Chapter 15
Network, Shared Flow and Multi-level DEA Models: A Critical Review

Lorenzo Castelli and Raffaele Pesenti

Abstract In the last two decades, complex and detailed DEA models that consider the internal structure of DMUs have been proposed by several authors. This chapter describes the mathematical formulations, along with their main variants, extensions and applications, of three large and popular model families: network (with special emphasis on multi-stage), shared flow (also known as multi-component or multi-activity), and multi-level models. Each family is a different generalization of the same elementary internal structure. This review extends and updates the classification presented in Castelli et al. (Ann Oper Res 173(1):207–235, 2010).

Keywords Data envelopment analysis • Network-DEA • Shared-flows • Multi-level • Multi-stage • Multi-component • Survey

15.1 Introduction

Data Envelopment Analysis (DEA) has been a standard tool for evaluating the relative efficiencies of Decision Making Units (DMUs) since the paper of Charnes et al. (1978) based on the seminal work of Farrell (1957). Some underlying assumptions are common to standard DEA models. The *efficiency* of a DMU is defined as the weighted ratio of the outputs (products or outcomes) yielded by the DMU over the inputs (resources used or consumed). DMUs are *homogeneous*, i.e., they all have the same types of inputs and outputs, and *independent*, i.e., no constraint binds input

L. Castelli (✉)
Dipartimento di Ingegneria e Architettura, Università degli Studi di Trieste,
Via A.Valerio 10, 34127 Trieste, Italy
e-mail: castelli@units.it

R. Pesenti
Dipartimento di Management, Università Ca' Foscari di Venezia,
San Giobbe – Cannaregio 837, 30121 Venezia, Italy

W.D. Cook and J. Zhu (eds.), *Data Envelopment Analysis*,
International Series in Operations Research & Management Science 208,
DOI 10.1007/978-1-4899-8068-7_15,

and output levels of a DMU with the inputs and outputs of other DMUs. Furthermore, DMUs are seen as *black boxes*, i.e., their internal structures are not considered. As a consequence, generally, there is no clear evidence of the transformations to which the inputs are subject to within the DMUs.

In the last two decades, several authors have explored the possibility of abandoning the black box perspective and of considering the internal structures of the DMUs (see, e.g., Sect. 3 of the paper by Cook and Seiford (2009) devoted to the major research thrusts over 30 years since the work of Charnes et al. (1978), or the dedicated chapter in the book by Cook et al. (2007) or, finally, the specific subsections in the citation-based DEA literature survey by Liu et al. (2013b)). These authors justify their approach by observing that, in some particular contexts, the knowledge of the internal structure of DMUs can give further insights for the DMU performance evaluation.

The aim of this chapter is to survey the models that consider internal structures of DMUs. The main rationale of the classification is driven by identifying three families of models as different generalizations of the same elementary formulation.

In particular, we analyze a specific model by comparing a set of homogeneous and independent DMUs, each composed of a set of Decision Making SubUnits (DMSUs). In the literature, subunit, component, activity, division, (sub)structure and (sub)process are synonyms of DMSU and are reported as such in this review. Each subunit is allowed to perform a unique function or activity. Only to keep the notation simple, we also assume that all the DMUs under comparison have the same internal structure.

All the models that we consider can be derived from an *elementary* one that assumes that each DMU internal structure complies with the following assumptions:

Assumptions

1. No intermediate flows among DMSUs exist. In other words, the output of a DMSU cannot be the input of another DMSU (and also cannot re-enter the same DMSU).
2. All the subunits of the DMU do not have shared inputs and shared outputs, i.e., the DMU does not have the opportunity to decide how to allocate its inputs or outputs among its subunits in order to maximize its efficiency (Cook et al. 2000).
3. Any input (output) of the DMU is also an input (output) of one of its subunits.

Here note that Assumption 2 implies that the components of an elementary DMU do not compete for the same resource and do not synergically yield the same product. It follows that the combined presence of Assumptions 1 and 2 makes all the DMSUs of an elementary DMU independent (Fig. 15.1).

By dropping one of the three above assumptions at a time, we obtain different families of DEA models. Specifically:

- We refer to *network* DEA models when Assumption 1 is neglected. Here DMUs have at least one output of a DMSU which is an input of a different DMSU (see Fig. 15.2). These models are of interest because they also allow to describe

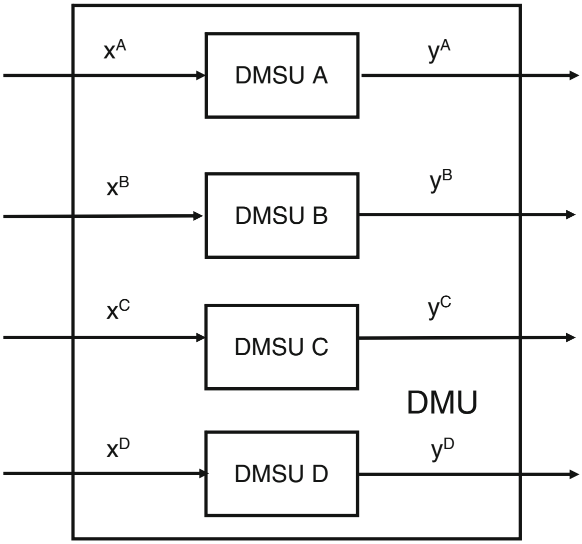

Fig. 15.1 Elementary DMU

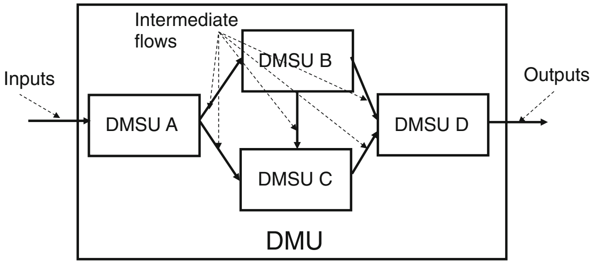

Fig. 15.2 Network DMU

systems where the DMUs are organized in networks so that the outputs of some of them become inputs for other ones. This framework may encompass manufacturing production systems, and in general supply chains, in which some DMUs yield intermediate products that feed other DMUs. The same framework may also include the dynamic DEA models in which some outputs of the DMUs at period t become their inputs in the next period, $t + 1$ (Färe and Grosskopf 2000). Finally, this framework may also possibly cover a further line of research (not discussed in this chapter) that is in fact not specifically devoted to just assessing the efficiency of DMUs but to considering DMUs as components of a greater structure which is interested in maximizing its future efficiency by either re–allocating resources or fixing targets to DMUs (see, e.g., Sect. 5 of the previous version of this survey (Castelli et al. 2010)).

- We refer to *shared flow (or multi-activity or multi-component)* DEA models when Assumption 2 no longer holds (see Fig. 15.4). As an example, this situation may occur when DMUs are divided into different components that require

common resources (e.g., money) or produce goods or services obtained through the synergy and collaboration among them (e.g., the quality of service provided to customers). Then, a DMU may maximize its efficiency also by choosing the most appropriate allocation of the common flows among the subunits and not only by optimizing the weight associated to each flow as it happens in standard DEA.
- We refer to *multi-level* DEA models when Assumption 3 is dropped, i.e., when DMU inputs (outputs) are not necessarily inputs (outputs) of its subunits (see Fig. 15.8).

In the following sections, we first describe the formulation to maximize the relative efficiency of an elementary DMU (Sect. 15.2). Then we introduce the basic reference models (typically with constant returns to scale) for network (Sect. 15.3), shared flow (Sect. 15.5), and multi-level (Sect. 15.6) DEA models. Section 15.4 is specifically devoted to the evaluation of multi-stage processes, a special class of network DEA models. We provide interpretations and applications proposed by different authors, and specify the possible variations from the basic model. In Sect. 15.7 conclusions are drawn. Throughout the paper we assume that the reader is familiar with at least the seminal works on DEA (see, e.g., Banker et al. 1984; Charnes et al. 1978), as we will not define or justify basic concepts such as, e.g., positive non-Archimedean value ε, slack variables, production set, virtual inputs and outputs, Constant or Variable Returns to Scale (CRS or VRS, respectively), allocative and technical efficiencies.

15.2 Elementary Model

In this section, we introduce a DEA model for assessing the efficiency of elementary DMUs (i.e., whose internal structure follows Assumptions 1–3). To this end, for each elementary DMU k, let us define

- i, j, r: the indexes of the generic input, output, and DMSU, respectively,
- $X_k^r = \{x_{ik}^r\}$: the vector of the inputs of DMSU r,
- $Y_k^r = \{y_{jk}^r\}$: the vector of the outputs of DMSU r,
- $\nu^r = \{\nu_i^r\}$: the vector of weights of the inputs of DMSU r
- $\mu^r = \{\mu_j^r\}$: the vector of weights of the outputs of DMSU r.

For an elementary DMU 0 belonging to a set of N homogeneous and independent DMUs with the same internal structure, the CRS input-oriented version of the envelopment-based DEA model can be written as:

$$\theta_0^* = \min \theta_0 - \varepsilon\left(\sum_r\left(\sum_i s_i^{r-} + \sum_j s_j^{r+}\right)\right) \tag{15.1a}$$

$$\sum_{k} \lambda_k^r x_{ik}^r = \theta_0 x_{i0}^r - s_i^{r-} \quad \forall i, r \tag{15.1b}$$

$$\sum_{k} \lambda_k^r y_{jk}^r = y_{j0}^r + s_j^{r+} \quad \forall j, r \tag{15.1c}$$

$$\lambda_k^r, s_i^{r-}, s_j^{r+} \geq 0 \quad \forall i, j, k, r \tag{15.1d}$$

where λ_k^r is the multiplier of DMSU r belonging to DMU k, and s_i^{r-}, s_j^{r+} are the slack variables.

The dual formulation of (15.1) is the following multiplier-based DEA model:

$$e_0^* = \max \sum_{j,r} \mu_j^r y_{j0}^r \tag{15.2a}$$

$$\sum_{i,r} \nu_i^r x_{i0}^r = 1 \tag{15.2b}$$

$$\sum_{j} \mu_j^r y_{jk}^r \leq \sum_{i} \nu_i^r x_{ik}^r \quad \forall k, r \tag{15.2c}$$

$$\nu_i^r, \mu_j^r \geq \varepsilon \quad \forall i, j, r. \tag{15.2d}$$

In Model (15.2) the maximum relative efficiency e_0^* is assessed by comparing DMU 0 with all the existing subunits. Then, as shown in Yang et al. (2000), Castelli et al. (2004), and Kao (2009b), e_0^* is equal to the maximum relative efficiency of its subunits, and DMU 0 is:

- Weakly efficient if and only if there exists at least one of its subunits which is weakly efficient relative to the corresponding subunits of other DMUs;
- CRS-efficient if and only if each of its subunits is CRS-efficient relative to the corresponding subunits of other DMUs.

A multiple input and single output elementary configuration is also proposed by Färe and Primont (1984). Specifically, the authors, relying on the Farrell (1957) output-based efficiency measure, construct a reference technology for DMUs using their subunit data. Next, they compare this efficient technology against the reference frontier of the subunits, i.e., as if the subunits were independent DMUs and not part of a larger DMU. Kao (2000) generalizes this model for cases of multiple outputs and multiple inputs.

15.3 Network DEA Models

In this section, we describe DEA models for DMUs that present intermediate flows between subunits. In this case, the subunits are neither independent nor homogeneous. They are interdependent in the sense that part of the output produced by some of them may be used as an input by other ones. In addition, their interdependency leads to their non-homogeneity as they may present different inputs and/or different outputs.

The basic network DEA models have been introduced by Färe (1991), Färe and Whittaker (1995) and Färe and Grosskopf (1996b). These models represent DMUs composed of two consecutive subunits with one intermediate flow: the output from the first subunit is used as input in the second one. Then, Färe and Grosskopf (2000) extend these models to consider DMUs made of more subunits (see also the book by Färe et al. 2007).

Since the above seminal papers, many different models, both envelopment- and multiplier-based, have appeared in the literature. Here, as an illustrative example, we provide a CRS envelopment-based (input oriented) model under the assumption that all DMUs have exactly the same internal structure in terms of DMSUs. Specifically, we assess the relative efficiency θ_0^* of the whole DMU 0 using the following notation: for each DMU k, r indicates a generic DMSU of k, then x_{ik}^r is the amount of the i-th external input of the DMU entering subunit r, y_{jk}^r is the amount of the j-th final output of the DMU produced by subunit r, and z_{lk}^{rt} is the l-th intermediate flow of DMU produced by subunit r and used by subunit t (Fig. 15.2); $pred(r)$ represents the set of predecessors of subunit r, i.e., the set of subunits which have at least one output used as input by subunit r, similarly, $succ(r)$ is the set of successors of subunit r; finally s_k^r are slack variables.

$$\theta_0^* = \min \theta_0 - \varepsilon \sum_r \sum_i s_i^{r-} \tag{15.3a}$$

$$\sum_k \lambda_k^r x_{ik}^r = \theta_0 x_{i0}^r - s_i^{r-} \quad \forall i, r \tag{15.3b}$$

$$\sum_k \lambda_k^r \sum_{t \in pred(r)} z_{lk}^{tr} = \sum_{t \in pred(r)} z_{l0}^{tr} - s_l^{r-} \quad \forall l, r \tag{15.3c}$$

$$\sum_k \lambda_k^r \sum_{t \in succ(r)} z_{lk}^{rt} = \sum_{t \in succ(r)} z_{l0}^{rt} + s_l^{r+} \quad \forall l, r \tag{15.3d}$$

$$\sum_k \lambda_k^r y_{jk}^r = y_{j0}^r + s_j^{r+} \quad \forall j, r \tag{15.3e}$$

$$s_i^{r-}, s_l^{r-}, s_l^{r+}, s_j^{r+}, \lambda_k^r \geq 0 \quad \forall k, i, j, l, r. \tag{15.3f}$$

As for standard envelopment-based DEA formulations, model (15.3) considers a radial measure of efficiency (as ε is a positive non-Archimedean parameter) and is based upon the definition of the Production Possibility Set (PPS) of DMU 0. Indeed, provided that $\theta_0 = 1$, constraints (15.3b)–(15.3f) describe the PPS of DMU 0 in the following terms. For each subunit r, constraints (15.3b) and (15.3c) indicate that the value of each external input flow i or intermediate input flow l cannot be less than the conic combination of the values of the corresponding input flows of the analogous DMSUs r from all the observed DMUs. Similarly, constraints (15.3d) and (15.3e) indicate that the value of each final output flow j or intermediate output flow l cannot exceed a conic combination of the values of the corresponding output flows of the analogous DMSUs r from all the observed DMUs. Model (15.3) describes a *closed* (network) process since each subunit either receives only external input flows or only intermediate flows and, analogously, it either produces only final output flows or only intermediate flows. Model (15.3) trivially generalizes the model proposed by Färe and Whittaker (1995), where the slack variables are omitted, and implies that the observed DMUs and their DMSUs exhibit constant returns to scale (CRS) and strong disposability of inputs and outputs (see Färe and Grosskopf 1996b).

In the rest of the paper, for both CRS and VRS situations we will introduce envelopment- and multiplier-based DEA models. Differently from the standard DEA models, the multiplier- and envelopment-based network DEA models are not, in general, dual of each others (Chen et al. 2010a, 2013b). They represent two different approaches that may produce different efficiency results. For this reason, Chen et al. (2010a, 2013b) suggest that envelopment-based network DEA models should be used for determining the frontier projection for inefficient DMUs. Differently, multiplier-based network DEA models should be used for determining the DMSU (called *division* by the authors) efficiency. In addition, the authors also point out that, contrary to what it is sometimes suggested, it is not sufficient to add convexity constraints to an envelopment-based network DEA model or free variables to a multiplier-based network DEA model to make these models capable of describing VRS network processes.

15.3.1 Non-radial Measures of Efficiency

Leaving aside the radial measure of efficiency considered in model (15.3), some authors propose different non-radial measures of efficiency for network DEA models.

Tone and Tsutsui (2009) introduce a VRS *Slack-Based Measure* (SBM) of efficiency. Following Pastor et al. (1999) for standard DEA models, this efficiency measure is a function of the slack variables and is appropriate when we employ flows, such as labor, materials and capital, that are substitutional and do not change proportionally. Specifically, Tone and Tsutsui (2009) substitute objective (15.3a) with

$$\theta_0^* = \min_{\lambda,\, s} \sum_r w_r \left(1 - \frac{1}{m_r} \sum_{i=1}^{m_r} \frac{s_i^{r-}}{x_{i0}^r}\right), \tag{15.4}$$

where, for each subunit r, w_r is a constant parameter that weighs the relative importance of the subunit and m_r is the number of its inputs. Even though objective (15.4) is adequate only for an input oriented model, Tone and

Tsutsui (2009) propose analogous measures of efficiencies for output oriented and non-oriented models. They also introduce a *discretionary formulation* that is applied when the DMU 0 under assessment may decide the values of its intermediate flows in the light of other DMUs' intermediate flow values.

In practice, the discretionary formulation requires the substitution of the two sets of constraints (15.3c) and (15.3d) with constraints

$$\sum_k \lambda_k^r \sum_{t \in pred(r)} z_{lk}^{tr} = \sum_k \lambda_k^r \sum_{t \in succ(r)} z_{lk}^{rt} \quad \forall l, r. \tag{15.5}$$

Tone and Tsutsui (2009) finally claim that their approach has the further advantage that it can be trivially modified to also model CRS processes.

Fukuyama and Weber (2010) introduce the network *directional* slack-based measures. In these measures, the values of the slack variables are normalized on the basis of user defined coefficients. For example, the coefficients ${x_{i0}}^r$ in objective (15.4) would be substituted by generic positive coefficients ${g_i}^r$, being the vector $g_x = \{{g_i}^r\}$ the desired direction of input contraction. These efficiency measures are then extended to account also for possible undesirable (or *bad*) outputs.

In a paper addressing sensitivity analysis in network DEA models, Avkiran and McCrystal (2012) introduce a *Range Adjusted Measure* (RAM) of efficiency. This measure builds upon the one by Cooper et al. (1999) for standard DEA models and, again, it is a function of the values of the slack variables. Then, the authors compare the results obtained with the application of sensitivity analysis to envelopment-based RAM network DEA models and to corresponding SBM network DEA models.

15.3.2 Simultaneous Evaluation of DMU and DMSU Efficiencies

Some authors specifically focus their work on developing models aiming at evaluating subunit efficiencies and at studying the influence of such values to the efficiency of the DMU the subunits belong to.

Their research is justified by the following facts. The knowledge of the internal structure of the observed DMUs allows to determine whether better performances could be obtained by a DMU that merged the technologies of the most efficient substructures of the observed DMUs. In addition, the assessment of the efficiency of each subunit might prevent that in a DMU the inefficiency of some of its DMSUs may be compensated by the efficiency of others.

Castelli et al. (2001) introduce a DEA-like model to compare non-homogenous and interdependent subunits belonging to the same DMU. A given subunit r may be evaluated according to three different sets: (a) all the subunits homogeneous to it, (b) all the subunits of the DMU, and (c) with respect to a given output, all the subunits yielding that output. In this last case, the rationale is that these subunits, although not necessarily homogeneous, have a certain degree of commonality because they can be considered as potential substitutes for each other, as far as the production of that output is concerned. Thus the interest in comparing them. As a possible limitation, Lewis and Sexton (2004) point out that this approach may lead to small reference sets. Castelli et al. (2001) also link the subunits' and DMU efficiencies by defining an efficiency value W obtained by maximizing the product of the efficiency of the subunit under evaluation and the efficiency of the DMU it belongs to. In this way, subunits not only maximize their own efficiency, but also positively contribute to the efficiency of the whole system they are part of. Indeed, the authors prove that a subunit seeking to optimize its W efficiency behaves with a benevolent attitude, i.e., being equal to other conditions, it also maximizes a combination of the efficiencies of the other subunits. In addition, the authors show that the whole DMU is efficient if and only if all its subunits are W efficient.

Sexton and Lewis (2003) and Lewis and Sexton (2004) explicitly compute the efficiencies of the subunits using both input and output oriented formulations. Their basic models can be seen as an adaptation of model (15.3), where the efficiency of subunit r belonging to DMU 0 is optimized and constraints (15.3b)–(15.3e) are adequately rewritten. In a simple case of DMUs composed of two subunits S_1 and S_2 in series, the authors show that DMU 0 is efficient when its output values are equal to the output values produced in the case that S_2 is efficient and uses the intermediate product levels that it would have received, had S_1 been efficient. Lewis and Sexton (2004) describe the internal structure of each DMU as an acyclic direct graph. This graph has a node for each subunit plus one origin and one destination node. In this case, the authors show that a necessary (but not sufficient) condition for a whole DMU to be efficient is the existence of a path from origin to destination along with every subunit is efficient. As a consequence, it is possible that, when considering the internal structure, all DMUs under evaluation are inefficient. Lewis et al. (2009) use the model presented in Lewis and Sexton (2004) to assess simultaneously organizational capability, efficiency, and effectiveness in Major League Baseball.

Kao (2009a) proposes a *relational* approach (see also Kao and Hwang 2008, Sect. 15.4.2.1), whose underlying concept is that some relationship exists between the measure of the overall DMU efficiency and the measure of its DMSUs' efficiencies, for example, a simple multiplication, as in Kao and Hwang (2008), or a weighted average, as in Chen et al. (2009a). The authors' assumptions imply that the relational network DEA models, when formulated with a multiplier-based measure of efficiency, are also characterized by the fact that the same flows have associated the same weights no matter which subunits these flows belong to. In other words, an intermediate flow presents the same weight both when is considered as an output flow of a DMSU and when is considered as an input flow of a different DMSU. In the same context, Lozano (2011) introduces an envelopment-based relational network DEA model to asses the technical, scale, cost and allocative efficiency scores of the DMUs. To this end, he proposes an axiomatic approach to define the PPS of a DMU through the composition of the PPS of each of the DMUs' subunits. Then, Lozano et al. (2013) generalize the previous model to take into account processes with undesirable outputs and apply this new model to assess airport performances.

Relying on work by Chen et al. (2010a), Fukuyama and Mirdehghan (2012) propose a two-phase slacks-based network model to assess the efficiency of a set of DMUs and their DMSUs. To this aim, the authors first consider an additive envelopment-based network DEA model that optimizes the slacks of exogenous inputs and final outputs. Then, they use a linear programming model to assess the efficiency status of each DMSU.

Most recently, Kao and Chan (2013) have introduced a multi-objective programming method that computes both the overall efficiencies of the DMUs and the divisional efficiencies of the DMSUs of network DEA models.

15.3.3 DMSU Ownership

Some authors have considered the consequences of having DMSUs of the same DMU controlled by different agents that may have different agendas (see also the game theory approach discussed in Sect. 15.4.2.5).

Chang et al. (2011) discuss the importance of taking into account the ownership of the different DMSUs composing the DMU under assessment. The authors argue that an agent interested in assessing the efficiency of a DMU cannot include in her DEA model the external inputs and final outputs of the DMSUs that she does not own. In fact, these flows are usually unknown to her. Differently, she can assume the knowledge of the internal flows if they are regulated by contracts between the different DMSUs. Accordingly, the authors introduce three ownership-specified (centralized, distributed and hybrid) network DEA models which take into account the different possibility of ownership of the DMSUs. Similar problems are also considered by Chen and Yan (2011). These latter authors are motivated by the necessity of assessing the efficiency of DMUs representing supply chains. As main result, they prove that a supply chain is weakly efficient only if there exists a path from the external inputs to the final outputs along which all DMSUs are weakly efficient. They also show that it is never appropriate to ignore the internal structure of the supply chain. In fact, standard DEA models may lead to overrate the efficiency of a supply chain not only when different agents pursuing their own agenda own the different DMSUs, but also when all the DMSUs are owned by a single agent and the DMSUs are then centralized controlled.

15.3.4 Applications

Not surprisingly, many network DEA models have been proposed to evaluate the performances of different processes, applied in particular to the top-five industries (transportation, banking, agriculture and farm, healthcare, and education) addressed by the standard DEA literature (Liu et al. 2013a).

15.3.4.1 Hospitality and Transportation Industries

Hsieh and Lin (2010) apply a relational network model to the tourist hotels in Taiwan and present a survey on the efficiency assessment in the hospitality industry. In the same context, Zhang and Ma (2011) apply a network DEA model to assess the business efficiency of Chinese hotel and tourism firms.

Yu (2008a) compares the results obtained through standard DEA and network DEA models for assessing the performances of 40 global railways in terms of technical efficiency, service effectiveness, and technical effectiveness. The author suggests to include the environmental factors as non-discretionary inputs and underlines how the network DEA model provides deeper insight regarding the sources of inefficiency. The importance of environmental factors is discussed also in Yu (2010) where a network DEA model is proposed to deal with both production and service efficiency in airports.

Sheth et al. (2007) apply network DEA models to the assessment of bus routes by expanding the Färe and Grosskopf (2000) approach to account for the different perspectives of operators and users, and for multiple goals. Hahn et al. (2011) propose a network DEA model to assess the efficiency of Seoul arterial bus routes. Zhao et al. (2011) assess the efficiency of a transportation system by considering the perspectives of the different stakeholders, such as transportation service providers, users, and the community. The authors propose a model that includes undesirable outputs and where the different perspectives are inter-related through intermediate flows. Finally, Li (2012) uses a network DEA model to assess the China's railway transport industry.

15.3.4.2 Production of Goods or Services

Färe and Whittaker (1995) apply a model similar to model (15.3) to a diary production problem and compare the result obtained with the ones obtained with a standard DEA model. The former model turns out to have greater discrimination power: only 23 % of the DMUs are on the efficiency frontier compared to 65 % when intermediate flows are not explicitly taken into account.

Liu et al. (2012) introduce a network DEA model to assess non-profit farmers associations in Taiwan. Lin and Chiu (2013) and Matthews (2013) propose SBM network DEA models (see Sect. 15.3.1) to improve Taiwan bank performance evaluation and assess Chinese bank income efficiency, respectively. Matthews (2013) uses metrics of risk management practice and risk management organization as intermediate inputs. Vaz et al. (2010) exploit a network DEA model to assess the performances of retail stores, and Lee and Johnson (2012) use a relational network model (see Sect. 15.3.2) to decompose the efficiency of profitability for a general production system.

Löthgren and Tambour (1999) estimate efficiency and productivity for a sample of Swedish pharmacies taking also into account customer satisfaction. The pharmacy technology is represented by a production and a consumption node. The production node yields (final) outputs (e.g., outpatient prescriptions) and also produces nonmarketable characteristics and attributes (e.g., the service level) that are considered as intermediate inputs of the consumption node. Together with external inputs (e.g., customer-service labor hours) the consumption node provides customer quality assessments on pharmacy service.

Chilingerian and Sherman (2004) study network DEA applications to health care systems.

15.3.4.3 Governmental Entities

Prieto and Zofio (2007) employ the network DEA model introduced by Färe and Grosskopf (2000) to assess the economies of a set of countries belonging to the Organisation for Economic Co-operation and Development with the aim of identifying best practices. Each national economy is described in terms of a network where different nodes use primary inputs to produce intermediate input and outputs, and satisfy final demand. Each node represents a basic economic sector, such as agriculture, manufacturing, construction, and services.

Guan and Chen (2010, 2012) and Chen and Guan (2012) apply network DEA to measure the efficiency of China's regional innovation systems, whereas Amatatsu et al. (2012) assess efficiency and returns-to-scale of Japanese local governments.

15.4 Multi-stage Network DEA Models

In this section, we introduce a particular type of network DEA models that thanks to their simple structure and wide applicability have been extensively studied in the past years. Specifically, we consider the two-stage, or in general multi-stage, network DEA models. Such models assume that each DMU is composed of two, or in general more, consecutive stages, each one being a single DMSU or a set of parallel DMSUs (Fig. 15.3). For example, the very first network DEA models by

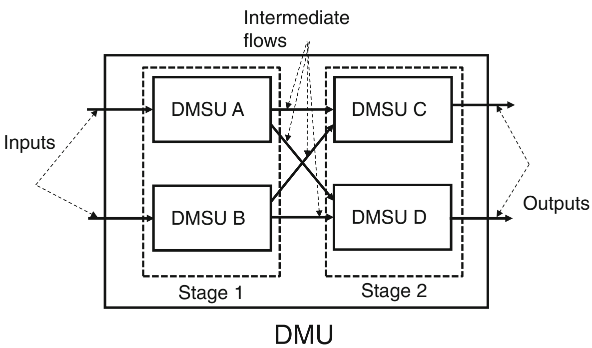

Fig. 15.3 Two-stage DMU

Färe (1991), Färe and Whittaker (1995) and Färe and Grosskopf (1996b) introduced in Sect. 15.3 are two-stage models. Multi-stage models are obviously used for the efficiency evaluation of multi-stage processes but they are also studied because they can model the evolution of processes over time. In this latter case the multi-stage models are referred to as dynamic network models and each stage represents the same DMU at different times (see Sect. 15.4.4).

Seiford and Zhu (1999) are among the first authors to deal with multi-stage processes. They consider each stage and the whole DMU as independent and evaluate the efficiencies of these structures using conventional DEA models. Differently, most of the papers that we analyze in this section take into account some form of interaction between consecutive stages, as it is usually done in network DEA models. In this context, Cook et al. (2010a) and Agrell and Hatami-Marbini (2013) propose two interesting surveys on the multi-stage network literature, respectively considering a game theoretic and a supply chain management perspective. Some alternative DEA models for two-stage process are also surveyed in Wang and Chin (2010).

As an illustrative example, we provide a general model for a two-stage process where each stage is made in turn of parallel DMSUs (see Fig. 15.3). To this aim, we extend the notation used in model (15.3). For each DMU k, x_{ik}^{f} is the amount of the i-th input of the DMU entering subunit f in the first stage; y_{jk}^{s} is the amount of the j-th output of the DMU produced by subunit s in the second stage; z_{lk}^{f} is the amount of the l-th intermediate flow of the DMU output of the subunit f of the first stage; whereas z_{lk}^{f} is the amount of the same flow input of the subunit s of the second stage; finally, z_{lk}^{fs} is the amount of flow l of subunit f that feeds subunit s, with $\sum_s z_{lk}^{fs} = z_{hk}^{f}$.

We define the efficiency of DMU 0 as

$$e_0^* = \frac{\sum_j \sum_s \mu_{js} y_{j0}^{s}}{\sum_i \sum_f \nu_{if} x_{i0}^{f}}. \tag{15.6}$$

Then a possible multiplier-based two-stage (input oriented) DEA model that assesses the relative efficiency of DMU 0 is

$$e_0^* = \max \sum_j \sum_s \mu_{js} y_{j0}^{s} \tag{15.7a}$$

$$\sum_i \sum_f \nu_{if} x_{i0}^{f} = 1 \tag{15.7b}$$

$$\sum_l \upsilon_{lf} z_{lk}^{f} \leq \sum_i \nu_{if} x_{ik}^{f} \quad \forall k, f \tag{15.7c}$$

$$\sum_j \mu_{js} y_{jk}^s \leq \sum_l \eta_{hl} z_{lk}^s \quad \forall k, s \tag{15.7d}$$

$$G(\upsilon_{lf}, z_{lk}^f, \eta_{ls}, z_{lk}^s) \geq 0 \quad \forall k, l, f, s \tag{15.7e}$$

$$\mu_{js}, \nu_{if}, \upsilon_{lf}, \eta_{ls} \geq \varepsilon \quad \forall i, j, l, f, s \tag{15.7f}$$

where conditions (15.7e) represent a set of constraints that link the output flows of the subunits of the first stage with the input flows of the subunits of the second stage. Note that the above formulation, as well as formulation (15.3) for general network structured DMUs, applies to closed processes.

15.4.1 Balancing Intermediate Flows

Castelli et al. (2004) consider two-stage processes where each second stage DMSU receives as input only intermediate flows and discuss two different kinds of constraints (15.7e): virtual *weights balancing constraints* and *flow balancing constraints*.

In the former case, the overall perceived values of the intermediate flows are balanced, i.e., virtual weights of the input flows of the second stage are equal to virtual weights of the feeding flows:

$$\eta_{ls} z_{lk}^s = \sum_f \upsilon_{lf} z_{lk}^{fs} \quad \forall k, l, s.$$

Under this assumption, the authors prove that the relative efficiency of the DMU under evaluation is equal to the product of the maximum relative efficiency of each single stage calculated according to model (15.2).

In the latter case, not only the perceived values but also the flows themselves are balanced:

$$\eta_{ls} = \upsilon_{lf} \quad \forall k, l, f, s \quad \text{and} \quad z_{lk}^s = \sum_f z_{lk}^{fs} \quad \forall k, l, s.$$

Under this second assumption, the authors prove that the relative efficiency of a DMU is assessed by comparing it with each observed DMUs together with all DMUs that could be obtained by composing their two stages with any possible combination of the subunits of the observed DMUs. Castelli et al. (2004) finally point out that the flow balancing constraints model can also be derived as dual of model (15.3) when specialized to a two-stage process, and claim that their results could be generalized to second stage DMSUs with multiple inputs.

15.4.2 Extensions

15.4.2.1 Relational Models

Kao and Hwang (2008) customize model (15.7) to multi-stage processes such that each stage includes a single DMSU, and each DMSU may have multiple inputs. In particular, in Kao and Hwang (2008) the relational approach (see Sect. 15.3.2) is introduced for the first time in the context of two-stage network DEA. In their paper, the authors also discuss possible solutions for dealing with multiple optimal weights. For instance, the authors suggest to choose as optimal the weights that maximize the efficiency of the first stage while maintaining the overall efficiency score of the DMU. Subsequently, Kao (2009a) extends the relational model in Kao and Hwang (2008) to series-parallel networks by utilizing dummy DMSUs such that a DMU structured as a network of DMSUs can be represented by a multi-stage structure where each stage can be composed of a set of parallel DMSUs. Here again, the flow balancing constraints are imposed and the same flow has the same weight all over the network, no matter if it is used as an input or as an output. Kao and Hwang (2010) apply the relational network model to assess information technology on firm performance in a banking industry.

Differently from Kao and Hwang (2008), which assess DMU and DMSU efficiencies of a two-stage DEA model in two separate and consecutive steps, Liu (2011) explains, in a short note, how to assess such efficiencies simultaneously. Liu and Lu (2012) introduce a network-based method for ranking of efficient units in two-stage DEA models. Specifically, each DMU is a node in a network and is linked with its peers. Links are weighted on the basis of the peer importance. Efficient DMUs are then ranked on the basis of their centrality in such a network.

Chen and Zhu (2004) propose a DEA framework that considers a two-stage process as efficient when each stage is efficient. Chen et al. (2009b) prove the equivalence between the CRS version of the Chen and Zhu (2004) model and the Kao and Hwang (2008) model. In this context, the interested reader is also referred to the survey by Agrell and Hatami-Marbini (2013). This paper consider the different two-stage models presented in the literature and points out which of them provides the equivalent results.

15.4.2.2 Variable Returns to Scale and Additive Measures of Efficiency

Chen et al. (2009a) observe that the multi-stage model by Kao and Hwang (2008) is applicable to CRS only. Indeed, it assesses the efficiency of the overall process as the product of the efficiencies of the different stages (i.e., the geometric mean of stage efficiencies). As an example, in the specific case of a two-stage process composed of a single DMSU f in the first stage and a single

DMSU s in the second stage, holding flow balancing constraints, the DMU efficiency (15.6) is:

$$e_0^* = \frac{\sum_j \mu_{js} y_{j0}^s}{\sum_i \nu_{if} x_{i0}^f} = \frac{\sum_j \mu_{js} y_{j0}^s}{\sum_l \upsilon_{ls} z_{l0}^s} \times \frac{\sum_l \upsilon_{ls} z_{l0}^s}{\sum_i \nu_{if} x_{i0}^f}. \tag{15.8}$$

To extend the two-stage models to VRS, Chen et al. (2009a), within the same relational model framework, measure the efficiency of the overall process as a weighted sum of the efficiencies of the two stages:

$$e_0^* = w_s \frac{\sum_j \mu_{js} y_{j0}^s + \omega^s}{\sum_l \upsilon_{ls} z_{l0}^s} + w_f \frac{\sum_l \upsilon_{ls} z_{l0}^s + \omega^f}{\sum_i \nu_{if} x_{i0}^f}, \tag{15.9}$$

where w_s and w_f are user-specified weights such that $w_s + w_f = 1$ and the terms ω^f and ω^s, free variables, express the scale efficiencies of the first and second stage, respectively. As pointed out by Cook et al. (2010b), Eq. (15.9) evaluates the overall performance of the network also in terms of the performances of the individual DMSUs.

Chen et al. (2009a) also show that efficiency measure (15.9) cannot be linearized in the same way efficiency measure (15.6) is turned into Eqs. (15.7a) and (15.7b), unless weights w_s and w_f are chosen to be proportional to the “sizes” of each stage, in terms of total resources devoted to each stage, that is,

$$w_s = \frac{\sum_l \upsilon_{ls} z_{l0}^s}{\sum_i \nu_{if} x_{i0}^f + \sum_l \upsilon_{ls} z_{l0}^s}, \quad w_f = \frac{\sum_i \nu_{if} x_{i0}^f}{\sum_i \nu_{if} x_{i0}^f + \sum_l \upsilon_{ls} z_{l0}^s}. \tag{15.10}$$

In a subsequent study, Chen et al. (2010a) point out that, differently from the standard DEA models, the multiplier and envelopment-based two-stage DEA models are not, in general, dual of each others, but represent two different approaches that provide different information and may produce different efficiency results (see also Chen et al. (2013b) in Sect. 15.3). Specifically, the authors show how some two-stage models in the literature may fail to provide the complete information on how to project inefficient DMUs on to the DEA frontier. Then, they develop two-stage models capable of determining these DEA frontier projections for inefficient DMUs at least in the CRS case. Finally, they indicate that further study is then needed to develop models capable of determining the DEA frontier projections for VRS inefficient DMUs since even their own previous model

(Chen et al. 2009a), which assesses correctly both the overall DMU efficiency and the efficiency of each stage, is not sufficient to yield these projections.

Chiou and Lan (2007) address two-stage VRS models, too. They propose an additive measure of efficiency equal to the one proposed in (15.9) when $w_s = w_f = 1$. They use this measure to asses both efficiency and effectiveness of a transportation system. In Chiou et al. (2010), the same authors discuss in detail the properties of their two-stage VRS model, that they call *integrated* DEA model, and generalize their efficiency measure to obtain exactly (15.9), where w_s and w_f are arbitrarily fixed. Differently form Chen et al. (2009a), these authors do not linearize their model but claim the existence and the uniqueness of optimal weights μ_{js}, υ_{ls}, ν_{if}, ω^f and ω^s. Unfortunately, Lim and Zhu (2013) show that such a conclusion is a false statement. They also show how the two-stage DEA model proposed in Chiou and Lan (2007) can be transformed into a parametric linear program.

Finally, Kao and Hwang (2011) propose a multiplier-based relational VRS two-stage model. By solving both an output-oriented and input-oriented model, the authors are able to separate the technical and the scale efficiencies of the DMUs.

15.4.2.3 Open Multi-stage Processes

Cook et al. (2010b) introduce multi-stage DEA models for *open* serial processes, i.e., where some outputs from a given stage may leave the system while new inputs can enter at any stage. As in Chen et al. (2009a), the authors represent the overall efficiency as an additive weighted average of the efficiencies of the DMSUs. These results are also applied to general series-parallel network structures. Open multi-stage processes are considered also by Golany et al. (2006). These latter authors assume that each stage is governed by a different manager that will not agree to "vertical integration" initiatives unless higher efficiency (with respect to separately applying conventional DEA) is achieved. For this reason these authors propose a measure that identifies a Pareto optimal point for the efficiency values of the DMSUs that compose their system. As multiple Pareto optimal point may exist, they discuss the properties of three different possible ways of choosing the Pareto efficient point of interest.

15.4.2.4 Unoriented Models

Holod and Lewis (2011) present a two-stage DEA unoriented model, i.e., a DEA model that seeks to simultaneously decrease input levels and increase output levels (the interested reader is referred to Färe et al. (2002) for standard hyperbolic/unoriented DEA models). The authors use this model to assess bank efficiency and address what they call the DEA literature "deposit dilemma", that is, the lack of agreement on whether deposits should be considered as an input or an output. The authors solve this dilemma by representing deposits as intermediate flows in a two-stage unoriented DEA model. A similar model is also introduced by Lewis et al. (2013), who show how to solve it through an iterative algorithm that alternates between an input-oriented push backward step and an output-oriented push forward step. The same authors are currently working on a general network DEA unoriented model (Mallikarjun et al., 2014)

Yu and Chen (2011) use also a similar measure of efficiency to assess the air routes performance of an airline in Taiwan. In their paper, the authors initially present an interesting discussion on the definition of the performances of airlines in term of production efficiency, service effectiveness and operational effectiveness and a critical analysis of their own previous works. Then, they compare the results obtained through their model with the ones yielded by a corresponding multi-stage DEA model proposed by Chiou and Chen (2006), even though Lin (2008) identifies in this last paper some methodological and terminological inaccuracies.

15.4.2.5 Game Theoretic Perspective

The assessment of two-stage processes has been studied also relying on game theory. In particular, Liang et al. (2006) compare a leader-follower and a cooperative relationship between DMSUs of a supply chain. Liang et al. (2008) show that in a cooperative contest, when different intermediate flows between the two stages are present, then multiple efficiency values for the two stages may emerge. Differently, in a non-cooperative context a two-stage network DEA model just produces the same results as applying a standard DEA model to the two stages consecutively. Li et al. (2012) generalize the result proposed in Liang et al. (2008) by also allowing external inputs to the second stage. Chen et al. (2006) propose a DEA game model in a two-stage supply chain and prove the existence of numerous Nash equilibria efficiency points for the DMSUs.

As already pointed out, recently Cook et al. (2010a) have published an interesting survey that analyzes the DEA models used to assess the efficiency of two-stage processes from a game theoretic perspective. The authors categorize this literature using either Stackelberg (leader-follower) or cooperative game concepts. In this framework, only the multi-stage processes referring to cooperative game or, equivalently, to centralized control concepts have their overall efficiencies assessed through network models like model (15.7) or its variations. Differently, the processes referring to leader-follower concepts have the efficiency of their two stages assessed through two separated non-network DEA models. In this work, Cook et al. (2010a) also point out the equivalence of different two-stage DEA models available in the literature.

Zha et al. (2008) propose a two-stage VRS DEA model where the measure of the overall efficiency is given by the geometric mean of the efficiencies of the two-stages. Specifically, the efficiency of the first stage is evaluated with the input-oriented VRS model and the second stage with the output-oriented VRS model. Then, the overall efficiency is evaluated in a cooperative manner. In the same context, Zha and Liang (2010) introduce a two-stage DEA model with shared inputs to be allocated among the two stages (see also Sect. 15.5). Again, the efficiency measure is in the product-form and the process overall efficiency is assessed assuming that the two stages participate in a cooperative game. Also Wu (2010) considers a two-stage DEA model where stages share some inputs. Here the author assumes that there exists a Stackelberg-game relationship between the two stages and proposes a bilevel programming DEA model, which is solved using a branch and bound algorithm. Wu (2010) provides as case studies the application of his model to a banking chain and a manufacturing supply chain.

15.4.2.6 Processes with Feedback

Liang et al. (2011) consider two-stage processes with feedback, that is, processes in which some of the final outputs of the second stage become inputs of the first stage. In this context, the authors propose two multiplier-based network DEA models. The first (and simpler) one aims at maximizing the average efficiency of the two individual stages. The second model instead ranks the two stages in accordance with their relative importance and is formulated as a bilevel model. In both cases, the authors assume that the weights applied to the intermediate and feedback flows are the same for both stages. In addition, they assume that the weights of the intermediate and feedback flows are fixed when they play the role of outputs of the associated stage. This latter assumption is important in the second model, which maximizes the efficiency of the first stage and let the efficiency of the second stage depend on the first stage's one. In fact, the efficiency of the first stage depends in turn on the value of the weights of the feedback flows, which are fixed when the efficiency of the second stage is assessed. Both models are nonlinear, but their nonlinearity is only due to one or two variables, respectively. Hence, they can be practically solved by iteratively and tentatively assigning values to such few variables.

15.4.3 Applications

Besides some exceptions as in Wei and Chang (2011) who face the problem of *designing* an efficient multi-stage process (the authors propose a DEA approach to support the optimal design of DMU external input, intermediate flow and final output portfolios), in most cases DEA models are used to assess the efficiency of existing processes. This section illustrates several applications of two-stage DEA models.

15.4.3.1 Banking Sector

Avkiran (2009) employs a two-stages DEA model to assess United Arab Emirates (UAE) banks using a slacks-based inefficiency measure. Similarly, Paradi et al. (2011) introduce a SBM two-stage DEA model to study the performance of banks when bad outputs are present. Bad outputs are also considered by Fukuyama and Weber (2010) that introduce a two-stage model to study Japanese banks' performances. This last model accounts for slacks in the input and output constraints defining the technology, and allows inefficiency to be measured with non-radial contractions in inputs and expansions in outputs, even when slack does not exist. This model is also applied by Fukuyama and Matousek (2011) to assess the efficiency of Turkish bank system. Akther et al. (2013) introduce bad outputs while assessing 19 Bangladesh banks and use a slacks-based inefficiency measure within a two-stages DEA model. Huang et al. (2009) assess the efficiency of Chinese banks with a relational two-stage model. Yang and Liu (2012) prove, by integrating a two-stage DEA model and a fuzzy multiobjective model, that in Taiwan mixed ownership banks are more efficient than the fully state-owned ones. Grigoroudis et al. (2013) present a three-stage DEA model to assess banks in terms of satisfaction, employee appraisal, and business performance. Their paper is also a good introduction to the literature that links operating efficiency and quality of service in the bank sector. Wu and Birge (2012) introduce what they call a two-stage serial-chain merger DEA model to evaluate mortgage banking operations. Premachandra et al. (2012) apply a two-stage model to assess the performance of mutual funds.

15.4.3.2 Production Processes and Supply Chains

Liu and Wang (2009) use a two-stage relational DEA model to assess the efficiency of printed circuit board industry in Taiwan. Lee and Johnson (2011) use a multi-stage DEA model to represent the production processes in the semiconductor manufacturing industry. Saranga and Moser (2010) apply what they call classical two-stage Value Chain DEA models to assess the performances of purchasing and supply management activities. Yang et al. (2011) propose a envelopment-based multi-stage DEA model for assessing the performances of supply chains. These authors state the novelty of their model affirming that, even though there is a rich literature on DEA models for supply chains, the exact definition for supply chain production possibility set is still unclear. For this reason, the authors propose two possible types of supply chain production possibility sets that then they prove equivalent. Mirhedayatian et al. (2013) propose a model for assessing "green" supply chains. Chen et al. (2012a) use two-stage DEA model for evaluating sustainable product design performances. They propose both centralized and decentralized models as in Chen and Yan (2011) to analyze the simultaneous, proactive, and reactive approaches adopted by firms for sustainable design.

Bai-Chen et al. (2012) apply a two-stage model to assess both economic benefits and carbon emissions of China's power plants. The author call their model "environmental" network DEA model as it takes into account environmental factors as non-discretionary inputs.

Cao and Yang (2011) measure the performance of Internet companies, whereas Asai (2011) employs a two-stages DEA model to assess Japanese broadcasters.

15.4.3.3 Transportation

Lu et al. (2012) use a two-stage additive DEA model based on the works of Chen et al. (2009a) and of Cook et al. (2010b) to assess the production and marketing efficiency of airline industry. The authors show that low-cost carriers, on average, are more efficient than the full-services ones from a production perspective, but they are less efficient marketers.

Chang and Yu (2012) also deal with low-cost carriers. Specifically, the authors use a SBM two-stage DEA model to assess production and consumption efficiencies. Yu (2010) adopt a SBM efficiency measure to model an open process and assess both production and service efficiency in airports. In this work, the author points out that environmental factors have an important influence in the performances of transportation systems and hence they must be taken into consideration even if that are beyond managerial control. For these reasons, on one side he models these factors as quasi-fixed/non-discretionary inputs; on the other side, he associates no slack variable to them and consequently he does not include them in the SBM efficiency.

Zhu (2011) applies the centralized model by Liang et al. (2008) to asses the efficiency of a set of airlines. Wanke (2013a) (respectively Wanke (2013b)) applies an analogous model to assess the physical infrastructure and flight (respectively shipment and consolidation) efficiency drivers in Brazilian airports (respectively ports). Adler et al. (2013) use a two-stage DEA model for benchmarking airports taking into account of both terminal and airside activities. These last authors point out how previous benchmarking studies based on standard DEA models may arrive to opposing conclusions, whereas a network DEA structure provide more meaningful benchmarks with comparable peer units and target values that are achievable in the medium term. To reach such results, the authors apply a dynamic clustering approach (Golany and Thore 1997) that, for each DMU_0, restricts the set of possible peers to include only DMUs with similar mixes of flows. The rationale of this choice is to set a target for an inefficient DMU_0 which is accessible in the short to medium term.

15.4.3.4 Sports

Moreno and Lozano (2012) introduce an interesting survey of DEA models to analyze sport performances and then compare the results of a standard DEA model with a generalized two-stage one to assess the efficiency of NBA teams. In both models they use SBM efficiency (Tone and Tsutsui 2009). The authors finally conclude that the two-stage DEA model has more discriminating power and provides more insight than the standard one.

15.4.4 Dynamic Networks

Dynamic networks DEA models are multi-stage models that describe the evolution of processes over time. A recent survey of this network DEA literature sub-area can be found in Fallah-Fini et al. (2013) which review all the literature (including non DEA works) on non-parametric dynamic efficiency measurement.

In the basic version of these models, each stage represents the same DMU, as a black box, at different times. Färe and Grosskopf (2000) consider the same production process in two successive periods/DMSUs with period-specific inputs and outputs. Some of the outputs produced in the first period, that is by the first DMSU, are used as inputs in the second period, that is by the second DMSU (see also Färe and Grosskopf 1996a). The authors model these time-intermediate products as intermediate flows of a (dynamic) network DEA model and, hence, they may evaluate the relative efficiency of the involved process using Model (15.3). An illustration of this kind of dynamic network DEA models can be found in Bogetoft et al. (2009). Another basic dynamic network DEA model is introduced by Troutt et al. (2001) who, strangely enough, do not present appropriate bibliography except for two seminal papers on standard DEA.

Nemoto and Goto (1999) use a dynamic network DEA model to describe the intertemporal behavior of a firm. The authors identify the intertemporal efficient cost frontier using an envelopment-based network DEA model. Their model includes both discretionary and quasi-fixed inputs. Discretionary inputs are period-specific, whereas quasi-fixed inputs are the only time-intermediate flows. Both kinds of inputs are assumed variable (instead of, e.g., being considered constant and possibly multiplied by variable scaling factors θ, as it is customary in standard DEA input oriented models) and a linear combination of them is minimized. In a subsequent work, Nemoto and Goto (2003) apply this model to Japanese electric utilities to show how to evaluate the efficiencies of quasi-fixed inputs and describe their adjustment processes. Sueyoshi and Sekitani (2005) propose the VRS formulation of the this model. Later, also Von Geymueller (2009) applies a variation of this model to assess the efficiency of electricity transmission operations.

Tone and Tsutsui (2010) introduce the slacks-based version of the above network dynamic model. In addition, the authors indicate how to deal with both discretionary and non-discretionary intermediate flows, and point out that these flows must be dealt with differently depending on their desirability. Earnings carried forward are a possible example of desirable intermediate flows; on the contrary, losses carried forward are a possible example of undesirable ones.

Kao (2012) proposes a relational approach for dynamic multi-stage processes and underlines that the previous methods described the literature for calculating the efficiency of these processes may produce over-estimated scores if their dynamic nature is disregarded.

In more complex dynamic models, each stage represents again the same DMU at different times, but now this DMU in turn models a multi-stage process. Chen (2009) introduces such a dynamic network DEA model to represent a production network. Let DMSU_r^k be the generic r-th DMSU of the k-th DMU. The author defines a (dynamic) network, the nodes of which are the subunits $\text{DMSU}_r{}^k$ at the different times t. Then, he assumes that, at each time t, only a fraction of the intermediate output flow of DMSU_r^k is received immediately as intermediate input flow by the successive $\text{DMSU}_{(r+1)}^k$. The complementary fraction of the intermediate output flow is stored and received by $\text{DMSU}_{(r+1)}^k$ in successive times, possibly with some losses if this intermediate flow consists of a perishable material. Tone and Tsutsui (2014) extend these kind of dynamic network DEA models to situations in which SBM efficiency is taken into consideration and the DMUs observed over time model general network process.

Finally, other authors (see, e.g., Chen and van Dalen 2010; Emrouznejad and Thanassoulis 2005; Sengupta 1995) consider dynamic DEA models in order to take into account input flows received at a time period t, e.g., capital, that may have a productive effect not only in the same time period t but also over future time periods. These models, however, usually do not consider time-intermediate flows between DMSUs. As an example, Chen and van Dalen (2010) propose an envelopment-based dynamic DEA model assuming that the input received at a time period t may have a productive effect not only in the same time period but also over a given time horizon of, say, length g. On the basis of this observation, for each time period t, they assess the process performances using an efficiency measure that considers the input flow x^t and a value $\tilde{y}^t$ function of the output flows, for $r = 0, \ldots, g$, produced between t and $t + g$.

15.5 Shared Flow DEA Models

In this section, we deal with DEA models for DMUs that include DMSUs that either share some of their inputs or their outputs. These models assume that the total amount of each input (or output) flow entering (or exiting) the whole DMU is known and a-priori fixed, as it is customary in standard DEA models. However, they also assume that the amount of shared flow allocated to each subunits may be considered as a decision variable to be used to maximize the DMU efficiencies (see Fig. 15.4). Even in this case, the subunits of the DMUs cannot be considered independent since they compete for the allocation of the flows that they share. Beasley (1995) introduces one of the first examples of a shared flow DEA model, even if it was not originally referred to as such. The model is applied to departments

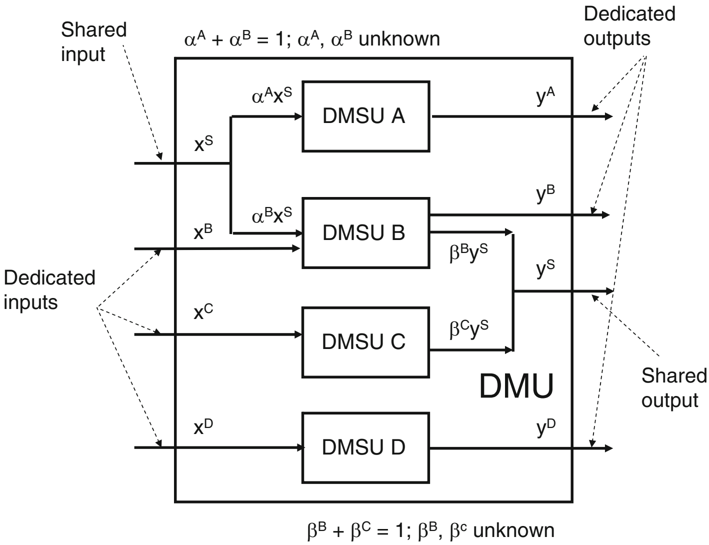

Fig. 15.4 A shared flow DMU: DMSUs A and B are not independent because they compete for the same shared resource. Similarly DMSUs B and C are not independent because of the shared output. DMSU D is independent of DMSUs A, B and C

of different universities devoted to the same disciplines. The departments are homogeneous and independent DMUs. Within each of them, the teaching and research activities clearly define two different separable functions. One of the DMU inputs, research income, is specifically *dedicated* to the research function. The other DMU inputs, general and equipment expenditure, are *shared (joined)* between the two functions. DMU outputs are split, i.e., no shared outputs exist: the number of undergraduates and of taught postgraduates are outputs of the teaching function; the number of research postgraduates, research income, and research rating are outputs of the research function. Kao and Lin (2012) extend this application to the situation in which some input/output data are fuzzy numbers.

15.5.1 Formulation of Shared Flow DEA Models

Referring to r as the generic component of DMU k, now vectors X_k^r, Y_k^r, ν^r, and μ^r introduced in Sect. 15.2 are defined as the vectors of dedicated inputs, dedicated outputs, weights of the dedicated inputs, and weights of dedicated outputs of component r, respectively. In addition, we define

- $X_k^S = \{x_{ik}^S\}$: the vector of shared inputs,
- $Y_k^S = \{y_{jk}^S\}$: the vector of shared outputs,
- $\nu^S = \{\nu_i^S\}$: the vector of weights of shared inputs,

- $\mu^S = \{\mu_j^S\}$: the vector of weights of shared outputs,
- $\alpha^r = \{\alpha_i^r\}$: the vector of proportions of the shared inputs allocated to component r,
- $\beta^r = \{\beta_j^r\}$: the vector of proportions of the shared outputs attributed to component r.

With a little abuse of notation we also define $\alpha^r X_k^S$ as the column vector whose generic entry is $\alpha_i^r x_{ik}^S$. In this context, $\alpha_i^r x_{ik}^S$ is the amount of shared input i allocated to component r by DMU k to maximize its efficiency. When a shared input cannot be clearly divided among functions (e.g., general expenditure), then α_i^r can be seen as the proportion of the (virtual) value of the input i allotted to component r. Similarly, we define $\beta^r Y_k^S$ as the column vector whose generic entry is $\beta_j^r y_{jk}^S$ where β_j^r is always seen as the proportion of the (virtual) value of output j that can be attributed to component r because it is assumed that no component can produce a shared output by itself but needs synergy with other components. As an example, the quality of service level provided by an organization to its customers depends on the degree of collaboration and integration among its subdivisions, each of them sharing with other subunits the responsibility for such output. When outputs common to different components are produced without the need of synergy among them, the literature refers to them as *overlapping* outputs (see Sect. 15.5.2.5 for details).

15.5.1.1 Primal Formulation

Consider, for the sake of simplicity, the case when shared outputs are not present. The efficiency of DMU k is expressed as

$$e_k = \frac{\sum_r \mu^r Y_k^r}{\sum_r \nu^r X_k^r + \sum_r \nu^S(\alpha^r X_k^S)},$$

the *partial* efficiency of the single component r is defined as

$$e_k^r = \frac{\mu^r Y_k^r}{\nu^r X_k^r + \nu^S(\alpha^r X_k^S)},$$

and the *aggregate* efficiency $\hat{e}_k = \sum_r q_k^r e_k^r$ as the weighted combination of the partial efficiencies of its components, where the weight q_k^r of each component r is

$$q_k^r = \frac{\nu^r X_k^r + \nu^S(\alpha^r X_k^S)}{\sum_p \nu^p X_k^p + \sum_p \nu^S(\alpha^p X_k^S)}.$$

Hence q_k^r is the fraction of DMU k total weighted inputs that are consumed by component r: $\sum_r q_k{}^r = 1\ \forall k$. Also Yang et al. (2000) introduced the concept of partial efficiency measures but they applied it on an elementary model (see Sect. 15.2). The general model proposed by Beasley (1995) is

$$e_0^* = \max e_0 \tag{15.11a}$$

$$e_k^r \leq 1 \quad \forall k, r \tag{15.11b}$$

$$\sum_r \alpha_i^r = 1 \quad \forall i \tag{15.11c}$$

$$\nu_i^r, \nu_i^S, \alpha_i^r, \mu_j^r \geq \varepsilon \quad \forall i, j, r. \tag{15.11d}$$

Condition (15.11b) imposes that the partial efficiency of each DMU component cannot exceed 1. Beasley (1995) proves that when each DMU is free to allocate the value of the shared inputs among its different components, the aggregate efficiency $\hat{e}_k$ and the efficiency e_k are coincident when maximized.

As for the standard DEA formulations, model (15.11) can be rewritten as follows

$$e_0^* = \max \sum_r \mu^r Y_0^r \tag{15.12a}$$

$$\sum_r \nu^r X_0^r + \sum_r \nu^S (\alpha^r X_0^S) = 1 \tag{15.12b}$$

$$\mu^r Y_k^r \leq \nu^r X_k^r + \nu^S (\alpha^r X_k^S) \quad \forall k, r \tag{15.12c}$$

$$\sum_r \alpha_i^r = 1 \quad \forall i \tag{15.12d}$$

$$\nu_i^r, \nu_i^S, \alpha_i^r, \mu_j^r \geq \varepsilon \quad \forall i, j, r. \tag{15.12e}$$

Model (15.12) is not linear because of inequalities (15.12b) and (15.12c). When no shared inputs exist, model (15.12) easily reduces to the elementary model (15.2) as $X_k^S = 0\ \forall k$. Hence the terms $\sum_r \nu^S (\alpha^r X_0^S)$ in constraint (15.12b) and $\nu^S (\alpha^r X_k^S)$ in constraint (15.12c), and constraint (15.12d) are no longer necessary.

15.5.1.2 Dual Formulation

Mar Molinero (1996) and Mar Molinero and Tsai (1997) propose an approach dual to model (15.11). In addition, the authors include *shared outputs*, i.e., outputs yielded synergically by two or more components. Their output oriented model for what they call a *multi-activity* process is

$$e_0^* = \max \sum_r q_0^r \theta_0^r + \varepsilon \left(\sum_i \left(s_i^{S-} + \sum_r s_i^{r-} \right) + \sum_j \left(s_j^{S+} + \sum_r s_j^{r+} \right) \right) \tag{15.13a}$$

$$\sum_k \lambda_k^r x_{ik}^r = x_{i0}^r - s_i^{r-} \quad \forall i, r \tag{15.13b}$$

$$\sum_k \sum_r \lambda_k^r (\alpha_i^r x_{ik}^S) = x_{i0}^S - s_i^{S-} \quad \forall i \tag{15.13c}$$

$$\sum_k \lambda_k^r y_{jk}^r = \theta_0^r y_{j0}^r + s_j^{r+} \quad \forall j, r \tag{15.13d}$$

$$\sum_k \sum_r \lambda_k^r (\beta_j^r y_{jk}^S) = \sum_r \theta_0^r (\beta_j^r y_{j0}^S) + s_j^{S+} \quad \forall j \tag{15.13e}$$

$$\sum_r \alpha_i^r = 1 \quad \forall i \tag{15.13f}$$

$$\sum_r \beta_j^r = 1 \quad \forall j \tag{15.13g}$$

$$\sum_r q_0^r = 1 \tag{15.13h}$$

$$\lambda_k^r, q_0^r, \alpha_i^r, \beta_j^r, s_i^{r-}, s_i^{S-}, s_j^{r+}, s_j^{S+} \geq 0 \quad \forall i, j, r, k. \tag{15.13i}$$

where q_0^r are positive weights representing the relative importance of each component r for DMU 0, and θ_0^r are measures of the inefficiencies of the components of DMU 0. Actually, θ_0^r are the reciprocals of the *distance functions* defined by Shephard (1970). Note that in the models proposed by Mar Molinero (1996) and Mar Molinero and Tsai (1997) the slack variables s_i^{r-}, s_i^{S-}, s_j^{r+}, s_j^{S+} are not present. Here they are imposed for coherence with the standard DEA dual models (see, e.g., Cooper et al. 2000).

When the values α_i^r, β_j^r, and q_0^r are not decision variables but are fixed, still satisfying conditions (15.13f), (15.13g) and (15.13i), the dual of model (15.13) is

$$e_0^* = \min \sum_r \nu^r \overset{r}{\underset{0}{X}} + \sum_r \nu^S (\alpha^r X_0^S) \tag{15.14a}$$

$$\mu^r Y_0^r + \mu^S (\beta^r Y_0^S) = q_0^r \quad \forall r \tag{15.14b}$$

$$\mu^r Y_k^r + \mu^S (\beta^r Y_k^S) \leq \nu^r X_k^r + \nu^S (\alpha^r X_k^S) \quad \forall k, r \tag{15.14c}$$

$$\nu_i^r, \nu_i^S, \mu_j^r, \mu_j^S \geq \varepsilon \quad \forall i, j, r. \tag{15.14d}$$

The above model parallels the output oriented version of model (15.12) when shared outputs are considered. Besides model (15.14) being linear, the main difference between the two models is the presence of the multiple constraints (15.14b) instead of the single one $\sum\limits_r \mu^r Y_0^r + \sum\limits_r \mu^S (\beta^r Y_0^S) = 1$. This latter constraint is a relaxation of the former ones because $\sum_r q_0^r = 1$. Conditions (15.14b) state a precise relationship between the relative importance attributed to a component and the optimal amount of outputs allocated to it (respectively, the optimal amount of allocated inputs if an input oriented model is considered). Then, conditions (15.14b) justify the choice in Beasley (1995) of expressing the weight q_k^r of the component r in the aggregated efficiency as equal to the fraction of DMU k total weighted inputs that are consumed by component r. Without conditions (15.14b), such a choice might appear arbitrary, although reasonable.

15.5.2 Extensions

Many authors have extended models (15.12) and (15.13). Common features of the different variants are that the aggregate efficiency of a DMU cannot exceed unity, and that a DMU is efficient if and only if it is efficient in all its components. In this section, we describe the peculiarity of each available modeling advance.

15.5.2.1 Weight Restrictions

Beasley (1995) himself does not present model (15.11), but he incorporates the additional constraints

$$(\nu^S, \nu^r, \forall r) \in \Omega_{in} \tag{15.15}$$

$$(\mu^r, \forall r) \in \Omega_{out} \tag{15.16}$$

where the sets Ω_{in} and Ω_{out} are *assurance regions* as defined in Thompson et al. (1990). Constraints (15.15) and (15.16) involve value judgements concerning the proportions α^r and the weights μ^r, and ν^r of the different DMU components. They are not strictly necessary for the definition of a shared flow DEA model, but might prevent the model from yielding unreasonable results. In this context, Beasley (1995) provides an example where, in the absence of constraints (15.15) and (15.16), one research postgraduate was worth about 880,000 undergraduates for a given department.

Assurance regions are also introduced by Yu (2012) to measure the performance of two-division international tourist hotels in Taiwan, which exhibit both shared inputs and shared outputs.

15.5.2.2 Variable Returns to Scale

Mar Molinero and Tsai (1997) prove that the feasible solutions of model (15.13) define a convex set and the objective (15.13a) is a convex function. Tsai and Mar Molinero (2002), considering the problem of assessing the performances of individual specialties of National Health Services Trusts in the UK, introduce and discuss a variable returns to scale version of model (15.13). The efficiency of each component r of DMU k is then defined as

$$e_k^r = \frac{\mu^r Y_k^r + \mu^S (\beta^r Y_k^S)}{\nu^r X_k^r + \nu^S (\alpha^r X_k^S) + \delta_k^r} \tag{15.17}$$

where the variable δ_k^r is unrestricted and its optimal value defines the component's returns to scale status. The aggregate efficiency of DMU k is

$$e^r = \frac{\sum_r \mu^r Y_k^r + \sum_r \mu^S (\beta^r Y_k^S)}{\sum_p \nu^p X_k^p + \sum_p \nu^S (\alpha^p X_k^S) + \sum_p \delta_k^p}. \tag{15.18}$$

Note that the optimal value of $\sum_p \delta_k^p$ may be zero even if some or all elements in the sum are different from zero. In this case, DMU k may appear to be operating under constant returns to scale and technically efficient when analyzed as a black box but, when its individual components are analyzed, it may be found scale inefficient in each of its activities (Tsai and Mar Molinero 2002). It follows that a DMU, that is efficient when considered as a black box, may be inefficient when its different components are taken into account, independently of its returns to scale status.

Variable returns to scale are also considered by Diez-Ticio and Mancebon (2002) to assess the efficiency of Spanish Police Service.

15.5.2.3 Different Weights on Shared Inputs

Cook et al. (2000) allow a same shared input i to be weighted differently by the subunits of the same DMU. The rationale behind such a choice is that different components may disagree on the importance of a same input. Consequently, the shared flow model as in Cook et al. (2000) includes in constraints (15.12b) and (15.12c) a set of vectors ν^{Sr}, one for each component r, instead of a single one. Also, a change of variables is proposed. In particular, let $i = 1, \ldots, s$ be the index of the shared inputs, then $\bar{\nu}_i^{Sr} = \nu_i^{Sr} \alpha_i^r$ for $i = 1, \ldots, s-1$ and $\bar{\nu}_s^{Sr} = \nu_s^{Sr} \left(1 - \sum_{i=1}^{s-1} \alpha_i^r\right)$. Because of these new variables, the authors obtain a linear model. The terms ${\nu_r}^S (\alpha^r {X_k}^S)$ in conditions (15.12b) and (15.12c) become $\bar{\nu}^{Sr} X_k^S$ and $\nu_r^S \geq \varepsilon$ in constraint (15.12e) turns $\bar{\nu}_i^{Sr} \geq \varepsilon \alpha_i^r$. Unfortunately, non-linearity may arise again when additional constraints concerning value judgements as constraints (15.15) and (15.16) are necessary. If such judgements are expressed also in terms of ν_r^S, the variable substitution may not lead to a linear model.

15.5.2.4 Additive Objective Function

Cook and Hababou (2001) and Cook and Zhu (2005, Chap. 6) present variables and constraints as in Cook et al. (2000) but differ in the objective function. They formulate an additive objective function representing an aggregate measure of the efficiencies of all the DMU subunits. In the classical additive DEA models (Charnes et al. 1985), a possible measure of the inefficiency of DMU k is given by the difference between the weighted sum of the inputs minus the weighted outputs of DMU k. Here Cook and Hababou (2001) suggest a multiobjective approach where the partial inefficiencies of all components are considered. For each subunit, the weighted sum of its inputs minus the weighted sum of its outputs is considered. In particular, the authors minimize the maximum partial inefficiency in order to give equal importance to each component, i.e., their objective function is

$$\min \max\{\nu^r X_k^r + \nu^{Sr}(\alpha^r X_k^S) - \mu^r Y_k^r : \quad \forall r \text{ subunit of } DMU_0\}. \tag{15.19}$$

Finally, the authors linearize their model with the same variable changes proposed in Cook et al. (2000).

15.5.2.5 Overlapping Outputs

Cook and Green (2004) deal with a manufacturing multi-plant company and point out that some outputs of different components of the same DMU can partially overlap, i.e., some outputs may be common to different components. In particular, each DMSU can yield a given amount of overlapping output j, with no need of synergy with the other components. Hence there is no possibility of attributing the considered amount to the other subunits. From this point of view, the overlapping outputs are different from the shared outputs considered in Mar Molinero and Tsai (1997) and Mar Molinero (1996). In fact, Cook and Green (2004) cannot approach what they call the *overlap problem* by introducing variables β^r as in model (15.13) to determine which proportions of shared outputs are attributed to each component: the efficiency of a single subunit r remains $e_k^r = \frac{\mu^r Y_k^r}{\nu^r X_k^r + \nu^{Sr}(\alpha^r X_k^S)}$ and, consequently, the aggregate efficiency of a whole DMU k is $e_k = \frac{\sum_r \mu^r Y_k^r}{\sum_r \nu^r X_k^r + \sum_r \nu^{Sr}(\alpha^r X_k^S)}$. However, the shared inputs are no longer allocated to the components because such task could hardly be performed without introducing some ambiguities due to the component overlapping. Shared inputs are allocated directly to the outputs. In particular, consider model (15.11) and the extension proposed in Cook et al. (2000). Cook and Green (2004) introduce a new set of variables α_i^j as the proportions of the shared inputs i allocated for outputs j. In addition, they replaced condition (15.11c) with $\sum_j \alpha_i^j = 1$, for all i. Finally, they defined α_i^r as $\alpha_i^r = \sum_{i \in O^r} \alpha_i^j$, where O^r is the set outputs of subunit r. Note that now, in general, $\sum_r \alpha_i^r \geq 1$.

The allocation of shared inputs directly to outputs was originally introduced in Färe et al. (1997). Even though the concept of DMSU is not explicitly mentioned, it can easily be inferred since one input can be allocated among various outputs.

15.5.2.6 Core Business Identification

Cook and Green (2004) and Cook and Zhu (2005, Chap. 11) address the problem of determining in which areas a DMU would perform better. Such areas form the *core business* of a DMU and should be privileged even at the cost of possibly forcing the DMU to abandon the components with less satisfactory performances. To this aim, Cook and Green (2004) modify the objective function of model (15.11) and add assignment constraints (each DMU must have at least one component assigned and each component must be assigned to at least one DMU).

To overcome the insufficiency associated with the black box approach that generally makes DMU inner data not to be available, Bi et al. (2012) consider DMUs with parallel structure and propose to divide the production activities within a DMU into two subsets or units. The first unit is termed as the core business unit (CBU), which includes the main production functions of DMU; the second unit is referred to as the non-core business unit (NCBU). The authors introduce a solution method that assumes that the information related to inner inputs/outputs is available for the DMU under evaluation. For the other DMUs, however, these data are generated by using the Pareto principle: as a rule of thumb, the CBU produces 80 % of total outputs of a DMU, while consumes only 20 % of total inputs. Accordingly, NCBU produces 20 % of the total outputs, while consumes 80 % of all inputs.

15.5.2.7 Resource Allocation

Shared flow models have also been used to allocate input costs among different subunits or activities. da Cruz et al. (2013) propose a model for estimating not only the overall efficiency of water utilities, but also the cost efficiency of drinking water and wastewater services. Using a shared input DEA methodology, the authors are also able to report estimates for the cost shares that correspond to each service. Similarly, Rogge and De Jaeger (2012) evaluate the cost efficiency of Flemish municipalities in the collection and processing of municipal solid waste by considering only one input ("waste cost") that is shared among different collection and treatment activities. In Rogge and De Jaeger (2013) this shared input DEA-model is further developed to make the partial and aggregate cost efficiency scores robust and also corrected for the impact of influences related to the operating environment and long-term policy variables. The same robust shared-input DEA approach has been applied by Broekel et al. (2013) to evaluate for multiple years the innovation efficiency of 150 German market labor regions, using as unique input "R&D employment" figures. Input costs are jointly allocated also by Salerno (2006) to estimate higher education institutions' per-student education costs in The Netherlands.

Barnum et al. (2011) introduce a DEA-based procedure for estimating the overall efficiency of metropolitan public transportation agencies in the United States. Specifically, the authors use a six-step shared flow model to allocate operating expenses among the agencies' organizational subunits that supply transit service. In each of the main step the authors use DEA to asses either the efficiency of the whole system or of each of the transportation modality. Finally, the sixth step, which involves a non-DEA mathematical program, estimates how inputs should be allocated among the target agency's subunits in order to minimize total expenses, while holding output constant.

15.5.2.8 Non-radial Measures of Efficiency

Chen et al. (2013a) describes an empirical study on Taiwan's farmers' cooperatives to offer policy suggestions as to how fixed resources can be effectively reallocated among different departments in a team production environment. The authors adopt Luenberger (1992)'s directional distance function to scale inputs and outputs, but not necessarily along the rays from the input and output origin (Fukuyama 2003). In such a way, the optimal input/output adjustment and the optimal allocation of shared inputs among different activities are taken into consideration simultaneously. Furthermore, the use of a directional distance function allows to easily incorporate an undesirable/bad output as a byproduct of desirable/good production activities. In fact, when we seek a reduction in the bad output and simultaneous increases in the good output, then the directional distance function will be a preferred method because it allows non-proportional adjustments of the good and bad outputs.

Yu and Lee (2009) use instead a *hyperbolic* network DEA model to evaluate the performances of hotels in Taiwan. Specifically, the authors extend the models introduced in Färe and Grosskopf (2000) and Färe and Whittaker (1995) by combining both the input and the output orientation in a non-linear fashion.

15.5.2.9 Two-Stage Networks

In the recent years, the integration between network and shared flow models has been addressed by some authors. Chen et al. (2010b) propose a DEA model to evaluate either the VRS or the CRS efficiency of a two-stage network process where some inputs are directly associated with both stages or shared by the two stages (Fig. 15.5). The DMU efficiency is computed as a convex combination of efficiency scores of the first and second stage, thus ensuring that a DMU is overall efficient if and only if each stage is efficient. In the case of an inefficient DMU, however, the decomposition of the overall DMU efficiency between the two stages may not be unique. Hence, following Kao and Hwang (2008), the authors propose, under both VRS and CRS, an approach to find a set of multipliers that maximize either the first or the second stage efficiency score while maintaining the overall efficiency score.

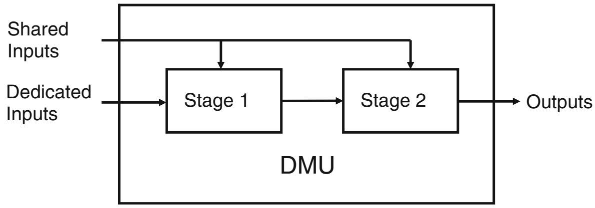

Fig. 15.5 Shared inputs in a two-stage network process

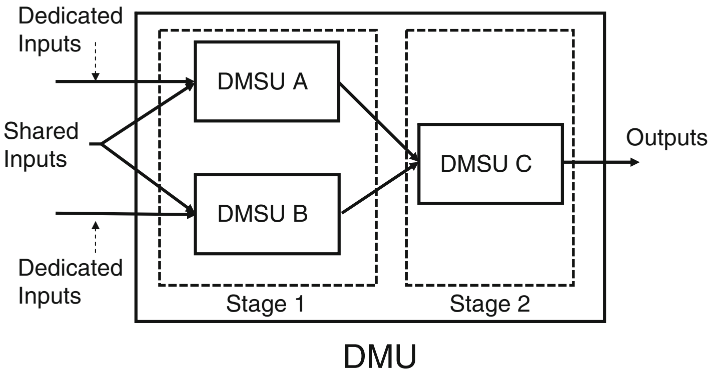

Fig. 15.6 Shared inputs in a open two-stage network process

Zha and Liang (2010) analyze the two-stage network process with shared inputs as in Fig. 15.5. Differently from Chen et al. (2010b), the authors propose to determine the overall DMU efficiency score as the product of the efficiency scores of the two stages, thus optimizing the overall efficiency through cooperation of the different stages, as suggested by Castelli et al. (2004).

To assess the efficiency of multimode bus transit systems, Yu and Fan (2006) introduce a two-stage shared input DEA model that incorporates both desirable and undesirable outputs, and also environmental (non-discretionary) inputs. Following Yu and Fan (2006) and Yu (2008b), a two-stage network with shared inputs between two parallel subunits of the first stage (see Fig. 15.6) has been proposed by Yu and Fan (2009) to simultaneously estimate the production efficiency, service effectiveness and operational effectiveness of Taiwan's bus transit system. Their network model, also called mixed structure network DEA model, extends both the network DEA model introduced by Färe and Grosskopf (2000) (series structure network) and the network DEA model developed by Mar Molinero (1996) (parallel structure network).

Chen et al. (2010b) show that their approach can be easily extended to open two-stage network processes where some inputs from the first stage do not become inputs to the second stage, and the second stage has its own inputs (Fig. 15.7).

Amirteimoori (2013) addresses the same two-stage network process with shared inputs as in Fig. 15.7 using the approach of Chen et al. (2010b), with the only

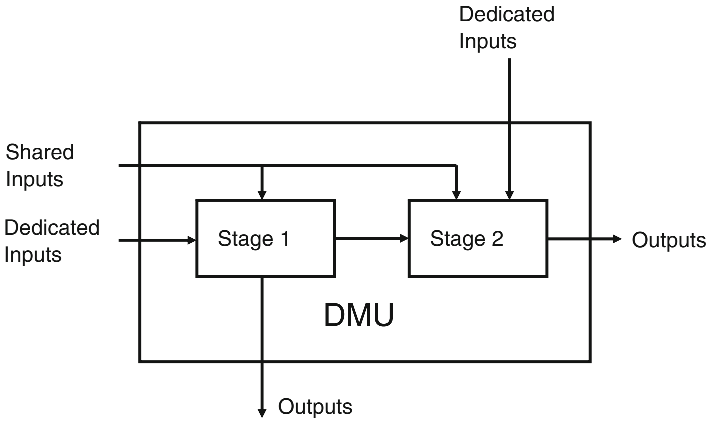

Fig. 15.7 Shared inputs in a two-stage network process with parallel subunits

difference that the intermediate flows are to be considered as undesirable outputs for the first stage. In the same two-stage setting, undesirable intermediate flows were earlier addressed by Yang (2009) who moreover simultaneously considers both DMU desirable and undesirable outputs to measure productive and environmental efficiency in farrow-to-finish pig production in Taiwan. Similarly, also Chen et al. (2012b) evaluate the relative performance of incineration plants in Taiwan by including desirable and undesirable outputs. To allow inputs and outputs to change non-proportionally the directional slacks-based inefficiency measure developed by Fukuyama and Weber (2009) is incorporated into their model (see also Sect. 15.5.2.8).

Two-Stage Network and Non-radial Measures of Efficiency

Sometimes the technology used to measure DMU efficiency has to deal with input excesses and output shortfalls simultaneously. In this case, the graph-oriented DEA model can be applied (Färe et al. 1985). In contrast to input-oriented and output-oriented DEA models, both inputs and outputs are allowed to vary by the same (or different) proportion, but inputs are proportionately decreased while outputs are simultaneously increased by the same (or different) proportion. Graph efficiency measurement has been used by Yu and Lin (2008) who present a multi-activity network DEA model to simultaneously estimate passenger and freight technical efficiency, service effectiveness, and technical effectiveness for 20 selected railways for the year 2002. This model extends the work from Mar Molinero and Tsai (1997). In particular, it generalizes model (15.13) for multi-stage processes and uses an objective function that penalizes all the external input and final output inefficiencies of all the components, with the exception of non-discretionary inputs (while the θ_0^r terms in model (15.13) penalize only the final outputs). For standard DEA models similar measures were proposed in Pastor et al. (1999).

Similarly, a graph-oriented DEA model is proposed by Chao et al. (2010) who apply the multi-activity DEA model to explore the relative efficiency of 12 financial holding companies in Taiwan.

Two-Stage and Dynamic Networks

Chen (2012) proposes a dynamic shared input DEA model to assess the efficiency of the swine production in Taiwan. The model is dynamic in the sense that a same DMU, made of two parallel DMSUs with shared inputs, is observed over time. Hence, some of the outputs of a period become some of the inputs of the following period. Efficiency is not measured radially. Instead, as in Chen et al. (2012b, 2013a), the directional Russell measure of slack-based inefficiency developed by Fukuyama and Weber (2009) is introduced to allow inputs and outputs with non-proportional changes.

15.6 Multi-level DEA Models

In this section, we deal with DEA models for DMUs exhibiting autonomous activities that cannot be associated to any of their subunits. In other words, these DMU present additional inputs/outputs not considered by their DMSUs. For example, in Cook et al. (1998), DMSUs are highway maintenance patrols and DMUs are the districts in which the maintenance patrols are grouped. The subunits have traffic and road conditions as possible inputs, while DMUs may include additional inputs that can be applied only to districts such as the extent of privatization and district engineers' experience. The same authors also introduced possible applications of their model to power plants and hospitals. These models are defined as *multi-level models* (Cook et al. 1998) where the top level, referred to as level n DMU, includes independent and homogeneous subunits, referred to as level $n-1$ DMUs. Recursively, the level $n-1$ DMUs include smaller independent and homogeneous subunits, level $n-2$ DMUs, and so on. Unlike shared flow models, the amount of input and output of each subunit is fixed. In this work, we introduce only two–level structures, and we simply refer to DMU for the level 2 DMU and to DMSU or subunit for level 1 DMUs (see Fig. 15.8).

By denoting i; j; k as the indexes of the generic input, output, and DMU, respectively, the following notation is introduced:

- $R_k = \{r_k\}$: the set of indexes r_k of all DMSUs belonging to DMU k,
- $X_k^r = \{x_{ik}^r\}$: the vector of the inputs of DMSU r_k,
- $X_k = \{x_{ik}\}$: the vector of the additional inputs of DMU k,
- $Y_k^r = \{y_{jk}^r\}$: the vector of the outputs of DMSU r_k,

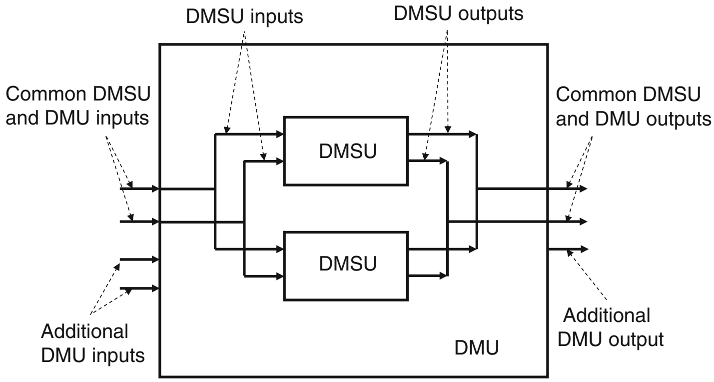

Fig. 15.8 A multi-level DMU: the DMU includes two homogeneous and independent subunits

- $Y_k = \{y_{jk}\}$: the vector of the additional outputs of DMU k,
- $\nu^1 = \{\nu_i^1\}$: the vector of weights of the inputs common to both DMSUs and DMUs,
- $\nu^2 = \{\nu_i^2\}$: the vector of weights of the additional inputs of DMUs,
- $\mu^1 = \{\mu_i^1\}$: the vector of weights of the outputs common to both DMSUs and DMUs,
- $\mu^2 = \{\mu_i^2\}$: the vector of weights of the additional outputs of DMUs.

Accordingly, the efficiency of a DMSU r_k is expressed as

$$e_k^r = \frac{\mu^1 Y_k^r}{\nu^1 X_k^r} \tag{15.20}$$

and the efficiency of a DMU k as

$$e_k = \frac{\mu^1 \sum\limits_{r_k \in R_k} Y_k^r + \mu^2 Y_k}{\nu^1 \sum\limits_{r_k \in R_k} X_k^r + \nu^2 X_k}. \tag{15.21}$$

Cook et al. (1998) present a unifying model for multi-level structures that assesses the efficiency of DMUs of different levels. The authors argue that the efficiency of a DMSU r_k should be evaluated only relative to those other subunits operating under the same conditions, in practice belonging to the same DMU k.

On the other hand, they also assert that the subunits in R_k should be taken into account when evaluating the efficiency of a DMU k. On the basis of these assumptions, Cook et al. (1998) propose that the efficiency of a DMSU 0_0 in DMU 0 is evaluated through the following model

$$e_0^{0*} = \max \mu^1 Y_0^0 \tag{15.22a}$$

$$\nu^1 X_0^0 = 1 \tag{15.22b}$$

$$\mu^1 Y_0^r \leq \nu^1 X_0^r \quad \forall r_0 \in R_0 \tag{15.22c}$$

$$\nu_i^1, \mu_j^1 \geq \varepsilon \quad \forall i, j. \tag{15.22d}$$

This is a standard DEA model that evaluates DMSU 0_0 relative only to subunits included in the same DMU 0. The efficiency of a DMU 0 is evaluated through the following model:

$$e_0^* = \max \mu^1 \sum_{r \in R_0} Y_0^r + \mu^2 Y_0 \tag{15.23a}$$

$$\nu^1 \sum_{r \in R_0} X_0^r + \nu^2 X_0 = 1 \tag{15.23b}$$

$$\mu^1 \sum_{r \in R_k} Y_k^r + \mu^2 Y_k \leq \nu^1 \sum_{r \in R_k} X_k^r + \nu^2 X_k \quad \forall k \tag{15.23c}$$

$$\mu^1 Y_k^r \leq \nu^1 X_k^r \quad \forall k, \forall r \in R_k \tag{15.23d}$$

$$\nu_i^2, \nu_i^1, \mu_j^2, \mu_j^1 \geq \varepsilon \quad \forall i, j. \tag{15.23e}$$

This model compares DMU 0 with all other DMUs. It is different from the linear programming model considering DMUs as black boxes due to the presence of constraints (15.23d). These constraints take into account the DMU internal structure by imposing that their efficiency is related to the efficiencies of their subunits. In particular, constraints (15.23d) force that the optimal values for weights ν_i^1, μ_j^1 are feasible for the DMSUs, i.e., the efficiency of each subunit should not exceed unity. Cook et al. (1998) present a unifying model for multi-level structures that includes both models (15.22) and (15.23). When the DMUs do not have additional inputs/outputs, model (15.23) reduces to the elementary model (15.2). In such case, constraints (15.23c) turn out to be redundant since they are implied by constraints (15.23d). Cook and Green (2005) apply the hierarchical model described in Cook et al. (1998) to the evaluation of power plants. These works are continued by Azadeh et al. (2008, 2011) who use hierarchical models for optimal location of solar plants and wind plants, respectively.

15.6.1 Comparing Subunits Belonging to Different DMUs

In model (15.22) any subunit is compared only against the other subunits belonging to the same DMU. The rationale is that inputs received and decisions taken by each DMU influence the efficiency of its subunits, then comparing subunits belonging to different DMUs would be questionable. In fact, DEA models assess the efficiency of a DMU as a function of its distance from the production frontier defined by the other observed DMUs. In a mathematical programming perspective, DEA models determine the efficiency of a DMU with respect to the other DMUs. In an econometric perspective, the observed DMUs are a sample of a larger population, and DEA is a biased estimator of the efficiency of a DMU with respect to the unknown real production set (Simar and Wilson 2000). In both situations, the larger the sample is, the more likely the DMU under assessment is inefficient. Also, the average efficiency of the DMUs of the sample decreases (Zhang and Bartles 1998). This is why Staat (2001) invites to interpret very carefully possible differences in the efficiencies of subunits belonging to different DMUs when the cardinalities of sets R_k vary. Cook et al. (1998) propose a way of correcting the possible biases by adjusting the efficiency of subunit r_k taking into account the size of the DMU k, the average efficiency of all the subunits in R_k, and the efficiency of DMU k. However, Staat (2002) points out that such a procedure returns different corrections for samples of equal size. He suggests to use bootstrap techniques (see, e.g., Simar and Wilson 2000) to overcome such deficiencies.

In a later paper Cook and Zhu (2007), always dealing with power plants, propose a different model to rectify the weaknesses in the one of Cook et al. (1998). In the new model, the efficiency of each DMSU is now assessed against all the other subunits even if they do not belong to the same DMU. Then, for each DMU, a common set of multipliers applicable to all its DMSUs is determined. Specifically, goal programming is used to identify the multipliers that minimize the maximum discrepancy among the DMSUs' efficiencies from their ideal levels computed in the previous step.

15.6.2 Shared and Multi-level Models

Wu et al. (2008) evaluate the efficiency and performance of the healthcare system in 23 counties and cities in Taiwan for the year 2003. In this paper, the authors propose that each county or city has a budget to produce all the different outputs that can be optimally distributed between such outputs. Hence they propose an input-shared flow model with respect to the available budget. However, Wu et al. (2008) also consider additional inputs (e.g., healthcare manpower and number of facilities) that are not explicitly linked to the different outputs, as in Fig. 15.8. It is then a multi-level model where the amount of input of each subunit is not fixed.

Let us conclude this section underlining that there exists an other DEA literature sub-area called "multi-level". The works in this sub-area deal with the presence of too many input or output flows. Then, they aggregate them in different groups and subgroups (see, e.g., Meng et al. 2008; Kao 2008; Eilat et al. 2008; Rezai and Davoodi 2011). We do not survey these works as they do non assume that DMUs present any internal structure.

15.7 Conclusions

In this work we provide a classification of the main DEA models assessing the efficiency of Decision Making Units when their internal structure is no longer considered as a black box, but insight on their inner processes is available. The interaction in each DMU among the input and output flows and its subunits identifies three broad categories of models. In particular, network DEA models are introduced when intermediate flows among the subunits are taken into account. Shared flow DEA models apply when it is possible to partition a DMU as a collection of components that contend their inputs and/or outputs to other components of the same DMU. Multi-level DEA models are referred to when some of the inputs (or outputs) of a DMU are also inputs (or outputs) of its subunits, and some other inputs (or outputs) are not. We show that these formulations are different generalizations of the same elementary model.

From a theoretical point of view, the knowledge of the internal structure of DMUs should spot the sources of organizational inefficiency by, e.g., preventing compensations among the subunits. In mathematical terms this translates into linking a DMU and its subunits' efficiencies. This relationship may vary across the different models. But, as a general result, a DMU cannot be efficient if none of its subunits are efficient. Furthermore, several applications show that the discrimination power of a DEA model which considers the internal structure of the DMU always increases with respect to the black box approach. As an extreme case, in some situations all DMUs may turn out to be inefficient.

There is large scope of research in the area of this type of DEA models both from a theoretical and application-oriented perspective. In the standard DEA literature, besides the original DEA formulation (Charnes et al. 1978) representing DMUs as black boxes in a CRS environment, many authors have proposed more sophisticated or alternative approaches taking into account, e.g., nonradial measures of efficiency, value judgments, economic measures of efficiency (see Fried et al. 2008, Chap. 3, for a comprehensive survey of such DEA models).

In the recent years, different works have devoted attention to these extensions even when DMUs have an internal structure. However, as pointed out by Chen et al. (2013b), issues still remain and need to be addressed. The presence of an internal structure in fact prevents from generlaizing some of the more obvious properties of the standard DEA models, as an example, the duality relationship between the multiplier-and envelopment-based DEA models. In the authors' opinion, other difficulties may also arise from the level of detail used to describe the internal structure of DMUs. In fact, the greater the detail of the internal structure of a DMU, the greater the discrimination power of a DEA model, but also usually the more difficult to find a sufficient number of homogeneous DMUs to compare. Another promising line of research considers the DEA models from a game theoretic or, in any case, multi-agent perspective. Indeed, from an applicative point of view, these DEA models may find application in the design of more efficient complex processes. In this context, the role of asymmetric information between DMSUs and their DMU (see, e.g., Bogetoft 2000) can be extended to the case when such asymmetry exists among DMSUs which make their decisions by means of a negotiation process.

References

Adler, N., Liebert, V., & Yazhemsky, E. (2013). Benchmarking airports from a managerial perspective. *Omega–International Journal of Management Science, 41*(2), 442–458.

Agrell, P. J., & Hatami-Marbini, A. (2013, to appear). Frontier-based performance analysis models for supply chain management: State of the art and research directions. *Computers & Industrial Engineering*. doi:10.1016/j.cie.2013.02.014.

Akther, S., Fukuyama, H., & Weber, W. L. (2013). Estimating two-stage network Slacks-based inefficiency: An application to Bangladesh banking. *Omega–International Journal of Management Science, 41*(1), 88–96.

Amatatsu, H., Ueda, T., & Amatatsu, Y. (2012). Efficiency and returns-to-scale of local governments. *Journal of the Operational Research Society, 63*(3), 299–305.

Amirteimoori, A. (2013). A DEA two-stage decision processes with shared resources. *Central European Journal of Operational Research, 21*(1), 141–151.

Asai, S. (2011). Efficiency of Japanese local broadcasters. *Journal of Media Economics, 24*(3), 158–173.

Avkiran, N. K. (2009). Opening the black box of efficiency analysis: An illustration with UAE banks. *Omega–International Journal of Management Science, 37*(4), 930–941.

Avkiran, N. K., & McCrystal, A. (2012). Sensitivity analysis of network dea: NSBM versus NRAM. *Applied Mathematics and Computation, 218*(22), 11226–11239.

Azadeh, A., Ghaderi, S., & Maghsoudi, A. (2008). Location optimization of solar plants by an integrated hierarchical DEA PCA approach. *Energy Policy, 36*(10), 3993–4004.

Azadeh, A., Ghaderi, S., & Nasrollahi, M. (2011). Location optimization of wind plants in Iran by an integrated hierarchical data envelopment analysis. *Renewable Energy, 36*, 1621–1631.

Bai-Chen, X., Ying, F., & Qian-Qian, Q. (2012). Does generation form influence environmental efficiency performance? An analysis of China's power system. *Applied Energy, 96*, 261–271.

Banker, R., Charnes, A., & Cooper, W. (1984). Some models for estimating technical and scale inefficiencies in DEA. *Management Science, 30*(9), 1078–1092.

Barnum, D. T., Karlaftis, M. G., & Tandon, S. (2011). Improving the efficiency of metropolitan area transit by joint analysis of its multiple providers. *Transportation Research Part E: Logistics and Transportation Review, 47*(6), 1160–1176.

Beasley, J. (1995). Determining teaching and research efficiencies. *Journal of the Operational Research Society, 46*(4), 441–452.

Bi, G., Feng, C., Ding, J., & Khan, M. R. (2012). Estimating relative efficiency of DMU: Pareto principle and Monte Carlo oriented DEA approach. *INFOR, 50*(1), 44–57.

Bogetoft, P. (2000). DEA and activity planning under asymmetric information. *Journal of Productivity Analysis, 13*(1), 7–48.

Bogetoft, P., Färe, R., Grosskopf, S., Hayes, K., & Taylor, L. (2009). Dynamic network DEA: An illustration. *Journal of the Operations Research Society of Japan, 52*(2), 147–162.

Broekel, T., Rogge, N., & Brenner, T. (2013). *The innovation efficiency of German regions – A shared-input DEA approach*. Technical report, Philipps-Universitët Marburg.

Cao, X., & Yang, F. (2011). Measuring the performance of internet companies using a two-stage data envelopment analysis model. *Enterprise Information Systems, 5*(2), 207–217.

Castelli, L., Pesenti, R., & Ukovich, W. (2001). DEA–like models for efficiency evaluations of specialized and interdependent units. *European Journal of Operational Research, 132*(2), 274–286.

Castelli, L., Pesenti, R., & Ukovich, W. (2004). DEA–like models for the efficiency evaluation of hierarchically structured units. *European Journal of Operational Research, 154*(2), 465–476.

Castelli, L., Pesenti, R., & Ukovich, W. (2010). A classification of DEA models when the internal structure of the Decision Making Units is considered. *Annals of Operations Research, 173*(1), 207–235.

Chang, Y.-C., & Yu, M.-M. (2012, to appear). Measuring production and consumption efficiencies using the slack-based measure network data envelopment analysis approach: The case of low-cost carriers. *Journal of Advanced Transportation*. doi:10.1002/atr.198.

Chang, T.-S., Tone, K., & Wei, Q. (2011, to appear). Ownership-specified network DEA models. *Annals of Operations Research*. doi:10.1007/s10479-011-0949-5.

Chao, C.-M., Yu, M.-M., & Chen, M.-C. (2010). Measuring the performance of financial holding companies. *The Service Industries Journal, 30*(6), 811–829.

Charnes, A., Cooper, W., & Rhodes, E. (1978). Measuring the efficiency of decision making units. *European Journal of Operational Research, 2*(6), 429–444.

Charnes, A., Cooper, W., Golany, B., Seiford, L., & Stutz, J. (1985). Foundation of data envelopment analysis for Pareto-Koopmans efficient empirical production functions. *Journal of Econometrics, 30*(12), 91–107.

Chen, C.-M. (2009). A network-DEA model with new efficiency measures to incorporate the dynamic effect in production networks. *European Journal of Operational Research, 194*(3), 687–699.

Chen, P.-C. (2012). Measurement of technical efficiency in farrow-to-finish swine production using multi-activity network data envelopment analysis: Evidence from Taiwan. *Journal of Productivity Analysis, 38*, 319–331.

Chen, K., & Guan, J. (2012). Measuring the efficiency of China's regional Innovation systems: Application of network data envelopment analysis (DEA). *Regional Studies, 46*(3), 355–377.

Chen, C.-M., & van Dalen, J. (2010). Measuring dynamic efficiency: Theories and an integrated methodology. *European Journal of Operational Research, 203*(3), 749–760.

Chen, C., & Yan, H. (2011). Network DEA model for supply chain performance evaluation. *European Journal of Operational Research, 213*(1), 147–155.

Chen, Y., & Zhu, J. (2004). Measuring information technology's indirect impact on firm performance. *Information Technology and Management, 5*(1–2), 9–22.

Chen, Y., Liang, L., & Yang, F. (2006). A DEA game model approach to supply chain efficiency. *Annals of Operations Research, 145*(1), 5–13.

Chen, Y., Cook, W. D., Ning, L., & Zhu, J. (2009a). Additive efficiency decomposition in two-stage DEA. *European Journal of Operational Research, 196*(3), 1170–1176.

Chen, Y., Liang, L., & Zhu, J. (2009b). Equivalence in two-stage DEA approaches. *European Journal of Operational Research, 193*(2), 600–604.

Chen, Y., Cook, W. D., & Zhu, J. (2010a). Deriving the DEA frontier for two-stage processes. *European Journal of Operational Research, 202*(1), 138–142.

Chen, Y., Du, J., Sherman, H. D., & Zhu, J. (2010b). DEA model with shared resources and efficiency decomposition. *European Journal of Operational Research, 207*(1), 339–349.

Chen, C., Zhu, J., Yu, J.-Y., & Noori, H. (2012a). A new methodology for evaluating sustainable product design performance with two-stage network data envelopment analysis. *European Journal of Operational Research, 221*(2), 348–359.

Chen, P.-C., Chang, C.-C., Yu, M.-M., & Hsu, S.-H. (2012b). Performance measurement for incineration plants using multi-activity network data envelopment analysis: The case of Taiwan. *Journal of Environmental Management, 93*(1), 95–103.

Chen, P.-C., Hsu, S.-H., Chang, C.-C., & Yu, M.-M. (2013a). Efficiency measurements in multi-activity data envelopment analysis with shared inputs: An application to Farmers' Organizations in Taiwan. *China Agricultural Economic Review, 5*(1), 24–42.

Chen, Y., Cook, W. D., Kao, C., & Zhu, J. (2013b). Network DEA pitfalls: Divisional efficiency and frontier projection under general network structures. *European Journal of Operational Research, 226*, 507–515.

Chilingerian, J., & Sherman, H. D. (2004). Health care applications: From hospitals to physician, from productive efficiency to quality frontiers. In W. W. Cooper, L. M. Seiford, & J. Zhu (Eds.), *Handbook on data envelopment analysis*. Boston: Springer.

Chiou, Y.-C., & Chen, Y.-H. (2006). Route-based performance evaluation of Taiwanese domestic airlines using data envelopment analysis. *Transportation Research Part E: Logistics and Transportation Review, 42*(2), 116–127.

Chiou, Y.-C., & Lan, L. W. (2007). Integrated data envelopment analysis models for measuring transport efficiency and effectiveness. *Journal of the Eastern Asia Society for Transportation Studies, 7*, 427–440.

Chiou, Y.-C., Lan, L. W., & Yen, B. T. (2010). A joint measurement of efficiency and effectiveness for non-storable commodities: Integrated data envelopment analysis approaches. *European Journal of Operational Research, 201*(2), 477–489.

Cook, W. D., & Green, R. (2004). Multicomponent efficiency measurement and core business identification in multiplant firms: A DEA model. *European Journal of Operational Research, 157*(3), 540–551.

Cook, W. D., & Green, R. (2005). Evaluating power plant efficiency: A hierarchical model. *Computers and Operations Research, 4*(9), 813–823.

Cook, W. D., & Hababou, M. (2001). Sales performance measurement in bank branches. *Omega–International Journal of Management Science, 29*(4), 299–307.

Cook, W. D., & Seiford, L. M. (2009). Data envelopment analysis (DEA) – Thirty years on. *European Journal of Operational Research, 192*(1), 1–17.

Cook, W. D., & Zhu, J. (2005). *Modeling performance measurement: Applications and implementation issues in DEA*. Berlin: Springer.

Cook, W. D., & Zhu, J. (2007). Within-group common weights in DEA: An analysis of power plant efficiency. *European Journal of Operational Research, 178*(1), 207–216.

Cook, W. D., Chai, D., Doyle, J., & Green, R. (1998). Hierarchies and groups in DEA. *Journal of Productivity Analysis, 10*(2), 177–198.

Cook, W. D., Hababou, M., & Tuenter, H. (2000). Multicomponent efficiency measurement and shared inputs in data envelopment analysis: An application to sales and service performance in bank branches. *Journal of Productivity Analysis, 14*(3), 209–224.

Cook, W. D., Liang, L., Yang, F., & Zhu, J. (2007). DEA models for supply chain or multi-stage structure. In J. Zhu & W. D. Cook (Eds.), *Modeling data irregularities and structural complexities in data envelopment analysis* (pp. 189–208). New York: Springer.

Cook, W. D., Liang, L., & Zhu, J. (2010a). Measuring performance of two-stage network structures by DEA: A review and future perspective. *Omega–International Journal of Management Science, 38*(6), 423–430.

Cook, W. D., Zhu, J., Bi, G., & Yang, F. (2010b). Network DEA: Additive efficiency decomposition. *European Journal of Operational Research, 207*(2), 1122–1129.

Cooper, W., Park, K., & Pastor, J. (1999). RAM: A range adjusted measure of inefficiency for use with additive models, and relations to other models and measures in DEA. *Journal of Productivity Analysis, 11*(1), 5–42.

Cooper, W., Seiford, L., & Tone, K. (2000). *Data envelopment analysis*. Boston: Kluwer.

da Cruz, N. F., Carvalho, P., & Marques, R. C. (2013). Disentangling the cost efficiency of jointly provided water and wastewater services. *Utilities Policy, 24*, 70–77.

Diez-Ticio, A., & Mancebon, M. (2002). The efficiency of the Spanish police service: An application of the multiactivity DEA model. *Applied Economics, 34*(3), 351–362.

Eilat, H., Golany, B., & Shtub, A. (2008). R&D project evaluation: An integrated DEA and balanced scorecard approach. *Omega–International Journal of Management Science, 36*(5), 895–912.

Emrouznejad, A., & Thanassoulis, E. (2005). A mathematical model for dynamic efficiency using data envelopment analysis. *Applied Mathematics and Computation, 160*(2), 363–378.

Fallah-Fini, S., Triantis, K., & Johnson, A. (2013, to appear). Reviewing the literature on non-parametric dynamic efficiency measurement: State-of-the-art. *Journal of Productivity Analysis*. doi:10.1007/s11123-013-0349-8.
Färe, R. (1991). Measuring Farrell efficiency for a firm with intermediate inputs. *Academia Economic Papers, 19*, 329–340.
Färe, R., & Grosskopf, S. (1996a). *Intertemporal production frontier: With dynamic DEA*. Boston: Kluwer.
Färe, R., & Grosskopf, S. (1996b). Productivity and intermediate products: A frontier approach. *Economics Letters, 50*(1), 65–70.
Färe, R., & Grosskopf, S. (2000). Network DEA. *Socio-Economic Planning Sciences, 34*(1), 35–49.
Färe, R., & Primont, D. (1984). Efficiency measures for multiplant firms. *Operations Research Letters, 3*(5), 257–260.
Färe, R., & Whittaker, G. (1995). An intermediate input model of dairy production using complex survey data. *Journal of Agricultural Economics, 46*(2), 201–213.
Färe, R., Grosskopf, S., & Lovell, C. (1985). *The measurement of efficiency of production*. Dordrecht: Kluwer-Nijhoff.
Färe, R., Grabowski, R., Grosskopf, S., & Kraft, S. (1997). Efficiency of a fixed but allocable input: A non-parametric approach. *Economics Letters, 56*(2), 187–193.
Färe, R., Grosskopf, S., & Zaim, O. (2002). Hyperbolic efficiency and return to the dollar. *European Journal of Operational Research, 136*(3), 671–679.
Färe, R., Grosskopf, S., & Whittaker, G. (2007). Network DEA. In J. Zhu & W D. Cook (Eds.), *Modeling data irregularities and structural complexities in data envelopment analysis* (pp. 209–240). New York: Springer.
Farrell, M. (1957). The measurement of productive efficiency. *Journal of the Royal Statistical Society, Series A (General), 120*(3), 253–281.
Fried, H., Knox Lovell, C., & Schmidt, S. (2008). *The measurement of productive efficiency and productive Growth*. New York: Oxford University Press.
Fukuyama, H. (2003). Scale characterizations in a DEA directional technology distance function framework. *European Journal of Operational Research, 144*(1), 108–127.
Fukuyama, H., & Matousek, R. (2011). Efficiency of Turkish banking: Two-stage network system. Variable returns to scale model. *Journal of International Financial Markets, Institutions and Money, 21*(1), 75–91.
Fukuyama, H., & Mirdehghan, S. (2012). Identifying the efficiency status in network DEA. *European Journal of Operational Research, 220*(1), 85–92.
Fukuyama, H., & Weber, W. L. (2009). A directional slack-based measure of technical inefficiency. *Socio-Economic Planning Sciences, 43*(4), 274–287.
Fukuyama, H., & Weber, W. L. (2010). A slacks-based inefficiency measure for a two-stage system with bad outputs. *Omega–International Journal of Management Science, 38*(5), 398–409.
Golany, B., & Thore, S. (1997). Restricted best practice selection in DEA: An overview with a case study evaluating the socio-economic performance of nations. *Annals of Operations Research, 73*, 117–140.
Golany, B., Hackman, S., & Passy, U. (2006). An efficiency measurement framework for multi-stage production systems. *Annals of Operations Research, 145*(1), 51–68.
Grigoroudis, E., Tsitsiridi, E., & Zopounidis, C. (2013). Linking customer satisfaction, employee appraisal, and business performance: An evaluation methodology in the banking sector. *Annals of Operations Research, 205*(1), 5–27.
Guan, J., & Chen, K. (2010). Measuring the innovation production process: A cross-region empirical study of China's high-tech innovations. *Technovation, 30*(5–6), 348–358.
Guan, J., & Chen, K. (2012). Modeling the relative efficiency of national innovation systems. *Research Policy, 41*(1), 102–115.

Hahn, J.-S., Kim, H.-R., & Kho, S.-Y. (2011). Analysis of the efficiency of Seoul Arterial Bus routes and its determinant factors. *KSCE Journal of Civil Engineering, 15*(6), 1115–1123.
Holod, D., & Lewis, H. F. (2011). Resolving the deposit dilemma: A new DEA bank efficiency model. *Journal of Banking and Finance, 35*(11), 2801–2810.
Hsieh, L.-F., & Lin, L.-H. (2010). A performance evaluation model for international tourist hotels in Taiwan-an application of the relational network dea. *International Journal of Hospitality Management, 29*(1), 14–24.
Huang, Y., Ge, H., & Feng, Y.-J. (2009). Relational network DEA model based on series system: Application of 14 commercial banks in China. *Xitong Gongcheng Lilun yu Shijian/System Engineering Theory and Practice, 29*(5), 106–114.
Kao, C. (2000). Short-run and long-run efficiency measures for multiplant firms. *Annals of Operations Research, 97*(1), 379–388.
Kao, C. (2008). A linear formulation of the two-level DEA model. *Omega–International Journal of Management Science, 36*, 958–962.
Kao, C. (2009a). Efficiency decomposition in network data envelopment analysis: A relational model. *European Journal of Operational Research, 192*(3), 949–962.
Kao, C. (2009b). Efficiency measurement for parallel production systems. *European Journal of Operational Research, 196*(3), 1107–1112.
Kao, C. (2012, to appear). Dynamic data envelopment analysis: A relational analysis. *European Journal of Operational Research* (0). doi:10.1016/j.ejor.2012.12.012.
Kao, H.-Y., & Chan, C.-Y. (2013). A discriminative multi-objective programming method for solving network DEA. *Smart Innovation, Systems and Technologies, 20*, 327–335.
Kao, C., & Hwang, S. (2008). Efficiency decomposition in two-stage data envelopment analysis: An application to non-life insurance companies in Taiwan. *European Journal of Operational Research, 185*(1), 418–419.
Kao, C., & Hwang, S.-N. (2010). Efficiency measurement for network systems: IT impact on firm performance. *Decision Support Systems, 48*(3), 437–446.
Kao, C., & Hwang, S.-N. (2011). Decomposition of technical and scale efficiencies in two-stage production systems. *European Journal of Operational Research, 211*(3), 515–519.
Kao, C., & Lin, P.-H. (2012). Efficiency of parallel production systems with fuzzy data. *Fuzzy Sets and Systems, 198*, 83–98.
Lee, C.-Y., & Johnson, A. (2011). A decomposition of productivity change in the semiconductor manufacturing industry. *International Journal of Production Research, 49*(16), 4761–4785.
Lee, C.-Y., & Johnson, A. L. (2012). Two-dimensional efficiency decomposition to measure the demand effect in productivity analysis. *European Journal of Operational Research, 216*(3), 584–593.
Lewis, H., & Sexton, T. (2004). Network DEA: Efficiency analysis of organizations with complex internal structure. *Computers and Operations Research, 31*(9), 1365–1410.
Lewis, H., Lock, K., & Sexton, T. (2009). Organizational capability, efficiency, and effectiveness in Major League Baseball: 1901–2002. *European journal of Operational Research, 197*, 731–740.
Lewis, H. F., Mallikarjun, S., & Sexton, T. R. (2013). Unoriented two-stage DEA: The case of the oscillating intermediate products. *European Journal of Operational Research, 229*(2), 529–539.
Li, W. (2012). Research on the organizational efficiency evaluation of China's railway transport industry with network DEA. *Advances in Information Sciences and Service Sciences, 4*(23), 772–778.
Li, Y., Chen, Y., Liang, L., & Xie, J. (2012). DEA models for extended two-stage network structures. *Omega–International Journal of Management Science, 40*(5), 611–618.
Liang, L., Yang, F., Cook, W., & Zhu, J. (2006). DEA models for supply chain efficiency evaluation. *Annals of Operations Research, 145*(1), 35–49.
Liang, L., Cook, W. D., & Zhu, J. (2008). DEA models for two-stage processes: Game approach and efficiency decomposition. *Naval Research Logistics, 55*(7), 643–653.

Liang, L., Li, Z.-Q., Cook, W., & Zhu, J. (2011). Data envelopment analysis efficiency in two-stage networks with feedback. *IIE Transactions, 43*, 309–322.

Lim, S., & Zhu, J. (2013). Integrated data envelopment analysis: Global vs. local optimum. *European Journal of Operational Research, 229*(1), 276–278.

Lin, E. T. (2008). Route-based performance evaluation of Taiwanese domestic airlines using data envelopment analysis: A comment. *Transportation Research Part E: Logistics and Transportation Review, 44*(5), 894–899.

Lin, T.-Y., & Chiu, S.-H. (2013). Using independent component analysis and network DEA to improve bank performance evaluation. *Economic Modelling, 32*(1), 608–616.

Liu, S.-T. (2011). A note on efficiency decomposition in two-stage data envelopment analysis. *European Journal of Operational Research, 212*(3), 606–608.

Liu, J. S., & Lu, W.-M. (2012). Network-based method for ranking of efficient units in two-stage DEA models. *Journal of the Operational Research Society, 63*, 1153–1164.

Liu, S.-T., & Wang, R.-T. (2009). Efficiency measures of PCB manufacturing firms using relational two-stage data envelopment analysis. *Expert Systems with Applications, 36* (3, Part 1), 4935–4939.

Liu, W.-B., Tso, C.-W., Peng, K.-C., Chiu, C.-C., & Shih, M.-L. (2012). Application of network DEA in the assessment of operating performances of farmers associations in Taiwan. *Journal of Interdisciplinary Mathematics, 15*(1), 65–81.

Liu, J., Lu, L., Lu, W.-M., & Lin, B. (2013a). A survey of DEA applications. *Omega–International Journal of Management Science, 41*(5), 893–902.

Liu, J. S., Lu, L. Y., Lu, W.-M., & Lin, B. J. (2013b). Data envelopment analysis 1978–2010: A citation-based literature survey. *Omega–International Journal of Management Science, 41*(1), 3–15.

Löthgren, M., & Tambour, M. (1999). Productivity and customer satisfaction in Swedish pharmacies. A DEA network model. *European Journal of Operational Research, 115*(3), 449–458.

Lozano, S. (2011). Scale and cost efficiency analysis of networks of processes. *Expert Systems with Applications, 38*(6), 6612–6617.

Lozano, S., Gutiérrez, E., & Moreno, P. (2013). Network DEA approach to airports performance assessment considering undesirable outputs. *Applied Mathematical Modelling, 37*(4), 1665–1676.

Lu, W.-M., Wang, W.-K., Hung, S.-W., & Lu, E.-T. (2012). The effects of corporate governance on airline performance: Production and marketing efficiency perspectives. *Transportation Research Part E: Logistics and Transportation Review, 48*(2), 529–544.

Luenberger, D. G. (1992). Benefit function and duality. *Journal of Mathematical Economics, 26*(5), 461–481.

Mallikarjun, S., Lewis, H., & Sexton, T. (2014). Operational Performance of US Public Rail Transit and Implications for Public Policy Socio-Economic Planning Sciences, Socio-Economic Planning Sciences (in press).

Mar Molinero, C. (1996). On the joint determination of efficiencies in a data envelopment analysis context. *Journal of the Operational Research Society, 47*(10), 1273–1279.

Mar Molinero, C., & Tsai, P. (1997). Some mathematical properties of a DEA model for the joint determination of efficiencies. *Journal of the Operational Research Society, 48*(1), 51–56.

Matthews, K. (2013). Risk management and managerial efficiency in Chinese banks: A network DEA framework. *Omega–International Journal of Management Science, 41*(2), 207–215.

Meng, W., Zhang, D., Qi, L., & Liu, W. (2008). Two-level DEA approaches in research evaluation. *Omega–International Journal of Management Science, 36*, 950–957.

Mirhedayatian, S., Azadi, M., & Farzipoor Saen, R. (2013, to appear). A novel network data envelopment analysis model for evaluating green supply chain management. *International Journal of Production Economics*. doi:10.1016/j.ijpe.2013.02.009.

Moreno, P., & Lozano, S. (2012, to appear). A network DEA assessment of team efficiency in the NBA. *Annals of Operations Research*. doi:10.1007/s10479-012-1074-9.

Nemoto, J., & Goto, M. (1999). Dynamic data envelopment analysis modeling intertemporal behavior of a firm in the presence of productive inefficiencies. *Economics Letters, 64*, 51–56.

Nemoto, J., & Goto, M. (2003). Measuring dynamic efficiency in production: An application of data envelopment analysis to Japanese electric utilities. *Journal of Productivity Analysis, 19*, 191–210.

Paradi, J., Rouatt, S., & Zhu, H. (2011). Two-stage evaluation of bank branch efficiency using data envelopment analysis. *Omega–International Journal of Management Science, 39*(1), 99–109.

Pastor, J., Ruiz, J., & Sirvent, I. (1999). An enhanced DEA Russell graph efficiency measure. *European Journal of Operational Research, 115*(3), 596–607.

Premachandra, I. M., Zhu, J., Watson, J., & Galagedera, D. U. A. (2012). Best-performing US mutual fund families from 1993 to 2008: Evidence from a novel two-stage DEA model for efficiency decomposition. *Journal of Banking & Finance, 36*(12), 3302–3317.

Prieto, A., & Zofio, J. (2007). Network DEA efficiency in input-output models: With an application to OECD countries. *European Journal of Operational Research, 178*(1), 292–304.

Rezai, H. Z., & Davoodi, A. (2011). Some remarks on the two-level DEA model. *Applied Mathematics Letters, 24*(6), 969–974.

Rogge, N., & De Jaeger, S. (2012). Evaluating the efficiency of municipalities in collecting and processing municipal solid waste: A shared input DEA-model. *Waste Management, 32*(10), 1968–1978.

Rogge, N., & De Jaeger, S. (2013). Measuring and explaining the cost efficiency of municipal solid waste collection and processing services. *Omega–International Journal of Management Science, 41*(4), 653–664.

Salerno, C. (2006). Using data envelopment analysis to improve estimates of higher education institution's per-student education costs. *Education Economics, 14*(3), 281–295.

Saranga, H., & Moser, R. (2010). Performance evaluation of purchasing and supply management using value chain DEA approach. *European Journal of Operational Research, 207*(1), 197–205.

Seiford, L., & Zhu, J. (1999). Profitability and marketability of the top 55 U.S. commercial banks. *Management Science, 45*(9), 1270–1288.

Sengupta, J. K. (1995). *Dynamics of data envelopment analysis: Theory of systems efficiency*. Dordrecht/Boston: Kluwer.

Sexton, T., & Lewis, H. (2003). Two–stage DEA: An application to Major League Baseball. *Journal of Productivity Analysis, 19*(2), 227–249.

Shephard, R. (1970). *Theory of cost and production frontier*. Princeton: Princeton University Press.

Sheth, C., Triantis, K., & Teodorović, D. (2007). Performance evaluation of bus routes: A provider and passenger perspective. *Transportation Research Part E: Logistics and Transportation Review, 43*, 453–478.

Simar, L., & Wilson, P. (2000). Statistical inference in non papametic frontier models: The state of the art. *Journal of Productivity Analysis, 13*(1), 49–78.

Staat, M. (2001). The effect of sample size on the mean efficiency in DEA: Comment. *Journal of Productivity Analysis, 15*(2), 129–137.

Staat, M. (2002). Bootstrapped efficiency estimates for a model for groups and hierarchies in DEA. *European Journal of Operational Research, 138*(1), 1–8.

Sueyoshi, T., & Sekitani, K. (2005). Returns to scale in dynamic DEA. *European Journal of Operational Research, 161*, 536–544.

Thompson, R., Langemeier, L., Lee, C., Lee, E., & Thrall, R. (1990). The role of multiplier bounds in efficiency analysis with application to Kansas farming. *Journal of Econometrics, 46*(1–2), 93–108.

Tone, K., & Tsutsui, M. (2009). Network DEA: A slacks-based measure approach. *European Journal of Operational Research, 197*(1), 243–252.

Tone, K., & Tsutsui, M. (2010). Dynamic DEA: A slacks-based measure approach. *Omega–International Journal of Management Science, 38*(3–4), 145–156.

Tone, K., & Tsutsui, M. (2014). Dynamic DEA with network structure: A slacks-based measure approach. *Omega–International Journal of Management Science, 42*(1), 124–131.
Troutt, M., Ambrose, P., & Chan, C. (2001). Optimal throughput for multistage input-output processes. *International Journal of Operations and Production Management, 21*, 148–158.
Tsai, P., & Mar Molinero, C. (2002). A variable returns to scale data envelopment analysis model for the joint determination of efficiencies with an example of the UK health service. *European Journal of Operational Research, 141*(1), 21–38.
Vaz, C. B., Camanho, A. S., & Guimaraes, R. C. (2010). The assessment of retailing efficiency using network data envelopment analysis. *Annals of Operations Research, 173*(1), 5–24.
Von Geymueller, P. (2009). Static versus dynamic DEA in electricity regulation: The case of US transmission system operators. *Central European Journal of Operational Research, 17*, 397–413.
Wang, Y.-M., & Chin, K.-S. (2010). Some alternative DEA models for two-stage process. *Expert Systems with Applications, 37*(12), 8799–8808.
Wanke, P. (2013a). Physical infrastructure and flight consolidation efficiency drivers in Brazilian airports: A two-stage network-DEA approach. *Journal of Air Transport Management, 31*, 1–5.
Wanke, P. F. (2013b). Physical infrastructure and shipment consolidation efficiency drivers in Brazilian ports: A two-stage network-DEA approach. *Transport Policy, 29*, 145–153.
Wei, Q. L., & Chang, T.-S. (2011). Optimal system design series-network DEA models. *Journal of the Operational Research Society, 62*, 1109–1119.
Wu, D. D. (2010). Bilevel programming data envelopment analysis with constrained resource. *European Journal of Operational Research, 207*, 856–864.
Wu, D. D., & Birge, J. R. (2012). Serial chain merger evaluation model and application to mortgage banking. *Decision Sciences, 43*(1), 5–36.
Wu, C.-H., Chang, C.-C., & Kuo, K. K. (2008). Evaluating the resource allocation efficiency of the healthcare system in Taiwan. *International Journal of Public Policy, 3*(5/6), 403–418.
Yang, C.-C. (2009). Productive efficiency, environmental efficiency and their determinants in farrow-to-finish pig farming in Taiwan. *Livestock Science, 126*(1–3), 195–205.
Yang, C., & Liu, H.-M. (2012). Managerial efficiency in Taiwan bank branches: A network DEA. *Economic Modelling, 29*(2), 450–461.
Yang, Y., Ma, B., & Koike, M. (2000). Efficiency–measuring DEA model for production system with k independent subsystems. *Journal of the Operations Research Society of Japan, 43*(3), 343–354.
Yang, F., Wu, D., Liang, L., Bi, G., & Wu, D. (2011). Supply chain DEA: Production possibility set and performance evaluation model. *Annals of Operations Research, 185*(1), 195–211.
Yu, M.-M. (2008a). Assessing the technical efficiency, service effectiveness, and technical effectiveness of the world's railways through NDEA analysis. *Transportation Research Part A: Policy and Practice, 42*(10), 1283–1294.
Yu, M.-M. (2008b). Measuring the efficiency and return to scale status of multi-mode bus transit – Evidence from Taiwan's bus system. *Applied Economics Letters, 15*(8), 647–653.
Yu, M.-M. (2010). Assessment of airport performance using the SBM-NDEA model. *Omega–International Journal of Management Science, 38*(6), 440–452.
Yu, M.-M. (2012). An integration of the multi-component DEA and GAR models to the measurement of hotel performance. *Current Issues in Tourism, 15*(5), 461–476.
Yu, M.-M., & Chen, P.-C. (2011). Measuring air routes performance using a fractional network data envelopment analysis model. *Central European Journal of Operational Research, 19*(1), 81–98.
Yu, M.-M., & Fan, C.-K. (2006). Measuring the cost effectiveness of multimode bus transit in the presence of accident risks. *Transportation Planning and Technology, 29*(5), 383–407.
Yu, M.-M., & Fan, C.-K. (2009). Measuring the performance of multimode bus transit: A mixed structure network DEA model. *Transportation Research Part E: Logistics and Transportation Review, 45*(3), 501–515.

Yu, M.-M., & Lee, B. C. (2009). Efficiency and effectiveness of service business: Evidence from international tourist hotels in Taiwan. *Tourism Management, 30*(4), 571–580.

Yu, M.-M., & Lin, E. (2008). Efficiency and effectiveness in railway performance using a multi-activity network DEA model. *Omega–International Journal of Management Science, 36*, 1005–1017.

Zha, Y., & Liang, L. (2010). Two-stage cooperation model with input freely distributed among the stages. *European Journal of Operational Research, 205*(2), 332–338.

Zha, Y., Liang, L., & Xu, C. y. (2008). Two-stage BCC model for cooperative efficiency evaluation using a geometric mean method. *Systems Engineering – Theory & Practice, 28* (10), 53–58.

Zhang, Y., & Bartles, R. (1998). The effect of sample size on the mean efficiency in DEA with an application to electricity distribution in Australia, Sweden and New Zealand. *Journal of Productivity Analysis, 9*(2), 187–204.

Zhang, Q., & Ma, J. b. (2011). Research on business efficiency of hotel and tourism enterprises based on the influence of innovation factors. *Energy Procedia, 5*, 742–746.

Zhao, Y., Triantis, K., Murray-Tuite, P., & Edara, P. (2011). Performance measurement of a transportation network with a downtown space reservation system: A network-DEA approach. *Transportation Research Part E: Logistics and Transportation Review, 47*(6), 1140–1159.

Zhu, J. (2011). Airlines performance via Two-stage network DEA approach. *Journal of CENTRUM Cathedra: The Business and Economics Research Journal, 4*(2), 260–269.