Chapter 1
Ranking Decision Making Units: The Cross-Efficiency Evaluation

José L. Ruiz and Inmaculada Sirvent

Abstract This chapter surveys the literature on the cross-efficiency evaluation, which is a methodology for ranking decision making units (DMUs) involved in a production process regarding their efficiency. Cross-efficiency evaluation has been developed in the context of analyses of relative efficiency carried out with Data Envelopment Analysis (DEA). It is usually claimed that the DEA efficiency scores cannot be used for ranking, because they result from a self-evaluation of units based on DMU-specific input and output weights. Cross-efficiency evaluation, in contrast, provides a peer-appraisal in which each DMU is evaluated from the perspective of all of the others by using their DEA weights. This makes it possible to derive an ordering. We make an exhaustive review of the existing work on the different issues related to the cross-efficiency evaluation. Other uses of this methodology different from the ranking of DMUs as well as the extensions that have been developed are also outlined.

Keywords Cross-efficiency evaluation • Ranking • DEA

1.1 Introduction

In decision making processes, ranking constitutes a crucial step for choosing among alternatives after their evaluation. In Multi-Attribute Decision Making (MADM) problems we have n alternatives which are assessed against m criteria. The evaluations that result from these assessments provide the final ranking values of the alternatives. Usually, the higher the ranking value the better the performance of the alternative, so the alternative with the highest ranking value is considered as the best of the alternatives.

Rankings have experienced an increasing popularity. An example of this can be found in Higher Education with the university rankings or league tables.

J.L. Ruiz (✉) • I. Sirvent
Centro de Investigación Operativa, Universidad Miguel Hernández, Avd. de la Universidad, s/n, Elche (Alicante) 03202, Spain
e-mail: jlruiz@umh.es; isirvent@umh.es

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Most visible international rankings are The Academic Ranking of World Universities (ARWU) by Shanghai Jiao Tung University, commonly known as the Shanghai index and the World University Ranking by Times Higher Education (THESQS). As has been widely acknowledged in the related literature, university rankings are controversial but influential. Despite their limitations, university rankings have some effect on decision making regarding higher education institutions: on the choice of a convenient place by students, on recruitment decisions by employers, on university policies, motivating the competitiveness among them, etc. See De Witte and Hudrlikova (2013) for a discussion on this issue and a review of the literature.

We here are concerned with the assessment of performance of DMUs involved in production processes. Specifically, the focus is on the evaluation of their relative efficiency in the use of several inputs to produce several outputs by means of DEA models. The DEA efficiency scores provide a self-evaluation of DMUs based on the inputs and output weights that show them in their best possible light. Thus, since the DMUs are evaluated with DMU-specific weights (which often differ across units), it is usually claimed that the efficiency scores that result from DEA models cannot be used for purposes of ranking DMUs.

The chapter is devoted to the so-called cross-efficiency evaluation. This methodology, as introduced in Sexton et al. (1986) and Doyle and Green (1994a), arose as an extension of DEA aimed at ranking DMUs. The idea behind the cross-efficiency evaluation is to apply one DMU's perspective to others, by using its DEA weights in the evaluations. That is, the efficiency of each unit is assessed with the weights of all the DMUs instead of with only its own weights. Each of these assessments, which are called the cross-efficiencies, is defined as the classical efficiency ratio of a weighted sum of outputs to a weighted sum of inputs. Eventually, the cross-efficiency score of a given unit is calculated as the average of the cross-efficiencies of such unit obtained with the weights of all the DMUs. Cross-efficiency evaluation provides thus a peer-evaluation of the DMUs, instead of a self-evaluation, which makes it possible to derive an ordering. We highlight the parallelism between the cross-efficiency evaluation and MADM problems. Cross-efficiency evaluation can be seen as a MADM problem in which the DMUs are the alternatives and the DEA weights of each of them act as the criteria used in the evaluations.

Cross-efficiency evaluation has received much attention in the related literature. In fact, "cross-efficiency evaluation and ranking" is identified as one of the four research fronts in DEA in the study carried out by Liu et al. (2016), which applies a network clustering method in order to group the DEA literature over the period 2000–2014. We also note that this methodology has been widely applied for ranking performance of DMUs in many different contexts. Sexton et al. (1986) included an evaluation of nursing homes while in Doyle and Green (1994b) an application to higher education can be found. See also Oral et al. (1991) for an application to R&D projects, Green et al. (1996) to preference voting, Baker and Talluri (1997) to industrial robot selection and Talluri and Yoon (2000) to the selection of advanced manufacturing technology (AMT). More recently, this methodology has been applied to the electricity distribution sector in Chen (2002), for the determination of the best labor assignment in a cellular manufacturing system in Ertay and Ruan (2005), to economic-environmental performance in Lu and Lo (2007), to sport in Wu et al. (2009a, b), Cooper et al. (2011), Ruiz et al. (2013) and Gutiérrez and Ruiz (2013a, b), to public procurement in Falagario et al. (2012) and to portfolio selection in Lim et al. (2014).

We review here the literature on the different issues related to the cross-efficiency evaluation. This includes the choice of DEA weights among alternate optima by using alternative secondary goals (Sect. 1.4) and the aggregation of cross-efficiencies (Sect. 1.5). Other uses of the cross-efficiency evaluation different from that concerned with rankings are discussed (Sect. 1.6), together with the extensions of the standard approach that have been developed and broaden the range of applicability of this methodology (Sect. 1.7). Previously, Sect. 1.2 summarizes the existing ranking methods in DEA and Sect. 1.3 briefly describes the standard approach to the cross-efficiency evaluation. Last section concludes.

1.2 Ranking Methods in DEA

The literature has widely dealt with the ranking of DMUs in the context of DEA. Adler et al. (2002) and Hosseinzadeh Lotfi et al. (2013) provide a couple of reviews, while the review of methods for improving discrimination in DEA in Angulo-Meza and Estellita Lins (2002) also considers some methods for ranking DMUs.

This body of research can be roughly described as follows. Firstly, we should mention the rankings that result from efficiency ratios obtained by using a common set of weights (CSW). CSW has the appeal of a fair and impartial evaluation in the sense that each variable is attached the same weight in the assessments of all the DMUs. This approach has been often followed in the efficiency analyses made in Economics and Engineering. Regarding that approach, Doyle and Green (1994a) point out that the choice itself of such weights often raises serious difficulties, and in many cases there is no universally agreed-upon the weights to be used. We note that there exist some DEA-based methods aimed at finding a CSW: see Ganley and Cubbin (1992), Roll and Golany (1993), Troutt (1997), Despotis (2002), Kao and Hung (2005), Liu and Peng (2008, 2009), Ramón et al. (2011) and Ramón et al. (2012).

The methods based on either the cross-efficiency evaluation or the super-efficiency score (Andersen and Petersen 1993) have been those that have received more attention in view of the number of published papers dealing with these issues. As said before, this chapter is devoted to the cross-efficiency evaluation, so it is described subsequently in detail. The super-efficiency score results from the evaluation of the DMUs with respect to the technology estimated by excluding the unit under assessment from the sample. This kind of scores (see Hashimoto 1997; Sueyoshi 1999 and Tone 2002) have been widely used for ranking DMUs, and their use has also been extended for the analysis of sensitivity and the detection of outliers. The infeasibility problems of the super-efficiency score are usually highlighted as a drawback of this efficiency measure, as well as the fact that it results from DMU-specific weights if it is used for purposes of ranking (as in DEA).

Some existing methods propose to rank DMUs through the benchmarking (see Sinuany-Stern et al. 1994 and Torgersen et al. 1996). The basic idea behind them is that a given DMU should rank high if it is frequently used as referent in the evaluation of the remaining units (obviously, these methods can only rank efficient DMUs). Other group of methods utilizes multivariate statistical techniques like canonical correlation analysis and discriminant analysis to rank the DMUs (see Sinuany-Stern et al. 1994 and Friedman and Sinuany-Stern 1997). These techniques are usually applied once the DEA classification into efficient and inefficient units has been obtained, and rank the units by using common weights. Empirically, non-parametric tests seem to show compatibility between the rank and the DEA dichotomic classification. Finally, we can mention a last group of papers that combine DEA and multi-criteria decision-making methods, such as AHP, fuzzy logic or multi-objective linear programming (see Halme et al. 1999; Li and Reeves 1999 and Kao and Liu 2000). Some of these approaches require the collection of additional, preferential information from relevant decision makers, which could be considered as the weakness of these methods.

Obviously, these methods have all their own attractive features and weaknesses, so no of them could be prescribed as the complete solution to the question of ranking.

1.3 The Cross-Efficiency Evaluation: The Standard Approach

Throughout the paper we assume that we have *n* DMUs that use *m* inputs to produce *s* outputs. These can be described by means of the vectors (X_j, Y_j), $j = 1, \ldots, n$, which are assumed to be non-negative. We also denote by X the $m \times n$ matrix of input vectors and by Y the $s \times n$ matrix of output vectors. The standard cross-efficiency evaluation is based on the CCR DEA model (Charnes et al. 1978), which is an oriented radial model. The following problem is the CCR model in its ratio form when used for the assessment of relative efficiency of a given DMU_0

$$
\begin{aligned}
\text{Max} \quad & \theta_0 = \frac{u' Y_0}{v' X_0} \\
\text{s.t.}: \quad & \frac{u' Y_j}{v' X_j} \leq 1 \qquad j = 1, \ldots, n \\
& v \geq 0_m, \quad u \geq 0_s
\end{aligned} \tag{1.1}
$$

In short, the optimal value of (1.1) is the DEA efficiency score of DMU_0 while the ratios in the constraints provide the cross-efficiencies of the remaining units calculated with the weights of DMU_0.

Model (1.1) is non-linear. Nevertheless, by using the results on linear fractional programming in Charnes and Cooper (1962), it can be converted into the following linear problem (which is the so-called dual multiplier formulation)

$$\begin{array}{ll} \text{Max} & u'Y_0 \\ \text{s.t.}: & v'X_0 = 1 \\ & -v'X_j + u'Y_j \leq 0 \quad j = 1, \ldots, n \\ & v \geq 0_m, \ \ u \geq 0_s \end{array} \tag{1.2}$$

Thus, if (v^d, u^d) is an optimal solution of (1.2) for a given DMU_d, then the cross-efficiency of DMU_j, $j = 1, \ldots, n$, obtained with the weights of DMU_d is the following

$$E_{dj} = \frac{u^{d'} Y_j}{v^{d'} X_j} \tag{1.3}$$

The E_{dj} ' s are usually collected in the so-called matrix of cross-efficiencies

$$E = \begin{pmatrix} E_{11} & \ldots & E_{1j} & \ldots & E_{1n} \\ \ldots. & & \ldots. & & \ldots. \\ E_{d1} & \ldots & E_{dj} & \ldots & E_{dn} \\ \ldots. & & \ldots. & & \ldots. \\ E_{n1} & \ldots & E_{nj} & \ldots & E_{nn} \end{pmatrix} \tag{1.4}$$

In each row d, we have the evaluations of the different units calculated with the DEA weights of DMU_d (so the main diagonal of the matrix contains the DEA efficiency scores). In each column j, we have the efficiencies of a given DMU_j calculated with the weights of all the DMUs. In fact, the cross-efficiency score of DMU_j, $j = 1, \ldots, n$, is usually defined as the average of the cross-efficiencies in the corresponding column. That is,

$$\overline{E}_j = \frac{1}{n} \sum_{d=1}^{n} E_{dj}, \quad j = 1, \ldots, n. \tag{1.5}$$

The cross-efficiency score $\bar{E}_j$ provides a peer-evaluation of DMU_j, and the DMUs can be ranked according to the values $\bar{E}_j$, $j = 1, \ldots, n$. The fact that the cross-efficiencies in each of the rows of E are obtained by using the same input and output weights is the reason why an ordering of DMUs can be derived on the basis of the cross-efficiency scores.

The literature has emphasized the following two as the principal advantages of the cross-efficiency evaluation: (1) it provides an ordering of the DMUs and (2) it eliminates unrealistic weighting schemes without requiring the elicitation of weight restrictions (see, for example, Anderson et al. 2002). Doyle and Green (1994a) have also highlighted the interpretation of the cross-efficiency evaluation as peer-appraisal. As a result, these authors suggest that cross-efficiency evaluation has less of the arbitrariness of additional constraints and has more of the right connotations of a democratic process, as opposed to authoritarianism (externally imposed weights, CSW) or egoism (self-appraisal, DEA).

But there are also some difficulties with the cross-efficiency evaluation. As it happens with other DEA-based approaches for ranking, for example with the super-efficiency score or even with the rankings provided by CSWs obtained by using DEA, there exists the possibility of rank reversal. That is, if a new DMU were added to the sample, then the ranking could change. Thus, the rank of a given DMU that results from the cross-efficiency scores should be seen as reflecting its relative position in presence of the DMUs considered in the sample. As discussed in Wang and Luo (2009), the rank reversal phenomenon occurs in many decision making approaches such as the Analytic Hierarchy Process (AHP), the Borda–Kendall (BK) method for aggregating ordinal preferences, the simple additive weighting (SAW) method and the technique for order preference by similarity to ideal solution (TOPSIS) method. These authors eventually claim that rank reversal "might be a normal phenomenon".

However, the problems with the alternate optima for the DEA weights have been the ones widely acknowledged as the main weakness of this methodology. The existence of alternative optimal solutions in (1.2) is a factor that may reduce the usefulness of the cross-efficiency evaluation, because we may have different cross-efficiency scores (and, consequently, different rankings) depending on the choice of DEA weights that is made. This is probably the issue related to the cross-efficiency evaluation that has received more attention in the literature. We discuss it in the next section.

1.4 The Choice of DEA Weights in Cross-Efficiency Evaluations

As a potential remedy to resolve the ambiguity of the multiple DEA weights, Sexton et al. (1986) already suggested making a choice among alternate optima by using some alternative secondary goal. They proposed the two well-known benevolent and aggressive approaches used to that end. The idea behind them is that $\mathrm{DMU_d}$ chooses among its optimal weights those that maximize/minimize in some way the cross-efficiencies of the other units. Some models were developed, which involve the use of different surrogates that try to avoid the non-linear formulations that result from the inclusion of the cross-efficiencies, which are ratios, in the problems. For example, instead of maximizing (minimizing) the sum of cross-efficiency ratios themselves, these authors suggested that an adequate surrogate is to minimize (maximize) the sum of the denominators of the fractions minus the sum of the numerators. Doyle and Green (1994a) implemented the benevolent/aggressive models below following those ideas

$$
\begin{array}{lll}
\mathrm{Max/Min} & \sum_{j\neq d}\left(u^{d'}Y_j - v^{d'}X_j\right) & \\
\mathrm{s.t.:} & v^{d'}X_d = 1 & \\
 & -\theta_d^* v^{d'}X_d + u^{d'}Y_d = 0 & \\
 & -v^{d'}X_j + u^{d'}Y_j \leq 0 & j=1,\ldots,n, \quad j\neq d \\
 & v^d \geq 0_m, \quad u^d \geq 0_s &
\end{array}
\tag{1.6}
$$

where θ_d^* is the DEA efficiency score of DMU_d.

In line with that approach, these authors also proposed the following two formulations

$$
\begin{array}{lll}
\mathrm{Max/Min} & u^{d'}\sum_{j\neq d} Y_j & \\
\mathrm{s.t.:} & v^{d'}\sum_{j\neq d} X_j = 1 & \\
 & -\theta_d^* v^{d'}X_d + u^{d'}Y_d = 0 & \\
 & -v^{d'}X_j + u^{d'}Y_j \leq 0 & j=1,\ldots,n, \quad j\neq d \\
 & v^d \geq 0_m, \quad u^d \geq 0_s &
\end{array}
\tag{1.7}
$$

which are two models that seek, as secondary goal, to maximize/minimize the efficiency of a composite DMU, while keeping unchanged the DEA efficiency score of DMU_d, θ_d^*.

Liang et al. (2008a) extend the work in Doyle and Green (1994a) by introducing various secondary objective functions, which are formulated in terms of the deviation variables $\alpha_j^d = v^{d'}X_j - u^{d'}Y_j$, $j = 1, \ldots, n$. The first secondary goal gives rise to the following model, which is equivalent to the benevolent formulation in (1.6)

$$
\begin{array}{lll}
\mathrm{Min} & \sum_{j=1}^{n}\alpha_j^d & \\
\mathrm{s.t.:} & v^{d'}X_d = 1 & \\
 & u^{d'}Y_d = 1 - \alpha_d^{d*} & \\
 & -v^{d'}X_j + u^{d'}Y_j + \alpha_j^d = 0 & j=1,\ldots,n \\
 & v^d \geq 0_m, \quad u^d \geq 0_s &
\end{array}
\tag{1.8}
$$

where $\alpha_d^{d*} = 1 - \theta_d^*$. This model minimizes the total deviation from the ideal point defined as the multiplier bundle for which every DMU is efficient, that is,

$\alpha_j^d = 0, j = 1, \ldots n$. The following two secondary goals are also proposed in that paper with the purpose of deriving weights for which the cross-efficiencies are as similar as possible:

1. Minimizing the maximum deviation variable

$$\text{Min Max } \alpha_d^j \tag{1.9}$$

   which is related to maximizing the minimum cross-efficiency among the n DMUs, and
2. Minimizing the mean absolute deviation

$$\text{Min } \frac{1}{n}\sum_{j=1}^{n}\left|\alpha_d^j - \overline{\alpha}_d\right| \tag{1.10}$$

   which is aimed at minimizing the variation among the cross-efficiencies of the DMUs, where $\overline{\alpha}_d = \frac{1}{n}\sum_{j=1}^{n}\alpha_d^j$.

The new models can be formulated by simply changing the objective of (1.8) with those in (1.9) and (1.10).

Wang and Chin (2010b) state that the three models above are established on the basis of an unrealistic ideal point and formulate some variants with the following differences: (1) the ideal point is associated with the multiplier bundle for which all the DMUs achieve their DEA efficiency scores $(\theta_1^*, \ldots, \theta_n^*)$, instead of using the value 1 as the target efficiency of each DMU, which is only achievable for the efficient units. As a result, the constraints $-v^{d'}X_j + u^{d'}Y_j + \alpha_j^d = 0, j = 1, \ldots, n$, in the Liang et al.'s models are replaced by $-v^{d'}\theta_j^* X_j + u^{d'}Y_j + \alpha_j^d = 0,\ j = 1, \ldots, n$; (2) the normalizing constraint is the same in the formulations associated with all the DMU_d's. In particular, they suggest the following constraint $v^{d'}\sum_{j=1}^{n}X_j + u^{d'}\sum_{j=1}^{n}Y_j = n$, and (3) aggressive formulations are also proposed (note that models (1.8)–(1.10) follow a benevolent approach).

Obviously, neither of the models we have just discussed is better than the others. The use of them in practice will depend on the circumstances. For instance, Liang et al. (2008a) suggest that minimizing the total deviation as in (1.8) would be an appropriate approach to the cross-efficiency evaluation when the DMUs are assumed to be in a non-cooperative and fully competitive mode. For example, in a supply chain where each member is acting in its own self-interest, without being concerned for the others. In contrast, minimizing the maximum deviation, (1.9), might be deemed appropriate in settings where a more cooperative situation prevails. For example, in the evaluation of bank branches under a single corporate head, where the worst performing units would be given the least gap possible between where they are and where they need to be. Minimizing the mean absolute deviation, (1.10), aims at equalizing the various efficiency scores. So, if we were concerned with an allocatable resource such as the equipment for the maintenance crews, this model might tend to result in the least amount of redistribution (to render the DMUs equally efficient) in regard to that resource.

Other approaches focus on the suitability of the profiles of DEA weights that are chosen without dealing directly with the cross-efficiencies. As said before, one of the advantages of the cross-efficiency evaluation is that it eliminates unrealistic weighting schemes without requiring the elicitation of weight restrictions. The idea is that the effects of unreasonable weights are cancelled out in the summary that the cross-efficiency evaluation makes (Anderson et al. 2002). However, as Ramón et al. (2010a) state, we may have more comprehensive cross-efficiency scores if we actually avoid unreasonable weights instead of expecting that their effects are eliminated in the amalgamation of weighting schemes. By unrealistic weighting schemes we often mean the profiles of weights with zeros. The literature has widely claimed the need to avoid zero weights because they imply that some of the inputs and/or outputs considered for the analysis are ignored in the assessments. But the literature has also claimed against the large differences usually found in the weights as a result of the DEA total weight flexibility. These include both the differences in the input weights and in the output weights used in the evaluation of a DMU (Cook and Seiford 2008 state that "the AR concept was developed to prohibit large differences in the values of multipliers") and the differences in the weights attached to the same variable by the different DMUs (see Roll et al. 1991; Pedraja-Chaparro et al. 1997 and Thanassoulis et al. 2004).

To prevent unrealistic weighting schemes in cross-efficiency evaluations different strategies have been followed. Ramón et al. (2010b) classify the DMUs in two sets NZ and Z. In NZ we have the DMUs that can make a choice of non-zero weights among their alternate optima, while Z consists of those that cannot. That is, NZ = E ∪E′ ∪ NE ∪ NE′ and Z = F ∪ NF according to the classification of DMUs in Charnes et al. (1991). Then, they propose that the DMU_d's in NZ choose among their alternate optima the profiles with the least dissimilar weights, by using the following model in Ramón et al. (2010a)

$$
\begin{array}{lll}
\text{Max} & \varphi_d & \\
\text{s.t.}: & & \\
& \sum_{i=1}^{m} \nu_i^d x_{id} = 1 & \\
& \sum_{r=1}^{s} \mu_r^d y_{rd} = \theta_d^* & \\
& -\sum_{i=1}^{m} \nu_i^d x_{ij} + \sum_{r=1}^{s} \mu_r^d y_{rj} \le 0 & j = 1, \ldots, n \\
& z_I \le \nu_i^d \le h_I & i = 1, \ldots, m \\
& z_O \le \mu_r^d \le h_O & r = 1, \ldots, s \\
& \dfrac{z_I}{h_I} \ge \varphi_d & \\
& \dfrac{z_O}{h_O} \ge \varphi_d & \\
& z_I, z_O \ge 0 &
\end{array}
\qquad (1.11)
$$

This model ensures in addition non-zero weights. As for the DMUs in Z, these are re-assessed with weights that cannot be more dissimilar than those of the DMU in NZ that needs to unbalance more its weights (as measured by $\varphi^* = \min_{d \in NZ} \varphi_d^*$) in order to achieve its CCR efficiency score. These are the weights used by the DMUs in Z in the cross-efficiency evaluation. See Wang et al. (2012), which also deals with the weight disparity, albeit it does not ensure non-zero weights.

A different strategy is followed in Ramón et al. (2011). The basic idea of the proposed approach is to ignore the profiles of weights of the inefficient DMUs in Z in the calculation of the cross-efficiency scores. That is, the cross-efficiency evaluation is carried out only with the weights of the DMUs in NZ, once these latter have made a choice among their alternate optima according to some suitable criterion. This approach is called "peer-restricted" cross-efficiency evaluation. Concerning the choice of weights that the DMUs in NZ make, the authors suggest to reduce as much as possible the differences between the profiles of weights selected. This criterion seeks, on one hand, to reduce the differences in the weights attached by the different DMUs to the same variable, and on the other, to reduce the dispersion in the samples of cross-efficiencies, so the cross-efficiency scores, which are the corresponding averages, are more representative of such cross-efficiencies. The choice of the profiles of weights to be used in the "peer-restricted" cross-efficiency evaluation is made by solving the following model

$$
\begin{array}{lll}
\text{Min} & \displaystyle\sum_{\substack{d, d' \in NZ \\ d < d'}} \left(\sum_{i=1}^{m} \left| v_i^d - v_i^{d'} \right| \bar{x}_i + \sum_{r=1}^{s} \left| u_r^d - u_r^{d'} \right| \bar{y}_r \right) & \\
\text{s.t.}: & & \\
& -\displaystyle\sum_{i=1}^{m} v_i^d x_{ij} + \sum_{r=1}^{s} u_r^d y_{rj} \le 0 & j = 1, \ldots, n; \quad d \in NZ \\
& -\theta_d^* \displaystyle\sum_{i=1}^{m} v_i^d x_{id} + \sum_{r=1}^{s} u_r^d y_{rd} = 0 & d \in NZ \\
& \displaystyle\sum_{i=1}^{m} v_i^d \bar{x}_i = 1 & d \in NZ \\
& v_i^d, u_r^d \ge 0 & \forall i, r, d
\end{array}
\tag{1.12}
$$

where $\bar{x}_i$, $i = 1, \ldots, m$, and $\bar{y}_r$, $r = 1, \ldots, s$, are the averages of input i and output r, respectively, across the DMUs in NZ. Note that model (1.12) includes a common normalizing constraint that makes the profiles of weights of the different DMUs comparable.

Model (1.12) can be extended to avoid zero weights (see the original paper for details).

1.4.1 Ranking Ranges and Cross-Efficiency Intervals

Liang et al. (2008a) state that the comparison of cross-efficiency scores obtained with different evaluation criteria allows us to obtain a better picture of cross-efficiency stability with respect to multiple DEA weights. However, this issue can be addressed more appropriately with an approach based on considering simultaneously all of the optimal solutions for the weights. Alcaraz et al. (2013) and Ramón et al. (2014) propose a couple of procedures for ranking DMUs based on the cross-efficiency evaluation which consider all the alternate optima for the DEA weights, thus avoiding the need to make a choice among them by using some alternative secondary goal. Instead of a single ranking, the former paper provides a range for the possible rankings of each DMU, while the latter deals with the cross-efficiency intervals that result from all the DEA weights and use some order relations for interval numbers in order to identify dominance relations between DMUs and rank them.

For each DMU_0, Alcaraz et al. (2013) find the range of its possible rankings that would result from considering all the DEA weights of all the DMUs. This range is determined by the best and the worst possible rankings of DMU_0. The best ranking of DMU_0 is defined as $r_0^b = \underset{(V,U)}{\mathrm{Min}}\{|H_0(V,U)|\} + 1$, where $H_0(V,U) = \left\{DMU_j, j = 1, \ldots, n / \overline{E}_j > \overline{E}_0\right\}$, V and U being the $m \times n$ and $s \times n$ matrices with the input weight vectors and the output weight vectors, respectively, of a given choice of DEA weights that each of the DMUs makes. It is shown that $r_0^b = n - LE_0^*$, where LE_0^* is the optimal value of the problem

$$\begin{array}{lll}
\text{Max} & \sum_{j \neq 0} I_j & \\
\text{s.t.}: & & \\
& \dfrac{u_d' Y_d}{v_d' X_d} = \theta_d^* \quad d = 1, \ldots, n & (13.1) \\
& \dfrac{u_d' Y_j}{v_d' X_j} \leq 1 \quad j = 1, \ldots, n; d = 1, \ldots, n & (13.2) \\
& E_{dj} = \dfrac{u_d' Y_j}{v_d' X_j} \quad j = 1, \ldots, n; d = 1, \ldots, n & (13.3) \\
& \overline{E}_j = \dfrac{1}{n} \sum_{d=1}^{n} E_{dj} \quad j = 1, \ldots, n & (13.4) \\
& \overline{E}_j - \overline{E}_0 \leq 1 - I_j \quad j = 1, \ldots, n, j \neq 0 & (13.5) \\
& V_d \geq 0_m, u_d \geq 0_s, \forall d, I_j \in \{0, 1\}, \forall j \neq 0 &
\end{array} \quad (1.13)$$

Likewise, the worst ranking is defined as $r_0^w = n - \underset{(V,U)}{\mathrm{Min}} \{|L_0(V,U)|\}$, where $L_0(V,U) = \{DMU_j, j = 1, \ldots, n / \overline{E}_j < \overline{E}_0\}$. Now, it is shown that $r_0^w = HE_0^* + 1$, where HE_0^* is the optimal value of the problem that results from replacing (13.5) with $\overline{E}_0 - \overline{E}_j \le M(1 - I_j)$, j = 1,. . .,n, $j \neq 0$, in (1.13).

The approach in Ramón et al. (2014) also considers simultaneously all the DEA weights of all the DMUs, but deals with the minimum and the maximum possible cross-efficiency scores, instead of with their best and worst rankings. For a given DMU_0, these are denoted by $\bar{E}_0^{L*}$ and $\bar{E}_0^{R*}$, and are obtained, respectively, as the optimal values of the problems

$$
\begin{array}{lll}
\mathrm{Min/Max} & \dfrac{1}{n}\left(\displaystyle\sum_{d=1}^{n} \dfrac{u_d' Y_0}{v_d' X_0}\right) & \\
\mathrm{s.t.:} & & \\
& \dfrac{u_d' Y_d}{v_d' X_d} = \theta_d^* & d = 1, \ldots, n \\
& \dfrac{u_d' Y_j}{v_d' X_j} \le 1 & d = 1, \ldots, n; \quad j = 1, \ldots, n \\
& v_d \ge 0_m, \quad u_d \ge 0_s & d = 1, \ldots, n
\end{array}
\tag{1.14}
$$

The authors show how to deal with the cross-efficiency intervals $[\bar{E}_j^{L*}, \bar{E}_j^{R*}]$, j = 1, . . ., n, in order to both identify dominance relations among DMUs and provide a ranking of units by using some order relations for interval numbers. Specifically, the following is an order relation between intervals often used in practice which may appropriately represent the DM's preferences in problems dealing with efficiency intervals: Let A and B be two intervals $A = [a^L, a^R]$ and $B = [b^L, b^R]$, then $A \le_{LR} B \Leftrightarrow a^L \le b^L$ and $a^R \le b^R$, and $A <_{LR} B \Leftrightarrow A \le_{LR} B$ and $A \neq B$. That is, $\le_{LR}$ represents the DM preference for the unit with the higher minimum cross-efficiency score and maximum cross-efficiency score. This order relation is actually a particular case of that originally introduced in Dubois and Prade (1980) for fuzzy numbers (based on the extension principle) when it is considered for interval numbers. The relation $\le_{LR}$ is however a partial order, so there may be pairs of intervals that cannot be compared, as is the case of those that are nested. Therefore, $\le_{LR}$ will usually not allow us to derive a full ranking of units, although it may yield useful information regarding dominance relations among DMUs in terms of cross-efficiency assessments. To be specific, if $\left[\overline{E}_j^{L*}, \overline{E}_j^{R*}\right] <_{LR} \left[\overline{E}_{j'}^{L*}, \overline{E}_{j'}^{R*}\right]$, then we say here that $DMU_{j'}$ dominates DMU_j. To derive a full ranking of units, the order relation $\le_\lambda$ proposed in Campos and Muñoz (1989), which takes into account the degree of optimism of the decision maker (λ), can be used (see the original papers for details).

Yang et al. (2012) also propose an approach to the cross-efficiency evaluation that avoids the need to make any choice of DEA weights. These authors deal with a $n \times n$ matrix of intervals of cross-efficiencies, which is assumed to be a matrix of stochastic variables, and use the stochastic multicriteria acceptability analysis

(SMAA-2) method proposed by Lahdelma and Salminen (2001) to derive a ranking of units. In order to do so, a probability distribution over the cross-efficiency intervals must thus be assumed, the uniform and the normal distributions being those used in that paper. This may eventually increase the computational burden needed by the proposed approach (note, in particular, that Monte-Carlo simulations are used to approximate values of the acceptability indices).

1.4.2 Illustrative Example

We use here the data in Zhu (1998) to illustrate some of the approaches to the cross-efficiency evaluation that have been previously described. The data consist of 18 Chinese cities, which are evaluated by using two inputs and three outputs. Table 1.1 records the data, the DEA efficiency scores, the interval cross-efficiency scores $[\bar{E}_j^{L*}, \bar{E}_j^{R*}]$, $j = 1, \ldots, 18$, the ranking that results from the order relation $\leq_{LR}$ and the ranking provided by the benevolent and the aggressive approaches obtained with (1.6).

The order relation $\leq_{LR}$ practically determines a full ranking of cities, in spite of being a partial order. Figure 1.1 depicts graphically the dominance relations among cities that can be identified by using it. It shows that only cities 5 and 13 cannot be compared to each other with $\leq_{LR}$, since their corresponding cross-efficiency intervals are nested. We can see that city 2 ranks first, followed by cities 6, 10 and 12. At the bottom, we have cities 17, 3, 15, 18 and 14. Although cities 5 and 13 cannot be ranked with $\leq_{LR}$, we can state that these two cities are placed in between cities 12 and 9, because they both dominate city 9 and are dominated by city 12. Therefore, they rank either fifth or sixth.

Note also that the ranking resulting from the use of $\leq_{LR}$ with the cross-efficiency intervals is to a large extent consistent with those that the benevolent and the aggressive formulations provide, which are two rankings that in return show many similarities. However, we should not think that the benevolent and aggressive approaches cover all the range of possible rankings, as Table 1.2 shows. This table reports the ranking ranges obtained with the approach in Alcaraz et al. (2013). We can see, for example, that 2, 6 and 10 are the top three cities in all the scenarios determined by all the possible choices of DEA weights that all the DMUs can make. Nevertheless, each of them can occupy any of the three positions at the top. These situations suggest therefore that the approach proposed can be a useful complement in the standard cross-efficiency evaluations, even when some specific alternative secondary goal has been used to the choice of DEA weights, because it allows us to gain insight into the robustness of the rankings provided against alternate optima.

In any case, the ranking ranges that have been found are not especially wide, so we can draw some interesting conclusions with confidence. For example, it can be stated that 2, 6, 10, 12, 5, 13, 9, 8, 1, 4 are the top ten cities, irrespective of the DEA weights that are chosen. This can be an interesting finding from the point of view of

Table 1.1 Data of illustrative example (*Source*: Zhu 1998)

City	x_1	x_2	y_1	y_2	y_3	DEA score	Cross-eff. interval	Ranking ($\preceq_{LR}$)	Benevolent	Aggressive
1	2874.8	16,738	160.89	80,800	5092	0.4691	[0.3807, 0.4462]	10	10	10
2	946.3	691	21.14	18,172	6563	1	[0.9044, 1.0000]	1	1	1
3	6854	43,024	375.25	144,530	2437	0.2779	[0.2084, 0.2436]	15	15	15
4	2305.1	10,815	176.68	70,318	3145	0.5022	[0.3895, 0.4522]	9	9	9
5	1010.3	2099	102.12	55,419	1225	0.6311	[0.5528, 0.6100]	5,6	6	5
6	282.3	757	59.17	27,422	246	1	[0.8873, 0.9722]	2	2	2
7	17478.6	116,900	1029.09	351,390	14,604	0.3580	[0.2577, 0.3047]	12	12	12
8	661.8	2024	30.07	23,550	1126	0.4959	[0.4028, 0.4635]	8	8	8
9	1544.2	3218	160.58	59,406	2230	0.6577	[0.5084, 0.5782]	7	7	7
10	428.4	574	53.69	47,504	430	1	[0.8511, 0.9247]	3	3	3
11	6228.1	29,842	258.09	151,356	4649	0.3010	[0.2429, 0.2820]	13	14	13
12	697.7	3394	38.02	45,336	1555	0.7866	[0.5754, 0.6871]	4	4	4
13	106.4	367	7.07	8236	121	0.7514	[0.5482, 0.6443]	5,6	5	6
14	4539.3	45,809	116.46	56,135	956	0.1382	[0.1091, 0.1291]	18	18	18
15	957.8	16,947	29.2	17,554	231	0.1867	[0.1405, 0.1694]	16	16	16
16	1209.2	15,741	65.36	62,341	618	0.4704	[0.3352, 0.4071]	11	11	11
17	972.4	23,822	54.52	25,203	513	0.3059	[0.2336, 0.2819]	14	13	14
18	2192	10,943	25.24	40,267	895	0.1953	[0.1334, 0.1616]	17	17	17

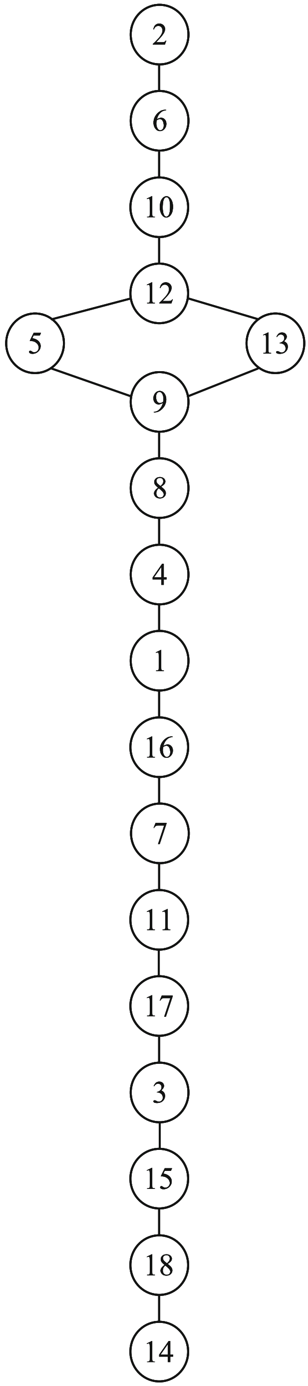

Fig. 1.1 Dominance relations

Table 1.2 Ranking ranges

	Ranking ranges	
DMU	r_0^b	r_0^w
1	9	10
2	1	3
3	15	15
4	8	10
5	4	6
6	1	3
7	12	12
8	8	9
9	6	7
10	1	3
11	13	14
12	4	5
13	5	6
14	18	18
15	16	17
16	11	11
17	13	14
18	16	17

management. We can go even further and state that 2, 6 and 10 are the top three (as said before), and that 5, 9, 12 and 13 will be always in between fourth and seventh, followed by 1, 4 and 8. Likewise, it has been found that cities 14, 15 and 18 are the poorest performers, city 14 always ranking at the bottom.

1.5 The Aggregation of Cross-Efficiencies

In MADM problems, the different criteria are often attached different weights for the evaluation of alternatives, as a way to consider their relative importance. These are usually determined on a subjective basis or trying to reflect the opinion of the decision makers (DMs). In the standard approach to the cross-efficiency evaluation, the cross-efficiency scores of the units are usually calculated as the averages of their cross-efficiencies. This means that the cross-efficiencies provided by the different DMUs are aggregated by attaching all of them the same importance. However, allowing for different aggregation weights may obviously introduce more flexibility into the analysis. In particular, in some situations, the DM could be interested in incorporating his/her preferences regarding the relative importance that should be attached to the cross-efficiencies provided by the different DMUs. For example, the DM might argue that the cross-efficiencies provided by the DMUs that globally rate the units better should be given more importance in the final aggregation. In other situations, it might be desirable that the cross-efficiencies provided by the DMUs that discriminate more among units are considered as more relevant and, accordingly, their associated aggregation weights should be larger. Furthermore, some authors have suggested that the choice of aggregation weights should be made in accordance with that of the DEA weights.

There is a number of papers that depart from the customary use of the arithmetic mean for the aggregation of cross-efficiencies and propose aggregation weights that result from models in which different conditions are imposed. In Wu et al. (2011) the aggregation weights reflect the entropy in the cross-efficiencies provided by the different DMUs. Wang and Chin (2011) use an approach based on ordered weighted averaged (OWA) operators in which, through the specification of the orness degree, they seek to reflect their particular belief that the self-evaluations should be attached more importance than that attached to the evaluations provided by the other DMUs. In Ruiz and Sirvent (2012), the aggregation weights are imposed to reflect the differences in the weights of the different profiles used to calculate the cross-efficiencies, specifically when the DEA weights are obtained from (1.11). Wang and Wang (2013) propose three weighted least-square models yielding aggregation weights which reflect in each case (1) the dissimilarity between the cross-efficiencies provided by different DMUs, (2) the deviations of these cross-efficiencies from the CCR efficiency scores, and (3) a combination of these two measures. And two game approaches are provided in Wu et al. (2008) and Wu et al. (2009c), where the DMUs are considered as players in a cooperative game and the aggregation weights of each DMU are defined, respectively, as the nucleolus solution of the game and from the Shapley value.

León et al. (2014) propose a general approach to the aggregation of cross-efficiencies which is based on Induced Ordered Weighted Averaging (IOWA) operators. In short, the idea is to rearrange the rows of the matrix of cross-efficiencies on the basis of an inducing variable and then attach the aggregation weights accordingly. As for the inducing variable, this should reflect the DM preferences regarding the relative importance of the cross-efficiencies provided by the different DMUs. Examples of inducing variables can be either $z_d = z(E_{dj}) = TE_d$, where TE_d is the total of the cross-efficiencies in row d of the matrix (1.4), $d = 1, \ldots, n$, or $z_d = z(E_{dj}) = 1 - e_d$, where $e_d = \frac{-1}{\log(n)} \sum_{j=1}^{n} E_{dj}^N \log\left(E_{dj}^N\right)$ (E_{dj}^N being $E_{dj} / \sum_{j=1}^{n} E_{dj}$), that is, the entropy in the cross-efficiencies of row d, $d = 1, \ldots, n$. Let $\widetilde{E} = \left(\widetilde{E}_{dj}\right)$ be the matrix of cross-efficiencies that results of re-arranging the rows of the cross-efficiency matrix E according to the ordering induced by $\mathbf{z}$. Thus, in the top rows of $\tilde{E}$ we have the cross-efficiencies that are most preferred by the DM, and consequently they should be attached larger aggregation weights, whereas in those in the bottom we have the least preferred cross-efficiencies, which will be attached the smaller aggregation weights. For example, if $z_d = TE_d$, then in the top rows we will have the cross-efficiencies provided by the DMUs that globally evaluate better the units, while if $z_d = 1 - e_d$, we will have those that discriminate more among units and therefore provide more information to derive a full ranking of units (note that if $E_{d1} = E_{d2} = \ldots = E_{dn}$ then DMU_d provides no information to rank the DMUs; in that case $z_d = 0$).

Concerning the aggregation weights, with this approach only vectors $(\omega_1, \ldots, \omega_n)$ such that $\omega_1 \geq \ldots \geq \omega_n$ are considered, where ω_d is the weight to be attached to the cross-efficiencies in the d-th row of $\tilde{E}$. Nevertheless, the DM can not only set an order of preference for the cross-efficiencies provided by the different DMUs but also he/she can adjust the degree of such preference by means of the so-called orness level, α. This measure was introduced by Yager (1988) and characterizes the degree to which the aggregation is like an "or" (Max) operation. For example, if $\alpha = 1$, then $\omega_1 = 1$ and $\omega_d = 0$, $d = 2, \ldots, n$, which means that the ultimate cross-efficiency scores are the cross-efficiencies in the first row of $\tilde{E}$. In other words, only the cross-efficiencies provided by the most preferred rating DMU are considered. The case $\alpha = 0.5$ is associated with the situation in which the DM has no preference on the cross-efficiencies provided by the different DMUs. Then, $\omega_1 = \ldots = \omega_n = 1/n$, which are the aggregation weights of the arithmetic mean used in the standard cross-efficiency evaluation. Values of the orness degree in between 0.5 and 1 would be associated with intermediate situations. As α gets closer to 1 the weight is progressively put on the rating DMUs in the top rows of $\tilde{E}$.

In order to calculate the aggregation weights ω_d, the minimax disparity problem proposed by Wang and Parkan (2005) can be used:

$$
\begin{array}{lll}
\text{Min} & \delta & \\
\text{s.t.}: & \dfrac{1}{n-1}\sum_{d=1}^{n}(n-d)\omega_d = \alpha & \\
& \sum_{d=1}^{n}\omega_d = 1 & \\
& \omega_d - \omega_{d+1} - \delta \leq 0 & d = 1, \ldots, n-1 \\
& \omega_d - \omega_{d+1} + \delta \geq 0 & d = 1, \ldots, n-1 \\
& \omega_d \geq 0 & d = 1, \ldots, n
\end{array}
\tag{1.15}
$$

where $\alpha \in [0, 1]$ is the orness degree, specified by the DM.

If $\alpha \geq 0.5$, model (1.15) ensures that the aggregation weights provided satisfy $\omega_1 \geq \ldots \geq \omega_n$, as required before. In addition, model (1.15) minimizes the maximum difference between pairs of adjacent weights, so this model somehow minimizes the differences between the aggregation weights.

Eventually, the cross-efficiency scores are calculated as $\overline{E}_j^{IOWA} = \sum_{d=1}^{n} \omega_d \widetilde{E}_{dj}, \quad j = 1, \ldots, n.$

It might be worth mentioning that, like in most approaches, we find here a common set of aggregation weights which is used in the evaluation of all the units. We believe that the ranking resulting from cross-efficiency scores calculated with common weights can be more widely accepted by users than one obtained when the aggregation weight of the cross-efficiencies provided by a given DMU is different in the evaluation of different units (that is, when the cross-efficiencies in a given row of the matrix are attached different weights), as it usually happens when using OWA operators for the aggregation like in Wang and Chin (2011). Moreover, León et al. (2014) suggest that the choice of the aggregation weights should be related to that of the DEA input and output weights. For example, if $z_d = TE_d$ is used as the inducing order variable, then a benevolent approach would be an appropriate strategy. Analogously, if $z_d = 1 - e_d$ then the DEA weights could be obtained by using the aggressive formulation.

1.5.1 Illustrative Example (Cont.)

Continuing the same example used in the previous section, we now illustrate the use of IOWA operators for the aggregation of cross-efficiencies. Specifically, the cross-efficiency evaluation is performed by using the order inducing variable based on the entropy, after the cross-efficiencies are obtained following an aggressive approach to the choice of DEA weights. The rows of the matrix of cross-efficiencies need to be re-arranged in descending order according to the values $1 - e_d$, and the aggregation weights are attached following that ordering. In this particular case, the cross-efficiencies provided by the three efficient cities, 2, 10 and 6, whose values of $1 - e_d$ are 0.495, 0.210 and 0.171, respectively, are attached the largest aggregation weights, while those provided by cities 1, 5, 8, 11, 14 and 15, with a value of $1 - e_d$ equal to 0.051, are attached the lowest ones. Table 1.3 records the

Table 1.3 Cross-efficiency scores for different orness values

	Orness degree					
DMU	0.5	0.6	0.7	0.8	0.9	1
1	0.381	0.346	0.304	0.247	0.141	0.032
2	0.921	0.889	0.850	0.798	0.698	1.000
3	0.208	0.191	0.169	0.137	0.086	0.006
4	0.390	0.358	0.320	0.265	0.170	0.031
5	0.559	0.533	0.503	0.464	0.381	0.061
6	0.887	0.843	0.789	0.716	0.576	0.034
7	0.258	0.235	0.206	0.165	0.096	0.013
8	0.403	0.372	0.337	0.293	0.198	0.059
9	0.519	0.497	0.469	0.428	0.354	0.073
10	0.873	0.850	0.828	0.815	0.740	0.079
11	0.243	0.222	0.198	0.167	0.105	0.016
12	0.578	0.524	0.463	0.394	0.231	0.048
13	0.552	0.508	0.459	0.409	0.278	0.035
14	0.109	0.098	0.085	0.067	0.036	0.002
15	0.141	0.125	0.106	0.083	0.039	0.001
16	0.335	0.299	0.257	0.211	0.105	0.004
17	0.234	0.207	0.175	0.132	0.056	0.002
18	0.134	0.122	0.108	0.093	0.055	0.009

cross-efficiency scores of the units for different orness degrees. The cross-efficiencies under $\alpha = 0.5$ are actually those of the standard aggressive approach, because that level of orness corresponds to the case of using the arithmetic mean for the aggregation of cross-efficiencies. The rankings remain quite stable as α increases, until we get $\alpha = 1$; then some changes occur. For $0.5 \leq \alpha \leq 0.9$, cities 2, 6 and 10 are the top three; cities 5, 9, 12 and 13 rank in between fourth and seventh, followed by 1, 4 and 8; and city 14 ranks at the bottom. These results coincide with those obtained with the approach by Alcaraz et al. (2013) we have previously commented. However, when $\alpha = 1$, that is, when the importance attached to the cross-efficiencies is not so far allocated between all the rows of the matrix of cross-efficiencies but we only use the profile of DEA weights that discriminate more among cross-efficiencies (that of city 2), we can see, for example, that city 6 falls outside the top three and rank eighth, while city 9 moves up to the third position. Besides, it is city 15 which ranks at the bottom.

1.6 Other Uses

The cross-efficiency evaluation has been used with other purposes different from the ranking of DMUs. These include the following.

1.6.1 Identification of Mavericks and All-Round Performers

The cross-efficiency scores allow us to discriminate between DMUs rated as DEA efficient. Nevertheless, the comparison between DEA efficiency scores and cross-efficiency scores can be exploited in a variety of other ways. For example, Doyle and Green (1994a) define the so-called Maverick index as follows (similarly, Baker and Talluri 1997 define the false positiveness index and Wang and Chin 2010a the efficiency disparity index):

$$M_j = \left(\theta_j^* - \bar{e}_j\right)/\bar{e}_j, \quad j = 1, \ldots, n \tag{1.16}$$

where $\bar{e}_j = 1/(n-1)\sum_{k \neq j} E_{kj}$. M_j measures the relative increment in the assessment of DMU_j when shifting from the model of peer-appraisal to that of self-appraisal. In that sense, it may identify mavericks as those that take advantage of the self-evaluation in their assessments. The higher M_j the more of a maverick is DMU_j. In practice, mavericks have often been identified as the efficient DMUs that appear in the reference sets of only a few other DMUs. Obviously, this counting procedure only applies to efficient DMUs, while M_j is applicable to all DMUs.

Note that the maverick index may also identify all-round performers. A DEA efficient DMU with a low value of M_j is a unit that is rated as efficient or near the efficiency with the profiles of weights of all the DMUs.

1.6.2 Classification of DMUs and Benchmarking

Doyle and Green (1994a) have also suggested the use of multivariate techniques, such as multi-dimensional scaling, principal component analysis or cluster analysis, for the classification of units into groups of DMUs on the basis of the information provided by the matrix of cross-efficiencies. Specifically, the correlation coefficient between a pair of columns tells us how similarly those two DMUs are appraised by their peers. Using these correlations as the elements in a matrix of resemblance and applying a clustering method yields clusters with similar DMUs. This could be of interest for purposes of benchmarking: the best peer-appraised DMU within each cluster, even though it may not be an efficient unit, is a suitable referent for other members of the cluster to compare against. These authors claim that this benchmark is inherently similar but "better" than other DMUs in the same cluster and, therefore, seems a more readily understable target to aim for than the linear combination of DMUs in the reference set provided by the conventional DEA, none of which may appear remotely similar to the unit under evaluation. This approach is followed in Wu et al. (2009b) for the benchmarking of countries at the Summer Olympics.

1.6.3 Fixed Cost and Resource Allocation

Du et al. (2014) use the cross-efficiency evaluation to approach cost and resource allocation problems. DEA has been successfully used to address those problems. However, the cross-efficiency evaluation provides a very reasonable and appropriate mechanism for allocating a shared resource/cost because it uses the concept of peer-appraisal. These authors claim that the allocations for fixed cost and resources resulting from their approach are more acceptable to the players involved because they are jointly determined by all DMUs rather than a specific one. All involved DMUs negotiate with one another to adjust the allocation plan for a better peer-evaluated performance until no one can improve further. A DEA-based iterative approach is developed, which is feasible and, especially for fixed cost allocation, ensures all DMUs to be efficient with the fixed cost allocated as an extra input measure.

1.7 Extensions

The standard approach to the cross-efficiency evaluation has been developed in the context of the CCR DEA model. Nevertheless, this methodology has been extended for use with non-oriented models—the models of directional distance functions (Chambers et al. 1998) and the multiplicative DEA model (Charnes et al. 1983)—, under variable returns to scale (VRS) and with fuzzy inputs and outputs. In addition, the original notion of cross-efficiency has also been generalized to deal with specific situations we sometimes find in DEA applications. These extensions broaden the range of applicability of the methodology.

1.7.1 Cross-Efficiency Evaluation with Directional Distance Functions

Ruiz (2013) explores the duality relations regarding the models of directional distance functions, which provide a measure of inefficiency in the sense of Farrell (1957), and establishes the equivalences with some fractional programming problems. This allows to defining the cross-efficiencies in the form of a ratio as follows

$$\mathrm{E}_{\mathrm{dj}}^{\beta} = \frac{\mathrm{v}^{\mathrm{d}'}\mathrm{X}_{\mathrm{j}} - \mathrm{u}^{\mathrm{d}'}\mathrm{Y}_{\mathrm{j}}}{\mathrm{v}^{\mathrm{d}'}\mathrm{X}_{\mathrm{j}} + \mathrm{u}^{\mathrm{d}'}\mathrm{Y}_{\mathrm{j}}}, \quad \mathrm{j} = 1, \ldots, \mathrm{n}. \tag{1.17}$$

It is shown that the cross-efficiencies (1.17) can actually be calculated by using the DEA weights $(\mathrm{v}^{\mathrm{d}}, \mathrm{u}^{\mathrm{d}})$ provided by the classical CCR model (1.2). The cross-efficiency score of a given $\mathrm{DMU_j}$, j = 1, . . ., n, is defined as the average of cross-efficiencies $\overline{\mathrm{E}}_{\mathrm{j}}^{\beta} = \frac{1}{\mathrm{n}}\sum_{\mathrm{d}=1}^{\mathrm{n}} \mathrm{E}_{\mathrm{dj}}^{\beta}$, j = 1, . . ., n, as usual. These scores can be used for ranking the DMUs.

Thus, this extension of the standard approach allows us to use the cross-efficiency evaluation with non-oriented measures of efficiency, i.e., which account for the inefficiency both in inputs and in outputs simultaneously.

1.7.2 Cross-Efficiency Evaluation with Multiplicative DEA Models

Cook and Zhu (2014) (see also Cook and Zhu 2015) develop an approach to the cross-efficiency evaluation based on the multiplicative DEA model. These authors define the cross-efficiency score of a given $\mathrm{DMU_j}$ as the geometric average of its cross-efficiencies. Then, they propose to evaluate each DMU_d with the so-called maximum log cross efficiency, which is the optimal value of the model

$$
\begin{aligned}
\text{Max} \quad & \left(\prod_{d=1}^{n} \frac{\prod_{r=1}^{s} y_{rj}^{u_r^d}}{\prod_{i=1}^{m} x_{ij}^{v_i^d}} \right)^{1/n} \\
\text{s.t.}: \quad & \\
& \frac{\prod_{r=1}^{s} y_{rj}^{u_r^d}}{\prod_{i=1}^{m} x_{ij}^{v_i^d}} \leq 1 \qquad d = 1, \ldots, n; \quad j = 1, \ldots, n \\
& \frac{\prod_{r=1}^{s} y_{rj}^{u_r^d}}{\prod_{i=1}^{m} x_{ij}^{v_i^d}} = \theta_d^{M*} \qquad d = 1, \ldots, n \\
& v_i^d, u_r^d \geq 1 \qquad i = 1, \ldots, m; \quad r = 1, \ldots, s; \quad d = 1, \ldots, n
\end{aligned}
\tag{1.18}
$$

where θ_d^{M*} is the efficiency score of DMU_d, $d = 1, \ldots, n$, provided by the multiplicative DEA model.

The attractive feature of this approach lies in that the cross-efficiency scores are uniquely determined with respect to the DEA weights, as they are defined as the optimal value of (1.18). Note, in any case, that this is not a conventional cross-efficiency approach in which the cross-efficiencies can be arranged in a matrix so that those in the same row are obtained with the same input and output weights: the weights associated with DMU_d in solving (1.18) for DMU_j are not necessarily the same as those that will be obtained when the model is solved for another $DMU_{j'}$.

1.7.3 Cross-Efficiency Evaluation Under VRS

Lim and Zhu (2015a) extend the cross-efficiency evaluation for use under VRS. To develop a way of resolving the problem of negative cross-efficiencies in the input-oriented VRS DEA model, they develop a geometric interpretation of the relationship between the VRS and CRS models. They show that, given an optimal solution (v^d, u^d, u_0^d) of an VRS-efficient DMU_d, a CRS efficiency score of DMU_d, measured under a translated Cartesian coordinate system defined by an adjusted origin determined by this optimal solution, is unity. This is interpreted as meaning that every DMU, via solving the VRS model, seeks for a translation of the Cartesian coordinate system and an optimal bundle of weights such that its CRS-efficiency score, measured under the chosen coordinate system, is maximized. Therefore, VRS cross-efficiency is related to the CRS cross-efficiency measures. In this context, Lim and Zhu (2015a) define the general concept of peer-evaluation in DEA as follows: “each DMU cross-evaluates other peer DMUs under its own best evaluation environment", where the best evaluation environment refers to the weights on the input–output factors as well as the new coordinate system that are most favourable to the DMU. Under this best evaluation environment, the DMU itself attains the highest efficiency score as well as the most productive scale size.

The cross-efficiencies are defined as follows

$$E_{dj}^{VRS} = \frac{u^{d'} Y_j}{v^{d'} X_j + u_0^d}, \quad j = 1, \ldots, n. \tag{1.19}$$

Note that E_{dd}^{VRS}, as defined in (1.19), does not coincide with the VRS efficiency score in the case of an inefficient DMU_d. Thus, for each DMU we have n cross-efficiencies and one (simple) efficiency score. Lim and Zhu (2015a) suggest to average the n cross-efficiencies to calculate an input-oriented VRS cross-efficiency score of the DMU. Alternatives are discussed in Lim and Zhu (2015b).

1.7.4 Fuzzy Cross-Efficiency Evaluation

Sirvent and León (2014) develop a fuzzy approach to the cross-efficiency evaluation, which allows us to extend the use of this methodology to the case of having imprecise data (that is, fuzzy inputs and outputs). These authors point out that the rankings of DMUs based on the ordering of fuzzy efficiencies can be criticized for the same reasons as those resulting from crisp DEA efficiency scores, which justifies the need of a fuzzy cross-efficiency evaluation. They also claim that, unlike in crisp DEA, it is not possible to set out a general approach to the cross-efficiency evaluation in Fuzzy Data Envelopment Analysis (FDEA), because there exist many different definitions of efficiency in FDEA. Thus, each fuzzy approach to the cross-efficiency evaluation will depend on the specific features of the FDEA model used for the measurement of the relative efficiency. In particular, they make some proposals to perform a cross-efficiency analysis based on the fuzzy DEA model by Guo and Tanaka (2001). These proposals are to be used in the case of fuzzy inputs and outputs being symmetrical triangular numbers, and the analysis is referred to a particular possibility level *h* in between 0 and 1 pre-specified by the decision-maker. The fuzzy cross-efficiencies are defined as non-symmetrical triangular fuzzy numbers, and the fuzzy cross-efficiency score of a given DMU is the average of its fuzzy cross-efficiencies obtained with the weights of all the DMUs. Once the fuzzy cross-efficiency scores of all the DMUs are obtained, they rank them by using a ranking index defined in Yager (1981), which eventually provides the ranking of DMUs. Since the FDEA model of Guo and Tanaka may have alternative optimal solutions for the input and output weights, Sirvent and León (2014) also propose, in a similar manner as in the crisp case, a fuzzy benevolent and a fuzzy aggressive formulation to choose among them. See Ruiz and Sirvent (2016) for a possibility approach to fuzzy cross-efficiency evaluation.

1.7.5 Game Cross Efficiency

Liang et al. (2008b) (see also Cook and Zhu 2015) claim that, in many DEA applications, some form of direct or indirect competition may exist among the DMUs under evaluation. To deal with this issue, they generalize the original cross-efficiency concept to the so-called DEA game-cross efficiency. Specifically, in that approach the DMUs are viewed as players in a game and the cross-efficiency scores as payoffs. Then, each DMU can choose to take a non-cooperative game stance to the extent that it will attempt to maximize its (worst possible) cross-efficiency under the condition that the cross-efficiency of each of the other DMUs does not deteriorate. The average game cross-efficiency score is obtained when the DMU's own maximized efficiency scores relative to each of the other units are averaged. To implement the DEA game cross-efficiency model proposed, an algorithm is derived which provides the wanted scores. Note again that this cannot be seen as a conventional cross-efficiency approach. One important difference lies in that the weights used to compute the cross-efficiencies of a given DMU are not necessarily an optimal solution of the CCR model (1.2). And, in addition, it cannot be ensured that the input and output weights used to calculate the game cross efficiencies for two units, say DMU_j and $DMU_{j'}$, relative to a given DMU_d, are the same, because each of the cross-efficiencies is the result of an independent optimization.

1.8 Conclusions

Liu et al. (2016) state that cross-efficiency evaluation and ranking "is a truly very focused subarea. Such a large coherent block of research studies indicates that many issues in cross-efficiency remain to be resolved and that there probably has not been a consensus on the method to address the issues in the original cross-efficiency concept". In our opinion, there was a need of an updated an organized survey of the literature on this methodology, and this chapter has tried to make a contribution to meeting such need. We have recapitulated the literature dealing with the two issues that have attracted more attention from the researchers: the use of alternative secondary goals to the choice of DEA weights among alternate optima and the aggregation of cross-efficiencies. And, in addition, it has been reviewed the existing work on other uses of the cross-efficiency evaluation different from the ranking of DMUs and the extensions of the standard approach, which still offer interesting directions for future research.

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