first stage, and DMU1 achieves its CRS efficiency score for the second stage.

Model (8.11) is also applied to the banking industry in Table 8.1. $h_{\min}^1$ and $h_{\min}^2$ are calculated as $h_{\min}^1 = \frac{1}{\theta_{\min}^1} = 12.9032$ and $h_{\min}^2 = \frac{1}{\theta_{\min}^2} = 19.4175$. Table 8.4 reports the results from (8.11). To make it comparable with the input-oriented bargaining results, we list the reciprocal of each output-oriented efficiency score, and the input-oriented results are listed in columns 2–4.

Seven DMUs, namely, DMUs 4, 10, 12, 13, 14, 22, and 23, have different efficiency decompositions under input- and output-orientations. This indicates that output-orientation can lead to different bargaining efficiency results from the input-oriented ones.

Table 8.3 Bargaining efficiency scores with breakdown points based upon the leader-follower model of Liang et al. (2008)

	Breakdown point {0.6345, 0.3094}			Breakdown point {0.3056, 0.4859}		
Bank	e_o^{1*}	e_o^{2*}	$e_o^{1*} \cdot e_o^{2*}$	e_o^{1*}	e_o^{2*}	$e_o^{1*} \cdot e_o^{2*}$
1	1.0000	0.4487	0.4487	0.8381	0.4859	0.4072
2	0.6823	0.5324	0.3633	0.6793	0.5331	0.3621
3	0.7946	0.5305	0.4215	0.6858	0.5669	0.3888
4	0.8721	0.4882	0.4258	0.8171	0.5221	0.4266
5	1.0000	0.6061	0.6061	1.0000	0.6061	0.6061
6	0.8180	0.5110	0.4180	0.6898	0.5881	0.4057
7	0.7842	0.5021	0.3937	0.6624	0.5546	0.3674
8	0.7451	0.6371	0.4747	0.7451	0.6371	0.4747
9	0.7022	0.6388	0.4486	0.7021	0.6389	0.4486
10	0.8110	0.4058	0.3291	0.4884	0.6946	0.3392
11	0.7413	0.5164	0.3828	0.5659	0.6955	0.3936
12	0.7089	0.6344	0.4497	0.6684	0.6756	0.4516
13	0.6809	0.6098	0.4152	0.5702	0.7807	0.4452
14	0.7139	0.5843	0.4171	0.6831	0.5938	0.4056
15	0.8478	0.7565	0.6414	0.8469	0.7582	0.6421
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
17	1.0000	0.7144	0.7144	1.0000	0.7144	0.7144
18	0.7974	1.0000	0.7974	0.7974	1.0000	0.7974
19	0.7484	0.7795	0.5834	0.7477	0.7811	0.5840
20	0.7541	0.7844	0.5915	0.7541	0.7844	0.5915
21	0.6550	0.8661	0.5673	0.6550	0.8661	0.5673
22	0.6732	0.7673	0.5165	0.6489	0.8007	0.5196
23	0.6429	0.6130	0.3941	0.6115	0.6479	0.3962
24	0.8711	0.9478	0.8256	0.8711	0.9478	0.8256
25	0.7403	1.0000	0.7403	0.7403	1.0000	0.7403
26	0.6345	0.8363	0.5306	0.6344	0.8363	0.5305
27	0.6573	0.9787	0.6433	0.6549	1.0000	0.6549
28	0.7736	0.8010	0.6197	0.7735	0.8012	0.6197
29	0.8092	1.0000	0.8092	0.8092	1.0000	0.8092
30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Note: The optimal value of parameter α represents the first-stage bargaining efficiency score (e_o^{1*}) for the corresponding DMU, and therefore we do not report α values in this table

8.6.2 Taiwanese Non-life Insurance Companies

Kao and Hwang (2008) describe a two-stage process where 24 non-life insurance companies use operational and insurance expenses to generate premiums in the first stage, and then underwriting and investment profits in the second stage. The inputs to the first stage are operational expenses and insurance expenses, and the outputs

Table 8.4 Output-oriented bargaining results for US commercial banks

	Input-oriented			Output-oriented		
Bank	e_o^{1*}	e_o^{2*}	$e_o^{1*} \cdot e_o^{2*}$	e_o^{1*}	e_o^{2*}	$e_o^{1*} \cdot e_o^{2*}$
1	1.0000	0.4487	0.4487	1.0000	0.4487	0.4487
2	0.6821	0.5327	0.3634	0.6821	0.5327	0.3634
3	0.7946	0.5305	0.4215	0.7946	0.5305	0.4216
4	0.8463	0.5050	0.4274	0.8172	0.5216	0.4262
5	1.0000	0.6061	0.6061	1.0000	0.6061	0.6061
6	0.8179	0.5111	0.4180	0.8180	0.5110	0.4180
7	0.7816	0.5042	0.3941	0.7816	0.5042	0.3940
8	0.7451	0.6371	0.4747	0.7451	0.6371	0.4747
9	0.7021	0.6389	0.4486	0.7022	0.6387	0.4485
10	0.5868	0.5735	0.3365	0.6909	0.4828	0.3336
11	0.6619	0.6281	0.4157	0.6619	0.6282	0.4158
12	0.6906	0.6576	0.4541	0.6996	0.6482	0.4535
13	0.5843	0.7640	0.4464	0.6617	0.6369	0.4214
14	0.7131	0.5852	0.4173	0.7139	0.5843	0.4172
15	0.8469	0.7582	0.6421	0.8469	0.7582	0.6421
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
17	1.0000	0.7144	0.7144	1.0000	0.7144	0.7144
18	0.7974	1.0000	0.7974	0.7974	1.0000	0.7974
19	0.7477	0.7811	0.5840	0.7477	0.7810	0.5840
20	0.7541	0.7844	0.5915	0.7542	0.7844	0.5916
21	0.6550	0.8660	0.5672	0.6550	0.8661	0.5673
22	0.6491	0.8005	0.5196	0.6732	0.7673	0.5166
23	0.6280	0.6330	0.3975	0.6430	0.6130	0.3941
24	0.8711	0.9478	0.8256	0.8711	0.9478	0.8257
25	0.7403	1.0000	0.7403	0.7403	1.0000	0.7403
26	0.6344	0.8363	0.5305	0.6345	0.8363	0.5306
27	0.6549	1.0000	0.6549	0.6549	1.0000	0.6549
28	0.7735	0.8012	0.6197	0.7736	0.8011	0.6198
29	0.8092	1.0000	0.8092	0.8093	1.0000	0.8093
30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

from the second stage are underwriting profit and investment profit. Direct written premiums and reinsurance premiums act as the intermediate measures connecting the two stages.

Table 8.5 shows the original data, and Table 8.6 reports the efficiency results obtained from both noncooperative (leader-follower) model and centralized model developed by Liang et al. (2008).

The same three breakdown points are considered in the bargaining game approach as in the previous bank application. First of all, from models (8.7) and (8.8), we get the CRS efficiency scores for stage 1's and stage 2's least ideal DMU as $\theta^1_{\min} = 0.001725$ and $\theta^1_{\min} = 0.001058$, respectively. Also the smallest leader-follower efficiency scores when either stage acts as the leader are calculated

Table 8.5 Taiwanese non-life insurance company data

Company	Operation expenses	Insurance expenses	Direct written premiums	Reinsurance premiums	Underwriting profit	Investment profit
1. Taiwan Fire	1,178,744	673,512	7,451,757	856,735	984,143	681,687
2. Chung Kuo	1,381,822	1,352,755	10,020,274	1,812,894	1,228,502	834,754
3. Tai Ping	1,177,494	592,790	4,776,548	560,244	293,613	658,428
4. China Mariners	601,320	594,259	3,174,851	371,863	248,709	177,331
5. Fubon	6,699,063	3,531,614	37,392,862	1,753,794	7,851,229	3,925,272
6. Zurich	2,627,707	668,363	9,747,908	952,326	1,713,598	415,058
7. Taian	1,942,833	1,443,100	10,685,457	643,412	2,239,593	439,039
8. Ming Tai	3,789,001	1,873,530	17,267,266	1,134,600	3,899,530	622,868
9. Central	1,567,746	950,432	11,473,162	546,337	1,043,778	264,098
10. The First	1,303,249	1,298,470	8,210,389	504,528	1,697,941	554,806
11. Kuo Hua	1,962,448	672,414	7,222,378	643,178	1,486,014	18,259
12. Union	2,592,790	650,952	9,434,406	1,118,489	1,574,191	909,295
13. Shingkong	2,609,941	1,368,802	13,921,464	811,343	3,609,236	223,047
14. South China	1,396,002	988,888	7,396,396	465,509	1,401,200	332,283
15. Cathay Century	2,184,944	651,063	10,422,297	749,893	3,355,197	555,482
16. Allianz President	1,211,716	415,071	5,606,013	402,881	854,054	197,947
17. Newa	1,453,797	1,085,019	7,695,461	342,489	3,144,484	371,984
18. AIU	757,515	547,997	3,631,484	995,620	692,731	163,927
19. North America	159,422	182,338	1,141,950	483,291	519,121	46,857
20. Federal	145,442	53,518	316,829	131,920	355,624	26,537
21. Royal & Sunalliance	84,171	26,224	225,888	40,542	51,950	6,491
22. Asia	15,993	10,502	52,063	14,574	82,141	4,181
23. AXA	54,693	28,408	245,910	49,864	0.1	18,980
24. Mitsui Sumitomo	163,297	235,094	476,419	644,816	142,370	16,976

according to the results from Table 8.6, which are ($\min_j\left\{\theta_j^1\right\} = 0.5895$, $\min_j\left\{e_j^2\right\} = 0.0870$) and ($\min_j\left\{e_j^1\right\} = 0.2507$, $\min_j\left\{\theta_j^2\right\} = 0.2795$).

Table 8.7 reports the bargaining results for both individual stages and the overall process associated with breakdown point $\{\theta_{\min}^1, \theta_{\min}^2\}$, ($\min_j\left\{\theta_j^1\right\} = 0.5895$, $\min_j\left\{e_j^2\right\} = 0.0870$) and ($\min_j\left\{e_j^1\right\} = 0.2507$, $\min_j\left\{\theta_j^2\right\} = 0.2795$) in columns 2 through 4, columns 5 through 7, and columns 8 through 10, respectively.

Table 8.6 Efficiency results for Taiwanese non-life insurance companies

	Stage 1 as the leader			Stage 2 as the leader			Centralized		
DMU	θ_j^1	e_j^2	$\theta_j^1 \cdot e_j^2$	e_j^1	θ_j^2	$e_j^1 \cdot \theta_j^2$	$e_j^{1,Centralized}$	$e_j^{2,Centralized}$	$e_j^{Centralized}$
1	0.9926	0.7045	0.6993	0.9260	0.7134	0.6606	0.9926	0.7045	0.6993
2	0.9985	0.6257	0.6248	0.9908	0.6275	0.6217	0.9985	0.6257	0.6248
3	0.6900	1.0000	0.6900	0.6900	1.0000	0.6900	0.6900	1.0000	0.6900
4	0.7243	0.4200	0.3042	0.4981	0.4323	0.2153	0.7243	0.4200	0.3042
5	0.8375	0.8060	0.6750	0.7376	1.0000	0.7376	0.8306	0.9234	0.7670
6	0.9637	0.4010	0.3864	0.9606	0.4057	0.3897	0.9606	0.4057	0.3897
7	0.7521	0.3522	0.2649	0.3000	0.5378	0.1613	0.6706	0.4124	0.2766
8	0.7256	0.3780	0.2743	0.3898	0.5114	0.1993	0.6631	0.4150	0.2752
9	1.0000	0.2233	0.2233	0.4388	0.2920	0.1281	1.0000	0.2233	0.2233
10	0.8615	0.5409	0.4660	0.2587	0.6736	0.1743	0.8615	0.5409	0.4660
11	0.7405	0.1677	0.1242	0.4718	0.3267	0.1541	0.6468	0.2534	0.1639
12	1.0000	0.7596	0.7596	1.0000	0.7596	0.7596	1.0000	0.7596	0.7596
13	0.8107	0.2431	0.1970	0.3384	0.5435	0.1839	0.6719	0.3093	0.2078
14	0.7246	0.3740	0.2710	0.3097	0.5178	0.1604	0.6699	0.4309	0.2887
15	1.0000	0.6138	0.6138	0.7102	0.7047	0.5005	1.0000	0.6138	0.6138
16	0.9072	0.3356	0.3045	0.5980	0.3848	0.2301	0.8856	0.3615	0.3201
17	0.7233	0.4557	0.3296	0.2507	1.0000	0.2507	0.6276	0.5736	0.3600
18	0.7935	0.3262	0.2588	0.6549	0.3737	0.2447	0.7935	0.3262	0.2588
19	1.0000	0.4112	0.4112	0.9787	0.4158	0.4069	1.0000	0.4112	0.4112
20	0.9332	0.5857	0.5466	0.4073	0.9014	0.3671	0.9332	0.5857	0.5466
21	0.7505	0.2623	0.1969	0.6918	0.2795	0.1934	0.7321	0.2743	0.2008
22	0.5895	1.0000	0.5895	0.5895	1.0000	0.5895	0.5895	1.0000	0.5895
23	0.8501	0.4509	0.3833	0.6812	0.5599	0.3814	0.8425	0.4989	0.4203
24	1.0000	0.0870	0.0870	0.3987	0.3351	0.1336	0.4287	0.3145	0.1348

Note that as with the previous bank data, in this application, the value for parameter α associated with the optimal solution is unique for each DMU throughout the entire searching range, which also leads to a unique pair of efficiency scores for both individual stages.

It can be seen from Table 8.7 that with breakdown point $\{\theta_{\min}^1, \theta_{\min}^2\}$, the bargaining efficiency results are exactly the same as those obtained from Liang et al. (2008)'s centralized model. Also from Table 8.7, note that for some DMUs, such as DMUs 1, 2, 3, 4, 5, 6, 10, 12, 15, 22, 23, their respective bargaining efficiency results remain unchanged under all three breakdown points, while for the rest DMUs, such as DMUs 7, 8, 9, 11, 13, 14, 16, 17, 18, 19, 20, 21, 24, their respective bargaining efficiency scores are varied according to different breakdown points.

Table 8.7 Bargaining efficiency scores with three breakdown points

	$\{\theta^1_{\min},\ \theta^2_{\min}\}$			{0.5895, 0.0870}			{0.2507, 0.2795}		
DMU	e_o^{1*}	e_o^{2*}	$e_o^{1*} \cdot e_o^{2*}$	e_o^{1*}	e_o^{2*}	$e_o^{1*} \cdot e_o^{2*}$	e_o^{1*}	e_o^{2*}	$e_o^{1*} \cdot e_o^{2*}$
1	0.9926	0.7045	0.6993	0.9926	0.7045	0.6993	0.9926	0.7045	0.6993
2	0.9985	0.6257	0.6248	0.9985	0.6257	0.6248	0.9985	0.6257	0.6248
3	0.6900	1.0000	0.6900	0.6900	1.0000	0.6900	0.6900	1.0000	0.6900
4	0.7243	0.4200	0.3042	0.7243	0.4200	0.3042	0.7243	0.4200	0.3042
5	0.8306	0.9234	0.7670	0.8306	0.9234	0.7670	0.8306	0.9234	0.7670
6	0.9606	0.4057	0.3897	0.9606	0.4057	0.3897	0.9606	0.4057	0.3897
7	0.6706	0.4124	0.2766	0.7521	0.3522	0.2649	0.6200	0.4317	0.2677
8	0.6631	0.4150	0.2752	0.7256	0.3780	0.2743	0.6630	0.4150	0.2751
9	1.0000	0.2233	0.2233	1.0000	0.2233	0.2233	0.4390	0.2920	0.1282
10	0.8615	0.5409	0.4660	0.8615	0.5409	0.4660	0.8615	0.5409	0.4660
11	0.6468	0.2534	0.1639	0.7292	0.2066	0.1507	0.4718	0.3267	0.1541
12	1.0000	0.7596	0.7596	1.0000	0.7596	0.7596	1.0000	0.7596	0.7596
13	0.6719	0.3093	0.2078	0.8107	0.2431	0.1971	0.4600	0.4344	0.1998
14	0.6699	0.4309	0.2887	0.7246	0.3740	0.2710	0.6699	0.4309	0.2887
15	1.0000	0.6138	0.6138	1.0000	0.6138	0.6138	1.0000	0.6138	0.6138
16	0.8856	0.3615	0.3201	0.8856	0.3615	0.3201	0.8687	0.3651	0.3172
17	0.6276	0.5736	0.3600	0.7231	0.4598	0.3325	0.6276	0.5736	0.3600
18	0.7935	0.3262	0.2588	0.7935	0.3262	0.2589	0.6551	0.3737	0.2448
19	1.0000	0.4112	0.4112	1.0000	0.4112	0.4112	0.9788	0.4158	0.4070
20	0.9332	0.5857	0.5466	0.9332	0.5857	0.5466	0.8159	0.6561	0.5353
21	0.7321	0.2743	0.2008	0.7505	0.2623	0.1969	0.6918	0.2795	0.1934
22	0.5895	1.0000	0.5895	0.5895	1.0000	0.5895	0.5895	1.0000	0.5895
23	0.8425	0.4989	0.4203	0.8425	0.4989	0.4203	0.8425	0.4989	0.4203
24	0.4287	0.3145	0.1348	0.7752	0.1390	0.1078	0.3987	0.3351	0.1336

Note: The optimal value of parameter α represents the first-stage bargaining efficiency score (e_o^{1*}) for the corresponding DMU, and therefore we do not report α values in this table

8.7 Conclusions

This chapter introduces the Nash bargaining game model as a way of addressing the conflict arising from intermediate measures, and presents an alternative approach to evaluate the efficiency scores for both stages and the overall process. Furthermore, it is proved that in the case of only one intermediate measure, the bargaining game approach yields the same efficiency results as obtained from the separately-applied standard DEA approach, and also with the non-cooperative and centralized approaches in Liang et al. (2008).

Different breakdown points can be used to calculate the bargaining efficiency scores. As a matter of fact, each DMU can use a specific breakdown point. For example, based upon the leader-follower model of Liang et al. (2008), both $\left(\min_j\left\{\theta_j^1\right\}, \min_j\left\{e_j^2\right\}\right)$ and $\left(\min_j\left\{e_j^1\right\}, \min_j\left\{\theta_j^2\right\}\right)$ can be used as breakdown points, where θ_j^1 and e_j^2 respectively represent the efficiency scores for stages 1 and 2 of DMU_j when Stage 1 is treated as the leader, whereas e_j^1 and θ_j^2 respectively represent the efficiency scores for stage 1 and 2 when Stage 2 takes the leader's role.

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