Chapter 18
Multicomponent Efficiency Measurement and Core Business Identification in Multiplant Firms

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Abstract As discussed in the previous chapters, the DMU may perform different types of functions. In that case it is desirable to derive a measure of performance not only at the DMU level but as well at the level of the particular functions within the DMU. In the current chapter we examine a set of manufacturing plants operating under a single umbrella, with the objective being to use the component or function measures to decide what might be considered as each plant's core business. It is proposed that this information can aid the company in any reorganization initiatives designed to capitalize on the strengths of each location (DMU).

Keywords Multicomponent • Core business • Shared inputs • Bundles

18.1 Introduction

The DEA model, developed by Charnes et al. (1978), provides a constant return to scale (CRS) methodology for evaluating the performance of a set of comparable decision making units (DMUs). In the usual setting, each DMU is evaluated in terms of a set of outputs that represent its accomplishments, and a set of inputs that represent the resources or circumstances at its disposal.

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In some application areas, it has been recognized that the DMU may perform different *types* of functions. In such situations, it is desirable to derive a measure of performance, not only at the level of the DMU, but, as well, at the level of the particular function within the DMU. Cook and Roll (1993) were the first to examine the idea of partial efficiency measures, where the separate components of the DMU possess their own bundles of outputs and inputs. These bundles were assumed to be mutually exclusive of one another. Beasley (1995) examined both teaching and research components within a set of universities in the UK, and presented a nonlinear programming model for measuring DMU performance. A similar situation is encountered in Cook et al. (2000), and Cook and Hababou (2001), where sales and service components are evaluated within a set of bank branches. They discuss linear models for providing both overall performance of a branch, as well as separate component performance measures. In that context, as with Beasley (1995) the input is a shared resource to be allocated to two production units. The complicating feature in each of these problem settings, that was not present in Cook and Roll (1993), is the presence of *shared resources*. The existence of *shared resources* means that the usual DEA structure must be modified to provide for a splitting of those resources among the various components.

In the current chapter we examine a set of manufacturing plants operating under a single corporate umbrella, with the objective of identifying how well each plant performs in each of its components thus identifying what might be considered each plant's a core business. Here, each component consists of a group of products selected from the totality of products offered, according to the specific interests of the corporate decision maker. Unlike the aforementioned dual-component applications (e.g., sales and service components in a bank branch), these components may overlap. Examples are (1) those products made from rolled steel of given dimensions; (2) those products servicing the automotive industry, . . ., etc. This setting is clearly similar to those discussed above in that product groups are functions of the business, and, as will be seen, there are resources that are shared among those components. The models proposed here represent a departure from the earlier work of Beasley (1995), Cook and Roll (1993), Cook et al. (2000), and Cook and Hababou (2001), in two respects. First, we examine the extension of the earlier models to a multi-component (two or more) setting. Second, using this multi-component structure as a point of departure, we develop models for *identifying the most appropriate* product groupings for each plant (DMU).

Section 18.2 presents the problem setting in more detail. In Sect. 18.3, extensions of the models of Cook et al. (2000, 2001) and Beasley (1995) are presented. Multiple, and potentially overlapping components are considered. These models are appropriate where the issue is one of identifying overall performance, as well as isolating particular areas (components) where the plant can be improved. Section 18.4 extends this idea to those situations wherein the organization wishes to identify the segment of the business that is performing *best* in any given DMU. In this way, the *core business* of each plant can be isolated, thus aiding the company in any reorganization initiatives designed to capitalize on the strengths of each location. Section 18.5 discusses the application of these models in the plant setting described earlier. Conclusions are given in Sect. 18.6.

18.2 Multicomponent Efficiency Measurement and Core Business Identification

In this chapter we examine multi-component efficiency measurement from two perspectives. In the *first* situation, we make the assumption that the purpose of the performance assessment exercise is to determine an aggregate measure of efficiency, as well as measures for each of the separate components. Such evaluation will aid management in identifying the extent to which overall performance can be improved. As well, for specific business areas, the measures can point to those that are doing well, as well as those that require attention. Section 18.3 addresses this setting.

In the *second* situation, it is assumed that the organization wishes to go beyond simply identifying the level of performance of specific subunits of the business. Rather, it is desirable to identify the area(s) where DMUs are performing best, hence defining what might reasonably be regarded as each DMU's *core business*. A given DMU may then wish to focus its energies on this selected part of the operation, while de-emphasizing, or in some cases, even abandoning those portions of the business where it performs at a less than satisfactory level. This development is undertaken in Sect. 18.4.

To illustrate these ideas we examine a company with several plants that operate in the rolled steel industry. The company manufactures steel products, both of the *finished* variety that are sold on the open market, and semi-finished items that are custom-ordered, and sold to other manufacturers. These latter products can, for example, be items such as slit steel, used by other firms that manufacture steel doors and door frames. Other products, such as cylindrical bearings, are further along the value chain, and are purchased by companies that manufacture such consumer products as lawn mowers, or outboard motors for boats. Anticipating the detail given in Sect. 18.5, it is convenient to view the company's operations in terms of nine distinct products, and in conventional DEA terms each of these products would be considered an output. However, corporate management as well as being interested in the overall efficiency of each plant, is also interested in performance with respect to four overlapping groupings of these nine products. In what follows we will refer to a defined group of products, variously and interchangeably, as a component, subunit or segment. In some cases products are grouped to represent a particular market segment, e.g., automotive manufacturers who source certain products from the company. In other cases they are grouped to represent an internally meaningful segment of the operation, e.g., all products both semi-finished and finished, but pertaining to a certain size or quality of steel, or products made on particular machines.

In the section to follow, we present model structures for evaluating both the aggregate performance of each of a set of DMUs, as well as the performance of the separate subunits or components within a DMU's operation. For purposes of this development, we utilize the problem setting discussed herein as a backdrop.

18.3 Multicomponent Model Structures

The conventional model structure for evaluating the relative efficiency of each member of a set of DMUs is the DEA model of Charnes et al. (1978). Specifically, given an output vector $Y_k = (y_{1k}, y_{2k} \ldots, y_{Rk})$, and input vector $X_k = (x_{1k}, x_{2k}, \ldots, x_{Ik})$, for each of a set of n DMUs $k = 1, \ldots, n$, the constant returns to scale model is given by

$$\begin{aligned} &\max \ \mu_o Y_o / v_o X_o, \\ &\text{subject to:} \\ &\mu_o Y_k / v_o X_k \le 1, \quad \text{all } k, \\ &\mu_o, v_o \ge \varepsilon. \end{aligned} \tag{18.1}$$

The structure in (18.1) presumes that one desires to measure the overall efficiency (e.g., operational efficiency) of each DMU, without consideration for the performance of subunits that may exist within the DMU. In the problem setting presented herein, we wish to provide for a more detailed performance evaluation, i.e., at the level of these subunits.

18.3.1 Multi-component Efficiency Measurement with Shared Inputs: Non-overlapping Subunits

Our point of departure for the discussion in this section, is the model structures of Cook et al. (2000), (see also Cook and Hababou 2001). There, the authors examine the problem of providing separate efficiency measures for both *sales* and *service* components of a set of bank branches for a major Canadian bank. Adopting the notation of Cook et al. (2000), and extending their model structure to "T" components, we have:

Parameters:

Y_k^t	= the R-dimensional vector of outputs included in the tth component of DMUk
R	= set of all outputs
R^t	= set of outputs generated by the tth component
X_k^t	= the I-dimensional vector of inputs dedicated to the tth component of DMUk
I	= set of all inputs
I^t	= set of inputs dedicated to the tth component
X_k^S	= the I^s-dimensional vector of inputs shared among the T components of DMU k
I^s	= set of shared inputs
L_i^t, U_i^t	= lower, upper limits on the portion of the ith shared resource, that can be assigned to the tth component of a DMU
T	= set of all components

Decision Variables:

μ_o^t	= vector of multipliers applied to outputs Y_o^t
ν_o^t	= vector of multipliers applied to inputs X_o^t
ν_o^{st}	= vector of multipliers applied to that portion of shared inputs X_o^S that are assigned to component t
α_o^t	= vector representing the proportion of shared inputs X_o^S allocated to the tth component

In the two-component problem addressed in Cook et al. (2000), the principal area of difficulty was the presence of shared inputs X_k^S.. Specifically, there are certain resources such as branch expenditure on computer technology and general branch staff, that are shared across the two components of the business. There is no well defined split of these resources across different functions, and the basic problem has to do with the allocation of these inputs among the components. To facilitate this, and at the same time extend the idea to the general case of T components, a decision vector α_k^t is introduced that permits the DMU k in question to apportion X_k^S among the T competing components. In Cook et al. (2000), this is done in a manner that optimizes the *aggregate* performance measure (of DMU "o") given by:

$$e_o^a = \sum_{t\varepsilon T} \mu_o^t Y_o^t \Big/ \left[\sum_{t\varepsilon T} \left(\nu_o^t X_o^t + \nu_o^{st} \left(\alpha_o^t X_o^S \right) \right) \right] \tag{18.2}$$

The component-specific performance measures e_o^t are given by:

$$e_o^t = \mu_o^t Y_o^t / \left(\nu_o^t X_o^t + \nu_o^{st} \left(\alpha_o^t X_o^S \right) \right) \tag{18.3}$$

It is pointed out that the notation $\alpha_o^t X_o^S$ represents the vector $\left(\alpha_{o1}^t x_{o1}^S, \alpha_{o2}^t x_{o2}^S, \ldots, \alpha_{oI^S}^t x_{oI^S}^S \right)$ of shared inputs allocated to component t by DMU "o".

In the discussion below, we distinguish between optimal performance measures and performance measures for a DMUk, evaluated in terms of the multipliers for a DMU "o" currently being considered. (Doyle and Green (1994) use the term *cross-evaluation* in this instance). For this purpose, we adopt the notation $\hat{e}_k^a, \hat{e}_k^t$ to denote the measures for DMUk that represent their optimal performance, while e_k^a, e_k^t denote performance relative to multipliers arising from the optimization of (some other) DMU "o".

The multi-component DEA model is given by:

$$\begin{aligned} \hat{e}_o^a &= max e_o^a \\ &\text{subject to} \end{aligned} \tag{18.4a}$$

$$e_k^t \le 1, \quad \text{all } t, k \tag{18.4b}$$

$$L_i^t \le \alpha_{oi}^t \le U_i^t \quad \text{all } t, \ i \ \varepsilon \ I^S, \tag{18.4c}$$

$$\sum_{t \varepsilon T} \alpha_{oi}^t = 1, \quad i \ \varepsilon I^S, \tag{18.4d}$$

$$\mu_o^t, \nu_o^t, \nu_o^{st} \ge \varepsilon, \quad \text{all } t. \tag{18.4e}$$

Here, the objective (18.4a) maximizes the overall performance measure for the DMU "o", in the spirit of the original DEA model of Charnes et al. (1978). Correspondingly, we restrict each component measure e_k^t by an upper bound of 1 in (18.4b). A permissible range on the proportion of the ith shared resource that can be allocated to the tth component by any DMU is given by (18.4c). Constraints (18.4d) specify that the proportional splits of any input i across the T components sum to unity. Finally, constraints (18.4e) restrict multipliers to be strictly greater than zero.

The limits L_i^t, U_i^t, on the proportions α_{oi}^t of the various inputs i to components t would need to be specified by the user. Such limits might generally arise from any information available at the plants regarding standard amounts of inputs i per unit of product in components t.

From the above discussion it is clear that problem (18.4) is a *restricted version* of problem (18.1). Specifically, any feasible solution to (18.4) is also feasible for (18.1). Problem (18.4) only permits multipliers which identify each component of the plant as a bona fide sub-DMU whose performance measure is captured at the same time as that of the entire plant. Problem (18.1), however, is focused purely at the plant level, with no recognition whatever of subunits.

Definition 18.1 A DMU "o" is said to be *efficient* if its aggregate score $\hat{e}_o^a = 1$.

Definition 18.2 A DMU "o" is said to be *efficient in its tth component* if $\hat{e}_o^t = 1$.

Theorem 18.1 *In model (18.4), the resulting aggregate performance measure* $\hat{e}_k^a$ *for any DMU*k, *does not exceed unity, i.e.,* $\hat{e}_k^a \le 1$.

Proof If we define

$$\beta_k^t = \left[\nu_o^t X_k^t + \nu_o^{st}\left(\alpha_o^t X_k^s\right)\right] \Big/ \sum_{t \varepsilon T} \left(\nu_o^t X_k^t + \nu_o^{st}\left(\alpha_o^t X_k^s\right)\right),$$

then, the aggregate measure (in terms of the (μ_o, ν_o) multipliers), is given by

$$\hat{e}_k^a = \sum_{t \varepsilon T} \beta_k^t \hat{e}_k^t.$$

Hence, e_k^a is a convex combination of the component measures, and as such $e_k^a \le 1$.

Q.E.D.

Theorem 18.2 *In model (18.4), a DMU is efficient if and only if it is efficient in each of its components.*

Proof Case 1: Assume all component measures $\hat{e}_k^t = 1$.

By definition,

$$\hat{e}_k^a = \sum_{t \varepsilon T} \beta_k^t \hat{e}_k^t$$

from Theorem 18.1, and since $\sum_t \beta_k^t = 1$, it follows that $\hat{e}_k^a = 1$.

Case 2: Assume $\hat{e}_k^a = 1$. Then, if any $\hat{e}_k^t < 1$, it must be the case that

$$\hat{e}_k^a = \sum_{t \varepsilon T} \beta_k^t \hat{e}_k^t < 1,$$

as well, in contradiction.

Q.E.D.

We now examine multi-component performance measurement when overlaps can occur.

18.3.2 Multi-component Efficiency Measurement with Overlapping Subunits

The models presented above presume a set of subunits that are mutually exclusive. Arguably, in the bank branch setting of Cook and Hababou (2001), and Cook et al. (2000), sales and service components meet the mutual exclusivity requirement. In many settings this restriction may not hold, however, as is the case with the business components described later.

In the case where mutual exclusivity prevails, it is sufficient to subdivide a shared input among the set of components. That is, α_{oi}^t represents the portion of input i assigned to component t. It is not necessary to address how this portion α_{oi}^t is distributed among the outputs comprising component t. In case there is *overlap* among the components due to the existence of common outputs, the manner in which the proportions $\{\alpha_{oi}^t\}_t{=}1^T$ behave, is no longer clear. It is, for example, not true that $\sum_{t\varepsilon T} \alpha_{oi}^t = 1$, due to the overlap.

In recognition of the *overlap problem*, we need to be more exacting as to how the shared input i is assigned to outputs $r \varepsilon \mathfrak{R}$. Specifically, we define variables α_{oir} that denote the proportion of shared input x_{oi}^s (the ith component of vector X_o^s) that is allocated to output y_{or}. As well, let L_i^r, U_i^r, denote lower and upper bounds, respectively, on α_{oir}, and impose the constraint

$$\sum_{r \in \Re} \alpha_{oir} = 1.$$

The proportion α^t_{oi} of input i allocated to component t is then the sum of the proportions α_{oir} of i allocated to those outputs comprising t, i.e.

$$\alpha^t_{oi} = \sum_{t \varepsilon \Re^t} \alpha_{oir}.$$

For brevity in modelling, we henceforth denote the feasible set of $\alpha = (\alpha^t)$ by

$$\begin{aligned} \Lambda_o = \{\alpha_o = \left(\alpha^t_o\right) : (1)\, \alpha^t_{oi} = \sum_{r \varepsilon \Re^t} \alpha_{oir}; \\ (2)\, L^r_i \leq \alpha_{oir} \leq U^r_i; (3) \sum_{r \varepsilon \Re} \alpha_{oir} = 1, \\ \alpha_{oir} \geq 0, \quad \text{all } i \,\varepsilon\, \mathrm{I}^s, \ \text{all}\, t\}. \end{aligned}$$

The multi-component DEA model is then given by:

$$\begin{aligned} &\text{Max } e^a_o, \\ &\text{subject to} \end{aligned} \tag{18.5a}$$

$$e^t_k \leq 1, \quad \text{all } t, k, \tag{18.5b}$$

$$\alpha_o \varepsilon \Lambda_o, \tag{18.5c}$$

$$\mu^t_o, v^t_o, v^{st}_o \geq \varepsilon, \quad \text{all } t. \tag{18.5d}$$

It is noted that the objective function (18.5a) credits the DMU for producing an output y^t_{or} as many times as that output appears as a member of a component's output set. For example, an output y_{or}, contained in both components $t = 1$ and $t = 2$, (i.e., $y^1_{or_1} = y^2_{or_2}$), would appear in (18.5a) twice, as $\mu^1_{or_1} y_{or_1}$ and $\mu^2_{or_2} y_{or_2}$.

We point out, however, that, as in the case of non-overlapping subunits, it is also true here that problem (18.5) is simply a restricted version of problem (18.1), if we view the inputs X in (18.1) as all being shared inputs. This is captured by the following theorem.

Theorem 18.3 *Any feasible solution to problem (18.5) is feasible to (18.1).*

Proof Define the R-dimensional multiplier vector $U^t = (u^t_r)$ by

$$u^t_r = \begin{cases} \mu^t_r & \text{if product r is in component t} \\ 0 & \text{otherwise} \end{cases}$$

and let $U = \sum_{t \varepsilon T} U^t$. Letting Y denote the R-dimensional vector of all outputs as used in (18.1), it follows that

$$\sum_{t\epsilon T} \mu^t Y^t = UY.$$

Similarly, one can replace the set of inputs $\{X^t\}$ by the I-dimensional vector $X(1) = (X^1, X^2, \ldots, X^T)$, and replace the set of "shared resource" vectors $\alpha^t X^s$ by the sum of these component shares to get X^s. Let $X = (X(1), X^s)$, the full vector of all inputs. Then, as with the output side, one can express the denominator of the performance measure as

$$\sum_{t\epsilon T} [\nu^t X^t + \nu^{st}(\alpha^t X^s)] = VX,$$

where V is defined in terms of the ν^t, ν^{st} in a manner analogous to the definition of U in terms of $\{\mu^t\}$. Hence e_o^a in (18.5) can be written as

$$e_o^a = UY/VX.$$

Since it is true that each component measure $e_k^t \le 1$, then it must also be true that the aggregate score $e_k^a \le 1$ as well. Thus, any feasible solution to (18.5) is also feasible for (18.1).

Q.E.D.

Hence, the overlap of the components does not lead to inconsistencies in regard to problem (18.1). Defining the aggregate measure in this manner results in the following theorem. The Proof is analogous to those of Theorems 18.1 and 18.2, and is, therefore, omitted.

Theorem 18.4

(*a*) *The aggregate measure of efficiency given by* (*18.5a*) *does not exceed unity.*

(*b*) *A DMU will be aggregate-efficient, (the objective function* (*18.5a*) *will equal unity), if and only if it is efficient in each component measure.*

Model (18.5), thus, allows one to examine the performance of a DMU in each business area. As well, it provides an overall or aggregate measure of performance across all business components.

Because the orientation of model (18.5) is toward evaluation of the DMU at an aggregate level, with component measures arising only as a by-product, it can be argued that the individual subunits of the business may not be shown in their most favorable light. In some cases, the strategic intent of the organization might be to identify the core business for each DMU, the purpose being to focus the attention of the DMU toward the areas of the business at which it performs best. In the section to follow, we present model structures wherein the intention is to choose a core business component on behalf of each DMU. It should be pointed out that the identification of a core business component will not necessarily imply the immediate termination of all activities at a plant that are not included in that component. Rather, a DMU would initially continue to service all existing activities, possibly phasing out non-core activities as these are redistributed to where they are best accomplished over some time horizon.

18.4 Modelling Selection of Core Business Components

A typical problem setting would be one where each of a set of plants for a given company produces a full product line, for sale and distribution to customers. There can be a number of reasons why it is cost effective for a certain product line, for example, to be manufactured in particular locations, but not in others. Certain manufactured items may, for instance, require specialized and expensive equipment that the company might prefer to make available in only one location. Alternatively, certain customers (e.g. farmers) may be highly concentrated in one geographical area, meaning that a plant close to that concentration should produce products related to that customer group. As well, simple economies of scale may dictate that the production for a product be concentrated in only a few plants, or even a single plant.

The problem then is to identify which collection of products or product lines should be handled by any given plant, thus defining that plant's core business.

The conventional DEA model does not readily lend itself to resource allocation (i.e. allocation of shared inputs). The DEA approach focuses attention on the performance of a particular DMU "o". If the objective is to allocate components to DMUs (plants), and to divide shared resources among products (and thus among components), one needs to view this allocation process from the perspective of the entire *collection* of DMUs, simultaneously rather than from the conventional DEA perspective, i.e. iteratively, one DMU at a time.

To facilitate the allocation of components to DMUs, define the bivalent variables $\{d_k^t\}_{t=1}^T$, for each DMU k,

$$d_k^t = \begin{cases} 1 & \text{if component } t \text{ is assigned to DMU } k, \\ 0 & \text{otherwise.} \end{cases}$$

The aggregate performance (ratio) measure for the collection of DMUs, given an allocation defined by a chosen set of d_k^t values, can be expressed as:

$$\frac{\sum_k \left[\sum_t d_k^t \mu^t Y_k^t\right]}{\sum_k \left[\sum_t d_k^t \left(\nu^t X_k^t + \nu^{st}\left(\alpha_k^t X_k^s\right)\right)\right]}$$

The *optimal* assignment of components to DMUs, as defined by the d_k^t, is arguably that for which the ratio of aggregate output to aggregate input is maximized. The set of d_k^t for which this maximum occurs can be determined by solving the fractional programming problem:

$$\max \sum_k \left[\sum_t d_k^t \mu^t Y_k^t \right] \Big/ \sum_k \left[\sum_t d_k^t \left(\nu^t X_k^t + \nu^{st} \left(\alpha_k^t X_k^s \right) \right) \right] \tag{18.6a}$$

subject to

$$\mu^t Y_k^t / \left(\nu^t X_k^t + \nu^{st} \left(\alpha_k^t X_k^s \right) \right) \leq 1, \quad \text{all } k, t \tag{18.6b}$$

$$\alpha \, \varepsilon \, \Lambda_0, \tag{18.6c}$$

$$\sum_t d_k^t \geq 1, \quad \text{all } k, \tag{18.6d}$$

$$\sum_k d_k^t \geq 1, \quad \text{all } t, \tag{18.6e}$$

$$\mu^t, \nu^t, \nu^{st} \geq \varepsilon, \quad \text{all } t, \quad d_k^t \, \varepsilon \, \{0, 1\}, \quad \text{all } k, t. \tag{18.6f}$$

Constraints (18.6b) restrict the ratio of outputs to inputs in *any* component to not exceed unity. (18.6c) requires that the resource splitting variables satisfy conditions as defined earlier in Λ_o. Constraints (18.6d) force each plant k to support *at least* one product group or component. Similarly, (18.6e) stipulates that each component must be produced at *one or more* of the plants.

It is conceivable that at the optimum, certain plants may be chosen to support several product groups, while other plants may service only one group.

Model (18.6a–f), assigns multipliers μ^t, ν^t, ν^{st} to each component t in each DMU k. While it is not the purpose of the model to measure the efficiency of the *entire* operation of each plant, the supplied (common set of) multipliers do in fact provide the basis for an efficiency score for each plant and the aggregate across all plants, should one want to extract these. That aggregate score clearly includes the contribution rendered by both core and non-core components of the plant. Admittedly, the set of multipliers is derived in a manner designed to display *core* components in their best light, and by implication, non-core components in a light less than best. Hence, non-core components may be represented in a disadvantageous manner. One might argue that this is appropriate since, over time, such non-core components will, in any event, be phased out. Thus, their estimated performance (by that stage) will be a non-issue. At the same time, the model does, in fact, recognize their existence, and the bounds $[L_i^r, U_i^r]$ appropriately force the allocation of shared resources across all components (both core and non-core). Thus, choice of these bounds by management affirms the continuing presence of non-core components in the operation.

Thus, the real purpose of the model is to single out those components of each plant on which that plant exhibits its best performance. It is these core components whose aggregate performance we wish to capture.

The implication of this is that when a set of plants exhibit inefficiency, it is often desirable to strive for *specialization*. The questions that management would like to answer are:

(1) In what parts of the operation should each plant specialize?
(2) If plant operations were reorganized to implement such specialization, what would be the anticipated performance of the resulting operation?
(3) How would each reorganized (future) plant perform?

Question 1: The purpose of the model is to extract those components at each plant that appear to be the ones in which the plant should specialize.

Question 2: While the model yields an aggregate performance across all core components in all plants, there is an implied measure of performance for each plant on a portion (core business portion) of that plant's operation. Specifically, using $\{\hat{d}_k^t\}_{t=1}^T$, for each k, the model yields a measure of performance for that subset of components in terms of the inputs that those components utilize, and the outputs generated by those components. This measure captures how the (reduced) plant *would* perform if non-core business elements were not present.

Question 3: In a reorganized structure, the essence of the model is that each plant would concentrate only on its core business activities. It is argued that if each plant were to scale up its operation such as to come to full capacity in its resource utilization, then it is hypothesized that the resulting output generated would be scaled up by the same factor.

To solve problem (18.6a–f), it can be shown that it is representable as a mixed integer linear programming problem. This is given by the following theorem.

Theorem 18.5 *Problem (18.6a–f) can be represented as a mixed integer (binary) linear problem.*

Proof Problem (18.6a–f) is equivalent to the mixed binary *nonlinear* programming model:

$$\max \sum_k \sum_t d_k^t \mu^t Y_k^t \tag{18.7a}$$

subject to

$$\sum_k \sum_t d_k^t \left(\nu^t X_k^t + \nu^{st}\left(\alpha_k^t X_k^s\right)\right) = 1 \tag{18.7b}$$

$$\mu^t Y_k^t - \left(\nu^t X_k^t + \nu^{st}\left(\alpha_k^t X_k^s\right)\right) \leq 0, \quad \text{all } k, t, \tag{18.7c}$$

$$\alpha \, \varepsilon \, \Lambda_o, \tag{18.7d}$$

$$\sum_t d_k^t \geq 1, \quad \text{all } k, \tag{18.7e}$$

$$\sum_k d_k^t \geq 1, \quad \text{all } t, \tag{18.7f}$$

$$\mu^t, \nu^t, \nu^{st} \geq \in, \quad \text{all } t, \; d_k^t \, \varepsilon \{0, 1\}, \quad \text{all } k, t. \tag{18.7g}$$

Make the change of variables:

$$\overline{\nu}^{st} = \nu^{st}\alpha^{t}, \mathrm{v}_k^{st} = d_k^t \overline{\nu}^{st}, \mathrm{v}_k^t = d_k^t \nu^t, u_k^t = d_k^t \mu^t.$$

It is noted that we can replace an expression such as $v_k^t = d_k^t \nu^t$ by the constraint set

$$v_k^t \le M d_k^t,$$
$$\nu^t \ge v_k^t,$$
$$\nu^t \le v_k^t + M\left(1 - d_k^t\right),$$

where M is a large positive number. Specifically, if $d_k^t = 0$, then $v_k^t = 0$; if $d_k^t = 1$, then $v_k^t = \nu^t$. A similar set of constraints can be imposed to replace the nonlinear expressions $u_k^t = d_k^t \mu^t$, and $v_k^{st} = d_k^t \upsilon^{st}$. Problem (18.7a–g) can then be written as the mixed binary linear programming model

$$
\begin{aligned}
&\max \sum_k \sum_{t\varepsilon T} u_k^t Y_k^t,\\
&\text{subject to}\\
&\sum_k \sum_{t\varepsilon T} \left(v_k^t X_k^t + v_k^{st} X_k^s\right) = 1, \sum_{t\varepsilon T} \left[u_k^t Y_k^t - \left(v_k^t X_k^t + v_k^{st} X_k^s\right)\right] \le 0, \text{all } k,\\
&\upsilon_k^t \le \mathrm{M} d_k^t, \text{all t},\\
&\nu^{\mathrm{t}} \ge \mathrm{v}_{\mathrm{k}}^{\mathrm{t}}, \text{all t},\\
&\nu^{\mathrm{t}} \le \mathrm{v}_{\mathrm{k}}^{\mathrm{t}} + \mathrm{M}\left(1 - \mathrm{d}_{\mathrm{k}}^{\mathrm{t}}\right), \text{all t},\\
&u_{\mathrm{k}}^{\mathrm{t}} \le \mathrm{M}\mathrm{d}_{\mathrm{k}}^{\mathrm{t}}, \text{all t},\\
&\mu^{\mathrm{t}} \ge u_{\mathrm{o}}^{\mathrm{t}}, \text{all t},\\
&\mu^{\mathrm{t}} \le u_{\mathrm{k}}^{\mathrm{t}} + \mathrm{M}\left(1 - \mathrm{d}_{\mathrm{k}}^{\mathrm{t}}\right), \text{all t},\\
&\upsilon_{\mathrm{k}}^{\mathrm{st}} \le \mathrm{M}\mathrm{d}_{\mathrm{k}}^{\mathrm{t}}, \text{all t},\\
&\overline{\nu}^{\mathrm{t}} \ge \mathrm{v}_{\mathrm{k}}^{\mathrm{st}}, \text{all t},\\
&\overline{\nu}^{\mathrm{t}} \le \mathrm{v}_{\mathrm{k}}^{\mathrm{st}} + \mathrm{M}\left(1 - \mathrm{d}_{\mathrm{k}}^{\mathrm{t}}\right), \text{all k, t},\\
&\alpha\, \varepsilon\, \Lambda,\\
&\sum_{\mathrm{t}} \mathrm{d}_{\mathrm{k}}^{\mathrm{t}} \ge 1, \text{all k},\\
&\sum_{\mathrm{k}} \mathrm{d}_{\mathrm{k}}^{\mathrm{t}} \ge 1, \text{all t},\\
&\overline{\nu}_{\mathrm{i}}^{\mathrm{st}} \ge \varepsilon \alpha_{\mathrm{i}}^{\mathrm{t}}, \text{all i, t},\\
&\mu_{\mathrm{r}}^{\mathrm{t}}, \nu_{\mathrm{i}}^{\mathrm{t}} \ge \varepsilon, \ \text{all i, r, t},\\
&u_{\mathrm{kr}}^{\mathrm{t}}, \mathrm{v}_{\mathrm{ki}}^{\mathrm{t}} \ge 0, \text{all i, r, k},\\
&\upsilon_{\mathrm{ki}}^{\mathrm{st}} \ge 0, \text{all r, t, i} = 1, \ldots, \mathrm{I}^{\mathrm{S}},\\
&\mathrm{d}_{\mathrm{k}}^{\mathrm{t}} \varepsilon \{0, 1\}, \text{all k, t.}
\end{aligned}
\tag{18.8}
$$

This completes the proof.

Q.E.D.

There are clearly variations of this model where, for example, it may be pertinent for certain product groupings or components to be manufactured in only certain plants that are perhaps in the best possible position to handle them. This might be due to equipment capability, proximity of the market, and so on. Thus, for a given component t_o, we might require that $d_k^{to} = 0,\ k\,\varepsilon\,\overline{K}^{to}$, where K^{to} is the set of allowable plants for manufacturing component t_o, and $\overline{K}^{to}$ is its compliment.

In the section to follow, this model is used to allocate business components to ten plants within the company under study.

18.5 Application of Core Business Selection Model to a Set of Plants

In the problem studied, 10 plants currently operate under a single corporate umbrella, producing a variety of steel products including bearings, pipes and sheet steel of various sizes. Clearly, some of these products are of the finished goods variety (e.g. pipes), while others are semi-finished, becoming components in other manufactured items (bearings), or are sold to other plants for further manufacturing (sheet steel).

As indicated earlier, it is convenient to view each plant's business as consisting of various components. While it is the case that there can be a large number of products to consider (e.g. different sizes of circular bearings), here items have been grouped by management under a few major categories. For purposes of this study we present the operation of any plant as consisting of four (overlapping) components, defined by their outputs y_r^t, the number of units of output r in the tth component:

Component #1:

- All solid bearings (y_1^1)
- Circular bearings (automotive) (y_2^1)
- Sheet steel $\leq$4 ft in length (y_3^1).

Component #2:

- Solid bearings (automotive) y_1^2)
- Steel pipes $\leq$8 ft in length y_2^2)
- Sheet steel 4–8 ft in length (y_3^2)

Component #3:

- Steel pipes >8 ft in length (y_1^3)
- Sheet steel >8 ft in length (y_2^3)

Table 18.1 Outputs for four components

Plant	y_1^1	y_2^1	y_3^1	y_1^2	y_2^2	y_3^2	y_1^3	y_2^3	y_1^4	y_2^4	y_3^4	y_4^4
1	50	30	70	30	60	50	40	80	30	50	50	70
2	45	35	60	25	50	50	40	75	35	55	45	60
3	75	25	50	35	55	40	50	70	25	60	75	50
4	60	40	80	40	40	30	70	50	40	50	60	80
5	35	25	25	20	25	20	35	20	25	30	35	25
6	55	60	40	40	60	45	60	50	60	50	55	40
7	120	100	100	100	80	120	120	60	100	110	120	60
8	60	80	25	50	100	20	80	35	80	80	60	25
9	25	75	65	20	25	80	100	70	75	70	25	65
10	100	55	40	70	35	65	35	45	55	60	100	40

Component #4:

- Circular bearings (automotive) (y_1^4)
- Circular bearing (non-auto) (y_2^4)
- All solid bearings (y_3^4)
- Sheet steel $\leq$4 ft in length (y_4^4)

Table 18.1 displays the data for all outputs for the 10 plants considered.

The resources committed to the production of these product lines can be grouped under four headings, namely

- Shop labour (x_1)
- Machine labour (x_2)
- Steel splitting equipment (x_3)
- Lathes (x_4)

Shop labour and machine labour are measured in full time equivalent (FTE) staff. Both equipment variables are expressed in hundreds of hours of capacity available per month. Given the manner in which the four components have been defined, with the inherent overlap of products, all four of these inputs should be viewed as shared resources.

Table 18.2 shows the amounts of the four resources corresponding to each plant.

The connection between the shared inputs and the product outputs (y_r^t) is quite complex, and must be reflected through the α_{ir}. If a given input such as lathes (x_4) does not impact on a particular output such as sheet steel ($\leq$4 ft) (y_3^1) then that particular variable α is set to zero. Figure 18.1 shows the input-to-output impact matrix.

In the figure, an "x" denotes the fact that the particular input contributes to the output shown. It must be noted as well, that when we have a product common to two or more components, the corresponding variables α_{ir} must be equated. For example, since sheet steel $\leq$4 ft is part of both components 1 and 4 (i.e., $y_3^1 = y_4^4$), then $\alpha_{1,3} = \alpha_{1,12}$.

Table 18.2 Shared resources

DMU	x_1	x_2	x_3	x_4
1	30	15	100	150
2	40	12	90	180
3	35	16	97	100
4	38	20	85	85
5	28	9	110	125
6	37	13	76	140
7	31	18	83	110
8	35	15	100	150
9	25	19	95	190
10	30	10	65	210

Fig. 18.1 Input versus output impact matrix

Input	y_1^1	y_2^1	y_3^1	y_1^2	y_2^2	y_3^2	y_1^3	y_2^3	y_1^4	y_2^4	y_3^4	y_4^4
x_1	—	—	x	—	x	x	x	x	—	—	—	x
x_2	x	x	—	x	—	—	—	—	x	x	x	—
x_3	—	—	x	—	x	x	x	x	—	—	—	x
x_4	x	x	—	x	—	—	—	—	x	x	x	—

For solution purposes we have restricted each α_{ir} to lie in the range 0.1–0.4. This means that for each shared input i, at least 10 % and not more than 40 % of that input would be dedicated to any given output r.. Although the decision on such bounds was difficult for management to pin down, the 0.1–0.4 range was deemed reasonable. As well, we impose both upper and lower limits on the numbers of plants to which any given component can be assigned. Specifically, we require for each component t:

$$1 \leq \sum_k d_k^t \leq 4.$$

Hence, at least one plant, and no more than four plants can be assigned component t.

Efficiency Results

Table 18.3 displays the optimal component assignment to plants. In summary:

Component #1 → Plants 5,7,10
Component #2 → Plants 6,8
Component #3 → Plants 1,3,9
Component #4 → Plants 2,4

The overall efficiency score corresponding to this assignment is 96.6 % (the value of objective function (18.7a)). Specifically, if plants are evaluated only on their core business components, their performance will be such that if viewed as a single entity, the aggregate score is 96.6 %. Table 18.4 displays both the current

Table 18.3 Assignment of components to plants

DMU	T_1	T_2	T_3	T_4
1	0	0	1	0
2	0	0	0	1
3	0	0	1	0
4	0	0	0	1
5	1	0	0	0
6	0	1	0	0
7	1	0	0	0
8	0	1	0	0
9	0	0	1	0
10	1	0	0	0

Table 18.4 Decomposition of DMU efficiency

	Assignment of components to plants				Partial efficiencies of component				
DMU	T_1	T_2	T_3	T_4	T_1	T_2	T_3	T_4	Aggregate efficiency
1	0	0	1	0	0.51	0.68	1.00	1.00	0.93
2	0	0	0	1	0.54	0.68	1.00	1.00	0.94
3	0	0	1	0	0.52	0.61	0.90	0.76	0.75
4	0	0	0	1	0.53	0.39	0.76	1.00	0.86
5	1	0	0	0	0.43	0.45	0.25	0.59	0.47
6	0	1	0	0	0.60	0.79	0.83	0.79	0.78
7	1	0	0	0	1.00	1.00	1.00	1.00	1.00
8	0	1	0	0	0.56	1.00	0.50	0.56	0.60
9	0	0	1	0	0.45	0.34	1.00	0.84	0.78
10	1	0	0	0	0.81	0.78	0.85	0.86	0.85

aggregate efficiencies for the 10 plants, as well as a decomposition of these scores into component efficiencies. For example, Plant #3 currently displays an overall performance score of 75 %. This is composed of partial efficiency scores of 52 %, 61 %, 90 % and 76 % for components 1, 2, 3 and 4, respectively. Recall that the measure of partial efficiency for a DMUk in its tth component is given by

$$e_k^t = \mu^t Y_k^t / \left[\nu^t X_k^t + \alpha_k^t \nu^{st} X_k^S \right].$$

It is noted, as well, that with the recommended component-to-plant assignments, plant #3 would be expected to have an efficiency of 90 % (up from 75 %), if it could ultimately phase out non-productive portions of its operation, and move its full emphasis to that part of the business defined by component #3. It must be emphasized that the component t_o assigned to a plant may not be the one whose partial efficiency is highest for that plant. Notice, for example, that component #2 is assigned to plant #6, with a partial efficiency of 79 %, yet component #3 actually performs better within that plant (at a partial efficiency of 83 %). This can occur because rather than minimizing a sum of efficiency ratios, we are optimizing the ratio of aggregate output (across all plants), to aggregate input.

18.6 Discussion

This chapter has examined the problem of identifying core business components for each of a set of comparable decision making units. In the context of a set of manufacturing plants, a modified version of the DEA model of Charnes et al. (1978) has been developed and demonstrated. Unlike conventional applications of DEA where the scope of the business (bundle of products produced) is assumed to remain fixed, the approach herein is intended to aid in making decisions pertaining to functional specialization in plants. An important by-product of the core-business selection process is the evaluation of efficiency of each component of the business as well as of the overall DMU. The result, as demonstrated by Table 18.4, is an efficiency profile that management can utilize in deciding where to aim for improvements and, as well, which components to de-emphasize or phase out.

We do not attempt to address issues relating to plant reorganization toward specialization. Rather, the model can aid management in choosing those (core) business activities to place within each plant. The logistics of restructuring and any change management considerations are beyond the scope of the current chapter.

One of the potential shortcomings of the model given here is the apparent absence of consideration of distribution costs on the input side. Specifically, in some settings, the choice of a particular plant as the location out of which a given component of the business will be operated, has distributional consequences. For example, manufacturing auto parts in a location remote from automobile plants (the customer) may be more costly than having them manufactured at a less efficient, but closer-to-market facility. In the application discussed herein, this issue was not highlighted as a major concern. Presumably, in situations where distribution is a major issue, one would need to augment the input bundle to include a provision for distribution costs.

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