International Series in Operations Research & Management Science

Shiuh-Nan Hwang
Hsuan-Shih Lee
Joe Zhu *Editors*

Handbook of Operations Analytics Using Data Envelopment Analysis

Springer

International Series in Operations Research & Management Science

Volume 239

More information about this series at http://www.springer.com/series/6161

Shiuh-Nan Hwang • Hsuan-Shih Lee • Joe Zhu
Editors

Handbook of Operations Analytics Using Data Envelopment Analysis

Springer

Editors
Shiuh-Nan Hwang
Department of Business Administration
Ming Chuan University
Taipei, Taiwan

Hsuan-Shih Lee
College of Maritime Science and Management
National Taiwan Ocean University
Keelung, Taiwan

Joe Zhu
School of Business
Worcester Polytechnic Institute
Worcester, MA, USA

ISSN 0884-8289 ISSN 2214-7934 (electronic)
International Series in Operations Research & Management Science
ISBN 978-1-4899-7703-8 ISBN 978-1-4899-7705-2 (eBook)
DOI 10.1007/978-1-4899-7705-2

Library of Congress Control Number: 2016941328

Printed on acid-free paper

This Springer imprint is published by Springer Nature
The registered company is Springer Science+Business Media LLC New York

Preface

In recent years, handbooks on Data Envelopment Analysis (DEA) have been published. They include *Handbook on Data Envelopment Analysis* (eds, W.W. Cooper, L.M. Seiford, and J. Zhu, 2011, Springer), *Data Envelopment Analysis: A Handbook of Modeling Internal Structures and Networks* (eds, W.D. Cook and J. Zhu, 2014, Springer), *Data Envelopment Analysis: A Handbook of Models and Methods* (ed. J. Zhu, 2015, Springer), and *Data Envelopment Analysis: A Handbook of Empirical Studies and Applications* (ed. J. Zhu, 2016, Springer). It is well known that DEA is a "data-oriented" approach for evaluating the performance of a set of entities called decision-making units (DMUs) whose performance is categorized by multiple metrics. These performance metrics are classified or termed as inputs and outputs. In general, DEA finds an envelopment for a set of data. This envelopment is called efficient frontier (in production theory) or best-practice frontier (in benchmarking terminology). While many DEA applications can be viewed as estimation of production functions, DEA can also be applied to manufacturing as well as service operations where DEA is used as a versatile tool for making various operational decisions.

To complement the existing DEA handbooks, the current handbook focuses on DEA applications in operations analytics which are fundamental tools and techniques for improving operation functions and attaining long-term competitiveness. In fact, the chapters in the handbook demonstrate that DEA can be viewed as Data Envelopment *Analytics*.

Chapter 1, by Ruiz and Sirvent, reviews cross-efficiency evaluation which provides a peer appraisal in which each DMU is evaluated from the perspective of all of the others by using their DEA weights.

Chapter 2, by Zhou, Poh, and Ang, presents a case study on measuring the environmental performance of OECD countries. Environmental performance measurement provides an analytical foundation for environmental policy analysis and decision making.

Chapter 3, by Serrano-Cinca, Mar-Molinero, and Fuertes Callén, demonstrates how to select a set of performance metrics (inputs and outputs) in DEA with an application to American banks.

Chapter 4, by Liu, proposes using a relational network model to take the operations of individual periods into account in measuring efficiencies, and the input and output data are treated as fuzzy numbers.

Chapter 5, by Avkiran and Zhu, shows how the efficient frontier methods DEA and stochastic frontier analysis (SFA) can be used synergistically. As part of the illustration, the authors directly compare locally incorporated foreign banks with Chinese domestic banks.

Chapter 6, by de la Torre, Sagarra, and Agasisti, integrates DEA and multidimensional scaling, with the aim to discuss the potential complementarities and advantages of combining both methodologies in order to reveal the efficiency framework and institutional strategies of the Spanish higher education system.

Chapter 7, by Wu, Kweh, Lu, Hung, and Chang, constructs a dynamic three-stage network DEA model which evaluates the R&D efficiency, technology-diffusion efficiency, and value-creation efficiency of Taiwanese R&D organizations over the period 2005–2009.

Chapter 8, by Wanke and Barros, presents a bootstrapping-based methodology to evaluate returns to scale and convexity assumptions in DEA.

Chapter 9, by Lozano, Hinojosa, Marmol, and Borrero, studies the possibilities of hybridizing DEA and cooperative games. Specifically, bargaining games and transferable utility games (TU games) are considered.

Chapter 10, by Fukuyama and Weber, uses DEA to represent the production technology and directional distance functions to measure bank performance. The performance measure allows the researcher to compare observed inputs and outputs, including undesirable outputs, with the outputs and inputs that might be produced if a producer were able to optimally choose production plans relative to a dynamic benchmark technology.

Chapter 11, by Ke, presents an input-specific Luenberger energy and environmental productivity indicator. DEA is utilized to estimate the directional distance function for composing the Luenberger energy and environmental productivity indicator.

Chapter 12, by Mehdiloozad and Sahoo, addresses the issue of reference set by differentiating between the uniquely found reference set, called the global reference set (GRS), and the unary and maximal types of the reference set for which the multiplicity issue may occur. The authors propose a general linear programming-based approach that is computationally more efficient than its alternatives. The authors define the returns to scale of an inefficient DMU at its projection point that is produced by all—but not some—of the units in its GRS.

Chapter 13, by Lim, Jahromi, Anderson, and Tudori, evaluates and compares the technological advancement observed in different hybrid electric vehicle (HEV) market segments over the past 15 years. The results indicate that the introduction of a wide range of midsize HEVs is posing a threat to the two-seaters and compact

HEV segments, while an SUV segment shows a fast adoption with a significant performance improvement.

Chapter 14, by Ding, Feng, and Wu, provides radial measurements of efficiency for the production process possessing multicomponents under different production technologies. Their approach is based on the construction of various empirical production possibility sets. Then the authors propose a procedure that is unaffected by multiple optima for estimating returns to scale.

Chapter 15, by Harrison and Rouse, considers issues around the use of accounting information in DEA with suggestions on how accounting data can be used in the modeling process and how DEA can be combined with other accounting approaches to improve performance evaluation.

Chapter 16, by Sueyoshi, explains how to use DEA environmental assessment to establish corporate sustainability and discusses that environmental assessment and protection are important concerns in modern business.

Chapter 17, by Sueyoshi and Yuan, summarizes previous works on the research efforts, including concepts and methodologies, on DEA environmental assessment applied to energy in the past three decades.

Chapter 18, by Sarkis, provides an overview of DEA and how it can be utilized alone and with other techniques to investigate corporate environmental sustainability questions. Some future DEA directions that could be used for research and application in corporate environmental sustainability are also defined.

We hope that this handbook, along with other aforementioned DEA handbooks, can serve as a reference for researchers and practitioners using DEA and as a guide for further development of DEA. We thank reviewers who provided valuable suggestions and comments to the chapters. We are also grateful to the authors who make important contributions toward advancing the DEA research frontier.

References

Cook, W. D., & Zhu, J. (2014). *Data envelopment analysis: A handbook of modeling internal structures and networks*. Springer.

Cooper, W. W., Seiford, L. M., & Zhu, J. (2011). *Handbook on data envelopment analysis* (2nd ed.). Springer.

Zhu, J. (2015). *Data envelopment analysis: A handbook of models and methods*. Springer.

Zhu, J. (2016). *Data envelopment analysis: A handbook of empirical studies and applications*. Springer.

Taipei, Taiwan — Shiuh-Nan Hwang
Keelung, Taiwan — Hsuan-Shih Lee
Worcester, MA, USA — Joe Zhu

Contents

1 **Ranking Decision Making Units: The Cross-Efficiency Evaluation** 1
José L. Ruiz and Inmaculada Sirvent

2 **Data Envelopment Analysis for Measuring Environmental Performance** 31
Peng Zhou, Kim Leng Poh, and Beng Wah Ang

3 **Input and Output Search in DEA: The Case of Financial Institutions** 51
C. Serrano Cinca, C. Mar Molinero, and Y. Fuertes Callén

4 **Multi-period Efficiency Measurement with Fuzzy Data and Weight Restrictions** 89
Shiang-Tai Liu

5 **Pitching DEA Against SFA in the Context of Chinese Domestic Versus Foreign Banks** 113
Necmi Kemal Avkiran and Yushu (Elizabeth) Zhu

6 **Assessing Organizations' Efficiency Adopting Complementary Perspectives: An Empirical Analysis Through Data Envelopment Analysis and Multidimensional Scaling, with an Application to Higher Education** 145
Eva M. de la Torre, Marti Sagarra, and Tommaso Agasisti

7 **Capital Stock and Performance of R&D Organizations: A Dynamic DEA-ANP Hybrid Approach** 167
Yueh-Cheng Wu, Qian Long Kweh, Wen-Min Lu, Shiu-Wan Hung, and Chia-Fa Chang

8 **Evaluating Returns to Scale and Convexity in DEA Via Bootstrap: A Case Study with Brazilian Port Terminals** 187
Peter F. Wanke and Carlos Pestana Barros

9 **DEA and Cooperative Game Theory** 215
Sebastián Lozano, Miguel Ángel Hinojosa
Amparo María Mármol, and Diego Vicente Borrero

10 **Measuring Bank Performance: From Static Black Box to Dynamic Network Models** 241
Hirofumi Fukuyama and William L. Weber

11 **Evaluation and Decomposition of Energy and Environmental Productivity Change Using DEA** 267
Ke Wang

12 **Identifying the Global Reference Set in DEA: An Application to the Determination of Returns to Scale** 299
Mahmood Mehdiloozad and Biresh K. Sahoo

13 **Technometrics Study Using DEA on Hybrid Electric Vehicles (HEVs)** 331
Dong-Joon Lim, Shabnam R. Jahromi, Timothy R. Anderson, and Anca-Alexandra Tudorie

14 **A Radial Framework for Estimating the Efficiency and Returns to Scale of a Multi-component Production System in DEA** 351
Jingjing Ding, Chenpeng Feng, and Huaqing Wu

15 **DEA and Accounting Performance Measurement** 385
Julie Harrison and Paul Rouse

16 **DEA Environmental Assessment (I): Concepts and Methodologies** 413
Toshiyuki Sueyoshi

17 **DEA Environmental Assessment (II): A Literature Study** 445
Toshiyuki Sueyoshi and Yan Yuan

18 **Corporate Environmental Sustainability and DEA** 483
Joseph Sarkis

Index 499

Contributors

Tommaso Agasisti Department of Management, Economics and Industrial Engineering, School of Management, Politecnico di Milano, Milano, Italy

Timothy R. Anderson Department of Engineering and Technology Management, Portland State University, Portland, OR, USA

Beng Wah Ang Department of Industrial and Systems Engineering, National University of Singapore, Singapore, Singapore

Necmi Kemal Avkiran UQ Business School, The University of Queensland, Brisbane, QLD, Australia

Carlos Pestana Barros UECE (Research Unit on Complexity and Economics), Instituto Superior de Economia e Gestão, Technical University of Lisbon, Lisbon, Portugal

Diego Vicente Borrero Pablo de Olavide University, Sevilla, Spain

Y. Fuertes Callén Department of Accounting and Finance, University of Saragossa, Zaragoza, Spain

Chia-Fa Chang Department of Financial Management, National Defense University, Taipei, Taiwan

C. Serrano Cinca Department of Accounting and Finance, University of Saragossa, Zaragoza, Spain

Jingjing Ding School of Management, Hefei University of Technology, Hefei, P.R. China

Chenpeng Feng School of Management, Hefei University of Technology, Hefei, P.R. China

Hirofumi Fukuyama Fukuoka University, Fukuoka, Japan

Julie Harrison Department of Accounting and Finance, The University of Auckland, Auckland, New Zealand

Miguel Ángel Hinojosa Pablo de Olavide University, Sevilla, Spain

Shiu-Wan Hung Department of Business Administration, National Central University, Jung-Li City, Taiwan

Shabnam R. Jahromi Department of Engineering and Technology Management, Portland State University, Portland, OR, USA

Qian Long Kweh Department of Accounting, College of Business Management and Accounting, Universiti Tenaga Nasional, Muadzam Shah, Pahang, Malaysia

Dong-Joon Lim Department of Engineering and Technology Management, Portland State University, Portland, OR, USA

Shiang-Tai Liu Graduate School of Business and Management, Vanung University, Chung-Li, Taiwan, ROC

Sebastián Lozano Sevilla University, Sevilla, Spain

Wen-Min Lu Department of Financial Management, National Defense University, Taipei, Taiwan

Amparo María Mármol Sevilla University, Sevilla, Spain

Mahmood Mehdiloozad Department of Mathematics, College of Sciences, Shiraz University, Shiraz, Iran

C. Mar Molinero Kent Business School, University of Kent, Canterbury, UK
Department of Management, Universitat Autònoma de Barcelona, Barcelona, Spain

Kim Leng Poh Department of Industrial and Systems Engineering, National University of Singapore, Singapore, Singapore

Paul Rouse Department of Accounting and Finance, The University of Auckland, Auckland, New Zealand

José L. Ruiz Centro de Investigación Operativa, Universidad Miguel Hernández, Elche (Alicante), Spain

Marti Sagarra Departament d'Economia, Facultat de Ciències Econòmiques i Empresarials, Universitat de Girona, Girona, Spain

Biresh K. Sahoo Xavier Institute of Management, Xavier University, Bhubaneswar, India

Joseph Sarkis Foisie School of Business, Worcester Polytechnic Institute, Worcester, MA, USA

Inmaculada Sirvent Centro de Investigación Operativa, Universidad Miguel Hernández, Elche (Alicante), Spain

Toshiyuki Sueyoshi Department of Management, New Mexico Institute of Mining and Technology, Socorro, NM, USA

Eva M. de la Torre Departamento de Economía y Hacienda Pública, Facultad de Ciencias Económicas y Empresariales, Universidad Autónoma de Madrid, Madrid, Spain

Anca-Alexandra Tudorie Department of Policy Analysis, Delft University of Technology, Delft, The Netherlands

Ke Wang School of Management and Economics, Beijing Institute of Technology, Beijing, China
Center for Energy and Environmental Policy Research, Beijing Institute of Technology, Beijing, China

Peter F. Wanke CELIG (Center for Studies in Logistics, Infrastrucutre and Management), Instituto COPPEAD de Administração, Universidade Federal do Rio de Janeiro, Rio de Janeiro, Brazil

William L. Weber Department of Economics and Finance, Southeast Missouri State University, Girardeau, MO, USA

Huaqing Wu School of Economics, Hefei University of Technology, Hefei, P.R. China

Yueh-Cheng Wu Department of Cultural Vocation Development, National Taipei University of Technology, Taipei, Taiwan

Yan Yuan Department of Management, New Mexico Institute of Mining and Technology, Socorro, NM, USA

Peng Zhou College of Economics and Management, Nanjing University of Aeronautics and Astronautics, Nanjing, China

Yushu (Elizabeth) Zhu UQ Business School, The University of Queensland, Brisbane, QLD, Australia

Chapter 1
Ranking Decision Making Units: The Cross-Efficiency Evaluation

José L. Ruiz and Inmaculada Sirvent

Abstract This chapter surveys the literature on the cross-efficiency evaluation, which is a methodology for ranking decision making units (DMUs) involved in a production process regarding their efficiency. Cross-efficiency evaluation has been developed in the context of analyses of relative efficiency carried out with Data Envelopment Analysis (DEA). It is usually claimed that the DEA efficiency scores cannot be used for ranking, because they result from a self-evaluation of units based on DMU-specific input and output weights. Cross-efficiency evaluation, in contrast, provides a peer-appraisal in which each DMU is evaluated from the perspective of all of the others by using their DEA weights. This makes it possible to derive an ordering. We make an exhaustive review of the existing work on the different issues related to the cross-efficiency evaluation. Other uses of this methodology different from the ranking of DMUs as well as the extensions that have been developed are also outlined.

Keywords Cross-efficiency evaluation • Ranking • DEA

1.1 Introduction

In decision making processes, ranking constitutes a crucial step for choosing among alternatives after their evaluation. In Multi-Attribute Decision Making (MADM) problems we have n alternatives which are assessed against m criteria. The evaluations that result from these assessments provide the final ranking values of the alternatives. Usually, the higher the ranking value the better the performance of the alternative, so the alternative with the highest ranking value is considered as the best of the alternatives.

Rankings have experienced an increasing popularity. An example of this can be found in Higher Education with the university rankings or league tables.

J.L. Ruiz (✉) • I. Sirvent
Centro de Investigación Operativa, Universidad Miguel Hernández, Avd. de la Universidad, s/n, Elche (Alicante) 03202, Spain
e-mail: jlruiz@umh.es; isirvent@umh.es

S.-N. Hwang et al. (eds.), *Handbook of Operations Analytics Using Data Envelopment Analysis*, International Series in Operations Research & Management Science 239, DOI 10.1007/978-1-4899-7705-2_1

Most visible international rankings are The Academic Ranking of World Universities (ARWU) by Shanghai Jiao Tung University, commonly known as the Shanghai index and the World University Ranking by Times Higher Education (THESQS). As has been widely acknowledged in the related literature, university rankings are controversial but influential. Despite their limitations, university rankings have some effect on decision making regarding higher education institutions: on the choice of a convenient place by students, on recruitment decisions by employers, on university policies, motivating the competitiveness among them, etc. See De Witte and Hudrlikova (2013) for a discussion on this issue and a review of the literature.

We here are concerned with the assessment of performance of DMUs involved in production processes. Specifically, the focus is on the evaluation of their relative efficiency in the use of several inputs to produce several outputs by means of DEA models. The DEA efficiency scores provide a self-evaluation of DMUs based on the inputs and output weights that show them in their best possible light. Thus, since the DMUs are evaluated with DMU-specific weights (which often differ across units), it is usually claimed that the efficiency scores that result from DEA models cannot be used for purposes of ranking DMUs.

The chapter is devoted to the so-called cross-efficiency evaluation. This methodology, as introduced in Sexton et al. (1986) and Doyle and Green (1994a), arose as an extension of DEA aimed at ranking DMUs. The idea behind the cross-efficiency evaluation is to apply one DMU's perspective to others, by using its DEA weights in the evaluations. That is, the efficiency of each unit is assessed with the weights of all the DMUs instead of with only its own weights. Each of these assessments, which are called the cross-efficiencies, is defined as the classical efficiency ratio of a weighted sum of outputs to a weighted sum of inputs. Eventually, the cross-efficiency score of a given unit is calculated as the average of the cross-efficiencies of such unit obtained with the weights of all the DMUs. Cross-efficiency evaluation provides thus a peer-evaluation of the DMUs, instead of a self-evaluation, which makes it possible to derive an ordering. We highlight the parallelism between the cross-efficiency evaluation and MADM problems. Cross-efficiency evaluation can be seen as a MADM problem in which the DMUs are the alternatives and the DEA weights of each of them act as the criteria used in the evaluations.

Cross-efficiency evaluation has received much attention in the related literature. In fact, "cross-efficiency evaluation and ranking" is identified as one of the four research fronts in DEA in the study carried out by Liu et al. (2016), which applies a network clustering method in order to group the DEA literature over the period 2000–2014. We also note that this methodology has been widely applied for ranking performance of DMUs in many different contexts. Sexton et al. (1986) included an evaluation of nursing homes while in Doyle and Green (1994b) an application to higher education can be found. See also Oral et al. (1991) for an application to R&D projects, Green et al. (1996) to preference voting, Baker and Talluri (1997) to industrial robot selection and Talluri and Yoon (2000) to the selection of advanced manufacturing technology (AMT). More recently, this methodology has been applied to the electricity distribution sector in Chen (2002), for the determination of the best labor assignment in a cellular manufacturing system in Ertay and Ruan (2005), to economic-environmental performance in Lu and Lo (2007), to sport in Wu et al. (2009a, b), Cooper et al. (2011), Ruiz et al. (2013) and Gutiérrez and Ruiz (2013a, b), to public procurement in Falagario et al. (2012) and to portfolio selection in Lim et al. (2014).

We review here the literature on the different issues related to the cross-efficiency evaluation. This includes the choice of DEA weights among alternate optima by using alternative secondary goals (Sect. 1.4) and the aggregation of cross-efficiencies (Sect. 1.5). Other uses of the cross-efficiency evaluation different from that concerned with rankings are discussed (Sect. 1.6), together with the extensions of the standard approach that have been developed and broaden the range of applicability of this methodology (Sect. 1.7). Previously, Sect. 1.2 summarizes the existing ranking methods in DEA and Sect. 1.3 briefly describes the standard approach to the cross-efficiency evaluation. Last section concludes.

1.2 Ranking Methods in DEA

The literature has widely dealt with the ranking of DMUs in the context of DEA. Adler et al. (2002) and Hosseinzadeh Lotfi et al. (2013) provide a couple of reviews, while the review of methods for improving discrimination in DEA in Angulo-Meza and Estellita Lins (2002) also considers some methods for ranking DMUs.

This body of research can be roughly described as follows. Firstly, we should mention the rankings that result from efficiency ratios obtained by using a common set of weights (CSW). CSW has the appeal of a fair and impartial evaluation in the sense that each variable is attached the same weight in the assessments of all the DMUs. This approach has been often followed in the efficiency analyses made in Economics and Engineering. Regarding that approach, Doyle and Green (1994a) point out that the choice itself of such weights often raises serious difficulties, and in many cases there is no universally agreed-upon the weights to be used. We note that there exist some DEA-based methods aimed at finding a CSW: see Ganley and Cubbin (1992), Roll and Golany (1993), Troutt (1997), Despotis (2002), Kao and Hung (2005), Liu and Peng (2008, 2009), Ramón et al. (2011) and Ramón et al. (2012).

The methods based on either the cross-efficiency evaluation or the super-efficiency score (Andersen and Petersen 1993) have been those that have received more attention in view of the number of published papers dealing with these issues. As said before, this chapter is devoted to the cross-efficiency evaluation, so it is described subsequently in detail. The super-efficiency score results from the evaluation of the DMUs with respect to the technology estimated by excluding the unit under assessment from the sample. This kind of scores (see Hashimoto 1997; Sueyoshi 1999 and Tone 2002) have been widely used for ranking DMUs, and their use has also been extended for the analysis of sensitivity and the detection of outliers. The infeasibility problems of the super-efficiency score are usually highlighted as a drawback of this efficiency measure, as well as the fact that it results from DMU-specific weights if it is used for purposes of ranking (as in DEA).

Some existing methods propose to rank DMUs through the benchmarking (see Sinuany-Stern et al. 1994 and Torgersen et al. 1996). The basic idea behind them is that a given DMU should rank high if it is frequently used as referent in the evaluation of the remaining units (obviously, these methods can only rank efficient DMUs). Other group of methods utilizes multivariate statistical techniques like canonical correlation analysis and discriminant analysis to rank the DMUs (see Sinuany-Stern et al. 1994 and Friedman and Sinuany-Stern 1997). These techniques are usually applied once the DEA classification into efficient and inefficient units has been obtained, and rank the units by using common weights. Empirically, non-parametric tests seem to show compatibility between the rank and the DEA dichotomic classification. Finally, we can mention a last group of papers that combine DEA and multi-criteria decision-making methods, such as AHP, fuzzy logic or multi-objective linear programming (see Halme et al. 1999; Li and Reeves 1999 and Kao and Liu 2000). Some of these approaches require the collection of additional, preferential information from relevant decision makers, which could be considered as the weakness of these methods.

Obviously, these methods have all their own attractive features and weaknesses, so no of them could be prescribed as the complete solution to the question of ranking.

1.3 The Cross-Efficiency Evaluation: The Standard Approach

Throughout the paper we assume that we have n DMUs that use m inputs to produce s outputs. These can be described by means of the vectors (X_j, Y_j), $j = 1, \ldots, n$, which are assumed to be non-negative. We also denote by X the $m \times n$ matrix of input vectors and by Y the $s \times n$ matrix of output vectors. The standard cross-efficiency evaluation is based on the CCR DEA model (Charnes et al. 1978), which is an oriented radial model. The following problem is the CCR model in its ratio form when used for the assessment of relative efficiency of a given DMU_0

$$
\begin{array}{lll}
\text{Max} & \theta_0 = \dfrac{u'Y_0}{v'X_0} & \\
\text{s.t.}: & \dfrac{u'Y_j}{v'X_j} \le 1 & j = 1, \ldots, n \\
& v \ge 0_m, \quad u \ge 0_s &
\end{array}
\tag{1.1}
$$

In short, the optimal value of (1.1) is the DEA efficiency score of DMU_0 while the ratios in the constraints provide the cross-efficiencies of the remaining units calculated with the weights of DMU_0.

Model (1.1) is non-linear. Nevertheless, by using the results on linear fractional programming in Charnes and Cooper (1962), it can be converted into the following linear problem (which is the so-called dual multiplier formulation)

$$\begin{array}{ll} \text{Max} & u'Y_0 \\ \text{s.t.}: & v'X_0 = 1 \\ & -v'X_j + u'Y_j \leq 0 \quad j = 1, \ldots, n \\ & v \geq 0_m, \ \ u \geq 0_s \end{array} \tag{1.2}$$

Thus, if (v^d, u^d) is an optimal solution of (1.2) for a given DMU_d, then the cross-efficiency of DMU_j, $j = 1, \ldots, n$, obtained with the weights of DMU_d is the following

$$E_{dj} = \frac{u^{d'} Y_j}{v^{d'} X_j} \tag{1.3}$$

The E_{dj} ' s are usually collected in the so-called matrix of cross-efficiencies

$$E = \begin{pmatrix} E_{11} & \ldots & E_{1j} & \ldots & E_{1n} \\ \ldots. & & \ldots. & & \ldots. \\ E_{d1} & \ldots & E_{dj} & \ldots & E_{dn} \\ \ldots. & & \ldots. & & \ldots. \\ E_{n1} & \ldots & E_{nj} & \ldots & E_{nn} \end{pmatrix} \tag{1.4}$$

In each row d, we have the evaluations of the different units calculated with the DEA weights of DMU_d (so the main diagonal of the matrix contains the DEA efficiency scores). In each column j, we have the efficiencies of a given DMU_j calculated with the weights of all the DMUs. In fact, the cross-efficiency score of DMU_j, $j = 1, \ldots, n$, is usually defined as the average of the cross-efficiencies in the corresponding column. That is,

$$\overline{E}_j = \frac{1}{n} \sum_{d=1}^{n} E_{dj}, \quad j = 1, \ldots, n. \tag{1.5}$$

The cross-efficiency score $\bar{E}_j$ provides a peer-evaluation of DMU_j, and the DMUs can be ranked according to the values $\bar{E}_j$, $j = 1, \ldots, n$. The fact that the cross-efficiencies in each of the rows of E are obtained by using the same input and output weights is the reason why an ordering of DMUs can be derived on the basis of the cross-efficiency scores.

The literature has emphasized the following two as the principal advantages of the cross-efficiency evaluation: (1) it provides an ordering of the DMUs and (2) it eliminates unrealistic weighting schemes without requiring the elicitation of weight restrictions (see, for example, Anderson et al. 2002). Doyle and Green (1994a) have also highlighted the interpretation of the cross-efficiency evaluation as peer-appraisal. As a result, these authors suggest that cross-efficiency evaluation has less of the arbitrariness of additional constraints and has more of the right connotations of a democratic process, as opposed to authoritarianism (externally imposed weights, CSW) or egoism (self-appraisal, DEA).

But there are also some difficulties with the cross-efficiency evaluation. As it happens with other DEA-based approaches for ranking, for example with the super-efficiency score or even with the rankings provided by CSWs obtained by using DEA, there exists the possibility of rank reversal. That is, if a new DMU were added to the sample, then the ranking could change. Thus, the rank of a given DMU that results from the cross-efficiency scores should be seen as reflecting its relative position in presence of the DMUs considered in the sample. As discussed in Wang and Luo (2009), the rank reversal phenomenon occurs in many decision making approaches such as the Analytic Hierarchy Process (AHP), the Borda–Kendall (BK) method for aggregating ordinal preferences, the simple additive weighting (SAW) method and the technique for order preference by similarity to ideal solution (TOPSIS) method. These authors eventually claim that rank reversal "might be a normal phenomenon".

However, the problems with the alternate optima for the DEA weights have been the ones widely acknowledged as the main weakness of this methodology. The existence of alternative optimal solutions in (1.2) is a factor that may reduce the usefulness of the cross-efficiency evaluation, because we may have different cross-efficiency scores (and, consequently, different rankings) depending on the choice of DEA weights that is made. This is probably the issue related to the cross-efficiency evaluation that has received more attention in the literature. We discuss it in the next section.

1.4 The Choice of DEA Weights in Cross-Efficiency Evaluations

As a potential remedy to resolve the ambiguity of the multiple DEA weights, Sexton et al. (1986) already suggested making a choice among alternate optima by using some alternative secondary goal. They proposed the two well-known benevolent and aggressive approaches used to that end. The idea behind them is that DMU_d chooses among its optimal weights those that maximize/minimize in some way the cross-efficiencies of the other units. Some models were developed, which involve the use of different surrogates that try to avoid the non-linear formulations that result from the inclusion of the cross-efficiencies, which are ratios, in the problems. For example, instead of maximizing (minimizing) the sum of cross-efficiency ratios themselves, these authors suggested that an adequate surrogate is to minimize (maximize) the sum of the denominators of the fractions minus the sum of the numerators. Doyle and Green (1994a) implemented the benevolent/aggressive models below following those ideas

$$
\begin{aligned}
\text{Max/Min} \quad & \sum_{j \neq d} \left(u^{d'} Y_j - v^{d'} X_j \right) \\
\text{s.t.:} \quad & v^{d'} X_d = 1 \\
& -\theta_d^* v^{d'} X_d + u^{d'} Y_d = 0 \\
& -v^{d'} X_j + u^{d'} Y_j \leq 0 \qquad j = 1, \ldots, n, \quad j \neq d \\
& v^d \geq 0_m, \quad u^d \geq 0_s
\end{aligned}
\tag{1.6}
$$

where θ_d^* is the DEA efficiency score of DMU_d.

In line with that approach, these authors also proposed the following two formulations

$$
\begin{aligned}
\text{Max/Min} \quad & u^{d'} \sum_{j \neq d} Y_j \\
\text{s.t.:} \quad & v^{d'} \sum_{j \neq d} X_j = 1 \\
& -\theta_d^* v^{d'} X_d + u^{d'} Y_d = 0 \\
& -v^{d'} X_j + u^{d'} Y_j \leq 0 \qquad j = 1, \ldots, n, \quad j \neq d \\
& v^d \geq 0_m, \quad u^d \geq 0_s
\end{aligned}
\tag{1.7}
$$

which are two models that seek, as secondary goal, to maximize/minimize the efficiency of a composite DMU, while keeping unchanged the DEA efficiency score of DMU_d, θ_d^*.

Liang et al. (2008a) extend the work in Doyle and Green (1994a) by introducing various secondary objective functions, which are formulated in terms of the deviation variables $\alpha_j^d = v^{d'} X_j - u^{d'} Y_j$, $j = 1, \ldots, n$. The first secondary goal gives rise to the following model, which is equivalent to the benevolent formulation in (1.6)

$$
\begin{aligned}
\text{Min} \quad & \sum_{j=1}^{n} \alpha_j^d \\
\text{s.t.:} \quad & v^{d'} X_d = 1 \\
& u^{d'} Y_d = 1 - \alpha_d^{d*} \\
& -v^{d'} X_j + u^{d'} Y_j + \alpha_j^d = 0 \quad j = 1, \ldots, n \\
& v^d \geq 0_m, \quad u^d \geq 0_s
\end{aligned}
\tag{1.8}
$$

where $\alpha_d^{d*} = 1 - \theta_d^*$. This model minimizes the total deviation from the ideal point defined as the multiplier bundle for which every DMU is efficient, that is,

$\alpha_j^d = 0, j = 1, \ldots n$. The following two secondary goals are also proposed in that paper with the purpose of deriving weights for which the cross-efficiencies are as similar as possible:

1. Minimizing the maximum deviation variable

$$\text{Min Max } \alpha_d^j \tag{1.9}$$

   which is related to maximizing the minimum cross-efficiency among the n DMUs, and
2. Minimizing the mean absolute deviation

$$\text{Min } \frac{1}{n}\sum_{j=1}^{n}\left|\alpha_d^j - \overline{\alpha}_d\right| \tag{1.10}$$

   which is aimed at minimizing the variation among the cross-efficiencies of the DMUs, where $\overline{\alpha}_d = \frac{1}{n}\sum_{j=1}^{n}\alpha_d^j$.

The new models can be formulated by simply changing the objective of (1.8) with those in (1.9) and (1.10).

Wang and Chin (2010b) state that the three models above are established on the basis of an unrealistic ideal point and formulate some variants with the following differences: (1) the ideal point is associated with the multiplier bundle for which all the DMUs achieve their DEA efficiency scores $(\theta_1^*, \ldots, \theta_n^*)$, instead of using the value 1 as the target efficiency of each DMU, which is only achievable for the efficient units. As a result, the constraints $-v^{d'}X_j + u^{d'}Y_j + \alpha_j^d = 0, j = 1, \ldots, n$, in the Liang et al.'s models are replaced by $-v^{d'}\theta_j^* X_j + u^{d'}Y_j + \alpha_j^d = 0,\ j = 1,\ \ldots,\ n$; (2) the normalizing constraint is the same in the formulations associated with all the DMU_d's. In particular, they suggest the following constraint $v^{d'}\sum_{j=1}^{n}X_j + u^{d'}\sum_{j=1}^{n}Y_j = n$, and (3) aggressive formulations are also proposed (note that models (1.8)–(1.10) follow a benevolent approach).

Obviously, neither of the models we have just discussed is better than the others. The use of them in practice will depend on the circumstances. For instance, Liang et al. (2008a) suggest that minimizing the total deviation as in (1.8) would be an appropriate approach to the cross-efficiency evaluation when the DMUs are assumed to be in a non-cooperative and fully competitive mode. For example, in a supply chain where each member is acting in its own self-interest, without being concerned for the others. In contrast, minimizing the maximum deviation, (1.9), might be deemed appropriate in settings where a more cooperative situation prevails. For example, in the evaluation of bank branches under a single corporate head, where the worst performing units would be given the least gap possible between where they are and where they need to be. Minimizing the mean absolute deviation, (1.10), aims at equalizing the various efficiency scores. So, if we were concerned with an allocatable resource such as the equipment for the maintenance crews, this model might tend to result in the least amount of redistribution (to render the DMUs equally efficient) in regard to that resource.

Other approaches focus on the suitability of the profiles of DEA weights that are chosen without dealing directly with the cross-efficiencies. As said before, one of the advantages of the cross-efficiency evaluation is that it eliminates unrealistic weighting schemes without requiring the elicitation of weight restrictions. The idea is that the effects of unreasonable weights are cancelled out in the summary that the cross-efficiency evaluation makes (Anderson et al. 2002). However, as Ramón et al. (2010a) state, we may have more comprehensive cross-efficiency scores if we actually avoid unreasonable weights instead of expecting that their effects are eliminated in the amalgamation of weighting schemes. By unrealistic weighting schemes we often mean the profiles of weights with zeros. The literature has widely claimed the need to avoid zero weights because they imply that some of the inputs and/or outputs considered for the analysis are ignored in the assessments. But the literature has also claimed against the large differences usually found in the weights as a result of the DEA total weight flexibility. These include both the differences in the input weights and in the output weights used in the evaluation of a DMU (Cook and Seiford 2008 state that "the AR concept was developed to prohibit large differences in the values of multipliers") and the differences in the weights attached to the same variable by the different DMUs (see Roll et al. 1991; Pedraja-Chaparro et al. 1997 and Thanassoulis et al. 2004).

To prevent unrealistic weighting schemes in cross-efficiency evaluations different strategies have been followed. Ramón et al. (2010b) classify the DMUs in two sets NZ and Z. In NZ we have the DMUs that can make a choice of non-zero weights among their alternate optima, while Z consists of those that cannot. That is, NZ = E ∪E′ ∪ NE ∪ NE′ and Z = F ∪ NF according to the classification of DMUs in Charnes et al. (1991). Then, they propose that the DMU_d's in NZ choose among their alternate optima the profiles with the least dissimilar weights, by using the following model in Ramón et al. (2010a)

$$
\begin{array}{lll}
\text{Max} & \varphi_d & \\
\text{s.t. :} & & \\
& \sum_{i=1}^{m} \nu_i^d x_{id} = 1 & \\
& \sum_{r=1}^{s} \mu_r^d y_{rd} = \theta_d^* & \\
& -\sum_{i=1}^{m} \nu_i^d x_{ij} + \sum_{r=1}^{s} \mu_r^d y_{rj} \le 0 & j = 1, \ldots, n \\
& z_I \le \nu_i^d \le h_I & i = 1, \ldots, m \\
& z_O \le \mu_r^d \le h_O & r = 1, \ldots, s \\
& \dfrac{z_I}{h_I} \ge \varphi_d & \\
& \dfrac{z_O}{h_O} \ge \varphi_d & \\
& z_I, z_O \ge 0 &
\end{array}
\tag{1.11}
$$

This model ensures in addition non-zero weights. As for the DMUs in Z, these are re-assessed with weights that cannot be more dissimilar than those of the DMU in NZ that needs to unbalance more its weights (as measured by $\varphi^* = \min_{d \in NZ} \varphi_d^*$) in order to achieve its CCR efficiency score. These are the weights used by the DMUs in Z in the cross-efficiency evaluation. See Wang et al. (2012), which also deals with the weight disparity, albeit it does not ensure non-zero weights.

A different strategy is followed in Ramón et al. (2011). The basic idea of the proposed approach is to ignore the profiles of weights of the inefficient DMUs in Z in the calculation of the cross-efficiency scores. That is, the cross-efficiency evaluation is carried out only with the weights of the DMUs in NZ, once these latter have made a choice among their alternate optima according to some suitable criterion. This approach is called "peer-restricted" cross-efficiency evaluation. Concerning the choice of weights that the DMUs in NZ make, the authors suggest to reduce as much as possible the differences between the profiles of weights selected. This criterion seeks, on one hand, to reduce the differences in the weights attached by the different DMUs to the same variable, and on the other, to reduce the dispersion in the samples of cross-efficiencies, so the cross-efficiency scores, which are the corresponding averages, are more representative of such cross-efficiencies. The choice of the profiles of weights to be used in the "peer-restricted" cross-efficiency evaluation is made by solving the following model

$$
\begin{array}{lll}
\text{Min} & \displaystyle\sum_{\substack{d,\, d' \in NZ \\ d < d'}} \left(\sum_{i=1}^{m} \left| v_i^d - v_i^{d'} \right| \bar{x}_i + \sum_{r=1}^{s} \left| u_r^d - u_r^{d'} \right| \bar{y}_r \right) & \\
\text{s.t.}: & & \\
& -\displaystyle\sum_{i=1}^{m} v_i^d x_{ij} + \sum_{r=1}^{s} u_r^d y_{rj} \le 0 & j = 1, \ldots, n; \quad d \in NZ \\
& -\theta_d^* \displaystyle\sum_{i=1}^{m} v_i^d x_{id} + \sum_{r=1}^{s} u_r^d y_{rd} = 0 & d \in NZ \\
& \displaystyle\sum_{i=1}^{m} v_i^d \bar{x}_i = 1 & d \in NZ \\
& v_i^d, u_r^d \ge 0 & \forall i, r, d
\end{array}
\tag{1.12}
$$

where $\bar{x}_i$, $i = 1, \ldots, m$, and $\bar{y}_r$, $r = 1, \ldots, s$, are the averages of input i and output r, respectively, across the DMUs in NZ. Note that model (1.12) includes a common normalizing constraint that makes the profiles of weights of the different DMUs comparable.

Model (1.12) can be extended to avoid zero weights (see the original paper for details).

1.4.1 Ranking Ranges and Cross-Efficiency Intervals

Liang et al. (2008a) state that the comparison of cross-efficiency scores obtained with different evaluation criteria allows us to obtain a better picture of cross-efficiency stability with respect to multiple DEA weights. However, this issue can be addressed more appropriately with an approach based on considering simultaneously all of the optimal solutions for the weights. Alcaraz et al. (2013) and Ramón et al. (2014) propose a couple of procedures for ranking DMUs based on the cross-efficiency evaluation which consider all the alternate optima for the DEA weights, thus avoiding the need to make a choice among them by using some alternative secondary goal. Instead of a single ranking, the former paper provides a range for the possible rankings of each DMU, while the latter deals with the cross-efficiency intervals that result from all the DEA weights and use some order relations for interval numbers in order to identify dominance relations between DMUs and rank them.

For each DMU_0, Alcaraz et al. (2013) find the range of its possible rankings that would result from considering all the DEA weights of all the DMUs. This range is determined by the best and the worst possible rankings of DMU_0. The best ranking of DMU_0 is defined as $r_0^b = \underset{(V,U)}{\text{Min}} \{|H_0(V,U)|\} + 1$, where $H_0(V,U) = \left\{DMU_j, j = 1, \ldots, n / \overline{E}_j > \overline{E}_0\right\}$, V and U being the $m \times n$ and $s \times n$ matrices with the input weight vectors and the output weight vectors, respectively, of a given choice of DEA weights that each of the DMUs makes. It is shown that $r_0^b = n - LE_0^*$, where LE_0^* is the optimal value of the problem

$$
\begin{array}{lll}
\text{Max} & \displaystyle\sum_{j \neq 0} I_j & \\
\text{s.t.}: & & \\
& \dfrac{u_d' Y_d}{v_d' X_d} = \theta_d^* \qquad d = 1, \ldots, n & (13.1) \\
& \dfrac{u_d' Y_j}{v_d' X_j} \leq 1 \qquad j = 1, \ldots, n; d = 1, \ldots, n & (13.2) \\
& E_{dj} = \dfrac{u_d' Y_j}{v_d' X_j} \qquad j = 1, \ldots, n; d = 1, \ldots, n & (13.3) \\
& \overline{E}_j = \dfrac{1}{n} \displaystyle\sum_{d=1}^{n} E_{dj} \qquad j = 1, \ldots, n & (13.4) \\
& \overline{E}_j - \overline{E}_0 \leq 1 - I_j \qquad j = 1, \ldots, n, j \neq 0 & (13.5) \\
& V_d \geq 0_m, u_d \geq 0_s, \forall d, I_j \in \{0, 1\}, \forall j \neq 0 &
\end{array}
\qquad (1.13)
$$

Likewise, the worst ranking is defined as $r_0^w = n - \underset{(V,U)}{\text{Min}}\{|L_0(V,U)|\}$, where $L_0(V,U) = \{DMU_j, j = 1, \ldots, n/\overline{E}_j < \overline{E}_0\}$. Now, it is shown that $r_0^w = HE_0^* + 1$, where HE_0^* is the optimal value of the problem that results from replacing (13.5) with $\overline{E}_0 - \overline{E}_j \le M(1 - I_j)$, j = 1,. . .,n, j ≠ 0, in (1.13).

The approach in Ramón et al. (2014) also considers simultaneously all the DEA weights of all the DMUs, but deals with the minimum and the maximum possible cross-efficiency scores, instead of with their best and worst rankings. For a given DMU_0, these are denoted by $\bar{E}_0^{L*}$ and $\bar{E}_0^{R*}$, and are obtained, respectively, as the optimal values of the problems

$$
\begin{array}{lll}
\text{Min/Max} & \dfrac{1}{n}\left(\displaystyle\sum_{d=1}^{n}\frac{u_d' Y_0}{v_d' X_0}\right) & \\
\text{s.t.}: & & \\
& \dfrac{u_d' Y_d}{v_d' X_d} = \theta_d^* & d = 1, \ldots, n \\
& \dfrac{u_d' Y_j}{v_d' X_j} \le 1 & d = 1, \ldots, n; \quad j = 1, \ldots, n \\
& v_d \ge 0_m, \quad u_d \ge 0_s & d = 1, \ldots, n
\end{array}
\tag{1.14}
$$

The authors show how to deal with the cross-efficiency intervals $[\bar{E}_j^{L*}, \bar{E}_j^{R*}]$, j = 1, . . ., n, in order to both identify dominance relations among DMUs and provide a ranking of units by using some order relations for interval numbers. Specifically, the following is an order relation between intervals often used in practice which may appropriately represent the DM's preferences in problems dealing with efficiency intervals: Let A and B be two intervals $A = [a^L, a^R]$ and $B = [b^L, b^R]$, then $A \le_{LR} B \Leftrightarrow a^L \le b^L$ and $a^R \le b^R$, and $A <_{LR} B \Leftrightarrow A \le_{LR} B$ and $A \ne B$. That is, $\le_{LR}$ represents the DM preference for the unit with the higher minimum cross-efficiency score and maximum cross-efficiency score. This order relation is actually a particular case of that originally introduced in Dubois and Prade (1980) for fuzzy numbers (based on the extension principle) when it is considered for interval numbers. The relation $\le_{LR}$ is however a partial order, so there may be pairs of intervals that cannot be compared, as is the case of those that are nested. Therefore, $\le_{LR}$ will usually not allow us to derive a full ranking of units, although it may yield useful information regarding dominance relations among DMUs in terms of cross-efficiency assessments. To be specific, if $\left[\overline{E}_j^{L*}, \overline{E}_j^{R*}\right] <_{LR} \left[\overline{E}_{j'}^{L*}, \overline{E}_{j'}^{R*}\right]$, then we say here that $DMU_{j'}$ dominates DMU_j. To derive a full ranking of units, the order relation $\le_\lambda$ proposed in Campos and Muñoz (1989), which takes into account the degree of optimism of the decision maker (λ), can be used (see the original papers for details).

Yang et al. (2012) also propose an approach to the cross-efficiency evaluation that avoids the need to make any choice of DEA weights. These authors deal with a n × n matrix of intervals of cross-efficiencies, which is assumed to be a matrix of stochastic variables, and use the stochastic multicriteria acceptability analysis

(SMAA-2) method proposed by Lahdelma and Salminen (2001) to derive a ranking of units. In order to do so, a probability distribution over the cross-efficiency intervals must thus be assumed, the uniform and the normal distributions being those used in that paper. This may eventually increase the computational burden needed by the proposed approach (note, in particular, that Monte-Carlo simulations are used to approximate values of the acceptability indices).

1.4.2 Illustrative Example

We use here the data in Zhu (1998) to illustrate some of the approaches to the cross-efficiency evaluation that have been previously described. The data consist of 18 Chinese cities, which are evaluated by using two inputs and three outputs. Table 1.1 records the data, the DEA efficiency scores, the interval cross-efficiency scores $[\bar{E}_j^{L*}, \bar{E}_j^{R*}]$, j = 1, . . ., 18, the ranking that results from the order relation $\leq_{LR}$ and the ranking provided by the benevolent and the aggressive approaches obtained with (1.6).

The order relation $\leq_{LR}$ practically determines a full ranking of cities, in spite of being a partial order. Figure 1.1 depicts graphically the dominance relations among cities that can be identified by using it. It shows that only cities 5 and 13 cannot be compared to each other with $\leq_{LR}$, since their corresponding cross-efficiency intervals are nested. We can see that city 2 ranks first, followed by cities 6, 10 and 12. At the bottom, we have cities 17, 3, 15, 18 and 14. Although cities 5 and 13 cannot be ranked with $\leq_{LR}$, we can state that these two cities are placed in between cities 12 and 9, because they both dominate city 9 and are dominated by city 12. Therefore, they rank either fifth or sixth.

Note also that the ranking resulting from the use of $\leq_{LR}$ with the cross-efficiency intervals is to a large extent consistent with those that the benevolent and the aggressive formulations provide, which are two rankings that in return show many similarities. However, we should not think that the benevolent and aggressive approaches cover all the range of possible rankings, as Table 1.2 shows. This table reports the ranking ranges obtained with the approach in Alcaraz et al. (2013). We can see, for example, that 2, 6 and 10 are the top three cities in all the scenarios determined by all the possible choices of DEA weights that all the DMUs can make. Nevertheless, each of them can occupy any of the three positions at the top. These situations suggest therefore that the approach proposed can be a useful complement in the standard cross-efficiency evaluations, even when some specific alternative secondary goal has been used to the choice of DEA weights, because it allows us to gain insight into the robustness of the rankings provided against alternate optima.

In any case, the ranking ranges that have been found are not especially wide, so we can draw some interesting conclusions with confidence. For example, it can be stated that 2, 6, 10, 12, 5, 13, 9, 8, 1, 4 are the top ten cities, irrespective of the DEA weights that are chosen. This can be an interesting finding from the point of view of

Table 1.1 Data of illustrative example (*Source*: Zhu 1998)

City	x_1	x_2	y_1	y_2	y_3	DEA score	Cross-eff. interval	Ranking ($\leq_{LR}$)	Benevolent	Aggressive
1	2874.8	16,738	160.89	80,800	5092	0.4691	[0.3807, 0.4462]	10	10	10
2	946.3	691	21.14	18,172	6563	1	[0.9044, 1.0000]	1	1	1
3	6854	43,024	375.25	144,530	2437	0.2779	[0.2084, 0.2436]	15	15	15
4	2305.1	10,815	176.68	70,318	3145	0.5022	[0.3895, 0.4522]	9	9	9
5	1010.3	2099	102.12	55,419	1225	0.6311	[0.5528, 0.6100]	5,6	6	5
6	282.3	757	59.17	27,422	246	1	[0.8873, 0.9722]	2	2	2
7	17478.6	116,900	1029.09	351,390	14,604	0.3580	[0.2577, 0.3047]	12	12	12
8	661.8	2024	30.07	23,550	1126	0.4959	[0.4028, 0.4635]	8	8	8
9	1544.2	3218	160.58	59,406	2230	0.6577	[0.5084, 0.5782]	7	7	7
10	428.4	574	53.69	47,504	430	1	[0.8511, 0.9247]	3	3	3
11	6228.1	29,842	258.09	151,356	4649	0.3010	[0.2429, 0.2820]	13	14	13
12	697.7	3394	38.02	45,336	1555	0.7866	[0.5754, 0.6871]	4	4	4
13	106.4	367	7.07	8236	121	0.7514	[0.5482, 0.6443]	5,6	5	6
14	4539.3	45,809	116.46	56,135	956	0.1382	[0.1091, 0.1291]	18	18	18
15	957.8	16,947	29.2	17,554	231	0.1867	[0.1405, 0.1694]	16	16	16
16	1209.2	15,741	65.36	62,341	618	0.4704	[0.3352, 0.4071]	11	11	11
17	972.4	23,822	54.52	25,203	513	0.3059	[0.2336, 0.2819]	14	13	14
18	2192	10,943	25.24	40,267	895	0.1953	[0.1334, 0.1616]	17	17	17

Fig. 1.1 Dominance relations

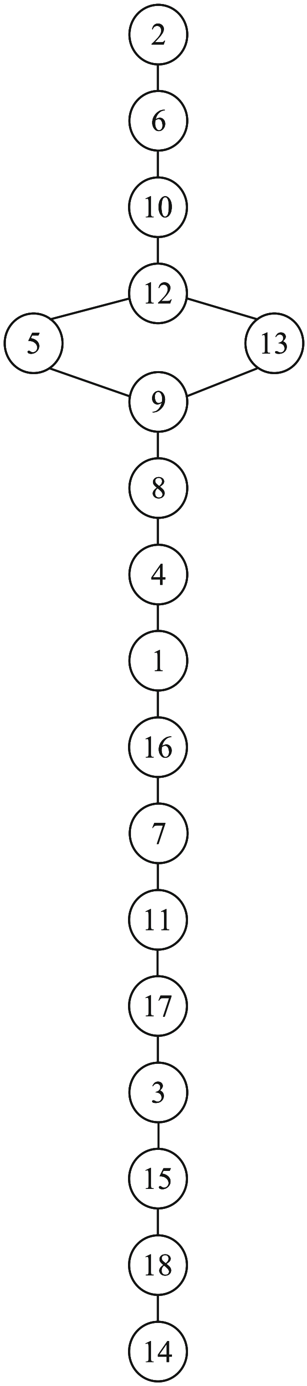


Table 1.2 Ranking ranges

DMU	Ranking ranges	
	r_0^b	r_0^w
1	9	10
2	1	3
3	15	15
4	8	10
5	4	6
6	1	3
7	12	12
8	8	9
9	6	7
10	1	3
11	13	14
12	4	5
13	5	6
14	18	18
15	16	17
16	11	11
17	13	14
18	16	17

management. We can go even further and state that 2, 6 and 10 are the top three (as said before), and that 5, 9, 12 and 13 will be always in between fourth and seventh, followed by 1, 4 and 8. Likewise, it has been found that cities 14, 15 and 18 are the poorest performers, city 14 always ranking at the bottom.

1.5 The Aggregation of Cross-Efficiencies

In MADM problems, the different criteria are often attached different weights for the evaluation of alternatives, as a way to consider their relative importance. These are usually determined on a subjective basis or trying to reflect the opinion of the decision makers (DMs). In the standard approach to the cross-efficiency evaluation, the cross-efficiency scores of the units are usually calculated as the averages of their cross-efficiencies. This means that the cross-efficiencies provided by the different DMUs are aggregated by attaching all of them the same importance. However, allowing for different aggregation weights may obviously introduce more flexibility into the analysis. In particular, in some situations, the DM could be interested in incorporating his/her preferences regarding the relative importance that should be attached to the cross-efficiencies provided by the different DMUs. For example, the DM might argue that the cross-efficiencies provided by the DMUs that globally rate the units better should be given more importance in the final aggregation. In other situations, it might be desirable that the cross-efficiencies provided by the DMUs that discriminate more among units are considered as more relevant and, accordingly, their associated aggregation weights should be larger. Furthermore, some authors have suggested that the choice of aggregation weights should be made in accordance with that of the DEA weights.

There is a number of papers that depart from the customary use of the arithmetic mean for the aggregation of cross-efficiencies and propose aggregation weights that result from models in which different conditions are imposed. In Wu et al. (2011) the aggregation weights reflect the entropy in the cross-efficiencies provided by the different DMUs. Wang and Chin (2011) use an approach based on ordered weighted averaged (OWA) operators in which, through the specification of the orness degree, they seek to reflect their particular belief that the self-evaluations should be attached more importance than that attached to the evaluations provided by the other DMUs. In Ruiz and Sirvent (2012), the aggregation weights are imposed to reflect the differences in the weights of the different profiles used to calculate the cross-efficiencies, specifically when the DEA weights are obtained from (1.11). Wang and Wang (2013) propose three weighted least-square models yielding aggregation weights which reflect in each case (1) the dissimilarity between the cross-efficiencies provided by different DMUs, (2) the deviations of these cross-efficiencies from the CCR efficiency scores, and (3) a combination of these two measures. And two game approaches are provided in Wu et al. (2008) and Wu et al. (2009c), where the DMUs are considered as players in a cooperative game and the aggregation weights of each DMU are defined, respectively, as the nucleolus solution of the game and from the Shapley value.

León et al. (2014) propose a general approach to the aggregation of cross-efficiencies which is based on Induced Ordered Weighted Averaging (IOWA) operators. In short, the idea is to rearrange the rows of the matrix of cross-efficiencies on the basis of an inducing variable and then attach the aggregation weights accordingly. As for the inducing variable, this should reflect the DM preferences regarding the relative importance of the cross-efficiencies provided by the different DMUs. Examples of inducing variables can be either $z_d = z(E_{dj}) = TE_d$, where TE_d is the total of the cross-efficiencies in row d of the matrix (1.4), $d = 1, \ldots, n$, or $z_d = z(E_{dj}) = 1 - e_d$, where $e_d = \frac{-1}{\log(n)} \sum_{j=1}^{n} E_{dj}^N \log\left(E_{dj}^N\right)$ (E_{dj}^N being $E_{dj} / \sum_{j=1}^{n} E_{dj}$), that is, the entropy in the cross-efficiencies of row d, $d = 1, \ldots, n$. Let $\widetilde{E} = \left(\widetilde{E}_{dj}\right)$ be the matrix of cross-efficiencies that results of re-arranging the rows of the cross-efficiency matrix E according to the ordering induced by **z**. Thus, in the top rows of $\tilde{E}$ we have the cross-efficiencies that are most preferred by the DM, and consequently they should be attached larger aggregation weights, whereas in those in the bottom we have the least preferred cross-efficiencies, which will be attached the smaller aggregation weights. For example, if $z_d = TE_d$, then in the top rows we will have the cross-efficiencies provided by the DMUs that globally evaluate better the units, while if $z_d = 1 - e_d$, we will have those that discriminate more among units and therefore provide more information to derive a full ranking of units (note that if $E_{d1} = E_{d2} = \ldots = E_{dn}$ then DMU_d provides no information to rank the DMUs; in that case $z_d = 0$).

Concerning the aggregation weights, with this approach only vectors $(\omega_1, \ldots, \omega_n)$ such that $\omega_1 \geq \ldots \geq \omega_n$ are considered, where ω_d is the weight to be attached to the cross-efficiencies in the d-th row of $\tilde{E}$. Nevertheless, the DM can not only set an order of preference for the cross-efficiencies provided by the different DMUs but also he/she can adjust the degree of such preference by means of the so-called orness level, α. This measure was introduced by Yager (1988) and characterizes the degree to which the aggregation is like an "or" (Max) operation. For example, if $\alpha = 1$, then $\omega_1 = 1$ and $\omega_d = 0$, $d = 2, \ldots, n$, which means that the ultimate cross-efficiency scores are the cross-efficiencies in the first row of $\tilde{E}$. In other words, only the cross-efficiencies provided by the most preferred rating DMU are considered. The case $\alpha = 0.5$ is associated with the situation in which the DM has no preference on the cross-efficiencies provided by the different DMUs. Then, $\omega_1 = \ldots = \omega_n = 1/n$, which are the aggregation weights of the arithmetic mean used in the standard cross-efficiency evaluation. Values of the orness degree in between 0.5 and 1 would be associated with intermediate situations. As α gets closer to 1 the weight is progressively put on the rating DMUs in the top rows of $\tilde{E}$.

In order to calculate the aggregation weights ω_d, the minimax disparity problem proposed by Wang and Parkan (2005) can be used:

$$\begin{array}{lll}
\text{Min} & \delta & \\
\text{s.t.}: & \dfrac{1}{n-1}\sum_{d=1}^{n}(n-d)\omega_d = \alpha & \\
& \sum_{d=1}^{n}\omega_d = 1 & \\
& \omega_d - \omega_{d+1} - \delta \leq 0 & d = 1, \ldots, n-1 \\
& \omega_d - \omega_{d+1} + \delta \geq 0 & d = 1, \ldots, n-1 \\
& \omega_d \geq 0 & d = 1, \ldots, n
\end{array} \tag{1.15}$$

where $\alpha \in [0, 1]$ is the orness degree, specified by the DM.

If $\alpha \geq 0.5$, model (1.15) ensures that the aggregation weights provided satisfy $\omega_1 \geq \ldots \geq \omega_n$, as required before. In addition, model (1.15) minimizes the maximum difference between pairs of adjacent weights, so this model somehow minimizes the differences between the aggregation weights.

Eventually, the cross-efficiency scores are calculated as $\overline{E}_j^{IOWA} = \sum_{d=1}^{n}\omega_d\widetilde{E}_{dj}, \quad j = 1, \ldots, n.$

It might be worth mentioning that, like in most approaches, we find here a common set of aggregation weights which is used in the evaluation of all the units. We believe that the ranking resulting from cross-efficiency scores calculated with common weights can be more widely accepted by users than one obtained when the aggregation weight of the cross-efficiencies provided by a given DMU is different in the evaluation of different units (that is, when the cross-efficiencies in a given row of the matrix are attached different weights), as it usually happens when using OWA operators for the aggregation like in Wang and Chin (2011). Moreover, León et al. (2014) suggest that the choice of the aggregation weights should be related to that of the DEA input and output weights. For example, if $z_d = TE_d$ is used as the inducing order variable, then a benevolent approach would be an appropriate strategy. Analogously, if $z_d = 1 - e_d$ then the DEA weights could be obtained by using the aggressive formulation.

1.5.1 Illustrative Example (Cont.)

Continuing the same example used in the previous section, we now illustrate the use of IOWA operators for the aggregation of cross-efficiencies. Specifically, the cross-efficiency evaluation is performed by using the order inducing variable based on the entropy, after the cross-efficiencies are obtained following an aggressive approach to the choice of DEA weights. The rows of the matrix of cross-efficiencies need to be re-arranged in descending order according to the values $1 - e_d$, and the aggregation weights are attached following that ordering. In this particular case, the cross-efficiencies provided by the three efficient cities, 2, 10 and 6, whose values of $1 - e_d$ are 0.495, 0.210 and 0.171, respectively, are attached the largest aggregation weights, while those provided by cities 1, 5, 8, 11, 14 and 15, with a value of $1 - e_d$ equal to 0.051, are attached the lowest ones. Table 1.3 records the

Table 1.3 Cross-efficiency scores for different orness values

	Orness degree					
DMU	0.5	0.6	0.7	0.8	0.9	1
1	0.381	0.346	0.304	0.247	0.141	0.032
2	0.921	0.889	0.850	0.798	0.698	1.000
3	0.208	0.191	0.169	0.137	0.086	0.006
4	0.390	0.358	0.320	0.265	0.170	0.031
5	0.559	0.533	0.503	0.464	0.381	0.061
6	0.887	0.843	0.789	0.716	0.576	0.034
7	0.258	0.235	0.206	0.165	0.096	0.013
8	0.403	0.372	0.337	0.293	0.198	0.059
9	0.519	0.497	0.469	0.428	0.354	0.073
10	0.873	0.850	0.828	0.815	0.740	0.079
11	0.243	0.222	0.198	0.167	0.105	0.016
12	0.578	0.524	0.463	0.394	0.231	0.048
13	0.552	0.508	0.459	0.409	0.278	0.035
14	0.109	0.098	0.085	0.067	0.036	0.002
15	0.141	0.125	0.106	0.083	0.039	0.001
16	0.335	0.299	0.257	0.211	0.105	0.004
17	0.234	0.207	0.175	0.132	0.056	0.002
18	0.134	0.122	0.108	0.093	0.055	0.009

cross-efficiency scores of the units for different orness degrees. The cross-efficiencies under $\alpha = 0.5$ are actually those of the standard aggressive approach, because that level of orness corresponds to the case of using the arithmetic mean for the aggregation of cross-efficiencies. The rankings remain quite stable as α increases, until we get $\alpha = 1$; then some changes occur. For $0.5 \leq \alpha \leq 0.9$, cities 2, 6 and 10 are the top three; cities 5, 9, 12 and 13 rank in between fourth and seventh, followed by 1, 4 and 8; and city 14 ranks at the bottom. These results coincide with those obtained with the approach by Alcaraz et al. (2013) we have previously commented. However, when $\alpha = 1$, that is, when the importance attached to the cross-efficiencies is not so far allocated between all the rows of the matrix of cross-efficiencies but we only use the profile of DEA weights that discriminate more among cross-efficiencies (that of city 2), we can see, for example, that city 6 falls outside the top three and rank eighth, while city 9 moves up to the third position. Besides, it is city 15 which ranks at the bottom.

1.6 Other Uses

The cross-efficiency evaluation has been used with other purposes different from the ranking of DMUs. These include the following.

1.6.1 Identification of Mavericks and All-Round Performers

The cross-efficiency scores allow us to discriminate between DMUs rated as DEA efficient. Nevertheless, the comparison between DEA efficiency scores and cross-efficiency scores can be exploited in a variety of other ways. For example, Doyle and Green (1994a) define the so-called Maverick index as follows (similarly, Baker and Talluri 1997 define the false positiveness index and Wang and Chin 2010a the efficiency disparity index):

$$M_j = \left(\theta_j^* - \bar{e}_j\right)/\bar{e}_j, \quad j = 1, \ldots, n \tag{1.16}$$

where $\bar{e}_j = 1/(n-1)\sum_{k \neq j} E_{kj}$. M_j measures the relative increment in the assessment of DMU_j when shifting from the model of peer-appraisal to that of self-appraisal. In that sense, it may identify mavericks as those that take advantage of the self-evaluation in their assessments. The higher M_j the more of a maverick is DMU_j. In practice, mavericks have often been identified as the efficient DMUs that appear in the reference sets of only a few other DMUs. Obviously, this counting procedure only applies to efficient DMUs, while M_j is applicable to all DMUs.

Note that the maverick index may also identify all-round performers. A DEA efficient DMU with a low value of M_j is a unit that is rated as efficient or near the efficiency with the profiles of weights of all the DMUs.

1.6.2 Classification of DMUs and Benchmarking

Doyle and Green (1994a) have also suggested the use of multivariate techniques, such as multi-dimensional scaling, principal component analysis or cluster analysis, for the classification of units into groups of DMUs on the basis of the information provided by the matrix of cross-efficiencies. Specifically, the correlation coefficient between a pair of columns tells us how similarly those two DMUs are appraised by their peers. Using these correlations as the elements in a matrix of resemblance and applying a clustering method yields clusters with similar DMUs. This could be of interest for purposes of benchmarking: the best peer-appraised DMU within each cluster, even though it may not be an efficient unit, is a suitable referent for other members of the cluster to compare against. These authors claim that this benchmark is inherently similar but "better" than other DMUs in the same cluster and, therefore, seems a more readily understable target to aim for than the linear combination of DMUs in the reference set provided by the conventional DEA, none of which may appear remotely similar to the unit under evaluation. This approach is followed in Wu et al. (2009b) for the benchmarking of countries at the Summer Olympics.

1.6.3 Fixed Cost and Resource Allocation

Du et al. (2014) use the cross-efficiency evaluation to approach cost and resource allocation problems. DEA has been successfully used to address those problems. However, the cross-efficiency evaluation provides a very reasonable and appropriate mechanism for allocating a shared resource/cost because it uses the concept of peer-appraisal. These authors claim that the allocations for fixed cost and resources resulting from their approach are more acceptable to the players involved because they are jointly determined by all DMUs rather than a specific one. All involved DMUs negotiate with one another to adjust the allocation plan for a better peer-evaluated performance until no one can improve further. A DEA-based iterative approach is developed, which is feasible and, especially for fixed cost allocation, ensures all DMUs to be efficient with the fixed cost allocated as an extra input measure.

1.7 Extensions

The standard approach to the cross-efficiency evaluation has been developed in the context of the CCR DEA model. Nevertheless, this methodology has been extended for use with non-oriented models—the models of directional distance functions (Chambers et al. 1998) and the multiplicative DEA model (Charnes et al. 1983)—, under variable returns to scale (VRS) and with fuzzy inputs and outputs. In addition, the original notion of cross-efficiency has also been generalized to deal with specific situations we sometimes find in DEA applications. These extensions broaden the range of applicability of the methodology.

1.7.1 Cross-Efficiency Evaluation with Directional Distance Functions

Ruiz (2013) explores the duality relations regarding the models of directional distance functions, which provide a measure of inefficiency in the sense of Farrell (1957), and establishes the equivalences with some fractional programming problems. This allows to defining the cross-efficiencies in the form of a ratio as follows

$$E_{dj}^{\beta} = \frac{v^{d'}X_j - u^{d'}Y_j}{v^{d'}X_j + u^{d'}Y_j}, \quad j = 1, \ldots, n. \tag{1.17}$$

It is shown that the cross-efficiencies (1.17) can actually be calculated by using the DEA weights (v^d, u^d) provided by the classical CCR model (1.2). The cross-efficiency score of a given DMU_j, $j = 1, \ldots, n$, is defined as the average of cross-efficiencies $\overline{E}_j^{\beta} = \frac{1}{n}\sum_{d=1}^{n} E_{dj}^{\beta}$, $j = 1, \ldots, n$, as usual. These scores can be used for ranking the DMUs.

Thus, this extension of the standard approach allows us to use the cross-efficiency evaluation with non-oriented measures of efficiency, i.e., which account for the inefficiency both in inputs and in outputs simultaneously.

1.7.2 Cross-Efficiency Evaluation with Multiplicative DEA Models

Cook and Zhu (2014) (see also Cook and Zhu 2015) develop an approach to the cross-efficiency evaluation based on the multiplicative DEA model. These authors define the cross-efficiency score of a given DMU_j as the geometric average of its cross-efficiencies. Then, they propose to evaluate each DMU_d with the so-called maximum log cross efficiency, which is the optimal value of the model

$$
\begin{aligned}
\text{Max} \quad & \left(\prod_{d=1}^{n} \frac{\prod_{r=1}^{s} y_{rj}^{u_r^d}}{\prod_{i=1}^{m} x_{ij}^{v_i^d}} \right)^{1/n} \\
\text{s.t. :} \quad & \\
& \frac{\prod_{r=1}^{s} y_{rj}^{u_r^d}}{\prod_{i=1}^{m} x_{ij}^{v_i^d}} \leq 1 \qquad d = 1, \ldots, n; \quad j = 1, \ldots, n \\
& \frac{\prod_{r=1}^{s} y_{rj}^{u_r^d}}{\prod_{i=1}^{m} x_{ij}^{v_i^d}} = \theta_d^{M*} \qquad d = 1, \ldots, n \\
& v_i^d, u_r^d \geq 1 \qquad i = 1, \ldots, m; \quad r = 1, \ldots, s; \quad d = 1, \ldots, n
\end{aligned}
\tag{1.18}
$$

where θ_d^{M*} is the efficiency score of DMU_d, $d = 1, \ldots, n$, provided by the multiplicative DEA model.

The attractive feature of this approach lies in that the cross-efficiency scores are uniquely determined with respect to the DEA weights, as they are defined as the optimal value of (1.18). Note, in any case, that this is not a conventional cross-efficiency approach in which the cross-efficiencies can be arranged in a matrix so that those in the same row are obtained with the same input and output weights: the weights associated with DMU_d in solving (1.18) for DMU_j are not necessarily the same as those that will be obtained when the model is solved for another $DMU_{j'}$.

1.7.3 Cross-Efficiency Evaluation Under VRS

Lim and Zhu (2015a) extend the cross-efficiency evaluation for use under VRS. To develop a way of resolving the problem of negative cross-efficiencies in the input-oriented VRS DEA model, they develop a geometric interpretation of the relationship between the VRS and CRS models. They show that, given an optimal solution (v^d, u^d, u_0^d) of an VRS-efficient DMU_d, a CRS efficiency score of DMU_d, measured under a translated Cartesian coordinate system defined by an adjusted origin determined by this optimal solution, is unity. This is interpreted as meaning that every DMU, via solving the VRS model, seeks for a translation of the Cartesian coordinate system and an optimal bundle of weights such that its CRS-efficiency score, measured under the chosen coordinate system, is maximized. Therefore, VRS cross-efficiency is related to the CRS cross-efficiency measures. In this context, Lim and Zhu (2015a) define the general concept of peer-evaluation in DEA as follows: "each DMU cross-evaluates other peer DMUs under its own best evaluation environment", where the best evaluation environment refers to the weights on the input–output factors as well as the new coordinate system that are most favourable to the DMU. Under this best evaluation environment, the DMU itself attains the highest efficiency score as well as the most productive scale size.

The cross-efficiencies are defined as follows

$$E_{dj}^{VRS} = \frac{u^{d'} Y_j}{v^{d'} X_j + u_0^d}, \quad j = 1, \ldots, n. \tag{1.19}$$

Note that E_{dd}^{VRS}, as defined in (1.19), does not coincide with the VRS efficiency score in the case of an inefficient DMU_d. Thus, for each DMU we have n cross-efficiencies and one (simple) efficiency score. Lim and Zhu (2015a) suggest to average the n cross-efficiencies to calculate an input-oriented VRS cross-efficiency score of the DMU. Alternatives are discussed in Lim and Zhu (2015b).

1.7.4 Fuzzy Cross-Efficiency Evaluation

Sirvent and León (2014) develop a fuzzy approach to the cross-efficiency evaluation, which allows us to extend the use of this methodology to the case of having imprecise data (that is, fuzzy inputs and outputs). These authors point out that the rankings of DMUs based on the ordering of fuzzy efficiencies can be criticized for the same reasons as those resulting from crisp DEA efficiency scores, which justifies the need of a fuzzy cross-efficiency evaluation. They also claim that, unlike in crisp DEA, it is not possible to set out a general approach to the cross-efficiency evaluation in Fuzzy Data Envelopment Analysis (FDEA), because there exist many different definitions of efficiency in FDEA. Thus, each fuzzy approach to the cross-efficiency evaluation will depend on the specific features of the FDEA model used for the measurement of the relative efficiency. In particular, they make some proposals to perform a cross-efficiency analysis based on the fuzzy DEA model by Guo and Tanaka (2001). These proposals are to be used in the case of fuzzy inputs and outputs being symmetrical triangular numbers, and the analysis is referred to a particular possibility level *h* in between 0 and 1 pre-specified by the decision-maker. The fuzzy cross-efficiencies are defined as non-symmetrical triangular fuzzy numbers, and the fuzzy cross-efficiency score of a given DMU is the average of its fuzzy cross-efficiencies obtained with the weights of all the DMUs. Once the fuzzy cross-efficiency scores of all the DMUs are obtained, they rank them by using a ranking index defined in Yager (1981), which eventually provides the ranking of DMUs. Since the FDEA model of Guo and Tanaka may have alternative optimal solutions for the input and output weights, Sirvent and León (2014) also propose, in a similar manner as in the crisp case, a fuzzy benevolent and a fuzzy aggressive formulation to choose among them. See Ruiz and Sirvent (2016) for a possibility approach to fuzzy cross-efficiency evaluation.

1.7.5 Game Cross Efficiency

Liang et al. (2008b) (see also Cook and Zhu 2015) claim that, in many DEA applications, some form of direct or indirect competition may exist among the DMUs under evaluation. To deal with this issue, they generalize the original cross-efficiency concept to the so-called DEA game-cross efficiency. Specifically, in that approach the DMUs are viewed as players in a game and the cross-efficiency scores as payoffs. Then, each DMU can choose to take a non-cooperative game stance to the extent that it will attempt to maximize its (worst possible) cross-efficiency under the condition that the cross-efficiency of each of the other DMUs does not deteriorate. The average game cross-efficiency score is obtained when the DMU's own maximized efficiency scores relative to each of the other units are averaged. To implement the DEA game cross-efficiency model proposed, an algorithm is derived which provides the wanted scores. Note again that this cannot be seen as a conventional cross-efficiency approach. One important difference lies in that the weights used to compute the cross-efficiencies of a given DMU are not necessarily an optimal solution of the CCR model (1.2). And, in addition, it cannot be ensured that the input and output weights used to calculate the game cross efficiencies for two units, say DMU_j and $DMU_{j'}$, relative to a given DMU_d, are the same, because each of the cross-efficiencies is the result of an independent optimization.

1.8 Conclusions

Liu et al. (2016) state that cross-efficiency evaluation and ranking "is a truly very focused subarea. Such a large coherent block of research studies indicates that many issues in cross-efficiency remain to be resolved and that there probably has not been a consensus on the method to address the issues in the original cross-efficiency concept". In our opinion, there was a need of an updated an organized survey of the literature on this methodology, and this chapter has tried to make a contribution to meeting such need. We have recapitulated the literature dealing with the two issues that have attracted more attention from the researchers: the use of alternative secondary goals to the choice of DEA weights among alternate optima and the aggregation of cross-efficiencies. And, in addition, it has been reviewed the existing work on other uses of the cross-efficiency evaluation different from the ranking of DMUs and the extensions of the standard approach, which still offer interesting directions for future research.

References

Adler, N., Friedman, L., & Sinuany-Stern, Z. (2002). Review of ranking in the data envelopment analysis context. *European Journal of Operational Research, 140*(2), 249–265.

Alcaraz, J., Ramón, N., Ruiz, J. L., & Sirvent, I. (2013). Ranking ranges in cross-efficiency evaluations. *European Journal of Operational Research, 226*(3), 516–521.

Andersen, P., & Petersen, N. C. (1993). A procedure for ranking efficient units in data envelopment analysis. *Management Science, 39*(10), 1261–1294.

Anderson, T. R., Hollingsworth, K., & Inman, L. B. (2002). The fixed weighting nature of a cross-evaluation model. *Journal of Productivity Analysis, 18*(1), 249–255.

Angulo-Meza, L., & Estellita Lins, M. P. (2002). Review of methods for increasing discrimination in data envelopment analysis. *Annals of Operations Research, 116*(1–4), 225–242.

Baker, R. C., & Talluri, S. (1997). A closer look at the use of data envelopment analysis for technology selection. *Computers and Industrial Engineering, 32*(1), 101–108.

Campos, L., & Muñoz, A. (1989). A subjective approach for ranking fuzzy numbers. *Fuzzy Sets and Systems, 29*, 145–153.

Chambers, R. G., Chung, Y., & Färe, R. (1998). Profit, directional distance functions and nerlovian efficiency. *Journal of Optimization Theory and Applications, 98*(2), 351–364.

Charnes, A., & Cooper, W. W. (1962). Programming with linear fractional functionals. *Naval Research Logistics Quarterly, 9*, 181–186.

Charnes, A., Cooper, W. W., Seiford, L. M., & Stutz, J. (1983). Invariant multiplicative efficiency and piecewise Cobb-Douglas envelopments. *Operations Research Letters, 2*, 101–103.

Charnes, A., Cooper, W. W., & Rhodes, E. (1978). Measuring the efficiency of decision making units. *European Journal of Operational Research, 2*(6), 429–444.

Charnes, A., Cooper, W. W., & Thrall, R. M. (1991). A structure for classifying and characterizing efficiency and inefficiency in data envelopment analysis. *Journal of Productivity Analysis, 2* (3), 197–237.

Chen, T. Y. (2002). An assessment of technical efficiency and cross-efficiency in Taiwan's electricity distribution sector. *European Journal of Operational Research, 137*(2), 421–433.

Cook, W. D., & Seiford, L. M. (2008). Data Envelopment Analysis (DEA)—Thirty years on. - *European Journal of Operational Research, 192*(1), 1–17.

Cook, W. D., & Zhu, J. (2014). DEA Cobb–Douglas frontier and cross-efficiency. *Journal of the Operational Research Society, 65*(2), 265–268.

Cook, W. D., & Zhu, J. (2015). DEA cross efficiency. In J. Zhu (Ed.), *Data envelopment analysis: A handbook of models and methods*. New York: Springer.

Cooper, W. W., Ramón, N., Ruiz, J. L., & Sirvent, I. (2011). Avoiding large differences in weights in cross-efficiency evaluations: Application to the ranking of basketball players. *Journal of Centrum Cathedra, 4*(2), 197–215.

De Witte, K., & Hudrlikova, L. (2013). What about excellence in teaching? A benevolent ranking of universities. *Scientometrics, 96*, 337–364.

Despotis, D. K. (2002). Improving the discriminating power of DEA: Focus on globally efficient units. *Journal of the Operational Research Society, 53*(3), 314–323.

Doyle, J. R., & Green, R. H. (1994a). Efficiency and cross-efficiency in DEA: Derivations, meanings and uses. *Journal of the Operational Research Society, 45*(5), 567–578.

Doyle, J. R., & Green, R. H. (1994b). Self and peer appraisal in Higher Education. *Higher Education, 28*(2), 241–264.

Du, J., Cook, W. D., Liang, L., & Zhu, J. (2014). Fixed cost and resource allocation based on DEA cross-efficiency. *European Journal of Operational Research, 235*(1), 206–214.

Dubois, D., & Prade, H. (1980). Systems of linear fuzzy constraints. *Fuzzy Sets and Systems, 3*(1), 37–48.

Ertay, T., & Ruan, D. (2005). Data envelopment analysis based decision model for optimal operator allocation in CMS. *European Journal of Operational Research, 164*(3), 800–810.

Falagario, M., Sciancalepore, F., Costantino, N., & Pietroforte, R. (2012). Using a DEA-cross efficiency approach in public procurement tenders. *European Journal of Operational Research, 218*(2), 523–529.

Farrell, M. J. (1957). The measurement of productive efficiency. *Journal of Royal Statistical Society, Series A, General., 120*(3), 253–281.

Friedman, L., & Sinuany-Stern, Z. (1997). Scaling units via the canonical correlation analysis and the data envelopment analysis. *European Journal of Operational Research, 100*(3), 629–637.

Ganley, J. A., & Cubbin, J. S. (1992). *Public sector efficiency measurement: Applications of data envelopment analysis*. Amsterdam: North-Holland.

Green, R., Doyle, J., & Cook, W. (1996). Preference voting and project ranking using DEA and cross-evaluation. *European Journal of Operational Research, 90*(3), 461–472.

Guo, P., & Tanaka, H. (2001). Fuzzy DEA: A perceptual evaluation method. *Fuzzy Sets and Systems, 119*, 149–160.

Gutiérrez, O., & Ruiz, J. L. (2013a). Data envelopment analysis and cross-efficiency evaluation in the management of sports teams: The assessment of game performance of players in the Spanish handball league. *Journal of Sport Management, 27*(3), 217–229.

Gutiérrez, O., & Ruiz, J. L. (2013b). Game performance versus competitive performance in the world championship of handball 2011. *Journal of Human Kinetics, 36*(1), 137–147.

Halme, M., Joro, T., Korhonen, P., Salo, S., & Wallenius, J. (1999). A value efficiency approach to incorporating preference information in data envelopment analysis. *Management Science, 45* (1), 103–115.

Hashimoto, A. (1997). A ranked voting system using a DEA/AR exclusion model: A note. *European Journal of Operational Research, 97*, 600–604.

Hosseinzadeh Lotfi, F., Jahanshahloo, G. R., Khodabakhshi, M., Rostamy-Malkhlifeh, M., Moghaddas, Z., & Vaez-Ghasemi, M. (2013). A review of ranking models in data envelopment analysis. *Journal of Applied Mathematics*, Article ID 492421, 20 p.

Kao, C., & Hung, H. T. (2005). Data envelopment analysis with common weights: The compromise solution approach. *Journal of the Operational Research Society, 56*(10), 1196–1203.

Kao, C., & Liu, S. T. (2000). Fuzzy efficiency measures in data envelopment analysis. *Fuzzy Sets and Systems, 113*, 427–437.

Lahdelma, R., & Salminen, P. (2001). SMAA-2: Stochastic multicriteria acceptability analysis for group decision making. *Operations Research, 49*(3), 444–454.

León, T., Ramón, N., Ruiz, J. L., & Sirvent, I. (2014). Using induced ordered weighted averaging (IOWA) operators for aggregation in cross-efficiency evaluations. *International Journal of Intelligent Systems, 29*(12), 1100–1116.

Li, X.-B., & Reeves, G. R. (1999). A multiple criteria approach to data envelopment analysis. *European Journal of Operational Research, 115*, 507–517.

Liang, L., Wu, J., Cook, W. D., & Zhu, J. (2008a). Alternative secondary goals in DEA cross-efficiency evaluation. *International Journal of Production Economics, 113*(2), 1025–1030.

Liang, L., Wu, J., Cook, W. D., & Zhu, J. (2008b). The DEA game cross-efficiency model and its Nash equilibrium. *Operations Research, 56*(5), 1278–1288.

Lim, S., Kwang, W. O., & Zhu, J. (2014). Use of DEA cross-efficiency evaluation in portfolio selection: An application to Korean stock market. *European Journal of Operational Research, 236*(1), 361–368.

Lim, S., & Zhu, J. (2015a). DEA cross-efficiency evaluation under variable returns to scale. *Journal of the Operational Research Society, 66*(3), 476–487.

Lim, S., & Zhu, J. (2015b). DEA cross efficiency under variable returns to scale. In J. Zhu (Ed.), *Data envelopment analysis: A handbook of models and methods*. New York: Springer.

Liu, J. S., Lu, L. Y. Y., & Lu, W.-M. (2016). Research fronts in data envelopment analysis. *Omega, 58*, 33–45.

Liu, F. H. F., & Peng, H. H. (2008). Ranking of units on the DEA frontier with common weights. *Computers and Operations Research, 35*(5), 1624–1637.

Liu, F. H. F., & Peng, H. H. (2009). A systematic procedure to obtain a preferable and robust ranking of units. *Computers and Operations Research, 36*(4), 1012–1025.

Lu, W. M., & Lo, S. F. (2007). A closer look at the economic-environmental disparities for regional development in China. *European Journal of Operational Research, 183*(2), 882–894.

Oral, M., Kettani, O., & Lang, P. (1991). A methodology for collective evaluation and selection of industrial R&D projects. *Management Science, 37*(7), 871–885.

Pedraja-Chaparro, F., Salinas-Jimenez, J., & Smith, P. (1997). On the role of weight restrictions in data envelopment analysis. *Journal of Productivity Analysis, 8*(2), 215–230.

Ramón, N., Ruiz, J. L., & Sirvent, I. (2010a). A multiplier bound approach to assess relative efficiency in DEA without slacks. *European Journal of Operational Research, 203*(1), 261–269.

Ramón, N., Ruiz, J. L., & Sirvent, I. (2010b). On the choice of weights profiles in cross-efficiency evaluations. *European Journal of Operational Research, 207*(3), 1564–1572.

Ramón, N., Ruiz, J. L., & Sirvent, I. (2011). Reducing differences between profiles of weights: A "peer-restricted" cross-efficiency evaluation. *Omega, 39*(6), 634–641.

Ramón, N., Ruiz, J. L., & Sirvent, I. (2012). Common sets of weights as summaries of DEA profiles of weights: With an application to the ranking of professional tennis players. *Expert Systems with Applications, 39*, 4882–4889.

Ramón, N., Ruiz, J. L., & Sirvent, I. (2014). Dominance relations and ranking of units by using interval number ordering with cross-efficiency intervals. *Journal of the Operational Research Society, 65*(9), 1336–1343.

Roll, Y., Cook, W. D., & Golany, B. (1991). Controlling factor weights in data envelopment analysis. *IEEE Transactions, 23*(1), 2–9.

Roll, Y., & Golany, B. (1993). Alternate methods of treating factor weights in DEA. *Omega, 21*(1), 99–109.

Ruiz, J. L. (2013). Cross-efficiency evaluation with directional distance functions. *European Journal of Operational Research, 228*(1), 181–189.

Ruiz, J. L., Pastor, D., & Pastor, J. T. (2013). Assessing professional tennis players using Data Envelopment Analysis (DEA). *Journal of Sports Economics, 14*(3), 276–302.

Ruiz, J. L., & Sirvent, I. (2012). On the DEA total weight flexibility and the aggregation in cross-efficiency evaluations. *European Journal of Operational Research, 223*(3), 732–738.

Ruiz, J.L., & Sirvent, I. (2016) Fuzzy cross-efficiency evaluation: a possibility approach. Fuzzy Optimization and Decision Making (DOI 10.1007/s10700-016-9240-1).

Sexton, T. R., Silkman, R. H., & Hogan, A. J. (1986). Data envelopment analysis: Critique and extensions. In R. H. Silkman (Ed.), *Measuring efficiency: An assessment of data envelopment analysis* (pp. 73–105). San Francisco: Jossey-Bass.

Sinuany-Stern, Z., Mehrez, A., & Barboy, A. (1994). Academic departments efficiency via data envelopment analysis. *Computers and Operations Research, 21*(5), 543–556.

Sirvent, I., & León, T. (2014). Cross-efficiency in Fuzzy Data Envelopment Analysis (FDEA): Some proposals. In A. Emrouznejad & M. Tavana (Eds.), *Studies in fuzziness and soft computing* (Vol. 309, pp. 101–116). Berlin Heidelberg: Springer.

Sueyoshi, T. (1999). DEA non-parametric ranking test and index measurement: Slack-adjusted DEA and an application to Japanese agriculture cooperatives. *Omega, 27*(3), 315–326.

Talluri, S., & Yoon, K. P. (2000). Cone-ratio DEA approach for AMT justification. *International Journal of Production Economics, 66*(2), 19–129.

Thanassoulis, E., Portela, M. C. S., & Allen, R. (2004). Incorporating value judgments in DEA. In W. W. Cooper, L. W. Seiford, & J. Zhu (Eds.), *Handbook on data envelopment analysis*. Norwell: Kluwer.

Tone, K. (2002). A slacks-based measure of super-efficiency in data envelopment analysis. *European Journal of Operational Research, 143*(1), 32–41.

Torgersen, A. M., Foørsund, F. R., & Kittelsen, S. A. C. (1996). Slack-adjusted efficiency measures and ranking of efficient units. *Journal of Productivity Analysis, 7*(4), 379–398.

Troutt, M. D. (1997). Derivation of the maximin efficiency ratio model from the maximum decisional efficiency principle. *Annals of Operations Research, 73*, 323–338.

Wang, Y. M., & Chin, K. S. (2010a). A neutral DEA model for cross-efficiency evaluation and its extension. *Expert Systems with Applications, 37*(5), 3666–3675.

Wang, Y. M., & Chin, K. S. (2010b). Some alternative models for DEA cross-efficiency evaluation. *International Journal of Production Economics, 128*(1), 332–338.

Wang, Y. M., & Chin, K. S. (2011). The use of OWA operator weights for cross-efficiency aggregation. *Omega, 39*(5), 493–503.

Wang, Y. M., Chin, K. S., & Wang, S. (2012). DEA models for minimizing weight disparity in cross-efficiency evaluation. *Journal of the Operational Research Society, 63*(8), 1079–1088.

Wang, Y. M., & Luo, Y. (2009). On rank reversal in decision analysis. *Mathematical and Computer Modelling, 49*(5–6), 1221–1229.

Wang, Y. M., & Parkan, C. (2005). A minimax disparity approach for obtaining OWA operator weights. *Information Sciences, 175*(1–2), 20–29.

Wang, Y. M., & Wang, S. (2013). Approaches to determining the relative importance weights for cross-efficiency aggregation in data envelopment analysis. *Journal of the Operational Research Society, 64*(1), 60–69.

Wu, J., Liang, L., & Chen, Y. (2009a). DEA game cross-efficiency approach to Olympic rankings. *Omega, 37*(4), 909–918.

Wu, J., Liang, L., & Yang, F. (2009b). Achievement and benchmarking of countries at the Summer Olympics using cross-efficiency evaluation method. *European Journal of Operational Research, 197*(2), 722–730.

Wu, J., Liang, L., & Yang, F. (2009c). Determination of the weights for the ultimate cross-efficiency using Shapley value in cooperative game. *Expert Systems with Applications, 36*(1), 872–876.

Wu, J., Liang, L., & Zha, Y. (2008). Determination of the weights of ultimate cross efficiency based on the solution of nucleolus in cooperative game. *Systems Engineering—Theory & Practice, 28*(5), 92–97.

Wu, J., Sun, J., Liang, L., & Zha, Y. (2011). Determination of weights for ultimate cross efficiency using Shannon entropy. *Expert Systems with Applications, 38*(5), 5162–5165.

Yager, R. R. (1981). A procedure for ordering fuzzy sets of the unit interval. *Information Sciences, 24*, 143–161.

Yager, R. R. (1988). On ordered weighted averaging aggregation operators in multi-criteria decision making. *IEEE Transactions on Systems, Man, and Cybernetics, 18*(1), 183–190.

Yang, F., Ang, S., Qiong, X., & Yang, C. (2012). Ranking DMUs by using interval DEA cross efficiency matrix with acceptability analysis. *European Journal of Operational Research, 223* (2), 483–488.

Zhu, J. (1998). Data envelopment analysis vs. principal component analysis: An illustrative study of economic performance of Chinese cities. *European Journal of Operational Research, 111* (1), 50–61.

Chapter 2
Data Envelopment Analysis for Measuring Environmental Performance

Peng Zhou, Kim Leng Poh, and Beng Wah Ang

Abstract Environmental performance measurement provides an analytical foundation for environmental policy analysis and decision making. As a popular performance evaluation tool, Data Envelopment Analysis (DEA) has been applied to construct environmental performance index in different ways, where modeling undesirable outputs and the choice/construction of efficiency measures are the main steps. This chapter gives an introductory text on applications of DEA to environmental performance measurement by describing the formulation of environmental DEA technologies as well as radial and non-radial DEA models for constructing pure environmental efficiency/productivity index. A case study on measuring the environmental performance of OECD countries is presented. Future directions of DEA applications to environmental modeling are discussed with reference to several recent developments in this area.

Keywords Data envelopment analysis • Environmental performance • Aggregation • Malmquist productivity index

2.1 Introduction

Environmental performance measurement has received increasing attention at different levels due to the global concern about environmental issues and sustainable development. At firm level, the improvement of environmental performance may lead to better financial performance and therefore bring stakeholders huge potential benefits. As such, the measurement of environmental performance has been regarded as the centre of the theoretical framework for business environmental

P. Zhou (✉)
College of Economics and Management, Nanjing University of Aeronautics and Astronautics, 29 Jiangjun Avenue, Nanjing 211106, China
e-mail: rocy_zhou@hotmail.com

K.L. Poh • B.W. Ang
Department of Industrial and Systems Engineering, National University of Singapore, 10 Kent Ridge Crescent, Singapore 119260, Singapore

S.-N. Hwang et al. (eds.), *Handbook of Operations Analytics Using Data Envelopment Analysis*, International Series in Operations Research & Management Science 239, DOI 10.1007/978-1-4899-7705-2_2

management (Tyteca 1996). At macro level, the measurement of environmental performance measurement helps to make environmental policy analysis and decision making more quantitative, empirically grounded and systematic (Hsu et al. 2014).

Technically speaking, the measurement of environmental performance often involves converting a set of indicators to a composite environmental index. Many studies have so far been reported on the construction of composite environmental index, which deal with cases that range from a specific environmental theme to the whole economic-energy-environmental system, and from a single country/region to multiple countries/regions (Zhou et al. 2006a). Of the various methods for constructing composite environmental index, data envelopment analysis (DEA) as a well-established non-parametric approach to efficiency evaluation has been widely employed. The review study by Cook and Seiford (2009) provides a sketch of the historical developments for DEA in the past three decades. As reported by Zhou et al. (2008a) in their survey study on the applications of DEA to energy and environmental analysis, about a quarter of studies dealt with environmental performance measurement.

This chapter discusses the use of DEA in environmental performance measurement. It is not our intention to provide a very comprehensive review of this area, which has been done by several earlier review studies like Zhou et al. (2008a) and Song et al. (2012). On the contrary, we only focus on the theoretical foundation (i.e. environmental DEA technology) and several basic DEA models for measuring environmental efficiency and productivity, while some recent developments have also been sketched. A case study is also presented in Sect. 2.4 to illustrate the applicability of the models. Finally, we end this chapter by some concluding remarks on the possible future research agenda about the use of DEA for environmental performance measurement.

2.2 Environmental DEA Technology

The application of DEA to environmental performance measurement starts from the incorporation of undesirable outputs into production technology (or production possibility set). It is aware that many production activities will inevitably generate undesirable (or bad) outputs as byproducts of desirable (or good) outputs. For instance, the emissions of carbon dioxide and sulphur dioxide are inevitable when coal is burned to generate electricity in a fossil-fired power plant. In DEA, the issue of undesirable outputs may be referred to as data irregularity (Zhu and Cook 2007). For the conventional DEA models, all the outputs are assumed to be of benefit type, i.e. more outputs are expected to be produced given the constraints of inputs. This assumption, however, does not hold for undesirable outputs in view of its 'undesirable' feature, which need to appropriately modeled into DEA framework.

A large number of methods for modeling undesirable outputs in the DEA framework have been proposed to deal with this situation (Scheel 2001). In addition to treating undesirable outputs as inputs, there are two popular groups of methods for handling undesirable outputs. One is based on the translation invariance property in DEA, for which we first multiply undesirable outputs by "-1" and then add a sufficiently large number to them to make all the undesirable outputs become positive. The study by Ali and Seiford (1990) found that additive DEA model and its variant are translation invariant under variable returns to scale (Banker et al. 1984). Later, Seiford and Zhu (2002) developed an approach to incorporating undesirable outputs into BCC–DEA model by using the concept of classification invariance. The other group uses the original data but is based on the concept of weak disposable reference technology introduced by Färe et al. (1989). The weak disposable reference technology, also known as environmental production technology or polluting technology, can be characterized in a nonparametric or parametric way. If it is characterized within a DEA framework, the resulting piece-wide linear production technology is often referred to as environmental DEA technology (Färe and Grosskopf 2004; Zhou et al. 2008b). Since the concept of environmental DEA technology is more frequently used for modeling environmental performance, in this chapter we shall only introduce environmental DEA technology and the relevant DEA models for environmental performance measurement.

Suppose that $\boldsymbol{x} \in \boldsymbol{R}_{+}^{N}$, $\boldsymbol{y} \in \boldsymbol{R}_{+}^{M}$ and $\boldsymbol{u} \in \boldsymbol{R}_{+}^{J}$ are respectively the vectors of inputs, desirable outputs and undesirable outputs. Then the environmental production technology can be represented by the following output set (Chung et al. 1997)

$$P(\boldsymbol{x}) = \{(\boldsymbol{y}, \boldsymbol{u}) : \boldsymbol{x} \text{ can produce } (\boldsymbol{y}, \boldsymbol{u})\} \tag{2.1}$$

According to Färe and Grosskopf (2004), $P(\boldsymbol{x})$ is often imposed the weak disposability and null-jointness assumptions as follows:

1. If $(\boldsymbol{y}, \boldsymbol{u}) \in P(\boldsymbol{x})$ and $0 \leq \theta \leq 1$, then $(\theta \boldsymbol{y}, \theta \boldsymbol{u}) \in P(\boldsymbol{x})$.
2. If $(\boldsymbol{y}, \boldsymbol{u}) \in P(\boldsymbol{x})$ and $\boldsymbol{u} = 0$, then $\boldsymbol{y} = 0$.

The first property says that a proportional reduction in desirable outputs and undesirable outputs is feasible. The second property implies that ceasing production activities is the only choice to eliminate all the undesirable outputs.

In application, the data on inputs and outputs for all the decision making units (DMUs) are required in order to make the environmental DEA technology be applicable. Assume that there are $k = 1, 2, \cdots, K$ DMUs and for DMU_k the observed data on the vectors of inputs, desirable outputs and undesirable outputs are respectively $\boldsymbol{x}_k = (x_{1k}, x_{2k}, \cdots, x_{Nk})$, $\boldsymbol{y}_k = (y_{1k}, y_{2k}, \cdots, y_{Mk})$ and $\boldsymbol{u}_k = (u_{1k}, u_{2k,} \cdots, u_{Jk})$. Under the constant returns to scale (CRS), the environmental DEA technology can be characterized by the following output set

$$
\begin{aligned}
P_{CRS}(\boldsymbol{x}) = \Big\{(\boldsymbol{y},\boldsymbol{u}) : &\sum_{k=1}^{K} z_k x_{nk} \le x_n, \quad n = 1,2,\cdots,N \\
&\sum_{k=1}^{K} z_k y_{mk} \ge y_m, \quad m = 1,2,\cdots,M \\
&\sum_{k=1}^{K} z_k u_{jk} = u_j, \quad j = 1,2,\cdots,J \\
&z_k \ge 0, \quad k = 1,2,\cdots,K \Big\}
\end{aligned}
\tag{2.2}
$$

As summarized in Zhou et al. (2008a), most of the studies on environmental performance measurement are based on the CRS environmental DEA technology. However, in actual situations the production technology may not always exhibit CRS and other cases like variable returns to scale (VRS) are likely to be observed (Tyteca 1996). Under the context of VRS, it is not appropriate to simply add the constraint of intensity variables being equal to one like the classical BCC–DEA model. As discussed in Färe and Grosskopf (2004) and Zhou et al. (2008b), the VRS environmental DEA technology may be characterized by the following production output set:

$$
\begin{aligned}
P_{VRS}(\boldsymbol{x}) = \Big\{(\boldsymbol{y},\boldsymbol{u}) : &\sum_{k=1}^{K} z_k x_{nk} \le x_n, \quad n = 1,2,\cdots,N \\
&\sum_{k=1}^{K} z_k y_{mk} \ge \alpha y_m, \quad m = 1,2,\cdots,M \\
&\sum_{k=1}^{K} z_k u_{jk} = \alpha u_j, \quad j = 1,2,\cdots,J \\
&\sum_{k=1}^{K} z_k = 1 \\
&\alpha \ge 1, \quad z_k \ge 0, \quad k = 1,2,\cdots,K \Big\}
\end{aligned}
\tag{2.3}
$$

where α is a parameter which allows the output set to satisfy the weak disposability assumption.

A graphical comparison between the CRS and VRS environmental DEA technologies is given by Zhou et al. (2008b). While (2.3) is theoretically consistent with the weak disposability and null-jointness properties, the resulting DEA models are nonlinear and difficult to solve. The study by Chen (2013) showed that the linear formulation of VRS environmental DEA technology given by Kuosmanen (2005), i.e. (2.4), is more appropriate for application, especially when the additive DEA models are used to measure environmental performance.

$$
\begin{aligned}
P_{VRS}(\boldsymbol{x}) = \Big\{(\boldsymbol{y}, \boldsymbol{u}) : & \sum_{k=1}^{K} (z_k + \lambda_k) x_{nk} \leq x_n, \quad n = 1, 2, \cdots, N \\
& \sum_{k=1}^{K} z_k y_{mk} \geq y_m, \quad m = 1, 2, \cdots, M \\
& \sum_{k=1}^{K} z_k u_{jk} = u_j, \quad j = 1, 2, \cdots, J \\
& \sum_{k=1}^{K} (z_k + \lambda_k) = 1 \\
& z_k \geq 0, \quad \lambda_k \geq 0, \quad k = 1, 2, \cdots, K\Big\}
\end{aligned}
\tag{2.4}
$$

It should be noted that in literature most of DEA models for environmental performance measurement are based on the CRS environmental DEA technology. Despite of the fact, several recent studies also adopt either (2.3) or (2.4) form of the VRS environmental DEA technology in their empirical analysis. An example is Chen (2013) who employs (2.4) to examine the energy efficiency of EU states.

2.3 Models for Measuring Environmental Performance

A large number of DEA models have been developed for environmental performance measurement under the constraint of environmental DEA technology. Most of them are built upon the CRS environmental DEA technology, e.g. Tyteca (1996, 1997), Färe et al. (2004), Zaim (2004), and Zhou et al. (2006a, b, 2007a, b, 2010a, b, 2012). Therefore, in this section we only introduce several typical DEA models for measuring environmental performance under the CRS environmental DEA technology. However, these models can be easily adapted to the case of VRS environmental DEA technology as mentioned above.

2.3.1 Environmental Efficiency Index

A standardized environmental efficiency index, which lies between zero and one, is often derived when multilateral comparison of environmental performance is concerned. Of the various DEA models for constructing environmental efficiency index, the undesirable outputs-oriented DEA model, i.e. (2.5), is particularly attractive (Tyteca 1997). In (2.5), undesirable outputs are reduced as much as possible by the same rate, while the constraint of environmental DEA technology is not violated.

$$
\begin{aligned}
EEI_1 = \min\ & \lambda \\
\text{s.t.}\ & \sum_{k=1}^{K} z_k x_{nk} \le x_n, \quad n = 1, 2, \cdots, N \\
& \sum_{k=1}^{K} z_k y_{mk} \ge y_m, \quad m = 1, 2, \cdots, M \\
& \sum_{k=1}^{K} z_k u_{jk} = \lambda u_j, \quad j = 1, 2, \cdots, J \\
& z_k \ge 0, \quad k = 1, 2, \cdots, K
\end{aligned}
\tag{2.5}
$$

Obviously, (2.5) offers a standardized index for evaluating the environmental efficiency of each DMU. A DMU with larger EEI_1 is believed to have a better environmental performance compared with other DMUs.

While (2.5) as a radial DEA model holds some desirable properties, it has relatively weak discriminating power. In addition, it cannot incorporate additional information offered by decision/policy makers regarding their individual preferences on different undesirable outputs. The weighted non-radial DEA model proposed by Zhu (1996) and Seiford and Zhu (1998) may be used to overcome the limitations. Zhou et al. (2007a, b) incorporate undesirable outputs into Zhu's non-radial DEA framework and develop the following non-radial DEA model for constructing an environmental efficiency index:

$$
\begin{aligned}
EEI_2 = \min\ & \sum_{j=1}^{J} w_j \lambda_j \\
\text{s.t.}\ & \sum_{k=1}^{K} z_k x_{nk} \le x_n, \quad n = 1, 2, \cdots, N \\
& \sum_{k=1}^{K} z_k y_{mk} \ge y_m, \quad m = 1, 2, \cdots, M \\
& \sum_{k=1}^{K} z_k u_{jk} = \lambda_j u_j, \quad j = 1, 2, \cdots, J \\
& z_k \ge 0, \quad \lambda_j \le 1, \quad k = 1, 2, \cdots, K; \quad j = 1, 2, \cdots, J
\end{aligned}
\tag{2.6}
$$

where w_j $(j = 1, 2, \cdots, J)$ refers to a set of normalized user-specified weights for adjusting the undesirable outputs, which may reflect the preference of decision/policy makers in adjusting each undesirable output. If there is only one undesirable output, (2.6) will be exactly the same as (2.5).

In (2.6), there is a constraint $\lambda_j \le 1$ which is not included by Zhou et al. (2007a, b). The additional constraint indicates that no undesirable outputs are allowed to increase, while in Zhou et al. (2007a, b) some undesirable outputs are allowed to be expanded in order to achieve higher overall reduction of undesirable outputs as a whole. In (2.6), the determination of the weights is also a controversial and difficult issue. In addition to the information on policy/decision makers' preference, the marginal abatement costs or shadow prices of undesirable outputs are also valuable in determining the weights. As shown in the recent study by Zhou et al. (2014b), DEA also plays a significant role in estimating the shadow prices of undesirable outputs.

Finally, it should be pointed out that a DMU with EEI_1 or EEI_2 equal to one may not be technical efficient since the slacks/surplus for certain inputs and desirable outputs may not be zero. As such, it might be appropriate to interpret EEI_1 and EEI_2 as pure environmental efficiency indexes.

2.3.2 Environmental Productivity Index

The environmental efficiency indexes described in Sect. 2.3.1 are static ones which are mainly for environmental performance comparisons between different DMUs at a certain point (period) of time. In addition to cross-section comparisons, decision/policy makers are also keen to track or monitor the trends in environmental performance of each DMU over time. While there are several formal time series analysis methods in DEA, a popular practice is to adapt the Malmquist productivity index initiated by Caves et al. (1982) and developed by Färe et al. (1994) to construct environmental productivity index. Originally, Malmquist productivity index is defined as a ratio of two distance functions. In the case of radial DEA models, the Shephard distance function is nicely the reciprocal of efficiency score. In virtue of this relationship, the environmental productivity index can be directly defined from efficiency scores with time index.

Let t and s ($t < s$) denote two time indexes. Suppose that EEI_p^q $(p, q = s, t)$ refers to the environmental efficiency index of a DMU derived from its input–output pairs for period of time p and the environmental DEA technology for period q. As described in Zhou et al. (2007b, 2010), the Malqmuist environmental productivity index (EPI) of DMU_0 can be calculated by

$$EPI = \left[\frac{EEI_s^t}{EEI_t^t}\frac{EEI_s^s}{EEI_t^s}\right]^{1/2} \tag{2.7}$$

Using EPI, we can then measure the environmental productivity change of each DMU to monitor its dynamic environmental performance over time. When $EPI > 1$, it indicates that an improvement of environmental performance from t to s is observed for the DMU being evaluated. When $EPI < 1$, a deterioration of environmental performance from t to s is identified for it.

Like the conventional Malmquist productivity index, we can also investigate the mechanism of environmental productivity changes by decomposing (2.7) into the following two contributing components:

$$EPI = \frac{EEI_s^s}{EEI_t^t} \times \left[\frac{EEI_s^t}{EEI_s^s}\frac{EEI_t^t}{EEI_t^s}\right]^{1/2} \tag{2.8}$$

where the first term of the right-hand side measures the environmental efficiency change (EEFCH), and the second term measures the shift of environmental DEA technology, i.e. technological change (TECH). It should be pointed that EPI and its contributing components can be derived from either radial or non-radial DEA model introduced in Sect. 2.3.1. For the latter, the studies by Zhou et al. (2007a, b) and Meng et al. (2013) provide two application examples. When there is only one undesirable output, choosing radial or non-radial DEA models will lead to the same environmental productivity index. Examples of such studies can be found in Zhou et al. (2010a, b) who developed a total factor carbon performance index for monitoring CO_2 emission performance over time.

2.3.3 Other Developments

The several DEA models mentioned above represent only the basic and typical ones for environmental performance measurement. Recent years have also seen a number of new developments in this field. Since the radial and non-radial DEA models described earlier do not incorporate slacks/surplus in inputs and desirable outputs, some scholars have attempted to model environmental performance by incorporating these slacks/surplus. For example, Zhou et al. (2006b) adapted the slacks-based measure (SBM) proposed by Tone (2001) to measure the economic-environmental performance and estimate the impacts of environmental regulations. Bian and Yang (2010) used the weighted SBM based on Shannon's entropy to measure energy and environmental performance simultaneously. Sueyoshi and Goto (2012) applied range-adjusted measure (RAM), a weighted sum of slack variables, to measure environmental performance under different disposability assumptions. Wang et al. (2013) employed the RAM–DEA model to examine the environmental performance of different provinces in China.

Directional distance function (DDF) as a new direction in efficiency and productivity analysis has also received increasing attention in environmental performance measurement (Färe and Grosskopf 2005). Chung et al. (1997) developed Malmquist-Luenberger productivity index by considering undesirable outputs in DDF. Boyd and McClelland (1999) used DDF to measure the impact of environmental regulations on productivity growth. Picazo-Tadeo et al. (2005) employed DDF to examine the impact of environmental regulation on firm's performance. The study by Managi and Jena (2008) analyzed the environmental productivity in India with DDF. Recently, Zhou et al. (2012) employed the non-radial directional distance function, which is closely linked to the slacks-based DEA models, to assess the energy and carbon performance in electricity generation.

Note that environmental efficiency or productivity index is basically a composite indicator aggregated from several underlying sub-indicators. DEA as a data weighting and aggregation tool has also received increasing attention in constructing composite indicators. Zhou et al. (2007a) developed a linear programming approach comprising two DEA-like models for constructing composite indicators. Since geometric aggregation is superior to arithmetic aggregation in terms of information loss, Zhou et al. (2010a, b) extended their earlier study and proposed a multiplicative DEA model for constructing composite indicators, which has been empirically used in a recent study by Blancard and Hoarau (2013).

Yet, there are still many other developments in the applications of DEA to environmental performance measurement, e.g. the incorporation of material balance condition into DEA models for measuring environmental performance (Coelli et al. 2007). However, it is not the purpose of this chapter to enumerate them in a complete way. Interested readers may refer to the survey study by Zhou et al. (2008a, b) and Song et al. (2012) for identifying more relevant studies.

2.4 Case Study

In this section, we apply the radial and non-radial DEA models as described in Sect. 2.3 to calculate the environmental efficiency index (EEI) and environmental productivity index (EPI) of 29 OECD countries from 2000 to 2011 under the CRS environmental DEA technology. It should be pointed out that the case study is mainly for illustrating purpose so that the policy and managerial implications of our modeling results as well as the data and model biases will not be discussed in detail.

2.4.1 Data

Capital stock and labor force are employed as two inputs and gross domestic production (GDP) is taken as desirable outputs. In the case of undesirable outputs, we choose carbon dioxide (CO_2) emissions, methane (CH_4) emissions and nitrous oxide (N_2O) emissions for use since they are major air pollutants causing global warming and having adverse health effects. The data on all the variables but capital stock were collected from the World Bank Group (WBG) and OECD Statistics (http://stats.oecd.org/). The data on capital stock are calculated by using perpetual inventory method based on the data of gross fixed capital formation. Table 2.1 shows the descriptive statistics of collected data for the six variables.

Table 2.1 Descriptive statistics of inputs and outputs for 29 OECD countries, 2000–2011

Indicators	Units	Max	Min	Mean	S.D.
Capital stock	Constant 2005 billion US$	213,138.49	71.36	13,873.82	31,290.39
Labor force	Ten thousand workers	15,800.00	16.60	1672.48	2933.48
Gross domestic product	Constant 2005 billion US$	138,000.00	98.40	11,389.07	24,105.53
Carbon dioxide (CO_2)	1000 tons	6,119,317.83	2773.28	414,084.18	1,058,109.36
Methane (CH_4)	1000 tons of CO_2 eq	610,114.02	437.00	43,949.50	106,083.55
Nitrous oxide (N_2O)	1000 tons of CO_2 eq	362,415.56	441.27	27,683.95	62,109.40

2.4.2 *Results and Discussions*

2.4.2.1 EEI Analysis

Table 2.2 shows the EEI results derived from the radial DEA model (2.5). It can be seen from Table 2.2 that the EEI values of nine countries are always equal to 1, which indicates that they had a better environmental performance than other countries. On the other hand, several countries like Slovak Republic, Czech Republic and Estonia have relatively lower EEI values, which might be an indication of poor environmental performance in these countries.

Table 2.3 shows the EEI results derived from the non-radial DEA model (2.6) by setting equal weights for undesirable outputs. It is not surprising that the EEI values from non-radial DEA model are lower than those from radial one since the former has a more relaxed constraint. Compared to the radial DEA model, non-radial DEA model has higher discriminating power since fewer countries had EEI values equal to one. Meanwhile, the EEI values for countries like Slovak Republic, Czech Republic and Estonia are still the lowest. However, there are two exception cases including Australia and New Zealand that have quite low non-radial EEI scores, whereas their radial EEI scores are equal to one as shown in Table 2.2. In addition to the data bias, one possible reason is that that the two countries did not perform well in certain dimension of undesirable outputs (like CH_4).

Figure 2.1 shows the average EEIs for OECD countries from both radial and non-radial DEA models over time. Not surprisingly, the average EEI_1 value is always above the average EEI_2 value, which comes from the fact that non-radial DEA model has a relaxed constraint than radial one. It could also be an indication that the radial DEA model may overestimate environmental efficiency since they only allow the reduction of undesirable outputs at the same rate. It can also be seen that the average EEIs for OECD countries are relatively stable, no matter whether radial or non-radial DEA model is used.

Table 2.2 Radial EEIs of 29 OECD countries, 2000–2011

Countries	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	Mean
Australia	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Austria	0.567	0.550	0.551	0.520	0.492	0.510	0.529	0.542	0.568	0.560	0.533	0.559	0.540
Belgium	1.000	1.000	1.000	1.000	1.000	0.845	0.645	0.582	0.599	0.622	0.586	0.554	0.786
Canada	0.471	0.463	0.460	0.412	0.380	0.386	0.399	0.335	0.301	0.310	0.302	0.291	0.376
Czech Republic	0.107	0.110	0.113	0.123	0.126	0.125	0.130	0.130	0.134	0.136	0.144	0.138	0.126
Denmark	1.000	1.000	1.000	0.903	0.977	0.691	0.599	0.639	0.673	0.638	0.667	1.000	0.816
Estonia	0.106	0.116	0.113	0.110	0.112	0.123	0.131	0.130	0.135	0.129	0.115	0.127	0.121
Finland	0.543	0.588	0.688	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.476	0.511	0.817
France	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Germany	0.628	0.599	0.615	0.608	0.603	0.608	0.639	0.641	1.000	1.000	1.000	1.000	0.745
Greece	0.351	0.338	0.331	0.338	0.330	0.326	0.332	0.266	0.264	0.276	0.274	0.238	0.305
Hungary	0.240	0.257	0.261	0.262	0.305	0.269	0.272	0.262	0.255	0.258	0.267	0.261	0.264
Iceland	0.946	1.000	0.847	0.921	0.935	1.000	1.000	1.000	1.000	1.000	0.648	0.647	0.912
Ireland	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Italy	0.607	0.558	0.533	0.503	0.484	0.465	0.482	0.449	0.454	0.468	0.491	0.473	0.497
Japan	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Luxembourg	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Netherlands	0.482	0.470	0.449	0.454	0.459	0.470	0.486	0.545	0.790	0.804	0.814	0.785	0.584
New Zealand	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Norway	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Portugal	0.404	0.447	0.430	0.520	0.479	0.476	0.510	0.473	0.524	0.559	0.674	0.677	0.514
Slovak Republic	0.150	0.147	0.156	0.169	0.176	0.185	0.198	0.214	0.219	0.234	0.231	0.286	0.197
Slovenia	0.256	0.246	0.264	0.287	0.296	0.297	0.297	0.295	0.307	0.296	0.307	0.288	0.286
Spain	0.424	0.428	0.391	0.422	0.393	0.376	0.389	0.361	0.403	0.438	0.479	0.412	0.410
Sweden	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

(continued)

Table 2.2 (continued)

Countries	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	Mean
Switzerland	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Turkey	0.659	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.972
United Kingdom	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
United States	0.417	0.368	0.365	0.394	0.375	0.329	0.330	0.305	0.331	0.334	0.328	0.325	0.350

Table 2.3 Non-radial EEIs of 29 OECD countries, 2000–2011

Countries	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	Mean
Australia	0.155	0.153	0.158	0.163	0.162	0.164	0.161	0.159	0.162	0.167	0.171	0.162	0.162
Austria	0.458	0.457	0.455	0.445	0.466	0.466	0.473	0.474	0.481	0.488	0.487	0.481	0.469
Belgium	0.487	0.468	0.489	0.517	0.516	0.508	0.514	0.513	0.510	0.500	0.464	0.496	0.498
Canada	0.293	0.278	0.278	0.263	0.246	0.241	0.238	0.208	0.198	0.207	0.191	0.184	0.235
Czech Republic	0.105	0.109	0.113	0.118	0.118	0.123	0.126	0.128	0.133	0.134	0.134	0.130	0.122
Denmark	0.617	0.582	0.562	0.542	0.561	0.589	0.541	0.512	0.495	0.454	0.456	0.473	0.532
Estonia	0.096	0.103	0.107	0.108	0.109	0.119	0.127	0.121	0.114	0.110	0.102	0.107	0.110
Finland	0.429	0.409	0.406	0.404	0.420	0.455	0.435	0.455	0.474	0.404	0.396	0.428	0.426
France	0.610	0.590	0.573	0.570	0.563	0.551	0.543	0.520	0.511	0.530	0.497	0.505	0.547
Germany	0.545	0.539	0.545	0.546	0.541	0.555	0.582	0.592	0.625	0.632	0.658	0.664	0.585
Greece	0.306	0.309	0.299	0.308	0.301	0.294	0.302	0.256	0.250	0.255	0.242	0.218	0.278
Hungary	0.142	0.143	0.152	0.155	0.156	0.159	0.165	0.146	0.154	0.154	0.154	0.191	0.156
Iceland	0.455	0.476	0.478	0.509	0.530	0.516	0.415	0.397	0.384	0.363	0.355	0.412	0.441
Ireland	0.237	0.261	0.291	0.317	0.319	0.322	0.326	0.330	0.312	0.312	0.306	0.309	0.303
Italy	0.571	0.547	0.516	0.492	0.470	0.456	0.467	0.436	0.437	0.439	0.446	0.431	0.476
Japan	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Luxembourg	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Netherlands	0.427	0.434	0.440	0.447	0.453	0.468	0.481	0.505	0.604	0.602	0.584	0.589	0.503
New Zealand	0.216	0.208	0.213	0.207	0.197	0.186	0.174	0.166	0.157	0.176	0.168	0.155	0.185
Norway	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Portugal	0.259	0.268	0.257	0.272	0.261	0.258	0.266	0.257	0.265	0.270	0.283	0.269	0.265
Slovak Republic	0.128	0.127	0.127	0.134	0.138	0.147	0.151	0.162	0.171	0.178	0.183	0.251	0.158
Slovenia	0.193	0.196	0.203	0.213	0.218	0.220	0.222	0.223	0.235	0.227	0.230	0.219	0.217
Spain	0.328	0.342	0.342	0.343	0.343	0.349	0.351	0.339	0.367	0.369	0.368	0.351	0.349
Sweden	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

(continued)

Table 2.3 (continued)

Countries	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	Mean
Switzerland	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Turkey	0.236	0.236	0.271	0.308	0.359	0.376	0.325	0.350	0.381	0.331	0.368	0.389	0.327
United Kingdom	0.692	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.974
United States	0.276	0.259	0.262	0.273	0.272	0.259	0.254	0.242	0.253	0.252	0.251	0.247	0.258

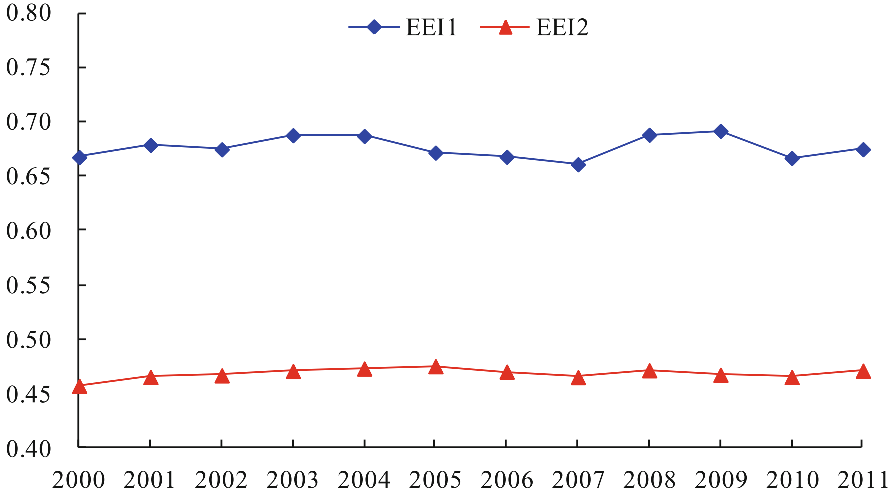


Fig. 2.1 Trends in the average EEI values from radial and non-radial DEA models

2.4.2.2 EPI Analysis

In order to assess how the environmental performance of OECD countries changed over time, we further employ (2.7) to calculate the EPI values for the countries. In this process, both radial and non-radial DEA models are employed. Table 2.4 shows the cumulative EPI values for all the countries in 2011 with 2000 as the base year. Equation (2.8) is used to compute the two contribution components of EPI and the cumulative results for 2011 are also shown in Table 2.4. If radial DEA model is used, the environmental productivity index decreased by 3.3 % from 2000 to 2011, which was mainly driven by the shift of environmental DEA technology. On the other hand, the environmental efficiency of OECD countries as a whole increased by 21.7 %. With non-radial DEA model, the environmental productivity of OECD countries decline by more during the period of time and the shift of environmental DEA technology was still the main contributing factor to the deterioration.

Table 2.5 shows the average EPI values as well as their contribution components derived from non-radial DEA models for OECD countries as a whole for every two consecutive years. It is found that from 2000 to 2009 the environmental productivity of OECD countries as a whole showed a declining trend mainly driven by the degeneracy of environmental production technology. However, in 2009–2011 the environmental productivity of these countries increased, which was mainly due to the growth of environmental efficiency while the situation of technology deterioration also became better. On the other hand, the environmental efficiency showed an increasing trend with the average growth rate equal to 1.9 % during the sample period.

Table 2.4 Cumulative EPI and their components, 2011

Countries	Radial DEA			Non-radial DEA		
	Cumulative EPI	EEFCH	TECH	Cumulative EPI	EEFCH	TECH
Australia	0.862	1.084	0.795	1.210	1.046	1.157
Austria	1.234	0.983	1.256	1.213	1.050	1.155
Belgium	0.578	0.761	0.760	0.455	1.301	0.350
Canada	0.191	1.102	0.174	0.227	1.003	0.226
Czech Republic	1.597	1.285	1.243	1.559	1.245	1.252
Denmark	1.221	1.000	1.221	0.339	1.076	0.315
Estonia	1.321	1.251	1.056	1.305	1.147	1.138
Finland	0.912	0.937	0.974	0.432	1.289	0.336
France	0.823	1.000	0.823	0.321	1.239	0.259
Germany	0.407	2.508	0.162	0.414	1.689	0.245
Greece	0.387	0.946	0.410	0.451	0.998	0.452
Hungary	1.428	1.085	1.316	1.103	1.486	0.742
Iceland	0.635	0.695	0.913	0.605	1.073	0.564
Ireland	0.713	1.166	0.612	1.619	1.301	1.245
Italy	0.705	0.979	0.720	0.598	1.023	0.584
Japan	1.012	1.000	1.012	1.012	1.000	1.012
Luxembourg	0.927	1.000	0.927	0.907	1.000	0.907
Netherlands	0.849	1.552	0.547	0.562	1.597	0.352
New Zealand	0.995	1.000	0.995	0.482	0.921	0.524
Norway	1.000	1.000	1.000	1.000	1.000	1.000
Portugal	1.930	1.672	1.154	1.302	1.040	1.252
Slovak Republic	2.379	1.900	1.252	2.396	1.921	1.247
Slovenia	1.352	1.125	1.201	1.391	1.139	1.221
Spain	1.242	0.972	1.278	1.334	1.070	1.246
Sweden	0.988	1.000	0.988	0.351	1.477	0.238
Switzerland	1.000	1.000	1.000	0.999	1.000	0.999
Turkey	0.373	2.562	0.146	0.397	2.305	0.172
United Kingdom	0.307	1.739	0.176	0.130	2.255	0.058
United States	0.679	0.985	0.689	0.919	0.984	0.934
Mean	0.967	1.217	0.855	0.863	1.265	0.730

2.5 Conclusion

DEA has been applied to model environmental performance at both macro and micro levels. This chapter first introduces the concepts and formulations of environmental DEA technologies, which highlights the fundamental role of modeling undesirable outputs in environmental performance measurement. We then present both radial and non-radial DEA models for measuring environmental efficiency and productivity. A case study of OECD countries for 2000–2011 is proposed to illustrate the use of different DEA models. While only several basic DEA models

Table 2.5 Non-radial EPI estimates and their components

	EPI	EFFCH	TECH
2000/2001	0.925	1.013	0.913
2001/2002	0.948	1.053	0.900
2002/2003	0.943	1.022	0.923
2003/2004	0.965	1.047	0.922
2004/2005	0.972	1.015	0.958
2005/2006	0.981	1.010	0.971
2006/2007	0.985	1.014	0.971
2007/2008	0.963	0.978	0.984
2008/2009	0.933	0.968	0.964
2009/2010	1.007	1.037	0.971
2010/2011	1.043	1.049	0.994
Mean	0.969	1.019	0.952

are introduced in this chapter, a quick summary of some other developments in this area is also provided.

This chapter is by far not a comprehensive review of DEA models for environmental performance measurement. In the last decade, many studies on the applications of DEA to environmental performance measurement have been reported. While a number of studies focused on the application aspect, others attempted to refine the existing DEA models in order to cater for their particular purposes, e.g. the incorporation of the slacks for specific variables. Indeed, DEA can easily handle different situations depending on the user targets, whereas the interpretation of the DEA results needs to be more careful. It is expected that this introductory text helps to invoke the attention of energy and environmental analysts to use DEA to model environmental issues for informing policy and decision making. In the future, one interesting topic is to examine the issue of carbon dioxide emissions with DEA as a result of the growing concern about climate change and global warming. This includes not only the measurement of carbon performance but also many other topics, e.g. the allocation of CO_2 emission allowance in emission trading. A recent example is Zhou et al. (2014a) in which the optimal path as well as policy strategies for controlling CO_2 emissions in China is derived through DEA modeling. Another interesting direction is to use DEA to benchmark corporate environmental performance as done by Chen and Delmas (2012), which helps to identify top performers with their competitive advantages for business strategy management. In this line of research, unpredicted data features may bring difficulty in the use of DEA and the interpretation of modeling results while are capable of generating more interesting works.

References

Ali, A. I., & Seiford, L. M. (1990). Translation invariance in data envelopment analysis. *Operations Research Letters, 9*, 403–405.

Banker, R. D., Charnes, A., & Cooper, W. W. (1984). Some models for estimating technical and scale inefficiencies in data envelopment analysis. *Management Science, 30*, 1078–1092.

Bian, Y., & Yang, F. (2010). Resource and environmental efficiency analysis of provinces in China: A DEA approach based on Shannon's entropy. *Energy Policy, 38*, 1909–1917.

Blancard, S., & Hoarau, J. F. (2013). A new sustainable human development indicator for small island developing states: A reappraisal from data envelopment analysis. *Economic Modelling, 30*, 623–635.

Boyd, G. A., & McClelland, J. D. (1999). The impact of environmental constraints on productivity improvement in integrated paper plants. *Journal of Environmental Economics and Management, 38*, 121–142.

Caves, D. W., Christensen, L. R., & Diewert, W. E. (1982). Multilateral comparisons of output, input and productivity using superlative index numbers. *The Economic Journal, 92*, 73–86.

Chen, C. M. (2013). A critique of non-parametric efficiency analysis in energy economics studies. *Energy Economics, 38*, 146–152.

Chen, C. M., & Delmas, M. (2012). Measuring eco-inefficiency: A new frontier approach. *Operations Research, 60*, 1064–1079.

Chung, Y. H., Fare, R., & Grosskopf, S. (1997). Productivity and undesirable outputs: A directional distance function approach. *Journal of Environmental Management, 51*, 229–240.

Coelli, T., Lauwers, L., & van Huylenbroeck, G. (2007). Environmental efficiency measurement and the materials balance condition. *Journal of Productivity Analysis, 28*, 3–12.

Cook, W. D., & Seiford, L. M. (2009). Data envelopment analysis (DEA)—Thirty years on. - *European Journal of Operational Research, 192*, 1–17.

Färe, R., & Grosskopf, S. (2004). Modeling undesirable factors in efficiency evaluation: Comment. *European Journal of Operational Research, 157*, 242–245.

Färe, R., & Grosskopf, S. (2005). *New directions: Efficiency and productivity*. Boston: Kluwer.

Färe, R., Grosskopf, S., Lovell, C. A. K., & Pasurka, C. (1989). Multilateral productivity comparisons when some outputs are undesirable: A nonparametric approach. *The Review of Economics and Statistics, 71*, 90–98.

Färe, R., Grosskopf, S., Norris, M., & Zhang, Z. (1994). Productivity growth, technical progress and efficiency change in industrialized countries. *American Economic Review, 84*, 66–83.

Färe, R., Grosskopf, S., & Hernandez-Sancho, F. (2004). Environmental performance: An index number approach. *Resource and Energy Economics, 26*, 343–352.

Hsu, A., Emerson, J., Levy, M., de Sherbinin, A., Johnson, L., Malik, O., Schwartz, J., & Jaiteh, M. (2014). *The 2014 environmental performance index*. New Haven, CT: Yale Center for Environmental Law & Policy. Retrieved from www.epi.yale.edu.

Kuosmanen, T. (2005). Weak disposability in nonparametric production analysis with undesirable outputs. *American Journal of Agricultural Economics, 87*, 1077–1082.

Managi, S., & Jena, P. R. (2008). Environmental productivity and Kuznets curve in India. *Ecological Economics, 65*, 432–440.

Meng, F. Y., Fan, L. W., Zhou, P., & Zhou, D. Q. (2013). Measuring environmental performance in China's industrial sectors with non-radial DEA. *Mathematical and Computer Modelling, 58*, 1047–1056.

Picazo-Tadeo, A. J., Reig-Martinez, E., & Hernandez-Sancho, F. (2005). Directional distance functions and environmental regulation. *Resource and Energy Economics, 27*, 131–142.

Scheel, H. (2001). Undesirable outputs in efficiency valuations. *European Journal of Operational Research, 132*, 400–410.

Seiford, L. M., & Zhu, J. (1998). Identifying excesses and deficits in Chinese industrial productivity (1953–1990): A weighted data envelopment analysis approach. *Omega, 26*, 279–296.

Seiford, L. M., & Zhu, J. (2002). Modeling undesirable factors in efficiency evaluation. *European Journal of Operational Research, 142*, 16–20.
Song, M. L., An, Q., Zhang, W., Wang, Z., & Wu, J. (2012). Environmental efficiency evaluation based on data envelopment analysis: A review. *Renewable and Sustainable Energy Reviews, 16*, 4465–4469.
Sueyoshi, T., & Goto, M. (2012). DEA environmental assessment of coal fired power plants: Methodological comparison between radial and non-radial models. *Energy Economics, 34*, 1854–1863.
Tone, K. (2001). A slacks-based measure of efficiency in data envelopment analysis. *European Journal of Operational Research, 130*, 498–509.
Tyteca, D. (1996). On the measurement of the environmental performance of firms—A literature review and a productive efficiency perspective. *Journal of Environmental Management, 46*, 281–308.
Tyteca, D. (1997). Linear programming models for the measurement of environmental performance of firms—Concepts and empirical results. *Journal of Productivity Analysis, 8*, 183–197.
Wang, K., Lu, B., & Wei, Y. M. (2013). China's regional energy and environmental efficiency: A range-adjusted measure based analysis. *Applied Energy, 112*, 1403–1415.
Zaim, O. (2004). Measuring environmental performance of state manufacturing through changes in pollution intensities: A DEA framework. *Ecological Economics, 48*, 37–47.
Zhou, P., Ang, B. W., & Han, J. Y. (2010a). Total-factor carbon emission performance: A Malmquist index analysis. *Energy Economics, 32*, 194–201.
Zhou, P., Ang, B. W., & Poh, K. L. (2006a). Comparing aggregating methods for constructing the composite environmental index: An objective measure. *Ecological Economics, 59*, 305–311.
Zhou, P., Ang, B. W., & Poh, K. L. (2006b). Slacks-based efficiency measures for modeling environmental performance. *Ecological Economics, 60*, 111–118.
Zhou, P., Ang, B. W., & Poh, K. L. (2007a). A mathematical programming approach to constructing composite indicators. *Ecological Economics, 62*, 291–297.
Zhou, P., Ang, B. W., & Poh, K. L. (2008a). A survey of data envelopment analysis in energy and environmental studies. *European Journal of Operational Research, 189*, 1–18.
Zhou, P., Ang, B. W., & Poh, K. L. (2008b). Measuring environmental performance under different environmental DEA technologies. *Energy Economics, 30*, 1–14.
Zhou, P., Ang, B. W., & Wang, H. (2012). Energy and CO_2 emission performance in electricity generation: A non-radial directional distance function approach. *European Journal of Operational Research, 221*, 625–635.
Zhou, P., Ang, B. W., & Zhou, D. Q. (2010b). Weighting and aggregation in composite indicator construction: A multiplicative optimization approach. *Social Indicators Research, 96*, 169–181.
Zhou, P., Poh, K. L., & Ang, B. W. (2007b). A non-radial DEA approach to measuring environmental performance. *European Journal of Operational Research, 178*, 1–9.
Zhou, P., Sun, Z. R., & Zhou, D. Q. (2014a). Optimal path for controlling CO_2 emissions in China: A perspective of efficiency analysis. *Energy Economics, 45*, 99–110.
Zhou, P., Zhou, X., & Fan, L. W. (2014b). On estimating shadow prices of undesirable outputs with efficiency models: A literature review. *Applied Energy, 130*, 799–806.
Zhu, J. (1996). Data envelopment analysis with preference structure. *Journal of the Operational Research Society, 47*, 136–150.
Zhu, J., & Cook, W. D. (2007). *Modeling data irregularities and structural complexities in data envelopment analysis*. New York: Springer.

Chapter 3
Input and Output Search in DEA: The Case of Financial Institutions

C. Serrano Cinca, C. Mar Molinero, and Y. Fuertes Callén

Abstract DEA has been extensively used but many problems remain unsolved. One of them is the selection of inputs and outputs. We propose here a methodology for input and output selection in the context of financial institutions. There are various views of what constitutes inputs and outputs in a financial institution. The paper uses multivariate statistical techniques in order to explore up to what point the various combinations of inputs and outputs are equivalent, and up to what point the efficiency score obtained by a given institution changes under the various combinations of inputs and outputs. This helps in the search for the best specification, and can guide other specification search tools such as the bootstrap. The extent to which two institutions that achieve the same efficiency score arrive at it following different strategies is explored with the aim of finding out what is behind a DEA score. By-products of the approach proposed here are the creation of league tables of financial institutions in terms of efficiencies and the possibility of assessing the strengths and weaknesses of individual institutions. This methodology is applied to the particular case of American banks efficiency.

Keywords Efficiency • Principal component analysis • Banking • Data envelopment analysis • Specification search • Bootstrap

3.1 Introduction

Efficiency is a key concept for financial institutions, and it has long been studied. A review of 130 such studies in 21 countries is given by Berger and Humphrey (1997). Berger and Humphrey classify papers according to the technical approach employed, which they identify as parametric—Stochastic Frontier Approach

C. Serrano Cinca (✉) • Y. Fuertes Callén
Department of Accounting and Finance, University of Saragossa, Zaragoza, Spain
e-mail: serrano@unizar.es

C. Mar Molinero
Kent Business School, University of Kent, Canterbury, UK

Department of Management, Universitat Autònoma de Barcelona, Barcelona, Spain

S.-N. Hwang et al. (eds.), *Handbook of Operations Analytics Using Data Envelopment Analysis*, International Series in Operations Research & Management Science 239, DOI 10.1007/978-1-4899-7705-2_3

(SFA), Distribution Free Approach (DFA), Thick Frontier Approach (TFA)—or non parametric—Data Envelopment Analysis (DEA), Free Disposal Hull (FDH), Index Numbers (IN), Mixed Optimal Strategy (MOS). By far the most popular technical approach is DEA, which was applied in 62 of the papers surveyed.

DEA is becoming widely used to assess the efficiency of organizations with multiple homogeneous decision units that produce several outputs with a variety of inputs. For an extensive bibliography of DEA see, for example, Emrouznejad and Thanassoulis (1996), and Seiford (1996). DEA is appropriate for sets of homogeneous units with similar inputs and similar outputs since it performs multiple comparisons using a Linear Programming based approach. The assumptions underlying DEA are minimal. Inputs and outputs can be measured in their own units, and these units can be different for the different inputs and outputs. A survey of the more restricted area of DEA applications to bank branch performance is given by Schaffnit et al. (1997). Some later references on the application of DEA to financial institutions are Athanassopoulos (1997), Pastor et al. (1997), Seiford and Zhu (1999), Saha and Ravisankar (2000), Dekker and Post (2001), Kuosmanen and Post (2001), Hartman et al. (2001), Luo (2003), and Wheelock and Wilson (2006).

For the purposes of this paper, it will be useful to make a distinction between model and specification in a DEA context. Different philosophical approaches as to what a financial institution does, and what is meant by efficiency will lead to different models. Two basic models are prevalent in the literature: intermediation and production; see Berger and Mester (1997) for a full discussion. Specification will refer to a more restricted concept: the particular set of inputs and outputs that enter into model definition.

The variety of models and specifications for financial efficiency analysis is reflected in practice. The selection of inputs and outputs varies from study to study; this is a complex subject on which there is much discussion. For example; a particular item, such as deposits, may be treated as an input or as an output according to whether the institution is modeled from the point of view of production or from the point of view of intermediation, see Athanassopoulos (1997). This is a matter of concern, as the level of efficiency of a financial institution may depend on the particular choice of inputs and outputs. It may be puzzling for the manager of a bank branch to discover that it is possible for different researchers to arrive at different conclusions about the efficiency of a bank branch when using the same technique (DEA) on the same data. However, this confusion may be more apparent than real, since alternative specifications may be equivalent. The study of the extent to which two different specifications are equivalent is one of the purposes of this paper. Specification search is complicated by the fact that there is no natural ordering that can be used to nest hypotheses test. This is a common problem in Econometrics to which there is not yet a satisfactory solution. Methodologies for specification searches in Econometrics have been extensively studied. Hendry and Mizon (1978) have argued that one should always start with a general model and check that simplifications are valid, rather than proceed in an "ad hoc manner". They further introduced the encompassing principle that the results obtained using a particular specification should be deductible from the general specification. This is, in broad lines, the philosophy that has guided our work.

Model and specification selection are not the only issues addressed in this paper. We wish to go behind the efficiency score. Two financial institutions may achieve the same DEA efficiency under a given model and under a common specification, but they may still be very different. Efficiency, being a mere score, may be compatible with a variety of management strategies. Imagine two institutions that achieve the same efficiency, one may have specialized in the production of a particular output and the other on the good use of a particular input. These differences will, of course, be reflected in different weight structures for inputs and outputs, and could be identified by means of such techniques as cross-efficiency analysis; Doyle and Green (1994). Here we apply a methodological approach based on the combination of DEA and multivariate statistical analysis; Serrano Cinca and Mar Molinero (2004), and Serrano Cinca et al. (2005). This approach has the advantage of visualizing the way in which a particular DEA score has been achieved by a financial institution, and how this score is related to the model selected.

The purpose of the paper is to discuss how to conduct a specification search, to describe how the method that we propose works in practice, and how the results can be interpreted. In this paper, efficiencies are calculated for a variety of DEA specifications. It is proposed that DEA modeling be embedded in a multivariate statistical framework.

This paper unfolds as follows. The next section contains a discussion of efficiency in financial institutions. The particular case study of American banks efficiency is introduced and presented in the next section. This is followed by a description of the model and its implementation. The paper is completed with a conclusions section.

3.2 Efficiency Modeling in Financial Institutions

For modeling purposes, financial institutions are often seen from the point of view of intermediation or from the point of view of production; see Athanassopoulos (1997), although other models also exist; Camanho and Dyson (2005). Under the intermediation model, institutions collect deposits and make loans in order to make a profit. Deposits and acquired loans are inputs. Institutions are interested in placing loans, which are traditional outputs in studies of this kind; see, for example Berger and Humphrey (1991). Under the production model, a financial institution uses physical resources such as labor and plant in order to process transactions, take deposits, lend funds, and so on. In the production model manpower and assets are treated as inputs and transactions dealt with—such as deposits and loans—are treated as outputs. See, for example, Vassiloglou and Giokas (1990), Schaffnit et al. (1997), Soteriou and Zenios (1999).

The mathematical models used to study the efficiency of financial institutions can be divided into two groups: those based on parametric frontier techniques, and those based on Data Envelopment Analysis (DEA). Berger and Humphrey (1991) find inconsistencies between the two approaches, although Ondrich and Ruggiero (2001) argue that both produce similar rankings, and conclude that there is no advantage in using parametric frontiers.

In this paper we focus on DEA models. Up to what point different DEA modeling approaches produce different efficiency results? This question can only be answered by looking at particular case studies. The selection of the correct set of inputs and outputs that enters a specification goes back a long way; Farrell (1957) observed that "this is a highly subjective matter, and one hesitates to attempt to lay down any objective criteria of plausibility". Oral and Yolalan (1990) found that a DEA model aimed at estimating service efficiency in bank branches in Turkey produced indistinguishable results from an alternative DEA model focused on profitability. A possible way out would be to create a general model that encompasses various modeling philosophies as particular cases. But care has to be exercised since the more inputs and outputs a model contains, the more units become efficient through specialization or, as Lovell and Pastor (1997) put it, "because they are self-identifiers". The relationship between efficiency and the number of inputs and outputs has been studied by Pedraja Chaparro et al. (1999).

Alternative specifications for inputs and outputs for a given model have been explored in many studies. Athanassopoulos (1997) observes a lack of consistency in the selection of inputs and outputs when studying bank branch efficiency. Oral and Yolalan (1990) experiment with various specifications and observe that efficiencies change according to the input/output mix chosen. Some times there is no choice, as the chosen specification is in part determined by the data that is available; Vassiloglou and Giokas (1990). Lovell and Pastor (1997) observe that alternative specifications may not give significantly different results, and apply the Pastor et al. (2002) methodology to choose a parsimonious specification. This approach is based on a sound mathematical model, but has a mechanical feel to it. Simar and Wilson (2000a, b) proposed an inferential methodology based on the bootstrap in order to test the validity of the inclusion or exclusion of a variable in a DEA specification. The consequences of mis-specification were explored by Smith (1997). A comparison of specification searches has been performed by Sirvent et al. (2005).

Different specifications are not totally equivalent, and it is difficult to assess what are the consequences for individual units of adding or removing an input/output without engaging in considerable extra work.

An alternative approach to specification search is applied in this paper; Serrano Cinca and Mar Molinero (2004), and Serrano Cinca et al. (2005). The distinctive features of a specification are revealed by embedding DEA efficiency results into a multivariate statistical framework. We use in particular Principal Components Analysis (PCA), multiple regression, and Hierarchical Cluster Analysis (HCA). PCA has been used as an alternative to DEA by Zhu (1998) and Premachandra

(2001). PCA as a data reduction technique to select inputs and outputs has been used by Adler and Golany (2001).

In this approach, PCA plays a fundamental role in specification and model selection. We do not attempt to find a “best” specification of inputs and outputs. A variety of possible specifications that offer combinations of inputs and outputs are estimated and efficiencies calculated for each financial institution under each specification. In this way, a matrix is obtained in which each column corresponds to a specification, and each row to a financial institution. This matrix is analyzed by means of Principal Components Analysis (PCA). Component scores are plotted to show the extent to which the efficiency of financial institutions remains unchanged under the various specifications. The plot is interpreted by means of Property Fitting (ProFit), a regression-based technique. The superimposition of the ProFit results on the scores plot will help to identify specification equivalence, guide model selection, identify outlying behavior, rank banks, and assess strategic behavior patterns in financial institutions that achieve the same efficiency score.

3.3 A Case Study: American Banks

US commercial banks are by far the best studied financial institutions from the point of view of efficiency. Their study has been undertaken from a variety of perspectives and using several methodologies: Stochastic Frontier Approach (SFA), Distribution Free Approach (DFA), Thick Frontier Approach (TFA), Data Envelopment Analysis (DEA) and Free Disposal Hull (FDH); see Berger and Humphrey (1997). Amongst the studies that have applied DEA to the analysis of efficiency in US banks we can list Aly et al. (1990), Barr et al. (1993), Berg et al. (1992, 1993), Charnes et al. (1990), Elyasiani and Mehdian (1995), Ferrier and Lovell (1990), Miller and Noulas (1996), Thompson et al. (1997), and Wheelock and Wilson (2006).

According to Barr et al. (2002), the environment in which U.S. commercial banks operate has undergone profound transformations, as has the way in which they conduct their business. They mention changes in the regulatory environment, in the impact of information technology, and in the way in which they assess risks. A very competitive industry has emerged. They further point out that such changes tend to result in a split between institutions that “perform relatively well and those that perform relatively poorly”, and that the different kinds of institutions can be identified using non-parametric methods.

This section will be divided into sub-sections. First, the data set will be described. The second subheading will concentrate on DEA and PCA. Empirical results will be interpreted in the third and fourth sub-sections.

3.3.1 The Data Set: Three Inputs and Three Outputs

Data was extracted from Standard & Poor's COMPUSTAT database for the year 2003, SIC code 6020, "Commercial Banks". All of them are incorporated in the US and are quoted in the New York stock exchange—or in Nasdaq. In total, 85 American banks met the conditions and all of them were selected. Having been extracted from annual accounts, all the data except number of employees are measured in monetary units. The list of all institutions is given in Table 3.1. Rather than use the full name of each institution, the "trading symbol" has been employed.

There is much agreement on what constitutes inputs and outputs under the production model and under the intermediation model, although not all authors use the same set of inputs and outputs. After a thorough survey of the inputs and outputs used in the literature,—see Table 3.2—the following inputs and outputs were selected. Inputs: labor, physical capital, and deposits. Outputs: interest and non-interest income; deposits; and loans.

3.3.1.1 Labor

This item measures the number of company workers as reported to shareholders. Labor has been used in banking efficiency studies by Sherman and Gold (1985), Ferrier and Lovell (1990), Aly et al. (1990), Berg et al. (1993), Seiford and Zhu (1999), Wheelock and Wilson (1999), and Luo (2003).

3.3.1.2 Physical Capital

This item measures the net cost or valuation of tangible fixed assets used in the regular business operations of the company, less accumulated depreciation, investment grants, and other deductions. It has been used by Berg et al. (1993), Seiford and Zhu (1999), Wheelock and Wilson (1999), Tortosa-Ausina (2002), Luo (2003), Barr et al. (2002), Aly et al. (1990) and Berg et al. (1991).

3.3.1.3 Deposits

This item measures the total demand, savings, and time deposits held on account for individuals, partnerships, and corporations plus deposits held on account for other banks. It is an issue whether deposits are inputs or outputs. See Pastor et al. (1997) for a discussion. Deposits are treated as inputs by Sealey and Lindley (1977), Mester (1989), Elyasiani and Mehdian (1992), Miller and Noulas (1996), Mester (1997), Brockett et al. (1997), and Casu and Girardone (2004); they are treated as outputs by Sherman and Gold (1985), Ferrier and Lovell (1990), Berger and Humphrey (1991). Berg et al. (1991), Berg et al. (1993), and Kumbhakar

Table 3.1 List of banks and the values of inputs and outputs

Trading symbol	Labor (number of employees)	Physical capital ($ millions)	Deposits ($ millions)	Interest and non-interest income ($ millions)	Loans ($ millions)
ASBC	4091	131.315	9792.840	972.499	10,225.957
ASO	12,385	964.692	30,440.352	2942.229	29,057.531
BAC	133,549	6036.000	414,113.000	49,006.000	365,300.000
BBT	26,300	1201.342	59,349.785	6243.924	61,791.606
BK	22,901	1398.000	56,406.000	6336.000	42,901.000
BNK	6700	264.818	17,901.184	1560.128	16,155.371
BOH	2700	160.005	7332.777	641.241	5792.040
BOKF	3449	175.901	9219.863	868.165	7355.250
BPFH	437	13.740	1658.461	158.764	1597.292
CBCF	2342	112.784	5442.266	500.796	5166.832
CBH	8200	811.451	20,701.398	1248.109	7371.285
CBSH	4967	336.366	10,206.207	919.077	8007.457
CBSS	7700	527.295	15,687.820	1801.343	17,120.918
CBU	1259	61.705	2725.488	228.710	2099.414
CFR	3268	168.611	8068.857	584.307	4510.035
CHZ	2058	75.179	4969.891	371.543	3692.482
CIH	212,400	6514.000	478,494.000	64,120.000	495,332.000
CMA	11,282	374.000	41,463.000	3299.000	39,499.000
CNB	3939	246.170	9768.590	908.257	11,845.233
CORS	468	26.313	2846.402	187.159	2397.323
CYN	2348	62.719	10,937.063	752.950	8122.501
EWBC	730	24.957	3312.667	211.322	3234.133
FBP	1983	85.269	6765.105	624.507	7624.077
FCF	1474	46.538	3288.275	289.176	2792.859
FCTR	1031	95.756	2427.897	240.210	2255.798
FHN	11,494	350.202	15,679.969	2670.886	16,808.412
FITB	18,899	1828.000	57,095.000	6474.000	53,419.000
FMBI	1646	91.535	4815.105	365.427	4012.998
FMER	3063	119.079	7502.781	777.415	6517.363
FNB	1682	199.735	6159.496	553.883	5657.201
FULT	2950	120.777	6751.781	572.518	6115.055
GBBK	1710	83.816	5312.664	579.261	4411.637
HBAN	7983	349.712	18,487.395	2361.797	22,260.658
HIB	5339	217.399	14,159.516	1260.388	12,878.136
HNBC	623	23.329	1979.081	146.838	1400.189
HU	1859	125.168	6243.355	527.174	4633.264
IBNK	886	54.563	1812.630	174.852	1674.527
IFC	3589	32.208	2899.662	700.283	3980.664
IFIN	2413	76.420	4207.117	582.256	199.530
JPM	93,453	6487.000	326,492.000	44,363.000	225,170.000
KEY	20,034	606.000	50,858.000	5730.000	61,305.000

(continued)

Table 3.1 (continued)

Trading symbol	Labor (number of employees)	Physical capital ($ millions)	Deposits ($ millions)	Interest and non-interest income ($ millions)	Loans ($ millions)
KRB	26,500	2676.597	31,836.078	11,684.361	35,998.091
LBAI	502	27.510	1325.682	79.705	834.637
MEL	20,900	668.000	20,843.000	4403.000	10,139.000
MI	12,244	438.485	22,270.105	2740.721	24,835.379
MRBK	3565	140.922	10,262.551	773.166	9146.329
MTB	14,000	398.971	33,114.945	2957.660	35,171.573
NARA	320	6.766	1061.415	81.802	988.795
NBY	477	14.768	815.839	72.222	582.933
NCC	33,331	1125.526	63,930.031	9593.821	93,521.063
NFB	2979	150.875	15,116.113	1255.091	12,222.539
NPBC	1074	43.653	2435.296	206.933	2223.667
NTRS	8056	498.300	26,270.000	2580.100	26,432.297
ONB	3019	181.398	6493.090	661.897	5496.387
PBKS	1629	49.575	3079.549	343.790	2754.023
PNC	23,200	1456.000	45,241.000	5969.000	34,848.000
PRK	1645	36.746	3414.249	320.152	2667.661
PVN	4525	84.198	10,101.055	2781.408	5655.070
PVTB	219	6.233	1547.359	101.442	1213.977
RF	16,180	629.638	32,732.535	3617.887	33,068.654
RIGS	1450	226.502	4286.230	346.932	3483.946
SBIB	1036	48.541	2418.369	200.944	2127.675
SIVB	969	14.999	3666.876	277.397	1924.729
SKYF	3546	154.242	8514.852	844.285	9361.842
SNV	10,909	791.439	15,941.609	2430.821	16,376.583
STI	27,578	1595.307	81,189.500	7071.841	85,358.766
STT	19,850	1212.000	47,516.000	4727.000	26,698.000
SUSQ	2065	62.961	4134.465	387.770	4220.598
SWBT	1760	117.951	4403.238	326.023	3545.564
TCB	8136	282.193	7611.746	1105.143	8606.531
TRMK	2356	108.374	5089.457	516.931	4958.336
TRST	488	20.168	2419.810	166.779	1113.527
TSFG	1918	142.705	6028.648	509.618	5690.583
UB	10,146	509.734	35,532.281	2563.916	25,636.939
UBSI	1585	46.354	4182.371	400.824	4264.482
UCBH	666	84.145	4483.520	283.465	3730.780
USB	51,377	1957.000	11,9052.000	14,571.000	117,299.000
VLY	2264	128.606	7162.965	605.695	6107.758
WABC	1003	35.748	3463.991	268.575	2269.420
WB	86,670	4619.000	221,225.000	24,474.000	165,375.000
WFC	140,000	3534.000	247,527.000	31,800.000	285,706.000
WL	2307	152.300	6577.199	633.000	6135.398

(continued)

Table 3.1 (continued)

Trading symbol	Labor (number of employees)	Physical capital ($ millions)	Deposits ($ millions)	Interest and non-interest income ($ millions)	Loans ($ millions)
WTFC	929	156.714	3876.621	276.083	3302.522
WTNY	2369	148.259	6158.582	427.573	4838.441
ZION	7896	407.825	20,896.695	1889.483	19,652.739

Physical capital, Deposits, Interest and non-interest income, and Loans in millions of U.S. dollars. Number of employees in units

et al. (2001); they are treated simultaneously as inputs and outputs by Aly et al. (1990), Lozano (1998), Wheelock and Wilson (1999) and Tortosa-Ausina (2002).

3.3.1.4 Interest and Non-interest Income

This item measures revenue received from all earning assets plus total revenue/income that cannot be attributed to interest and dividends received from earning assets. Used by Miller and Noulas (1996), Thompson et al. (1997), Brockett et al. (1997) and Seiford and Zhu (1999).

3.3.1.5 Loans

This item measures the monetary value of all outstanding loans, claims, and advances made to individual, commercial, and industrial borrowers, less reserves for possible credit losses and unearned income. Used by Sherman and Gold (1985), Ferrier and Lovell (1990), Aly et al. (1990), Berger and Humphrey (1991), Berg et al. (1991), Berg et al. (1993), English et al. (1993), Miller and Noulas (1996), Mester (1997), Wheelock and Wilson (1999), Brockett et al. (1997) and Casu and Girardone (2004).

The values of all inputs and outputs for all the banks are given in Table 3.1.

Notation will be introduced in order to simplify the discussion of the various specifications. Inputs are referred to by means of capital letters, in such a way that the first input is represented by the letter A, the second input by the letter B, and the third one by the letter C. Outputs are referred to by means of numbers. The first output is associated with number 1, the second output with number 2, and the third output with number 3. In this way a specification that treats a bank as an institution whose employees (input A) take deposits (output 2) and place loans in the market (output 3) would be labeled A23. If this specification is augmented with physical assets (input B) and income (output 1), the specification becomes AB123. Specification AB123 treats a bank as a production unit that employs manpower (A) and plant (B) in order to generate income, deposits, and loans. An intermediation model would be described by a specification such as AC13, in which deposits (C) are

Table 3.2 Inputs and outputs

Variable symbol	Variable name	Description and COMPUSTAT acronimous	Used by
Input A	Number of employees	Employees "EMP"	Sherman and Gold (1985), Ferrier and Lovell (1990), Aly et al. (1990), Berg et al. (1993), Seiford and Zhu (1999), Wheelock and Wilson (1999), and Luo (2003)
Input B	Physical capital	Fixed assets (net) "PPENT"	Berg et al. (1993), Seiford and Zhu (1999), Wheelock and Wilson (1999), Tortosa-Ausina (2002), Luo (2003), Barr et al. (2002), Aly et al. (1990), and Berg et al. (1991)
Input C	Deposits	Deposits customer "DPTC" + Deposits banks "DPTB"	Sealey and Lindley (1977), Mester (1989), Elyasiani and Mehdian (1992), Miller and Noulas (1996), Mester (1997), Brockett et al. (1997), and Casu and Girardone (2004)
Output 1	Interest and non-interest income	Interest & div inc total "IDIT" + Income noninterest tot bank "INITB"	Miller and Noulas (1996), Thompson et al. (1997), Brockett et al. (1997) and Seiford and Zhu (1999)
Output 2	Deposits	Deposits customer "DPTC" + Deposits banks "DPTB"	Sherman and Gold (1985), Ferrier and Lovell (1990), Berger and Humphrey (1991). Berg et al. (1991), Berg et al. (1993), and Kumbhakar et al. (2001)
Output 3	Loans	Loans/claims/advances banks & Govt "LCABG" + Loans/claims/advances customers "LCACU"	Sherman and Gold (1985), Ferrier and Lovell (1990), Aly et al. (1990), Berger and Humphrey (1991), Berg et al. (1991), Berg et al. (1993), English et al. (1993), Miller and Noulas (1996), Mester (1997), Wheelock and Wilson (1999), Brockett et al. (1997), and Casu and Girardone (2004)

treated as an input. Under the AC13 specification a bank is an institution whose employees collect deposits in order to make loans and generate income.

Other possible views of the way in which a bank operates can be generated by using different combinations of inputs and outputs. Efficiency ratios are generated by choosing a specification with only one output and one input. It is, of course, possible to use all possible combinations of inputs with all possible combinations of outputs. The total number of specifications that could possibly be generated with n inputs and m outputs is given by the formula

$$\sum\nolimits_{i=1}^{n} C_n^i * \sum\nolimits_{i=1}^{m} C_m^i \quad \text{where} \quad \mathrm{C}_\mathrm{n}^\mathrm{i} = \binom{\mathrm{n}}{\mathrm{i}} = \frac{\mathrm{n}!}{\mathrm{i}!(\mathrm{n}-\mathrm{i})!}$$

In general, it will not be necessary to calculate efficiencies under all possible specifications, as some of them can be discarded on a priori grounds. The first step in any modeling exercise is to discard nonsensical specifications. What are nonsensical specifications is a matter of judgment. We could have started by studying the 19 different specifications suggested by the authors that we reference in Table 3.2, but we did not have full information on the details of the specification used by each of the authors, and we took the pragmatic view that we would consider all possible combinations, since there were not very many. In general, we would recommend not proceeding in a mechanical way without thinking about the specifications being estimated. This is also good advice when the number of inputs/outputs is large but the number of DMUs is not, since we fall in what Simar and Wilson call “the curse of dimensionality”. However methodical our procedure is, we can never avoid the exercise of judgment.

In our case there are three inputs and three outputs, giving a possible total number of 49 specifications. Specifications that treat deposits both as inputs and outputs have been excluded, reducing their total number to 33. Some authors include deposits both as inputs and as outputs; examples are Maudos et al. (2002), and Camanho and Dyson (2005). Such specifications do not use exactly the same variable as input and as output. Camanho and Dyson (2005), following the “value added” modeling approach, use as input “interest costs from deposits” and as output “total value of deposits”.

The complete list of specifications and the inputs and outputs that they contain can be found in Table 3.3.

DEA efficiencies, on a scale from 0 to 100 %, for all banks were calculated under Variable Returns to Scale (VRS) for all specifications. The results are given in Table 3.4.

Only American listed commercial banks were included in the data in an attempt to preserve homogeneity, but there are wide variations in size, and this can result in extreme or discordant behavior. Discordant behavior has a negative impact on efficiency estimates. To explore the presence of outliers, we have applied the super-efficiency approach of Andersen and Petersen (1993) as modified by Banker et al. (1989); Wilson’s (1993) approach; and we have also taken into account Banker and Chang (2006). This procedure revealed that the outliers depend on the specification used. For example, under the specification AB13 we detected the following outliers: BAC, CIH, CYN, CMA, CORS, EWBC, IFC, JPM, KEY, PVTB, PVN, and UBCH. The process was repeated with the 33 specifications and the results are summarized in Table 3.5. Table 3.5 shows the number of specifications under which a particular bank is classified as outlier. It is to be

Table 3.3 The 33 specifications and their definitions

Specification	Input	Output
A1	Employees	Income
A12	Employees	Income, deposits
A123	Employees	Income, deposits, loans
A13	Employees	Income, loans
A23	Employees	Deposits, loans
A2	Employees	Deposits
A3	Employees	Loans
B1	Physical assets	Income
B12	Physical assets	Income, deposits
B123	Physical assets	Income, deposits, loans
B13	Physical assets	Income, loans
B23	Physical assets	Deposits, loans
B2	Physical assets	Deposits
B3	Physical assets	Loans
AB1	Employees, physical assets	Income
AB12	Employees, physical assets	Income, deposits
AB123	Employees, physical assets	Income, deposits, loans
AB13	Employees, physical assets	Income, loans
AB23	Employees, physical assets	Deposits, loans
AB2	Employees, physical assets	Deposits
AB3	Employees, physical assets	Loans
C1	Deposits	Income
C13	Deposits	Income, loans
C3	Deposits	Loans
AC1	Employees, deposits	Income
AC13	Employees, Deposits	Income, Loans
AC3	Employees, Deposits	Loans
BC1	Physical Assets, Deposits	Income
BC13	Physical Assets, Deposits	Income, Loans
BC3	Physical Assets, Deposits	Loans
ABC1	Employees, Physical Assets, Deposits	Income
ABC13	Employees, Physical Assets, Deposits	Income, Loans
ABC3	Employees, Physical Assets, Deposits	Loans

noticed that only one bank, CIH, displays systematic super-efficiency outlying behavior in all 33 specifications.

The standard reason for removing outliers in DEA is because efficiency estimates can be significantly affected by their presence or absence. But discordant behavior depends on the specification used, and a bank that is an outlier under one of the specifications may not appear to be an outlier under a different specification. We are unhappy about removing a bank for outlying behavior under some specifications and not under others. This is clearly, very much a matter of judgment as

Table 3.4 Efficiency results under all specifications

	A1	A12	A123	A13	A23	A2	A3	B1	B12	B123	B13	B23	B2	B3	AB1	AB12	AB123	AB13	AB23	AB2	AB3	C1	C13	C3	AC1	AC13	AC3	BC1	BC13	BC3	ABC1	ABC13	ABC3
ASBC	40	51	60	60	60	45	60	24	45	64	64	64	42	64	32	52	69	69	68	47	68	33	73	73	42	80	79	33	78	78	33	78	78
ASO	39	60	69	69	65	60	65	10	28	29	29	28	27	28	25	49	51	51	47	46	47	28	66	66	37	75	74	24	50	50	24	50	50
BAC	91	100	100	100	100	100	100	82	93	93	82	93	93	77	99	100	100	100	100	100	97	84	85	81	89	100	99	85	85	79	85	85	79
BBT	45	60	74	74	73	60	73	37	54	55	53	55	52	51	48	63	67	67	67	59	64	29	71	71	40	77	77	49	66	66	49	66	66
BK	53	65	66	65	66	65	55	33	44	44	37	42	42	29	49	62	62	57	57	57	42	31	55	52	46	68	59	49	53	44	49	53	44
BNK	38	57	62	62	62	57	62	18	52	55	55	54	51	54	28	61	64	64	61	58	61	27	63	63	36	73	73	26	65	65	26	65	65
BOH	40	52	52	52	48	48	45	14	25	28	28	27	24	27	27	45	47	47	43	41	43	32	56	56	42	69	63	27	45	45	27	45	45
BOKF	42	55	55	55	49	49	48	16	31	34	34	32	29	32	29	48	51	51	46	43	46	32	56	56	42	70	63	28	49	49	28	49	49
BPFH	71	71	76	76	66	54	66	58	58	70	70	68	49	68	71	71	79	79	72	55	72	63	87	87	88	99	93	72	87	87	88	99	93
CBCF	37	45	48	48	45	37	45	16	25	34	34	34	24	34	26	41	49	49	47	37	47	36	68	68	43	73	72	31	55	55	31	55	55
CBH	25	56	56	30	56	56	20	5	20	20	8	20	20	7	15	41	41	21	41	41	16	19	26	25	25	35	28	14	19	17	14	19	17
CBSH	31	41	41	41	38	38	37	9	18	19	19	18	17	18	20	34	35	35	32	31	32	30	55	55	35	59	56	24	40	40	24	40	40
CBSS	38	49	60	60	58	41	58	11	23	30	30	29	21	29	24	40	49	49	46	35	46	35	76	76	41	78	77	27	56	56	27	56	56
CBU	34	38	38	37	32	32	30	16	19	23	23	22	18	22	34	39	39	38	36	36	33	45	64	64	50	63	60	37	45	45	50	63	60
CFR	30	44	44	36	44	44	27	12	26	26	21	26	26	20	21	40	40	33	39	39	28	27	40	40	34	49	43	23	32	32	23	32	32
CHZ	32	40	40	39	38	38	32	19	31	36	36	35	31	35	32	47	47	45	46	46	41	33	55	55	39	60	58	30	47	47	39	60	58
CIH	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
CMA	50	94	100	100	100	94	100	37	100	100	100	100	100	100	46	100	100	100	100	100	100	23	65	65	39	94	94	35	91	91	35	91	91
CNB	38	51	73	73	73	46	73	12	24	40	40	40	22	40	25	42	63	63	63	38	63	31	85	85	40	94	94	26	68	68	26	68	68
CORS	76	91	92	92	92	89	92	33	44	60	60	60	44	60	76	99	100	100	100	99	100	39	68	68	70	97	97	43	69	69	70	100	99
CYN	54	88	88	79	88	88	79	40	100	100	100	100	100	100	54	100	100	100	100	100	100	24	52	52	44	81	79	36	88	88	44	92	92
EWBC	54	67	79	79	79	67	79	38	54	91	91	91	54	91	54	82	100	100	100	82	100	36	74	74	56	99	99	44	89	89	56	100	100
FBP	53	68	87	87	87	59	87	25	44	69	69	69	41	69	53	70	95	95	95	66	95	34	80	80	50	100	100	34	84	84	50	100	100
FCF	35	40	41	41	34	33	34	25	31	42	42	41	29	41	35	45	49	49	44	41	44	42	66	66	49	67	65	40	57	57	49	68	66
FCTR	43	46	48	48	39	34	39	11	12	16	16	15	10	15	43	46	48	48	39	34	39	52	76	76	60	77	74	38	47	47	60	77	74
FHN	38	43	49	49	38	27	38	23	38	50	50	43	32	43	30	43	53	53	42	32	42	49	78	74	50	73	64	43	73	64	43	73	64
FITB	66	80	93	93	87	80	86	26	35	35	32	32	32	29	51	65	67	67	60	60	59	31	65	64	53	88	82	46	52	49	46	52	49
FMBI	39	49	50	50	45	45	45	15	25	32	32	32	25	32	39	52	53	53	53	52	53	33	61	61	44	72	70	29	48	48	44	72	70

(continued)

Table 3.4 (continued)

FMER	43	53	55	55	47	43	47	22	36	43	43	42	34	42	32	50	56	56	51	43	51	36	62	62	46	73	66	34	57	57	34	57	57
FNB	56	71	75	75	71	61	71	10	16	21	21	21	16	21	33	51	56	56	54	46	54	34	66	66	52	90	85	27	46	46	27	46	46
FULT	33	43	45	45	45	39	45	17	30	38	38	38	29	38	25	42	49	49	48	39	48	32	64	64	38	69	68	29	56	56	29	56	56
GBBK	58	67	68	68	50	50	50	24	34	42	41	39	31	39	58	69	71	71	59	57	58	41	60	60	58	83	71	38	52	52	58	83	71
HBAN	48	60	80	80	76	50	76	21	44	61	61	58	40	58	34	58	74	74	69	49	69	38	83	83	48	92	91	34	78	78	34	78	78
HIB	39	54	60	60	60	52	60	18	46	52	52	51	44	51	29	56	62	62	60	53	60	28	63	63	38	74	73	27	64	64	27	64	64
HNBC	47	50	50	48	46	46	40	32	35	36	35	35	34	33	47	51	51	48	49	49	41	51	66	66	66	69	65	51	55	55	66	69	65
HU	49	62	62	60	56	56	49	15	26	29	28	27	25	27	32	52	52	52	48	48	46	33	54	54	48	74	66	28	43	43	28	43	43
IBNK	38	38	41	41	34	29	34	15	15	19	19	18	13	18	38	38	41	41	34	29	34	60	82	82	64	74	73	47	52	52	64	74	73
IFC	33	33	36	36	20	12	20	73	73	100	100	90	37	90	73	73	100	100	90	37	90	86	100	100	69	85	73	92	100	100	92	100	100
IFIN	41	44	44	41	26	26	9	26	30	30	26	24	24	8	41	45	45	41	33	33	9	52	52	19	55	55	19	46	46	17	55	55	19
JPM	100	100	100	100	100	100	86	68	68	68	68	68	68	42	100	100	100	100	92	92	71	95	95	58	100	100	80	84	84	50	84	84	50
KEY	54	67	96	96	95	67	95	65	91	100	100	100	84	100	69	88	100	100	100	78	100	31	83	83	46	96	96	65	100	100	65	100	100
KRB	89	89	89	89	39	30	39	38	38	38	38	13	10	13	72	72	72	72	26	21	26	100	100	78	100	100	63	100	100	47	100	100	47
LBAI	44	44	44	44	44	44	44	23	23	23	23	23	23	23	44	44	44	44	44	44	44	63	73	73	72	72	72	52	52	52	72	72	72
MEL	38	38	38	38	22	22	12	38	38	38	38	25	25	12	43	43	43	43	25	25	13	59	59	34	54	54	26	73	73	25	73	73	25
MI	36	47	60	60	56	41	56	19	44	55	55	52	41	52	28	50	60	60	55	43	55	36	77	77	41	77	75	32	71	71	32	71	71
MRBK	36	54	60	60	60	54	60	18	42	52	52	52	41	52	27	53	63	63	63	52	63	26	63	63	36	74	74	26	65	65	26	65	65
MTB	35	59	71	71	71	59	71	26	74	83	83	83	72	83	31	70	77	77	77	68	77	26	73	73	34	81	81	30	87	87	30	87	87
NARA	68	68	68	68	68	68	68	92	92	92	92	92	92	92	92	92	92	92	92	92	92	79	100	100	100	100	100	100	100	100	100	100	100
NBY	46	46	46	46	46	46	46	42	42	42	42	42	42	42	46	46	46	46	46	46	46	100	100	100	99	99	99	88	88	88	99	99	99
NCC	57	58	93	93	92	51	92	71	72	92	92	91	61	91	77	77	98	98	98	60	98	41	100	100	53	100	100	84	100	100	84	100	100
NFB	70	100	100	100	100	100	100	26	71	71	71	70	70	69	48	99	99	98	95	95	93	26	56	56	52	98	94	30	70	70	30	70	70
NPBC	36	40	42	42	37	33	37	21	24	33	33	33	23	33	36	41	44	44	43	37	43	48	75	75	54	73	72	42	57	57	54	73	72
NTRS	52	77	94	94	91	77	91	16	46	50	50	49	44	49	34	69	75	75	71	64	71	29	69	69	44	92	92	26	65	65	26	65	65
ONB	37	45	47	47	38	37	38	12	20	24	24	23	19	23	25	37	41	41	36	32	36	37	61	61	43	67	62	30	44	44	30	44	44
PBKS	37	40	42	42	30	28	30	27	30	40	40	38	25	38	37	42	48	48	40	34	40	50	70	70	54	70	65	46	57	57	54	71	66
PNC	49	54	55	55	51	51	43	29	35	35	30	29	29	23	44	51	51	49	42	42	33	37	57	53	46	64	53	52	54	41	52	54	41
PRK	35	39	39	39	31	31	29	34	42	53	52	49	38	49	35	48	53	53	49	44	49	43	61	61	48	63	59	46	59	59	48	69	66

PVN	100	100	100	100	42	42	26	100	100	100	100	68	68	51	100	100	100	100	68	68	51	80	80	40	100	100	41	100	100	50	100	100	53
PVTB	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	58	78	78	100	100	100	95	100	100	100	100	100
RF	39	50	62	62	58	50	58	27	49	52	52	49	45	49	36	56	60	60	54	50	54	31	70	69	39	73	71	39	66	65	39	66	65
RIGS	42	51	51	51	44	44	43	6	9	11	11	11	8	11	24	35	36	36	35	34	34	36	61	61	48	72	69	25	36	36	25	36	36
SBIB	37	40	42	42	37	34	37	19	21	29	29	28	20	28	37	41	43	43	41	37	41	48	73	73	55	71	70	41	53	53	55	71	70
SIVB	52	62	62	55	56	56	36	76	100	100	89	100	100	80	76	100	100	91	100	100	80	37	45	45	55	62	50	65	72	69	65	75	70
SKYF	40	51	62	62	62	44	62	18	32	49	49	49	30	49	29	46	62	62	62	41	62	34	77	77	42	84	83	31	70	70	31	70	70
SNV	36	43	48	48	39	30	39	9	17	21	21	18	14	18	23	32	38	38	30	25	30	45	74	71	46	71	63	33	53	47	33	53	47
STI	50	80	100	100	100	80	100	33	58	61	58	61	58	58	48	75	85	85	85	75	85	24	72	72	40	91	91	43	68	68	43	68	68
STT	44	62	62	50	62	62	37	24	40	40	27	38	38	20	38	56	56	43	53	53	29	28	42	39	39	54	43	38	41	32	38	41	32
SUSQ	33	38	42	42	39	30	39	23	30	49	49	49	29	49	33	43	54	54	52	38	52	40	74	74	45	74	74	38	65	65	45	76	76
SWBT	33	41	41	41	37	37	36	11	17	21	21	21	17	21	22	36	38	38	37	35	35	34	60	60	41	65	64	27	42	42	27	42	42
TCB	23	25	29	29	25	17	25	13	17	26	26	24	14	24	18	23	31	31	27	17	27	47	80	80	41	64	60	36	55	55	36	55	55
TRMK	38	44	48	48	42	34	42	17	24	35	35	34	22	34	27	39	48	48	45	34	45	39	70	70	46	74	72	34	55	55	34	55	55
TRST	66	75	75	66	72	72	45	40	49	49	40	49	49	31	66	81	81	66	81	81	45	44	47	47	69	69	54	51	51	43	69	69	54
TSFG	46	58	63	63	63	52	63	13	22	30	30	30	21	30	29	47	56	56	55	44	55	33	67	67	46	83	81	27	52	52	27	52	52
UB	41	88	88	72	88	88	70	15	62	62	48	62	62	46	28	78	78	63	78	78	59	21	50	50	34	69	69	21	54	54	21	54	54
UBSI	44	51	56	56	51	39	51	32	42	68	68	67	40	67	44	58	72	72	70	51	70	41	74	74	51	84	82	43	75	75	51	88	87
UCBH	77	100	100	100	100	100	100	14	24	32	32	32	24	32	77	100	100	100	100	100	100	31	61	61	62	100	100	27	49	49	62	100	100
USB	58	65	79	79	77	65	77	67	76	76	73	75	75	68	77	80	84	83	81	75	79	47	78	73	56	85	80	76	81	73	76	81	73
VLY	45	60	61	61	58	55	58	16	31	36	36	36	30	36	31	52	58	58	55	49	55	31	61	61	45	78	73	28	53	53	28	53	53
WABC	49	58	58	54	51	51	41	31	40	46	43	45	40	42	49	65	65	57	63	63	48	39	53	53	54	68	60	41	51	51	54	68	60
WB	59	73	75	71	75	73	67	51	63	63	51	63	63	43	66	74	74	68	74	74	60	64	68	60	64	78	69	65	65	52	65	65	52
WFC	48	50	74	74	74	50	74	88	93	100	100	100	93	100	81	83	94	94	94	78	94	82	100	100	67	95	95	91	100	100	91	100	100
WL	47	58	62	62	57	49	57	14	24	31	31	31	22	31	30	47	55	55	52	41	51	35	66	66	48	81	76	30	52	52	30	52	52
WTFC	54	66	67	67	64	62	64	7	10	15	15	15	10	15	31	47	50	50	49	47	48	35	64	64	54	85	83	26	40	40	26	40	40
WTNY	31	43	43	42	43	43	41	11	21	24	24	24	21	24	21	38	40	40	40	38	39	29	57	57	36	63	62	24	43	43	24	43	43
ZION	39	59	67	67	66	59	66	14	41	44	44	43	40	43	27	56	60	60	57	53	57	27	65	65	37	76	76	24	60	60	24	60	60

Table 3.5 The number of specifications under which a particular bank is classified as outlier

CIH	PVTB	PVN	CMA	UCBH	BAC	CYN	JPM	KEY	NARA	IFC	WFC	KRB	NCC	SIVB	EWBC	NFB	CORS	FBP	STI	NBY
33	29	18	16	16	12	12	12	12	11	10	10	8	8	8	6	6	5	4	4	3

well as a matter of applying the correct technical approach. We think that discordant observations may have a story to tell and it is possible that more can be learned by leaving them in the sample than by removing them. We are of the opinion that it is better to identify outliers, and try to explain why they are extreme values, and to assess how influential they are. We have, obviously, started by checking that there were no transcription or recording errors in the data. In order to do this, we conducted checks on the annual accounts of the relevant banks. Discussion on this subject will be resumed below.

Visual examination of Table 3.4 reveals some important features. Only one bank, Citicorp (CIH), is efficient under all specifications, highlighting the fact that the selection of inputs and outputs and, therefore, the view of what constitutes efficiency in the financial sector, is a matter of importance. This was one of the conjectures that guided this research. Some banks (PVTB, PVN, CMA, UCBH) are 100 % efficient under many specifications. In the same way, some banks achieve low scores under most specifications. Take, for example, PVN, which is 100 % efficient under 18 specifications, implying that this is an excellent institution. However, its efficiency drops to 26 % under A3. This suggests the presence of some weakness in PVN, a subject that will be further explored below. A counter example is NBY, whose DEA scores tend to be low, but becomes 100 % efficient under three specifications: C1, C3, C13. This indicates that, although NBY can take action to improve its efficiency, it has some strong points that deserve further attention.

Consider now the case of two institutions that achieve the same DEA score under a given specification. An example would be CYN and IFC. They both are 100 % efficient under AB123. But differences appear if other specifications are considered. For example, under AB2 CYN achieves 100 % efficiency while the same score for IFC is 37 %. Under specification C13 CYN is 52 % efficient while IFC is 100 % efficient. This indicates that the two institutions follow two different paths to efficiency. What is behind their strategies? Answering such a question was another of the objectives of this research.

In summary, the level of efficiency achieved by a particular financial institution depends on the chosen specification, indicating that specification search is delicate and important. In addition, if two financial institutions achieve the same efficiency score under a given specification they may do so following very different patterns of behavior: there is no single path to efficiency in financial institutions. Exploring what is behind a DEA score is the objective of the next three subsections.

3.3.2 DEA Specification Searches Using Multivariate Methods

Although visual inspection of Table 3.4 is a source of important insights, a more formal analysis of the information it contains will be performed. Table 3.4 will be treated as a matrix with 85 cases, the banks, and 33 variables, the specifications, and analyzed using multivariate statistical methods. The methodological approach will combine PCA, HCA, and ProFit.

PCA discloses how many independent aspects of efficiency are revealed by the specifications. ProFit is, in essence, regression with principal components. In this regression the issue of multicollinearity does not arise, as the regressors are orthogonal by design. The results of ProFit are represented graphically. ProFit gives an immediate visual understanding of the relationship between efficiency and specification. The reason why we apply HCA is a pragmatic one. PCA and ProFit produce representations in more than two dimensions, and it is very difficult to see what is going on. HCA answers the question: which points are close to each other in the space?

There are several reasons why we apply the tools of multivariate analysis to the matrix of efficiencies.

1. It is acknowledged that banking efficiency is a multidimensional concept. Berger and Humphrey (1997) state that: "Neither of these two approaches is perfect because neither fully captures the dual roles of financial institutions." Several authors (Berger and Humphrey 1997; Thanassoulis 1999; Oral and Yolalan 1990; Denizer et al. 2000; Camanho and Dyson 2005) suggest that bank efficiency should be simultaneously addressed under a variety of models such as production, intermediation, value added, and user cost. Our methodology does just this. Furthermore, Thanassoulis (1999) suggests that the production and the intermediation models are complementary rather than mutually exclusive. Here we use them simultaneously. It is precisely the fact that our approach treats efficiency as a multidimensional variable, that we can address the multiplicity of modeling approaches within one single framework.
2. A second advantage is that our approach permits an overall efficiency ranking over all the specifications. This extends to the ranking of efficient units. We will argue that to rank the banks all we need to do is to rank the first component loadings in PCA. We would like to quote Banker and Chang (2006) who argue that: the super-efficiency is a procedure for outlier identification, not for ranking efficient units. They find that the Andersen–Petersen procedure does not perform satisfactorily in ranking efficient units.
3. Our approach detects multivariate outliers, and explains why they show outlying behavior.
4. Our approach combines a strong data analysis methodology with the exercise of judgment, since it visualizes the results and this helps in explaining the main features of the data. It can guide the efficient application of bootstrap based inference procedures.
5. When, for a particular specification, two different DMUs achieve similar efficiencies, our approach explains the reasons why this level of efficiency has been achieved, revealing the strategic behavior of the decision makers.

The results of applying PCA to Table 3.4 are shown in Table 3.6. Four eigenvalues take values larger than one, accounting for 93.89 % of the total variance. The

Table 3.6 PCA results

Component	Eigenvalue	% of variance	Cumulative
PC1	20.833	63.132	63.132
PC2	4.687	14.203	77.335
PC3	3.097	9.384	86.719
PC4	2.367	7.174	93.893
PC5	0.608	1.843	95.737
PC6	0.555	1.683	97.420
PC7	0.223	0.677	98.097

first principal component accounts for 63.13 % of the variance. The second principal component is also of importance, as it accounts for a further 14.20 %. The variance accounted for drops to 9.38 % in the case of the third component, and to 7.17 % in the case of the fourth component. Component loadings are given in Table 3.7. In what follows the discussion will be based on these four components.

Component scores were calculated for each bank. The plot of the first and second component loadings for each bank is shown in Fig. 3.1. The plot of the third and fourth component loadings for each bank is shown in Fig. 3.2.

The banks that achieved full 100 % efficiency under a majority of specifications (CIH, PVTB, PVN, CMA, and UCBH) plot towards the right hand side of Fig. 3.1.

Super-efficient outliers identified in Table 3.5 clearly appear in an extreme of the representation in Figs. 3.1 and 3.2. What is the impact that the outliers have in the analysis? In an attempt to answer this question, we produced efficiency estimates both with and without outliers. We next applied the Simar and Wilson bootstrap procedure and we observed that frontier bias decreases when the outliers are removed. For example, under specification AB13, bootstrap bias is reduced from 0.282 to 0.230. For this same specification, bootstrap variance decreases from 0.00353 to 0.00324. However, we are reluctant to remove data just in order to improve statistical estimates, without any other justification. What matters is up to what point a bank is an influential observation. We studied the efficiencies when all the banks were included in the data and when some of the banks were removed, and we correlated both sets of numbers. Figure 3.1 was not affected by the removal of outliers, other than in the sense that the banks that had been removed ceased to appear. We calculated Pearson's correlation coefficients between efficiencies before and after outliers were removed, and these were high. For example, in the case of specification AB13, the correlation between the efficiencies calculated before and after outlier removal was 0.916. For these reasons, we kept the outliers in the analysis.

The banks that consistently under perform plot towards the left hand side of Fig. 3.1, and the ones that consistently have high efficiencies plot towards the right of the same figure. It is, therefore, clear that the first principal component can be interpreted as a "global efficiency score". Thus, an efficiency ranking of banks can be obtained by simply looking at the ordering on the first component. Usually, efficiency rankings are based on the concept of super-efficiency introduced by Andersen and Petersen (1993). However, Banker and Chang (2006) conclude that

Table 3.7 Component score matrix

	PC1	PC2	PC3	PC4
AB13	.961	−.109	−.014	−.031
AB123	.949	−.160	.057	−.101
AB12	.917	−.204	.264	−.117
AB23	.916	−.280	−.083	−.126
AB3	.904	−.227	−.298	.007
B13	.903	.084	−.162	−.350
B123	.900	.048	−.103	−.397
B23	.894	−.029	−.188	−.387
B12	.881	.005	.082	−.414
B3	.872	−.006	−.361	−.307
B2	.861	−.124	.036	−.396
BC13	.860	.346	−.167	−.084
AB1	.849	.256	.401	.000
AB2	.844	−.388	.180	−.133
AC13	.824	.061	−.128	.490
A13	.822	−.364	.148	.312
ABC13	.821	.375	−.131	.032
B1	.816	.377	.254	−.258
BC3	.810	.167	−.536	−.063
A123	.803	−.433	.213	.236
ABC3	.766	.184	−.491	.058
A3	.756	−.486	−.177	.349
A1	.752	.008	.549	.297
A23	.749	−.578	.013	.214
A12	.730	−.425	.446	.211
AC3	.693	−.110	−.477	.492
A2	.672	−.611	.250	.155
BC1	.659	.634	.334	−.054
ABC1	.641	.635	.340	.026
AC1	.589	.554	.457	.272
C1	.402	.780	.340	.174
C13	.556	.622	−.281	.389
C3	.477	.453	−.591	.393

the super-efficiency procedure is inappropriate in order to rank efficient units. Other ranking methods have also been proposed; Doyle and Green (1994), Sinuany-Stern and Friedman (1998), and Raveh (2000). The advantage of the ranking procedure applied here is that it embeds results from many different specifications, while the alternatives produce a ranking for each specification. Furthermore, this method permits a ranking of all the banks, whether efficient or inefficient, while under the alternative methodologies only efficient banks can be ranked.

Concentrating now on the second component, the North–South direction in Fig. 3.1, it can be observed that IFC plots towards the top of the figure, while

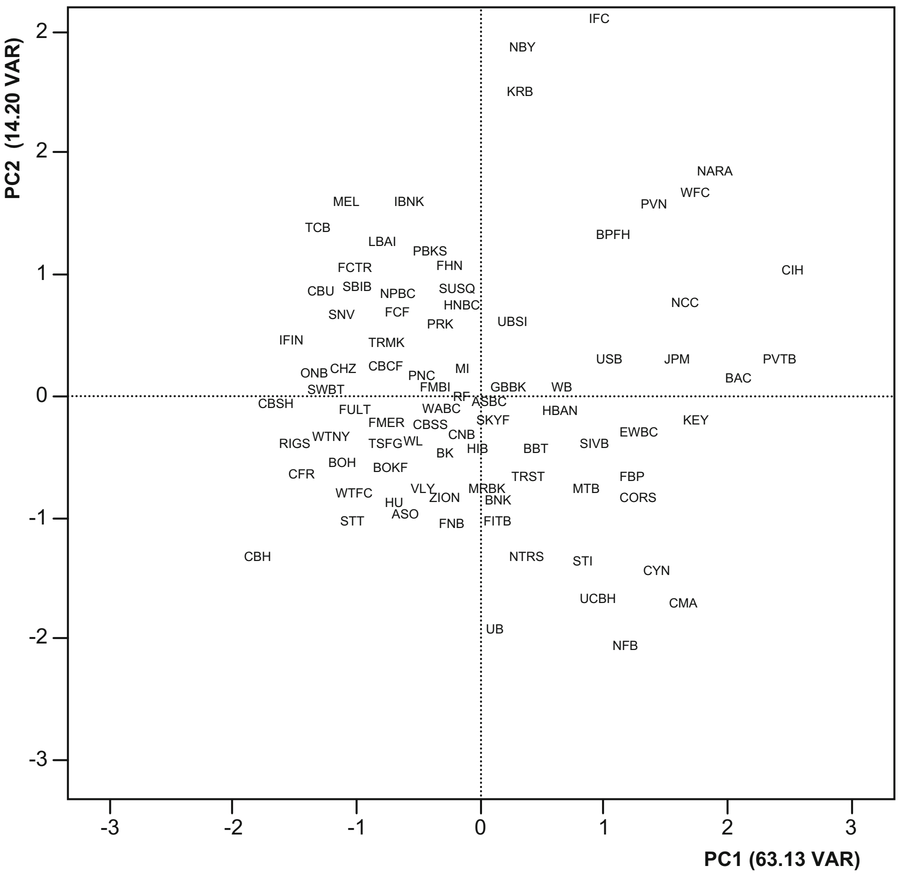


Fig. 3.1 Plot of the first and the second principal component scores. The banks that are discussed in detail in the text have been highlighted in bold

CYN plots towards the bottom. Both are 100 % efficient under many specifications. In which way they are different, and what accounts for their achieving full efficiency, will be revealed by attaching meaning to the second principal component. In the same way, interpretation of the position of banks in Fig. 3.2 requires that meaning be attached to the third and the fourth principal components.

A standard way of attaching meaning to principal components is to analyze component loadings. These are given in Table 3.7. It can be seen there that all loadings associated with the first component are positive, supporting the view that this component gives an overall measure of efficiency. The specification that achieves the highest first component loading is AB13. The efficiencies produced by AB13 have the highest Pearson's correlation with respect to the first principal component (0.961). If a combination of inputs and outputs were to be selected in order to produce a global assessment of efficiency, AB13 would be the most appropriate.

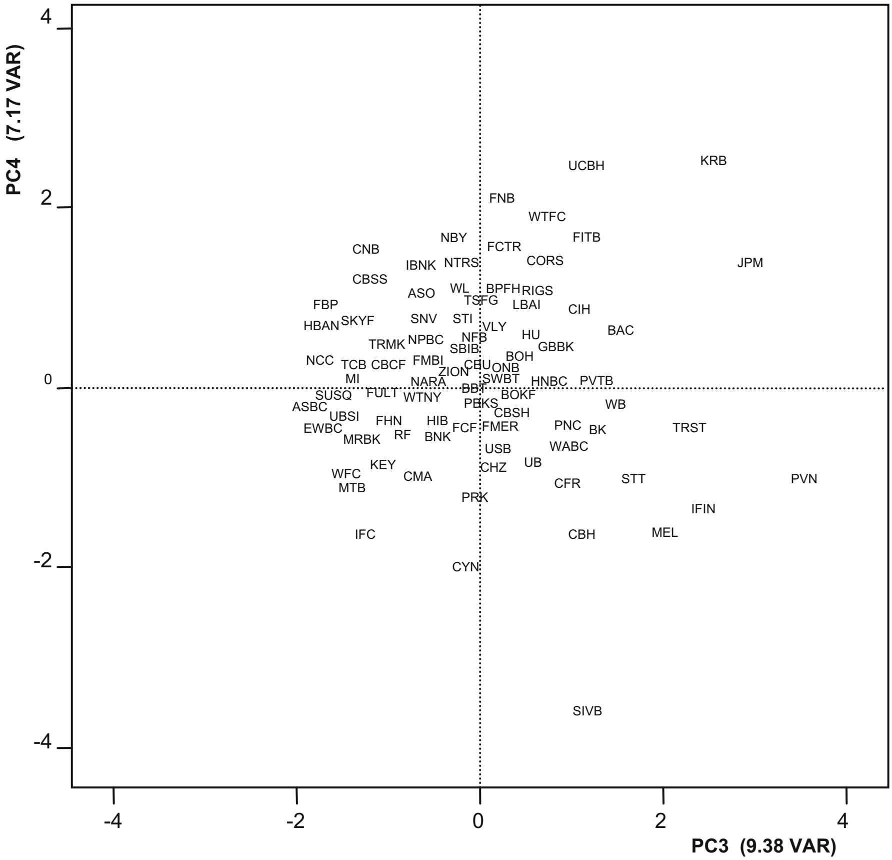


Fig. 3.2 Plot of the third and the fourth principal component scores. The banks that are discussed in detail in the text have been highlighted in bold

Specifications that include deposits as an input (C) are salient in the second component, in the sense that they achieve high positive component loadings. The third component appears to be associated with output 3—loans—versus outputs 2 and 1—deposits and income—, and the fourth one with input B—physical assets—versus inputs A and C—employees and deposits—.

These results can be visualized by means of ProFit and Cluster analysis. This will be done in the next subsection.

3.3.3 Results Visualization and Strategic Pattern Identification

Each specification generates a DEA score for each bank, and each bank is located in Figs. 3.1 and 3.2 by means of its component scores. It has just been observed that efficient banks plot towards the right hand side of Fig. 3.1 while inefficient banks plot towards the left hand side of the same figure. This appears to be the case for most specifications. Thus, there appears to be a link between the position of a bank in the figure, as given by the component scores, and efficiency. The relationship between DEA efficiency and component scores can be assessed by means of regression analysis and visualized. A linear regression is calculated for each specification. In the regression, banks are treated as observations. The dependent variable is the efficiency achieved by the bank under a given DEA specification. The regression assumes that the position of a bank in the space of the principal components is related to the efficiency achieved under the given DEA model, hence the explanatory variables are the coordinates of the bank in the space. These are, in fact, the four component loadings. Each coordinate, or component loading, is a regressor. Next, the results of the regression are represented graphically. The mathematical basis for this procedure is given in Mar Molinero and Mingers (2006).

In total, 33 regressions were performed. This procedure is known as Property Fitting (ProFit) analysis; see Schiffman et al. (1981). For a given specification, ProFit produces a directional vector on Figs. 3.1 and 3.2 in such a way that DEA efficiencies grow in the direction of the vector. Directional vectors were calculated for each one of the 33 specifications. Being regression-based, the quality of the representation can be assessed by means of the coefficient of determination, R^2, and the F statistic. These are shown in Table 3.8. It is to be noticed that values of R^2 are very high, all of them above 0.8, indicating that there is a strong linear relationship between DEA scores and the position of the bank in Figs. 3.1 and 3.2. The directional vectors are located in Figs. 3.1 and 3.2 by means of their standardized directional cosines, γ. The standardization transforms the vectors into unit length, using the formula:

$$\gamma_i = \frac{\beta_i}{\sqrt{\sum_{i=1}^{n} {\beta_i}^2}}$$

The value of the standardized directional cosines,—γ_1, γ_2, γ_3, and γ_4—and of their level of significance, are also shown in Table 3.8. ProFit vectors have been superimposed on component plots in Figs. 3.3 and 3.4.

When two ProFit lines are at an acute angle, the correlation between the efficiencies obtained under the associated specifications are positive, the smaller the angle the higher the correlation. When two ProFit lines are orthogonal, the two approaches to efficiency are independent. Thus, ProFit gives an immediate visual

Table 3.8 ProFit analysis: Linear regression results

Specification	*Directional cosines*				F	Adj R^2
	γ_1	γ_2	γ_3	γ_4		
A1	13.05	0.15	9.52	5.15	432.66	0.954
	(32.016)**	(0.359)	(23.359)**	(12.643)**		
A12	13.61	−7.92	8.31	3.94	441.61	0.955
	(31.366)**	(−18.263)**	(19.148)**	(9.077)**		
A123	16.05	−8.65	4.26	4.72	280.16	0.930
	(27.821)**	(−15.003)**	(7.382)**	(8.188)**		
Aa13	16.69	−7.38	3.00	6.34	255.02	0.924
	(27.266)**	(−12.056)**	(4.893)**	(10.361)**		
A23	16.47	−12.71	0.28	4.70	322.98	0.939
	(27.759)**	(−21.411)**	(0.474)	(7.920)**		
A2	14.09	−12.82	5.25	3.24	207.06	0.908
	(20.249)**	(−18.428)**	(7.544)**	(4.662)**		
A3	17.56	−11.29	−4.11	8.11	479.58	0.958
	(33.784)**	(−21.710)**	(−7.907)**	(15.591)**		
B1	19.41	8.96	6.04	−6.14	309.09	0.936
	(29.614)**	(13.667)**	(9.211)**	(−9.368)**		
B12	22.01	0.13	2.05	−10.36	413.14	0.952
	(36.650)**	(0.217)	(3.409)*	(−17.253)**		
B123	22.89	1.22	−2.62	−10.09	959.08	0.979
	(56.302)**	(2.994)*	(−6.451)**	(−24.817)**		
B13	22.93	2.14	−4.12	−8.90	676.81	0.970
	(47.663)**	(4.458)**	(−8.567)**	(−18.499)**		
B23	22.67	−0.74	−4.77	−9.82	1372.77	0.985
	(66.740)**	(−2.167)	(−14.050)**	(−28.893)**		
B2	21.09	−3.04	0.89	−9.71	216.55	0.911
	(26.489)**	(−3.823)*	(1.114)	(−12.194)**		
B3	22.03	−0.16	−9.12	−7.77	1301.55	0.984
	(63.386)**	(−0.472)	(−26.238)**	(−22.356)**		
AB1	18.78	5.68	8.87	−0.01	354.90	0.944
	(32.861)**	(9.932)**	(15.527)**	(−0.015)		
AB12	19.23	−4.28	5.54	−2.45	554.80	0.963
	(43.951)**	(−9.780)**	(12.664)**	(−5.610)**		
AB123	0.98	−0.16	0.06	−0.10	314.36	0.937
	(34.715)**	(−5.848)**	(2.093)	(−3.708)*		
AB13	0.99	−0.11	−0.01	−0.03	295.09	0.933
	(34.118)**	(−3.856)*	(−0.485)	(−1.115)		
AB23	0.94	−0.29	−0.09	−0.13	314.15	0.937
	(33.492)**	(−10.219)**	(−3.049)*	(−4.601)**		
AB2	0.88	−0.41	0.19	−0.14	212.03	0.909
	(25.723)**	(−11.830)**	(5.477)**	(−4.061)**		

(continued)

Table 3.8 (continued)

Specification	Directional cosines				F	Adj R^2
	γ_1	γ_2	γ_3	γ_4		
AB3	0.92	−0.23	−0.31	0.01	442.24	0.955
	(38.851)**	(−9.740)**	(−12.828)**	(0.308)		
C1	0.42	0.82	0.36	0.18	217.51	0.912
	(12.381)**	(24.042)**	(10.483)**	(5.371)**		
C13	0.58	0.65	−0.29	0.40	250.41	0.922
	(18.293)**	(20.443)**	(−9.237)**	(12.796)**		
C3	0.49	0.47	−0.61	0.41	294.21	0.933
	(16.926)**	(16.054)**	(−20.950)**	(13.918)**		
AC1	0.61	0.57	0.47	0.28	298.46	0.934
	(21.030)**	(19.770)**	(16.320)**	(9.717)**		
AC13	0.85	0.06	−0.13	0.51	307.40	0.936
	(29.815)**	(2.204)	(−4.631)**	(17.730)**		
AC3	0.71	−0.11	−0.49	0.50	513.40	0.961
	(32.019)**	(−5.084)**	(−22.043)**	(22.729)**		
BC1	0.68	0.65	0.34	−0.06	385.39	0.948
	(26.533)**	(25.533)**	(13.450)**	(−2.170)		
BC13	0.91	0.37	−0.18	−0.09	170.52	0.890
	(23.751)**	(9.557)**	(−4.614)**	(−2.311)		
BC3	0.82	0.17	−0.54	−0.06	816.18	0.975
	(46.866)**	(9.667)**	(−31.009)**	(−3.648)*		
ABC1	0.66	0.66	0.35	0.03	270.70	0.928
	(21.869)**	(21.656)**	(11.608)**	(0.890)		
ABC13	0.90	0.41	−0.14	0.03	100.17	0.825
	(18.010)**	(8.221)**	(−2.874)*	(0.692)		
ABC3	0.82	0.20	−0.53	0.06	128.76	0.859
	(18.690)**	(4.487)**	(−11.982)**	(1.423)		

*Significant at the 0.05 level, **Significant at the 0.01 level

understanding of the relationship between efficiency and specification. The length of the projection of the ProFit vector reflects its relevance in the interpretation of the particular figure. The longer the vector, the more agreement there is between the ordering of the banks in the representation and the efficiency values obtained from the specification.

ProFit vectors form a fan in Fig. 3.3. All vectors point in the direction in which efficiency grows. Each one of the 33 vectors indicates the way in which efficiency grows under a particular specification. Most vectors point in the direction of the first principal component. This confirms the observation that the first principal component gives an “overall measure of the efficiency” of a bank, and that an ordering along the first principal component produces an efficiency ranking of institutions. Since the first principal component accounts for 63.13 % of the variance, we

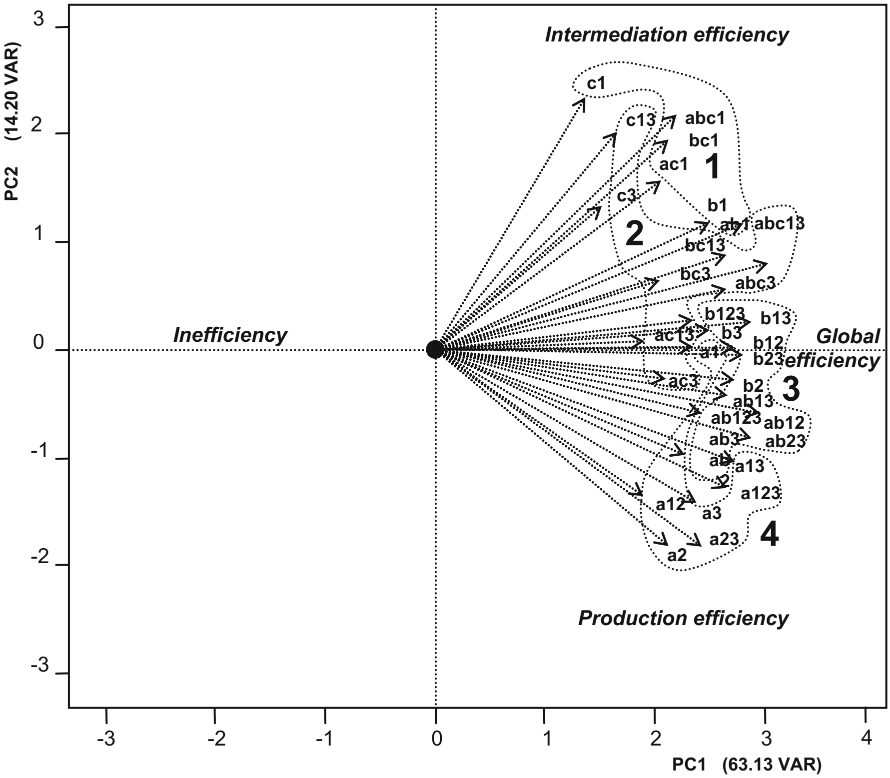


Fig. 3.3 ProFit lines and cluster results in the first two component plot

conclude that the ranking of banks along this component, which is a ranking of overall efficiency, is the most salient characteristic of the data.

A set of vectors, containing deposits as an input, is clearly associated with the second principal component, as they all point towards the top of Fig. 3.3. Deposits are treated as inputs under the production modeling philosophy. We conclude that the second principal component distinguishes between the two basic approaches to banking efficiency: intermediation and production. The second principal component, it has to remembered, accounts for 14.20 % of the variance. Other authors have addressed if there is a relationship between production and intermediation, Berger and Humphrey (1997), Thanassoulis (1999), Oral and Yolalan (1990), Denizer et al. (2000), Camanho and Dyson (2005). In general, these authors estimate a specification derived from each model and calculate the correlation between the efficiencies obtained. Their results tend not to be conclusive, as they find no significant relationship. Our approach clearly explains the positive but low correlations that the authors we have mentioned did find. Whether a bank opts for intermediation or for production, appears very clearly in the second component.

Using similar considerations, we can associate the value of the third principal component—that accounts for 9.38 % of the variance—with the decision to use

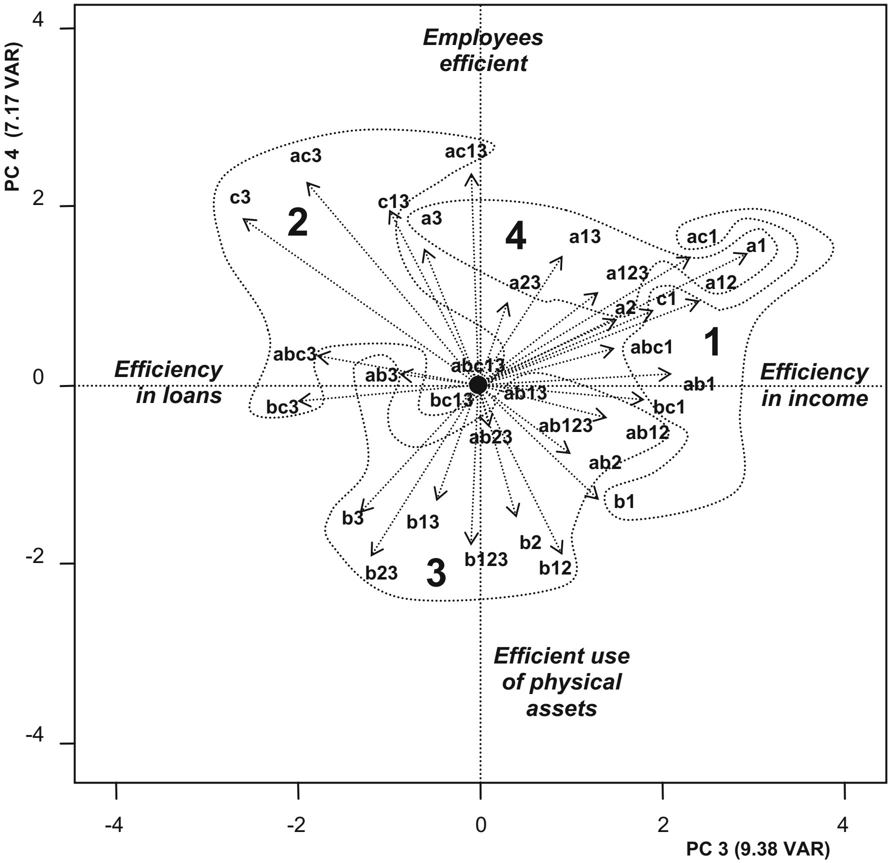


Fig. 3.4 ProFit lines and cluster results on the third and the fourth components plot

specifications containing as a sole output the value of loans (3), since a fan that includes only 3 as an output can clearly be discerned on the left hand side of Fig. 3.4. On the right hand side of this figure we find vectors associated with deposits as an output (2) and with income (1). This suggests that the third principal component distinguishes between two banking strategies: collecting deposits, or making loans.

Finally, in Fig. 3.4 it can be seen that the fourth principal component discriminates between specifications that include physical assets as an input (input B) and those contain the number of employees (input A). It is clear that vectors that contain input B in their definition point towards the bottom of Fig. 3.4, while those that contain input A (employees) in their definition point towards the top of the figure. We, therefore, interpret the fourth component as asset utilization efficiency, distinguishing between an orientation towards efficient use of human resources and efficient use of physical assets. This component explains 7.17 % of the variance.

Summarizing, in order to describe how a bank achieves a given level of efficiency, we must take into account four independent aspects: first, and most importantly, an overall measure of efficiency; second, we make a distinction between those banks whose efficiency is best seen in the light of the production model, and the banks whose efficiency is best seen under the intermediation model; third, we must take into account whether the bank follows a strategy directed to making loans or collecting deposits; finally, some banks specialize in the efficient use of human resources while others specialize in the efficient use of physical assets. There may be other features that help to discriminate between the various banks in terms of efficiency, but they have not been explored, as these four characteristics account for 93.89 % of the variation in efficiency.

The literature shows that there is no single definition of efficiency, and we show empirically that, given the inputs and outputs selected, we should examine several specifications. This substantially enriches the quality of the analysis that can be carried out. For example, the IFC bank is 100 % efficient under the ABC13 specification, but only achieves 12 % efficiency under specification A2, one of the lowest estimated efficiencies, indicating that, in this bank, the amount of deposits per employee is very low. In Fig. 3.1 we see that the vector associated with specification ABC13 points towards this bank, while the vector associated with specification A2 points away from this bank.

The graphical representation in Fig. 3.1 will not only disclose the approach to efficiency followed by a bank, but also the success with which this approach is being followed. The Property Fitting line that shows a bank under the best light will disclose the policy that the bank has adopted.

There are various paths that lead to the same place. For example, a bank that puts its weight into the improvement of employee efficiency will probably go for electronic banking or automatic tellers, and will invest in infrastructures (physical assets) while, at the same time, reducing the number of its staff and keeping the level of deposits. Such a bank will improve its efficiency under specification A2 (its stated objective), but will worsen its performance under specification B2.

All the above discussion has been based on the interpretation of two dimensional projections of a four dimensional data set. Each ProFit vector is plotted in a four dimensional space, and it would be appropriate to assess if the groups that are observed on the projections are true reflections of the groups that exist in the space. For this reason ProFit analysis has been supplemented with Hierarchical Cluster Analysis (HCA).

Efficiencies in Table 3.4 have been taken as inputs for HCA and clustered using Ward's method with Euclidean distances. This method maximizes within group homogeneity and between group heterogeneity. Since we are interested in finding out up to what point two different specifications are equivalent, we clustered variables (specifications) and not cases (banks). The dendrogram can be seen in Fig. 3.5.

Specifications group neatly into four clusters in Fig. 3.5. These clusters have been superimposed in Figs. 3.3 and 3.4, and have been labeled 1, 2, 3, and

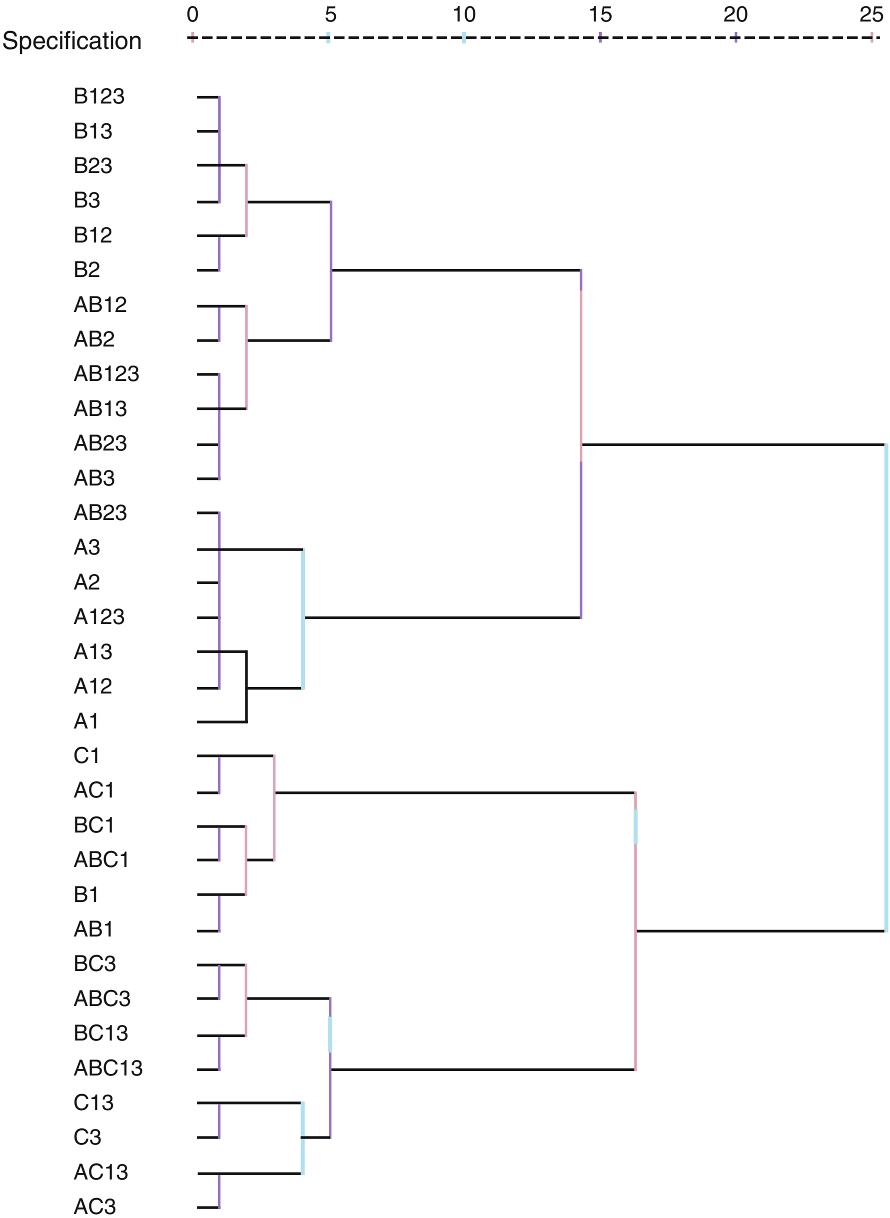


Fig. 3.5 Ward's method. Dendrogram

4. Clusters 1 and 2 group together at a higher level of clustering. Clusters 3 and 4 also group together at a higher level of clustering.

Clusters 1 and 2 are located at the North of Fig. 3.3, grouping specifications whose ProFit vectors point up and to the right of the figure. With the exception of specifications B1 and AB1, the 14 specifications that form clusters 1 y 2 contain deposits as input (C). Deposits as an input are a standard feature of intermediation models. Cluster 3 y 4 are located on the lower part of Fig. 3.3, and none of the specifications includes deposits (C) as an input. This is consistent with production models of efficiency.

The differences that exist between cluster 1 and cluster 2, on the one hand, and cluster 3 and cluster 4, on the other, are made apparent in Fig. 3.4. All the specifications contained in cluster 1 are located at the right hand side of the figure (positive end of the third principal component) and include income (1) as an output. Specifications associated with cluster 2 are located at the left of Fig. 3.4 and include loans (3) as an output. We conclude that cluster 1 contains specifications that are consistent with an intermediation model oriented to income, and that cluster 2 contains specifications that are consistent with an intermediation model oriented towards making loans.

The differences between Clusters 3 and 4 are also apparent in Fig. 3.4. Both clusters are associated with production type models. Cluster 3 is located at the bottom of Fig. 3.4, on the negative side of the fourth principal component. All the specifications contained in it contain input B, physical assets. Cluster 4 is located at the top of Fig. 3.4. All the specifications contain number of employees (A) as an input. We conclude that Cluster 3 contains specifications consistent with production models oriented towards efficiency use of physical assets, and that cluster 4 groups specifications consistent with a production model oriented to efficient use of human resources.

If equivalent specifications exist, they will group into clusters, and if specifications within a cluster share something in common, the analysis will reveal it, with the added bonus that model simplification will naturally follow. For example, A1 and AC3 appear very close to each other in Fig. 3.3, and one would conclude that the efficiencies that they produce are very similar, but they belong to different clusters, indicating that a bank obtains quite different efficiencies under each one of these two specifications. The reverse is also true: specifications AB123 and AB13 belong to the same cluster and appear next to each other in the representation, suggesting that AB13 is a valid simplification of ABC13, and this is something that can be tested using the bootstrap. In this case, high levels of correlation between the efficiencies produced by the different specifications will result in membership of the same cluster, and this would guide inferential procedures for specification search.

It can be argued that specifications contained in a given cluster are largely equivalent in the sense that they produce similar efficiency scores for the various banks. This can guide the search for simplifications in the specification. Each cluster can be represented by a single specification, reducing the total number of possible specifications from 33 to 4. The selected specification could be the most parsimonious one or the most central one within the cluster. Ganley and Cubbin (1992) suggest that one should perform sensitivity analysis in order "to test the robustness of DEA results to changes in the methods and data used". For Parkin and

Hollingsworth (1997), the way one test validity is to calculate the correlations that exist between the efficiencies obtained under the various specifications.

The question of up to what point two different specifications are equivalent is addressed here using the bootstrap procedure. Simar and Wilson (1998, 2000a, b), Richmond (2005) explain how to use the bootstrap for inference purposes in DEA. The bootstrap was used to test the equivalence of the two specifications that appear to be equivalent in the sense that their associated ProFit lines belong to the same cluster and are at a small acute angle in the representation. In order to test the power of this test, an issue raised by Cherchye and Post (2003), we also checked that two specifications that appear to be very different in the visualization can indeed be shown to be different by means of the bootstrap. We have used the software FEAR, Wilson (2007).

Our way of thinking was as follows. Given a particular specification, the bootstrap produced a series of possible replications of the efficiencies for each bank. We treated each replication as a set of efficiencies that could have been observed in practice. We produced 250 replications, mainly because the results indicated that this was a sufficient number for the purposes of this analysis. For a given specification, each replication produced a vector of bootstrap efficiencies, one for each bank. We calculated the average vector and treated it as a reference point for the other replications. We next calculated the squared distance between each replication and the average vector. In this way we obtained 250 distances that could have been observed by chance. For a one-sided test we are interested in the distance that is only surpassed in 5 % of the cases. This distance was 0.539. The next step was to calculate the distance between two original specifications, our preferred specification (AB13) and an alternative one (AB123). We found the distance between specification AB13 and AB123 to be 0.130. Under the hypothesis that AB123 is only random deviation away from AB13, we have observed nothing unusual, therefore we cannot reject the view that AB13 is a valid simplification of AB123, and adopted the more parsimonious specification. We repeated the test with two specifications that our methodology identifies as very different from AB13, C1 and A2. We found the distance between AB13 and C1 to be 8.736, well beyond our one-sided 95 % limit, and well beyond the one-sided 98 % limit (0.640). We concluded that specification C1 produces efficiency results that are significantly different from the results produced by specification AB13. When specification A2 was compared with AB13, the distance obtained was 3.596, also significantly different at the 98 % confidence level. This leads to the conclusion that if we want to study the efficiency of a bank, we should not proceed by choosing only one model and only one specification, as this may miss important features of its operations.

Given the present study, we think that we can only limit our conclusions to the data we have analyzed. In order to give policy advice we would like to perform more studies, using a larger sample and, perhaps, a time series, so that our conclusions could be deemed to be generally valid. Nevertheless, one could tentatively say that a good specification for global efficiency is AB13. This should be supplemented with the partial view of efficiency provided by specifications A2, and C1.

3.3.4 Dissecting the Efficiency Score

It has been argued that there is no single definition of efficiency in the context of banks. Different views of the way in which banks operate, as reflected in the different modeling philosophies will produce different efficiency scores. The combination of PCA, ProFit, and HCA sheds light into the reasons why a particular bank achieves a certain efficiency level. This subject will be further examined in what follows.

Take CYN and IFC, two previously discussed institutions. They both achieve 100 % efficiency under four specifications: B13, B123, AB123 and AB13. They both appear on the extreme right hand side of the first principal component in Fig. 3.1. They would both come at the top of an efficiency ranking based on the first principal component. We could just conclude that they are excellent institutions and leave it at that. But it is also to be noticed that under specifications C13, C3, BC13, BC3, ABC13, and ABC3 IFC is 100 % efficient but not CYN. All these contain deposits as an input, and are specifications that would be developed under the intermediation modeling philosophy. The specifications that make CYN 100 % efficient but not IFC are B12, B23, B2, B3, AB12, AB23, AB2, and AB3. All these specifications contain physical assets (B) in their definition, or employees (A) which leads to the conclusion that CYN owes its position in the league table to the efficient use of its physical and human assets, and that CYN is a good institution from the intermediation point of view.

This discussion can be extended to the differences and similarities of IFC and CYN under the third and fourth principal components. We see that their scores under principal components 3 and 4 are very similar, and this suggests that their only difference is in principal component 2, and this has just been discussed.

Systematic analysis of Figs. 3.1 and 3.2, together with the interpretations provided with the help of Figs. 3.3 and 3.4 makes it possible to assess the global efficiency of an institution and the strategies under which such global efficiency was achieved. Strengths and weaknesses become apparent. Take, for example, a previously mentioned case: NBY. In Fig. 3.1 NBY plots towards the center of the first component, indicating that its global efficiency is mediocre. Indeed, it only achieves 46 % efficiency under specification AB123. It is located at the top of the second principal component, which is consistent with being 100 % efficient under specifications C1, C3, and C13, all of them with deposits as an input, and implying that NBY would be only identified as efficient under an intermediation approach. NBY is located very near IFC. It has just been argued that IFC also appeared in a good light under an intermediation approach. The differences between NBY and IFC appear when we examine Figs. 3.2 and 3.4. In Fig. 3.2, IFC is located towards the most negative side of the fourth principal component, while NBY is located towards the top of the figure, at the positive end of the fourth principal component. In line with the interpretation of the fourth principal component, IFC appears as efficient under specifications that include physical assets (B) as an input and not very efficient when the specification contains the number of employees (A). This is confirmed by the observation that IFC achieves an efficiency of only 12 % under specification A2. NBY, on the other hand, appears at its best under specifications that contain input A, the number of employees.

Finally, PVN, another previously discussed bank, appears on the right hand side of Fig. 3.1, implying that it is efficient from the global point of view. Its location in this figure is consistent with intermediation efficiency. In Fig. 3.2, PVN is located towards the extreme right hand side. We notice that in Fig. 3.4, vectors associated with specifications that contain loans (output 3) point on the whole towards the left hand side. This implies that PVN under performs in specifications that contain loans as an output, something that is coherent with the results shown in Table 3.4. If we were to advise this bank we would recommend more efficiency in the granting of loans.

3.4 Conclusions

There has been much interest and debate on how to model DEA efficiency in financial institutions. This has extended over the type of model (intermediation or production) that is appropriate, as well as to the selection of inputs and outputs once a modeling philosophy has been selected. We have suggested a specification search strategy that highlights the extent to which two different DEA specifications produce similar results and the reasons why this happens.

The methodology proposed relies on estimating a variety of input/output mixtures and analyzing the results by means of multivariate statistical methods. Particular emphasis is given to data visualization, which is achieved by combining Principal Components Analysis, Property Fitting, and Hierarchical Cluster Analysis. Our work mimics the work of the econometrician in the estimation of a regression model. Economic theory leads to the selection of the variables that enter the regression equation and the way in which they should impact on the variable to be explained, but the work of the econometrician is basically of a statistical (technical) nature. After the statistical work is over, there comes the interpretation of the results. We have proceeded in the same way, by showing in an explicit way which banks are efficient under the intermediation and under the production model, and why they are efficient under a model and not under another one.

This approach has been applied to the particular case of American banks. A Principal Component Analysis has made it possible to identify a ranking of banks in terms of global efficiency, which is nothing else than a ranking along the first principal component. Furthermore, we have been able to identify four different views of what constitutes efficiency in a bank. The treatment of deposits as either inputs or outputs—a feature that distinguishes intermediation models from production models—has proven to be a key feature in the modeling of financial institutions, and this information has been captured by the second principal component. Another relevant aspect in the assessment of the differences in banking efficiency is the emphasis on inputs (physical assets versus employees), which is captured by the third principal component. The fourth principal component highlights the institution's orientation towards outputs and separates those institutions that are efficient at granting loans from those that are efficient at taking deposits.

The standard procedure in the assessment of banking efficiency, which starts with an a priori view of what inputs and outputs should be included in the calculation of efficiency should be revised, as different models and specifications can produce different efficiency results for a given institution. A more realistic view would be to accept that efficiency is a multidimensional concept, and that several models ought to be estimated and combined before managerial action is taken to improve the way in which a financial institution works.

Framing DEA results in a multivariate statistical context has allowed us to go behind efficiency as a mere score. It has been possible to offer a global view of the efficiency of an institution which encompasses many specifications; it has made it possible to assess why a particular institution has achieved a given level of efficiency under a given choice of inputs and outputs; and has made it possible to identify the various paths to efficiency followed by different institutions which would, under most studies, have been classified as equivalent but that differ in important aspects of their operations.

By showing which combinations of inputs and outputs generate similar efficiency results, the multivariate approach proposed in this paper can guide the application of inferential tools such as the bootstrap, and these can be used for hypothesis testing and specification selection.

Further advantages of the method proposed here is that it creates a natural ranking of institutions in terms of efficiency, and that it highlights the strengths and weaknesses of each institution.

References

Adler, N., & Golany, B. (2001). Evaluation of deregulated airline networks using data envelopment analysis combined with principal component analysis with an application to Western Europe. *European Journal of Operational Research, 132*(2), 18–31.

Aly, H., Grabowsky, R., Pasurka, C., & Rangan, N. (1990). Technical, scale, and allocative efficiencies in U.S. banking: An empirical investigation. *Review of Economics and Statistics, 72*, 211–219.

Andersen, P., & Petersen, N. C. (1993). A procedure for ranking efficient units in data envelopment analysis. *Management Science, 39*, 1261–1264.

Athanassopoulos, A. D. (1997). Service quality and operating efficiency synergies for management control in the provision of financial services: Evidence from Greek bank branches. *European Journal of Operational Research, 98*(2), 300–313.

Banker, R. D., & Chang, H. (2006). The super-efficiency procedure for outlier identification, not for ranking efficient units. *European Journal of Operational Research, 175*(2), 1311–1320.

Banker, R. D., Das, S., & Datar, S. M. (1989). Analysis of cost variances for management control in hospitals. *Research in Governmental and Nonprofit Accounting, 5*, 269–291.

Barr, R. S., Kory, A. K., Siems, T., & Zimmel, S. (2002). Evaluating the productive efficiency and performance of U.S. commercial banks. *Managerial Finance, 28*(8), 3–25.

Barr, R. F., Seiford, L., & Siems, T. (1993). Envelopment analysis approach to measuring managerial quality of banks. *Annals of Operations Research, 45*, 1–19.

Berg, S. A., Forsund, F. R., Hjalmarsson, L., & Suominen, M. (1993). Banking efficiency in the Nordic countries. *Journal of Banking & Finance, 17*(2–3), 371–388.

Berg, S. A., Forsund, F. R., & Jansen, E. S. (1991). Bank output measurement and the construction of best practice frontiers. *Journal of Productivity Analysis, 2*, 127–142.

Berg, S. A., Forsund, F. R., & Jansen, E. S. (1992). Technical efficiency of Norwegian banks: The non-parametric approach to efficiency measurement. *Journal of Productivity Analysis, 2*(2), 127–142.

Berger, A. N., & Humphrey, D. B. (1991). The dominance of inefficiencies over scale and product mix economies in banking. *Journal of Monetary Economics, 28*, 117–148.

Berger, A. N., & Humphrey, D. B. (1997). Efficiency of financial institutions: International survey and directions for future research. *European Journal of Operational Research, 98*(2), 175–212.

Berger, A. N., & Mester, L. J. (1997). Inside the black box: What explains differences in the efficiencies of financial institutions? *Journal of Banking and Finance, 21*, 895–947.

Brockett, P. L., Charnes, A., Cooper, W. W., Huang, Z. M., & Sun, D. B. (1997). Data transformations in DEA cone ratio envelopment approaches for monitoring bank performances. *European Journal of Operational Research, 98*(2), 250–268.

Camanho, A. S., & Dyson, R. G. (2005). Cost efficiency, production and value-added models in the analysis of bank branch performance. *Journal of the Operational Research Society, 56*, 483–494.

Casu, B., & Girardone, C. (2004). *An analysis of the relevance of OBS items in explaining productivity change in European banking*. EFMA 2004 Basel Meetings Paper.

Charnes, A., Cooper, W. W., Huang, Z. M., & Sun, D. B. (1990). Polyhedral cone-ratio DEA models with an illustrative application to large commercial banks. *Journal of Econometrics, 46*, 73–91.

Cherchye, L., & Post, T. (2003). Methodological advances in DEA: A survey and an application for the Dutch electricity sector. *Statistica Neerlandica, 57*(4), 410–438.

Dekker, D., & Post, T. (2001). A quasi-concave DEA model with an application for bank branch performance evaluation. *European Journal of Operational Research, 132*, 296–311.

Denizer, C. A., Dinc, M., & Tarimcilar, M. (2000). *Measuring banking efficiency in the pre- and post-liberalization environment: Evidence from the Turkish banking system*. Working Paper, World Bank. Retrieved from http://econ.worldbank.org/view.php?type=5&id=1286

Doyle, J., & Green, R. (1994). Efficiency and cross-efficiency in DEA. Derivations, meanings and uses. *Journal of the Operational Research Society, 45*, 567–578.

Elyasiani, E., & Mehdian, S. M. (1992). Productive efficiency performance of minority and nonminority-owned banks. A nonparametric approach. *Journal of Banking and Finance, 16*, 933–948.

Elyasiani, E., & Mehdian, S. M. (1995). The comparative efficiency performance of small and large U.S. commercial banks in the pre- and post-deregulation eras. *Applied Economics, 27* (November), 1069–1079.

Emrouznejad, A., & Thanassoulis, E. (1996). *An extensive bibliography of data envelopment analysis*. Journal Papers. Warwick Business School.

English, M., Grosskopf, S., Hayes, K., & Yaisawarng, S. (1993). Output allocative and technical efficiency of banks. *Journal of Banking and Finance, 17*(April), 349–366.

Farrell, M. J. (1957). The measurement of productive efficiency. *Journal of the Royal Statistical Society. Series A (General), 120*(3), 253–290.

Ferrier, G., & Lovell, C. A. K. (1990). Measuring cost efficiency in banking. Econometric and linear programming evidence. *Journal of Econometrics, 46*, 229–245.

Ganley, J. A., & Cubbin, J. S. (1992). *Public sector efficiency measurement. Applications of data envelopment analysis*. Amsterdam: North Holland.

Hartman, T. E., Storbeck, J. E., & Byrnes, P. (2001). Allocative efficiency in branch banking. *European Journal of Operational Research, 134*, 232–242.

Hendry, D. F., & Mizon, G. F. (1978). Serial correlation as a convenient simplification, not a nuisance: A comment on a study of the demand for money by the bank of England. *The Economic Journal, 88*, 549–563.

Kumbhakar, S. C., Lozano Vivas, A., Knox Lovell, C., & Hasan, I. (2001). The effects of deregulation on the performance of financial institutions. The case of Spanish savings banks. *Journal of Money, Credit and Banking, 33*(1), 101–120.

Kuosmanen, T., & Post, T. (2001). Measuring economic efficiency with incomplete price information. With an application to European commercial banks. *European Journal of Operational Research, 134*, 43–58.

Lovell, C. A. K., & Pastor, J. T. (1997). Target setting. An application to a bank branch network. *European Journal of Operational Research, 98*, 290–299.

Lozano, A. (1998). Profit efficiency for Spanish savings banks. *European Journal of Operational Research, 98*(2), 381–394.

Luo, X. (2003). Evaluating the profitability and marketability efficiency of large banks. An application of data envelopment analysis. *Journal of Business Research, 56*, 627–635.

Mar Molinero, C., & Mingers, J. (2006). An evaluation of the limitations of, and alternatives to, the Co-Plot methodology. *Journal of the Operational Research Society*, advance online publication 31 May 2006.

Maudos, J., Pastor, J. M., & Perez, F. (2002). Competition and efficiency in the Spanish banking sector: The importance of specialization. *Applied Financial Economics, 12*, 505–516.

Mester, L. J. (1989). Testing for expense preference behaviour. Mutual versus stock savings and loans. *The Rand Journal of Economics, 4*, 483–498.

Mester, L. J. (1997). Measuring efficiency at U.S. banks. Accounting for heterogeneity is important. *European Journal of Operational Research, 98*(2), 230–242.

Miller, S. M., & Noulas, A. G. (1996). The technical efficiency of large bank production. *Journal of Banking and Finance, 20*(April), 495–509.

Ondrich, J., & Ruggiero, J. (2001). Efficiency measurement in the stochastic frontier model. *European Journal of Operational Research, 129*, 434–442.

Oral, M., & Yolalan, R. (1990). An empirical study on measuring operating efficiency and profitability of bank branches. *European Journal of Operational Research, 46*, 282–294.

Parkin, D., & Hollingsworth, B. (1997). Measuring production efficiency of acute hospitals in Scotland, 1991–94: Validity issues in data envelopment analysis. *Applied Economics, 29*(11), 1425–1433.

Pastor, J. M., Perez, F., & Quesada, J. (1997). Efficiency analysis in banking firms. An international comparison. *European Journal of Operational Research, 98*, 395–407.

Pastor, J., Ruiz Gomez, J. L., & Sirvent, I. (2002). A statistical test for nested radial DEA models. *Operations Research, 50*(4), 728–735.

Pedraja Chaparro, F., Salinas Jimenez, J., & Smith, P. (1999). On the quality of the data envelopment analysis model. *Journal of the Operational Research Society, 50*, 636–645.

Premachandra, I. M. (2001). A note on DEA vs principal component analysis. An improvement to Joe Zhu's approach. *European Journal of Operational Research, 132*, 553–560.

Raveh, A. (2000). The Greek banking system. Reanalysis of performance. *European Journal of Operational Research, 120*, 525–534.

Richmond, J. (2005). Slack and net technical efficiency measurement: A bootstrap approach. *International Journal of Information Technology and Decision Making, 4*(3), 395–410.

Saha, A., & Ravisankar, T. S. (2000). Rating of Indian commercial banks. A DEA approach. *European Journal of Operational Research, 124*, 187–203.

Schaffnit, C., Rosen, D., & Paradi, J. C. (1997). Best practice analysis of bank branches. An application of DEA in a large Canadian bank. *European Journal of Operational Research, 98* (2), 270–290.
Schiffman, J. F., Reynolds, M. L., & Young, F. W. (1981). *Introduction to multidimensional scaling. Theory, methods and applications*. London: Academic.
Sealey, C. W., & Lindley, J. T. (1977). Inputs, outputs, and theory of production cost at depository financial institutions. *Journal of Finance, 32*, 1251–1266.
Seiford, L. M. (1996). Data envelopment analysis: The evolution of State of Art (1978–1995). *Journal of Productivity Analysis, 7*, 99–137.
Seiford, L. M., & Zhu, J. (1999). Profitability and marketability of the top 55 U.S. commercial banks. *Management Science, 45*(9), 1270–1288.
Serrano Cinca, C., Fuertes Callén, Y., & Mar Molinero, C. (2005). Measuring DEA efficiency in internet companies. *Decision Support Systems, 38*(4), 557–573.
Serrano Cinca, C., & Mar Molinero, C. (2004). Selecting DEA specifications and ranking units via PCA. *Journal of the Operational Research Society, 55*, 521–528.
Sherman, D., & Gold, F. (1985). Branch operating efficiency. Evaluation with data envelopment analysis. *Journal of Banking and Finance, 9*, 297–315.
Simar, L., & Wilson, P. W. (1998). Sensitivity analysis of efficiency scores: How to bootstrap in nonparametric frontier models. *Management Science, 44*, 49–61.
Simar, L., & Wilson, P. W. (2000a). Statistical inference in nonparametric frontier models: The state of the art. *Journal of Productivity Analysis, 13*, 49–78.
Simar, L., & Wilson, P. W. (2000b). A general methodology for bootstrapping in non-parametric frontier models. *Journal of Applied Statistics, 27*, 779–802.
Sinuany-Stern, Z., & Friedman, L. (1998). DEA and the discriminant analysis of ratios for ranking units. *European Journal of Operational Research, 111*, 470–478.
Sirvent, I., Ruiz, J. L., Borras, F., & Pastor, J. T. (2005). A Monte Carlo evaluation of several tests for the selection of variables in DEA models. *International Journal of Information Technology and Decision Making, 4*(3), 325–343.
Smith, P. (1997). Model misspecification in data envelopment analysis. *Annals of Operations Research, 73*, 233–252.
Soteriou, A., & Zenios, S. A. (1999). Operations, quality and profitability in the provision of banking services. *Management Science, 45*(9), 1221–1238.
Thanassoulis, E. (1999). Data envelopment analysis and its use in banking. *Interfaces, 29*(3), 1–13.
Thompson, R. G., Brinkmann, E. J., Dharmapala, P. S., Gonzalez-Lima, M. D., & Thrall, R. M. (1997). DEA/AR profit-ratios and sensitivity of 100 largest U.S. commercial banks. *European Journal of Operational Research, 98*(2), 213–229.
Tortosa-Ausina, E. (2002). Exploring efficiency differences over time in the Spanish banking industry. *European Journal of Operational Research, 139*, 643–664.
Vassiloglou, M., & Giokas, D. (1990). A study of the relative efficiency of bank branches. An application of data envelopment analysis. *The Journal of the Operational Research Society, 41*, 591–597.
Wheelock, D. C., & Wilson, P. W. (1999). Technical progress, inefficiency, and productivity change in US banking, 1984–1993. *Journal of Money, Credit, Banking, 31*(2), 212–234.
Wheelock, D. C., & Wilson, P. W. (2006). *Robust non-parametric quantile estimation of efficiency and productivity change in U.S. commercial banking, 1985–2004*. Research Division Federal Reserve Bank of St. Louis, Working Paper 2006-041A. Retrieved from http://research.stlouisfed.org/wp/2006/2006-041.pdf
Wilson, P. W. (1993). Detecting outliers in deterministic nonparametric frontier models with multiple outputs. *Journal of Business and Economic Statistics, 11*, 319–323.
Wilson, P. W. (2007). FEAR: A package for Frontier efficiency analysis with R. *Socio-Economic Planning Sciences, 42*(4), 247–254.
Zhu, J. (1998). Data envelopment analysis vs. principal component analysis. An illustrative study of economic performance of Chinese cities. *European Journal of Operational Research, 111*, 50–61.

Chapter 4
Multi-period Efficiency Measurement with Fuzzy Data and Weight Restrictions

Shiang-Tai Liu

Abstract In measuring the overall efficiency of a set of decision making units (DMUs) in a time span covering multiple periods, the conventional approach is to use the aggregate data of the multiple periods via a data envelopment analysis (DEA) technique, ignoring the specific situation of each period. In the real word, there are situations that the observations are inexact and imprecise in nature and they have to be estimated. This study proposes using a relational network model to take the operations of individual periods into account in measuring efficiencies, and the input and output data are treated as fuzzy numbers. Moreover, the assurance region approach is utilized in the model to reduce the weight flexibility for the prevention of overly optimistic, even unrealistic, measures of efficiency. The overall and period efficiencies of a DMU can be calculated at the same time, and since the observations are fuzzy, the derived overall and period efficiencies are fuzzy as well. A pair of two-level mathematical programs is developed to calculate the lower and upper bounds of the α-cut of the fuzzy efficiencies. It is shown that the fuzzy overall efficiency is still a weighted average of the fuzzy period efficiencies. Fuzzy measures obtained from fuzzy observations are more informative than crisp measures obtained from fuzzy observations to be precise.

Keywords Data envelopment analysis • Fuzzy sets • Network system • Weight restriction

4.1 Introduction

Efficiency, defined as the ratio of the minimal input required to the actual input used to produce the same amount of output, or the ratio of the actual output produced to the maximal output that can be produced from the same amount of input, is an effective measure for identifying production systems with unsatisfactory

S.-T. Liu (✉)
Graduate School of Business and Management, Vanung University, Chung-Li, Tao-Yuan 320, Taiwan, ROC
e-mail: stliu@vnu.edu.tw

S.-N. Hwang et al. (eds.), *Handbook of Operations Analytics Using Data Envelopment Analysis*, International Series in Operations Research & Management Science 239, DOI 10.1007/978-1-4899-7705-2_4

performance, so that necessary amendments can be made to either reduce wasteful resources or increase expected outputs. Data envelopment analysis (DEA) is an important and widely used nonparametric technique for evaluating the performance of decision making units (DMUs) in a multiple-input multiple-output production technology. DEA was initially proposed by Charnes et al. (1978). Due to the sound economic theory it is based on and ease of calculation, hundreds of real world applications and methodological advances have been reported since its first appearance in the literature (see, for example, the review of Cook and Seiford 2009).

For cases in which the period of time being examined is composed of clearly defined time units, such as years, the total inputs consumed and total outputs produced in all of the periods are aggregated for efficiency measurement. More often, the average input and output data of each period are used. Since DEA has a unit-invariant property (Lovell and Pastor 1995), the efficiencies calculated from these two types of data, total and average, are the same. When the aggregate data over all the period is used, the resulting efficiency is an overall measure of the performance of the specified period of time, and the specific efficiency of individual periods remains unknown. In this case, the result that a DMU is overall efficient does not necessarily imply that every period is efficient. In fact, it is possible that one period is abnormally inefficient while it is overall efficient, and the abnormal performance may sometimes provide clues about the likelihood of certain events, such as bankruptcy. Therefore, it would be helpful if the period-specific efficiencies could also be known. Kao and Liu (2014) propose using a relational network model to take the operations of individual periods into account in measuring efficiencies. The overall and period efficiencies of a DMU can be calculated at the same time. Interestingly, the overall efficiency is a weighted average of the period efficiencies, and the weights are the most favorable ones for the DMU being evaluated.

One characteristic of the DEA approach is that the efficiency measure is sensitive to data variations. Unfortunately, in the real world, observations are usually difficult to measure precisely (Guo et al. 2000), observations are missing and need to be estimated (Kao and Liu 2000b), or the data need to be predicted (Kao and Liu 2004). One way to manipulate imprecise data directly is to represent the uncertain values by membership functions of the fuzzy set theory (Bellman and Zadeh 1970; Zadeh 1978; Zimmermann 1996). Under the framework of DEA, different approaches for measuring efficiency in fuzzy environments have been proposed (Kao and Liu 2000a; Lertworasirkul et al. 2003; Shokouhi et al. 2010), and some applications have been reported (Guo 2009). A good review of these approaches and applications can be found in Hatami-Marbini et al. (2011) and Emrouznejad and Tavana (2014). Recently, fuzzy network DEA approaches receive more attentions (Kao and Liu 2011, 2012; Lozano 2014a, b). However, these approaches do not take the weight restrictions into consideration.

Basically, crisp and fuzzy DEA models do not require a priori specification of input and output weights (or multipliers) and by letting these weights run freely to obtain optimal weights for all inputs and outputs of each DMU without imposing any constraints on these weights. In performing DEA, we sometimes encounter extreme values or zeroes in input or output weights. When the values of weights of inputs or outputs are in the zero cases, they do not contribute to interpret the performance results. In addition, the multipliers, if left to run freely, may lead to overly optimistic, even unrealistic, measures of efficiency (Asmild et al. 2007). Saati and Memariani (2005) propose a procedure to find a common set of weights in the fuzzy DEA model. By assessing upper bounds on factor weights and compacting the resulting intervals, a common set of weights is determined. Liu (2008) investigates a fuzzy DEA model with assurance region that is able to evaluate the performance of flexible manufacturing system. Liu (2014) develops a methodology for a fuzzy two-stage DEA model considering weight restrictions when the input and output data are represented as fuzzy numbers. The assurance region approach is adopted to restrict weight flexibility in this fuzzy two-stage DEA model. Nevertheless, both Liu's methods (2008, 2014) cannot be applied to measure the multi-period efficiencies of DMUs.

The purpose of this study is to develop a model, based on the network DEA approach, to measure the overall and period efficiencies of a set of DMUs in a period of time considering weight restrictions when the input and output data are fuzzy numbers. Based on Zadeh's extension principle (Zadeh 1978), a pair of two-level mathematical programs is formulated to calculate the upper bound and lower bound of the fuzzy efficiency score. We then transform this pair of two-level mathematical programs into a pair of one-level mathematical programs. Solving this pair of mathematical programs, we obtain the fuzzy overall and period-specific efficiencies at the same time, and a relationship in which the former is a weighted average of the latter is also derived.

This paper is organized as follows. In the next section, a conventional network DEA model with weight restriction is introduced. Then we propose a methodology to calculate the fuzzy multi-period and period-specific efficiencies for the fuzzy network DEA with weight restrictions. After this, we utilize one example with fuzzy observations and weight restrictions to illustrate the idea proposed in this paper. Finally, some conclusions are drawn based on the discussion.

4.2 Crisp Network DEA with Weight Restrictions

Let X_{ij} and Y_{rj} denote the ith input, $i = 1, \ldots, m$, and rth output, $r = 1, \ldots, s$, respectively, of the jth DMU, $j = 1, \ldots, n$. The CCR model of DEA for calculating the efficiency of DMU k is (Charnes et al. 1978):

$$
\begin{aligned}
E_k^{CCR} = \max & \sum_{r=1}^{s} u_r Y_{rk} \Big/ \sum_{i=1}^{m} v_i X_{ik} \\
\text{s.t.} \quad & \sum_{r=1}^{s} u_r Y_{rj} \Big/ \sum_{i=1}^{m} v_i X_{ij} \le 1, \quad j = 1, \ldots, n, \\
& u_r, v_i \ge \varepsilon, \quad r = 1, \ldots, s, \ i = 1, \ldots, m,
\end{aligned}
\tag{4.1}
$$

where u_r and v_i are virtual multipliers and ε is a small non-Archimedean number imposed for avoiding ignorance of any factor (Charnes et al. 1978; Charnes and Cooper 1984).

Consider a multi-period system composed of q periods, where the superscript p in $X_{ij}^{(p)}$ and $Y_{rj}^{(p)}$ denotes the corresponding period. The total quantities of the ith input and the rth output in all q periods for DMU j are $X_{ij} = \sum_{p=1}^{q} X_{ij}^{(p)}$ and $Y_{rj} = \sum_{p=1}^{q} Y_{rj}^{(p)}$, respectively. For the multi-period system, if each period is viewed as a process of a network system, then it resembles the structure of a parallel production system with q processes. The relational model proposed by Kao (2009) for measuring the efficiency of parallel systems can thus be adopted to calculate the efficiency of multi-period systems.

The relational network model has two special features. One is that the same factor has the same multiplier associated with it, regardless of the period it corresponds to. This requirement can also be derived from the terms of $u_r Y_{rj}$ and $v_i X_{ij}$ in Model (4.1): $u_r Y_{rj} = u_r \sum_{p=1}^{q} Y_{rj}^{(p)} = \sum_{p=1}^{q} u_r Y_{rj}^{(p)}$ and $v_i X_{ij} = v_i \sum_{p=1}^{q} X_{ij}^{(p)} = \sum_{p=1}^{q} v_i X_{ij}^{(p)}$. In this expression, $Y_{rj}^{(p)}$ and $X_{ij}^{(p)}$ of different periods p, $p = 1, \ldots, q$, have the same multipliers, u_r and v_i, respectively. The other is that in calculating the overall efficiency of the system, not only must the system inputs and outputs be considered, but also the period-specific ones. That is, the constraints of $\sum_{r=1}^{s} u_r Y_{rj}^{(p)} - \sum_{i=1}^{m} v_i X_{ij}^{(p)} \le 0$, $p = 1, \ldots, q$, $j = 1, \ldots, n$, are added to Model (4.1). Kao and Liu (2014) taking these two points into account, the relational network model for the multi-period system as:

$$
\begin{aligned}
E_k = \max\ & \sum_{r=1}^{s} u_r Y_{rk} \\
\text{s.t.}\ & \sum_{i=1}^{m} v_i X_{ik} = 1, \\
& \sum_{r=1}^{s} u_r Y_{rj} - \sum_{i=1}^{m} v_i X_{ij} \le 0, \quad j = 1, \ldots, n, \\
& \sum_{r=1}^{s} u_r Y_{rj}^{(p)} - \sum_{i=1}^{m} v_i X_{ij}^{(p)} \le 0, \quad p = 1, \ldots, q,\ \ j = 1, \ldots, n, \\
& u_r, v_i \ge \varepsilon, \quad r = 1, \ldots, s,\ \ i = 1, \ldots, m.
\end{aligned}
\tag{4.2}
$$

Obviously, the overall efficiency of this model, E_k, will not exceed that calculated from the aggregate one, E_k^{CCR}, because of the additional third constraint set.

The sum of the constraints corresponding to the q periods of a DMU in Model (4.2), $\sum_{r=1}^{s} u_r Y_{rj}^{(p)} - \sum_{i=1}^{m} v_i X_{ij}^{(p)} \le 0$, $p = 1, \ldots, q$, is just the constraint corresponding to the system of this DMU, $\sum_{r=1}^{s} u_r Y_{rj} - \sum_{i=1}^{m} v_i X_{ij} \le 0$; therefore, the latter are redundant and can be removed. For this reason, Kao and Liu (2014) reformulate Model (4.2) as:

$$
\begin{aligned}
E_k = \max \ & \sum_{r=1}^{s}\sum_{p=1}^{q} u_r Y_{rk}^{(p)} \\
\text{s.t.} \ & \sum_{i=1}^{m}\sum_{p=1}^{q} v_i X_{ik}^{(p)} = 1, \\
& \sum_{r=1}^{s} u_r Y_{rj}^{(p)} - \sum_{i=1}^{m} v_i X_{ij}^{(p)} \leq 0, \quad p = 1, \ldots, q, \ j = 1, \ldots, n, \\
& u_r, v_i \geq \varepsilon, \quad r = 1, \ldots, s, \ i = 1, \ldots, m.
\end{aligned} \tag{4.3}
$$

In this case, the constraints of Model (4.3) are those corresponding to the q periods of the n DMUs, indicating that each period is considered as an independent DMU in constructing the efficiency frontier, and the objective is to find the multipliers that will produce the maximum overall efficiency for the DMU being evaluated. In other words, all q periods are assumed to have the same technology.

After the optimal solutions u_r^* and v_i^* are obtained, the overall efficiency E_k and period efficiencies $E_k^{(p)}$, $p = 1, \ldots, q$, for DMU k, are calculated as (Kao and Liu 2014):

$$
\begin{aligned}
E_k &= \sum_{p=1}^{q}\left(\frac{\sum_{r=1}^{s} u_r^* Y_{rk}^{(p)}}{\sum_{i=1}^{m} v_i^* X_{ik}}\right) = \frac{\sum_{r=1}^{s} u_r^* Y_{rk}}{\sum_{i=1}^{m} v_i^* X_{ik}} == \sum_{r=1}^{s} u_r^* Y_{rk} \\
E_k^{(p)} &= \frac{\sum_{r=1}^{s} u_r^* Y_{rk}^{(p)}}{\sum_{i=1}^{m} v_i^* X_{ik}^{(p)}}, \quad p = 1, \ldots, q.
\end{aligned} \tag{4.4}
$$

It is shown in Kao and Hwang (2008) and Kao (2009) that, by setting the weight $w^{(p)}$ to the proportion of the aggregate input consumed in period p in that of all periods, $\sum_{i=1}^{m} v_i^* X_{ik}^{(p)} / \sum_{i=1}^{m} v_i^* X_{ik}$, the overall efficiency is the average of the q period efficiencies weighted by $w^{(p)}$:

$$
\begin{aligned}
\sum_{p=1}^{q} w^{(p)} E_k^{(p)} &= \sum_{p=1}^{q}\left(\frac{\sum_{i=1}^{m} v_i^* X_{ik}^{(p)}}{\sum_{i=1}^{m} v_i^* X_{ik}} \times \frac{\sum_{r=1}^{s} u_r^* Y_{rk}^{(p)}}{\sum_{i=1}^{m} v_i^* X_{ik}^{(p)}}\right) = \sum_{p=1}^{q}\left(\frac{\sum_{r=1}^{s} u_r^* Y_{rk}^{(p)}}{\sum_{i=1}^{m} v_i^* X_{ik}}\right) \\
&= \frac{\sum_{r=1}^{s} u_r^* Y_{rk}}{\sum_{i=1}^{m} v_i^* X_{ik}} = E_k
\end{aligned}
$$

Note that the set of weights selected by each DMU is the most advantageous one to calculate the overall efficiency, and they may not be the same for all DMUs. Model

(4.4) is thus able to not only calculate the overall and period efficiencies of the multi-period system, but also obtain a mathematical relationship between them.

The multipliers u_r and v_i are treated as unknown variables in (4.3). Therefore, each DMU may select any multipliers as it wants for its inputs and outputs. An important extension of DEA models is restricting weight flexibility. We can find in the literature different approaches to setting bounds in weight restrictions of DEA models (Thompson et al. 1986, 1990; Charnes et al. 1990; Bal et al. 2008; Ramón et al. 2010; Korhonen et al. 2011). The assurance region approach, first proposed by Thompson et al. (1986), is originally developed with the purpose of incorporating value judgments into the analysis, that is, prior information, expert opinions, or preferences concerning the underlying process of assessing efficiency. Additionally, this approach is very comprehensive to bounding the DEA multipliers. Therefore, we adopt the assurance region approach to impose weight restrictions in this paper.

Suppose the relative importance elicited from the experts range from L_{I_1} to U_{I_1} for input item 1 and from L_{I_2} to U_{I_2} for input item 2. The associated constraints are $L_{I_1}/U_{I_2} \le v_1/v_2 \le U_{I_1}/L_{I_2}$. Generalizing to all inputs and outputs, respectively, gives

$$\begin{aligned} &L_{I_a}/U_{I_b} \le v_a/v_b \le U_{I_a}/L_{I_b}, \quad a < b = 2, \ldots, m \\ &L_{O_a}/U_{O_b} \le u_a/u_b \le U_{O_a}/L_{O_b}, \quad a < b = 2, \ldots, s. \end{aligned} \tag{4.5}$$

To simplify the notation, let $F_{ab}^L = L_{I_a}/U_{I_b}$, $F_{ab}^U = U_{I_a}/L_{I_b}$, $G_{ab}^L = L_{O_a}/U_{O_b}$, and $G_{ab}^U = U_{O_a}/L_{O_b}$. Including assurance region in (4.3) gives the following mathematical form:

$$\begin{aligned} E_k = \max & \sum_{r=1}^{s}\sum_{p=1}^{q} u_r Y_{rk}^{(p)} \\ \text{s.t.} \quad & \sum_{i=1}^{m}\sum_{p=1}^{q} v_i X_{ik}^{(p)} = 1 \\ & \sum_{r=1}^{s} u_r Y_{rj}^{(p)} - \sum_{i=1}^{m} v_i X_{ij}^{(p)} \le 0, \quad p = 1, \ldots, q, \ j = 1, \ldots, n, \\ & -v_a + F_{ab}^L v_b \le 0, \quad v_a - F_{ab}^U v_b \le 0, \ a < b = 2, \ldots, m, \\ & -u_a + G_{ab}^L u_b \le 0, \quad u_a - G_{ab}^U u_b \le 0, \ a < b = 2, \ldots, s. \end{aligned} \tag{4.6}$$

In the next section, we shall develop a fuzzy multi-period efficiency measurement model considering weight restrictions.

4.3 Fuzzy Multi-period Efficiency with Weight Restrictions

Without loss of generality and for simplicity of notation, suppose all observations are described by fuzzy numbers; then crisp numbers can be considered as degenerate fuzzy numbers with only one value in the domain of fuzzy sets. When the observations are fuzzy numbers, the resulting efficiencies will also be fuzzy numbers.

Denote $\widetilde{X}_{ij}$ and $\widetilde{Y}_{rj}$, as the fuzzy counterparts of X_{ij} and Y_{rj}, respectively, in the deterministic case. Conceptually, Model (4.6) for fuzzy observations can be formulated as:

$$\begin{aligned} \widetilde{E}_k = \max\ & \sum_{r=1}^{s}\sum_{p=1}^{q} u_r \widetilde{Y}_{rk}^{(p)} \\ \text{s.t.}\ & \sum_{i=1}^{m} v_i \sum_{p=1}^{q} \widetilde{X}_{ik}^{(p)} = 1 \\ & \sum_{r=1}^{s} u_r \widetilde{Y}_{rj}^{(p)} - \sum_{i=1}^{m} v_i \widetilde{X}_{ij}^{(p)} \le 0, \quad p = 1, \ldots, q,\ \ j = 1, \ldots, n, \\ & -v_a + F_{ab}^{L} v_b \le 0, \quad v_a - F_{ab}^{U} v_b \le 0,\ \ a < b = 2, \ldots, m, \\ & -u_a + G_{ab}^{L} u_b \le 0, \quad u_a - G_{ab}^{U} u_b \le 0,\ \ a < b = 2, \ldots, s. \end{aligned} \tag{4.7}$$

Since the observations are fuzzy numbers, the resulting efficiency $\widetilde{E}_k$ should also be a fuzzy number. To obtain the membership function $\mu_{\widetilde{E}_k}$ for $\widetilde{E}_k$, one may rely on Zadeh's extension principle (Zadeh 1978; Zimmermann 1996), which describes the relationship between the membership function of $\widetilde{E}_k$ and the membership functions of $\widetilde{X}_{ij}^{(p)}$ and $\widetilde{Y}_{rj}^{(p)}$.

$$\mu_{\widetilde{E}_k}(e) = \sup_{x,y} \min\left\{ \mu_{\widetilde{X}_{ij}^{(p)}}\left(x_{ij}^{(p)}\right), \mu_{\widetilde{Y}_{rj}^{(p)}}\left(y_{rj}^{(p)}\right), \forall i, r, p, j \,\middle|\, e = E_k(\boldsymbol{x}, \boldsymbol{y}) \right\}, \tag{4.8}$$

where $E_k(\boldsymbol{x},\boldsymbol{y})$ is defined by Model (4.3).

Nguyen (1978) indicates that the application of the extension principle to $\widetilde{E}_k$ can be viewed as the application of this extension principle to the α-cuts of $\widetilde{E}_k$. This idea is adopted for calculating the fuzzy efficiency $\widetilde{E}_k$. Denote $(X_{ij}^{(p)})_\alpha = [(X_{ij}^{(p)})_\alpha^L, (X_{ij}^{(p)})_\alpha^U]$ and $(Y_{rj}^{(p)})_\alpha = [(Y_{rj}^{(p)})_\alpha^L, (Y_{rj}^{(p)})_\alpha^U]$, as the α-cuts of $\widetilde{X}_{ij}$ and $\tilde{Y}_{rj}$, respectively. To find the membership function $\mu_{\widetilde{E}_k}(e)$, it suffices to find the lower and upper bounds of the α-cut of $\widetilde{E}_k$, $(E_k)_\alpha = [(E_k)_\alpha^L, (E_k)_\alpha^U]$.

The upper bound $(E_k)_\alpha^U$ is equal to $\max\{e | \mu_{\widetilde{E}_k}(e) \ge \alpha\}$, and the lower bound $(E_k)_\alpha^L$ is equal to $\min\{e | \mu_{\widetilde{E}_k}(e) \ge \alpha\}$. In symbols, they can be expressed as:

$$(E_k)_\alpha^U = \max_{\substack{\left(X_{ij}^{(p)}\right)_\alpha^L \le x_{ij} \le \left(X_{ij}^{(p)}\right)_\alpha^U \\ \left(Y_{rj}^{(p)}\right)_\alpha^L \le y_{rj} \le \left(Y_{rj}^{(p)}\right)_\alpha^U \\ \forall i,r,p,j}} E_k(\boldsymbol{x},\boldsymbol{y}) \tag{4.9a}$$

$$(E_k)_\alpha^L = \min_{\substack{\left(X_{ij}^{(p)}\right)_\alpha^L \le x_{ij} \le \left(X_{ij}^{(p)}\right)_\alpha^U \\ \left(Y_{rj}^{(p)}\right)_\alpha^L \le y_{rj} \le \left(Y_{rj}^{(p)}\right)_\alpha^U \\ \forall i,r,p,j}} E_k(\boldsymbol{x},\boldsymbol{y}) \tag{4.9b}$$

Note that $E_k(\boldsymbol{x}, \boldsymbol{y})$ is a mathematical program with maximization as the objective function. Therefore, Models (4.9a) and (4.9b) are two-level programs, with $E_k(\boldsymbol{x},\boldsymbol{y})$ as the inner program. $(E_k)_\alpha^U$ and $(E_k)_\alpha^L$ can be calculated via the following two-level programming models:

$$(E_k)_\alpha^U = \max_{\substack{\left(X_{ij}^{(p)}\right)_\alpha^L \le x_{ij}^{(p)} \le \left(X_{ij}^{(p)}\right)_\alpha^U \\ \left(Y_{rj}^{(p)}\right)_\alpha^L \le y_{rj}^{(p)} \le \left(Y_{rj}^{(p)}\right)_\alpha^U \\ \forall i,r,p,j}} \cdot \left\{\begin{array}{l} \max \sum_{r=1}^{s}\sum_{p=1}^{q} u_r y_{rk}^{(p)} \\ \text{s.t.} \sum_{i=1}^{m}\sum_{p=1}^{q} v_i x_{ik}^{(p)} = 1, \\ \quad \sum_{r=1}^{s} u_r y_{rj}^{(p)} - \sum_{i=1}^{m} v_i x_{ij}^{(p)} \le 0,\ j = 1, \ldots, n,\ \ p = 1, \ldots, q, \\ \quad - v_a + F_{ab}^{\mathrm{L}} v_b \le 0,\ v_a - F_{ab}^{\mathrm{U}} v_b \le 0,\ \ \mathrm{a} < b = 2, \ldots, m, \\ \quad - u_a + G_{ab}^{\mathrm{L}} u_b \le 0,\ u_a - G_{ab}^{\mathrm{U}} u_b \le 0,\ \ \mathrm{a} < b = 2, \ldots, s. \end{array}\right. \tag{4.10a}$$

$$(E_k)_\alpha^L = \min_{\substack{\left(X_{ij}^{(p)}\right)_\alpha^L \le x_{ij}^{(p)} \le \left(X_{ij}^{(p)}\right)_\alpha^U \\ \left(Y_{rj}^{(p)}\right)_\alpha^L \le y_{rj}^{(p)} \le \left(Y_{rj}^{(p)}\right)_\alpha^U \\ \forall i,r,p,j}} \cdot \left\{\begin{array}{l} \max \sum_{r=1}^{s}\sum_{p=1}^{q} u_r y_{rk}^{(p)} \\ \text{s.t.} \sum_{i=1}^{m}\sum_{p=1}^{q} v_i x_{ik}^{(p)} = 1, \\ \quad \sum_{r=1}^{s} u_r y_{rj}^{(p)} - \sum_{i=1}^{m} v_i x_{ij}^{(p)} \le 0,\ j = 1, \ldots, n,\ p = 1, \ldots, q, \\ \quad - v_a + F_{ab}^{\mathrm{L}} v_b \le 0,\ v_a - F_{ab}^{\mathrm{U}} v_b \le 0,\ \mathrm{a} < b = 2, \ldots, m, \\ \quad - u_a + G_{ab}^{\mathrm{L}} u_b \le 0,\ u_a - G_{ab}^{\mathrm{U}} u_b \le 0,\ \mathrm{a} < b = 2, \ldots, s. \end{array}\right. \tag{4.10b}$$

For each set of $x_{ij}^{(p)}$ and $y_{rj}^{(p)}$ values defined by the respective α-cuts in the outer program (first level), the efficiency is calculated in the inner program (second level). The sets of $x_{ij}^{(p)}$ and $y_{rj}^{(p)}$ values, which produce the largest and smallest efficiencies, are determined at the first level by Models (4.10a) and (4.10a), respectively.

The inner and outer programs of Model (4.10a) have the same direction for optimization, maximization. Therefore, they can be combined into one level, with the objective function of the inner program as the overall objective function and the constraints at the two levels as the overall constraints, which yields:

$$
\begin{aligned}
(E_k)_\alpha^U = \max \ & \sum_{r=1}^{s}\sum_{p=1}^{q} u_r y_{rk}^{(p)} \\
\text{s.t.} \ & \sum_{i=1}^{m}\sum_{p=1}^{q} v_i x_{ik}^{(p)} = 1, \\
& \sum_{r=1}^{s} u_r y_{rj}^{(p)} - \sum_{i=1}^{m} v_i x_{ij}^{(p)} \le 0, \quad p = 1, \ldots, q, \ j = 1, \ldots, n, \\
& \left(X_{ij}^{(p)}\right)_\alpha^L \le x_{ij}^{(p)} \le \left(X_{ij}^{(p)}\right)_\alpha^U, \quad i = 1, \ldots, m, \ p = 1, \ldots, q, j = 1, \ldots, n, \\
& \left(Y_{rj}^{(p)}\right)_\alpha^L \le y_{rj}^{(p)} \le \left(Y_{rj}^{(p)}\right)_\alpha^U, \quad r = 1, \ldots, s, \ p = 1, \ldots, q, \ j = 1, \ldots, n, \\
& -v_a + F_{ab}^L v_b \le 0, \quad v_a - F_{ab}^U v_b \le 0, \ a < b = 2, \ldots, m, \\
& -u_a + G_{ab}^L u_b \le 0, \quad u_a - G_{ab}^U u_b \le 0, \ a < b = 2, \ldots, s.
\end{aligned}
\tag{4.11}
$$

This program is nonlinear due to the nonlinear terms $u_r y_{rj}^{(p)}$ and $v_i x_{ij}^{(p)}$. However, by substituting the former by $c_{rj}^{(p)}$ and the latter by $d_{ij}^{(p)}$, Model (4.11) can be transformed into the following linear program:

$$
\begin{aligned}
(E_k)_\alpha^U = \max \ & \sum_{r=1}^{s}\sum_{p=1}^{q} c_{rk}^{(p)} \\
\text{s.t.} \ & \sum_{i=1}^{m}\sum_{p=1}^{q} d_{ik}^{(p)} = 1, \\
& \sum_{r=1}^{s} c_{rj}^{(p)} - \sum_{i=1}^{m} d_{ij}^{(p)} \le 0, \quad p = 1, \ldots, q, \ j = 1, \ldots, n, \\
& v_i\left(X_{ij}^{(p)}\right)_\alpha^L \le d_{ij}^{(p)} \le v_i\left(X_{ij}^{(p)}\right)_\alpha^U, \quad i = 1, \ldots, m, \ p = 1, \ldots, q, \ j = 1, \ldots, n, \\
& u_r\left(Y_{rj}^{(p)}\right)_\alpha^L \le c_{rj}^{(p)} \le u_r\left(Y_{rj}^{(p)}\right)_\alpha^U, \quad r = 1, \ldots, s, \ p = 1, \ldots, q, \ j = 1, \ldots, n, \\
& -v_a + F_{ab}^L v_b \le 0, \quad v_a - F_{ab}^U v_b \le 0, \ a < b = 2, \ldots, m, \\
& -u_a + G_{ab}^L u_b \le 0, \quad u_a - G_{ab}^U u_b \le 0, \ a < b = 2, \ldots, s.
\end{aligned}
\tag{4.12}
$$

After an optimal solution $(u_r^*, v_i^*, c_{rk}^{(p)*}, d_{ik}^{(p)*})$ is obtained, we have $y_{rk}^{(p)*} = c_{rk}^{(p)*}/u_r^*$ and $x_{ik}^{(p)*} = d_{ik}^{(p)*}/v_i^*$. Then the overall and period efficiencies and the associated weights are calculated as:

$$\begin{aligned}
(E_k)_\alpha^U &= \sum_{r=1}^{s}\sum_{p=1}^{q} u_r^* y_{rk}^{(p)*} \Big/ \sum_{i=1}^{m}\sum_{p=1}^{q} v_i^* x_{ik}^{(p)*} \\
\left(E_k^{(p)}\right)_\alpha^U &= \sum_{p=1}^{q} u_r^* y_{rk}^{(p)*} \Big/ \sum_{p=1}^{q} v_i^* x_{ik}^{(p)*} \quad p = 1, \ldots, q \\
w_k^{(p)} &= \sum_{p=1}^{q} v_i^* x_{ik}^{(p)*} \Big/ \sum_{i=1}^{m}\sum_{p=1}^{q} v_i^* x_{ik}^{(p)*}.
\end{aligned} \tag{4.13}$$

The overall efficiency $(E_k)_\alpha^U$ is the average of the period efficiencies $(E_k^{(p)})_\alpha^U$ weighted by $w_k^{(p)}$ for all α values. The overall efficiency, $(E_k)_\alpha^U$, is the same as the result of Kao and Lin (2012). The only difference is that the weight restrictions are considered in this study.

The conversion of Program (10b) to a one-level program is not so straightforward, because the directions for optimization for the inner and outer programs are different. The inner program is a linear program when the values $x_{ij}^{(p)}$ and $y_{rj}^{(p)}$ are assigned by the outer program. According to the duality theorem (Dantzig 1963), the primal and dual programs have the same objective value at optimality. Hence, the inner program can be replaced by its dual to change the objective function from maximization to minimization. The dual form of the inner program of (10b) can be formulated as

$$\begin{aligned}
\min \quad & \theta - \varepsilon\left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+\right) \\
\text{s.t.} \quad & \theta\sum_{p=1}^{q} x_{ik}^{(p)} - \sum_{p=1}^{q}\sum_{j=1}^{n} \lambda_j^{(p)} x_{ij}^{(p)} + \sum_{b>i}\left(-\alpha_{ib}^L + \alpha_{ib}^U\right) + \sum_{a<i}\left(\alpha_{ai}^L F_{ai}^L - \alpha_{ai}^U F_{ai}^U\right) - s_i^+ = 0, \\
& i = 1, \ldots, m, \\
& \sum_{p=1}^{q}\sum_{j=1}^{n} \lambda_j^{(p)} y_{rj}^{(p)} + \sum_{b>r}\left(-\beta_{rb}^L + \beta_{rb}^U\right) + \sum_{a<r}\left(\beta_{ar}^L G_{ar}^L - \beta_{ar}^U G_{ar}^U\right) - s_r^- = \sum_{p=1}^{q} y_{rk}^{(p)}, \\
& r = 1, \ldots, s, \\
& \lambda_j^{(p)}, s_i^+, s_r^- \geq 0, \quad i = 1, \ldots, m, \;\; r = 1, \ldots, s, \;\; p = 1, \ldots, q, \;\; j = 1, \ldots, n, \\
& \theta \text{ unrestricted in sign.}
\end{aligned} \tag{4.14}$$

After this replacement, both the inner and outer programs have the same direction of minimization; thus, they can be combined into the same one level. The resulting program of (10b) becomes the following one-level program:

$$
\begin{aligned}
(E_k)^L_\alpha = \min\ & \theta - \varepsilon\left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+\right) \\
\text{s.t.}\ & \theta\sum_{p=1}^{q} x_{ik}^{(p)} - \sum_{p=1}^{q}\sum_{j=1}^{n}\lambda_j^{(p)} x_{ij}^{(p)} + \sum_{b>i}\left(-\alpha_{ib}^L + \alpha_{ib}^U\right) + \sum_{a<i}\left(\alpha_{ai}^L F_{ai}^L - \alpha_{ai}^U F_{ai}^U\right) - s_i^+ = 0, \\
& i = 1, \ldots, m, \\
& \sum_{p=1}^{q}\sum_{j=1}^{n}\lambda_j^{(p)} y_{rj}^{(p)} + \sum_{b>r}\left(-\beta_{rb}^L + \beta_{rb}^U\right) + \sum_{a<r}\left(\beta_{ar}^L G_{ar}^L - \beta_{ar}^U G_{ar}^U\right) - s_r^- = \sum_{p=1}^{q} y_{rk}^{(p)}, \\
& r = 1, \ldots, s, \\
& \left(X_{ij}^{(p)}\right)_\alpha^L \le x_{ij}^{(p)} \le \left(X_{ij}^{(p)}\right)_\alpha^U, \quad i = 1, \ldots, m,\ p = 1, \ldots, q,\ j = 1, \ldots, n, \\
& \left(Y_{rj}^{(p)}\right)_\alpha^L \le y_{rj}^{(p)} \le \left(Y_{rj}^{(p)}\right)_\alpha^U, \quad r = 1, \ldots, s,\ p = 1, \ldots, q,\ j = 1, \ldots, n, \\
& \lambda_j^{(p)}, s_i^+, s_r^- \ge 0, \quad i = 1, \ldots, m, r = 1, \ldots, s,\ p = 1, \ldots, q,\ j = 1, \ldots, n, \\
& \theta \text{ unrestricted in sign.}
\end{aligned}
\tag{4.15}
$$

This program is nonlinear due to the nonlinear terms $\lambda_j^{(p)} x_{ij}^{(p)}$ and $\lambda_j^{(p)} y_{rj}^{(p)}$, and these cannot be linearized by variable substitutions, as occurs in Model (4.11). Since this program has only $m + s$ nonlinear constraints, which is of small size in the standard of nonlinear programming, most commercial nonlinear programming solvers can be used to derive a solution.

The objective value of (4.15) is just the lower bound of the system efficiency at the α level, $(E_k)^L_\alpha$. At optimality, the reduced costs of s_i^+ and s_r^- are the values of multipliers v_i and u_r, respectively, of the primal program. The overall and period efficiencies and the corresponding weights subsequently yield

$$
\begin{aligned}
(E_k)^L_\alpha &= \sum_{r=1}^{s}\sum_{p=1}^{q} u_r^* y_{rk}^{(p)*} \Big/ \sum_{i=1}^{m}\sum_{p=1}^{q} v_i^* x_{ik}^{(p)*} \\
\left(E_k^{(p)}\right)_\alpha^L &= \sum_{p=1}^{q} u_r^* y_{rk}^{(p)*} \Big/ \sum_{p=1}^{q} v_i^* x_{ik}^{(p)*} \quad p = 1, \ldots, q \\
w_k^{(p)} &= \sum_{p=1}^{q} v_i^* x_{ik}^{(p)*} \Big/ \sum_{i=1}^{m}\sum_{p=1}^{q} v_i^* x_{ik}^{(p)*}.
\end{aligned}
\tag{4.16}
$$

Similar to the upper-bound case, $(E_k)^L_\alpha$ is the average of $(E_k^{(p)})^L_\alpha$ weighted by $w_k^{(p)}$ for all α values. Together with the upper bound of the efficiencies derived from (4.12), the bounds of the fuzzy efficiencies at a specific α-level are obtained. Enumerating various values of α, the membership functions of $\widetilde{E}_k$ and $\widetilde{E}_k^{(p)}$ can be approximated numerically.

Table 4.1 Triangular fuzzy numbers for efficiency measurement

DMU	Period	X_1	X_2	Y_1	Y_2
1	1	(16, 22, 28)	(60, 65, 70)	(19, 25, 31)	(24, 30, 36)
	2	(24, 26, 28)	(76, 82, 88)	(20, 24, 28)	(18, 21, 24)
2	1	(18, 26, 34)	(70, 78, 86)	(24, 27, 30)	(28, 32, 36)
	2	(27, 30, 33)	(65, 69, 73)	(11, 14, 17)	(16, 19, 22)
3	1	(14, 23, 32)	(66, 75, 84)	(21, 26, 31)	(20, 26, 32)
	2	(17, 28, 39)	(57, 62, 67)	(31, 38, 45)	(36, 39, 42)
4	1	(16, 19, 22)	(78, 83, 88)	(18, 22, 26)	(12, 15, 18)
	2	(26, 31, 36)	(75, 83, 91)	(16, 20, 24)	(20, 23, 26)
5	1	(12, 19, 26)	(86, 92, 98)	(19, 28, 37)	(26, 29, 32)
	2	(27, 32, 37)	(90, 94, 98)	(15, 25, 35)	(20, 23, 26)
6	1	(22, 26, 30)	(84, 90, 96)	(18, 22, 26)	(20, 24, 28)
	2	(24, 28, 32)	(77, 80, 83)	(25, 29, 33)	(22, 26, 30)
7	1	(18, 20, 22)	(80, 86, 92)	(19, 23, 27)	(20, 26, 32)
	2	(32, 38, 44)	(48, 53, 58)	(17, 22, 27)	(18, 22, 26)
8	1	(24, 28, 32)	(88, 92, 96)	(19, 23, 27)	(22, 27, 32)
	2	(21, 29, 37)	(71, 77, 83)	(22, 27, 32)	(16, 20, 24)
9	1	(24, 28, 32)	(74, 78, 82)	(16, 19, 22)	(24, 28, 32)
	2	(26, 30, 34)	(81, 86, 91)	(18, 23, 28)	(32, 36, 40)
10	1	(10, 15, 20)	(64, 72, 80)	(30, 36, 42)	(24, 31, 38)
	2	(16, 23, 30)	(48, 53, 58)	(29, 33, 37)	(42, 47, 52)
11	1	(18, 24, 30)	(84, 90, 96)	(16, 21, 26)	(28, 31, 34)
	2	(27, 34, 41)	(77, 84, 91)	(23, 27, 31)	(20, 23, 26)
12	1	(14, 19, 24)	(82, 88, 94)	(27, 31, 35)	(20, 24, 28)
	2	(24, 29, 34)	(50, 56, 62)	(13, 16, 19)	(26, 29, 32)
13	1	(16, 21, 26)	(86, 92, 98)	(32, 34, 36)	(26, 30, 34)
	2	(29, 35, 41)	(85, 89, 93)	(26, 30, 34)	(28, 30, 32)
14	1	(20, 24, 28)	(86, 92, 98)	(18, 22, 26)	(36, 39, 42)
	2	(33, 36, 39)	(64, 69, 74)	(16, 19, 22)	(28, 32, 36)
15	1	(22, 25, 28)	(78, 84, 90)	(21, 25, 29)	(22, 28, 34)
	2	(31, 36, 41)	(48, 53, 58)	(15, 20, 25)	(30, 34, 38)
16	1	(22, 26, 30)	(66, 70, 74)	(24, 30, 36)	(29, 33, 37)
	2	(32, 35, 38)	(56, 61, 66)	(16, 19, 22)	(26, 31, 36)

4.4 Example

In this section we utilize an example to illustrate the idea proposed in this study. Suppose we have two periods both of which consume two inputs and produce two outputs. The inputs consumed and outputs produced by each period of the 16 DMUs are represented by symmetric triangular fuzzy numbers, which are shown in Table 4.1. The notation used in this paper is (a, b, c) for a triangular fuzzy number with a, b, and c as the coordinates of the three vertices of the triangle. The values of a, b, and c indicate, respectively, the most pessimistic, most possible, and most optimistic values of a fuzzy number.

The relative importance of input item X_1 is a range lying between 0.0102 and 0.0205, that is, $v_1 = (0.0102, 0.0205)$, in a scale of 1.0. For X_2, Y_1, and Y_2, they are also represented in the ranges of $v_2 = (0.0018, 0.0063)$, $u_1 = (0.0071, 0.0105)$, and $u_2 = (0.0083, 0.0115)$, respectively. Following (4.5), the assurance regions generated for the input- and output-type multipliers, respectively, are

$$\frac{0.0102}{0.0063} \leq \frac{v_1}{v_2} \leq \frac{0.0205}{0.0018} \text{and} \frac{0.0071}{0.0115} \leq \frac{u_1}{u_2} \leq \frac{0.0105}{0.0083}.$$

According to (4.12) and (4.15), the data contained in Table 4.1 are used to measure the fuzzy overall efficiency $\widetilde{E}_k$ and two period-specific efficiencies $\widetilde{E}_k^{(1)}$ and $\widetilde{E}_k^{(2)}$ with weight restrictions of the 16 DMUs. Table 4.2 lists the results for $\alpha = 0$, $0.1, 0.2, \ldots, 1.0$, where O, P1, and P2 refer to overall, Period 1, and Period 2, respectively. Since the fuzzy efficiency score lies in a range, the different value of possibility shows the different interval of the efficiency score. Moreover, the greater value of the possibility level, the narrower the interval is. Specifically, the possibility level $\alpha = 0$ shows the range that the efficiency score will definitely appear and the possibility level $\alpha = 1.0$ shows the efficiency score that is most likely to be. For example, while the overall efficiency score of DMU 1 is fuzzy, its value is impossible to exceed 0.929 or fall below 0.275. At the other extreme end of $\alpha = 1$, the single value of 0.556 indicates that this value is definitely possible for the efficiency score of this DMU.

As noted previously, the relational network model can not only calculate the overall and period efficiencies, but also establish a weighted average relationship between them. This can be verified by multiplying the period efficiencies of each DMU in Table 4.2 by their associated weights, the values shown in Table 4.3, and summing over these two periods to get the overall efficiency. For example, the lower bound efficiencies of the overall, Period 1, and Period 2 of DMU 1 at $\alpha = 0$ are 0.275, 0.312, and 0.243, respectively, and the associated weights of Period 1 and Period 2 are 0.464 and 0.536, individually. The weighted average of DMU 1 is $0.312 \times 0.464 + 0.243 \times 0.536$, or 0.275, which is exactly its overall efficiency. Similarly, the upper bound efficiencies of the overall, Period 1, and Period 2 of DMU 1 are 0.929, 1.0, and 0.852, respectively; the corresponding weights of Period 1 and Period 2 are, respectively, 0.519 and 0.481. The weighted average is $1.0 \times 0.519 + 0.852 \times 0.481 = 0.929$. Clearly, this value is exactly the upper bound of the overall efficiency. This relationship is true for the lower bound and upper bound efficiencies of every DMU at all α values.

At $\alpha = 0$, only the overall, Period 1, and Period 2 efficiencies of DMUs 3 and 10 have an upper value of 1.0, indicating that these two DMUs have the possibility of being evaluated as efficient. Moreover, the $\alpha = 0$ cut also indicates that the overall efficiency of DMU 10 is clearly better than the other DMUs, as its $\alpha = 0$ cut does not overlap with those of others. This is also true form the viewpoint of its efficiency at $\alpha = 1$ cut because all the overall, Period 1, and Period 2 efficiencies have a perfect efficiency score 1.0. On the contrary, the overall efficiency of DMU 4 is worse than the other DMUs from the same viewpoint of the $\alpha = 0$ and $\alpha = 1$ cuts.

Table 4.2 α-Cuts of the fuzzy efficiency scores with weight restrictions

DMU		$\alpha=0$	$\alpha=0.1$	$\alpha=0.2$	$\alpha=0.3$	$\alpha=0.4$	$\alpha=0.5$	$\alpha=0.6$	$\alpha=0.7$	$\alpha=0.8$	$\alpha=0.9$	$\alpha=1$
1	O	(.275, .929)	(.293, .901)	(.314, .873)	(.338, .845)	(.363, .817)	(.390, .789)	(.419, .737)	(.450, .686)	(.483, .640)	(.518, .596)	(.556, .556)
	P1	(.312, 1.00)	(.335, 1.00	(.352, 1.00)	(.382, 1.00)	(.414, 1.00)	(.449, 1.00)	(.494, .925)	(.527, .853)	(.571, .786)	(.618, .726)	(.670, .670)
	P2	(.243, .852)	(.257, .801)	(.280, .753)	(.298, .708)	(.318, .666)	(.338, .626)	(.374, . 589)	(.383, .554)	(.408, .521)	(.434, .490)	(.461, .461)
2	O	(.247, .795)	(.262, .771)	(.278, .747)	(.295, .724)	(.313, .691)	(.331, .644)	(.351, .600)	(.374, .559)	(.400, .523)	(.428, .489)	(.457, .457)
	P1	(.309, 1.00)	(.329, 1.00)	(.349, 1.00)	(.371, 1.00)	(.393, .975)	(.417, .900)	(.443, .830)	(.488, .766)	(.524, .701)	(.563, .651)	(.605, .605)
	P2	(.177, .591)	(.188, .555)	(.200, .521)	(.212, .490)	(.225, .461)	(.239, .433)	(.253, .406)	(.262, .381)	(.280, .360)	(.298, .338)	(.317, .317)
3	O	(.343, 1.00)	(.367, 1.00)	(.393, 1.00)	(.421, 1.00)	(.451, 1.00)	(.483, .976)	(.516, .934)	(.553, .893)	(.600, .836)	(.651, .768)	(.707, .707)
	P1	(.256, 1.00)	(.277, 1.00)	(.298, 1.00)	(.321, 1.00)	(.346, 1.00)	(.372, .944)	(.400, .854)	(.455, .774)	(.496, .706)	(.541, .645)	(.591, .591)
	P2	(.432, 1.00)	(.462, 1.00)	(.493, 1.00)	(.526, 1.00)	(.561, 1.00)	(.598, 1.00)	(.638, 1.00)	(.646, 1.00)	(.697, .957)	(.754, .883)	(.815, .815)
4	O	(.207, .847)	(.222, .787)	(.239, .732)	(.256, .681)	(.275, .635)	(.295, .591)	(.316, .551)	(.339, .514)	(.364, .480)	(.390, .447)	(.418, .418)
	P1	(.225, .967)	(.242, .892)	(.260, .827)	(.304, .767)	(.328, .712)	(.354, .661)	(.347, .615)	(.372, .572)	(.400, .532)	(.429, .495)	(.461, .461)
	P2	(.194, .763)	(.208, .711)	(.223, .664)	(.236, .619)	(.255, .578)	(.275, .540)	(.294, .505)	(.315, .472)	(.337, .441)	(.360, .412)	(.385, .385)

5	O	(.225, .942)	(.246, .905)	(.267, .869)	(.291, .833)	(.316, .798)	(.343, .764)	(.373, .714)	(.404, .658)	(.438, .606)	(.475, .559)	(.516, .516)
	P1	(.281, 1.00)	(.307, 1.00)	(.358, 1.00)	(.364, 1.00)	(.432, 1.00)	(.474, 1.00)	(.470, .945)	(.511, .863)	(.557, .789)	(.607, .722)	(.662, .662)
	P2	(.179, .884)	(.196, .818)	(.212, .757)	(.233, .701)	(.255, .649)	(.278, .601)	(.298, .557)	(.323, .516)	(.350, .477)	(.378, .442)	(.409, .409)
6	O	(.258, .959)	(.274, .904)	(.293, .843)	(.313, .787)	(.335, .736)	(.357, .688)	(.382, .644)	(.407, .603)	(.435, .565)	(.464, .529)	(.496, .496)
	P1	(.221, .914)	(.241, .853)	(.259, .795)	(.288, .730)	(.310, .680)	(.334, .634)	(.359, .592)	(.366, .552)	(.392, .515)	(.420, .481)	(.449, .449)
	P2	(.298, 1.00)	(.307.954)	(.327, .890)	(.347, .843)	(.373, .790)	(.399, .741)	(.428, .696)	(.449, .653)	(.478, .614)	(.509, .576)	(.542, .542)
7	O	(.243, .875)	(.260, .837)	(.278, .797)	(.296, .744)	(.316, .696)	(.337, .650)	(.359, .608)	(.382, .568)	(.406, .531)	(.433, .496)	(.463, .463)
	P1	(.258, 1.00)	(.275, 1.00)	(.293, .995)	(.312, .926)	(.332, .863)	(.353, .805)	(.375, .750)	(.397, .700)	(.422, .652)	(.529, .609)	(.568, .568)
	P2	(.229, .770)	(.245, .710)	(.262, .653)	(.281, .612)	(.300, .573)	(.321, .537)	(.342, .502)	(.365, .470)	(.390, .440)	(.360, .412)	(.385, .385)
8	O	(.231, .959)	(.247, .898)	(.263, .831)	(.280, .771)	(.300, .715)	(.323, .664)	(.347, .617)	(.372, .574)	(.400, .533)	(.430, .496)	(.462, .462)
	P1	(.233, .923)	(.248, .863)	(.263, .793)	(.279, .740)	(.317, .691)	(.329, .645)	(.352, .603)	(.377, .564)	(.403, .527)	(.431, .493)	(.461, .461)
	P2	(.229, 1.00)	(.246, .939)	(.263, .875)	(.281, .805)	(.287, .742)	(.316, .685)	(.341, .632)	(.368, .584)	(.397, .540)	(.429, .500)	(.463, .463)
9	O	(.272, .941)	(.287, .892)	(.304, .840)	(.321, .790)	(.341, .743)	(.364, .699)	(.389, .657)	(.416, .617)	(.444, .578)	(.475, .541)	(.507, .507)
	P1	(.250, .876)	(.265, .824)	(.280, .775)	(.296, .729)	(.312, .685)	(.334, .644)	(.358, .604)	(.384, .567)	(.408, .532)	(.436, .497)	(.465, .465)
	P2	(.291, 1.00)	(.308, .955)	(.326, .899)	(.344, .847)	(.369, .797)	(.393, .750)	(.422, .705)	(.452, .663)	(.478, .622)	(.511, .582)	(.545, .545)

(continued)

Table 4.2 (continued)

DMU		α = 0	α = 0.1	α = 0.2	α = 0.3	α = 0.4	α = 0.5	α = 0.6	α = 0.7	α = 0.8	α = 0.9	α = 1
10	O	(.567, 1.00)	(.608, 1.00)	(.651, 1.00)	(.698, 1.00)	(.767, 1.00)	(.842, 1.00)	(.901, 1.00)	(.937, 1.00)	(.975, 1.00)	(1.00, 1.00)	(1.00, 1.00)
	P1	(.458, 1.00)	(.495, 1.00)	(.534, 1.00)	(.680, 1.00)	(.752, 1.00)	(.830, 1.00)	(.797, 1.00)	(1.00, 1.00)	(1.00, 1.00)	(1.00, 1.00)	(1.00, 1.00)
	P2	(.681, 1.00)	(.727, 1.00)	(.775, 1.00)	(.712, 1.00)	(.779, 1.00)	(.851, 1.00)	(1.00, 1.00)	(1.00, 1.00)	(1.00, 1.00)	(1.00, 1.00)	(1.00, 1.00)
11	O	(.241, .921)	(.256, .886)	(.272, .844)	(.289, .783)	(.310, .726)	(.332, .674)	(.356, .625)	(.382, .581)	(.409, .541)	(.439, .505)	(.470, .470)
	P1	(.250, 1.00)	(.266, 1.00)	(.282, 1.00)	(.300, .938)	(.339, .871)	(.364, .810)	(.415, .754)	(.421, .655)	(.453, .607)	(.487, .564)	(.524, .524)
	P2	(.233, .852)	(.248, .793)	(.263, .725)	(.279, .666)	(.286, .616)	(.305, .570)	(.323, .527)	(.349, .522)	(.373, .488)	(.399, .456)	(.427, .427)
12	O	(.289, .952)	(.307, .921)	(.326, .891)	(.346, .861)	(.371, .831)	(.397, .793)	(.425, .737)	(.454, .685)	(.486, .639)	(.520, .597)	(.557, .557)
	P1	(.305, 1.00)	(.324, 1.00)	(.343, 1.00)	(.401, 1.00)	(.471, 1.00)	(.462, .980)	(.548, .901)	(.533, .829)	(.573, .769)	(.616, .714)	(.663, .663)
	P2	(.271, .902)	(.288, .846)	(.306, .793)	(.295, .744)	(.297, .698)	(.336, .649)	(.343, .610)	(.382, .573)	(.407, .525)	(.434, .493)	(.462, .462)
13	O	(.318, .952)	(.335, .921)	(.356, .890)	(.378, .860)	(.402, .830)	(.428, .801)	(.455, .751)	(.484, .704)	(.515, .661)	(.547, .620)	(.583, .283)
	P1	(.355, 1.00)	(.374, 1.00)	(.425, 1.00)	(.452, 1.00)	(.522, 1.00)	(.559, 1.00)	(.599, .932)	(.584, .870)	(.623, .812)	(.665, .759)	(.710, .710)
	P2	(.286, .907)	(.301, .850)	(.303, .797)	(.321, .748)	(.334, .703)	(.358, .661)	(.382, .621)	(.408, .585)	(.433, .550)	(.460, .518)	(.488, .488)

14	O	(.299, .868)	(.316, .844)	(.333, .820)	(.350, .794)	(.369, .759)	(.390, .717)	(.415, .676)	(.442, .638)	(.470, .600)	(.499, .565)	(.531, .531)
	P1	(.325, 1.00)	(.342, 1.00)	(.360, 1.00)	(.379, 1.00)	(.399, .972)	(.475, .915)	(.522, .861)	(.542, .810)	(.593, .758)	(.619, .708)	(.662, .662)
	P2	(.273, .752)	(.288, .714)	(.304, .676)	(.321, .639)	(.338, .603)	(.320, .570)	(.334, .539)	(.360, .509)	(.378, .481)	(.405, .454)	(.429, .429)
15	O	(.282, .950)	(.300, .910)	(.319, .856)	(.339, .802)	(.360, .751)	(.382, .703)	(.406, .659)	(.431, .618)	(.457, .579)	(.484, .543)	(.513, .513)
	P1	(.255, 1.00)	(.272, 1.00)	(.290, .949)	(.308, .888)	(.328, .830)	(.364, .775)	(.387, .712)	(.410, .666)	(.434, .622)	(.460, .582)	(.487, .487)
	P2	(.312, .905)	(.331, .834)	(.351, .779)	(.373, .732)	(.396, .687)	(.402, .646)	(.427, .613)	(.454, .576)	(.482, .542)	(.511, .509)	(.542, .542)
16	O	(.316, .908)	(.335, .884)	(.355, .856)	(.377, .823)	(.399, .787)	(.422, .741)	(.447, .697)	(.473, .656)	(.500, .621)	(.528, .589)	(.558, .558)
	P1	(.362, 1.00)	(.385, 1.00)	(.408, 1.00)	(.433, 1.00)	(.459, .999)	(.486, .935)	(.515, .875)	(.545, .819)	(.576, .718)	(.609, .680)	(.644, .644)
	P2	(.271, .818)	(.288, .774)	(.305, .721)	(.323, .670)	(.342, .618)	(.361, .584)	(.382, .552)	(.404, .522)	(.427, .529)	(.451, .502)	(.476, .476)

Table 4.3 α-Cuts of the paired weights (P1, P2) for Period 1 and Period 2

DMU		$\alpha = 0$	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	$\alpha = 0.4$	$\alpha = 0.5$	$\alpha = 0.6$	$\alpha = 0.7$	$\alpha = 0.8$	$\alpha = 0.9$	$\alpha = 1$
1	L	(.464, .536)	(.462, .538	(.473, .527)	(.471, .529)	(.468, .532)	(.466, .534)	(.464, .536)	(.461, .539)	(.458, .542)	(.455, .545)	(.453, .547)
	U	(.519, .481)	(.502, .498)	(.486, .514)	(.469, .531)	(.453, .547)	(.436, .564)	(.439, .561)	(.443, .557)	(.446, .554)	(.449, .551)	(.453, .547)
2	L	(.527, .473)	(.525, .475)	(.523, .477)	(.521, .479)	(.519, .481)	(.517, .483)	(.515, .485)	(.498, .502)	(.494, .506)	(.490, .510)	(.486, .514)
	U	(.500, .500)	(.486, .514)	(.472, .528)	(.458, .542)	(.448, .552)	(.453, .547)	(.458, .542)	(.463, .537)	(.477, .523)	(.482, .518)	(.486, .514)
3	L	(.511, .489)	(.511, .489)	(.511, .489)	(.511, .489)	(.511, .489)	(.511, .489)	(.511, .489)	(.484, .516)	(.484, .516)	(.483, .517)	(.483, .517)
	U	(.455, .545)	(.473, .527)	(.423, .577)	(.425, .575)	(.423, .577)	(.427, .573)	(.450, .550)	(.472, .528)	(.482, .518)	(.482, .518)	(.483, .517)
4	L	(.427, .573)	(.427, .573)	(.427, .573)	(.427, .573)	(.427, .573)	(.427, .573)	(.427, .573)	(.427, .573)	(.426, .574)	(.426, .574)	(.426, .574)
	U	(.412, .588)	(.418, .582)	(.419, .581)	(.420, .580)	(.421, .579)	(.422, .578)	(.423, .577)	(.424, .576)	(.424, .576)	(.425, .575)	(.426, .574)
5	L	(.450, .550)	(.448, .552)	(.446, .554)	(.443, .557)	(.441, .559)	(.438, .562)	(.435, .565)	(.432, .568)	(.429, .571)	(.425, .575)	(.422, .578)
	U	(.498, .502)	(.479, .521)	(.461, .539)	(.443, .557)	(.425, .575)	(.408, .592)	(.404, .596)	(.409, .591)	(.414, .586)	(.418, .582)	(.422, .578)
6	L	(.517, .483)	(.505, .495)	(.504, .496)	(.504, .496)	(.503, .497)	(.503, .497)	(.502, .498)	(.501, .499)	(.501, .499)	(.500, .500)	(.499, .501)
	U	(.483, .517)	(.489.511)	(.489, .511)	(.493, .507)	(.494, .506)	(.495, .505)	(.496, .504)	(.497, .503)	(.498, .502)	(.498, .502)	(.499, .501)
7	L	(.497, .503)	(.498, .502)	(.499, .501)	(.500, .500)	(.501, .499)	(.502, .498)	(.503, .497)	(.505, .495)	(.506, .494)	(.429, .517)	(.429, .571)
	U	(.459, .541)	(.438, .562)	(.420, .580)	(.421, .579)	(.422, .578)	(.424, .576)	(.425, .575)	(.426, .574)	(.427, .573)	(.428, .572)	(.429, .571)

8	L	(.508, .492)	(.510, .490)	(.512, .488)	(.513, .487)	(.498, .502)	(.500, .500)	(.502, .498)	(.504, .496)	(.506, .494)	(.508, .492)	(.510, .490)
	U	(.531, .469)	(.534, .466)	(.532, .468)	(.528, .472)	(.525, .475)	(.522, .478)	(.520, .480)	(.517, .483)	(.515, .485)	(.512, .488)	(.510, .490)
9	L	(.478, .522)	(.478, .522)	(.478, .522)	(.478, .522)	(.481, .519)	(.481, .519)	(.481, .519)	(.481, .519)	(.481, .519)	(.481, .519)	(.481, .519)
	U	(.476, .524)	(.480, .520)	(.480, .520)	(.480, .520)	(.480, .520)	(.480, .520)	(.481, .519)	(.481, .519)	(.481, .519)	(.481, .519)	(.481, .519)
10	L	(.513, .487)	(.514, .486)	(.514, .486)	(.435, .565)	(.435, .565)	(.435, .565)	(.489, .511)	(.475, .525)	(.462, .538)	(.456, .544)	(.441, .559)
	U	(.437, .563)	(.445, .555)	(.502, .498)	(.486, .514)	(.490, .510)	(.494, .506)	(.450, .550)	(.486, .514)	(.467, .533)	(.471, .529)	(.441, .559)
11	L	(.479, .521)	(.479, .521)	(.479, .521)	(.479, .521)	(.456, .544)	(.455, .545)	(.454, .546)	(.453, .547)	(.453, .547)	(.452, .548)	(.451, .549)
	U	(.466, .534)	(.448, .552)	(.432, .568)	(.431, .569)	(.432, .568)	(.433, .567)	(.433, .567)	(.447, .553)	(.448, .552)	(.449, .551)	(.451, .549)
12	L	(.532, .468)	(.532, .468)	(.532, .468)	(.485, .515)	(.483, .517)	(.482, .518)	(.480, .520)	(.479, .521)	(.477, .523)	(.475, .525)	(.472, .528)
	U	(.506, .494)	(.489, .511)	(.473, .527)	(.456, .544)	(.440, .560)	(.435, .565)	(.436, .564)	(.437, .563)	(.467, .533)	(.470, .530)	(.472, .528)
13	L	(.468, .532)	(.467, .533)	(.437, .563)	(.436, .564)	(.434, .566)	(.433, .567)	(.431, .569)	(.430, .570)	(.428, .572)	(.427, .573)	(.425, .575)
	U	(.489, .511)	(.474, .526)	(.459, .541)	(.443, .557)	(.428, .572)	(.414, .586)	(.416, .584)	(.418, .582)	(.421, .579)	(.423, .577)	(.425, .575)
14	L	(.511, .489)	(.511, .489)	(.510, .490)	(.510, .490)	(.509, .491)	(.452, .548)	(.449, .551)	(.447, .553)	(.444, .556)	(.441, .559)	(.438, .562)
	U	(.468, .532)	(.456, .544)	(.443, .557)	(.430, .570)	(.423, .577)	(.425, .575)	(.427, .573)	(.428, .572)	(.431, .569)	(.434, .566)	(.438, .562)

(continued)

Table 4.3 (continued)

DMU		$\alpha = 0$	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	$\alpha = 0.4$	$\alpha = 0.5$	$\alpha = 0.6$	$\alpha = 0.7$	$\alpha = 0.8$	$\alpha = 0.9$	$\alpha = 1$
15	L	(.521, .479)	(.522, .478)	(.522, .478)	(.523, .477)	(.524, .476)	(.524, .476)	(.525, .475)	(.526, .474)	(.526, .474)	(.527, .473)	(.528, .472)
	U	(.479, .521)	(.458, .542)	(.449, .551)	(.448, .552)	(.447, .553)	(.447, .553)	(.465, .535)	(.466, .534)	(.467, .533)	(.468, .532)	(.528, .472)
16	L	(.490, .510)	(.490, .510)	(.490, .510)	(.489, .511)	(.489, .511)	(.489, .511)	(.489, .511)	(.489, .511)	(.488, .512)	(.488, .512)	(.488, .512)
	U	(.494, .506)	(.486, .514)	(.482, .518)	(.464, .536)	(.444, .556)	(.447, .553)	(.449, .551)	(.451, .549)	(.487, .513)	(.488, .512)	(.488, .512)

4.5 Conclusion

In measuring the efficiencies of DMUs, when the time span for efficiency measurement covers multiple periods, the average data of all periods is often used to get the glance of performance. However, one shortcoming of this aggregate method is that it ignores the operations of individual periods. As a result, many inefficient DMUs will be evaluated as efficient. Additionally, an evaluation process usually comprises complicated inputs and outputs, where some factors cannot be precisely measured. We can represent the imprecise data as fuzzy numbers. Traditionally, DEA allows the flexibility in the determination of the weights (multipliers) on inputs and outputs when assessing the relative efficiency of a DMU. One can restrict the weight flexibility to derive more realistic measures of efficiency.

In this paper we develop a methodology to find the fuzzy efficiency measures of a fuzzy network DEA model with weight restrictions when the observations are fuzzy numbers. Different from the conventional network model, which treats each process independently, this paper uses a relational approach to link individual periods together. By viewing the multi-period system as a parallel network one, the operation of each period can then be taken into account in evaluating the overall efficiency. Since the assurance region approach is comprehensive to bounding the multipliers, this approach is adopted to impose weight restrictions on the model.

When input and output data are fuzzy numbers, the derived efficiencies become fuzzy as well. We develop a pair of two-level mathematical programs to calculate the lower and upper bounds of the α-cut of the fuzzy efficiency. The program for calculating the upper bound efficiency is transformed into a conventional one-level linear program such that optimal solutions could be obtained easily. On the other hand, the program for calculating the lower bound efficiency is transformed into a one-level nonlinear program. Since the nonlinearity of the program is not strong, optimal solutions can be obtained by most nonlinear programming solvers. The relational network model proposed in this paper is able to measure the fuzzy overall and period efficiencies at the same time, and the fuzzy overall efficiency is a weighted average of the fuzzy period efficiencies. This relationship always exists for the lower and upper bound efficiencies of every DMU at all α values.

Fuzzy efficiency measures are more informative than crisp ones because they provide not only the most likely values, but also the range that all possible values can appear within. This prevents the decision maker from being over-confident with results that are making inappropriate decisions. With the ability to calculate fuzzy period efficiencies and to find the most favorable weights to calculate the fuzzy overall efficiency, the idea proposed in this work should be more convincible for measuring fuzzy multi-period efficiencies.

Acknowledgment This research was supported by the Ministry of Science and Technology of the Republic of China (Taiwan), under grant MOST 103-2410-H-238-005.

References

Asmild, M., Paradi, J. C., Reese, D. N., & Tam, F. (2007). Measuring overall efficiency and effectiveness using DEA. *European Journal of Operational Research, 178*, 305–321.

Bal, H., Örckü, H. H., & Çelebioğlu, S. (2008). A new method based on the dispersion of weights in data envelopment analysis. *Computers & Industrial Engineering, 54*, 502–512.

Bellman, R., & Zadeh, L. A. (1970). Decision making in a fuzzy environment. *Management Science, 17B*, 141–164.

Charnes, A., & Cooper, W. W. (1984). The non-Archimedean CCR ratio for efficiency analysis: A rejoinder to Boyd and Färe. *European Journal of Operational Research, 15*, 333–334.

Charnes, A., Cooper, W. W., Huang, Z. M., & Sun, D. B. (1990). Polyhedral cone-ratio models with an illustrative application to large commercial banks. *Journal of Econometrics, 46*, 73–91.

Charnes, A., Cooper, W. W., & Rhodes, E. (1978). Measuring the efficiency of decision making units. *European Journal of Operational Research, 2*, 429–444.

Cook, W. D., & Seiford, L. M. (2009). Data envelopment analysis (DEA)—Thirty years on. *European Journal of Operational Research, 192*, 1–17.

Dantzig, G. B. (1963). *Linear programming and extensions*. Princeton: Princeton University Press.

Emrouznejad, A., & Tavana, M. (2014). *Performance measurement with fuzzy data envelopment analysis*. Heidelberg: Springer.

Guo, P. J. (2009). Fuzzy data envelopment analysis and its application to location problems. *Information Sciences, 179*, 820–829.

Guo, P., Tanaka, H., & Inuiguchi, M. (2000). Self-organizing fuzzy aggregation models to rank the objects with multiple attributes. *IEEE Transactions on Systems, Man and Cybernetics, Part A, 30*, 573–580.

Hatami-Marbini, A., Emrouznejad, A., & Tavana, M. (2011). A taxonomy and review of the fuzzy data envelopment analysis literature: Two decades in the making. *European Journal of Operational Research, 214*, 457–472.

Kao, C. (2009). Efficiency measurement for parallel production systems. *European Journal of Operational Research, 196*, 1107–1112.

Kao, C., & Hwang, S. N. (2008). Efficiency decomposition in two-stage data envelopment analysis: An application to non-life insurance companies in Taiwan. *European Journal of Operational Research, 185*, 418–429.

Kao, C., & Lin, P. H. (2012). Efficiency of parallel production systems with fuzzy data. *Fuzzy Sets and Systems, 198*, 83–98.

Kao, C., & Liu, S. T. (2000a). Fuzzy efficiency measures in data envelopment analysis. *Fuzzy Sets and Systems, 113*, 427–437.

Kao, C., & Liu, S. T. (2000b). Data envelopment analysis with missing data: An application to university libraries in Taiwan. *Journal of the Operational Research Society, 51*, 897–905.

Kao, C., & Liu, S. T. (2004). Predicting bank performance with financial forecasts: A case of Taiwan commercial banks. *Journal of Banking and Finance, 28*, 2353–2368.

Kao, C., & Liu, S. T. (2011). Efficiencies of two-stage systems with fuzzy data. *Fuzzy Sets and Systems, 176*, 20–35.

Kao, C., & Liu, S. T. (2014). Multi-period efficiency measurement in data envelopment analysis: The case of Taiwanese commercial banks. *Omega, 47*, 90–98.

Korhonen, P. J., Soleimani-damaneh, M., & Wallenius, J. (2011). Ratio-based RTS determination in weight-restricted DEA models. *European Journal of Operational Research, 215*, 431–438.

Lertworasirkul, S., Fang, S. C., Joines, J. A., & Nuttle, H. L. W. (2003). Fuzzy data envelopment analysis (DEA): A possibility approach. *Fuzzy Sets and Systems, 139*, 379–394.

Liu, S. T. (2008). A fuzzy DEA/AR approach to the selection of flexible manufacturing systems. *Computers & Industrial Engineering, 54*, 66–76.

Liu, S. T. (2014). Restricting weight flexibility in fuzzy two-stage DEA. *Computers & Industrial Engineering, 54*, 66–76.

Lovell, C. A. K., & Pastor, J. T. (1995). Units invariant and translation invariant DEA models. *Operations Research Letters, 18*, 147–151.

Lozano, S. (2014a). Computing fuzzy process efficiency in parallel systems. *Fuzzy Optimization and Decision Making, 13*, 73–89.

Lozano, S. (2014b). Process efficiency of two-stage systems with fuzzy data. *Fuzzy Sets and Systems, 243*, 36–49.

Nguyen, H. T. (1978). A note on the extension principle for fuzzy sets. *Journal of Mathematical Analysis and Applications, 64*, 369–380.

Ramón, N., Ruiz, J. L., & Sirvent, I. (2010). A multiplier bound approach to assess relative efficiency in DEA without slacks. *European Journal of Operational Research, 203*, 261–269.

Saati, S., & Memariani, A. (2005). Reducing weight flexibility in fuzzy DEA. *Applied Mathematics and Computation, 161*, 611–622.

Shokouhi, A. H., Hatami-Marbini, A., Tavana, M., & Satti, S. (2010). A robust optimization approach for imprecise data envelopment analysis. *Computers & Industrial Engineering, 59*, 387–397.

Thompson, R. G., Langemeier, L. N., Lee, C. T., & Thrall, R. M. (1990). The role of multiplier bounds in efficiency analysis with application to Kansas farming. *Journal of Econometrics, 46*, 93–108.

Thompson, R. G., Singleton, F. D., Thrall, R. M., & Smith, B. A. (1986). Comparative site evaluations for locating high energy lab in Texas. *Interfaces, 16*, 1380–1395.

Zadeh, L. A. (1978). Fuzzy sets as a basis for a theory of possibility. *Fuzzy Sets and Systems, 1*, 3–28.

Zimmermann, H. Z. (1996). *Fuzzy set theory and its applications* (3rd ed.). Boston: Kluwer-Nijhoff.

Chapter 5
Pitching DEA Against SFA in the Context of Chinese Domestic Versus Foreign Banks

Necmi Kemal Avkiran and Yushu (Elizabeth) Zhu

Abstract The primary motivation is to show how the efficient frontier methods data envelopment analysis (DEA) and stochastic frontier analysis (SFA) can be used synergistically. As part of the illustration, we directly compare locally incorporated foreign banks with Chinese domestic banks. Both DEA and SFA reveal that foreign banks are *less* efficient. DEA shows the main source of inefficiency for foreign banks as managing *interest income*, whereas domestic banks are inefficient in managing *non-interest income* and *interest expense*. SFA reveals contextual variables such as interbank ratio, loan-to-deposit ratio and cost-to-income ratio are significant in explaining inefficiency. The correspondence of rankings based on DEA vs. SFA is positive and moderate in strength but efficiency estimates do not belong to the same distribution. Using DEA and SFA side-by-side can encourage more rigorous and in-depth bank efficiency studies where each method's limitation can be overcome by the other.

Keywords Technical efficiency • Scale efficiency • Data Envelopment Analysis • Stochastic frontier analysis • Single-output Translog function • Multi-output Translog distance function • Cobb-Douglas function • Robustness testing • Chinese banks • Efficiency spillovers • Profitability • Potential improvements • Efficiency contribution measure

5.1 Introduction

The *primary motivation* of this chapter is to compare and contrast the well-established efficient frontier methods data envelopment analysis (DEA) and stochastic frontier analysis (SFA) in generating efficiency estimates. In efficient frontier literature on banking, the choice between DEA and SFA is often based on authors' preferences and the complementary nature of these methods makes a final compelling argument in favor of one or the other difficult. We set out to

N.K. Avkiran (✉) • Y. Zhu
UQ Business School, The University of Queensland, Brisbane 4072, QLD, Australia
e-mail: n.avkiran@business.uq.edu.au; e.zhu@business.uq.edu.au

S.-N. Hwang et al. (eds.), *Handbook of Operations Analytics Using Data Envelopment Analysis*, International Series in Operations Research & Management Science 239, DOI 10.1007/978-1-4899-7705-2_5

explore whether DEA and SFA can be used in a synergistic manner to allay various research design concerns such as potential distortion of results by measurement error or mis-specification of assumed functional relationships. The research design includes various robustness tests such as sensitivity of results to majority state-owned large banks, and stability of results to modelled outputs and functional specification.

Briefly, DEA is a peer benchmarking method used in comparing performance of organizations of similar operations and identifying relative inefficiencies that may detract from performance. As a non-parametric efficient frontier method, DEA calculates a comparative ratio of weighted outputs to weighted inputs that defines performance—reported as a relative efficiency estimate. On the other hand, SFA is a parametric efficiency measurement method that explains the variation in organizational performance in terms of managerial efficiency, operating environment and statistical noise. SFA efficiency estimates are based on parameter values of regression. In Sect. 5.3, further details on DEA and SFA and formal definitions are provided, including a discussion of how firm-specific factors (i.e. contextual variables) can be used to explain inefficiency.

The primary motivation of this chapter is executed in the context of how foreign banks in China perform when compared against domestic banks as well as each other in the post-2007 period. Between 2002 and 2006, China further opened up its domestic financial markets to foreign financial institutions through various reforms that expanded the scope of business in foreign exchange and renminbi (RMB). Business engaged in by domestic and locally incorporated foreign banks (hereon, foreign banks) include such activities as receiving deposits from the general public; granting loans of short, medium or long term; handling negotiable instruments; trading bonds; issuing letters of credit and guarantees; handling domestic and foreign settlements; issuing bank cards; interbank lending, etc., all effective as of 11 December 2006.[1] That is, foreign banks are allowed to conduct the same types of RMB business as their domestic counterparts and have essentially been granted equal status as of December 2006 (Xu 2011). Consequently, as of 2007, foreign banks have been in competition with domestic banks, and these two cohorts can be analyzed together in benchmarking studies to enable a more direct comparison. Recent examples of applications of DEA to Chinese banking data include Chen et al. (2005), Ariff and Can (2008), Hu et al. (2008), Yao et al. (2008), Avkiran and Morita (2010) and Avkiran (2011). Others who have used SFA instead include Fu and Heffernan (2007) and Jiang et al. (2009). Luo et al. (2011) use DEA as well as SFA in a study of Chinese domestic banks only.

[1] See ‘Regulations of the People’s Republic of China on Administration of Foreign-funded Banks’ (CBRC 2006). The same regulations also apply to the banking institutions established on Chinese mainland by financial institutions originating from the Hong Kong Special Administrative Region, the Macao Special Administrative Region, or Taiwan. For example, in our sample, Hang Seng bank (China) Ltd, and CITIC Ka Wah Bank (China) Ltd with home groups from the Hong Kong Special Administrative Region are treated as foreign banks rather than Chinese domestic banks (see Article 72).

Key findings of this study for the period 2008–2010 show foreign banks to be generally less efficient compared to domestic banks based on DEA as well as SFA. An examination of the sources of inefficiency reveals management of *interest income* as an area in need of closer examination by foreign banks. On the other hand, domestic bank operations appear to be primarily inefficient in managing *non-interest income* and *interest expense*. Other findings suggest that contextual variables such as interbank ratio, loan-to-deposit ratio and cost-to-income ratio are significant in explaining inefficiency. However, significance tests show that efficiency estimates from these different methods do not belong to the same distribution. Furthermore, lower SFA efficiency estimates are better in separating the more efficient domestic banks from foreign banks. Under SFA, single-output Translog functional form emerges as a better specification compared to the Cobb-Douglas specification or the two-output Translog distance function. Overall, intuitive findings from bank performance analysis pave the way for use of DEA and SFA side-by-side without the researcher having to justify one method at the expense of the other. We expect such an inclusive approach to bring stronger rigor to applications of frontier methods in banking and encourage more in-depth studies.

The rest of the chapter is organized as follows. Section 5.2 begins by briefly discussing the Chinese banking sector. It continues to further discuss efficiency spillovers that bring foreign and domestic banks closer and details the performance models used for estimating bank efficiency including firm-specific factors. Section 5.3 describes the data, followed by a discussion of DEA and SFA methods that includes formulations. Section 5.4 reports results and analyses based on DEA and SFA and corresponding robustness tests, ending with a comparison of DEA versus SFA efficiency estimates. Section 5.5 concludes the chapter with a summary of main findings and managerial implications.

5.2 Conceptual Framework

5.2.1 Chinese Banking Sector

The Chinese banking sector has been offering a wider range of products and services as a result of the ongoing deregulation which gained momentum since China joined the World Trade Organization in December 2001. Main examples of successful listings among the Chinese domestic banks include the Agricultural Bank of China Ltd., Bank of China Ltd., Bank of Communications Ltd., China Construction Bank Corp., and Industrial and Commercial Bank of China Ltd. These majority state-owned commercial banks keep a large branch network throughout China, and thus, hold a greater share of the retail banking market. Other domestic banks include joint-stock commercial banks with minority state or government ownership, city commercial banks, rural commercial banks, and wholly state-owned banks known as policy banks. The China Banking Regulatory Commission (CBRC)—established in 2003—is the main entity responsible for monitoring implementation of regulations and reforms, and the People's Bank of China is the central bank. A more extensive historical background to the development of the Chinese banking sector can be read in Berger et al. (2009) and in Asmild and Matthews (2012).

Foreign banks in China have a history of slow entry—representative offices being allowed for the first time in 1979—followed by branches a few years later in special economic zones. It was not until 1996 that foreign banks—under individual licenses—were permitted to engage in business with local enterprises by accepting deposits and writing loans in renminbi. Lin (2011) maintains that the predominant form of foreign bank entry into China is *green field* investment where new branches are established from ground up, rather than the *brown field* approach that requires taking over or building on an existing branch. Green field investments are likely to be more expensive because such an exercise would include recruiting and training staff while working on building reputation. Furthermore, such costs would have to be allocated across multiple periods, and at least during the initial years of operation, cost control is likely to be treated as of secondary importance because the focus would be on expanding market share.

The basic motivation of policy makers and regulators for encouraging foreign bank presence revolves around anticipated enhancement of structure and competitive efficiency of a country's banking system. For example, foreign banks are often credited with contributing to improvement of domestic banking through efficiency spillovers. Spillovers may take the form of emulation of innovative products and services of foreign banks by domestic banks as seen in personal banking, and relocation of talent from foreign to domestic banks (see Deng et al. 2011 and Xu 2011). *Such spillovers bring foreign and domestic bank operations closer*, *thus enabling benchmarking against a common frontier*. Nevertheless, foreign banks are still in a stage of growth as they open more branches and employ more people. According to PricewaterhouseCoopers (2012, p. 21) "They are yet to benefit from increases in operational efficiency and economies of scale". It is this expectation of substantial differences in performance that further encourages this chapter to pitch DEA against SFA in the context of measuring the operational or technical efficiency of foreign and domestic banks.

The initial anticipated finding based on the comment by PricewaterhouseCoopers (2012) is more efficient domestic banks for the period 2008–2010, which can be explained by the progress made by the domestic banks since the early days of foreign ownership (see preceding discussion on efficiency spillovers). Yet, an earlier study by Berger et al. (2009) based on SFA efficiency estimates of thirty-eight Chinese banks across 1994–2003 state that, on average, in developing nations foreign banks are usually *more efficient* than or at least as efficient as private domestic banks, and more efficient than state-owned banks. In contrast, Lensink et al. (2008) who use SFA on a much larger sample of 2095 banks across 105 countries (1998–2003) report a general finding of *less efficient* foreign banks. Therefore, in the presence of these potentially conflicting findings, we compare and contrast our chosen non-parametric and parametric methods with a view to using one method as a robustness test for the other.

5.2.2 Modeling Performance to Estimate Bank Efficiency

There is no consensus on how to model bank performance, particularly in the context of evaluating technical efficiency. A recent study of major DEA applications in banking literature in top journals across 2004–2009 concludes, ". . .there is no clear agreement amongst the selection of inputs and outputs beyond the general observance of the intermediation approach to bank behavior" (Avkiran 2011, p. 326). The traditional intermediation executed by banks as part of their regular operations include incurring *interest expense* and *non-interest expense* to generate deposits (bank liabilities) and writing loans (bank assets) to generate *interest income*, as well as generating *non-interest income* from service fees and sales commissions. Hence, in this performance benchmarking exercise where we pitch DEA against SFA, the objective of banks is considered as implementing this intermediation process efficiently in order to operate profitably. Since we are looking at two main expense categories and two main revenue categories as the potential key variables, we are in fact proposing to measure profitability when we treat them as inputs and outputs, respectively.

One of the basic operations of banks is to make profits by selling liabilities with one set of features (e.g. liquidity, risk, size and return) and using the proceeds to buy assets with a different set of features. For example, term deposit accounts (liabilities) held in the name of a number of individuals can provide the underlying funds needed to write a mortgage loan (asset). In fact, there is no need to look at different types of assets and liabilities and sacrifice discrimination unless the purpose is to comment on specific products/services, and the researcher has a very large sample. Therefore, the performance modeling in this study begins with a parsimonious set of two discretionary key inputs and one output (where we collapse interest income and non-interest income into *total income*) designed to generate a technical efficiency estimate for each bank. In the second stage, we model all four key variables without aggregation and note whether findings on comparing DEA and SFA are still similar when dimensionality rises. Yao et al. (2008), Jiang et al. (2009) and Avkiran (2011) use similar variables involving Chinese banks. Others who have also used these variables with banks from other countries include Miller and Noulas (1996), Bhattacharyya et al. (1997), Brockett et al. (1997), Leightner and Lovell (1998), and Sturm and Williams (2004).

5.2.3 Contextual Variables

According to Banker and Natarajan (2008), OLS or Tobit regression can be used in order to understand the impact of various factors or contextual variables on DEA efficiency estimates. On the other hand, McDonald (2009) concludes that while Tobit may not be appropriate in this context, OLS is a consistent estimator when used in second stage DEA efficiency analyses (see Greene 2012 regarding Tobit regression and the further discussion at the end of Sect. 5.3.2 of this chapter). In SFA, firm-specific factors or contextual variables are incorporated into the regression equation. For example, we can explore the relationship between efficiency and a selection of key traditional financial performance ratios. Potential candidates include cost-to-income as an overall efficiency ratio used by industry analysts; impaired loans-to-gross loans (or, non-performing loans ratio, NPL) as a measure of credit or asset quality; and interbank ratio as a measure of liquidity (ratio of due from banks to due to banks).[2]

Historically, domestic banks have shown limited appetite for efficient operations or lending purely based on risk-return analysis because of their closer ties with governments. For example, in the past many politically directed lending decisions have contributed to high non-performing loans, although such practices may gradually be in decline—at least as evidenced by substantially lower non-performing loans (e.g., according to the China Banking Regulatory Commission, in 2005 the NPL ratio was 4.2 %, whereas by 2009—midway through this study—it had fallen to 1.58 %).[3] Similarly, because of domestic banks' larger branch networks and more captive customer base—where workers' wages are deposited—such banks have a larger deposits base although this does not necessarily imply a larger interbank ratio if lending to other banks is limited.

Another financial ratio of potential interest is the loan-to-deposit. This ratio can also be used as a firm-specific factor to acknowledge the impact of regulation on efficiency. For example, the loan-to-deposit ratio is decreed not to exceed 75 % for all banks operating in China, yet the foreign banks appear to be handicapped by a smaller branch network in raising deposits, with flow-on limitations on lending (the grace period for meeting the 75 % threshold ended in December 2011). Another related confounding factor is the practice by the regulators of accepting only one branch application at a time. All else the same, these conditions are likely to make efficient revenue generation more difficult because lower deposit raising capacity is expected to limit revenue generation from traditional lending activities. Thus, this study also investigates whether regulation of the loan-to-deposit ratio is likely to have an impact on the efficiency estimates. Summing up, we explore to what extent

---

[2] The interbank ratio is the ratio of funds lent to other banks divided by funds borrowed from other banks. A ratio greater than 1 indicates that the bank is a net lender in the interbank market and is therefore more liquid.

[3] http://www.cbrc.gov.cn/EngdocView.do?docID=B22DBFC5175C4AC0AC7926AD7AFEEE27.

a small selection of key traditional financial ratios (firm-specific factors or contextual variables) are likely to play a significant role in explaining inefficiency.

5.3 Data and Method

5.3.1 Data

This study spans 2008–2010 in an effort to measure the performance of banks in China against their peers and excludes the three wholly state-owned policy banks the Agricultural Development Bank of China, the China Development Bank and the China Exim Bank. Remaining commercial banks with varying degrees of state ownership are included based on panel data availability across the variables of interest. Essentially, 2008 marks the first reporting period that captures the operations for foreign banks when they are considered as offering a range of products and services similar to domestic banks (data were collected in late 2012 and early 2013 but data for 2011 were mostly unavailable). The 3-year study period is also appropriate for the common efficient frontier constructed with the pooled data. The primary data source was Wharton's Research Data Services.

After accounting for missing data, we were left with 16 foreign banks and 37 domestic banks that consistently had data across all the variables for the 3-year study period (see Table 5.1). The sample represents about 75 % of the market as measured by bank assets. We were also able to collect data for this sample for the firm-specific factors of cost-to-income ratio, impaired loans-to-gross loans, interbank ratio and loan-to-deposit ratio. Overall, the data collection effort produces a sample of 159 bank-year observations in a balanced panel data set, and enables setting up an efficient frontier common across 3 years. In this sample, four of the Big Six foreign banks and eight countries and domestic commercial banks are well represented (see Table 5.1).

Descriptive statistics and correlations between performance variables and firm-specific factors are shown in Table 5.2. All of the firm-specific factors are correlated at low levels with the performance variables, and all of the NPL and interbank ratio correlations are statistically insignificant. The extensive testing in Banker and Natarajan (2008, p. 56) demonstrates that two-stage methods become unreliable in explaining the impact of contextual variables (i.e. firm-specific factors) when such variables are highly correlated with performance variables; the correlations in the second half of Table 5.2 are all low and mostly insignificant.

Once the foreign and domestic banks are benchmarked against the common frontier, it is easier to compare how these different cohorts perform against each other. This approach is appropriate as long as the panel data do not cover too many years because it assumes no substantial changes in the production technology during the study period. Various applications of the common frontier in banking can be found in Dietsch and Lozano-Vivas (2000), Hasan and Marton (2003),

Table 5.1 Banks in the study (53 banks, or 159 bank-years across 2008–2010)

	Sorted by home country	
Foreign banks in China (N = 16)	Crédit Agricole CIB (China)	France
	Société Générale (China)	France
	CITIC Ka Wah Bank (China)[a]	Hong Kong
	Hang Seng Bank (China)[b]	Hong Kong
	Nanyang Commercial Bank (China)	Hong Kong
	Bank of Tokyo Mitsubishi UFJ (China)	Japan
	Mizuho Corporate Bank (China)	Japan
	Hana Bank (China)	Korea
	Woori Bank (China)	Korea
	Bank International Ningbo	Singapore
	United Overseas Bank (China)	Singapore
	Fubon Bank (Hong Kong)	Taiwan
	HSBC Bank (China)[b]	United Kingdom
	Royal Bank of Scotland (China)	United Kingdom
	Standard Chartered Bank (China)[b]	United Kingdom
	Citibank (China)[b]	United States of America
Chinese domestic banks (N = 37)	*Sorted alphabetically*	
	Agricultural Bank of China	China Merchants Bank
	Bank of Beijing	China Minsheng Banking
	Bank of China	China Zheshang Bank
	Bank of Communications	Chong Hing Bank
	Bank of Dongguan	Fudian Bank
	Bank of Fuxin	Fujian Haixia Bank
	Bank of Guangzhou	Guangzhou Rural Commercial Bank
	Bank of Hangzhou	Hankou Bank
	Bank of Jilin	Harbin Bank
	Bank of Nanjing	Huaxia Bank
	Bank of Ningbo	Huishang Bank
	Bank of Qingdao	Industrial and Commercial Bank of China
	Bank of Shanghai	Industrial Bank
	Bank of Wenzhou	Nanchong City Commercial Bank
	Beijing Rural Commercial Bank	Shanghai Pudong Development Bank
	China CITIC Bank	Shanghai Rural Commercial Bank
	China Construction Bank	Shengjing Bank
	China Everbright Bank	Shenzhen Development Bank[c]
	China Guangfa Bank	

[a]This bank's new name is CITIC Bank International (China) Ltd.
[b]Belongs to the group of Big Six foreign banks
[c]This bank's new name is Ping An Bank Co Ltd.

Table 5.2 Descriptive statistics on performance data adjusted for GDPD and firm-specific factors (N = 159)[a]

	Mean	Median	SD[b]	CV[c]	Minimum	Maximum	Skewness
Interest expense[d]	2184.12	257.75	4914.24	2.25	0.28	23,581.23	3.00
Non-interest expense	1617.06	203.77	3806.12	2.35	5.16	16,422.91	3.00
Interest income	5689.89	669.63	13,237.00	2.33	4.73	65,203.36	3.07
Non-interest income	821.65	64.10	2203.35	2.68	0.00	10,348.24	3.32
Total income	6511.53	770.21	15,369.30	2.36	8.65	75,497.37	3.07
Impaired loans-to-gross loans (or, NPL)[e]	1.77	1.02	4.45	2.52	0.05	38.22	7.38
Interbank ratio	350.84	191.09	671.87	1.92	15.16	7280.19	7.69
Loan-to-deposit ratio	65.62	62.63	20.70	0.32	25.91	209.82	3.21
Cost-to-income ratio	49.26	41.41	30.65	0.62	23.06	350.83	6.70
Correlation matrix	Interest expense	Non-interest expense	Interest Income	Non-interest income	Total income[f]		
Impaired loans-to-gross loans (or, NPL)[e]	−0.013 (0.869)	−0.012 (0.876)	−0.014 (0.862)	−0.012 (0.885)	−0.014 (0.864)		
Interbank ratio	−0.058 (0.464)	−0.046 (0.561)	−0.052 (0.514)	−0.045 (0.575)	−0.051 (0.521)		
Loan-to-deposit ratio	−0.182 (0.022)	−0.172 (0.030)	−0.177 (0.026)	−0.161 (0.043)	−0.175 (0.027)		
Cost-to-income ratio	−0.180 (0.023)	−0.164 (0.038)	−0.177 (0.026)	−0.154 (0.053)	−0.174 (0.028)		

[a]The Gross Domestic Product Deflator (GDPD) used in this study is considered technically more accurate than the Consumer Price Index (CPI). GDPD is not based on a fixed basket of goods and services; that is, the basket is allowed to change with people's consumption and investment patterns. Thus, new expenditure patterns are allowed to show up in the deflator as people respond to changing prices. Annual GDPD for the study years were: 7.8 % (2008), −0.6 % (2009) and 6.7 % (2010) Data were not winsorized because of the relatively small sample of banks with complete data on variables of interest. For example, the minimum of 0.00 reported for non-interest income is actually .000001 and belongs to Bank of Guangzhou in 2009. Similarly, the maximum of 7280.19 reported for interbank ratio belongs to Royal Bank of Scotland (China) in 2010

[b]*SD* standard deviation

[c]*CV* coefficient of variation (SD/Mean)

[d]All the financial items are in USD million

[e]Significance levels are in brackets

[f]Sum of interest income and non-interest income

Avkiran (2009) and Chortareas et al. (2013). Next, we proceed to outline the principles of DEA and SFA—the two efficient frontier methods at the heart of this study—where the primary motivation of the chapter calls for close attention to designing tests in a comparable manner.

5.3.2 Data Envelopment Analysis (DEA)

Before providing a formal definition of the DEA model used, we begin with an intuitive introduction to this non-parametric method. DEA informs the user whether performance can be improved relative to observed benchmark performance in a peer group. Under standard DEA, the relative efficiency estimate (a scalar value) is expressed as a number between 0 and 1, where a decision-making unit (DMU) with an estimate of less than 1 is considered inefficient. Benchmark units on the efficient frontier determine the potential improvements or projections for the various inefficient units not on the frontier. DEA follows the condition of Pareto optimality for efficient operations, where a DMU or a production unit is not efficient if an output can be raised without raising any of the inputs and without lowering any other output. Similarly, a DMU is not efficient if an input can be lowered without decreasing any of the outputs and without increasing any other input (Charnes et al. 1981).

Key strengths of DEA include the property that no preconceived functional structure is imposed on the data in determining the efficient units. That is, DEA does not assume a particular production technology common to all DMUs. This means a unit's efficiency can be assessed based on other observed performance by benchmarking similar organizations that are better at executing various processes. As an efficient frontier method, DEA identifies the inefficiency in a particular DMU by comparing it to efficient DMUs, rather than trying to associate a DMU's performance with statistical averages that may not be applicable to that DMU. Another strength of DEA is its ability to handle related multiple inputs and multiple outputs in producing a scalar estimate. That is, the optimization process embedded in the linear program behind DEA accounts for the trade-off between multiple variables before reporting a single efficiency estimate for a unit. As Gelade and Gilbert (2003) underline, individual ratios looking at different aspects of an organization's effectiveness cannot depict a full picture because ratios are unlikely to be independent. Alongside the various strengths already mentioned, standard DEA's main limitation is the assumption that data are free of measurement error, thus making DEA more sensitive than stochastic methods to the presence of measurement error. That is, DEA is often considered deterministic where the method assumes random variations cancel out one another (for an opposite argument where DEA is set up as a stochastic frontier estimation method, see Banker and Natarajan 2008).

Historically, DEA literature has been dominated by radial models that can be traced to publication of the seminal article, Charnes et al. (1978). In this study, we use the output-oriented, variable returns-to-scale version of radial DEA (often abbreviated as BCC after Banker et al. 1984). Output-orientation is used because we are primarily interested in identifying overall revenue generating inefficiencies, i.e. measuring to what extent banks are maximizing their revenues for given levels of expenses. Next we briefly provide a formal definition of radial DEA (Coelli et al. 2005, Cooper et al. 2007, 2011 provide authoritative expositions of DEA with extensive detail).

Efficiency can be defined as the ratio of weighted sum of outputs to weighted sum of inputs. Efficiency of a DMU, h_o, assuming controllable inputs and constant returns-to-scale, can thus be written as

$$h_o = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}} \tag{5.1}$$

where s = number of outputs
u_r = weight of output r
y_{ro} = amount of output r produced by the observed DMU
m = number of inputs
v_i = weight of input i
x_{io} = amount of input i used by the observed DMU

While outputs and inputs can be measured and entered in this equation without standardization, determining a common set of weights can be problematic. DMUs may well value outputs and inputs quite differently. This potential problem was addressed through optimization in the CCR model by Charnes et al. (1978) by allowing a DMU to adopt a set of weights that will maximize its efficiency ratio without the same ratio for other DMUs exceeding 1. Introduction of this constraint converts the productivity ratio into a measure of *relative* efficiency. Thus, we re-write (5.1) in the form of a fractional programming problem:

$$\begin{aligned} &\max \ h_o = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}} \\ &\text{subject to} \\ &\qquad \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1 \ \text{ for each DMU in the sample} \\ &\text{where } \mathrm{j} = 1, \ldots, \mathrm{n} \ \text{(number of DMUs).} \end{aligned} \tag{5.2}$$

Equation (5.2) represents the *ratio form* DEA. However, (5.2) has an infinite number of solutions. To avoid this problem, we convert (5.2) to the more familiar components of a linear programming problem. In (5.3), known as the *multiplier form*, we set the denominator to a constant and maximize the numerator.

$$\begin{aligned} \max \quad & h_o = \sum_{r=1}^{s} u_r y_{ro} \\ \text{subject to} \quad & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \\ & \sum_{i=1}^{m} v_i x_{io} = 1 \\ & u_r, v_i \geq \varepsilon \succ 0 \end{aligned} \tag{5.3}$$

In order to prevent an output or an input being mathematically omitted in calculation of efficiency, the smallest values weights u and v are permitted to have are non-zero small positive numbers (ε). Equation (5.3) represents constant returns-to-scale with controllable inputs. It is a primal linear programming problem that models *input contraction* (i.e. input-oriented). The output-oriented CCR model is represented by (5.4):

$$\begin{aligned} \min \quad & h_o = \sum_{i=1}^{m} v_i x_{io} \\ \text{subject to} \quad & \sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} \geq 0 \\ & \sum_{r=1}^{s} u_r y_{ro} = 1 \\ & u_r, v_i \geq \varepsilon \succ 0 \end{aligned} \tag{5.4}$$

The BCC model used in this study to measure pure technical efficiency is derived by introducing a convexity constraint $\sum_{j=1}^{n} \lambda_j = 1$ into (5.4), thus ensuring that an inefficient DMU is benchmarked against DMUs of similar size.

The radial models defined above generate bounded efficiency estimates. As such, Tobit regression of firm-specific factors on DEA efficiency estimates can be regarded appropriate in explaining their impact because estimates are bounded or censored (Grosskopf 1996). However, given the doubts raised by McDonald (2009) about using Tobit in second stage DEA efficiency analyses, we focus on OLS regression and compare findings to Tobit regression. According to Banker and Natarajan (2008), particularly when there is no direct production correspondence between inputs and outputs, DEA may have an advantage over parametric methods where efficiency estimates are generated in the first stage and inefficiencies are explained in the second stage by introducing contextual variables via regression (see Simar and Wilson 2011 for a *caveat emptor* on two-stage DEA).

5.3.3 Stochastic Frontier Analysis (SFA)

Aigner et al. (1977) and Meeusen and van den Broeck (1977) devised Stochastic Frontier Analysis (SFA) independently, and SFA is often regarded as the parametric equivalent of DEA. SFA is a type of regression in which the asymmetric (non-negative) managerial inefficiency effects can be separated from the symmetric error term component, i.e. statistical noise. Examples of statistical noise include errors in measuring variables in the model, or omitted variables; instances of managerial inefficiency include inadequately trained personnel.

We consider the well-established Cobb-Douglas and Translog (Transcendental Logarithmic) functions. Translog is a generalization of the Cobb-Douglas function and includes second order input terms; Translog is a flexible functional form that allows partial elasticities of substitution between inputs to vary. To bring confidence to the choice of functional specification, we initially investigate both options and perform a likelihood ratio (LR) test to compare the fit of the two functional specifications. Based on the LR test results (see second last paragraph in Sect. 5.4.3.1), we find that the Translog function is more appropriate. An additional argument as to why Cobb-Douglas would be inappropriate in a competitive industry such as banking is the non-concave Cobb-Douglas output dimensions (Klein 1953, p. 227).

In the core SFA analysis using the Translog function with pooled data, the sum of outputs *interest income* and *non-interest income* (i.e. total income) becomes the dependent variable. The input variables and the firm-specific factors that may impact efficiency are the same as those used in DEA. The general equation using the Translog function with two inputs is as follows:

$$\begin{aligned} \text{Production function: } ln(y_i) &= \beta_0 + \beta_1(\ln x_{1,i}) + \beta_2 \ln(x_{2,i}) + \frac{1}{2}\beta_3(\ln x_{1,i})^{\mathbf{2}} \\ &\quad + \frac{1}{2}\beta_4(\ln x_{2,i})^2 + \beta_5 \ln x_{1,i} {*} \ln x_{2,i} - z_i\delta + W_i + v_i \\ \text{Inefficiency function: } E[\mu_i] &= z_i\delta \qquad u_i \sim N^+\left(\mu_i, \sigma_u^2\right) \end{aligned} \tag{5.5}$$

where $ln(y_i)$, is the natural logarithm of the output *total income*, $\ln(x_{1i})$, $\ln(x_{2i})$, are the logarithm of the inputs *interest expense* and *non-interest expense*, respectively, followed by three variables which are the second order of the input variables and their interaction term. The Translog function provides a broader format to describe the relationship between the output and input levels than the Cobb-Douglas function because the output variable may be correlated with higher order input variables—a relationship not considered in a Cobb-Douglas function; Cobb-Douglas also makes the simplistic assumption that all production units have the same elasticities. v_i is the two-sided i.i.d. error term. u_i is the inefficiency term comprised of two parts where W_i is defined by the truncation of the normal distribution with zero mean and the variance of σ^2, and $z_i\delta$ is the mean of inefficiencies modeled as a linear function of the firm-specific factors.

Altogether, there are five variables in the inefficiency equation (including the foreign bank dummy variable) to explain bank inefficiencies by firm characteristics—discussed earlier under the heading of firm-specific factors. In summary, we expect to observe that the output production of total income can be explained by the input variables of *interest expense* and *non-interest expense* and their second order approximations. The inefficiency function allows us to test the association between inefficiencies and bank characteristics. While in the single-output Translog function the two outputs are aggregated into one output as *total income*, it is also possible to test the two-output case. Therefore, later in the chapter we explore the two-input two-output extended model which has greater dimensionality.

A distance function can handle the case of multiple outputs (see Coelli and Perelman 1999, 2000). The output distance function (Shephard 1970) is defined on the output set $P(x)$ as follows:

$$D_O(x, y) = \min\{\theta : (y/\theta) \in P(x)\} \tag{5.6}$$

where θ is the scalar distance, and $D_O(x, y)$ is non-decreasing, positively linearly homogenous and convex in y and decreasing in x (Lovell et al. 1994). The above output distance function can be represented in Translog form:

$$\begin{aligned} \ln D_{Oi} = \alpha_0 &+ \sum_{m=1}^{M} \alpha_m lny_{mi} + \frac{1}{2}\sum_{m=1}^{M}\sum_{n=1}^{M} \alpha_{mn} lny_{mi} lny_{ni} + \sum_{k=1}^{K} \beta_k lnx_{ki} \\ &+ \frac{1}{2}\sum_{k=1}^{K}\sum_{l=1}^{K} \beta_{kl} lnx_{ki} lnx_{li} + \sum_{k=1}^{K}\sum_{m=1}^{M} \gamma_{km} lnx_{ki} lny_{mi} \end{aligned} \tag{5.7}$$

where $i = 1, 2, \ldots, N$, denotes bank-years in the data set. We choose the output of *interest income* as the Mth output, y_{Mi}, and derive the multiple-output Translog distance function for SFA:

$$\begin{aligned} -lny_{Mi} = \alpha_0 &+ \sum_{m=1}^{M-1} \alpha_m lny^*_{mi} + \frac{1}{2}\sum_{m=1}^{M-1}\sum_{n=1}^{M-1} \alpha_{mn} lny^*_{mi} lny^*_{ni} + \sum_{k=1}^{K} \beta_k lnx_{ki} \\ &+ \frac{1}{2}\sum_{k=1}^{K}\sum_{l=1}^{K} \beta_{kl} lnx_{ki} lnx_{li} + \sum_{k=1}^{K}\sum_{m=1}^{M-1} \gamma_{km} lnx_{ki} lny^*_{mi} + v_i - u_i \end{aligned} \tag{5.8}$$

where $y^*_{mi} = y_{mi}/y_M, y^*_{ni} = y_{ni}/y_M$.

The SFA regression does not require specification of the direction of impact of firm-specific factors and these can be observed from the signs of the emerging parameters. Neither is it essential to assume a functional form although it is common practice. SFA enables hypothesis testing and estimation of standard errors using maximum-likelihood methods (Coelli et al. 1998). Similar to the studies by Jiang et al. (2009) and Deng et al. (2011) on Chinese bank efficiency, this study also relies on the one-step approach proposed in Battese and Coelli (1995) where non-negative technical inefficiencies are a function of firm-specific factors (contextual variables). Banker and Natarajan (2008) also consider appropriate for the parametric approach a one-step procedure that jointly estimates inefficiency and the impact of contextual variables; further support for the one-step procedure can be found in Wang and Schmidt (2002) who provide evidence based on Monte Carlo testing. Inefficiencies are independently distributed as truncations of normal distribution with constant variance but mean values that are a linear function of the observed variables. We use FRONTIER 4.1 (by Tim Coelli) to estimate the parameters of SFA regressions.

SFA efficiency estimates based on regression are not highly sensitive to large data changes—a potential advantage over DEA when substantial measurement errors are suspected. Fries and Taci (2005) claim SFA to be more appropriate in situations where measurement errors are more likely—such as transition economies. On the other hand, SFA may be inappropriate if the structural form assumed or the distributional assumptions made for random errors or inefficiencies are not representative of the organizations studied. For example, Luo and Donthu (2005) report that management prefer DEA and regard it as a more reliable frontier method.

In summary, DEA and SFA both have some key assumptions that may become the main weaknesses of these methods. That is, standard DEA assumes no measurement error, whereas SFA studies commonly assume a particular structure which may not be appropriate for the whole sample. Thus, this study compares and contrasts results from both methods in an analysis where an industry best-practice frontier is determined under each approach. We unfold the comparison in two stages where we initially use a single output (core model) but later move to a two-output benchmarking model (extended model)—assuming variable returns-to-scale in acknowledgement of the nature of the sample (see the next section for formal tests of scale inefficiency).

5.4 Results and Analysis

5.4.1 Testing for Scale Inefficiency Using DEA

In general, assuming variable-returns-to-scale would acknowledge the often different scale of operations anticipated among banks operating across China. A quick look at the minima and maxima in Table 5.2 suggests the presence of substantial differences in the scale of operations. Therefore, we explore this issue through the radial DEA formulations of CCR (Charnes et al. 1978) and BCC (Banker et al. 1984) which permit calculation—rather than inference—of scale inefficiencies, i.e. scale efficiency equals the ratio of CCR to BCC efficiency estimates. We compute rank correlations between output-oriented CCR and BCC estimates (two inputs and two outputs) and measure statistical differences. Spearman's rho 0.7340 between CCR and BCC estimates are significant at the 0.000 level. However, when we test for statistical differences between radial CCR and BCC efficiency estimates, Mann-Whitney *U* test rejects the null at the 0.000 significance level.

The above finding suggests there are significant differences between efficiency estimates that assume constant returns-to-scale vs. variable returns-to-scale, i.e. there is substantial scale inefficiency despite a statistically significant rank correlation. We quantify such differences by computing scale efficiencies. While the mean scale efficiency is reasonably high at 0.9029, there is a wide range of estimates (0.3350–1.0000) that are substantially skewed at −2.35. When we rank the bank-years on descending scale efficiency, we find that the last fourteen places are occupied by foreign banks with Société Générale (China) representing the bottom two bank-years (ranked results are available from the authors). The overall conclusion is one of substantial scale inefficiency at least in some of the banks, but to a greater extent with the foreign banks when mean scale efficiency estimates are compared across the two cohorts (domestic 0.9357 vs. foreign 0.8269). Thus, we conclude that using the variable returns-to-scale specification is better in order to rule out any impact of scale inefficiency in the overall analysis; this choice is also in line with Translog SFA (see last paragraph in Sect. 5.4.3.1), thus enabling a meaningful comparison between DEA and SFA.

5.4.2 Main DEA Results

5.4.2.1 Core Model (Single-Output BCC-O)

The analysis begins with the radial, output-oriented BCC which assumes variable returns-to-scale. In order to facilitate a more systematic comparison between DEA and SFA, we begin with a simple *core model* comprised of one output (total income) and two inputs (interest expense and non-interest expense). Instead of simply listing ranked 159 bank-years obtained from DEA, we provide a summary of the information extracted (the ranked list is available from the authors). Results indicate a wide range of efficiency estimates (0.4867–1.0000). Mean efficiency estimates (foreign 0.7900, domestic 0.8672) and mean ranks (foreign 95, domestic 72) point to a *less efficient* foreign bank cohort. Mann-Whitney *U* test for foreign versus domestic banks efficiency estimates rejects the null that the estimates come from the same distribution at the 0.004 level. The three most frequently referenced or emulated efficient bank-years by DEA algorithm in determining the relative efficiency estimates for others in the sample are: 77 times for Huishang Bank 2010 (domestic), 63 times for Bank International Ningbo 2008 (foreign) and 46 times for Bank of Beijing 2008 (domestic)—highlighting the dominance of domestic banks.

Next, following the example set by Banker and Natarajan (2008) and McDonald (2009), we report OLS regression of firm-specific factors on the core performance model DEA efficiency estimates, which suggests, all else the same, a 1 percentage point drop in the loan-to-deposit or cost-to-income ratios could lead to a 0.0708 percentage point and 0.2159 percentage point rise in overall bank efficiency significant at the 0.0459 and 0.0017 levels, respectively, where the residuals are distributed normally (Tobit regression results are very similar to OLS and available from the authors). These relationships are robust to various additional tests such as logging variables or removing outliers.[4]

An additional robustness test of the sample involves removing the five majority state-owned large banks from the data set of the core model (i.e. 15 bank-year data points) and checking the difference between the two cohorts' efficiency estimates. Mean efficiency estimates (foreign 0.7900, domestic 0.8676) are almost identical to those of the full sample—indicating little if any distortion caused by the large majority state-owned banks. Once again, Mann-Whitney *U* test on foreign versus domestic banks rejects the null that the estimates come from the same distribution at the 0.004 level for the core model. Similarly, when we regress firm-specific factors on efficiency estimates from the reduced sample, the same factors emerge as statistically significant in explaining efficiency with almost identical coefficients and significance levels (available from the authors).

5.4.2.2 Extended Model (Two-Output BCC-O)

The extended model takes advantage of two outputs (i.e. interest income and non-interest income that were summed to create total income under the core model), and the same two inputs. The extended model approach is designed to explore whether similar findings can be observed in the presence of increased dimensionality. Once again, results indicate a wide range of efficiency estimates (0.5444–1.0000). Mean efficiency estimates (foreign 0.8258, domestic 0.9156) and mean ranks (foreign 94, domestic 70) still point to a *less efficient* foreign bank cohort. Mann-Whitney *U* test rejects the null that the estimates come from the same distribution at the 0.002 level. The three most frequently emulated efficient bank-years are: 56 times for Nanchong City Commercial Bank 2010 (domestic), 55 times for Bank of Beijing 2009 (domestic) and 55 times for Bank of Jilin 2010 (domestic)—once again highlighting the dominant domestic banks where Bank of Beijing perseveres across both models. OLS regression of firm-specific factors on the extended model DEA efficiency estimates reveal similar results to that of the core model where a 1 percentage point drop in the loan-to-deposit or cost-to-income ratios could lead to a 13.5167 percentage points and 0.1168 percentage point rise in overall bank efficiency significant at the 0.0001 and 0.0498 levels, respectively (Tobit regression results are very similar to OLS and available from the authors). Once again, tests of robustness reveal that the above relationships hold after removal of outliers or logging of variables.

[4] SFA is even less sensitive to the presence of any outliers because it estimates the efficient frontier by fitting a regression line to the production possibilities set, rather than relying on extreme performers to define the frontier.

The additional robustness test of removing the five majority state-owned large banks from the data set results in mean estimates almost identical to those of the full sample. Mann-Whitney *U* test on foreign versus domestic banks rejects the null that the estimates come from the same distribution at the 0.006 level for the extended model. Regressing firm-specific factors on efficiency estimates shows the same factors as statistically significant in explaining efficiency with almost identical coefficients and significance levels (details available from the authors). In short, we have no reason to believe that including the majority state-owned banks in either the core or the extended model are distorting our main findings.

5.4.2.2.1 Overall Potential Improvements Identified by DEA Using the Extended Model

Figure 5.1 summarizes the overall potential improvements identified by DEA using the extended model, i.e. radial inefficiencies or under-produced outputs, as well as slacks or over-utilized inputs. In the full sample of 159 bank-years, most of the inefficiencies are embedded in non-interest income—which suggests that some banks are falling substantially behind their peers in generating income from less traditional banking activities. The second largest source of inefficiency is interest expense and this can be construed as a reflection of the regulated interest rates in China. The second pie-chart also identifies non-interest income, followed by interest expense, as the main sources of inefficiency among domestic banks. Finally, the third pie-chart points to interest income as the major source of inefficiency among foreign banks. This observation can be interpreted as an outcome of their limited branch networks and the general position of domestic banks across China as favored institutions, in particular, with regards to government or state related loans. As intuitively expected, compared to foreign banks the extent of inefficiencies embedded in interest expense is much greater with domestic banks because of their operations' emphasis on handling deposits.

In summary, the pie-charts indicate that the main source of inefficiency among the foreign banks is interest income, whereas the domestic banks appear to be mostly inefficient in managing their non-interest incomes and interest expenses. The inefficiencies seen with foreign banks are a reflection of limited access such institutions have to potential borrowers. At the same time, the inefficiency embedded in non-interest income of domestic banks highlights the potential for growth as such banks become more skilled in providing less traditional banking services. Similarly, as market deregulation unfolds at a steady pace, inefficiencies in interest expenses are also likely to lessen.

5.4.2.2.2 Assessing the Marginal Role of the Output Variables in DEA: Efficiency Contribution Measures (ECM) for the Extended Model

We implement the method outlined in Pastor et al. (2002) on the extended model using the full sample, i.e. N = 159. The approach calls for making an *inefficient*

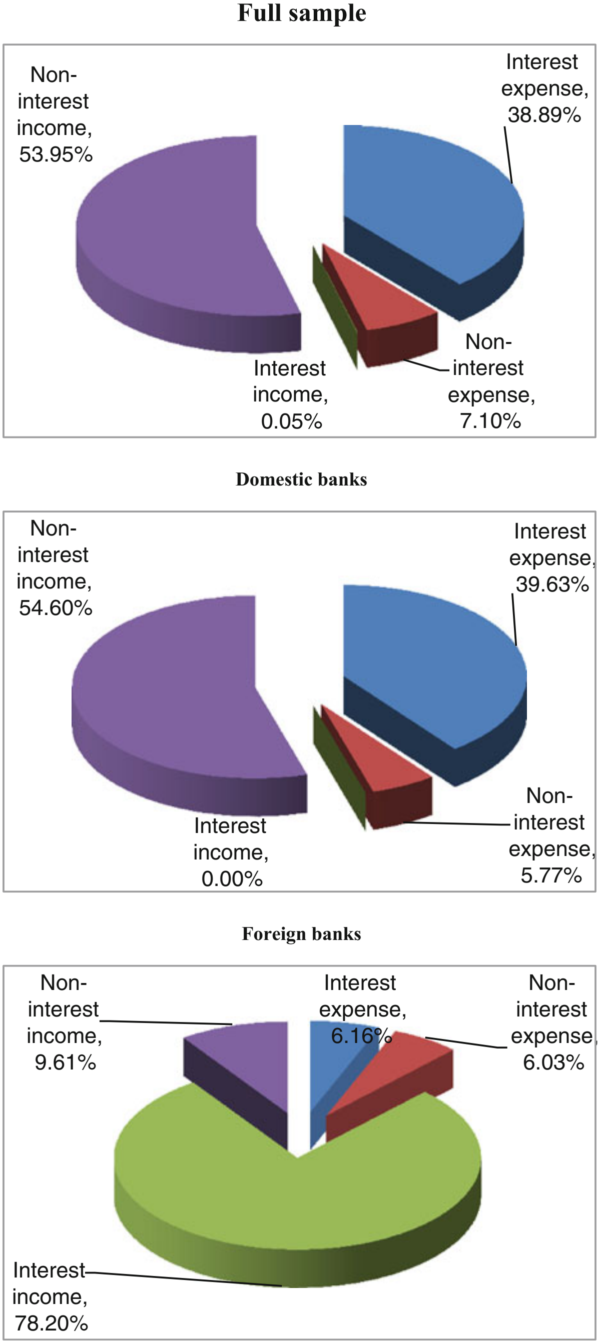


Fig. 5.1 Potential improvements identified by output-oriented DEA for the variables in the extended model with two outputs and two inputs

DMU *efficient* by increasing actual output levels to their projected levels determined by the efficient frontier, and re-running output-oriented DEA without the variable under scrutiny (i.e. the candidate). Calculation of ECM for each of the two candidates follows the steps outlined below:

1. The initial DEA with the full-complement of variables identifies the projected output levels for the inefficient DMUs.
2. Actual output levels for inefficient DMUs are replaced by projections, i.e. virtual DMUs are created.
3. DEA is repeated without the candidate output variable but in the presence of virtual DMUs.
4. The ratio of the efficiency estimate from the reduced model to the estimate from the original full-complement model yields ECM or ρ_o.
5. If $\rho_o = 1$, then the candidate has no marginal effect on the observed DMUs' efficiencies. Alternatively, if $\rho_o > 1$, then the candidate variable has some effect.

Pastor et al. (2002) develop a non-parametric statistical test to evaluate the significance of ECM. From the full set of ECM (ρ) values generated using the sample, a random sample of ρ values are drawn. If a candidate is not relevant, efficiency estimates are unlikely to be affected by its presence. This means corresponding random ρ values are also unlikely to be high. This idea is formalized by introducing two parameters, namely, $\bar{\rho}$ ($\bar{\rho} \succ 1$) representing tolerance for changes in efficiency estimates due to the candidate, and $p_o(0 < p_o < 1)$ representing the proportion of units with efficiency changes that exceed the tolerance. Hence, the marginal impact of a candidate on efficiency estimates would be deemed statistically significant when $P[\Gamma \succ \bar{\rho}] \succ p_o$ where Γ is the random ρ. For example, if $p_o = 0.20$ and $\bar{\rho} = 1.15$, the above relationship would indicate the candidate as relevant if more than 20 % of the DMUs had associated efficiency change greater than 15 % when the variable is omitted. Using Monte Carlo experiments, Pastor et al. (2002) report that parameters of $p_o = 0.15$ and $\bar{\rho} = 1.1$ provide a good performance of the significance test. We adopt these parameters to evaluate the significance of ECM scores for candidate variables. Results indicate that when *interest income* is treated as the candidate, 3.77 % of the DMUs have ECM above 1.1. Alternatively, when *non-interest income* is the candidate, a significant 44.03 % (i.e. greater than 15 %) of the DMUs have ECM greater than 1.1, i.e. non-interest income plays a greater role in efficiency evaluation or discriminating between the DMUs. This finding ties in well with the insight previously gained from Figure 5.1 where the largest potential improvement (inefficiency) across the full sample was indicated for non-interest income.

5.4.3 SFA Results

5.4.3.1 Core Model (Single-Output Translog Function)

We start with the Translog function SFA and report the results in Table 5.3 using the dependent variable of *total income* (the logarithm of the sum of interest income

Table 5.3 SFA parameters for the core model with one output[a]

		Sample robustness test
	Translog function (N = 159) (5.1)	Translog function without large majority state-owned banks (N = 144) (5.2)
Dependent variable: Total income		
Production function[b]		
Intercept[c]	1.664***(0.000)	1.641***(0.000)
Interest expense	0.322***(0.000)	0.310***(0.000)
Non-interest expense	0.628***(0.000)	0.648***(0.000)
0.5 * Interest expense squared	0.129***(0.000)	0.126***(0.000)
0.5 * Non-interest expense squared	0.087*(0.047)	0.077(0.051)
Interest expense × Non-interest expense	−0.105**(0.004)	−0.099***(0.001)
Inefficiency function (firm-specific factors)		
Impaired loans-to-gross loans (asset quality)[d]	−0.211(0.179)	−0.232(0.189)
Interbank ratio (liquidity)	0.002*(0.019)	0.002**(0.01)
Loan-to-deposit ratio (regulation)	0.076**(0.004)	0.072*(0.038)
Cost-to-income ratio (overall efficiency)	0.535***(0.000)	0.534***(0.000)
Foreign bank dummy	0.116***(0.000)	0.115***(0.000)
Sigma-squared $(\sigma_u^2 + \sigma_v^2)$	0.005***(0.000)	0.005***(0.000)
Gamma $\left(\frac{\sigma_u^2}{\sigma_u^2+\sigma_v^2}\right)$	0.999***(0.000)	0.999***(0.000)
Log likelihood	199.649	179.678
LR test of the one-sided error	273.147	253.730
Mean efficiency estimate	0.7168	0.7113

[a]SFA model assumes a truncated normal distribution of inefficiencies. *P*-values are in parentheses
[b]All the variables take logarithm values in the production functions
[c]***Significant at 0.1 %; **significant at 1 %; *significant at 5 %
[d]Also known as the non-performing loans ratio (NPL)

and non-interest income). The Translog function is well-specified because all the input variables are highly significant. A positive relationship between the output variable and the first order of two input variables (the logarithm of *interest expense* and the logarithm of *non-interest expense*) suggests that total income rises with an increase in different expense components that are part of the intermediation process undertaken by banks. The second order approximation input variables are also shown to be significant (with one exception under robustness testing) and the magnitudes of the coefficient estimates are non-negligible which indicates the second order approximation is also significantly related to the output variable.

Next, we focus on the inefficiency function results detailed in column 1 of Table 5.3 and the statistically significant coefficients therein. The estimated coefficients in the inefficiency function reveal how firm-specific factors impact on bank technical efficiency. For example, the positive coefficient for cost-to-income ratio is consistent with the expectation that higher costs would be found in less efficient operations (a relationship already observed under the regression of firm-specific factors on DEA efficiency estimates). This is a highly anticipated finding and brings further confidence to the analysis because the cost-to-income ratio is the banking industry's standard overall efficiency ratio. A positive coefficient for interbank ratio (the liquidity measure) is also consistent with conventional wisdom. That is, a higher interbank ratio suggests that a bank having difficulty in converting deposits to commercial or consumer loans would lend to other banks in the wholesale market instead, thus enjoying narrower interest margins in the process. This reduction in margins manifests itself as inefficiency in generating income. Similarly, the positive loan-to-deposit ratio signals that regulation handicaps banks' ability to generate income as this ratio approaches the 75 % threshold (see Sect. 5.2.3). On the other hand, the positive and significant coefficient of the foreign bank dummy variable brings confidence to the overall finding already reported using DEA that foreign banks are less efficient than domestic banks.

Finally, the insignificant coefficient for the impaired loans-to-gross loans ratio indicates that non-performing loans in Chinese banking are well managed and do not impact on efficiencies in generating income. This is a reflection of the high-growth Chinese economy where authorities regard non-performing loans as an acceptable price to pay for growth; in fact, there is a thriving market where NPL are removed from bank books through purchases made by asset management companies originally established by government in 1999.

Other parameters reported include the gamma value, $\gamma = \frac{\sigma_u^2}{\sigma_u^2+\sigma_v^2}$, that is, the variance of the normal distribution scaled by the sum of the variance of the normal distribution and variance of the two-sided disturbance term. In theory, the gamma value can range between 0 and 1, where a higher value indicates inefficiencies playing a greater role in the total residual terms. The high gamma values (0.999) across both samples imply negligible noise; this insight also brings more confidence to DEA reported earlier as the presence of high levels of noise in data can potentially distort DEA efficiency estimates—highlighting how the two methods can complement each other. Coupled with mostly statistically significant production and inefficiency function variables, results indicate that the presence of inefficiency is non-negligible and dominate the variance of the total residual terms; therefore, the two-sided noise v_i has little impact on total variance. The null hypothesizing the absence of inefficiency is rejected at the 0.001 level of significance with a log likelihood ratio of 199.6 along with the high LR test of the one-sided error at 273.1. These observations indicate that the model is well specified and significant at the equation level.

Focusing on the efficiency estimates for all bank-years using the Translog function, once again, instead of listing the ranked 159 bank-years obtained from

SFA (core model), we summarize our key observations (the ranked list is available from the authors). Results indicate a wide range of efficiency estimates (0.1516–0.9706). Examining the two cohorts' mean efficiency estimates (foreign 0.5818, domestic 0.7752) and mean ranks (foreign 127, domestic 60) indicates that SFA also estimates the foreign bank cohort to be less efficient but in a more discriminating manner than DEA. Independent samples *t*-test and Mann-Whitney *U* test on foreign versus domestic bank efficiency estimates both reject the null that the estimates come from the same distribution at the 0.0001 level. The top performing three bank-years in the sample in descending order are all domestic banks represented by the Bank of Guangzhou (2008, 2010), and Huishan Bank (2010) and there is a very clear congregation of domestic bank-years in the top half of the sample sorted by descending SFA efficiency estimates. A comparison of SFA and DEA is offered in Sect. 5.4.4.

We continue by implementing the same sample robustness test previously undertaken with DEA. That is, we exclude the five majority state-owned large banks to see whether the results of our core SFA test will vary. We find that leaving out the 15 bank-years (five banks for three consecutive years) does not change the main results (see results in column two of Table 5.3). The input variables of interest expense and non-interest expense are still significantly positively correlated with the output variable of total income, foreign banks remain less efficient, and the associations originally observed in the inefficiency equation are retained.

We also test the Cobb-Douglas function first mentioned in Sect. 5.3.3. To determine which functional form fits the data better, other factors such as the dependent variable and firm-specific factors are kept the same. The Cobb-Douglas is a special case of the Translog function where all the coefficients of the second order terms are restricted to be 0, i.e. $\beta_3 = \beta_4 = \beta_5 = 0$ in (5.5). Hence, Cobb-Douglas imposes more stringent assumptions on data than the Translog function. In choosing between Cobb-Douglas and Translog specifications, such restrictions are tested using the likelihood ratio test (LR test) with the null hypothesis that Cobb-Douglas is nested in Translog. The null is strongly rejected at the level of 0.001 with the LR ratio of 141.94, thus adding another formal argument in favor of the Translog function first visited in paragraph 2 of Sect. 5.3.3.

In DEA, we have already established that the appropriate assumption on the elasticity of scale is variable returns-to-scale (VRS). In the spirit of ensuring DEA and SFA analyses are comparable, we need to establish that VRS also holds in SFA. Hence, the null hypothesis of constant returns-to-scale (CRS) in Translog SFA is tested. The returns-to-scale can be estimated as the sum of interest expense and non-interest expense coefficients (see Sect. 8.4 in Coelli et al. 2005). The assumption of CRS is equivalent to the null hypotheses that the first order coefficients add up to 1 and rows and columns of the matrix of the second order coefficients sum up to zero. In order to test the restrictions *jointly*, we employ the Wald test. The unreported results (available from the authors) show that the null hypothesis of CRS is strongly rejected at the level of 0.000 with the Chi(3)-square value of 424.72. Hence, we are confident that efficiency estimates from DEA and SFA are based on the same assumption of variable returns-to-scale.

5.4.3.2 Extended Model (Two-Output Translog Function)

Column 1 of Table 5.4 presents the two-output Translog function results on the full sample of 159 bank-years. Nine regressors of output and input items are used in the right hand side of the Translog function and their signs vary. Interpreting the coefficients of these regressors is difficult at best where inputs and outputs interact with each other; thus, normally emphasis is placed on efficiency estimates.

We next turn to the inefficiency function results in Table 5.4 with two outputs. The observed signs correspond to the findings using the single-output Translog function and results suggest the higher cost-to-income ratio is associated with less efficiency and foreign banks are less efficient than domestic banks. However, the other three firm-specific variables are shown to be unrelated to bank technical inefficiencies. We also test the robustness of the two-output model using a smaller sample of 144 bank-years in which the five large majority state-owned banks are removed. The sample robustness test results reported in column 2 of Table 5.4 are quantitatively similar to that of column 1 with the exception of an insignificant gamma. The gamma value is an important measure of the presence of inefficiencies and the robustness test suggests the component of inefficiency is now negligible in relation to the total residual terms—a most unlikely scenario given what we already know about the sample. The above observations suggest that the single-output Translog function provides a better fit for our data than the two-output model.

5.4.4 *Comparing DEA and SFA Results*

We now return to the primary motivation of this study. Theory points out that DEA efficiency estimates are expected to be greater than SFA efficiency estimates because DEA efficiency estimates are upwardly biased in comparison to the unobserved true efficiency estimates, in particular with small samples (Badin et al. 2014). On the other hand, SFA may provide more consistent estimates. Descriptive statistics in Table 5.5 on the full sample indicate that the mean and median DEA efficiency estimates are higher than SFA efficiency estimates. We run a series of statistical tests to further compare DEA efficiency estimates with those generated by SFA. For the core model, Spearman's *rho* 0.590 significant at the 0.01 level indicates that the correspondence of rankings between the two methods is moderate rather than high; for the extended model, the rank correlation is 0.538 also significant at the 0.01 level. These correlations compare favorably to the Spearman's *rho* of 0.480 (significant at the 0.10 level) reported by Luo et al. (2011) on a sample of Chinese commercial banks across 1999–2008. More importantly, Mann-Whitney *U* test rejects the null that the estimates come from the same distribution at the 0.05 level for both the core and extended models—highlighting the different distributions of efficiency estimates created by a non-parametric versus parametric efficient frontier method. In summary, DEA

Table 5.4 SFA parameters for the extended model with two outputs

		Sample robustness test
	Translog function (N = 159) (1)	Translog functionwithout large majority state-owned banks (N = 144) (2)
Dependent variable: Log(*Interest income*)		
Production function		
Intercept[a]	1.252*** (0.000)	1.318 ***(0.000)
Log(Non-interest income/Interest income)	0.125**(0.005)	0.120***(0.001)
Log(Interest expense)	−0.431** (0.000)	−0.381***(0.000)
Log(Non-interest expense)	−0.528*** (0.000)	−0.550***(0.000)
Log(Interest expense) * Log (Non-interest expense)	0.128**(0.004)	0.095*(0.024)
0.5Log(Interest expense) * Log (Non-interest income/Interest income)	−0.072* (0.036)	−0.060*(0.042)
0.5Log(Non-interest expense) * Log (Non-interest income/Interest income)	0.054(0.171)	0.046(0.169)
0.5Log(Interest expense) * Log(Interest expense)	−0.159*** (0.000)	−0.129***(0.000)
0.5Log(Non-interest expense) * Log (Non-interest expense)	−0.104(0.052)	−0.073(0.165)
0.5Log(Non-interest income) * Log (Non-interest income)	0.004(0.123)	0.005*(0.014)
Inefficiency function (firm-specific factors)		
Impaired loans-to-gross loans (asset quality)[b]	−0.082(0.633)	−0.179(0.236)
Interbank ratio (liquidity)	0.002(0.208)	0.002(0.062)
Loan-to-deposit ratio (regulation)	0.002(0.957)	0.005(0.906)
Cost-to-income ratio (overall efficiency)	0.511*** (0.000)	0.495***(0.000)
Foreign bank dummy	0.172*** (0.000)	0.181***(0.000)
Sigma-squared $(\sigma_u^2 + \sigma_v^2)$	0.006*** (0.000)	0.006***(0.000)
Gamma $\left(\frac{\sigma_u^2}{\sigma_u^2+\sigma_v^2}\right)$	0.999*** (0.000)	0.014 (0.997)
Log likelihood	183.265	165.891
Mean efficiency estimate	0.7475	0.7414

[a]***Significant at 0.1 %; **significant at 1 %; *significant at 5 %
[b]Also known as the non-performing loans ratio (NPL)

Table 5.5 Descriptive statistics on DEA and SFA efficiency estimates (N = 159)

	DEA, core model (single-output BCC-O)[a]	SFA, core model (single-output Translog function)	DEA, extended model (two-output BCC-O)	SFA, extended model (two-output Translog function)
Mean	0.8439	0.7168	0.8885	0.7475
Median	0.8520	0.7426	0.9129	0.7790
Standard deviation	0.1157	0.1275	0.1112	0.1382
Coefficient of variation[b]	0.1371	0.1779	0.1251	0.1849
Maximum	1.0000	0.9706	1.0000	0.9997
Minimum	0.4867	0.1516	0.5444	0.1692
Skewness	−0.6064	−0.9699	−1.0550	−0.7566
Kurtosis	−0.0055	2.2913	0.5435	1.2988
Number of efficient bank-years	17	n/a	29	n/a

[a]Core model has one output; extended model has two outputs. BCC-O; Banker, Charnes and Cooper radial DEA, output-oriented
[b]Ratio of standard deviation to mean

and SFA efficiency estimates that use Chinese data from 2008 to 2010 are statistically different.

Similarly, a visual examination of the distribution of bank-years across the unreported sample sorted on SFA efficiency estimates reveals a stronger congregation of foreign banks in the bottom half of the table compared to the DEA's sorted sample (available from the authors). In fact, under SFA the more efficient bank-years are almost entirely populated by domestic banks. The more noticeable congregation of domestic versus foreign banks in the sorted SFA sample suggests that SFA efficiency estimates are more discriminating. The non-parametric Kolmogorov-Smirnov test (K-S test) can also be used as a formal test to establish whether the efficiency estimates from SFA and DEA differ significantly. The null hypothesis of 'no difference' or 'same distribution' is rejected for efficiency estimates from both the single-output and two-output performance models at a *D-statistic* of 0.4717 (0.000) and 0.5157 (0.000), respectively; the K-S test also reports that the efficiency estimates are unlikely to be normally or log normally distributed.[5]

[5] The higher number of efficient bank-years under DEA with the extended model reflects the impact of greater dimensionality when a second output is introduced; the impact of increased dimensionality is equally easily discernible when core model means and medians are compared against those from the extended model (see Table 5.5). Clearly, there is a loss of discrimination as dimensionality rises for a given sample size – better known as the curse of dimensionality.

We extend the comparison by examining the bank-years common to the fourth (top 25 %) and first (bottom 25 %) quartiles across DEA and SFA with the aim of observing the degree of agreement between these methods at the two extremes when results are ranked (N = 159). Twenty-two of the forty bank-years found in the fourth quartile in the single-output Translog SFA are also found in DEA, and 24 are found in the first quartile—a rather poor correspondence confirming the distributional test reported earlier in this section. Similarly, 19 of the bank-years found in the fourth quartile in SFA based on the two-output Translog function are found in DEA, and 23 are found in the first quartile. A closer look at the membership of the top ten bank-years ranked in the single-output Translog SFA finds only four corresponding to those identified as efficient under DEA; the same approach yields a correspondence of five bank-years when the two-output Translog function results are compared to DEA—once again highlighting the distributional differences between parametric and non-nonparametric results.

The above observations highlight the risks involved in exclusively relying on DEA or SFA for ranking purposes. Does the researcher have to favor one method over the other? The answer can be found in time-series forecasting literature which suggests that a single set of efficiency estimates can be constructed by taking the geometric means of the estimates to emerge from DEA and SFA (Coelli and Perelman 1999). It has been argued that taking simple average of estimates from multiple methods can reduce bias by averaging out individual biases (Palm and Zellner 1992).

5.5 Concluding Remarks

The primary motivation of this study is to compare and contrast the popular DEA and SFA methods in a bank benchmarking exercise and explore the possibility of using these rival methods in a complementary manner. This motivation is actioned in the context of how foreign banks in China perform when compared against domestic banks.

It is worth summarizing the complementarity between DEA and SFA. In particular, when the non-parametric and parametric methods lead to the same key findings as seen in this chapter, researchers can rely on DEA to identify the main potential improvements (see Fig. 5.1), while SFA can be relied upon to directly explain the role of firm-specific factors on inefficiency (see Table 5.3). On the other hand, when DEA and SFA produce significantly different rankings, then the researcher may consider other ranking approaches, e.g. constructing geometric means based on efficiency estimates from DEA and SFA before ranking. In situations where measurement error cannot be reliably assessed, SFA can act as a test of robustness for DEA. Similarly, when the functional structure assumed by SFA may not apply equally across the sample, DEA can become the test of robustness for SFA. Interestingly, sample robustness testing suggests that the presence of large majority state-owned commercial banks do not distort the main findings. Furthermore, at least in the case of Chinese banking data, the single-output

Translog function estimated by SFA is a better fitting functional specification than the Cobb-Douglas or the two-output Translog functions.

According to DEA, in general, foreign banks are less efficient than domestic banks. A break-down of the sources of inefficiency in the modeled performance variables point to management of interest income among the foreign banks as a key area for potential improvement, whereas the domestic banks appear to suffer mainly from inefficiencies in managing non-interest income and interest expense. The inefficiencies identified in this study with foreign bank operations can be construed as a consequence of limited access they have to potential depositors and borrowers. Similarly, the inefficiency found in generation of non-interest income by domestic banks points to the potential for expansion as domestic banks become more adept in less traditional banking services. These are intuitive findings based on what we know about Chinese bank regulation and well-accepted strengths and weaknesses of foreign versus domestic bank operations.

SFA reports similar yet more discriminating results to that of DEA regarding the less efficient foreign banks. Parameters of the inefficiency function in SFA reveal mostly anticipated relationships. For example, the liquidity measure, regulation measure, and the industry ratio for overall efficiency show a significant but negative impact on total income. Overall, results point to the use of parsimonious benchmarking models and a Translog function as appropriate choices for discriminating among performance of banks.

DEA and SFA efficiency estimates based on the study's performance modeling are significantly correlated but they do not belong to the same distribution. Overall, the intuitive findings from these methods from opposing camps indicate that efficiency estimates are not simply manifestations of specific assumptions that underlie DEA or SFA, thus bringing confidence to using either method or both in benchmarking bank performance. That is, similar to the conclusion reached by Weill (2004) for European banking, we also conclude that neither method can be categorically identified as the most suitable for Chinese banking.

The two methods illustrated in this study can be used by regulators for checking against in-house performance evaluation systems and identifying those banks that may need closer scrutiny. For regulatory purposes, the comparison of the two efficient frontier techniques can be further expanded by following the six consistency conditions identified by Bauer et al. (1998). Revealed potential improvements can also be used by bank management who may be interested in developing a better understanding of their weaknesses and strengths against their industry peers. Other compelling reasons to undertake multivariate benchmarking can be found within the framework of Basel III expected to be fully implemented by 2019. Given the greater awareness of the interconnectedness of the global financial system since the global financial crisis of 2007–2009, comparisons with peers are likely to be more important than simply mechanically checking a list of regulatory boxes for a given institution. For example, two ratios proposed within the Basel III framework, namely, the 30-day liquidity coverage ratio (LCR), and the net stable funding ratio (NSFR), deserve special attention and can be included in future benchmarking exercises when data become available.

Acknowledgements We wish to extend our sincere thanks to Chris O'Donnell and Valentin Zelenyuk for their guidance on SFA, Mette Asmild for general comments, Keay-shen See for his unwavering assistance in data collection (others include Jay Shen, Danielle Chang and Jimmy Leung), and Martin Cvelbar for his prompt service on all matters related to the use of library services. We also appreciate the pre-submission critical reading of the chapter by Tom Smith, Terry Walter, Barry Oliver and Ghon Rhee, as well as the suggestions made by the workshop participants at the Griffith University and the World Finance Conference.

References

Aigner, D., Lovell, C. A. K., & Schmidt, P. (1977). Formulation and estimation of stochastic frontier production function models. *Journal of Econometrics, 6*, 21–37.

Ariff, M., & Can, L. (2008). Cost and profit efficiency of Chinese banks: A non-parametric analysis. *China Economic Review, 19*, 260–273.

Asmild, M., & Matthews, K. (2012). Multi-directional efficiency analysis of efficiency patterns in Chinese banks 1997–2008. *European Journal of Operational Research, 219*, 434–441.

Avkiran, N. K. (2009). Removing the impact of environment with units-invariant efficient frontier analysis: An illustrative case study with intertemporal panel data. *Omega—International Journal of Management Science, 37*, 535–544.

Avkiran, N. K. (2011). Association of DEA super-efficiency estimates with financial ratios: Investigating the case for Chinese banks. *Omega—International Journal of Management Science, 39*, 323–334.

Avkiran, N. K., & Morita, H. (2010). Benchmarking firm performance from a multiple-stakeholder perspective with an application to Chinese banking. *Omega—International Journal of Management Science, 38*, 501–508.

Badin, L., Daraio, C., & Simar, L. (2014). Explaining inefficiency in nonparametric production models: The state of the art. *Annals of Operations Research, 214*, 5–30.

Banker, R. D., Charnes, A., & Cooper, W. W. (1984). Some models for estimating technical and scale inefficiencies in data envelopment analysis. *Management Science, 30*(9), 1078–1092.

Banker, R. D., & Natarajan, R. (2008). Evaluating contextual variables affecting productivity using data envelopment analysis. *Operations Research, 56*(1), 48–58.

Battese, G. E., & Coelli, T. J. (1995). A model for technical inefficiency effects in a stochastic frontier production function for panel data. *Empirical Economics, 20*, 325–332.

Bauer, P. W., Berger, A. N., Ferrier, G. D., & Humphrey, D. B. (1998). Consistency conditions for regulatory analysis of financial institutions: A comparison of frontier efficiency methods. *Journal of Economics and Business, 50*, 85–114.

Berger, A. N., Hasan, I., & Zhou, M. M. (2009). Bank ownership and efficiency in China: What will happen in the world's largest nation? *Journal of Banking & Finance, 33*, 113–130.

Bhattacharyya, A., Lovell, C. A. K., & Sahay, P. (1997). The impact of liberalization on the productive efficiency of Indian commercial banks. *European Journal of Operational Research, 98*, 332–345.

Brockett, P. L., Charnes, A., Cooper, W. W., Huang, Z. M., & Sun, D. B. (1997). Data transformations in DEA cone ratio envelopment approaches for monitoring bank performances. *European Journal of Operational Research, 98*, 250–268.

Charnes, A., Cooper, W. W., & Rhodes, E. (1978). Measuring the efficiency of decision-making units. *European Journal of Operational Research, 2*, 429–444.

Charnes, A., Cooper, W. W., & Rhodes, E. (1981). Evaluating program and managerial efficiency—An application of data envelopment analysis to program follow through. *Management Science, 27*, 668–697.

Chen, X. G., Skully, M., & Brown, K. (2005). Banking efficiency in China: Application of DEA to pre- and post-deregulation eras: 1993–2000. *China Economic Review, 16*, 229–245.
China Banking Regulatory Commission. (2006). *Regulations of the People's Republic of China on Administration of Foreign-funded Banks*. Retrieved from http://www.cbrc.gov.cn/EngdocView.do?docID=20080116D0F770B46656D8DAFF4313B4992E4E00
Chortareas, G. E., Girardone, C., & Ventouri, A. (2013). Financial freedom and bank efficiency: Evidence from the European Union. *Journal of Banking & Finance, 37*, 1223–1231.
Coelli, T., & Perelman, S. (1999). A comparison of parametric and non-parametric distance functions: With application to European railways. *European Journal of Operational Research, 117*(2), 326–339.
Coelli, T., & Perelman, S. (2000). Technical efficiency of European railways: A distance function approach. *Applied Economics, 32*, 1967–1976.
Coelli, T. J., Prasada, R. D. S., O'Donnell, C. J., & Battese, G. E. (2005). *An introduction to efficiency and productivity analysis* (2nd ed.). New York: Springer.
Coelli, T. J., Prasada, R. D. S., & Battese, G. E. (1998). *An introduction to efficiency and productivity analysis*. Boston: Kluwer Academic Publishers.
Cooper, W. W., Seiford, L. M., & Tone, K. (2007). *Data envelopment analysis: A comprehensive text with models, applications, references and DEA-Solver software* (2nd ed.). New York: Springer.
Cooper, W. W., Seiford, L. M., & Zhu, J. (2011). *Handbook on data envelopment analysis* (2nd ed.). Norwell, MA: Kluwer Academic Publishers.
Deng, Z., Guo, H., & Kong, G. (2011). Efficiency spillovers of foreign direct investment in the Chinese banking system. *Global Economic Review, 40*, 179–191.
Dietsch, M., & Lozano-Vivas, A. (2000). How the environment determines banking efficiency: A comparison between French and Spanish industries. *Journal of Banking & Finance, 24*, 985–1004.
Fries, S., & Taci, A. (2005). Cost efficiency of banks in transition: Evidence from 289 banks in 15 post-communist countries. *Journal of Banking & Finance, 29*, 55–81.
Fu, X., & Heffernan, S. (2007). Cost X-efficiency in China's banking sector. *China Economic Review, 18*, 35–53.
Gelade, G., & Gilbert, P. (2003). Work climate and organizational effectiveness: The application of data envelopment analysis in organizational research. *Organizational Research Methods, 6*, 482–501.
Greene, W. H. (2012). *Econometric analysis*. Boston: Prentice Hall.
Grosskopf, S. (1996). Statistical inference and nonparametric efficiency: A selective survey. *Journal of Productivity Analysis, 7*, 161–176.
Hasan, I., & Marton, K. (2003). Development and efficiency of the banking sector in a transitional economy: Hungarian experience. *Journal of Banking & Finance, 27*, 2249–2271.
Hu, J.-L., Su, Y.-Y., & Chen, C.-P. (2008). Efficiency of nationwide banks in China. *The Journal of American Academy of Business, 13*(2), 84–90.
Jiang, C. X., Yao, S. J., & Zhang, Z. Y. (2009). The effects of governance changes on bank efficiency in China: A stochastic distance function approach. *China Economic Review, 20*, 717–731.
Klein, L. R. (1953). *A textbook of econometrics*. New York: Row Peterson.
Leightner, J. E., & Lovell, C. A. K. (1998). The impact of financial liberalization on the performance of Thai banks. *Journal of Economics and Business, 50*, 115–131.
Lensink, R., Meesters, A., & Naaborg, I. (2008). Bank efficiency and foreign ownership: Do good institutions matter? *Journal of Banking & Finance, 32*, 834–844.
Lin, H. D. (2011). Foreign bank entry and firms' access to bank credit: Evidence from China. *Journal of Banking & Finance, 35*, 1000–1010.
Lovell, C. A. K., Richardson, S., Travers, P., & Wood, L. L. (1994). Resources and functioning: A new view of inequality in Australia. In W. Eichhorn (Ed.), *Models and measurement of welfare and inequality*. Berlin: Springer.

Luo, X. M., & Donthu, N. (2005). Assessing advertising media spending inefficiencies in generating sales. *Journal of Business Research, 58*, 28–36.

Luo, D., Yao, S. J., Chen, J., & Wang, J. L. (2011). World financial crisis and efficiency of Chinese commercial banks. *The World Economy, 34*, 805–825.

McDonald, J. (2009). Using least squares and tobit in second stage DEA efficiency analyses. *European Journal of Operational Research, 197*, 792–798.

Meeusen, W., & van den Broeck, J. (1977). Efficiency estimation from Cobb-Douglas production functions with composed error. *International Economic Review, 18*, 435–445.

Miller, S. M., & Noulas, A. G. (1996). The technical efficiency of large bank production. *Journal of Banking & Finance, 20*, 495–509.

Palm, F. C., & Zellner, A. (1992). To combine or not to combine? Issues of combining forecasts. *Journal of Forecasting, 11*, 687–701.

Pastor, J. T., Ruiz, J. L., & Sivent, I. (2002). A statistical test for nested radial DEA models. *Operations Research, 50*(4), 728–735.

PricewaterhouseCoopers. (2012). Foreign banks in China.

Shephard, R. W. (1970). *Theory of cost and production functions*. Princeton: Princeton University Press.

Simar, L., & Wilson, P. W. (2011). Two-stage DEA: Caveat emptor. *Journal of Productivity Analysis, 36*, 205–218.

Sturm, J. E., & Williams, B. (2004). Foreign bank entry, deregulation and bank efficiency: Lessons from the Australian experience. *Journal of Banking & Finance, 28*, 1775–1799.

Wang, H. J., & Schmidt, P. (2002). One-step and two-step estimation of the effects of exogenous variables on technical efficiency levels. *Journal of Productivity Analysis, 18*, 129–144.

Weill, L. (2004). Measuring cost efficiency in European banking: A comparison of frontier techniques. *Journal of Productivity Analysis, 21*, 133–152.

Xu, Y. (2011). Towards a more accurate measure of foreign bank entry and its impact on domestic banking performance: The case of China. *Journal of Banking & Finance, 35*, 886–901.

Yao, S. J., Han, Z. W., & Feng, G. F. (2008). Ownership reform, foreign competition and efficiency of Chinese Commercial Banks: A non-parametric approach. *The World Economy, 31*, 1310–1326.

Chapter 6
Assessing Organizations' Efficiency Adopting Complementary Perspectives: An Empirical Analysis Through Data Envelopment Analysis and Multidimensional Scaling, with an Application to Higher Education

Eva M. de la Torre, Marti Sagarra, and Tommaso Agasisti

Abstract In this chapter we integrate Data Envelopment Analysis (DEA) and Multidimensional Scaling (MDS) with the aim to discuss the potential complementarities and advantages of combining both methodologies in order to reveal the efficiency framework and strategies of organisations. To do so we use the example of the Spanish HE system. MDS empowers efficiency analysis, by means of defining areas through which universities and their ratios and efficiency indicators can be grouped and clustered, contributing to the understanding of those potential factors that are behind efficiency—and helping in explaining it. In this sense, MDS sheds light on the 'process' that leads to higher/lower levels of efficiency, conditional to universities' characteristics. We complete the study with a discussion of efficiency in three institutions.

Keywords Performance evaluation • Benchmarking • Efficiency • Data Envelopment Analysis (DEA) • Multidimensional Scaling (MDS) • Higher education

E.M. de la Torre (✉)
Departamento de Economía y Hacienda Pública, Facultad de Ciencias Económicas y Empresariales, Universidad Autónoma de Madrid, Avda. Francisco Tomás y Valiente, 5, Campus Cantoblanco, Madrid 28049, Spain
e-mail: eva.torre@uam.es

M. Sagarra
Departament d'Economia, Facultat de Ciències Econòmiques i Empresarials, Universitat de Girona, C/ de la Universitat de Girona, 10, Campus de Montilivi, Girona 17071, Spain
e-mail: marti.sagarra@udg.edu

T. Agasisti
Department of Management, Economics and Industrial Engineering, School of Management, Politecnico di Milano, Piazza Leonardo da Vinci, 32, Milano 20133, Italy
e-mail: tommaso.agasisti@polimi.it

S.-N. Hwang et al. (eds.), *Handbook of Operations Analytics Using Data Envelopment Analysis*, International Series in Operations Research & Management Science 239, DOI 10.1007/978-1-4899-7705-2_6

6.1 Introduction

Higher Education (HE) systems produce large amounts of data that need to be analysed and interpreted, and desirably understood by the wide public (e.g. citizens, policy makers and other stakeholders) in order to answer the increasing demand of accountability and transparency in the use of public funds. Combining and comparing differentiated methodologies which aggregate multiple inputs and outputs in the efficiency's evaluation of Decision Making Units (DMUs—universities in the case of this work), can result in several advantages such as obtaining a more complete understanding of the data, surpassing the inherent shortcomings of each methodology when applied separately, or making the results of the analysis more intuitive and accessible to the non-specialist.

In this chapter we combine two methodologies: (i) Data Envelopment Analysis (DEA), a non-statistical efficiency technique that applies linear programming to weight the inputs and outputs and to rank the efficiency of DMUs; and (ii) Multidimensional Scaling (MDS), a distance-based multivariate analysis technique in the production of statistical maps. The aim is to discuss the potential complementarities and advantages of combining both methodologies in order to reveal the efficiency framework and institutional strategies of the Spanish HE system. In the last years an increasing interest in the efficiency and characterisation of the HE sector has arisen because it is one of the main national driving forces for economic growth (Johnes 2008). From the point of view of efficiency, the Spanish HE system is largely a public sector, being the public universities those that account for the majority of the resources employed and the outputs produced in the sector. As a consequence of the economic crisis, the public resources have decreased and their efficient use has become a matter of major relevance, also in the case of HE. Finally, universities are increasingly acting as strategic actors, making it necessary to address the also increasing heterogeneity of the sector in order to perform unbiased assessment exercises (international rankings and evaluation agencies), and MDS is a valuable tool for addressing such heterogeneity.

Several authors have combined DEA with a multivariate statistical technique, usually Principal Component Analysis (PCA). In DEA, DMUs may become fully efficient or not depending on the inputs and outputs employed in the model specification, and so a methodology aimed at guiding model selection in DEA is desirable (Serrano-Cinca and Mar-Molinero 2004). Consequently, some authors use PCA as a method to overcome these DEA shortcomings or to compare the results of both methodologies. For instance, Zhu (1998) evaluates the economic performance of Chinese cities, Adler and Golany (2001) analyse the West European air transportation industry, and Bruce Ho and Dash Wu (2009) assess the online banking performance combining both techniques.

Instead, the joint utilization of DEA and a multivariate analysis technique is fairly new in the educational context. With the aim of showing the potential of the combination of these methods, we show the results of an empirical application of DEA and MDS to the Spanish HE system. In so doing, we formulate the specific research question: *which aspects of the Spanish universities' efficiency can be described, when analysed through the two methods—and which cannot be adequately interpreted when considering a single one?* In the empirical exercise, we combine two data sources on the Spanish HE System: the Integrated University Information System (*Sistema Integrado de Información Universitaria*—SIIU) of the Spanish Ministry of Education; and the Observatory of the Research Activity of Spanish Universities (*Observatorio de la Actividad Investigadora en la Universidad Española*—IUNE). On the one hand, the SIIU is a new platform for the collection, processing and analysis of data on the Spanish HE system. It contains extensive data on the academic activity of universities and their financial statements. On the other hand, the IUNE Observatory was established in 2012. It counts on a bibliometric team responsible for producing the IUNE's bibliometric indicators based on the Web of Science (ISI) and gathers data on research and innovation activities from Spanish administrative sources (Sanz Casado et al. 2011).

The remainder of the chapter is organized as follows. In Sect. 6.2, the two methodologies are briefly presented, and in Sect. 6.3 the dataset is described. In Sect. 6.4, DEA is applied on the data about Spanish universities, while Sect. 6.5 combines DEA and MDS methods in an empirical exercise. Section 6.6 contains some concluding remarks.

6.2 DEA and MDS Methodologies: A Brief Overview

6.2.1 The Data Envelopment Analysis Method

DEA (Farrell 1957; Charnes et al. 1978; Deprins et al. 1984) is a non-parametric method that estimates the frontier of production possibilities (or frontiers) from the observations (DMUs) in the sample through linear programming. The DMUs positioned in the frontier (universities in our example) are identified as efficient units while the level of inefficiency of those under the frontier is measured as the distance from the frontier itself. To approximate the frontier it is assumed that any university $\mathrm{j} = 1, \ldots, \mathrm{k}, \ldots, \mathrm{n}$ consumes $x = \left(x_{1j}, \ldots, x_{ij}, \ldots x_{mj}\right) \in R_{+}^{m}$ inputs to produce $y = \left(y_{1j}, \ldots, y_{rj}, \ldots, y_{sj}\right) \in R_{+}^{s}$ outputs. The different level of input consumption and output production of the universities is used to estimate an underlying technology $T^{*} = \left\{(x, y) \in R_{+}^{m} \times R_{+}^{s} \middle| x \; can \; produce \; y\right\}$ common for all universities. This underlying technology defines the production possibilities set. Finally, the efficiency score, or distance of each DMU to the frontier, is calculated as a weighted ratio between inputs and outputs.

Since in this work we focus in the public sector of the Spanish HE system, we employ an output-oriented DEA, because the resources of public institutions are strongly influenced by external political and economic factors, being the objective of institutions (and of the output oriented DEA) that one of maximizing the output given the resources available (Tone and Sahoo 2003; Duch 2006; Agasisti and Perez-Esparrells 2010; Berbegal-Mirabent et al. 2013). We also take care of the strong variability of the Spanish universities in terms of size employing the variable returns to scale (VRS) version of the DEA method, which compares DMUs with those of the same relative size when calculating the efficiency frontier. In a DEA VRS model the following assumptions are made: free disposability of inputs and outputs, or the possibility of producing less with more; convexity, which accounts for the feasibility of the weighted average of feasible production plans; no possible rescaling; and positive weights for inputs and outputs that when added are equal to 1.

Descriptions of the use of DEA in the HE sector can be found in the literature. In particular, Johnes (2006) comments on the different uses of this methodology in HE and Berbegal-Mirabent and Solé-Parellada (2012) classify the range of proxies used in DEA empirical studies. Regarding the analysis of the efficiency of the wider education sector, Worthington (2001) and De Witte and Lopez-Torres (2015) perform a literature revision of the different efficiency methodologies used, including DEA.

6.2.2 The Multidimensional Scaling Method

Multidimensional Scaling (MDS) is a non-parametric and distance-based multivariate analysis technique, which produces statistical maps from the main characteristics of the data, thus having the advantage of making the results accessible to the non-specialist in an intuitive way (Sagarra et al. 2015a, b). As Mar-Molinero and Serrano-Cinca (2001) point out, MDS implementation does not need of any sophisticated statistics, and it offers a different paradigm, a different way to look at the problem, lying its power in its accessibility. Examples of the use of MDS in the HE sector are Stenberg and Davis (1978), Mar-Molinero (1989, 1990), Mar-Molinero and Mingers (2007) and Sagarra et al. (2015a, b, 2016).

Given a set of distances, or similarities, between pairs of points, MDS has two procedures or variants to construct the statistical maps in order to locate the points in the space. The first procedure is the metric or classical scaling, which relies on the absolute value of the similarities (Chatfield and Collins 1992). The second version of the algorithm, the one used in this work, is the non-metric or ordinal scaling, which relies on the relative ordering of the similarities. It attempts to place the points in the map in such a way that if the similarity between two points is "large" they are placed next to each other, and if the similarity is "small" they are placed far apart. In this way, a simple visual inspection of the map may provide insights into the information contained in the distance (similarity) matrix. MDS generates reference scales, hence its name.

In MDS proximities can be worked out: (i) between cases (in our case universities, see for example Sagarra et al. 2015a, b), or (ii) between variables (in this case, ratios and DEA specifications). In this work, proximities are calculated between variables, not just because the number of cases can be very large and the number of variables stays normally between more reasonable limits, but because we want to explore the reasons why a university has achieved a particular DEA value, and to assess the combinations of DEA specifications with the traditional ratios. Furthermore, obtaining a complete image of the Spanish HE system structure will allow us to analyse the different strategies adopted by each university, and the formation of strategic groups.

An additional advantage of MDS is that complementary methodologies can be easily applied to add meaning to the statistical map or configuration. In this work we complement MDS results with Hierarchical Cluster Analysis and Property Fitting (a technique which relies on linear regression), so this subject is discussed in full below.

Because scaling maps are built using relationships of order, the problem of discordant or extreme observations is minimised (Coxon 1982). For the same reason, MDS is more general than PCA, but both are closely related when the data are multivariate normal and correlations are used as measures of distance (Chatfield and Collins 1992).

6.3 Data and Selection of Indicators

6.3.1 Our Sample

The exercise of complementation of the DEA and MDS methodologies presented in this work is applied to data for the academic year 2010–2011 on 47 Spanish universities, all of them public universities offering on-campus education.[1] The 47 universities considered in the analyses gather most of the resources and production of the Spanish HE system (see Table 6.1).

DEA is a methodology sensible to outliers, a limitation that does not apply to the case of MDS. Therefore, we have excluded from the sample three public universities: Universidad Internacional Menéndez Pelayo (UIMP) and Universidad Internacional de Andalucía (UNIA), which are special universities with no academic staff but able to issue university degrees; as well as Universidad Nacional de Educación a Distancia (UNED), the public university in Spain providing distance education which confers it with special characteristics. We also exclude all private universities, since they would add too much heterogeneity to the sample.

[1] A list of the universities included in the analysis and their acronyms is available in Appendix.

Table 6.1 Resources and production of the universities included and excluded in the sample

	Universities in the sample			Universities excluded from the analysis		
	Total	%	Obs.	Total	%	Obs.
Enrolled students	1,181,526	77.23	47	348,336	22.77	30
Academic staff HC	98,930	89.59	47	11,499	10.41	30
Academic staff FTE	72,598.7	89.85	47	8198.6	10.15	30
Graduates	216,061	80.90	47	51,008	19.10	30
Publications	50,605	95.31	47	2492	4.69	30
Citations	396,729	96.92	47	12,991	3.08	30
Patents	838	98.36	47	14	1.64	30

Source: Authors' elaboration

6.3.2 Inputs and Outputs Employed in the DEA Analysis

As we have already mentioned, the DEA methodology assumes that $x = (x_{1j}, \ldots, x_{ij}, \ldots x_{mj}) \in R_{+}^{m}$ inputs are used to produce $y = (y_{1j}, \ldots, y_{rj}, \ldots, y_{sj}) \in R_{+}^{s}$ outputs. In particular, our DEA specification considers that universities use two inputs to produce three outputs (see Table 6.2). This entails a very simplified framework of the production process of universities, but useful to explain our methodological choices.

As inputs, we consider the total enrolment and total academic staff full time equivalent (FTE), both of them widely used in the DEA literature (e.g.: Johnes 1996; Fandel 2007; Agasisti and Dal Bianco 2009; Katharaki and Katharakis 2010; Kempkes and Pohl 2010; Rayeni and Saljooghi 2010; Kuah and Wong 2011; Lee 2011; Duh et al. 2014; or Johnes 2014). They represent the human capital of the university, being strongly correlated to the current expenditures and the facilities available at the institution. The academic staff is the labour force of the university performing the core activities of the institution.

Similar to Thanassoulis et al. (2011), we consider three different outputs: the number of graduated students (teaching output), the number of publications (research output) and the number of patents granted (third mission output). Regarding the teaching output, several studies approximate it through the number of graduates (e.g. Wolszczak-Derlacz and Parteka 2011; Lu and Chen 2013; Duh et al. 2014). The DEA literature considers the numbers of students both, aggregated and separated by degree level. In our case, our proxy aggregates bachelor and master level students, since in 2011 in Spain there were still students taking and graduating in pre-Bologna degrees (Agasisti and Perez-Esparrells 2010). As for the number of publications, it is the most important factor for the evaluation of the academic staff in Spain, and it is a proxy of the research output widely employed in DEA studies on HEIs, being the following the most recent ones: Lee (2011), Berbegal-Mirabent et al. (2013) or Duh et al. (2014). Finally, we approximate the third mission output through the number of patents, because they recognise the legal rights to commerce inventions and "an increase in the number of university

Table 6.2 Definition of the variables employed in the DEA analysis

Type of variable	Code	Variable	Description
Inputs	A	Enrolled students[a]	Enrolled students in undergraduate and Master studies (units)
	B	Academic staff FTE[a]	Total academic staff (Full-time-equivalent) excluding those in the affiliated institutions (units)
Outputs	1	Graduates[a]	Students graduated in undergraduate and Master studies (units)
	2	Publications[b]	Number of publications in the Web of Science—ISI (units)
	3	Patents granted[c]	Number of national patents and patents with a PCT (Patent Cooperation Treaty[d]) international extension (units)

Source: Authors' elaboration
[a]*Source*: SIIU. Data on enrolment refers to the 15th of March of 2011. Data on graduates and the academic staff refers to the 31st of December of 2010
[b]*Source*: Observatorio IUNE—ISI. 2014. Data on publications refers to 2011
[c]*Source*: Observatorio IUNE—Red OTRI. 2014. Data on third mission refers to 2011
[d]The Patent Cooperation Treaty (1970) is an international patent law treaty, which allows to protect inventions in each of its contracting states with a single application, the so-called international application or PCT application. http://www.wipo.int/pct/en/

patents is therefore an indicator of transfer improvement" (Kim 2013, p. 187). Our proxy double weights those patents with an international extension PCT and most probably underestimates the total number of patents produced by universities since inventions are not always disclosed to the university Technology Transfer Office (Thursby et al. 2001; Di Gregorio and Shane 2003; Markman et al. 2007; Siegel et al. 2007).

6.3.3 Indicators Included in the MDS Analysis

The variables included in the MDS analysis are 18 ratios related to the three missions of the university (teaching, research and knowledge transfer—see Table 6.3) and the efficiency scores for the 21 possible DEA specifications given the two inputs and three outputs used in this work (Table 6.4), where specification is to be understood as a particular combination of inputs and outputs. The 18 ratios (Table 6.3) were calculated from the raw data for each university, in order to describe each Spanish university and to create rankings based on one or multiple indicators. The ratios correct the size effects of the raw data and attempt to describe: (i) the employment structure of faculty staff (fte_hc); (ii) the teaching activity: composition of the student body in terms of the discipline studied (under_enrol, humsc_grad, sci_grad and med_grad), and teaching productivity (grad_enrol); (iii) the research activity: research by disciplines (humsc_pub, sci_pub and med_pub) and the level of citation by publication (cit_pub); (iv) different productivity measures of faculty staff (enrol_fte, grad_fte, pub_fte and pat_fte); (v) the relation

Table 6.3 Definition of the 18 ratios calculated from the raw data

Ratio	Ratio description
fte_hc	Academic staff (FTE)/Academic staff (HC)
under_enrol	N. undergraduate students enrolled/N. students enrolled
humsc_grad	Graduates (Social Sciences and Humanities)/Total graduates
sci_grad	Graduates (Sciences)/Total graduates
med_grad	Graduates (Medicine)/Total graduates
humsc_pub	Publications (Social Sciences and Humanities)/Total publications
sci_pub	Publications (Sciences)/Total publications
med_pub	Publications (Medicine)/Total publications
enrol_fte	N. students enrolled/Academic staff (FTE)
grad_fte	N. graduates/Academic staff (FTE)
pub_fte	N. publications/Academic staff (FTE)
pat_fte	N. patents/Academic staff (FTE)
pat_pub	N. patents/N. publications
grad_enrol	N. graduates/N. students enrolled
pub_enrol	N. publications/N. students enrolled
pat_enrol	N. patents/N. students enrolled
cit_pub	N. citations/N. publications
size	Total enrolment (Bachelor + Master)

Source: Authors' elaboration

between the three missions of universities (pat_pub, pub_enrol and pat_enrol); and (vi) a measure of university size (size) in order to account for the possibility of non-linear effects. Through including the efficiency scores of the 21 possible DEA specifications (Table 6.4) we go beyond the efficiency score that a single DEA analysis that includes all the inputs and outputs assigns to each university and we explore the reasons why a university has achieved a particular score (Sagarra et al. 2016).

6.4 Studying HEIs' Efficiency by Means of Data Envelopment Analysis: Results

In an output oriented analysis efficient units take efficiency scores equal to 1 and inefficient units higher than 1 (the production of outputs can be increased without increasing the inputs level). However, to make the interpretation of results easier we present the inverse of the efficiency scores, so efficient units keep taking efficiency scores equal to 1 but inefficient units score under 1. Table 6.5 contains a ranking of the Spanish universities based on their efficiency scores and Table 6.6 shows some descriptive statistics on these scores.

This analysis labels 16 universities as efficient (34.04 % of the sample). According to our results, the average efficiency of the Spanish HE system is

Table 6.4 DEA specifications[a]

Specification	Inputs	Outputs
A1	A	1
AB1	A, B	1
B1	B	1
A2	A	2
AB2	A, B	2
B2	B	2
A3	A	3
AB3	A, B	3
B3	B	3
A12	A	1, 2
AB12	A, B	1, 2
B12	B	1, 2
A13	A	1, 3
AB13	A, B	1, 3
B13	B	1, 3
A23	A	2, 3
AB23	A, B	2, 3
B23	B	2, 3
A123	A	1, 2, 3
AB123	A, B	1, 2, 3
B123	B	1, 2, 3

Source: Authors' elaboration
[a]The meaning of the codes A, B, 1, 2, and 3 can be consulted in Table 6.2

0.911 and the standard deviation is 0.108. Despite the heterogeneity regarding the size and productivity of the Spanish public universities, the Spanish HE sector is a fairly efficient system and the variability of the efficiency scores is quite low. Among the universities with lower efficiency levels there are those located in the Canary Islands (ULL and ULPGC) and the efficient universities are mostly located in Catalonia (UAB, UB, UPC and UPF) and Madrid (UAH, UAM, UCM and UPM), which indicates that the special location of these universities confers them with significant differences in efficiency with respect to those universities located in other regions. Additionally, Table 6.7 shows that, on average, that efficient universities use more inputs than the inefficient ones to produce more output. Moreover, if we consider the weights endogenously assigned by the DEA method to each institution as proxies of their "implicit" strategy, Table 6.8 reveals that the Spanish HE system is strongly teaching oriented (weights are specially high for the numbers of students) and that the efficient sector of the system shows a production structure relatively more oriented to research and third mission activities than the less efficient sector.

Table 6.5 Ranking of Spanish public universities according to the efficiency scores

N.	Univ.	Eff. scores	N.	Univ.	Eff. scores	N.	Univ.	Eff. scores
1	UAB	1	17	EHU	0.997	33	UVA	0.876
2	UAH	1	18	UNEX	0.992	34	UA	0.867
3	UAM	1	19	UM	0.990	35	UHU	0.860
4	UB	1	20	UPO	0.989	36	UAL	0.859
5	UCM	1	21	UCLM	0.967	37	UIB	0.856
6	UMA	1	22	USC	0.956	38	UNIOVI	0.854
7	UNAVARRA	1	23	USAL	0.945	39	UBU	0.840
8	UNILEON	1	24	URJC	0.938	40	UMH	0.834
9	UNIRIOJA	1	25	URV	0.931	41	UJAEN	0.797
10	UPC	1	26	UNICAN	0.926	42	UDG	0.791
11	UPCT	1	27	UCA	0.925	43	UDC	0.744
12	UPF	1	28	UPV	0.922	44	UCO	0.734
13	UPM	1	29	UNIZAR	0.910	45	ULL	0.650
14	US	1	30	UGR	0.906	46	UJI	0.589
15	UV	1	31	UC3M	0.897	47	ULPGC	0.589
16	UVIGO	1	32	UDL	0.891			

Source: Author's elaboration

Table 6.6 Descriptive statistics on the efficiency scores by DEA specification. 2010–2011

	Min	Q1	Median	Mean	Q3	Max	SD	N. of efficient universities	% of efficient universities
Eff. scores	0.589	0.859	0.938	0.911	1	1	0.108	16	34.04 %

Source: Author's elaboration

Table 6.7 Average use of inputs and average production of outputs for efficient and inefficient universities

	A. staff (FTE)	Enrolm.	Grads.	Pubs.	Patents
efficient	1855.0	31,165.9	5742.8	1554.4	26.9
inefficient	1384.5	22,028.1	4005.7	830.2	13.1
difference	**470.5**	**9137.8**	**1737.1**	**724.2**	**13.8**
difference %	**34.0 %**	**41.5 %**	**43.4 %**	**87.2 %**	**105.3 %**

Source: Authors' elaboration

6.5 Combining DEA and MDS Methodologies: Results

6.5.1 Preliminary Insights

As we have already mentioned, we apply MDS to a data set of 39 variables: 18 ratios related to the three missions of the university and the efficiency scores for 21 DEA specifications. Applying MDS we reduce the dimensionality of the data. As it is

Table 6.8 Average weights assigned to inputs and outputs

	N		A. staff	Enrolment	Graduates	Publications	Patents
All universities	47	Average	**0.270**	**0.709**	**0.822**	**0.044**	**0.135**
		SD	0.429	0.523	0.261	0.141	0.190
Efficient universities	16	Average	**0.210**	**0.582**	**0.670**	**0.109**	**0.222**
		SD	0.367	0.525	0.377	0.227	0.272
Inefficient universities	31	Average	**0.301**	**0.774**	**0.900**	**0.010**	**0.090**
		SD	0.459	0.499	0.109	0.021	0.102

Source: Authors' elaboration

Table 6.9 Stress_1 and dimensionality

Dimension	Stress_1
1	0.2808
2	0.1526
3	0.1017
4	0.0633
5	0.0387
6	0.0326

Source: Authors' elaboration

common practice in MDS, we assess this dimensionality using the Stress_1 statistic (Kruskal and Wish 1978). Table 6.9 and Fig. 6.1 show the Stress_1 statistic for each dimensionality. The configuration in six dimensions returns a Stress_1 value of 0.0326, which is considered as "excellent" in Kruskal's (1964) verbal classification. Since the more dimensions in the configuration, the better the fit, we have performed the analysis with an additional (seventh) dimension but it contributes very little to reducing the stress and the results are equivalent to the six-dimensional configuration, indicating that a configuration in six dimensions is appropriate.

Therefore, each variable has been represented through a set of coordinates in a six-dimensional space. However, it is not possible to visualise a six-dimensional map and we are forced to work with projections onto pairs of dimensions. The projection of the configuration on Dimensions 1 and 2 can be seen in Fig. 6.2, and the projection of the configuration on Dimensions 2 and 3 can be seen in Fig. 6.3. The scales in Figs. 6.2 and 6.3 range from -3 to $+3$ because the algorithm automatically standardises the dimensions to mean zero and unit variance.

The next step in MDS methodology is to interpret the configuration. We have selected Property Fitting and Hierarchical Cluster Analysis to do so. Property Fitting (ProFit) is a technique which relies on linear regression and that comes under the general umbrella of biplots (Gower and Hand 1996; Mar-Molinero and Mingers 2007). It explores, with a series of vectors through the configuration, if a particular characteristic of the data grows in a given direction. Following the procedure given by Sagarra et al. (2016), the first step is to standardise the 39 variables of the study to mean zero and unit variance, in order to make the results unit-independent. The hypothesis is that the values of the 39 standardised variables (for the 47 universities) can be explained in relation to the statistical configuration

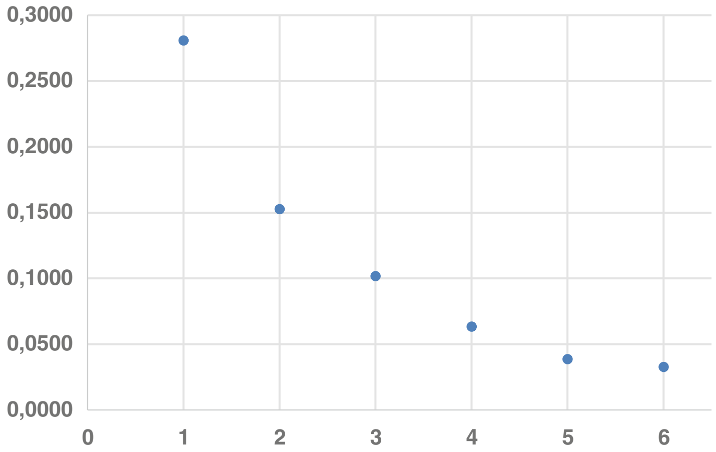


Fig. 6.1 Elbow diagram. *Source*: Authors' elaboration

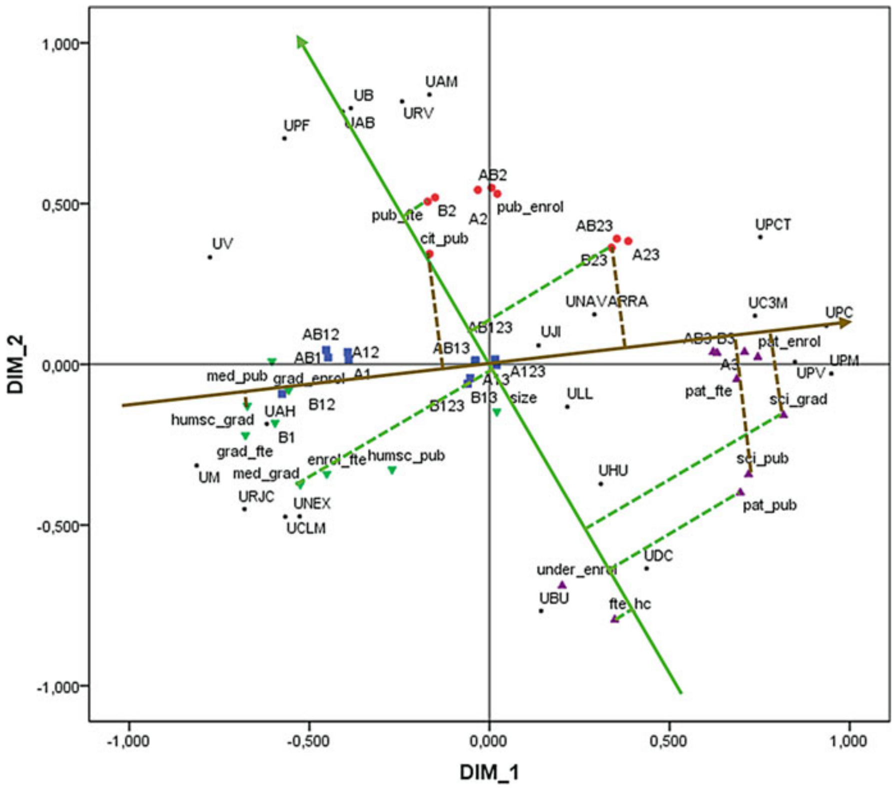


Fig. 6.2 Multidimensional scaling configuration in Dimensions 1 and 2. *Source*: Authors' elaboration

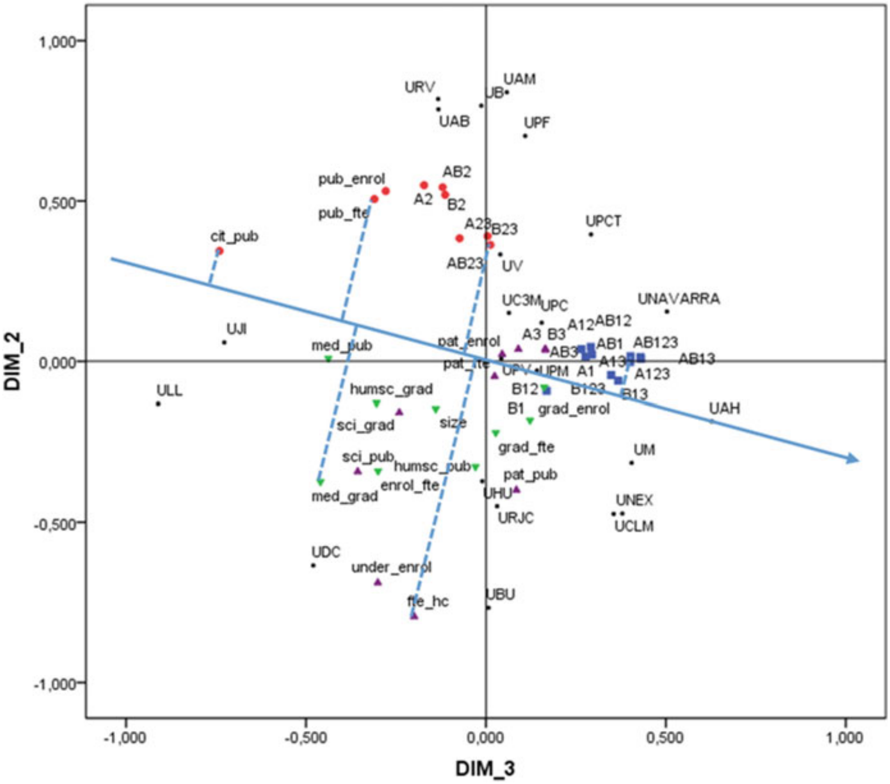


Fig. 6.3 Multidimensional scaling configuration in Dimensions 2 and 3. *Source*: Authors' elaboration

derived from the analysis, meaning that the position of the variables in the configuration explains the values of the 39 standardised variables for each university. We run as many linear regressions as universities we have, using each university as dependent variables, and the coordinates of the variables in the space (dimensions) for that university as independent variables, in order to plot a column vector for each university, whose elements are the standardised values of the 39 variables for that university. Table 6.10 shows the directional cosines for the universities in the study. Since we use linear regression, we measure the quality of fit by means of the coefficient of determination, the adjusted R^2, also shown in Table 6.10.

We have not completely represented all the universities with R^2 values higher than 0.5, in order not to clutter the representation. We have plotted in Figs. 6.2 and 6.3 the points of 22 representative universities from Table 6.10 (with very high R^2 values and with different sizes and very well differentiated strategies), and we have drawn the vector lines of three universities, UAB and UPC (Fig. 6.2) and UAH (Fig. 6.3), in order to explain the interpretation of the technique. Although both the configuration and the lines belong to a space of six dimensions (i.e. the higher the

Table 6.10 Results of ProFit analysis

University	Dim1	Dim2	Dim3	Dim4	Dim5	Dim6	Adjusted R^2
EHU	−0.118	−0.116	0.260	−0.661	−0.284	0.238	0.607
UA	0.045	−0.604	−0.058	0.545	0.103	0.314	0.734
UAB	−0.408	0.786	−0.132	−0.115	0.212	−0.003	0.833
UAH	−0.618	−0.185	0.627	0.067	0.054	−0.132	0.803
UAL	−0.530	−0.481	−0.043	0.358	−0.080	−0.308	0.696
UAM	−0.167	0.839	0.058	−0.202	0.016	−0.155	0.764
UB	−0.385	0.797	−0.013	−0.135	0.354	0.115	0.930
UBU	0.143	−0.767	0.007	0.094	−0.413	0.232	0.812
UC3M	0.737	0.151	0.064	0.321	−0.331	−0.074	0.747
UCA	−0.039	−0.773	0.354	0.136	0.012	0.022	0.696
UCLM	−0.567	−0.474	0.355	−0.328	−0.168	0.115	0.788
UCM	−0.539	0.164	0.291	−0.142	0.340	0.412	0.653
UCO	0.195	−0.013	−0.815	0.198	0.271	−0.032	0.782
UDC	0.436	−0.635	−0.480	0.130	0.073	0.058	0.821
UDG	−0.146	0.179	−0.688	0.120	−0.282	0.098	0.561
UDL	−0.543	0.368	−0.075	−0.097	−0.146	−0.182	0.406
UGR	−0.328	−0.058	−0.203	0.026	0.549	0.298	0.458
UHU	0.309	−0.372	−0.010	0.719	−0.308	−0.174	0.854
UIB	−0.428	0.144	−0.442	0.516	0.154	−0.015	0.631
UJAEN	0.204	−0.733	−0.400	0.321	0.027	−0.056	0.816
UJI	0.136	0.059	−0.727	0.593	−0.083	0.030	0.892
ULL	0.216	−0.132	−0.911	−0.058	−0.041	0.094	0.891
ULPGC	0.190	−0.303	−0.767	−0.134	−0.146	0.094	0.720
UM	−0.813	−0.315	0.405	−0.051	0.088	−0.073	0.928
UMA	0.299	−0.449	0.441	0.415	0.310	0.073	0.714
UMH	−0.466	0.248	−0.397	0.069	0.333	−0.286	0.565
UNAVARRA	0.291	0.155	0.503	−0.191	−0.653	−0.223	0.850
UNEX	−0.527	−0.473	0.379	−0.465	−0.036	−0.144	0.861
UNICAN	0.120	0.593	−0.398	−0.487	−0.320	−0.090	0.848
UNILEON	−0.601	−0.486	0.233	−0.375	0.008	−0.239	0.821
UNIOVI	−0.103	−0.165	−0.591	−0.600	−0.191	0.063	0.748
UNIRIOJA	−0.105	0.051	0.464	0.121	−0.308	0.388	0.393
UNIZAR	0.259	0.375	−0.100	−0.626	−0.288	0.148	0.661
UPC	0.935	0.120	0.155	0.032	0.185	−0.131	0.959
UPCT	0.752	0.396	0.292	−0.200	−0.226	0.043	0.881
UPF	−0.569	0.703	0.109	0.321	−0.116	−0.035	0.939
UPM	0.949	−0.029	0.142	−0.001	0.183	−0.024	0.947
UPO	−0.443	−0.099	0.542	0.371	−0.373	−0.003	0.735
UPV	0.848	0.008	0.043	0.002	0.271	−0.074	0.763
URJC	−0.680	−0.450	0.031	0.243	0.343	−0.076	0.819
URV	−0.243	0.818	−0.133	−0.047	−0.230	−0.209	0.816
US	0.604	−0.236	0.322	0.082	0.439	0.260	0.752

(continued)

Table 6.10 (continued)

University	Dim1	Dim2	Dim3	Dim4	Dim5	Dim6	Adjusted R^2
USAL	−0.704	−0.325	0.092	−0.388	−0.203	0.100	0.776
USC	0.679	0.204	−0.099	−0.116	0.174	−0.250	0.547
UV	−0.776	0.333	0.040	0.023	0.401	0.067	0.860
UVA	0.182	−0.565	0.022	−0.517	−0.312	0.208	0.716
UVIGO	−0.202	0.032	0.587	0.067	0.053	−0.307	0.392

Source: Authors' elaboration

inclination of the unit vector with respect to the plane, the shorter will be the projection of the vector), we deal with the projections into two dimensions by drawing both the line and the projection of the unit vector associated with the line. For every 'vectorised' university we have projected, as an example, only six variables on to the vector.

Following Sagarra et al. (2016), if we consider a scale associated with every of the three represented lines, the zero of the scale is in the centre of coordinates of the six-dimensional configuration. As we move towards one end of the line, the scale increases in general from 0 to +3, and as we move towards the other end, the scale decreases in general from 0 to −3.

Finally, as the points representing variables in Figs. 6.2 and 6.3 are just projections of a six-dimensional configuration on a two-dimensional space, it is possible for two variables to appear near to each other in Fig. 6.2 and/or Fig. 6.3 while being far apart in the six-dimensional space. We resort to Hierarchical Cluster Analysis to find out which variables are closer to each other in the full six-dimensional space. The clustering algorithm is the one suggested by Ward, which maximises homogeneity within clusters and heterogeneity between clusters. For visualization purposes, the variables contained in each of the resultant clusters have been distinguished in Figs. 6.2 and 6.3 by using different geometric shapes (circles, squares and two types of triangles).

6.6 Results

Figure 6.2 shows the first and second dimensions of the MDS analysis. In the specific case of UAB, moving from the top to the bottom of the line we find the following variables: pub_fte, B23, med_grad, sci_grad, pat_pub and fte_hc, taking the values 3.74, 1.69, 1.16, −0.49, −0.89 and −1.55 respectively. In the case of UPC, we find sci_grad, sci_pub, pat_fte, B23, cit_pub and humsc_grad taking the values 3.30, 2.32, 3.26, 1.69, −0.42 and −3.09 respectively. In the case of UAH, we find AB123, fte_hc, B23, med_grad, pub_fte and cit_pub taking the values 0.81, −0.90, −0.37, 0.16, −0.29 and −0.80 respectively. The ordering of the projections over the line is related with the ordering of the original standardised values of the variables, although the match is not perfect. If the match had been perfect, we would have found a value of the adjusted R^2 much closer to unity.

If we analyse the position of the variables in Fig. 6.2 it is to be noticed that at the far right of the first dimension we find those efficiency (DEA) models and the ratio that contain output 3 (patents) in relation with input B (academic staff FTE), as well as ratios related to the participation of the science area in the overall university productivity and to the relationship between the production of patents and the production on teaching and research. We find in this area of the figure the polytechnic universities and UC3M, a university not classified as polytechnic but with a strong technical profile. On the opposite side, at the left of the first dimension, we find those DEA models that contain outputs 1 and 2 (graduates and publications) in relation with inputs A and B (enrolled students and academic staff FTE), and several ratios relating these inputs and outputs, as well as the importance of medicine and social science & humanities areas in the global activity of universities. The universities located at this end of the first dimension are more regional (e.g. UM, UCLM, UNEX, URJC, among others). This suggests that Dimension 1 could be labelled as "orientation towards efficiency in the third mission vs. orientation towards the traditional missions of teaching and research".

Analysing the second dimension in Fig. 6.2, it is to be noticed that at the top of the "map" are located those DEA models relating output 2 (publications) mainly to input B (academic staff FTE) but also to input A (enrolled students). We find together with these DEA specifications different ratios related to the publication productivity, such as pub_fte, pub_enrol or cit_pub. The kind of universities we find here are UPF, UAB, UB, UAM, and a more regional one, URV, being most of them located in Catalonia and Madrid (as in the case of the efficient universities according to the previous DEA analysis). This suggests that Dimension 2 could be interpreted as "orientation towards efficiency in research". Finally, we note that some variables, most of them efficiency (DEA) models, are located in the centre of Fig. 6.2. An inspection of the six-dimensional coordinates of all the variables shows us that the meaning of Dimension 3 could be explained by these variables. Figure 6.3 represents the projection of the six-dimensional configuration into Dimension 2 and 3. The third dimension appears to be associated with DEA specifications which join all three outputs 1, 2 and 3 (graduates, publications and patents) in combination with both inputs A and B (enrolled students and academic staff FTE), emphasizing here the presence of the full DEA model AB123, which joins all the inputs and outputs. This suggests that Dimension 3 could be labelled as "orientation towards overall efficiency", a strategy that seems appropriate to define the priorities of universities like UAH or UNAVARRA. We have not attempted to find meaning to the rest of dimensions since, for the purposes of this study; efficiency related effects can be well described using Fig. 6.2 and the following Fig. 6.3.

Figure 6.4 shows the dendrogram of the cluster analysis performed to identify the proximity between variables in the six-dimensional space. Although the selection of the number of clusters is at analysts' discretion, the dendrogram shows a clear division among four clusters, in line with the findings obtained from the ProFit analysis and from the interpretation of each dimension. A simple inspection of Fig. 6.4 adds meaningful findings to the previous analyses: in the first place, it confirms the full coherence between the location of ratios and their corresponding

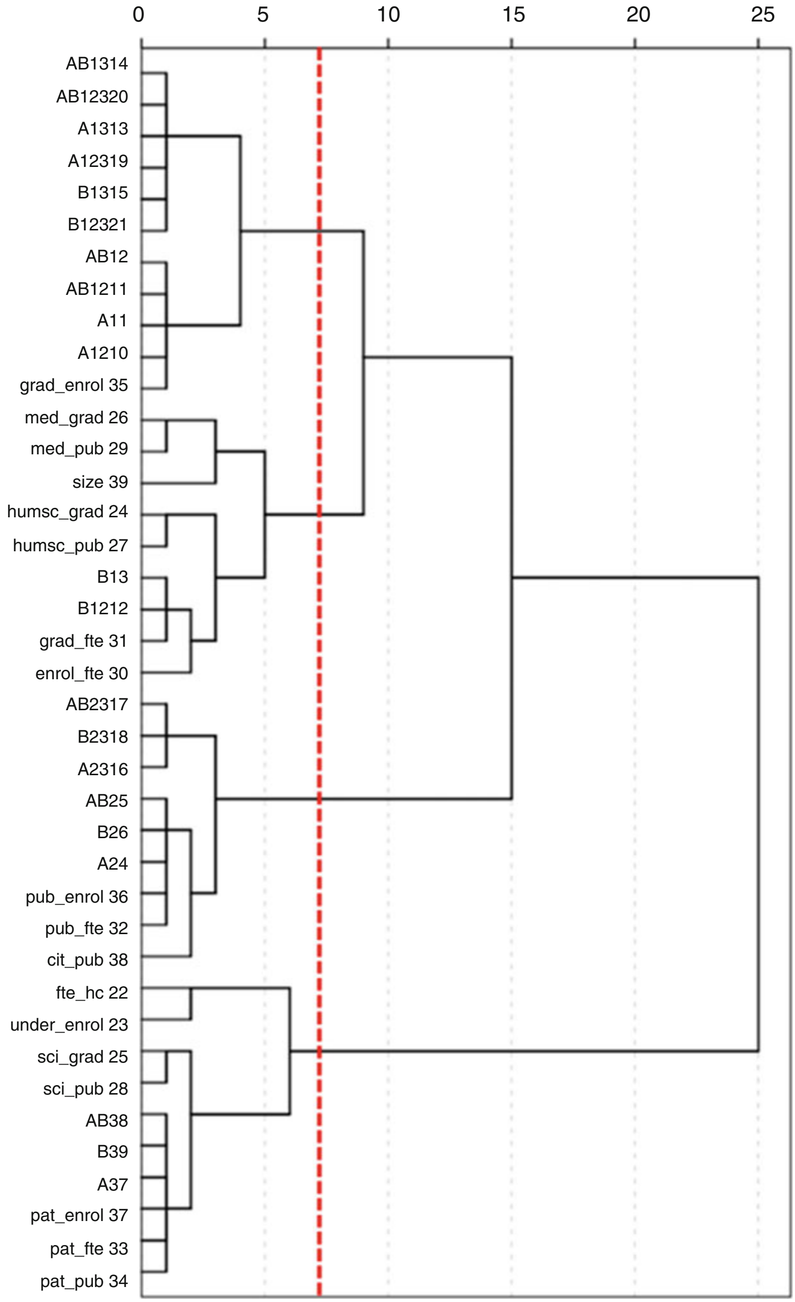


Fig. 6.4 Dendrogram for cluster analysis of variables. *Source*: Authors' elaboration

(or nearest) DEA models. In the second place, it sheds light on the mechanisms and influential factors behind the different strategies adopted by each university.

6.7 Concluding Remarks

In this contribution we have analysed the efficiency of the Spanish public HE sector through a DEA analysis, and the strategies and characteristics of its universities through the MDS method. MDS empowers efficiency analysis, by means of defining areas through which universities and their ratios and efficiency indicators can be grouped and clustered, contributing to the understanding of those potential factors that are behind efficiency—and helping in explaining it. In this sense, MDS sheds light on the 'process' that leads to higher/lower levels of efficiency, conditional to universities' characteristics. Combining two different methods to assess the performance (and characteristics) of the organizations is essential to look at the phenomena under different perspectives. Single indicators are not able to describe efficiency, while studying efficiency ratios is not sufficient to cluster universities according to their characteristics.

These results can be employed by policy makers for multiple uses. For making an example, it is advisable to take into account both efficiency scores and universities' characteristics to design incentive schemes, such as competitive fund allocations. Considering different dimensions can shed light on the strategies adopted by single universities, and can be a reference to verify ex-ante and ex-post whether the objectives set are pursued or not, by devoting efforts and resources in directions and activities which are coherent with the strategy. Also, the policy-maker can set targets of efficiency improvements to be reached—by controlling for institutions' strategies and factors used by them.

Both efficiency scores and MDS graphs may be used also by managers of single institutions. In this perspective, the two methods are an example of benchmarking tools. Through DEA, universities' managers can identify potential peers, and set targets for improving their efficiency by leveraging their inputs and outputs' weights. Through MDS, each university can individuate its own positioning in the various clusters created by the (multi)dimensional space. Combining the information, thus, each manager can decide which are the strategies to be developed, the corrections to be implemented, and the indicators to be monitored to check whether performance is improved over time in an efficient way. In other words, efficiency analyses and MDS can be some ingredients of an evolved management control system at organizational level.

However, a transversal point that must be stressed is that to unfold the potential of both policy and managerial use of data in developing evidence based policies and strategies, it is necessary to provide the involved actors with training to fully understand the potential of these—as well as their limitations.

Finally, we recall two limits of the current chapter that pave the way for future, potential extensions of this research: (i) the integration of Stochastic Frontier

Analysis with DEA and MDS would make the overall analysis more robust methodologically and (ii) the extension of the time span for considering more than 1 year would allow to study how efficiency and institutional strategies vary over time.

Appendix: List of Universities Included in the Analysis and Their Acronyms

N.	Abbreviation	University name
1	EHU	Universidad del País Vasco/Euskal Herriko Unibertsitatea
2	UA	Universidad de Alicante
3	UAB	Universitat Autònoma de Barcelona
4	UAH	Universidad de Alcalá
5	UAL	Universidad de Almería
6	UAM	Universidad Autónoma de Madrid
7	UB	Universitat de Barcelona
8	UBU	Universidad de Burgos
9	UC3M	Universidad Carlos III de Madrid
10	UCA	Universidad de Cádiz
11	UCLM	Universidad de Castilla-La Mancha
12	UCM	Universidad Complutense de Madrid
13	UCO	Universidad de Córdoba
14	UDC	Universidad de A Coruña
15	UDG	Universitat de Girona
16	UDL	Universitat de Lleida
17	UGR	Universidad de Granada
18	UHU	Universidad de Huelva
19	UIB	Universitat de les Illes Balears
20	UJAEN	Universidad de Jaén
21	UJI	Universidad Jaume I de Castellón
22	ULL	Universidad de La Laguna
23	ULPGC	Universidad de Las Palmas de Gran Canaria
24	UM	Universidad de Murcia
25	UMA	Universidad de Málaga
26	UMH	Universidad Miguel Hernández de Elche
27	UNAVARRA	Universidad Pública de Navarra
28	UNEX	Universidad de Extremadura
29	UNICAN	Universidad de Cantabria
30	UNILEON	Universidad de León
31	UNIOVI	Universidad de Oviedo
32	UNIRIOJA	Universidad de La Rioja
33	UNIZAR	Universidad de Zaragoza

(continued)

N.	Abbreviation	University name
34	UPC	Universitat Politécnica de Catalunya
35	UPCT	Universidad Politécnica de Cartagena
36	UPF	Universitat Pompeu Fabra
37	UPM	Universidad Politécnica de Madrid
38	UPO	Universidad Pablo de Olavide
39	UPV	Universidad Politécnica de Valencia
40	URJC	Universidad Rey Juan Carlos
41	URV	Universitat Rovira i Virgili
42	US	Universidad de Sevilla
43	USAL	Universidad de Salamanca
44	USC	Universidad de Santiago de Compostela
45	UV	Universitat de València (Estudi General)
46	UVA	Universidad de Valladolid
47	UVIGO	Universidad de Vigo

Source: Authors' elaboration

References

Adler, N., & Golany, B. (2001). Evaluation of deregulated airline networks using data envelopment analysis combined with principal components analysis with an application to Western Europe. *European Journal of Operational Research, 132*, 260–273.

Agasisti, T., & Dal Bianco, A. (2009). Reforming the university sector: Effects on teaching efficiency—Evidence from Italy. *Higher Education, 57*, 477–498.

Agasisti, T., & Perez-Esparrells, C. (2010). Comparing efficiency in a cross-country perspective: The case of Italian and Spanish state universities. *Higher Education, 59*, 85–103.

Berbegal-Mirabent, J., Lafuente, E., & Solé, F. (2013). The pursuit of knowledge transfer activities: An efficiency analysis of Spanish universities. *Journal of Business Research, 66*, 2051–2059.

Berbegal-Mirabent, J., & Solé-Parellada, F. (2012). What are we measuring when evaluating universities' efficiency? *Regional and Sectoral Economics Studies, 12*(3), 31–46.

Bruce Ho, C.-T., & Dash Wu, D. (2009). Online banking performance evaluation using data envelopment analysis and principal component analysis. *Computers & Operations Research, 36*, 1835–1842.

Charnes, A., Cooper, W., & Rhodes, E. (1978). Measuring the efficiency of decision making units. *European Journal of Operational Research, 2*(6), 429–444.

Chatfield, C., & Collins, A. J. (1992). *Introduction to multivariate analysis*. London: Chapman and Hall.

Coxon, A. P. M. (1982). *The user's guide to multidimensional scaling*. London: Heinemann Educational Books.

De Witte, K., & Lopez-Torres, L. (2015). *Efficiency in education. A review of literature and a way forward*. TIER Working Paper Series. Mimeo

Deprins, D., Simar, L., & Tulkens, H. (1984). Measuring labor efficiency in post offices. In M. Marchand, P. Pestieau, & H. Tulkens (Eds.), *The performance of public enterprises: Concepts and measurements*. Amsterdam: North Holland.

Di Gregorio, D., & Shane, S. (2003). Why do some universities generate more start-ups than others? *Research Policy, 32*(2), 209–227.

Duch, N. (2006) La eficiencia de las universidades españolas. In *Informe C y D 2006* (pp. 310–325). Barcelona: Fundación CyD.

Duh, R.-R., Chen, K.-T., Lin, R.-C., & Kuo, L.-C. (2014). Do internal controls improve operating efficiency of universities? *Annals of Operations Research, 221*, 173–195.

Fandel, G. (2007). On the performance of universities in North Rhine-Westphalia, Germany: Government's redistribution of funds judged using DEA efficiency measures. *European Journal of Operational Research, 176*(1), 521–533.

Farrell, M. J. (1957). The measurement of productive efficiency. *Journal of the Royal Statistical Society, A/120/3,* 253–290.

Gower, J. C., & Hand, D. J. (1996). *Biplots*. London: Chapman & Hall.

Johnes, J. (1996). Performance assessment in higher education in Britain. *European Journal of Operational Research, 89*, 18–33.

Johnes, J. (2006). Data envelopment analysis and its application to the measurement of efficiency in higher education. *Economics of Education Review, 25*(3), 273–288.

Johnes, J. (2008). Efficiency and productivity change in the English higher education sector from 1996/97 to 2004/5. *The Manchester School, 76*(6), 653–674.

Johnes, J. (2014). Efficiency and mergers in English higher education 1996/97 to 2008/09: Parametric and non-parametric estimation of the multi-input multi-output distance function. *The Manchester School, 82*(4), 465–487.

Katharaki, M., & Katharakis, G. (2010). A comparative assessment of Greek universities' efficiency using quantitative analysis. *International Journal of Educational Research, 49*(4–5), 115–128.

Kempkes, G., & Pohl, C. (2010). The efficiency of German universities—Some evidence from nonparametric and parametric methods. *Applied Economics, 42*(16), 2063–2079.

Kim, Y. (2013). The ivory tower approach to entrepreneurial linkage: Productivity changes in university technology transfer. *The Journal of Technology Transfer, 38*(2), 180–197. doi:10.1007/s10961-011-9217-8.

Kruskal, J. B. (1964). Multidimensional scaling by optimizing goodness of fit to a non-metric hypothesis. *Psychometrika, 29*(1), 1–27.

Kruskal, J. B., & Wish, M. (1978). *Multidimensional scaling*. London: Sage.

Kuah, C. T., & Wong, K. Y. (2011). Efficiency assessment of universities through data envelopment analysis. *Procedia Computer Science, 3*, 499–506.

Lee, B. L. (2011). Efficiency of research performance of Australian Universities: A reappraisal using a bootstrap truncated regression approach. *Journal of Economic Analysis and Policy, 41* (3), 195–203.

Lu, Y.-H., & Chen, K.-H. (2013). Appraising the cost efficiency of higher technological and vocational education institutions in Taiwan using the metafrontier cost-function model. *Research in Higher Education, 54*(6), 627–663.

Markman, G., Gianiodis, P., & Phan, P. (2007). An agency theoretic study of the relationship between knowledge agents and university technology transfer offices. *IEEE Transactions on Engineering Management, 55*, 29–36.

Mar-Molinero, C. (1989). A multidimensional scaling analysis of the 1986 ratings of universities in the UK. *Higher Education Review, 21*, 7–25.

Mar-Molinero, C. (1990). Understanding university undergraduate applications. *Higher Education Review, 22*, 7–23.

Mar-Molinero, C., & Mingers, J. (2007). Mapping MBA Programmes: An alternative analysis. *Journal of the Operational Research Society, 58*(1), 874–886.

Mar-Molinero, C., & Serrano-Cinca, C. (2001). Bank failure: A multidimensional scaling approach. *The European Journal of Finance, 7*, 165–183.

Rayeni, M. M., & Saljooghi, F. H. (2010). Network data envelopment analysis model for estimating efficiency and productivity in universities. *Journal of Computer Science, 6*(11), 1252–1257.

Sagarra, M., Mar-Molinero, C., & Agasisti, T. (2016). *Exploring the efficiency of Mexican universities: Integrating data envelopment analysis and multidimensional scaling*. *Omega*. In press.

Sagarra, M., Mar-Molinero, C., & García-Cestona, M. (2015). Spanish savings banks in the credit crunch: Could distress have been predicted before the crisis? A multivariate statistical analysis. *The European Journal of Finance, 21*(3), 195–214.

Sagarra, M., Mar-Molinero, C., & Rodríguez-Regordosa, H. (2015). Evaluating the success of educational policy in Mexican Higher Education. *Higher Education, 69*(3), 449–469.

Sanz Casado, E., De Filippo, D., García Zorita, C., & Efraín-García, P. (2011). Observatorio IUNE: una nueva herramienta para el seguimiento de la actividad investigadora del sistema universitario español. *Bordon, 63*(2), 101–115.

Serrano-Cinca, C., & Mar-Molinero, C. (2004). Selecting DEA specifications and ranking units via PCA. *Journal of the Operational Research Society, 55*, 521–528.

Siegel, D. S., Veugelers, R., & Wright, M. (2007). Technology transfer offices and commercialization of university intellectual property: Performance and policy implications. *Oxford Review of Economic Policy, 23*(4), 640–660.

Stenberg, R. J., & Davis, J. C. (1978). Student perceptions of Yale and its competitors. *College and University, 53*, 262–279.

Thanassoulis, E., Kortelainen, M., Johnes, G., & Johnes, J. (2011). Costs and efficiency of higher education institutions in England: A DEA analysis. *Journal of the Operational Research Society, 62*(7), 1282–1297.

Thursby, J. G., Jensen, R., & Thursby, M. C. (2001). Objectives, characteristics and outcomes of university licensing: A survey of major US universities. *Journal of Technology Transfer, 26*, 59–72.

Tone, K., & Sahoo, B. (2003). Scale, indivisibilities and production function in data envelopment analysis. *International Journal of Production Economics, 84*(2), 165–192.

Wolszczak-Derlacz, J., & Parteka, A. (2011). Efficiency of European public higher education institutions: A two-stage multicountry approach. *Scientometrics, 89*, 887–917.

Worthington, A. C. (2001). An empirical survey of frontier efficiency measurement techniques in education. *Education Economics, 9*(3), 245–268.

Zhu, J. (1998). Data envelopment analysis vs. principal component analysis: An illustrative study of economic performance of Chinese cities. *European Journal of Operational Research, 111*, 50–61.

Chapter 7
Capital Stock and Performance of R&D Organizations: A Dynamic DEA-ANP Hybrid Approach

Yueh-Cheng Wu, Qian Long Kweh, Wen-Min Lu, Shiu-Wan Hung, and Chia-Fa Chang

Abstract Assessing resource allocation in R&D organizations is an important issue that requires a comprehensive measure to characterize it. To provide a greater picture, we first construct a dynamic three-stage network DEA model, which evaluates the R&D efficiency, technology-diffusion efficiency, and value-creation efficiency of Taiwanese R&D organizations over the period 2005–2009. Before integrating window analysis and network data envelopment analysis (DEA) to estimate dynamic efficiencies, we apply Analytic Network Process (ANP) to determine the relative importance of each stage. Subsequently, we employ panel data regression to examine whether the capital stock of patents, quality of human resources, and capability of service support affect the dynamic efficiencies of the R&D organizations. Our findings show that the mean R&D efficiency score is greater than that of the technology-diffusion efficiency, with the value-creation efficiency score being the lowest, suggesting that R&D organizations have to firstly work on improving the technology-diffusion inefficiency, and finally improving the value-creation inefficiency. Our panel data regression analysis indicates that the

Y.-C. Wu
Department of Cultural Vocation Development, National Taipei University of Technology, 1, Sec. 3, Zhongxiao E. Rd., Taipei 10608, Taiwan
e-mail: ywu@ntut.edu.tw

Q.L. Kweh
Department of Accounting, College of Business Management and Accounting, Universiti Tenaga Nasional, Sultan Haji Ahmad Shah Campus, Muadzam Shah, Pahang 26700, Malaysia
e-mail: qlkweh@gmail.com

W.-M. Lu (✉) • C.-F. Chang
Department of Financial Management, National Defense University, No. 70, Sec. 2, Zhongyang North Rd., Beitou, Taipei 112, Taiwan
e-mail: wenmin.lu@gmail.com; chiafa.chang@gmail.com

S.-W. Hung
Department of Business Administration, National Central University, No. 300, Jung-Da Road, Jung-Li City 320, Tao-Yuan, Taiwan
e-mail: shiuwan@mgt.ncu.edu.tw

S.-N. Hwang et al. (eds.), *Handbook of Operations Analytics Using Data Envelopment Analysis*, International Series in Operations Research & Management Science 239, DOI 10.1007/978-1-4899-7705-2_7

capital stock of patents do affect the efficiencies of the R&D organizations, even including the quality of human resources and capability of service support. That is, managers should focus on technological development and innovation to improve their corporate performance.

Keywords Network data envelopment analysis • Analytic Network Process • Window analysis • R&D organizations • Patents

7.1 Introduction

From the perspective of a dynamic and three-stage data envelopment analysis (DEA) procedure, this study integrates window analysis and network DEA as well as Analytic Network Process (ANP) to evaluate the research and development (R&D) efficiency, technology-diffusion efficiency, and value-creation efficiency of Taiwanese R&D organizations over the period 2005–2009. This study further investigates changes in the efficiency scores of the R&D organizations in different industries from a long-term perspective. Furthermore, from the viewpoint of organizational innovation, this study examines the impacts of the capital stock of patents, quality of human resources, and capability of service support on the performance of the R&D organizations. This relation is a key input into the continuing discussion on the role of innovation in corporate performance. Recent years have seen a shift in attention from a focus on labor-intensive environment to an emphasis on emphasizing knowledge-intensive environment (Efrat 2014), whereby technological development has become a key factor in a country's competitiveness. That is, countries around the world formulate policies to encourage the development of science and technology as well as their innovation in order to sustain economic growth. In this regard, R&D organizations play a vital role in achieving technological innovation in a country (Lu and Hung 2011).

In this study, we focus our analysis on Taiwan because it serves as a suitable setting to examine the above-stated purposes. In 2007, the Science, Technology and Industry Scoreboard released by the Organization for Economic Cooperation and Development (OECD) documents that most of the OECD countries including Taiwan prioritize technology and innovation in stimulating economic growth. In fact, Taiwan has progressed from a labor-intensive economy to a capital-intensive and technology-intensive economy since the 1950s. Taiwan has long emphasized the development of technology and innovation in its modernization and economic development plans. In today's challenging world, Taiwan continues to focus on developing a knowledge-intensive economy to cope well in the intense global competitive environment. According to the 2009 World Economic Forum, Taiwan has moved into the innovation-oriented period from innovation-oriented transitional period. Among the initiatives implemented by the Taiwanese government are: (i) promoting the collaboration between players in the practice and academicians, (ii) providing small and medium enterprises (SME) with consultations on innovative R&D technology, (iii) developing new technological and innovative services, and (iv) reducing the gap on technology among industrial player, to name but a few of the ventures by the country.

With the increasing emphasis on technological development and innovation, requirements for performance evaluation of R&D organizations have become more critical. Despite its obvious importance, academic studies to date do not adequately address the question of how to objectively quantify and benchmark the performance of national R&D organizations. This study addresses the issue, making several important contributions to the literature. Through analyses on R&D efficiency, technology-diffusion efficiency, and value-creation efficiency, we provide insights to assist governments in implementing performance improvement strategies to enhance competitive advantage of R&D organizations. Note also that we employ ANP analysis to obtain the relative weights for each stage of efficiency from the average scores given by five R&D managers, which are then used in the DEA analysis. Furthermore, this paper examines changes in the efficiency performance of the R&D organizations in different industries from a long-term perspective.

To effectively evaluate efficiency changes over time, a researcher can employ several data envelopment analysis (DEA) models such as window analysis (Klopp 1985), the Malmquist index (Färe et al. 1994), and the dynamic slacks-based measure (SBM) (Tone and Tsutsui 2010). DEA is a non-parametric method that utilizes mathematical programming to evaluate the relative efficiency of decision making units (DMUs) via simultaneous handling of multiple variables (Cooper et al. 2006). Note that performance evaluation is a complex process that requires more than a single criterion to characterize it, suggesting that a uni-dimensional performance measure is not capable of comprehensively assess an organization's performance evaluation (Hung et al. 2013; Zhu 2009). However, the traditional DEA approach not only neglect changes in efficiency across several periods, but also disregard intermediate measures or linking activities (Chen and Zhu 2004; Tone and Tsutsui 2009). To address the problem, we integrate window analysis (Klopp 1985) and a network DEA model (Tone and Tsutsui 2009). Specifically, we evaluate the performance of Taiwanese R&D organizations through a hybrid approach based on a dynamic network DEA and ANP.

For the first time, as far as we know, we also document the impact of the capital stock of patents on R&D efficiency, technology-diffusion efficiency, and value-creation efficiency. This effect is present even including quality of human resources and capability of service support in our panel data regression models, with the exception of value-creation efficiency. In the last decade, we have seen mounting evidence of the usefulness of the capital stock of patents. For example, Guellec and Bruno (2004) and Wang and Huang (2007) argue that the capital stock of patents serves as an indicator to understand the competitive advantage and ultimately the performance of an organization.

The remainder of this study is organized as follows. Section 7.2 discusses the literature on the current status of Taiwanese R&D organizations and DEA applications in R&D organizations. Section 7.3 describes the research design of this study. Section 7.4 presents the results. A final section concludes the paper.

7.2 Literature Review

7.2.1 Current Status of Taiwanese R&D Organizations

R&D organizations are research institutes established by government to develop effective technology improvement plans and to transfer their technological development and innovation to industries (Edquist 1997). R&D organizations play an important role in the innovation system of a country, whereby they coordinate and execute R&D activities in the country. To ensure ordered allocation of national resources and to rapidly grow SMEs' skills and knowledge in their industries, R&D organizations are also responsible to help SMEs to engage in R&D projects of government-owned corporations.

In Taiwan, the Department of Industrial Technology of the Ministry of Economic Affairs has established many R&D organizations like Institute for Information Industry, Development Center for Biotechnology, Metal Industries Research & Development Center, Food Industry Research & Development Institute, Taiwan Textile Research Institute, Cycling & Health Industry R&D Center, United Ship Design & Development Center, Stone & Resource Industry R&D Center, Printing Technology Research Institute, Plastics Industry Development Center, Precision Machinery Research Development Center, Medical and Pharmaceutical and Development Center, Footwear & Recreation Technology Research Institute, and Animal Technology Institute Taiwan, all of which are to support industrial development, to build the high-tech industries in Taiwan, to achieve technological development and innovation, and ultimately to improve the nation's competitive advantage.

Another R&D organization in Taiwan is Chung Shan Institute of Science and Technology (CSIST) under the Ministry of Defense. Since 1969, CSIST have been developing many systems and architects of national defense; even though it is no longer a military unit, it is still an important resource of defense technology of Taiwan. Its key R&D activities include the areas of electronics, information warfare, and advanced weapon system. It positively interacts with other major research institutes in Taiwan, and expands its R&D activities to many universities in order to boost academic involvement in the national defense technology. Under government policy, CSIST actively joins research projects on technological development and focuses their target on technologies that are beneficial to military and civilian.

Although various works have been completed to investigate the operating performance of R&D organizations, there is little convincing evidence that examines the dynamic performance of R&D organizations. In the first stage, this study integrates window analysis and network DEA to evaluate the performance of R&D organizations in terms of R&D efficiency, technology-diffusion efficiency, and value-creation efficiency, a three-stage DEA analysis. Through this innovative approach, we not only can understand differences in managerial performance of R&D organizations, but also can find out efficiency changes of R&D organizations over long-term periods. This study aims to provide such information with insights into resource allocation that could help managers in making strategic decision to improve their competitive advantage.

7.2.2 DEA Applications in R&D Organizations

Research into the effects of R&D investment on improving productivity has a long history (see for example, González and Gascón 2004; Griliches 1988; Hartmann 2003; Mansfield 1980, 1988; Saiki et al. 2006; Walwyn 2007). Lee and Park (2005) argue that measuring R&D productivity is a prerequisite for improving R&D productivity. Using the DEA approach, the authors measure the relative R&D efficiency of Asian countries. Since R&D policy is an important national agenda, we contend that evaluation of resources allocation and value creation of R&D organizations in a country should be highlighted, in line with the study by Lee et al. (2009) that evaluates the efficiency of national R&D programs. Other studies have also applied DEA to examine relative R&D efficiency across countries, including the US and Japan (Co and Chew 1997), European countries (Rousseau and Rousseau 1998), and developed and developing countries (Sharma and Thomas 2008).

Among all, Rousseau and Rousseau (1997) are among the first scholars to recommend the use of DEA for estimating the relative national/inter-countries R&D efficiencies. After that, the two similar scholars apply DEA again to gauge the R&D efficiency of European countries (Rousseau and Rousseau 1998). Another early study by Co and Chew (1997) on applying DEA to take into consideration R&D expenditures is another effective study that answers the question of whether firms in the U.S. or those in Japan perform better. Other subsequent DEA-application studies (for example, Nasierowski and Arcelus 2003; Wang and Huang 2007) apply a two-step approach, in which they regress environmental factors on R&D efficiency. Besides individual efficiency analysis, they also answer what factors are contributive to productivity.

Guan and Chen (2012) introduce an innovated concept to further enrich the R&D performance measurement research, whereby they separate the R&D process into two stages of efficiency measures, namely knowledge production process and knowledge commercialization process. After estimating efficiency, approximating to Nasierowski and Arcelus (2003), the authors also analyze a regression model to examine the effects of environmental factors on the efficiency. Lu et al. (2014) also apply the same concept in studying the national innovation systems in 30 countries. Other relevant DEA-application studies include Zhang et al. (2003) who employ stochastic frontier analysis (SFA) to examine the R&D efficiency and productivity of firms in mainland China; Cherchye and Abeele (2005) gauge the R&D efficiency of universities in Finland and the Netherlands, respectively.

While the two-stage process model is useful, it is subject to one limitation, that is, the weights given to each stage of process model are subjective. To date, a number of studies have estimate efficiency based on a hybrid approach of DEA and

ANP (Sipahi and Timor 2010). As ANP is able to categorize and analyze complicated decision makings, a researcher may first use the technique to obtain relative weights for two or more stages of process model, which would be next used in the efficiency analysis. Furthermore, extant DEA-application studies in the R&D field may not be sufficient as they generally ignore changes in efficiency over times or dynamic performance in today's dynamic world. In summary, although several studies have been carried out to explore R&D efficiency, this study identifies a gap that should be filled.

7.3 Research Design

7.3.1 Three-Stage Value-Creation Process of R&D Organizations

Most of the existing performance evaluation studies on R&D organizations depend on efficiency specifications of R&D projects. They divide organizational efficiency into input, output, and result application and economic efficiency, which are the four major processes of R&D activities. According to the Execution Efficiency Report on Science Projects of Artificial Person Institutions, standards to evaluate the efficiencies of R&D organizations are organization development, R&D development, and industry efficiency as at the year of 2010. In line with prior studies, this study applies DEA, particularly the evaluation model created by Tone and Tsutsui (2009) to build our three-stage value-creation process of R&D organizations. Our specifications of value-creation process of R&D organizations are consistent with that of Lu et al. (2010). See Fig. 7.1 regarding the building specifications of value-creation process of R&D organizations.

In terms of the selection of input and output variables, we follow prior studies (Hsu 2005; Liu and Lu 2010; Wu et al. 2006) and base our selection on the data availability in the annual report of Execution of R&D Projects of 2009. In the stage of R&D efficiency, we examine the efficiency of R&D organizations in utilizing human resources, time and funds to generate research outputs and intellectual properties. That is, we use manpower, research time, and research funds as input variables, and patents, technology acquired, research reports, research publications, and outsourced research as output variables. In the stage of technology-diffusion efficiency, we evaluate how well research outputs and intellectual properties are disseminated. At this stage, input variables are the outputs from the stage of R&D efficiency, while output variables are patents transferred, technology transferred, technology services, and seminar. The third stage, the stage of value-creation efficiency, discusses the generation of value from the technology diffusion. The ultimate outputs include investment and production value. Table 7.1 summarizes definitions of input and output variables.

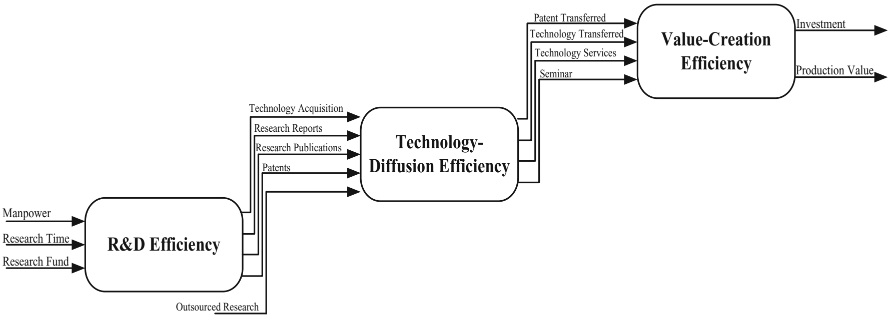


Fig. 7.1 Value-creation process of R&D organizations

Table 7.1 Definitions of the input and output variables

Item	Definition	Unit
Manpower	Manpower is the total number of personnel engaged in each project	Number
Time	Time is the total executive time of each project	Month
Budget	Budget includes all money invested in each project	Thousand
Patents	Patents are the number of patents produced by each project	Number
Technology Acquisition	Technology acquisition includes the planned, selective, focalized	Number
Research Reports	Completed the implementation of the project study report number of articles, including technical, research, training and other reports	Number
Publications	Publications include all papers and reports published by each project	Number
Sub-study	Research activities, some of the work plans by the industry or academia responsible	Number
Patent Transfer	Technology plan, through technology transfer, licensing patents to manufacturers to use the license and royalty income	Thousand
Technology Transfer	Technology and patent transfer include all technology and patent transferred to the firms by each project	Number
Technology Services	Technology services are the services provided by each project for product development, equipment calibration and maintenance, technical supports, etc., to the firms	Number
Seminar	Will result in an open manner to explain the activities of the industry, including technical seminars, training workshops, technical seminars, presentations	Number
Investment	Firm investments are the investments made by various firms for new technologies or production due to each project	Thousand
Production Value	The key results achieved through the transfer of production technology, to promote the industry to expand the production scale of the original	Thousand

Source: Definitions from the Ministry of Economic Affairs that are available in the 2009 Annual Report

7.3.2 Data Selection and Description

According to the annual reports of Execution of R&D Projects from the Department of Industrial Technology, the numbers of R&D projects executed by artificial persons are far more than industries and academics between 2005 and 2009. In other words, artificial person institutes are the major R&D power in Taiwan. We thus use 29 artificial person R&D organizations as our study objects, including eight R&D units under ITRI, six units under CSIST of Ministry of National Defense and Institute for Information Industry, Development Center for Biotechnology, Metal Industries Research & Development Center, Food Industry Research & Development Institute, Taiwan Textile Research Institute, Cycling & Health Industry R&D Center, United Ship design & Development Center, Stone & Resource Industry R&D Center, Printing Technology Research Institute, Plastics Industry Development Center, Precision Machinery Research Development Center, Medical and Pharmaceutical and Development Center, Footwear & Recreation Technology Research Institute, Animal Technology Institute Taiwan, and INER. Each R&D unit is regarded as a DMU. Furthermore, as R&D projects executed by artificial person institutes have different orientations due to their different R&D specifications, we divide the sample into two types, namely 'ordinary elements and environment establishments' and 'innovations and R&D services and compatibilities'. Note that DEA necessitates homogenous sample organizations. Therefore, we only discuss projects belong to 'ordinary elements and environment establishments'.

This study uses annual reports of Execution of R&D Projects from the Department of Industrial Technology as secondary data resource. Considering the date of publication and sources of data, we only choose samples for the period 2005–2009. These annual reports were prepared by Taiwan Institute of Economic Research (TIER), a delegate of the Department of Industrial Technology. The reports are of high credibility and completeness; they fully cover the execution results of all R&D projects run by the artificial person institutes.

The descriptive statistics of input and output variables of our research samples are shown at Table 7.2. During the sample period, the average human resource is 166 persons; the average working time is 38 months; the average of research funds is NTD404.85 millions; the average number of patents is 68; the average number of technology acquired is 20; the average number of research reports is 137; the average number of research publications is 90; the average number of research outsourced are 20; the average patent authorization fee is NTD15.75 millions; the average technology transferred is 35; the average number of technology services is 56; the average number of seminars is 22; the mean investment amount during execution time is NTD877.63 millions; the mean production value of execution time is NTD2882.34 millions. In summary, we could infer that the Taiwanese government has allocated much effort and budget in the long-term investment on R&D projects.

Table 7.3 shows the correlation coefficients among the input and output variables. The results show that the inputs and outputs are all positively and

Table 7.2 Descriptive statistics of the input and output variables

Variable	Mean	Q1	Q3	Std. dev.
Research Fund	404,851	64,302	573,850	582,475
Manpower	166	34	239	193
Research Time	38	12	48	28
Outsourced research	23	7	29	23
Investment	880,630	134,000	1,330,450	1,178,813
Production value	2,885,347	135,000	4,095,000	4,649,439
Patents	71	10	77	134
Technology acquired	4	0	5	3
Research reports	137	22	163	189
Research publications	90	16	115	132
Patent transferred	16,050	700	12,757	33,917
Technology transferred	32	11	43	33
Technology services	51	14	67	70
Seminar	25	7	29	32

significantly related, with few exceptions. It can thus be concluded that the inputs and outputs used in this study have "isotonicity" relationships. That is, the correlation analysis justifies our selection of the variables in the model (Golany and Roll 1989).

7.3.3 Dynamic Extension of Network Slack-Based Measure DEA Model

Traditional network DEA models utilize a radial measure to estimate the relative efficiency for each DMU in a multi-stage value-creation process. However, objectivity in radial models could be lacking in that they are not able to reveal the real input/output conditions for each organization, and stand on the assumption that inputs or outputs undergo proportional changes. Furthermore, the network DEA analysis is cross sectional, neglecting the efficiency changes of organizations over several periods. In this regard, we apply window analysis for the longitudinal performance measure as it is able to analyze efficiency changes across periods. Put differently, we are able to analyze the multidimensional performance of R&D organizations from a dynamic view. To overcome the shortcomings discussed above, we combine the SBM network data envelopment analysis (Tone and Tsutsui 2009) and the window analysis (Klopp 1985) to ensure enhanced estimates of efficiency across periods with internal linking activities in a single implementation for every DMU.

This study deals with n R&D organizations ($j = 1, \ldots, n$) consisting of K stages ($k = 1, \ldots, K$) in T periods ($t = 1, \ldots, T$); m_k and r_k are the numbers of inputs and outputs to stage k, respectively; $z_{dj}^{t(f,h)}$ is the amount of linking intermediate product

Table 7.3 Correlation coefficients among the input and output variables

	x_1	x_2	x_3	z_1	z_2	z_3	z_4	z_5	y_1	y_2	y_3	y_4	u_1	u_2
Manpower (x_1)	1													
Research Time (x_2)	0.270**	1												
Research Fund (x_3)	0.974**	0.185*	1											
Patents (z_1)	0.922**	0.089	0.964**	1										
Technology Acquired (z_2)	0.593**	0.206*	0.591**	0.548**	1									
Research Reports (z_3)	0.950**	0.216**	0.935**	0.887**	0.650**	1								
Research Publications (z_4)	0.946**	0.136	0.953**	0.943**	0.589**	0.908**	1							
Outsourced Research (z_5)	0.768**	0.180*	0.732**	0.684**	0.570**	0.730**	0.762**	1						
Patent Transferred (y_1)	0.413**	0.469**	0.404**	0.427**	0.114	0.303**	0.312**	0.105	1					
Technology Transferred (y_2)	0.405**	0.505**	0.350**	0.304**	0.433**	0.383**	0.401**	0.431**	0.238**	1				
Technology Services (y_3)	0.481**	0.278**	0.467**	0.416**	0.355**	0.457**	0.495**	0.293**	0.170*	0.332**	1			
Seminar (y_4)	0.822**	0.064	0.852**	0.831**	0.576**	0.831**	0.865**	0.762**	0.220**	0.350**	0.367**	1		
Investment (u_1)	0.446**	0.574**	0.352**	0.281**	0.198*	0.399**	0.357**	0.427**	0.415**	0.656**	0.238**	0.364**	1	
Production value (u_2)	0.291**	0.476**	0.196*	0.151	0.092	0.206*	0.226**	0.288**	0.404**	0.382**	0.146	0.140	0.617**	1

Note: ***$P < 0.01$; **$P < 0.05$; *$P < 0.1$

d from stage f to stage h to organization j in period t; The window starting at time t, $1 \le t \le T$ and with the width w, $1 \le w \le T - t$, has $n \times w$ observations. $T - w + 1$ is the number of windows $(p = 1, \ldots, T - w + 1)$. The dynamic extensions of network SBM DEA model for the observed organization in period t with the width w under a variable returns to scale assumption and the free link activities program problem is as follows:

$$\eta_o^p = Min \frac{\sum_{k=1}^{K} \omega_k \sum_{p=1}^{T-w+1} \left[1 - \frac{1}{m_k}\left(\sum_{i=1}^{m_k} \frac{s_i^{p,k^-}}{x_{io}^{p,k}}\right)\right]}{\sum_{k=1}^{K} \omega_k \sum_{p=1}^{T-w+1} \left[1 + \frac{1}{r_k}\left(\sum_{r=1}^{r_k} \frac{s_r^{p,k^+}}{y_{ro}^{p,k}}\right)\right]}$$

$$\begin{aligned}
&S.T.\\
&x_{io}^{p,k} = \sum_{p=1}^{T-w+1} \sum_{j=1}^{n\times w} x_{ij}^{p,k} \lambda_j^{p,k} + s_i^{p,k^-}, \quad i = 1, \ldots, m_k,\ p = 1, \ldots T - w + 1,\\
&y_{ro}^{p,k} = \sum_{p=1}^{T-w+1} \sum_{j=1}^{n\times w} y_{rj}^{p,k} \lambda_j^{p,k} - s_r^{p,k^+}, \quad r = 1, \ldots, r_k,\ p = 1, \ldots, T - w + 1,\\
&\quad \sum_{p=1}^{T-w+1} \sum_{j=1}^{n\times w} z_{dj}^{(f,h)} \lambda_j^{p,h} = \sum_{p=t}^{t=w-1} \sum_{j=1}^{n} z_{dj}^{(f,h)} \lambda_j^{p,f},\ \forall (f,h),\\
&\quad \sum_{p=1}^{T-w+1} \sum_{j=1}^{n\times w} \lambda_j^{p,k} = 1, \quad k = 1, \ldots, K,\ p = 1, \ldots, T - w + 1,\\
&\quad \lambda_j^{p,k} \ge 0,\ s_i^{p,k^-} \ge 0,\ s_r^{p,k^+} \ge 0;\ j = 1, \ldots, nw,
\end{aligned} \tag{7.1}$$

where s_i^{p,k^-} and s_r^{p,k^+} are the optimal input slacks and output slacks at stage k; ω_k is the relative weight of stage k which is determined corresponding to its importance and $\sum_{i=1}^{k} \omega_k = 1, \omega_k \ge 0 (\forall k)$. $\sum_{p=1}^{T-w+1} \sum_{j=1}^{n\times w} \lambda_j^{p,k} = 1$ constructed best practice frontier exhibits variable returns to scale technology at stage k with window p. Transforming this program problem into a linear program using the Charnes and Cooper transformation (Charnes et al. 1978) will solve the problem itself.

If $\eta_o^{p^*} = 1$ in (7.1), the observed organization is called overall efficient in window p. The efficiency of observed organization at stage k in window p can be defined by

$$\tau_{ko}^{p*} = \frac{\sum_{p=1}^{T-w+1} \left[1 - \frac{1}{m_k}\left(\sum_{i=1}^{m_k} \frac{s_i^{p,k^{-*}}}{x_{io}^{p,k}}\right)\right]}{\sum_{p=1}^{T-w+1} \left[1 + \frac{1}{r_k}\left(\sum_{r=1}^{r_k} \frac{s_r^{p,k^{+*}}}{y_{ro}^{p,k}}\right)\right]}, \quad k = 1, \ldots, K, \tag{7.2}$$

$$p = 1, \ldots, T - w + 1,$$

where $s_i^{p,k^{-*}}$ and $s_r^{p,k^{+*}}$ are the optimal input slacks and output slacks in (7.1). If $\tau_{ko}^{p*} = 1$, then the observed organization is technically efficient at stage k. If τ_{ko}^{p*} is smaller than one, then the observed organization is technically inefficient.

7.4 Results and Discussions

7.4.1 Performance Analysis in Value-Creation Process

Many researchers have utilized the ANP techniques for multi-criteria decision analyses (Sipahi and Timor 2010) as this method is particularly useful in analyzing complicated decisions. In this study, three stages of efficiencies are developed to comprehensively measure the dynamic performance of R&D organizations in Taiwan. Using ANP, we are able to find the relative importance of each stage of the value-creation process. Specifically, five R&D managers are randomly chosen from the sample R&D organizations and asked to evaluate the relative importance of value-creation process based on the first evaluation part of ANP by Saaty (1996). Table 7.4 shows the results of the weights obtained for each stage of the value-creation process. The relative weights in Table 7.4 are calculated as follows. First, we ask the five managers to express their viewpoints on the relative importance of each stage through the Likert Scale of 1–9 as in Saaty (1996). With that, the relative weights are obtained accordingly. Second, we calculated the geometric mean of the scores obtained in the first step. These mean values are used as the input weights for the efficiency estimates of the value-creation process.

For understanding the connectivity of inner economical activities of R&D organizations over long-term periods, we integrate window analysis and network DEA to evaluate the R&D efficiency, technology-diffusion efficiency, and value creation efficiency of Taiwanese R&D organizations for the period 2005–2009. In order to understand dynamic performance of these 5 years, we calculate a 5-year average performance value of each R&D organization. We also use standard deviation to determine the stability of the 5-year performance.

Table 7.4 Input weights for value-creation process

Expert	R&D efficiency	Technology-diffusion efficiency	Value-creation efficiency	Sum
Manager 1	0.467	0.256	0.277	1.000
Manager 2	0.415	0.342	0.243	1.000
Manager 3	0.473	0.243	0.284	1.000
Manager 4	0.513	0.212	0.275	1.000
Manager 5	0.455	0.282	0.263	1.000
Mean	0.465	0.265	0.270	1.000

Dynamic performance of each stage is shown in Table 7.5. For R&D efficiency, the overall average efficiency score is 0.459. There are five R&D organizations that achieve an efficiency value of at least 0.8, namely SRIRDC, PTRI, PIDC, FRTRI, and INER. We further check the standard deviations of the five organizations; the results show that their variation levels are generally and relatively smaller than those of other organizations, implying that their performance is more stable as compared to others over the sample period. The top-performing R&D organization is PTRI, which has the highest R&D efficiency score and the lowest value of standard deviation. This finding means that this organization is the best learning benchmark for other R&D organizations.

To remove the R&D inefficiency, we suggest that the R&D organizations should apply patent for valuable key technologies, design global patent map, and increase creation of intellectual properties. In terms of technology acquisition, the organizations should reinforce international cooperation and technology authorization, so that their R&D level could improve and reach international level. More importantly, they are able to possess their core technologies. As for research reports and research publications, the organizations should work harder on preparing technical reports and publishing academic studies to show their achievement in R&D and accomplishment in academic research. In summary, the R&D organizations should focus on key profitable technology, patent map layout, intellectual property, and academic achievement.

On the front of technology-diffusion efficiency, the overall average efficiency score is 0.608. There are 11 organizations that achieve an efficiency value of at least 0.8. We further check their standard deviations, which shows that the results of these organizations are again better than those of others. Among the organizations, two organizations, APIT and INER, are considered as efficient in terms of technology-diffusion efficiency. Therefore, other organizations should take the two organizations as the best learning model for diffusing technology.

Our suggestions for organizations that need improvement in terms of technology-diffusion efficiency are as follows. First, organizations should actively transfer their research results to industries and increase their profit from patent authorization fees (technology transfer and patent authorization); organizations should hold technology forums, training camps, and exhibitions to present their research reports with the purpose of helping industries to enhance their technology capabilities (seminars). To accelerate the technological development in the industries, R&D organizations should fully utilize technology services, international standard authentications, and technology platform exchange mechanisms to provide technical help to industries in R&D activities (technology and industry services). In summary, R&D organizations should enforce the proliferation effect of their R&D results, and establish transfer mechanisms of technologies, patents, and industry services.

As for the results on the value-creation efficiency, the overall average efficiency value is 0.176. There is only one R&D organization with an efficiency value above 0.8, viz. ITRI_CCRL. For this stage, we also provide some suggestions for improvement. First, R&D organizations should actively transfer their research

Table 7.5 Mean efficiencies of R&D organizations for the 3-year windows during 2005–2009

R&D organizations	R&D efficiency		Technology-diffusion efficiency		Value-creation efficiency		Overall efficiency	
	Mean	Std. dev.	Mean	Std. dev.	Mean	Std. dev.	Mean	Std. dev.
ITRI_HQ	0.320	0.140	0.188	0.025	0.050	0.047	0.212	0.062
ITRI_EORL	0.149	0.024	0.645	0.225	0.090	0.088	0.265	0.054
ITRI_CCRL	0.192	0.060	0.466	0.208	0.806	0.290	0.431	0.036
ITRI_MSRL	0.246	0.309	0.373	0.353	0.473	0.296	0.341	0.315
ITRI_MCRL	0.254	0.177	0.485	0.165	0.566	0.327	0.399	0.132
ITRI_EERL	0.318	0.082	0.657	0.242	0.195	0.119	0.375	0.087
ITRI_BERL	0.271	0.034	0.425	0.115	0.018	0.024	0.243	0.047
ITRI_STC	0.415	0.076	0.610	0.233	0.137	0.158	0.392	0.084
Mean	0.271	0.059	0.481	0.109	0.292	0.073	0.332	0.064
CSIST_RL1	0.601	0.256	0.550	0.209	0.034	0.027	0.434	0.166
CSIST_RL2	0.395	0.103	0.521	0.273	0.035	0.021	0.331	0.110
CSIST_ERL	0.338	0.122	0.527	0.227	0.096	0.049	0.323	0.102
CSIST_CTI	0.222	0.036	0.177	0.046	0.154	0.061	0.192	0.021
CSIST-RL4	0.407	0.031	0.536	0.172	0.124	0.112	0.365	0.057
CSIST-RL5	0.179	0.024	0.187	0.010	0.450	0.312	0.254	0.096
Mean	0.357	0.048	0.416	0.078	0.149	0.068	0.316	0.044
III	0.162	0.109	0.155	0.047	0.432	0.178	0.233	0.098
DCB	0.257	0.063	0.926	0.166	0.216	0.439	0.423	0.069
MIRDC	0.212	0.082	0.280	0.117	0.370	0.120	0.273	0.058
FIRDI	0.272	0.035	0.766	0.226	0.122	0.080	0.363	0.067
TTRI	0.192	0.016	0.233	0.026	0.208	0.089	0.207	0.020
CHIRDC	0.661	0.132	0.787	0.147	0.048	0.035	0.529	0.077
USDDC	0.655	0.021	0.584	0.159	0.080	0.031	0.481	0.048
SRIRDC	0.875	0.151	0.975	0.056	0.032	0.012	0.674	0.064
PTRI	1.000	0.001	0.976	0.053	0.228	0.432	0.785	0.121
PIDC	0.938	0.082	0.864	0.304	0.071	0.076	0.684	0.109
PMRDC	0.635	0.061	0.939	0.076	0.023	0.014	0.550	0.025
MPDC	0.607	0.173	0.874	0.117	0.009	0.016	0.516	0.112
FRTRI	0.889	0.168	0.932	0.069	0.019	0.014	0.666	0.061
APIT	0.717	0.076	1.000	0.000	0.019	0.034	0.604	0.039
INER	0.921	0.033	1.000	0.000	0.005	0.006	0.695	0.017
Mean	0.600	0.026	0.753	0.033	0.125	0.046	0.512	0.014
Total average	0.459	0.028	0.608	0.042	0.176	0.033	0.422	0.024

results to the industries, get their patents authorized, and utilize delegations and industry services. This is because through the proliferation of their technology results to value-added applications in the industries, they are able to fully bring positive effects from direct and indirect investments, such as an increase in the production value, newly created industries. In summary, R&D organizations have to concern about increasing the values of the industries, creating new profit basis for the industries, and growing Taiwanese companies from 'technology followers' to 'value creators'.

From the above-discussed results, we find that the average efficiency score of the technology-diffusion efficiency (0.608) is better than that of the R&D efficiency (0.459) and the value-creation efficiency (0.176). The findings suggest that R&D organizations should (i) enforce the proliferation effect at the stage of technology-diffusion efficiency, (ii) transfer their technologies and timely get their patents authorized, and (iii) disseminate their research results through the technology services and consultancy services.

7.4.2 The Relationship Between Capital Stock and R&D Organizations Performance

The purpose of establishing R&D organizations is to raise the technology level of the industries and to accelerate the innovation in the industries, which could create values. As a result, it is important that R&D organizations invest in the capital stock of patents. Currently, a patent in Taiwan will be protected by law for 10 years; the economical benefits of research outcomes are also protected by patent law for up to a maximum of a decade. In other words, a researcher should evaluate patents from the perspective of the capital stock of patents.

This study uses the number of patents as the proxy of the capital stock of patents, consistent with prior studies that also use the number of patents to gauge the capability of R&D and innovation (Griliches 2007; Hall and Bagchi-Sen 2007; Trajtenberg 1990). It has been argued that the more patents a R&D organization has, the stronger its power is at technological development and innovation (Griliches 2007; Trajtenberg 1990). In measuring the capital stock of patents, a researcher can amortize the capital stock of patents of a R&D organization at 15 %. Specifically, the formula to calculate the capital stock of patents (*PAT*) for *i* R&D unit at year *t* is as follows:

$$PAT_{i,t} = PAT_{i,t-1} \times (1 - 15\,\%) + P_{\mathrm{t}} \tag{7.3}$$

where P is the ratio of the number of patents acquired to the number of patents applied of each R&D organization (the patent acquired ratio).

In addition to the capital stock of patents, prior studies also indicate that the quality of human resources and capability of service support could affect the performance of a R&D organization. The quality of human resources can be evaluated by their education and working experiences (Souitaris 2002). Therefore, we define the quality of human resources as the number of employees with doctoral degrees in a R&D organization because better qualified human resources would possess higher quality of research capability. In other words, talented employees are the cornerstone of technological development and innovation, as well as the core of knowledge-based economic development. Consistent with Souitaris (2002) who finds that the quality of R&D human resources are highly related to technical innovation, we predic that the quality of R&D human resources is positively related to the performance of R&D organizations.

In implementing R&D projects or developing new products, R&D organizations use their existing technologies and equipments to provide short-term services such as maintenance and technical consultancy. In this study, we use the average charged amount of contracted service for the industries to proxy for the capability of service support. A higher value of the variable indicates that an organization has better capability at providing service support. Specifically, R&D organizations are able to provide better services to their customers through innovating their services (Chakravarty et al. 1995; Upton 1995) because the capability of service support is the key element in achieving competitive advantage. Therefore, we predict that there is a positive relationship between the capability of service support and the performance of R&D organizations.

To determine the relationship between the capital stock of patents, quality of human resources, and capability of service support, and the performance of R&D organizations, we apply panel data regression models. Banker and Natarajan (2008) have documented that the use of a two-stage procedure involving DEA followed by an ordinary least squares (OLS) regression analysis yields consistent estimators of the regression coefficients. Note that panel data estimation procedures are superior to the simply-pooled OLS procedures. An advantage of panel data regression is that it could adjust for organization-specific and year-specific effects.

Table 7.6 presents the panel data regression. The results show that the capital stock of patents is positively and significantly related to the R&D efficiency and technology-diffusion efficiency. Although the capital stock of patents is negatively related to the value-creation efficiency, the coefficient doesn't reach the conventional significance level. In the technological development and innovation process, patents are the key to performance. That is, the number of patents acquired reflects the degree of competitiveness of a R&D organization (Deeds and Hill 1996; Mowery et al. 1996). As noted earlier, we study patents acquired by the R&D projects that are executed by 29 artificial person institutes for the period 2005–2009. The untabulated statistics show that approximately 6000 units of patents were acquired by these organizations. While the results imply that the R&D organizations have been planning their global patent policies, building complete lines of patents, enforcing the quality of patents, and generating competitive advantages of their research results, the negative association between the capital stock of patents and value-creation efficiency indicates that R&D organizations should continue (i) to introduce new and advanced technologies, (ii) to learn higher level key technologies, and (iii) to integrate superior resources in the organizations. These methods could ensure that the R&D organizations are able to demonstrate their capabilities at design, production and management, to build up their irreplaceable specialization, and ultimately create values through competitive advantages.

Table 7.6 Results of panel data regression

	Dependent variable		
	R&D efficiency	Technology-diffusion efficiency	Value-creation efficiency
Independent variables	Fixed-effect model	Random-effect model	Random-effect model
Constant		0.400	0.201
Patent capital stock	0.352*	0.778***	−0.111
Quality of HR	0.145	0.220	−0.461
Ability of service support	0.297***	0.190*	0.788
R^2	0.640	0.311	0.536

Note: ***P < 0.01; **P < 0.05; *P < 0.1

However, the regression results in Table 7.6 show that there is no significant relationship between the quality of human resources and the performance of R&D organizations. A R&D organization should be able to create future competitiveness by acquiring new technical knowledge, developing new technologies through cooperation with other countries, exchanging knowledge and personnel with foreign institutes, and transforming human resources into value. That is, it might mean that R&D organizations concentrate too much on their capabilities of R&D but pay less attention the important roles played of their employees.

As for the capability of service support, the coefficients are all positive, but only those for the R&D and technology-diffusion efficiencies are significant. These outcomes support our prediction that the better the capability of service support, the better the performance of R&D organizations, suggesting that customers are satisfied if organizations can provide valuable service support. During the sample period, the accumulated number of industry services cases reached 6911 and the revenue was NTD8000 millions, which means that R&D organizations actively deploy their core technologies and help the industries to raise their values.

7.5 Conclusions

In today's dynamic economic environment, evaluating the performance of R&D organizations is a process that requires a comprehensive measure to characterize it. In this study, we develop a dynamic three-stage network DEA model through the combination of window analysis and network DEA. The innovative DEA model evaluates the R&D efficiency, technology-diffusion efficiency, and value-creation efficiency of Taiwanese R&D organizations over the period 2005–2009. Before performing DEA analysis, we first apply the ANP technique to define the relative importance of each stage of efficiency. The DEA analysis suggests that managers should first focus on removing the technology-diffusion inefficiency, then eliminating the value-creation inefficiency, and finally improving the R&D efficiency.

In the second stage, panel data regression is employed to examine the impacts of the capital stock of patents, quality of human resources, and capability of service support on the dynamic performance of the R&D organizations. The panel data regression outcomes shows that the capital stock of patents and capability of service support positively affect the performance of R&D organizations.

Despite the innovative application of the dynamic and network DEA models in this study, we highlight that future studies may apply the dynamic network SBM model by Tone and Tsutsui (2014) to account for dynamic efficiencies. Note that, however, this study looks at R&D organizations that can hardly be characterized by carry-overs, which are permanent accounts that are accumulated over periods, used in the dynamic SBM model. Therefore, future research may apply the similar innovative approach to examine organizations in a different industry or even the dynamic network SBM model for the same industry when more data become available to the public.

References

Banker, R. D., & Natarajan, R. (2008). Evaluating contextual variables affecting productivity using data envelopment analysis. *Operations Research, 56*(1), 48–58.

Chakravarty, S., Feinberg, R., & Widdows, R. (1995). What do consumers want from banks? *Journal of Retail Banking, 12*, 15–19.

Charnes, A., Cooper, W. W., & Rhodes, E. (1978). Measuring the efficiency of decision making units. *European Journal of Operational Research, 2*(6), 429–444.

Chen, Y., & Zhu, J. (2004). Measuring information technology's indirect impact on firm performance. *Information Technology and Management, 5*(1), 9–22.

Cherchye, L., & Abeele, P. V. (2005). On research efficiency: A micro-analysis of Dutch university research in Economics and Business Management. *Research Policy, 34*(4), 495–516.

Co, H., & Chew, K. (1997). Performance and R&D expenditures in American and Japanese manufacturing firms. *International Journal of Production Research, 35*(12), 3333–3348.

Cooper, W. W., Seiford, L. M., & Tone, K. (2006). *Introduction to data envelopment analysis and its uses: With DEA-solver software and references*. New York: Springer Science.

Deeds, D. L., & Hill, C. W. (1996). Strategic alliances and the rate of new product development: An empirical study of entrepreneurial biotechnology firms. *Journal of Business Venturing, 11* (1), 41–55.

Edquist, C. (1997). *Systems of innovation: Technologies, institutions, and organizations*. London: Pinter Publishers/Cassell Academic.

Efrat, K. (2014). The direct and indirect impact of culture on innovation. *Technovation, 34*(1), 12–20.

Färe, R., Grosskopf, S., Norris, M., & Zhang, Z. (1994). Productivity growth, technical progress, and efficiency change in industrialized countries. *American Economic Review, 84*(1), 66–83.

Golany, B., & Roll, Y. (1989). An application procedure for DEA. *Omega, 17*(3), 237–250.

González, E., & Gascón, F. (2004). Sources of productivity growth in the Spanish pharmaceutical industry (1994–2000). *Research Policy, 33*(5), 735–745.

Griliches, Z. (1988). Productivity puzzles and R&D: Another nonexplanation. *The Journal of Economic Perspectives, 2*(4), 9–21.

Griliches, Z. (2007). *R&D, Patents and Productivity*. Chicago: University of Chicago Press.

Guan, J., & Chen, K. (2012). Modeling the relative efficiency of national innovation systems. *Research Policy, 41*(1), 102–115.

Guellec, D., & Bruno, V. P. P. (2004). From R&D to productivity growth: Do the institutional settings and the source of funds of R&D matter? *Oxford Bulletin of Economics and Statistics, 66*(3), 353–378.

Hall, L. A., & Bagchi-Sen, S. (2007). An analysis of firm-level innovation strategies in the US biotechnology industry. *Technovation, 27*(1), 4–14.

Hartmann, G. C. (2003). Linking R&D spending to revenue growth. *Research-Technology Management, 46*(1), 39–46.

Hsu, K. H. (2005). Using balanced scorecard and fuzzy data envelopment analysis for multinational R&D project performance assessment. *Journal of American Academy of Business, 7*(1), 189–196.

Hung, C.-L., Kuo, S.-J., & Dong, T.-P. (2013). The relationship between team communication, structure, and academic R&D performance: Empirical evidence of the national telecommunication program in Taiwan. *R&D Management, 43*(2), 121–135.

Klopp, G. (1985). *The analysis of the efficiency of productive systems with multiple inputs and outputs.* Ph.D. thesis, University of Illinois at Chicago, IL, USA. Retrieved from http://search.proquest.com/docview/303474777?accountid=15184, ProQuest Dissertations & Theses A&I database.

Lee, H. Y., & Park, Y. T. (2005). An international comparison of R&D efficiency: DEA approach. *Asian Journal of Technology Innovation, 13*(2), 207–222.

Lee, H., Park, Y., & Choi, H. (2009). Comparative evaluation of performance of national R&D programs with heterogeneous objectives: A DEA approach. *European Journal of Operational Research, 196*(3), 847–855.

Liu, J. S., & Lu, W.-M. (2010). DEA and ranking with the network-based approach: A case of R&D performance. *Omega, 38*(6), 453–464.

Lu, W.-M., & Hung, S.-W. (2011). Exploring the operating efficiency of technology development programs by an intellectual capital perspective—A case study of Taiwan. *Technovation, 31*(8), 374–383.

Lu, W.-M., Kweh, Q. L., & Huang, C.-L. (2014). Intellectual capital and national innovation systems performance. *Knowledge-Based Systems, 71*, 201–210.

Lu, W.-M., Wang, W.-K., Tung, W.-T., & Lin, F. (2010). Capability and efficiency of intellectual capital: The case of fabless companies in Taiwan. *Expert Systems with Applications, 37*(1), 546–555.

Mansfield, E. (1980). Basic research and productivity increase in manufacturing. *The American Economic Review, 70*(5), 863–873.

Mansfield, E. (1988). Industrial R&D in Japan and the United States: A comparative study. *The American Economic Review, 78*(2), 223–228.

Mowery, D. C., Oxley, J. E., & Silverman, B. S. (1996). Strategic alliances and interfirm knowledge transfer. *Strategic Management Journal, 17*, 77–91.

Nasierowski, W., & Arcelus, F. (2003). On the efficiency of national innovation systems. *Socio-Economic Planning Sciences, 37*(3), 215–234.

Rousseau, S., & Rousseau, R. (1997). Data Envelopment Analysis as a tool for constructing scientometric indicators. *Scientometrics, 40*(1), 45–56.

Rousseau, S., & Rousseau, R. (1998). The scientific wealth of European nations: Taking effectiveness into account. *Scientometrics, 42*(1), 75–87.

Saaty, T. L. (1996). *Decision making with dependence and feedback: The analytic network process* (Vol. 4922). Pittsburgh: RWS Publications.

Saiki, T., Akano, Y., Watanabe, C., & Tou, Y. (2006). A new dimension of potential resources in innovation: A wider scope of patent claims can lead to new functionality development. *Technovation, 26*(7), 796–806.

Sharma, S., & Thomas, V. (2008). Inter-country R&D efficiency analysis: An application of data envelopment analysis. *Scientometrics, 76*(3), 483–501.

Sipahi, S., & Timor, M. (2010). The analytic hierarchy process and analytic network process: An overview of applications. *Management Decision, 48*(5), 775–808.

Souitaris, V. (2002). Firm–specific competencies determining technological innovation: A survey in Greece. *R&D Management, 32*(1), 61–77.

Tone, K., & Tsutsui, M. (2009). Network DEA: A slacks-based measure approach. *European Journal of Operational Research, 197*(1), 243–252.

Tone, K., & Tsutsui, M. (2010). Dynamic DEA: A slacks-based measure approach. *OMEGA—The International Journal of Management Science, 38*(3), 145–156.

Tone, K., & Tsutsui, M. (2014). Dynamic DEA with network structure: A slacks-based measure approach. *OMEGA—The International Journal of Management Science, 42*(1), 124–131.

Trajtenberg, M. (1990). A penny for your quotes: Patent citations and the value of innovations. *The Rand Journal of Economics, 21*, 172–187.

Upton, D. M. (1995). What really makes factories flexible? *Harvard Business Review, 73*(4), 74–84.

Walwyn, D. (2007). Finland and the mobile phone industry: A case study of the return on investment from government-funded research and development. *Technovation, 27*(6), 335–341.

Wang, E. C., & Huang, W. (2007). Relative efficiency of R&D activities: A cross-country study accounting for environmental factors in the DEA approach. *Research Policy, 36*(2), 260–273.

Wu, W.-Y., Tsai, H.-J., Cheng, K.-Y., & Lai, M. (2006). Assessment of intellectual capital management in Taiwanese IC design companies: Using DEA and the Malmquist productivity index. *R&D Management, 36*(5), 531–545.

Zhang, A., Zhang, Y., & Zhao, R. (2003). A study of the R&D efficiency and productivity of Chinese firms. *Journal of Comparative Economics, 31*(3), 444–464.

Zhu, J. (2009). *Quantitative models for performance evaluation and benchmarking: DEA with spreadsheets*. Boston: Springer.

Chapter 8
Evaluating Returns to Scale and Convexity in DEA Via Bootstrap: A Case Study with Brazilian Port Terminals

Peter F. Wanke and Carlos Pestana Barros

Abstract This paper presents a simple methodology, built upon the bootstrapping technique originally developed by Simar and Wilson (Handbook on Data Envelopment Analysis, Kluwer International Series, Boston, 2004), in order to evaluate, unambiguously, returns to scale and convexity assumptions in DEA. The basic idea is to use confidence intervals and bias corrected central estimates to test for significant differences on distance functions and returns-to-scale indicators provided by different DEA models. This methodology is illustrated by means of a case study in the Brazilian port sector, where anecdotal evidence regarding an eventual capacity shortfall is corroborated.

Keywords Data Envelopment Analysis • Bootstrapping • Returns-to-scale • Convexity • Ports • Brazil

8.1 Introduction

Nonparametric efficiency estimators such as Data Envelopment Analysis (DEA) typically rely on linear programming techniques for computation of estimates, and are often characterized as deterministic, as if to suggest that the methods lack any statistical underpinnings (Simar and Wilson 2004). Applied studies that have used these methods have typically presented point estimates of inefficiency, with no measure or even discussion of uncertainty surrounding these estimates (Cesaro

P.F. Wanke (✉)
CELIG (Center for Studies in Logistics, Infrastrucutre and Management), Instituto COPPEAD de Administração, Universidade Federal do Rio de Janeiro,
Rua Paschoal Lemme 355; 21.949-900, Rio de Janeiro, Brazil
e-mail: peter@coppead.ufrj.br

C.P. Barros
UECE (Research Unit on Complexity and Economics), Instituto Superior de Economia e Gestão, Technical University of Lisbon, Rua Miguel Lupi 20; 1249-078, Lisbon, Portugal
e-mail: cbarros@iseg.utl.pt

S.-N. Hwang et al. (eds.), *Handbook of Operations Analytics Using Data Envelopment Analysis*, International Series in Operations Research & Management Science 239, DOI 10.1007/978-1-4899-7705-2_8

et al. 2009). Indeed, many papers contain statements where efficiency is described as being computed or calculated as opposed to being estimated, and results are frequently referred to as efficiencies rather than efficiency estimates (e.g. Ray 2010; Zarepisheh et al. 2010).

The choice of terminology in describing the nonparametric efficiency approaches and their results is perhaps understandable given (until very recently) the lack of a "tool box" with aids for diagnostics, inference etc, such as the one available to researches using parametric approaches (Simar and Wilson 2004). To solve these problems, bootstrap techniques have been introduced into DEA analysis (Cesaro et al. 2009). The bootstrap technique permits the sensitivity of efficiency scores relative to the sampling variation of the frontier to be analyzed, avoiding problems of asymptotic sampling distributions.

DEA results, in fact, may be affected by sampling variation in the sense that distances to the frontier are underestimated if the best performers in the population are not included in the sample. To account for this, Simar and Wilson (1998, 2000) originally proposed a bootstrapping method allowing the construction of confidence intervals for DEA efficiency scores which relies on smoothing the empirical distribution. This technique consists of a simulation of a true sampling distribution by mimicking a data generating process, using the outputs from DEA. In this way, a new dataset is created and DEA is re-estimated using this dataset. Repeating the process many times allows a good approximation to be achieved of the true distribution of the sampling (Cesaro et al. 2009).

Generally speaking, statistical inference based on a non-parametric frontier approach may be useful to determine whether a productive unit is actually operating at its most productive scale size or not. When a productive unit is found to be operating in the region of increasing returns to scale, an implied judgment is that it is smaller than its optimal size (Ray 2010). Similarly, a firm operating in the region of diminishing returns to scale is considered to be too large.

In applied research, the common practice is to select either an input or an output-orientation projection and to draw conclusions about the returns to scale at observed bundle solely on the basis of the selected projection (Zarepisheh et al. 2010). Seldom, if ever, is there any attempt to crosscheck whether the other projection also leads to the same conclusions about returns to scale (Ray 2010). Because returns to scale can be different at the other projection, this practice can be misleading. Indeed, this can be a source of confusion about whether the firm is too small or large.

Convexity assumptions may be also assessed via statistical inference, testing whether a given production frontier actually encompasses or is embedded by another one (Simar and Wilson 2004). It is possible that, even when a given orientation is assumed, the violation of the convexity assumption imposed by DEA may lead to ambiguous returns to scale characterizations (Daraio and Simar 2007).

This paper presents a simple methodology, built upon the bootstrapping technique originally developed by Simar and Wilson (2004), in order to evaluate, unambiguously, returns to scale and convexity assumptions in DEA. The basic idea is to use confidence intervals and bias corrected central estimates as cornerstone tools, not only to test for significant differences on efficiency scores and their reciprocals (that is, their distance functions), but also on returns to scale indicators provided by different DEA models. A final contribution of the paper lies in its empirical application which considers the Brazilian port terminals. Inspired by the current debate in the Brazilian port sector, in which anecdotal evidences suggest a capacity shortfall (Agência Brasil 2004; Doctor 2003; Sales 2001), returns to scale are examined in the sector.

The remainder of the paper unfolds as follows. In Sect. 8.2, efficiency measurement and returns-to-scale characterization issues are discussed in light of DEA models. Sect. 8.3 provides additional information on estimation and bootstrapping in DEA. Section 8.4 presents a brief review on the Brazilian port industry and summarizes the data collection process. In Sect. 8.5, the respective empirical application illustrates the methodology proposed. Conclusions are given in Sect. 8.6.

8.2 Efficiency Measurement and RTS Characterization

8.2.1 Measuring Efficiency Scores Under Different Orientations and Frontiers

DEA is a non-parametric model first introduced by Charnes et al. (1978). It is based on linear programming (LP) and is used to address the problem of calculating relative efficiency for a group of Decision Making Units (DMUs) by using multiple measures of inputs and outputs. Given a set of DMUs, inputs and outputs, DEA determines for each DMU a measure of efficiency obtained as a ratio of weighted outputs to weighted inputs. There are several variations of the technique (Cooper et al. 2007). They differ not only with regard to the orientation and how the distance to the frontier is calculated for inefficient DMUs, but also with respect to efficiency change over time, undesirable outputs, resource congestion, disposability of outputs and inputs, just to mention some possible variations.

Compared with the stochastic parametric frontier approach, DEA imposes neither a specific functional relationship between production outputs and inputs, nor any assumptions on the specific statistical distribution of the error terms (Cullinane et al. 2006). An efficient frontier is on the boundary of a convex poly tope created in the space of inputs and outputs, and in which each vertex is an efficient DMU (Dulá and Helgason 1996). Another feature of DEA is that the relative weights of the inputs and the outputs do not need to be known a priori, that is, these weights are determined as part of the solution of the linear problem (Zhu 2003).

Consider a set of n observations on the DMUs. Each observation, DMU_j ($j = 1, \ldots, n$) uses m inputs x_{ij} ($i = 1, \ldots, m$) to produce s outputs y_{rj} ($r = 1, \ldots, s$).

Table 8.1 DEA envelopment models

Frontier type	Input-oriented	Output-oriented
Constant Returns to Scale (CRS), also known as CCR (Charnes et al. 1978)	$\min \theta$ $s.t.$ $\sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{io}, \forall i$ $\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}, \forall r$ $\lambda_j \geq 0, \forall j$ (8.1)	$\max \phi$ $s.t.$ $\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{io}, \forall i$ $\sum_{j=1}^{n} \lambda_j y_{rj} \geq \phi y_{ro}, \forall r$ $\lambda_j \geq 0, \forall j$ (8.2)
Varying Returns to Scale (VRS), also known as BCC (Banker et al. 1984)	Add $\sum_{j=1}^{n} \lambda_j = 1$	Add $\sum_{j=1}^{n} \lambda_j = 1$

Table 8.1 summarizes the envelopment models with respect to the orientations and frontier types (Zhu 2003), where DMU_o represents one of the n DMUs under evaluation, and x_{io} and y_{ro} are the ith input and rth output for DMU_o, respectively.

Thus, the BCC model differs from the CCR model only in the adjunction of the constraint $\sum_{j=1}^{n} \lambda_j = 1$. Together with the constraints $\lambda_j \geq 0, \forall j$, this imposes a convexity assumption on allowable ways in which the observations for the s DMUs may be combined within an efficient frontier, as is illustrated in Fig. 8.1.

As regards the model orientation, whether input or output-oriented, the two measures provide the same scores under constant returns to scale (CRS), but are unequal when varying returns to scale (VRS) are assumed as the efficient frontier (Cooper et al. 2004). Essentially, one should select an orientation according to which quantities (inputs or outputs) the decision-makers have most control over (Coelli 1996). However, given that LP cannot suffer from such statistical problems as simultaneous equation bias, the choice of an appropriate orientation is not as crucial as it is in the econometric estimation case (Coelli 1996). Furthermore, the choice of orientation will have only minor influences upon the scores obtained and their relative ranks (Coelli and Perelman 1999).

The dual LP problems to the envelopment models are called multiplier models (Zhu 2003). They are shown in Table 8.2. The weighted input and output of $\sum_{i=1}^{m} v_i x_{ij}$ and $\sum_{r=1}^{s} u_r y_{rj}$ are called virtual input and virtual output, respectively. Seiford and Thrall (1990) provide a detailed discussion on these models.

As least as important as DEA, even though not so popular, is the Free Disposal Hull (FDH) model. As seen, the non-parametric literature has extensively discussed efficiency measurement in convex frontier models as DEA (e.g. Banker et al. 2004 and reference therein). However, the convexity assumption where DEA relies on may be difficult to argue in real world, as it implies additivity and divisibility.

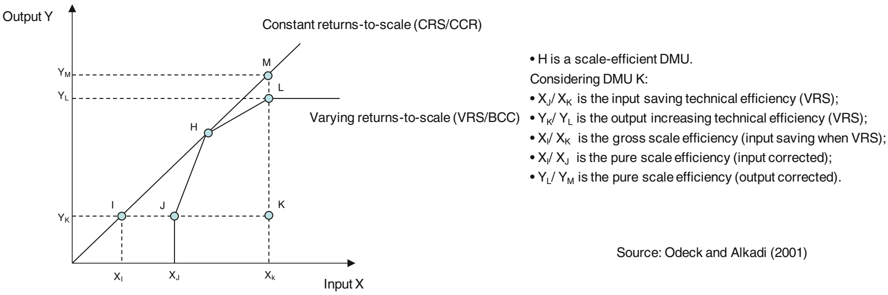


Fig. 8.1 Efficiency measurement illustrated: DEA-CCR and BCC frontiers

Table 8.2 DEA multiplier models

Frontier type	Input-oriented	Output-oriented
	$\max \sum_{r=1}^{s} u_r y_{ro} + u_o$ $s.t.$ $\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + u_o \leq 0$ $\sum_{i=1}^{m} v_i x_{io} = 1$ $u_r, v_i \geq 0$ (8.3)	$\min \sum_{i=1}^{m} v_i x_{io} + v_o$ $s.t.$ $\sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} + v_o \geq 0$ $\sum_{r=1}^{s} u_r y_{ro} = 1$ $u_r, v_i \geq 0$ (8.4)
CRS	$u_o = 0$	$v_o = 0$
VRS	u_o free in sign	v_o free in sign

Table 8.3 FDH models

Frontier type	Input-oriented	Output-oriented
VRS only (Kerstens and Vanden Eeckaut 1999; Deprins et al. 1984)	$\min \theta$ $s.t.$ $\sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{io}, \forall i$ $\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}, \forall r$ $\sum_{j=1}^{n} \lambda_j = 1$ $\lambda_j \in \{0, 1\}$ (8.5)	$\max \phi$ $s.t.$ $\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{io}, \forall i$ $\sum_{j=1}^{n} \lambda_j y_{rj} \geq \phi y_{ro}, \quad \forall r$ $\sum_{j=1}^{n} \lambda_j = 1$ $\lambda_j \in \{0, 1\}$ (8.6)

Therefore, its non-convex generalization, the FDH model, would be more adequate (De Witte and Marques 2008).

Although LP is typically not used to compute the efficiency scores within the FDH model (Simar and Wilson 2004), Table 8.3 presents its mixed integer formulation for the input and output oriented cases.

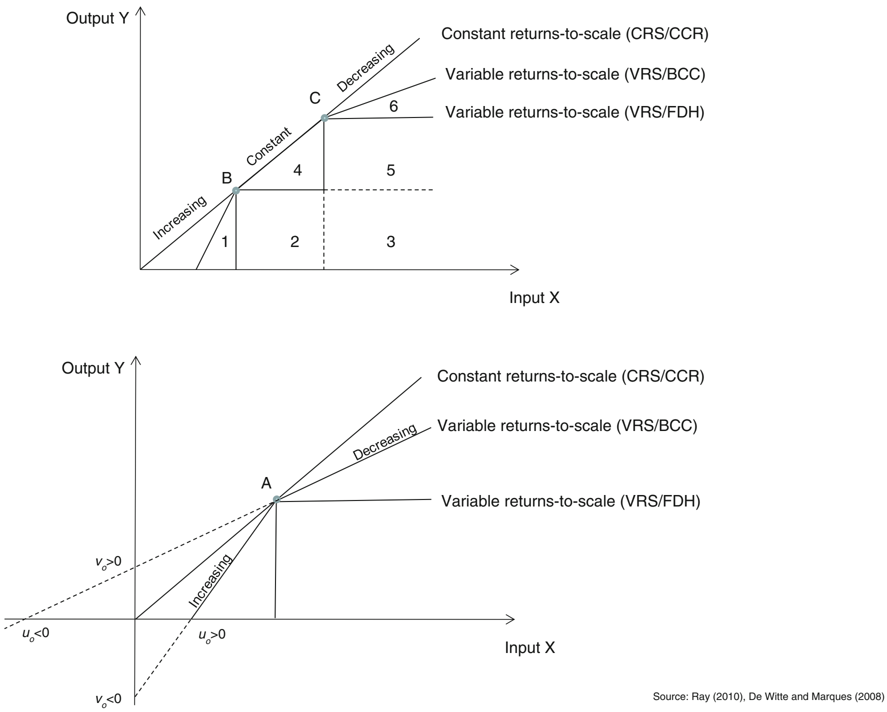


Fig. 8.2 Returns-to-scale under DEA and FDH frontiers

The binary value for λ_j, combined with the constraint $\sum_{j=1}^{n} \lambda_j = 1$ ensures that the efficiency score is only affected from actually observed production units, in contrast to a convex combination of quantities in DEA (De Witte and Marques 2008). This is illustrated in Fig. 8.2. The efficiency score also varies between 0 and 1, where a value of 1 denotes an efficient observation.

8.2.2 *Scaling or RTS Characterization*

Scale inefficiency is due to either increasing or decreasing returns-to-scale (RTS). Although the constraint on $\sum_{j=1}^{n} \lambda_j$ actually determines the prevalent RTS type of an efficient frontier (Zhu 2003)—CRS or VRS—scale inefficiency at a given DMU can be assessed under both models. As pointed out by Cooper et al. (2007), while the CCR model simultaneously evaluates RTS and technical inefficiency, the BCC model separately evaluates technical efficiency—with efficiency scores from the envelopment models (8.1) and (8.2)—and RTS—with u_o and v_o, respectively, from models (8.3) and (8.4).

As seen in Fig. 8.1, if $\sum_{j=1}^{n} \lambda_j = 1$ is omitted from models (8.1) and (8.2), a CRS efficient frontier is obtained. Figure 8.2 (Top) exemplifies the optimal solution for the CCR model, consisting of all points upon the ray from the origin that intersect the Most Productive Scale Size (MPSS) region, which are represented by the line segment BC. According to Banker et al. (2004), if the point being evaluated falls within the MPSS, it can be expressed as a convex combination of its extreme points so that $\sum_{j=1}^{n} \lambda_j = 1$ and constant RTS prevail. If the point is above this region, its coordinate values will be larger than their corresponding coordinates in MPSS so that $\sum_{j=1}^{n} \lambda_j > 1$ and decreasing RTS prevail. On the other hand, if the point is below the MPSS region then $\sum_{j=1}^{n} \lambda_j < 1$, with the prevalence of increasing RTS.

As noted by Odeck and Alkadi (2001), the term $\sum_{j=1}^{n} \lambda_j$ is also known as Scale Indicator (SI_o) within the CCR model. So, even though the term CRS is used to characterize the CCR model, this model may be used to determine whether increasing, decreasing or constant RTS prevail at a given DMU, by making the input and output slacks explicit in the LP formulation. For instance, if its "input saving" efficiency is greater than its "output increasing" efficiency, increasing RTS prevails (Odeck and Alkadi 2001).

Now with respect to the BCC model, since its efficient frontier is strictly concave, the optimal solution will necessarily designate a given DMU as being in the region of constant, decreasing, or increasing RTS. Figure 8.2 (Bottom) exemplifies the RTS evaluation under the BCC model for an input orientation: increasing RTS prevail if $u_o > 0$; decreasing, if $u_o < 0$; and constant, if $u_o = 0$.

8.2.3 *Orientation Impact on RTS Characterization*

Although the choice of orientation will have only minor influences upon the efficiency scores obtained and their relative ranks (Coelli and Perelman 1999), it should be noted, however, that input and output oriented models may give different results in their RTS findings (Banker et al. 2004). Thus the result secured may depend on the orientation used (Ray 2010). Increasing RTS may result from an input-oriented model, for example, while an application of an output oriented model may produce a decreasing RTS characterization from the same data.

One can easily verify, based upon Fig. 8.2 (Top) that both horizontal (input-oriented) and vertical (output-oriented) projections of DMUs located within regions 1, 4, and 6 upon CCR and BCC frontiers lead to the same conclusions: increasing, constant, and decreasing RTS, respectively. However, if output-oriented projection were used instead input-oriented projections on DMUs located within regions 2, 3, and 5 different conclusions would be drawn (cf. Table 8.4). This is due to the fact that input-oriented and the output-oriented models CCR/BCC models yield

Table 8.4 Orientation and RTS characterization

	Model	CCR		BCC	
	Orientation	Input	Output	Input	Output
Region	1	IRS	IRS	IRS	IRS
	2	IRS	CRS	IRS	CRS
	3	IRS	DRS	IRS	DRS
	4	CRS	CRS	CRS	CRS
	5	CRS	DRS	CRS	DRS
	6	DRS	DRS	DRS	DRS

IRS increasing RTS, *DRS* decreasing RTS, *CRS* constant RTS

different projection points on the CCR/BCC efficient frontier, upon which RTS is determined (Zarepisheh et al. 2010).

The fact that different orientations may lead to different RTS characterizations, regardless of the DEA model adopted—CCR or BCC—is well-discussed in literature (see, for instance, Zarepisheh et al. 2010; Ray 2010; Banker et al. 2004). However, under what conditions CCR and BCC models generate different RTS characterizations, observing the same orientation, is a frontier estimation issue that shall be further explored (Daraio and Simar 2007), where the "real life" violation of the convexity assumption may or may not be involved.

For instance, according to Fig. 8.2 (Top and Bottom) even that convexity assumptions hold, horizontal and vertical projections upon points A, B, and C cannot be unambiguously defined as CRS, IRS, or DRS without further testing, since different classifications may emerge from CCR and BCC models, specially if $m + s > 2$ (multiple inputs and outputs). Different scaling conclusions may also be derived when convexity assumptions are violated. For instance, this might happen if the DRS region preceded the IRS region. Scale Indicator values close to 1 and/or u_o values close to 0 would possibly indicate such ambiguities for a given DMU. These issues are addressed next.

8.3 Estimation and Bootstrapping in DEA

8.3.1 Estimation

Thus far, according to Simar and Wilson (2004), none of the theoretical models presented in the previous sections are actually observed, including the efficient frontier (CCR, BCC, or FDH) and its respective distance function to each DMU ($D(x, y/\cdot)$). More precisely, distance functions can be viewed as applied tools in efficiency measurement. For example, the reciprocal of the input distance function is equivalent to what is called in DEA/FDH terminology as the input oriented measure of technical efficiency/efficiency score (Färe et al. 2004), that is

$D(x, y/\cdot) = \theta_{\bullet}^{-1}$, where • represents a given efficient frontier and $^{-}$ symbolizes an estimate.

Thus, all these elements must be estimated. Simar and Wilson (2004) advocate that the terminology often used in the DEA/FDH literature is sometimes confusing and misleading. For instance, the terms "CCR model", "BCC model", and "FDH model" are, as a matter of fact, misnomers, since they constitute different estimators of the efficient frontier, not different models. In other words, the term "DEA model" should not obfuscate the fact that DEA is a class of efficient frontier estimators, characterized by, among other things, convexity assumptions (Cesaro et al. 2009). Departing from Tables 8.1 and 8.3, it is possible to compute estimates for $D(x, y/\cdot)$, writing them as linear programs. In particular, if the input-orientation is considered, it follows that:

$$\left[D\left(x, y/\overline{CCR}\right)\right]^{-1} = \min\{\theta/y \leq Y\lambda, \theta x \geq X\lambda, \lambda \geq 0\}, \tag{8.7}$$

$$\left[D\left(x, y/\overline{BCC}\right)\right]^{-1} = \min\{\theta/y \leq Y\lambda, \theta x \geq X\lambda, e\lambda = 1, \lambda \geq 0\}, \tag{8.8}$$

$$\left[D\left(x, y/\overline{FDH}\right)\right]^{-1} = \min\{\theta/y \leq Y\lambda, \theta \geq X\lambda, \lambda \in \{0, 1\}\}, \tag{8.9}$$

where, again, X and Y are, respectively, vectors of observed inputs and outputs and e is a vector of ones.

Taking Fig. 8.2 as reference, and considering that the actual, non-observable, efficient frontier is, in fact, convex, it is possible to affirm that (Simar and Wilson 2004):

$$\overline{FDH} \subseteq \overline{BCC} \subseteq \overline{CCR}. \tag{8.10}$$

The differences between the actual, non-observable, efficient frontier and any of the estimators $\overline{FDH}$, $\overline{BCC}$, and $\overline{CCR}$ are of utmost importance, for these differences determine the differences between $D(x, y/\cdot)$ and any of the corresponding estimators. The focus of interest is $D(x, y/\cdot)$, fixed, but unknown. Estimators, on the other hand, are necessarily random variables upon which statistical tests, or at least, confidence intervals (CI) can be built to derive useful conclusions.

The importance of bootstrap-based approaches, such as those presented in Simar and Wilson (2004) and Wilson (2008), for estimation on the efficiency frontier, should be put into perspective. As seen in previous sections, the discussions on RTS in different DEA models have been confined to "qualitative" characterizations in the form of identifying whether they are increasing, decreasing, or constant (Banker et al. 2004; Cooper et al. 2007). These bootstrap approaches, however, which are also useful to deal with the asymptotic distribution of DEA/FDH estimators, can be used to implement statistical tests of constant returns to scale versus varying returns to scale, convexity among other things (Wilson 2009). For example, Daraio and Simar (2007) developed several conditional measures of efficiency, which also provide indicators for the type of RTS. The bootstrap methodology used in this study is detailed next.

Putting into a broader perspective, it is important for the practitioner to be able to test empirically hypotheses relating to the DGP and having to do with the shape of the frontier (e.g., convexity, returns to scale, etc.). This is important not only for economic considerations, but also for statistical reasons, since as shown above there is much to be gained in terms of statistical precision by assuming convexity of the production frontier or constant-returns to scale if such assumptions are appropriate (Simar and Wilson 2013).

8.3.2 Bootstrapping Method

Rather than using the inconsistent naive bootstrap, Simar and Wilson (1998) propose using a smooth bootstrap. In this early study, Simar and Wilson implement the smoothed bootstrap in a simple model under the assumption that the distribution of the inefficiencies along the chosen direction (input rays or output rays) is homogeneous in the input-output space. Hence the smoothing operates only on the estimation of the univariate density of the efficiencies, making the problem much easier to handle. Simar and Wilson (2000) extend this idea to a more general heterogeneous case where the distribution of efficiency is allowed to vary over the production frontier. Results from intensive Monte-Carlo experiments described in both papers suggest that these bootstrap procedures give reasonable approximations for correcting the bias of the efficiency estimates and for building individual confidence intervals for the efficiency of any fixed point (Simar and Wilson 2013).

The method used in this study departs from the one developed by Simar and Wilson (2004), which adapted the bootstrap methodology to the case of DEA/FDH efficiency estimators, and uses an Gaussian kernel density function for random data generation. The algorithm is detailed next.

Algorithm

1. For each DMU_j, apply all distance function estimators in (8.7)–(8.9) to obtain the reciprocal estimates of $D\left(x_j, y_j/\overline{FDH}\right)$, $D\left(x_j, y_j/\overline{CCR}\right)$, and $D\left(x_j, y_j/\overline{BCC}\right)$.
2. Reflect the n reciprocal estimates for each efficient frontier (FDH, BCC, and CCR) about the unity, and determine the respective bandwidth parameters h_{FDH}, h_{BCC}, and h_{CCR} via ordinary least squares.
3. Use step [4] to draw n bootstrap values $\overline{D_j}$, $j = 1,\ldots,n$, from the respective kernel density function, for each one of the efficiency estimates from step [1] and their reflected values from step [2].
4. Let $\{\varepsilon_j\}_{j=1}^{n}$ be a set of iid draws from the probability density function used to define the respective kernel function; let $\{d_j\}_{j=1}^{n}$ be a set of values drawn independently, uniformly, and with replacement from the respective set of reflected distance function estimates $R = \left\{D\left(x_j, y_j/\cdot\right), 2 - D\left(x_j, y_j/\cdot\right)\right\}$; and

let $\overline{d} = n^{-1}\sum_{j=1}^{n} d_j$. Then compute $\overline{d_j} = \overline{d} + \left(1 + h_{\bullet}^2/s^2\right)^{1/2}\left(d_j + h_{\bullet}\varepsilon_j - \overline{d}\right)$, where s^2 is the sample variance of the values $d_j + h_{\bullet}\varepsilon_j$. If $\overline{d_j} \le 1$, then $\overline{D_j} = \overline{d_j}$, otherwise $\overline{D_j} = 2 - \overline{d_j}$.

5. Construct a pseudo dataset of inputs and outputs ($\overline{S_n^{\bullet}} = \left\{\left(\overline{x_j}, \overline{y_j}\right)\right\}_{j=1}^{n}$) for each efficient frontier (*FDH*, *BCC*, and *CCR*) with elements $\left(\overline{x_j}, \overline{y_j}\right)$ given by $\overline{x_j} = \overline{D_j}$ $x_j/D\left(x_j, y_j/^j\cdot\right)$ and $\overline{y_j} = y_j$ (if input-oriented).
6. Use the respective distance function estimators in (8.7)–(8.9) to compute the bootstrap estimate $D(\overline{x_o}, \overline{y_o}/\cdot)$.
7. Repeat steps [3] to [5] B times to obtain a set of B bootstrap estimates $\{D_b(\overline{x_o}, \overline{y_o}/\cdot)\}_{b=1}^{B}$.

As a matter of fact, once the B pseudo datasets of inputs and outputs for the n DMUs have been obtained, it is straightforward to estimate CIs on a given DMU_o, not only for the actual distance functions, but also for the efficiency scores and the RTS indicators. Table 8.5 summarizes the pseudo datasets of additional estimates that can be possibly generated from the pseudo datasets of inputs and outputs, using the algorithm previously presented.

8.4 Case Study: Brazilian Port Terminals

Transportation has increased in importance for the economy and firms in the globalization scenario. In order to support trade oriented economic development, port authorities have increasingly been under pressure to improve port efficiency, ensuring that port services are provided on an internationally competitive basis. There is a consensus that ports form a vital link in the overall trading chain by contributing to a nation's international competitiveness (Tongzon 1989; Chin and Tongzon 1998).

In Brazil, one of the so-called "emerging countries" or "BRICs" (Wilson and Purushothaman 2003)—acronym that stands for Brazil, Russia, India, and China—exports in nominal prices more than doubled in the period between 2002 and 2008, reaching almost US$ 200 bn (Fleury and Hijjar 2008). About half of this volume was due to primary commodities and partly processed commodities (soy, iron ore, oil, frozen orange juice, petrochemicals, coffee, sugar, ethanol, pulp, etc.); the other half was due to manufactured products (processed meat, automobiles, steel, aircraft, appliances, auto parts, etc.).

According to Curcino (2007), the Brazilian Federal Law 8630—edited in 1993 and also known as "Port Modernization" Law—was the path for port privatization, leasing of terminals, installation of local port authorities, and labor deregulation, breaking up with the state monopoly on the sector. Although investments in capacity expansion were minimal from that period to these days, the comparison

Table 8.5 Pseudo datasets of estimates

Inputs and outputs	Distance functions	Efficiency scores	RTS indicators
$\left\{\left(\overline{x_o}, \overline{y_o}/\overline{FDH}\right)\right\}_{b=1}^{B}$	$\left\{D_b\left(\overline{x_o}, \overline{y_o}/\overline{FDH}\right)\right\}_{b=1}^{B}$ using (8.9)	$\left\{\theta_b\left(\overline{x_o}, \overline{y_o}/\overline{FDH}\right)\right\}_{b=1}^{B}$ using (8.5)	NA
$\left\{\left(\overline{x_o}, \overline{y_o}/\overline{BCC}\right)\right\}_{b=1}^{B}$	$\left\{D_b\left(\overline{x_o}, \overline{y_o}/\overline{BCC}\right)\right\}_{b=1}^{B}$ using (8.8)	$\left\{\theta_b\left(\overline{x_o}, \overline{y_o}/\overline{BCC}\right)\right\}_{b=1}^{B}$ using (8.1) + convexity constraint on $\sum_{j=1}^{n} \lambda_j$	$\left\{u_{o,b}\left(\overline{x_o}, \overline{y_o}/\overline{BCC}\right)\right\}_{b=1}^{B}$ using (8.3)
$\left\{\left(\overline{x_o}, \overline{y_o}/\overline{CCR}\right)\right\}_{b=1}^{B}$	$\left\{D_b\left(\overline{x_o}, \overline{y_o}/\overline{CCR}\right)\right\}_{b=1}^{B}$ using (8.7)	$\left\{\theta_b\left(\overline{x_o}, \overline{y_o}/\overline{CCR}\right)\right\}_{b=1}^{B}$ using (8.1)	$\left\{SI_{o,b}\left(\overline{x_o}, \overline{y_o}/\overline{CCR}\right)\right\}_{b=1}^{B}$ using (8.1)

NA non-applicable

of several ports in terms of their overall efficiency has become an essential part of the Brazilian microeconomic reform agenda for sustaining economic growth based on foreign trade (Fleury and Hijjar 2008). For example, in 2006 a federal authority linked to the Transportation Ministry was created to monitor operational bottlenecks and to allocate investments among Brazilian ports and terminals.

Traditionally, the performance of ports and terminals has been variously evaluated by numerous attempts at calculating and seeking to optimize the operational productivity of cargo handling at the berth and in the terminal area (see Cullinane et al. 2006 for a comprehensive list of references). In recent years approaches such as DEA and FDH (Cullinane et al. 2005) have been increasingly utilized to analyze production and performance of ports and terminals. It must be noted, however, that FDH is less frequently used than DEA, the technique that presents the largest amount of applications in this sector (see Panayides et al. 2009 for a comprehensive list of references until that date).

As regards model orientation, some authors, such as Rios and Maçada (2006), Barros (2003), Barros and Athanassiou (2004), and Park and De (2004) used input-oriented models. The basic idea behind this choice is that the output increasing potential should be interpreted with more care, unless there is demand for it and, therefore, decision-makers should focus on “stressing” production inputs for a given level of output that may not necessarily be maximal (Odeck and Alkadi 2001). On the other hand, however, port inputs are strictly seen by some authors as fixed assets, long-term investments, which are difficult to demobilize in the short-term (Cullinane et al. 2006) and, thus, decision-makers should focus on maximizing outputs for a given level of production inputs.

Due to the scarcity of official data and due to confidentiality regarding physical resources allocated to terminal operations, a questionnaire was sent via e-mail at the beginning of 2009, covering a convenience sample of 25 terminals that had previously agreed to provide such information. This sample size is comparable to similar DEA applications: the comprehensive literature review presented in Panayides et al. (2009) indicates that the number of DMUs (ports/terminals) researched ranges from 6 to 104 (mean 28). If the work of Wang and Cullinane (2006) was excluded, the average number of DMUs would be 19.8. Since there are 46 ports and 124 terminals operating in Brazil (Fleury and Hijjar 2008), the response rate of this survey is around 20 % (25/124).

If alternative criteria were used to evaluate the representativeness of this sample size of 25 terminals, results of similar magnitude—at least—would be obtained. Along with ANTAQ (Brazilian National Waterborne Transportation Agency—www.antaq.gov.br), the total amount of cargo handled by ports in Brazil during 2008 was 284.8 million tons, implying a share of 42.6 % for these 25 selected terminals. ANTAQ also provides data regarding the total number of berths in Brazilian ports (244 in 2008). Compared with the total number of berths of these 25 terminals on that date (49), it follows that they represented 20.1 % of the total national amount. Lastly, data provided by ANTAQ with respect to the frequency of shipments per year in Brazilian ports (15,183 in 2008) suggest that these 25 terminals represent 48.5 % of this total amount.

Table 8.6 Terminals researched

Port	Terminal	Port	Terminal
Aratu	Porto de Aratu	Itaguaí	TECAR
Rio de Janeiro	Terminal da Ilha Guaíba	Itaguaí	Terminal de Alumina do Porto de Itaguaí
Rio de Janeiro	Rio TPS	Vitória	Terminal Portuário PEIU
Santos	Terminal ADM	Aratu	Terminal Marítimo Dow Brasil Industrial
Santos	Terminal XXXIX	Aratu	TEGAL
Santos	Teaçu Armazéns Gerais	Aratu	Tequimar
Santos	Citrosuco Serviços Portuários	Paranaguá	Cattalini Terminais Marítimos
Santos	TIS—Terminal Intermodal de Santos	Paranaguá	União Vopak Armazéns Gerais
Santos	União Terminais	Rio Grande	Terminal Santa Clara
Santos	Vopak Terminal Alemoa	Rio Grande	TERIG
Santos	Terminal 37	Manaus	Super Terminais Comércio e Indústria
Santos	Tecondi	Paranaguá	TCP—Terminal de Contêineres de Paranaguá
Suape	Tecon Suape		

It is methodologically noteworthy that, although ANTAQ yearly presents several statistics on terminal production levels—throughputs, frequencies, and loading hours—data regarding physical resources such as berths, areas, and parking lots are presented aggregated on the port level, not reflecting the their consumption, usage, or even allocation between terminals. Therefore, it was decided, as part of the research strategy, to collect this data directly from the field in order to get accurate figures in terms of the terminal operation level.

The terminals researched, as well as their respective ports of origin, are listed in Table 8.6. The three variable inputs collected from each terminal are: terminal area (in square meters), size of parking lot for incoming trucks (in number of trucks), and number of shipping berths. As regards the outputs, two variables were collected: aggregate throughput per year (in tons) and number of loaded shipments per year. Their descriptive statistics are presented in Table 8.7.

Before proceeding, it is worth commenting that, although international, peer-reviewed papers dealing with the application of these techniques in Brazilian ports are scarce, empirical and anecdotal evidences suggest that Brazilian terminals present increasing returns to scale. Put in other words, it seems that the capacity of the Brazilian terminals is too small relative to the tasks that it performs or, literally, that these productive units are running short in capacity due to the foreign trade boom verified over the last few years.

Table 8.7 Summary statistics for the sample

	Inputs measured			Outputs measured		
	Number of berths	Terminal area (sq. m)	Parking lot (# of trucks)	Aggregate throughput (tons/year)	Loaded shipments (per year)	Terminal type (Container = 0/ bulk = 1)
Mean	1.96	214,189.12	59.67	4,850,909.60	294.24	0.80
St. dev.	0.93	329,956.02	49.41	18,062,163.68	303.65	0.41
CV	0.48	1.54	0.83	3.72	1.03	0.51

As a matter of fact, Rios and Maçada (2006) point out that, until that date, no studies developed in Brazil were found. The authors analyzed the relative efficiency of 20 container terminals located in Mercosur during the years of 2002, 2003, and 2004 by means of an input-oriented BCC model. Results indicate that 60 % of the terminals were found to be efficient in this three-year period, probably reflecting the fact that the Brazilian ports had reached record rates of cargo traffic, including higher value added products, such as automobiles. According to these authors, container traffic had increased 23.1 % by that time. In Argentina, the container sector had an increase of almost 17 %. No further international peer-reviewed studies, on the efficiency of Brazilian ports or terminals, were found since 2006 until 2010.

In order to evaluate the adequacy of the convexity assumption imposed by DEA models and to characterize the prevalent RTS within the collected sample of Brazilian terminals, the methodological framework presented in Sect. 8.3 was applied. More precisely, 95 % CIs were determined, not only for the set of estimators $\left\{D_b\left(\overline{x_o},\overline{y_o}/\overline{FDH}\right)\right\}_{b=1}^{B}$, $\left\{D_b\left(\overline{x_o},\overline{y_o}/\overline{BCC}\right)\right\}_{b=1}^{B}$ and $\left\{D_b\left(\overline{x_o},\overline{y_o}/\overline{CCR}\right)\right\}_{b=1}^{B}$—to accept/reject the convexity assumption at a given DMU_o/terminal—but also for $\left\{SI_{o,b}\left(\overline{x_o},\overline{y_o}/\overline{CCR}\right)\right\}_{b=1}^{B}$ and $\left\{u_{o,b}\left(\overline{x_o},\overline{y_o}/\overline{BCC}\right)\right\}_{b=1}^{B}$—to assess how its rejection impacts the RTS characterization under the same input-orientation. These analyses were implemented in Maple 12, with 1000 bootstrap replications, generated upon Gaussian kernel density functions, for each efficient frontier. Their results are discussed next.

8.5 Results

8.5.1 *Initial Estimates*

The efficiency rankings calculated using DEA/FDH input-oriented models are given in Table 8.8, as well as the RTS scores (SI and u_o) for each DMU. As one would expect, the FDH model yields higher average efficient estimates than do both DEA models (an index value of 1.00 equates to maximum efficiency). Specifically, the CCR model yields lower average efficiency estimates than the BCC model, with

Table 8.8 Initial estimates[a]

DMU		Input slacks			Output slacks		Efficiency scores—input-oriented			RTS characterizations			
#	Terminal	Number of berths	Terminal area (sq. m)	Parking lot (# of trucks)	Aggregate throughput (tons/yr)	Loaded shipments (per year)	θ_{FDH}	θ_{BCC}	θ_{CCR}	SI	RTS-CCR	u_o	RTS-BCC
1	Super Terminais Comércio e Indústria	–	–	25.11	635,762.83	–	0.72	0.51	0.12	0.09	Increasing	0.45	Increasing
2	TCP—Terminal de Contêineres de Paranaguá	–	–	–	–	–	1.00	1.00	1.00	1.00	Constant	–	Constant
3	Terminal 37	–	132,667.31	77.13	2,388,844.15	–	0.60	0.59	0.54	1.13	Decreasing	(0.37)	Decreasing
4	Tecondi	–	–	42.99	1,734,657.60	–	1.00	0.74	0.55	0.41	Increasing	0.33	Increasing
5	Tecon Suape	–	–	37.35	1,223,029.39	–	1.00	0.77	0.64	0.55	Increasing	0.30	Increasing
6	Porto de Aratu	–	–	–	–	–	1.00	1.00	0.87	0.44	Increasing	1.00	Increasing
7	**Terminal da Ilha Guaíba**	–	–	–	–	–	**1.00**	**1.00**	**1.00**	**1.00**	**Constant**	**0.63**	**Increasing**
8	**Rio TPS**	–	–	–	–	–	**1.00**	**1.00**	**1.00**	**1.00**	**Constant**	**0.97**	**Increasing**
9	Terminal ADM	–	–	–	–	–	1.00	1.00	0.28	0.08	Increasing	1.00	Increasing
10	Terminal XXXIX	–	32,703.79	59.74	–	147.40	1.00	1.00	0.22	0.04	Increasing	1.00	Increasing
11	Teaçu Armazéns Gerais	–	178,000.00	120.00	195,380.00	111.00	1.00	1.00	0.19	0.10	Increasing	1.00	Increasing
12	TECAR	–	324,784.50	–	–	101.87	0.96	0.34	0.05	0.08	Increasing	1.00	Increasing
13	Terminal de Alumina do Porto de Itaguaí	–	–	–	–	–	1.00	1.00	0.11	0.05	Increasing	1.00	Increasing

14	Terminal Portuário PEIU	–	–	50.00	114,024.00	3.00	1.00	1.00	0.04	0.01	Increasing	0.39	Increasing
15	Terminal Marítimo Dow Brasil Industrial	–	–	–	457,523.39	0.07	1.00	1.00	0.73	0.36	Increasing	0.28	Increasing
16	TEGAL	–	–	–	–	–	1.00	1.00	0.95	0.54	Increasing	0.50	Increasing
17	Tequimar	–	–	6.80	–	–	1.00	0.76	0.64	0.56	Increasing	0.49	Increasing
18	Cattalini Terminais Marítimos	–	64,688.87	93.70	–	32.25	1.00	0.50	0.18	0.17	Increasing	0.29	Increasing
19	**União Vopak Armazéns Gerais**	–	–	**31.62**	**150,758.96**	–	**1.00**	**0.58**	**0.16**	**0.11**	**Increasing**	**(0.00)**	**Decreasing**
20	Terminal Santa Clara	–	–	–	3,112,359.26	–	1.00	1.00	0.77	0.77	Increasing	1.00	Increasing
21	**TERIG**	–	–	–	–	–	**1.00**	**1.00**	**1.00**	**1.00**	**Constant**	**0.43**	**Increasing**
22	Citrosuco Serviços Portuários	–	133,000.00	20.00	895,380.00	147.00	1.00	1.00	0.10	0.05	Increasing	0.31	Increasing
23	TIS – Terminal Intermodal de Santos	–	–	14.82	354,848.10	–	0.64	0.52	0.17	0.12	Increasing	0.30	Increasing
24	**União Terminais**	–	–	**11.02**	–	–	**0.67**	**0.45**	**0.24**	**0.24**	**Increasing**	–	**Constant**
25	Vopak Terminal Alemoa	–	–	12.40	220,666.76	–	0.50	0.38	0.15	0.15	Increasing	0.33	Increasing

[a]It should be noted that the parking lot of DMUs 2, 6, 7, 8, 13, 15, and 20 is equal to zero. These zero values were substituted by 0.01—according to the “zero replacement” feature now offered by some DEA softwares (Barr 2004)—in order to proceed with the analyses within the ambit of input-oriented models

respective average values of 0.47 and 0.81, and. In other words, the CCR model identifies more inefficient terminals (21 vs. 11) than the BCC model does. This result is not surprising, as the CCR model fits a linear production technology, whereas the BCC model features variables returns to scale, which are more flexible and reflect managerial efficiency apart from purely technical limits.

The vast majority (20 out 25) of the Brazilian terminals analyzed seems to be unambiguously experiencing IRS under both RTS characterizations. Only one terminal appears to be unambiguously experiencing DRS (DMU 3, Terminal 37 at Port of Santos). Discrepancies between RTS characterizations were found in five cases (DMUs 7, 8, 19, 21 and 24), three of them scale efficient, that is, located at the MPSS.

According to Odeck and Alkadi (2001) and Ross and Droge (2004), a DMU may be scale inefficient if it experiments decreasing returns to scale by being too large in size, or if it is failing to take full advantage of increasing returns to scale by being too small. So far, these initial results suggest that most Brazilian port terminals are running short in capacity. Put in other words, the capacity of the terminal is too small relative to the tasks that it performs.

Considering the results presented in Table 8.8, the terminal area and the truck parking lot tend to be less efficiently used than the shipping berths under idle capacity at the inefficient terminals. As a matter of fact, the output of these terminals may indeed be higher, not only in terms of aggregate throughput in tons per year but also in terms of the number of shipments loaded. It is interesting to note that as these outputs increase, the level of shipment consolidation at each terminal is impacted. In general, these results help to explain the increasing returns to scale observed in the majority of the terminals analyzed: although there are no berths left, which may suggest that capacity is strangled, the slacks of terminal areas and parking lots can be used to increase the aggregate throughput per shipment per berth.

8.5.2 Preliminary Statistics Tests on Initial Estimates

Before proceeding, it is relevant to ensure that the different terminals presented in Table 8.6 consist, as matter of fact, of a group of homogenous DMUs, upon which valid conclusions can be derived regarding returns-to-scale and convexity. There are particular analyses that may help characterizing such homogeneity, for which hypothesis tests could be performed on the initial estimates, prior to bootstrapping. These analyses frequently involve the identification of: (a) the adequacy of a given model assumption—that is, returns to scale—to the whole data set; (b) the eventual differences between two groups—container vs. bulk terminals—in terms of their returns-to-scale and efficiency levels (Banker and Natarajan 2004); (c) the relevant and irrelevant inputs used (Wagner and Shimshak 2007); and (d) the most influential observations—or outliers—in the data set (Pastor et al. 1999; Wilson 2008). These issues are discussed next.

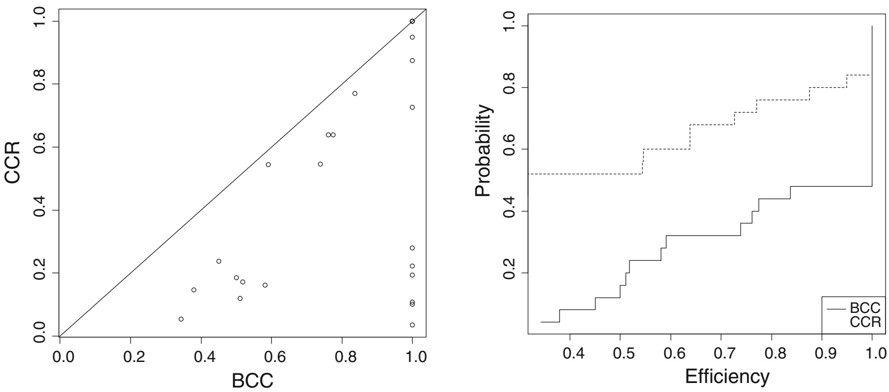


Fig. 8.3 Efficiency comparison between different technology sets

8.5.2.1 Testing for Model Specification

In order to distinguish between the two possible technology sets—constant vs. varying returns to scale—the non-parametric Kruskal-Wallis test was performed on DEA CCR and BCC efficiency estimates (Banker and Natarajan 2004). Technology assumptions within Brazilian port terminals are evaluated, therefore, by testing whether or not efficiency estimates are the same under the two technologies (Bogetoft and Otto 2010). Results presented in Fig. 8.3 indicate that the null hypothesis of a constant returns-to-scale technology should be rejected in favor of a varying one (p-value = 0.0008518; Chi-squared = 11.1248; df = 1). Thus, it is possible to affirm that varying returns to scale prevails within Brazilian port terminals.

8.5.2.2 Testing for Differences Between Container and Bulk Terminals

Here it will be distinguished not between two sets of model assumptions, but rather between two groups of observations—container and bulk terminals—according to the discussion presented in Bogetoft and Otto (2010). The idea is to check whether different production processes could lead to significant differences in efficiency and scale estimates, thus jeopardizing the basic DEA assumption of homogeneous DMUs comparison. Again, the non-parametric Kruskal-Wallis test was performed on DEA efficiency estimates for each group. Results presented in Fig. 8.4 indicate that the null hypothesis of identical efficiency levels between container and bulk Brazilian terminals cannot be rejected (p-value = 0.3409; Chi-squared = 0.907; df = 1).

Similarly, taking the scale indicator between both groups into account, results presented in Fig. 8.5 for the Kruskal-Wallis test also indicate that the null hypothesis of identical returns-to-scale between container and bulk Brazilian terminals cannot be rejected (p-value = 0.1023; Chi-squared = 2.6687; df = 1).

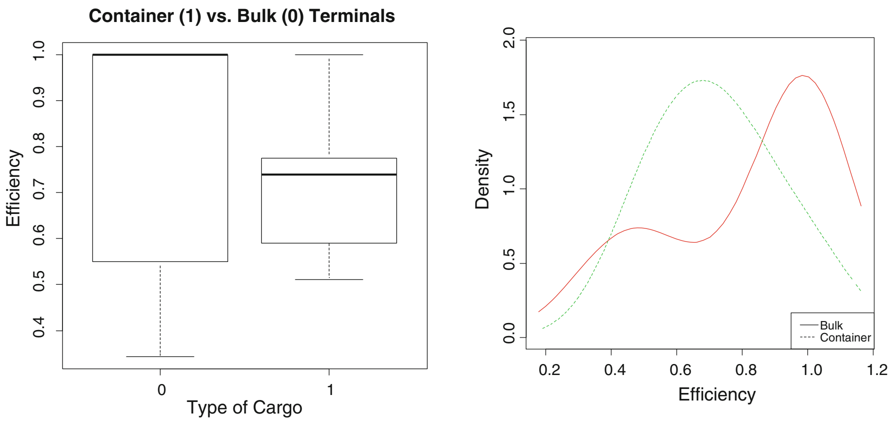


Fig. 8.4 Efficiency comparison between container and bulk terminals (box plot and kernel densities)

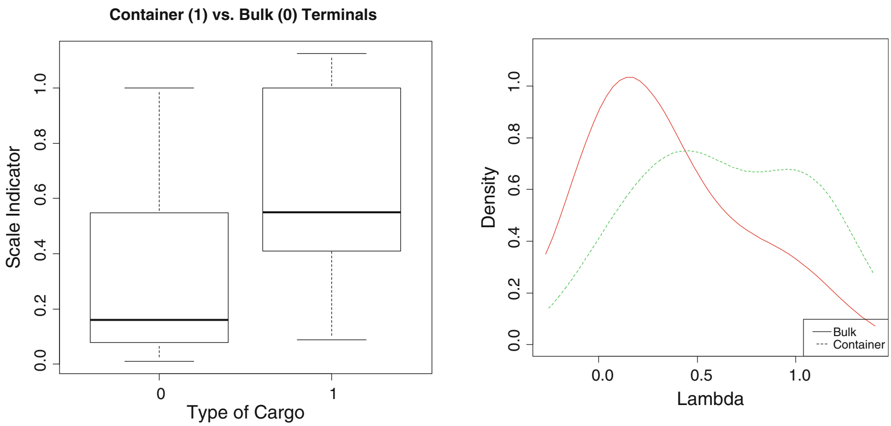


Fig. 8.5 Scale indicator comparison between container and bulk of terminals (box plot and kernel densities)

8.5.2.3 Testing for Relevant Inputs and Outputs

According to Simar and Wilson (2013), reducing the dimensionality of the problem by testing the relevance of certain inputs and outputs or the possibility of aggregating inputs or outputs is always of interest when applying statistical approaches into nonparametric frontier models. In this research, although different production processes between container and bulk terminals did not lead to significant differences in terms of efficiency levels and returns-to-scale, it is possible that some input/output variable may reflect the specifics of such processes, turning out to be irrelevant to the whole technology set. For instance, if pipes or railroads are the

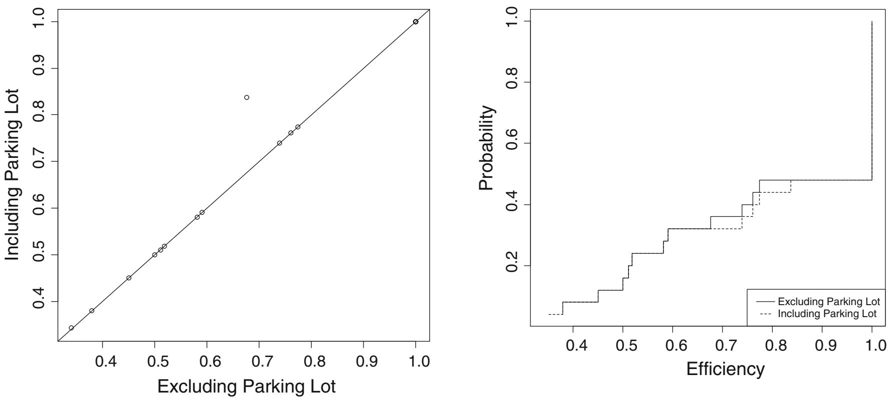


Fig. 8.6 Efficiency comparison when the parking lot input is dropped-off

transportation modes used for feeding a bulk terminal, the parking lot for trucks may be irrelevant for this production process and, therefore, should be discarded from the analysis.

Hence, the non-parametric Kruskal-Wallis test is performed to test whether fewer inputs should be included, thus moving the analysis towards what is called a "core production model" (Wagner and Shimshak 2007). More precisely, the number of shipments per berth and the throughput per berth capture the most fundamental essence of port production, both when its definition and its operations planning are taken into consideration. For example, Alderton (2008) provides a legal definition for ports: "they are areas within which ships are loaded and/or discharged of cargoes and includes the usual places where ships wait or their turn". On the other hand, Meisel (2009) discusses the berth allocation problem, affirming that "this problem is to assign a berthing position and a berthing time to each vessel, such that a given objective function is optimized".

Results presented in Fig. 8.6 for the Kruskal-Wallis test indicate that the null hypothesis of identical efficiency levels between the original analysis and the other one, where the parking lot variable is discarded, cannot be rejected (p-value = 0.1023; Chi-squared = 2.6687; df = 1). Since these two technology sets are only slightly different in terms of efficiency, the parking lot variable is not an influential input and, therefore, should be discarded from subsequent analyses.

8.5.2.4 Testing for Outliers

The approach developed by Wilson (2008) to identify the most influential observations (outliers) within the ambit of DEA analyses—was conducted with the support of the general purpose statistical software R using the function *ap* from the FEAR library. It is based on the data cloud method, which is briefly described

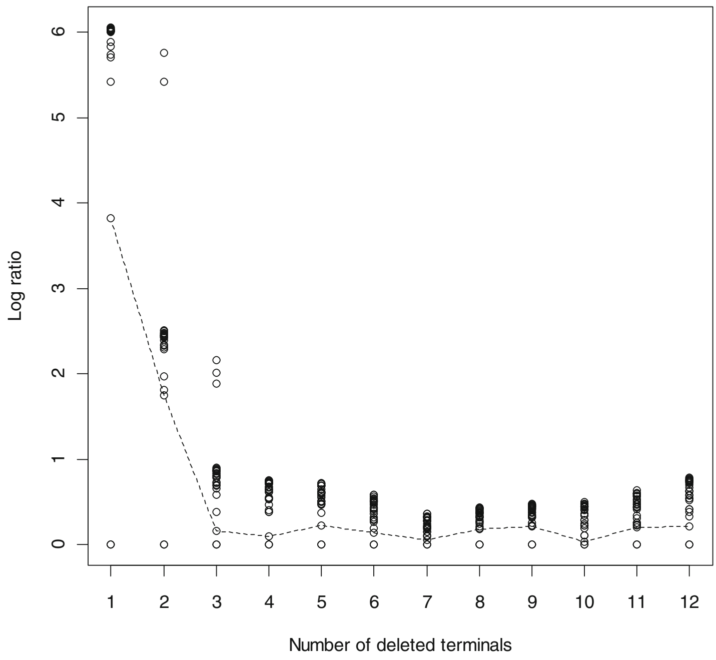


Fig. 8.7 Log-ratio plot vs. number of deleted terminals from the sample

by Bogetoft and Otto (2010). Figure 8.7 indicates that the dashed lines peaks at two deleted terminals from the original sample, that is, there are two outliers. Additional results derived from this analysis indicate that these outliers are DMU 7 (Terminal da Ilha Guaíba) and DMU 12 (TECAR). They are, therefore, discarded from subsequent analysis.

8.5.3 Bootstrapped Efficiency Scores and Convexity Assumption

The lower and upper bounds for the 95 % CIs for the FDH, BCC, and CCR efficiency scores, as well as their respective bias corrected central estimates, are presented in Fig. 8.8 for each terminal, after discarding two outliers and one irrelevant input variable. The procedures for computing these estimates, based upon 1000 bootstrap replications for each efficient frontier, followed the discussions detailed in Simar and Wilson (2004), Bogetoft and Otto (2010), Curi et al. (2010). The asymptotic nature of the CIs should also be noted, as their lower and upper bounds are not symmetrical around the central estimate.

With respect to the convexity assumption, the upper bounds for the 95 % CIs for the FDH, BCC, and CCR distance functions are given in Fig. 8.9. Readers should

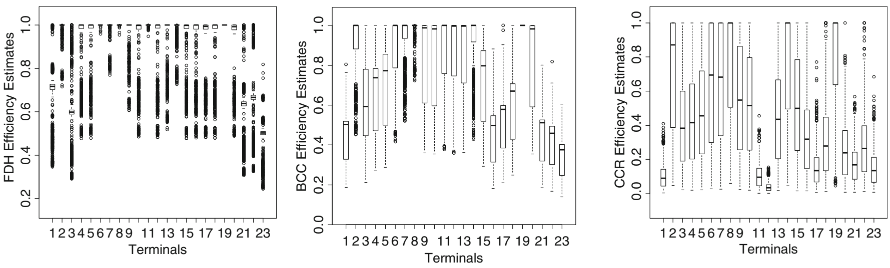
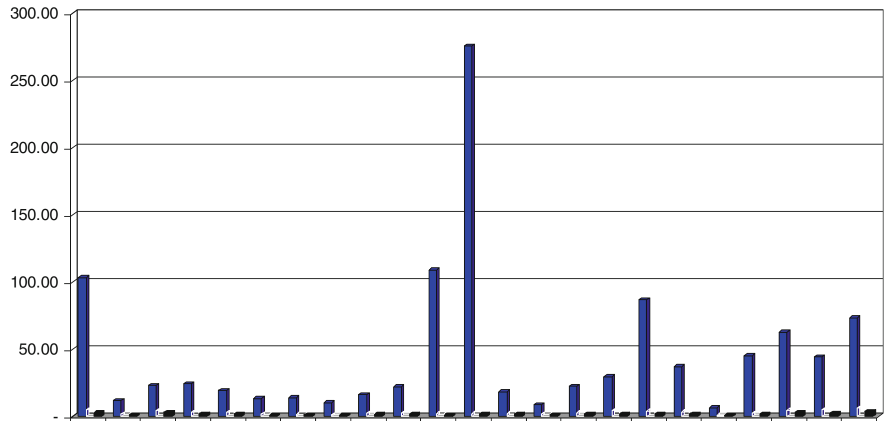


Fig. 8.8 Bootstrapped efficiency scores



	Super Terminais	TCP - Terminal de	Terminal 37	Tecondi	Tecon Suape	Porto de Aratu	Rio TPS	Terminal ADM	Terminal XXXIX	Teaçu Armazéns	Terminal de Alumi	Terminal Portu	Terminal Maríti	TEGAL	Tequimar	Cattalini Termi	União Vopak Arma	Terminal Santa	TERIG	Citrosuco Serviç	TIS - Terminal	União Terminais	Vopak Terminal
Upper CCR	103.4	11.72	23.10	24.40	19.19	13.45	14.16	10.28	16.43	22.14	109.0	275.6	18.54	8.76	22.37	29.67	87.00	37.09	6.45	45.21	62.76	44.53	73.34
Upper BCC	4.97	1.95	4.21	3.10	2.99	1.99	1.64	1.27	2.50	2.57	2.09	2.21	2.20	1.80	2.88	4.55	4.17	3.38	1.00	2.57	4.76	5.07	6.25
Upper FDH	2.73	1.29	3.27	1.94	1.89	1.00	1.20	1.00	1.54	1.91	1.00	1.89	1.74	1.30	1.74	1.95	1.94	1.91	1.00	1.97	3.03	2.53	3.81

Fig. 8.9 Upper bounds for the 95 % CIs—FDH, BCC, and CCR distance functions

note that taking the reciprocals of the CI estimates, for the case of analyzing input distance functions instead of efficiency scores, requires reversing the order of the bounds; that is, the reciprocal of the upper bound for the input distance function measure gives the lower bound for the efficient score measure, and vice-versa (Wilson 2009). Considering the conditions stated in (8.10), it follows that the convexity assumption is statistically supported within this data set.

8.5.4 *RTS Characterizations: CIs for* SI *and* u_o

The lower and upper bounds for the 95 % CIs for the *SI* and u_o RTS indicators, as well as their respective bias corrected central estimates, are given in Fig. 8.10. The methodology used to analyze these results is synthesized in Fig. 8.11. Within the

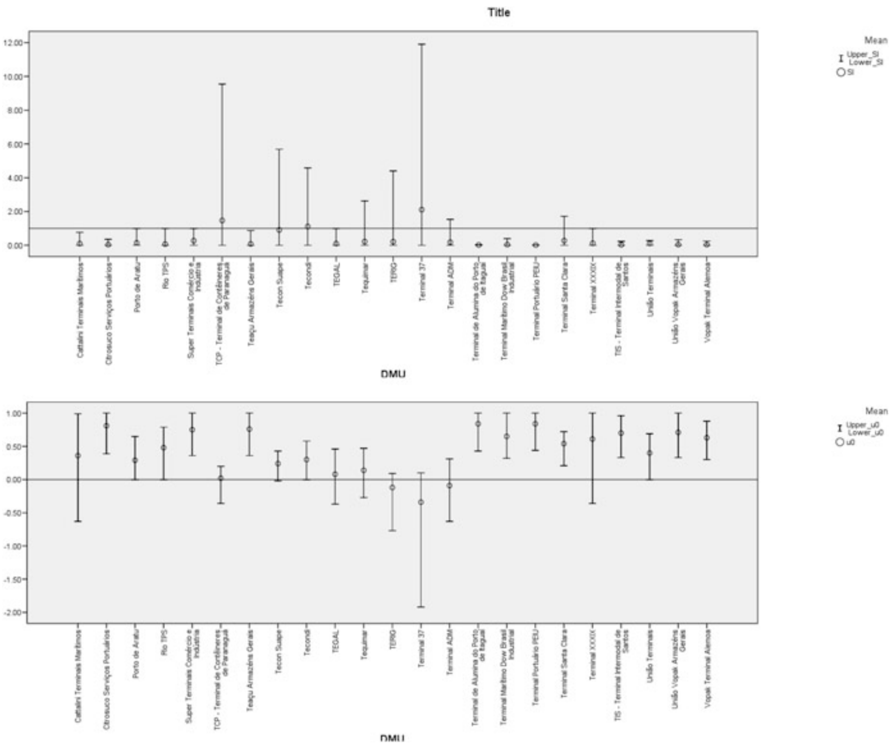


Fig. 8.10 95 % CIs for *SI* and u_o

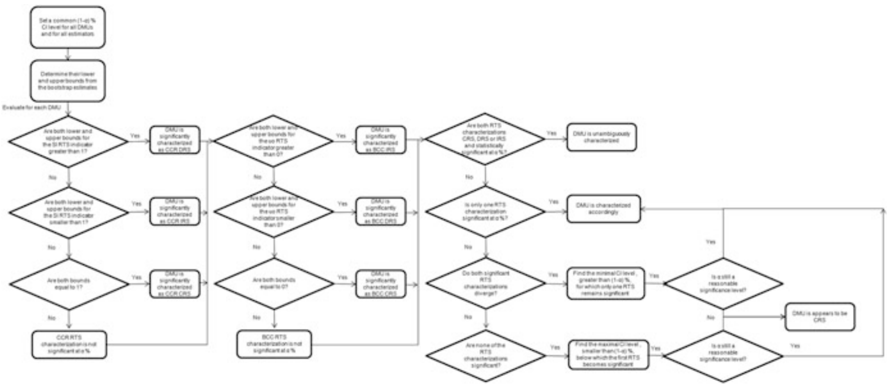


Fig. 8.11 Simple methodology to assess RTS characterization based upon CIs

CCR case, a given RTS characterization is considered to be statistically significant only if the lower and upper bounds of the confidence interval for the *SI* indicator are both greater than 1 (DRS) or smaller than 1 (IRS). On the other hand, if the BCC case is considered, only if the lower and upper bounds of the confidence interval for the u_o indicator are both greater than 0 (IRS) or smaller than 0 (DRS). Both bounds equal to 1 or 0, respectively, strongly suggests CRS at a given significance level. As argued by Bogetoft and Otto (2010), since the connection between a given RTS characterization and its estimates is uncertain or stochastic, the hypotheses of a given characterization should be rejected if at least one of the estimated scale indicators falls outside such critical values.

Eight out of 23 Brazilian terminals—after discarding the two outliers—seem to be unambiguously experiencing IRS under both RTS characterizations at 5 % of significance. In six other cases, only one RTS characterization was found to be statistically significant at 5 %. No terminal appears to be unambiguously experiencing DRS at 5 % of significance. Only one terminal was found to be experiencing DRS under a given characterization. The remainder eight terminals are experiencing CRS. Discrepancies between RTS characterizations, originally found in five cases except for one outlier, were eliminated.

8.5.5 Discussion

Although terminal production is well known for its complexity, this research not only shows that different Brazilian container and bulk terminals can be described in terms of a core production process, but also that they constitute a homogenous group of DMUs in terms of efficiency levels and economies of scale. The findings confirm that the majority of Brazilian terminals present increasing returns to scale, thus providing an argument for their upgrading. In other words, this means that the size of Brazilian terminals should be scaled-up in order to deal with constantly growing demand requirements.

These results may be useful for the purpose of implementing public policies or, at least, for helping in defining investment priorities. Differently from several Asian countries, for instance, in Brazil, since early 1990, there was relatively little investment in new ports, not only because of federal budget constraints but also because additional capacity could be gained by improving the existing ports via terminal privatization, deregulation etc. Now, after some successful reforms, has come the time for effective capacity expansion, as further reforms of existing terminals seem to be not enough to cater for future growth.

8.6 Conclusions

This paper illustrates how the bootstrapping methodology proposed by Simar and Wilson (2004) may be used to characterize, unambiguously, returns-to-scale under different DEA models, when a given input/output orientation is assumed. This methodology, which was implemented in Maple 12, allows the inference of confidence intervals and bias corrected central estimates, not only for returns-to-scale indicators, but also for the efficiency scores and their reciprocals, that is, their distance functions. Specifically, the latter may be also used to test for the convexity assumption imposed by DEA efficient frontiers at a given DMU altogether with FDH.

The purposes of the numerical example conducted within Brazilian port terminals are twofold. First, it was useful to corroborate empirical evidences regarding the fact that these terminals are running short in capacity, i.e., that increasing returns-to-scale prevail within this industry. Second, it served as a basis to illustrate the analytical developments of the proposed methodology, given that the researched Brazilian container and bulk terminals were proven to constitute a homogeneous group of DMUs in terms of efficiency levels and returns-to-scale that could be described by the same core production process.

References

Alderton, P. (2008). *Port management and operations*. London: Informa.

Banker, R. D., Charnes, A., & Cooper, W. W. (1984). Some models for estimating technical scale inefficiencies in Data Envelopment Analysis. *Management Science, 30*, 1078–1092.

Banker, R. D., Cooper, W. W., Seiford, L. M., & Zhu, J. (2004). Returns to scale in DEA. In W. W. Cooper, L. M. Seiford, & J. Zhu (Eds.), *Handbook on data envelopment analysis*. Boston: Kluwer International Series.

Barr, R. (2004). DEA software tools and technology—A state-of-the-art survey. In W. W. Cooper, L. M. Seiford, & J. Zhu (Eds.), *Handbook on data envelopment analysis*. Boston: Kluwer International Series.

Barros, C. P. (2003). The measurement of efficiency of Portuguese sea port authorities with DEA. *International Journal of Transport Economics, 30*, 335–354.

Barros, C. P., & Athanassiou, M. (2004). Efficiency in European seaports with DEA: Evidence from Greece and Portugal. *Maritime Economics & Logistics, 6*, 122–140.

Bogetoft, P., & Otto, L. (2010). *Benchmarking with DEA, SFA, and R*. New York: Springer.

Brasil, A. (2004). *Caminhões no porto de Paranaguá formam filas de 85 quilômetros*. Retrieved from www.ceol.com.br

Cesaro, L., Marongiu, S., Arfini, F., Donati, M., & Capelli, M. G. (2009). *Methodology for analysing competitiveness, efficiency and economy of scale—Use and applications of DEA*. FACEPA Deliverable D5.1.3 (April). Retrieved from http://www.ekon.slu.se/facepa

Charnes, A., Cooper, W. W., & Rhodes, E. (1978). Measuring efficiency of decision making units. *European Journal of Operational Research, 2*, 429–444.

Chin, A., & Tongzon, J. (1998). Maintaining Singapore as a major shipping and air transport hub. In T. Toh (Ed.), *Competitiveness of the Singapore economy* (pp. 83–114). Singapore: Singapore University Press.

COELLI T. (1996). *A guide to DEAP Version 2.1: A data envelopment analysis (computer) program*. Retrieved from http://www.une.edu.au/econometrics/cepa.htm

Coelli, T., & Perelman, S. (1999). A comparison of parametric and non-parametric distance functions: With application to European railways. *European Journal of Operational Research, 117*, 326–339.

Cooper, W. W., Seiford, L. M., & Tone, K. (2007). *Data envelopment analysis: A comprehensive text with models, applications, references and DEA-solver software*. New York: Springer.

Cooper, W. W., Seiford, L. M., & Zhu, J. (2004). *Handbook on data envelopment analysis*. Boston: Kluwer Academic.

Cullinane, K., Song, D., & Wang, T. (2005). The application of mathematical programming approaches to estimating container port production efficiency. *Journal of Productivity Analysis, 24*, 73–92.
Cullinane, K., Wang, T., Song, D., & Ji, P. (2006). The technical efficiency of container ports: Comparing data envelopment analysis and stochastic frontier analysis. *Transportation Research Part A, 40*, 354–374.
Curcino, G. A. (2007). *Análise de adequabilidade de portos às novas teorias e práticas portuárias: um estudo de caso no porto de Belém.* Master in Sciences dissertation, Department of Civil Engineering, University of Brasília.
Curi, C., Gitto, S., & Mancuso, P. (2010). New evidence on the efficiency of Italian airports: A bootstrapped DEA analysis. *Socio-Economic Planning Sciences, 45*, 84–93.
Daraio, C., & Simar, L. (2007). Conditional nonparametric frontier models for convex and nonconvex technologies: A unifying approach. *Journal of Productivity Analysis, 28*, 13–32.
De Witte, K., & Marques, R. C. (2008). *Big and beautiful? On non-parametrically measuring scale economies in non-convex technologies*. Retrieved from http://www.econ.kuleuven.be/ces/discussionpapers/default.htm
Deprins, D., Simar, L., & Tulkens, H. (1984). Measuring labor efficiency in post offices. In M. Marchand, P. Pestieau, & H. Tulkens (Eds.), *The performance of public enterprises: Concepts and measurement*. Amsterdam: Elsevier.
Doctor, M. (2003). Institutional modernization and the legacy of corporatism: The case of port reform in Brazil. *Journal of Latin American Studies, 35*, 341–367.
Dulá, J. H., & Helgason, R. V. (1996). A new procedure for identifying the frame of the convex hull of a finite collection of points in multidimensional space. *European Journal of Operational Research, 92*, 352–367.
Färe, R., Grosskopf, S., & Whittaker, G. (2004). Distance functions. In W. W. Cooper, L. M. Seiford, & J. Zhu (Eds.), *Handbook on data envelopment analysis*. Boston: Kluwer International Series.
Fleury, P. F., & Hijjar, M. F. (2008). *Logistics overview in Brazil 2008*. Retrieved from http://www.ilos.com.br/index2.php?option=com_docman&task=doc_view&gid=31&Itemid=44
Kerstens, K., & Vanden Eeckaut, P. (1999). Estimating returns to scale using non-parametric deterministic technologies: A new method based on goodness-of-fit. *European Journal of Operational Research, 113*, 206–214.
Meisel, F. (2009). *Seaside operations planning in container terminals*. Berlin: Physica-Verlag.
Odeck, J., & Alkadi, A. (2001). Evaluating efficiency of the Norwegian bus industry using data envelopment analysis. *Transportation, 28*, 211–232.
Panayides, P. M., Maxoulis, C. N., Wang, T. F., & Ng, K. (2009). A critical analysis of DEA applications to seaport economic efficiency measurement. *Transport Reviews, 29*, 183–206.
Park, K. R., & De, p. (2004). An alternative approach to efficiency measurement of seaports. *Maritime Economics & Logistics, 6*, 53–69.
Pastor, J. T., Ruiz, J. L., & Sirvent, I. (1999). A statistical test for detecting influential observations in DEA. *European Journal of Operational Research, 115*, 542–554.
Ray, S. C. (2010). *A one-step procedure for returns to scale classification of decision making units in data envelopment analysis*. Working paper 2010-07. Retrieved from http://www.econ.uconn.edu/
Rios, L. R., & Maçada, A. C. C. (2006). Analyzing the relative efficiency of container terminals of Mercosur using DEA. *Maritime Economics & Logistics, 8*, 331–346.
Ross, A. D., & Droge, C. (2004). An analysis of operations efficiency in large-scale distribution systems. *Journal of Operations Management, 21*, 673–688.
Sales, A. (2001). O Brasil competitivo passa pelos portos. *Revista Tecnologística, 69*.
Seiford, L. M., & Thrall, R. M. (1990). Recent developments in DEA: The mathematical programming approach to frontier analysis. *Journal of Econometrics, 46*, 7–38.
Simar, L., & Wilson, P. W. (1998). Sensitivity analysis of efficiency scores: How to bootstrap in nonparametric frontier models. *Management Science, 44*, 49–61.

Simar, L., & Wilson, P. W. (2000). A general methodology for bootstrapping in nonparametric frontier models. *Journal of Applied Statistics, 27*, 779–802.

Simar, L., & Wilson, P. W. (2004). Performance of the bootstrap for DEA estimators and the iterating principle. In W. W. Cooper, L. M. Seiford, & J. Zhu (Eds.), *Handbook on data envelopment analysis*. Boston: Kluwer International Series.

Simar, L., & Wilson, P. (2013). *Statistical approaches for nonparametric frontier models: A guided tour*. Discussion paper. 2013/47, University Catholic of Louvain. Retrieved March 26, 2014, from http://www.uclouvain.be/cps/ucl/doc/stat/documents/DP2013_47_ALL.pdf

Tongzon, J. (1989). The impact of wharfage costs on Victoria's export-oriented industries. *Economic Papers, 8*, 58–64.

Wagner, J. M., & Shimshak, D. G. (2007). Stepwise selection of variables in data envelopment analysis: Procedures and managerial perspectives. *European Journal of Operational Research, 180*, 57–67.

Wang, T. F., & Cullinane, K. P. B. (2006). The efficiency of European container terminals and implications for supply chain management. *Maritime Economics & Logistics, 8*, 82–99.

Wilson, P. W. (2008). FEAR 1.0: A software package for frontier efficiency analysis with R. *Socio-Economic Planning Sciences, 42*, 247–254.

Wilson, P. W. (2009). *FEAR 1.12 user's guide*. Retrieved from http://www.clemson.edu/economics/faculty/wilson/Software/FEAR/fear.html

Wilson, D., & Purushothaman, R. (2003). *Dreaming with BRICs: The path to 2050*. Global Economic Paper No: 99. Retrieved from http://www.gs.com

Zarepisheh, M., Khorram, E., & Jahanshahloo, G. R. (2010). Returns to scale in multiplicative models in data envelopment analysis. *Annals of Operations Research, 173*, 195–206.

Zhu, J. (2003). *Quantitative models for performance evaluation and benchmarking: Data envelopment analysis with spreadsheets and DEA excel solver*. New York: Springer.

Chapter 9
DEA and Cooperative Game Theory

Sebastián Lozano, Miguel Ángel Hinojosa, Amparo María Mármol, and Diego Vicente Borrero

Abstract In this chapter the possibilities of hybridizing data envelopment analysis (DEA) and cooperative games are studied. Specifically, bargaining games and transferable utility games (TU games) are considered. There are already a number of different DEA approaches that are based on these types of cooperative games but, more importantly, there is the potential for further cooperation from both techniques.

Keywords Game theory • Bargaining solutions • Core concepts • Data envelopment analysis (DEA) • Process efficiency • Series production system • DEA games • DEA production games

9.1 Introduction

An increasing number of game theoretical approaches to DEA problems have recently appeared in the literature. Specially important are those involving bargaining game theory and cooperative TU game approaches.

Previous research involving bargaining game theory deals with problems such as the determination of a common set of weights for the assessment of the efficiency of a set of decision making units (DMUs); the computation of the process efficiency in a network; the assessment of efficiency when two or more groups of inputs are considered, or alternatively different perspectives are used to compute the efficiency.

On the other hand, cooperative TU game theory approaches have also been used to analyze situations within the framework or concepts developed in DEA. One well-known example of these problems is that of consensus-making among individuals or organizations that use multiple criteria to evaluate their performance.

S. Lozano • A.M. Mármol
Sevilla University, Sevilla, Spain
e-mail: slozano@us.es; amarmol@us.es

M.Á. Hinojosa (✉) • D.V. Borrero
Pablo de Olavide University, Sevilla, Spain
e-mail: mahinram@upo.es; dvbormol@upo.es

S.-N. Hwang et al. (eds.), *Handbook of Operations Analytics Using Data Envelopment Analysis*, International Series in Operations Research & Management Science 239, DOI 10.1007/978-1-4899-7705-2_9

Another group of approaches deals with the idea of cooperation among several organizations by sharing data about the feasible input/output operating points. Apart from the above approaches, other possibilities to apply cooperative TU game theory to DEA exist, such as the measuring of the importance of the variables in DEA applications.

In this chapter, we revise the mentioned approaches and propose some others. The structure of this chapter is the following. First, in Sect. 9.2, the basic concepts about cooperative games are introduced and explained. In Sects. 9.3 and 9.4, the various existing DEA approaches that use cooperative games are reviewed. These include bargaining approaches to DEA and DEA-based TU games. Finally, in Sect. 9.5, some potential further applications of cooperative games in a DEA context are proposed.

9.2 Cooperative Game Theory

In cooperative game theory it is assumed that players can commit to behave in a way that is socially optimal. The main issue is how to share the benefits arising from cooperation. The set of players is denoted by $N := \{1, 2, \ldots, n\}$. For each $S \subset N$, we refer to S as a *coalition*, with $|S|$ denoting the number of players in S. Coalition N is often referred to as the *grand coalition*. It is assumed that players can make binding agreements and, hence, notions like *fairness* and *equity* are taken into account when finding allocations of the total amount that the grand coalition can obtain.

We deal with two important subclasses of cooperative games: *bargaining problems* and *transferable utility games (TU games)*.

9.2.1 Bargaining Problems

An n-agent bargaining problem is a pair (F, d) where F and d are respectively a subset and a point of the n-dimensional Euclidean space, verifying

1. F is bounded and closed (i.e., it contains its boundary),
2. there is at least one point of F strictly dominating d ($x \in F$ exists, such that $x_i > d_i$ for each $i = 1, \ldots, n$),
3. (F, d) is d-comprehensive (i.e., if $x \in F$ and $x \geq y \geq d$, then $y \in F$),
4. $F \subset \mathrm{IR}_+^n$ and $d \in F$.

The set F represents the vectors of possible results to which the n agents have access if they cooperate. Their preferences over these vectors of results differ. If they agree on a particular alternative, then that is what they get. Otherwise, they end up at the pre-specified alternative in the feasible set, d, called the *disagreement point or breakdown point*. Therefore, the only feasible alternatives that matter are those over the disagreement point d. Let $\mathscr{B}^N$ be the class of these n-agent bargaining problems. Moreover, for each bargaining problem $(F, d) \in \mathscr{B}^N$, we define the compact set $F_d := \{x \in F\colon x \geq d\}$.

In the axiomatic theory of bargaining originated in the fundamental paper by Nash (1950), F is assumed to be convex. The reason is that each point of F is interpreted as the utility levels (measured in some von Neumann-Morgerstern scale) reached by the agents through the choice of one of the alternatives, or as a randomization among the available alternatives. Convexity of F is due to the possibility of randomization. Non-convex bargaining problems have also been studied in the literature (see, for instance, Herrero 1989, Conley and Wilkie 1991, and Conley and Wilkie 1996).

A solution, φ, on the class of bargaining problems, $\mathscr{B}^N$, associates with each problem $(F, d) \in \mathscr{B}^N$ an unique point of F, $\varphi(F, d)$, interpreted as a prediction, or a recommendation, for that problem. We focus here on two relevant solutions or allocation rules, namely the *Nash solution* and the *Kalai-Smorodinsky solution*.

9.2.1.1 The Nash Solution

The best-known bargaining solution for convex problems, introduced in Nash (1950), consists of a compromise obtained by maximizing the product of the utility gains from the disagreement point: $N(F, d)$ is the maximizer of $\prod_{i=1}^{n} (x_i - d_i)$ for $x \in F$, $x \geq d$. Figure 9.1 represents the Nash solution in a two-agent bargaining problem, where u_1 and u_2 are the utilities of agents 1 and 2, respectively.

In the class of convex bargaining problems the Nash solution is characterized by four well-known properties (axioms), namely *Pareto-optimality* (PO), *symmetry* (SYM), *scale invariance* (SI) and *contraction independence* (CI) (also referred to in the literature as *independence of irrelevant alternatives*). These properties for a solution φ are formalized as:

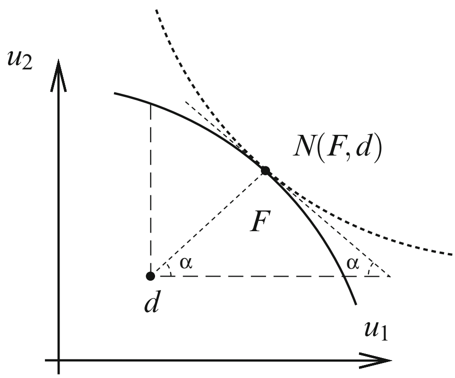


Fig. 9.1 The Nash bargaining solution

PO: $\varphi(F,d)$ is in the Pareto frontier of F.

SYM: If F is invariant under all exchanges of agents, then $\varphi_i(F,d) = \varphi_j(F,d)$ for all $i,j \in 1,2,\ldots,n$.

SI: $\lambda_i\varphi_i(F,d) = \varphi_i(\lambda \times F, \lambda \times d)$, where $\lambda \times d = (\lambda_i d_i)_{i=1,2,\ldots,P}$, and $\lambda \times F = \{\lambda \times s : s \in F\}$, being $\lambda \in \mathrm{IR}^n_{++}$ (i.e., the solution does not depend on the scale in which the utilities are measured).

CI: if $F' \subseteq F$ and $\varphi(F,d) \in F'$, then $\varphi(F',d) = \varphi(F,d)$.

Several generalizations of the Nash bargaining solution for non-convex problems have been proposed in the literature. An interesting one is the so-called *Nash extension solution*, proposed by Conley and Wilkie (1996). This solution is continuous, single-valued, coincides with the Nash solution if the problem is convex, and approximates the Nash solution otherwise. To construct the Nash extension solution, the convex hull of F, $con(F)$, is considered, and the line segment, $L(F,d)$, connecting the disagreement point with the point obtained by applying the Nash solution to the problem $(con(F),d)$, $N(con(F),d)$, is drawn. The Nash extension solution, denoted by NE, is defined as: for each (F,d), $NE(F,d)$ is the maximal element, x, with respect to the partial order on IR^n, such that $x \in L(F,d) \cap F$ (see Fig. 9.2).

The Nash extension solution, NE, shares some of the properties of the Nash solution. In fact, on the whole class $\mathscr{B}^N$, it is characterized by Pareto-optimality (PO), symmetry (SYM), scale invariance (SI), together with *ethical monotonicity* (EM) and *continuity* (CONT). That is, in relation with the characterization of the Nash solution for convex problems, in the characterization of the Nash extension solution, contraction independence (CI) is substituted with these last two properties. Continuity is a very natural property, meaning that similar problems should be solved similarly, whereas Ethical Monotonicity is a modification of CI which applies to non-convex problems. Formally, these two properties are defined for a bargaining solution, φ, as:

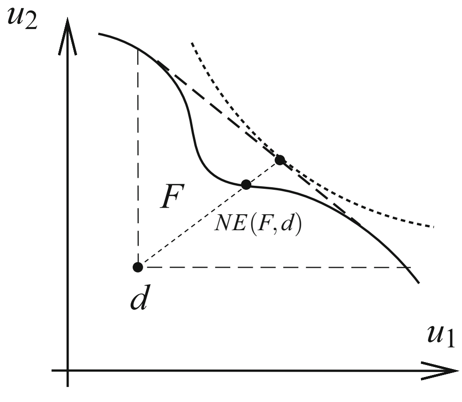


Fig. 9.2 The Nash extension solution

EM: If $F' \subset F$ and $\varphi(con(F), d) \in con(F')$, then $\varphi(F, d) \geq \varphi(F', d)$.

CONT: For all sequence of problems $\{F^{\nu}, d^{\nu}\}_{\nu=1}^{\infty}$, $\varphi^{\nu} \to F$ in the Hausdorff topology, and $d^{\nu} \to d$, then $\varphi(F^{\nu}, d^{\nu}) \to \varphi(F, d)$.

The Nash extension solution, *NE*, is the only solution on the whole class $\mathscr{B}^N$ satisfying PO, SYM, SI, EM and CONT.

9.2.1.2 The Kalai-Smorodinsky Solution

The Kalai-Smorodinsky solution was introduced for two-player bargaining problems as an alternative allocation rule in Kalai and Smorodinsky (1975). This solution strongly depends on the aspiration levels of the players. These levels give rise to the so-called *utopia point* of a bargaining problem $(F, d) \in \mathscr{B}^N$, given by the vector $b(F, d) \in \mathrm{IR}^N$, where, for each $i \in N, b_i(F, d) = \max_{x \in F_d}\{x_i\}$. The Kalai-Smorodinsky solution, KS, is defined, for each $(F, d) \in \mathscr{B}^N$, by $KS(F, d) := d + \bar{t}(b(F, d) - d)$, where $\bar{t} := \max\{t \in \mathrm{IR} : \ d + t(b(F, d) - d) \in F_d\}$. Note that the compactness of F_d ensures that $\bar{t}$ is well defined. Figure 9.3 represents the Kalai-Smorodinsky solution in a two-agent example:

Kalai and Smorodinsky (1975) provided a characterization of their allocation rule for two-player bargaining problems that is based on the following monotonicity property, called *individual monotonicity* (IM):

IM: Let (F, d), $(\widehat{F}, d) \in \mathscr{B}^N$ be a pair of bargaining problems such that $\widehat{F}_d \subset F_d$. Let $i \in N$ be such that, for each $j \neq i$, $b_j(\widehat{F}, d) = b_j(F, d)$. If φ is an allocation rule for n-player bargaining problems that satisfies IM, then $\varphi_i(\widehat{F}, d) \leq \varphi_i(F, d)$.

The Kalai-Smorodinsky solution is the unique allocation rule for two-player bargaining problems that satisfies PO, SYM, SI, and IM. Unfortunately, this characterization cannot be generalized to $n \geq 3$ because there is no solution for n-player bargaining problems ($n \geq 3$) satisfying PO, SYM, and IM (see

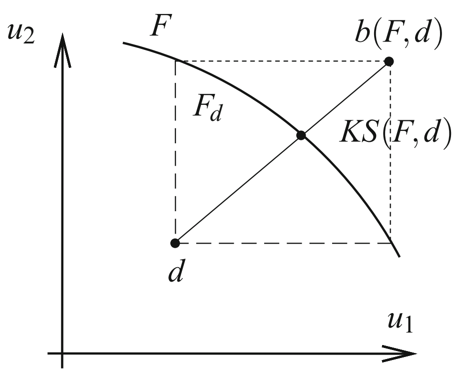


Fig. 9.3 Kalai-Smorodinsky solution

Roth 1979). In spite of this negative result, Thomson (1980) showed that PO, SYM and IM characterize the Kalai-Smorodinsky solution if we restrict attention to a certain (large) domain of n-player bargaining problems.

9.2.2 Transferable Utility Games

In a transferable utility game, in short, TU-game, the different coalitions that can be formed among the players in N can enforce certain cooperation (possibly through binding agreements); the problem is to decide how the benefits generated by the cooperation of the players have to be shared among them. The transferable utility assumption has important implications. It implicitly assumes that there is a *numeraire* good (money, for instance) such that the utilities of all the players are linear with respect to it and that this good can be freely transferred among players. Formally a TU-game is a pair (N, v), where N is the set of players and $v: \ 2^N \to \mathrm{IR}$ is the characteristic function of the game, which represent, for each coalition $S \subset N$, the worth of the coalition, that is, what each coalition can guarantee without the collaboration of the players outside the coalition. By convention, $v(\theta) = 0$. When no confusion arises, we denote the game (N, v) by v. Let G^N be the class of TU-games with n players.

A TU-game $v \in G^N$ is

- *monotonic* if, for each pair $S, T \subset N$, with $S \subset T$, we have $v(S) \leq v(T)$.
- *zero-normalized* if, for each $i \in N$, $v(\{i\}) = 0$. Given $v \in G^N$, the game $w \in G^N$ defined, for each $S \subset N$, by $w(S) := v(S) - \sum_{i \in S} v(\{i\})$ is zero normalized.
- *additive* if, for each $i \in N$ and each $S \subset N \setminus \{i\}$, $v(S \cup \{i\}) = v(S) + v(\{i\})$.
- *weakly superadditive* if, for each $i \in N$ and each $S \subset N \setminus \{i\}$, $v(S \cup \{ i\}) \geq v(S) + v(\{i\})$.
- *superadditive* if, for each pair $S, T \subset N$, with $S \cap T = \theta$, $v(S \cup T) \geq v(S) + v(T)$.
- *zero-monotonic* if its zero-normalization is a monotonic game. It can be easily checked that $v \in G^N$ is weakly super superadditive if and only if it is zero-monotonic.

The main goal of the theory of TU-games is to define *solutions* that select, for each TU-game, a set of allocations that are admissible for the players.[1] There are two possible approaches in developing a solution concept. One of them is based on *stability*, where the objective is to find solutions that choose sets of allocations that are stable according to different criteria. This is the approach underlying, for instance, the *core* (Gillies, 1953), the *stable sets* (von Neumann and Mongerstern, 1944), and the *bargaining set* (Aumann and Maschler, 1964). The second approach is based on *fairness*: it aims to find allocation rules that propose,

[1] When the solution recommends a singleton we will refer to it as an allocation rule.

for each TU-game, an allocation that represents a fair compromise for the players. This is the approach underlying, for instance, the *Shapley value* (Shapley, 1953), the nucleolus (Schmeidler, 1969), and the τ-value (Tijs, 1981). Below we refer to some of these solution concepts, namely, the core, the Shapley value, the least core and the nucleolus.

9.2.2.1 The Core and Related Concepts

Let $v \in G^N$, and $x \in \mathrm{IR}^N$. Then

- x is *efficient* if $\sum_{i \in N} x_i = v(N)$. Hence, provided that v is a superadditive game, efficiency just requires that the total benefit from cooperation is actually shared among the players.
- x is *individually rational* if, for each $i \in N$, $x_i \geq v(\{i\})$. The set of *imputations* of a TU-game, $I(v)$, consists of all the efficient and individually rational allocations.
- x is *coalitionally rational* if, for each $S \subset N$, $\sum_{i \in S} x_i \geq v(S)$. The set of efficient and coalitionally rational allocations is the *core* of the game, denoted by $C(v)$. Its elements are called *core allocations*. Core allocations are stable in the sense that no coalition has incentives to block any of them.

A necessary and sufficient condition for a game to have a nonempty core was independently proved by Bondareva (1963) and Shapley (1967), and is known as the Bondareva-Shapley theorem.

- A family of coalitions $\mathscr{F} \subset 2^N \backslash \theta$ is *balanced* if there are positive real numbers $\{\alpha_S : S \in \mathscr{F}\}$ such that, for each $i \in N$, $\sum_{S \in \mathscr{F};\, i \in S} \alpha_S = 1$. The numbers $\{\alpha_S : S \in \mathscr{F}\}$ are called *balanced coefficients*.
- A TU-game $v \in G^N$ is *balanced* if, for each balanced family $\mathscr{F}$, with balanced coefficients $\{\alpha_S : S \in \mathscr{F}\}$, $\sum_{S \in \mathscr{F}} \alpha_S v(S) \leq v(N)$. A TU-game $v \in G^N$ is *totally balanced* if, for each $S \subset N$, the TU-subgame[2] (S, v_S) is balanced.
- (Bondareva-Shapley theorem) $C(v) \neq \theta$ if and only if v is balanced.

9.2.2.2 The Shapley Value

An allocation rule for n-players TU-games is a map $\varphi : G^N \rightarrow \mathrm{IR}^N$. Probably the most important allocation rule is the *Shapley value*. The Shapley value, ϕ, is defined, for each $v \in G^N$ and each $i \in N$, by

[2] The restriction of (N, v) to the coalition S is the TU-game (S, v_S), where, for each $T \subset S$, $v_S(T) := v(T)$.

$$\phi_i(v) = \sum_{S \subset N \setminus \{i\}} \frac{|S|!(n-|S|-1)!}{n!}(v(S \cup \{i\}) - v(S)).$$

In the Shapley value, each player gets a weighted average of the contributions he makes to the different coalition.

An alternative definition of the Shapley value, based on the so-called *vectors of marginal contributions* is

$$\phi_i(v) = \frac{1}{n!} \sum_{\pi \in \Pi(N)} m_i^{\pi}(v),$$

where $\Pi(N)$ denote the set of all permutations of the elements in N and the vector of marginal contributions associated with π, $m^{\pi}(v)$, is defined, for each $\pi \in \Pi(N)$, by $m_i^{\pi}(v) := v(P^{\pi}(i) \cup \{i\}) - v(P^{\pi}(i))$, where $P^{\pi}(i)$ denote the set of predecessors of i under the ordering given by π, i.e., $j \in P^{\pi}(i)$ if and only if $\pi(j) < \pi(i)$.

Shapley (1953) characterizes the Shapley value by means of four appealing properties that an allocation rule should satisfy. To establish these properties, the following concepts need to be introduced. Given a game $v \in G^N$,

1. A player $i \in N$ is a *null player* if, for each $S \subset N$, $v(S \cup \{i\}) - v(S) = 0$
2. Two players i and j are *symmetric* if, for each coalition $S \subset N \setminus \{i, j\}$, $v(S \cup \{i\}) = v(S \cup \{j\})$

In the class of TU-games with n players, the Shapley value is characterized by four well-known properties (axioms), namely *efficiency* (EFF), *null player property* (NPP), *symmetry* (SYM) and *additivity* (ADD). These properties for an allocation rule φ are formalized as:

EFF: φ satisfies EFF if, for each $v \in G^N$, $\sum_{i \in N} \varphi_i(v) = v(N)$ (EFF requires that φ allocates the total worth of the grand coalition, $v(N)$, among the players).

NPP: φ satisfies NPP if, for each $v \in G^N$, and each null player $i \in N$, $\varphi_i(v) = 0$ (NPP says that players that contribute zero to every coalition, i.e., they do not generate any benefit, should receive nothing).

SYM: φ satisfies SYM if, for each pair $i, j \in N$ of symmetric players, $\varphi_i(v) = \varphi_j(v)$ (SYM asks φ to treat these players equally).

ADD: φ satisfies ADD if, for each pair $v, w \in G^N$, $\varphi(v + w) = \varphi(v) + \varphi(w)$ (despite of being a natural requirement, ADD is not motivated by any fairness notion).

For the class of superadditive games, the Shapley value belongs to the set of imputations. However, the Shapley value for a game may lie outside its core, even when the latter is nonempty. A class of superadditive games that satisfy that the Shapley value is always a core allocation is the class of *convex games*. A TU-game

$v \in G^N$ is convex if, for each $i \in N$ and each pair $S, T \subset N \setminus \{i\}$, with $S \subset T$, $V(S \cup \{i\}) - v(S) \leq V(T \cup \{i\}) - v(T)$. The most important result for convex games relates convex games, vectors of marginal contributions and the core, and establishes that, given a game $v \in G^N$, the following statements are equivalent:

(i) v is convex.
(ii) For each $\pi \in \Pi(N)$, $m^{\pi}(v) \in C(v)$.
(iii) $C(v) = con\{m^{\pi}(v) \ : \ \pi \in \Pi(N)\}$, where *con* denotes the convex hull of the set.

The second part of the equality in (iii) is the convex hull of the set of vectors of marginal contributions. It is commonly known as the *Weber set*, formally introduced as a solution concept by Weber (1988).

As a consequence of the above result, for convex games, the Shapley value is always a core allocation.

9.2.2.3 The Least Core and the Nucleolus

A very important allocation rule, besides the Shapley value, is *the nucleolus* (Schmeidler, 1969).

Let $v \in G^N$ and let $x \in \mathrm{IR}^N$ an allocation. Given a coalition $S \subset N$, the *excess of coalition S with respect to x* is defined by $e(S, x) := v(S) - \sum_{i \in S} x_i$. This a measure of the degree of dissatisfaction of coalition S when allocation x is realized. Note that for each $x \in C(v)$, and each $S \subset N$, $e(S, x) \leq 0$. The least core of the game consists of those allocations that minimize the maximum excess among the coalitions, that is, those allocations for which the player or the coalition with the higher degree of dissatisfaction can not be better.

Now define the *vector of ordered excesses*, $\theta(x) \in \mathrm{IR}^{2^N}$ as the vector whose components are the excesses of the coalitions in 2^N arranged in non increasing order. The nucleolus consists of those imputations that minimize the vector of non-increasing ordered excesses according to the lexicographic order within the set of imputations. The nucleolus is actually a singleton, i.e., it is never empty and it contains an unique allocation. We refer to the nucleolus of v as $\nu(v)$. Moreover, for TU-games with non-empty core, the nucleolus is a core element.

9.3 Nash Bargaining Approaches to DEA

A number of DEA studies have used Nash bargaining game theory (NBGT) approaches to solve different types of problems. In this group of studies we include those that apply the Nash Bargaining (NB) solution plus others than use the Kalai-Smorodinsky (KS) solution or the Nash extension (NE) solution. In addition, as we will see in this section, some researchers do not apply the original NB approach but an ad hoc variant. One thing that all these DEA models share is that they use a multiplier formulation. Another common feature is that the resulting optimization models are always non-linear. Table 9.1 shows a summary of the below-reviewed papers.

The DEA problem most frequently solved using NBGT is the computation of a common set of weights with which to assess the efficiency of all DMUs. Thus, while in conventional DEA each DMU chooses its own set of weights for the inputs and outputs (and does so seeking to appear under the best possible light), in this type of DEA models, the same set of weights is used to compute the efficiency scores of all DMUs. In all these applications the players represent the DMUs and their payoffs are their respective efficiency scores. The first paper to propose NBGT for this sort of situations was Wu et al. (2009a) which use a NB variant that maximizes the product, for all DMU, of two differences: on one hand the difference between the CCR efficiency score (imposed as upper bound) and, on the other hand, the difference between the efficiency score and the cross efficiency score (imposed as lower bound). They apply the method to the classical dataset of 37 R&D projects from Green et al. (1996).

Wang and Li (2014) propose an improvement over Wu et al. (2009a) to overcome the drawback that the cross efficiency of a DMU may not be unique. Hence, they propose to use as lower bound a so-called minimum cross efficiency, which is obtained by using a target aggression model. They present an application to supplier evaluation.

Wu et al. (2013) also use a common set of weights approach for efficiency assessment after the reallocation of a reduced, fixed input. They do not apply the original NB solution but a variant with no disagreement point (therefore assumed to be zero) and with special constraints. The approach is applied to reallocate the greenhouse gases emissions of 15 European Union members.

Sugiyama and Sueyoshi (2014) propose to use the KS bargaining solution (over the comprehensive hull of the feasible efficiency scores) for finding a common set of weights. The resulting model, although still non-linear, is relatively simple. The approach is applied to a dataset of 9 Japanese electric power companies.

Omrani et al. (2015) use a different approach in which each DMU chooses its own set of inputs and outputs weights. Therefore, the number of players is equal to the number of DMU but, in addition, the game is played as many times as the number of DMUs. For each player, the payoff is the efficiency of the DMU being assessed (labeled DMU 0) and the disagreement point is the minimum cross efficiency each DMU could obtain in a conventional approach. This disagreement point is supposedly imposed as lower bound on the efficiency score of DMU 0. They apply this approach to a sample of 37 Iranian electricity distribution companies.

A completely different type of DEA problems to which NBGT has been applied is to compute the process efficiency in a Network DEA context. The players here are the processes whose efficiency score is to be computed, and the game is played for each DMU separately. So far only simple network topologies have been studied. Thus, Du et al. (2011) and Zhou et al. (2013) considered a two stage in series

Table 9.1 Summary of NBGT approaches to DEA

DEA problem type	#Times game played	#Players	Reference	Solution concept	Remarks
Common set of weights	Once	#DMUs	Wu et al. (2009a)	NB variant	CRS efficiency score as upper bound, cross-efficiency score as lower bound
			Wu et al. (2013)	NB	Reallocation of fixed input, zero disagreement point
			Wang and Li (2014)	NB variant	CRS efficiency score as upper bound, minimum cross-efficiency score as lower bound
			Sugiyama and Sueyoshi (2014)	KS	Comprehensive hull of feasible efficiency scores
	#DMUs	#DMUs	Omrani et al. (2015)	NB	Efficiency of DMU 0 as payoff for all players, minimum cross-efficiency score as lower bound
Network DEA process efficiency	#DMUs	#Processes	Du et al. (2011)	NB	Two-stage series system
			Zhou et al. (2013)	NB	Two-stage series system
			Jalali-Naini et al. (2013)	NB variant	First stage (leader) + two parallel processes (followers), minimization product of differences w.r.t. upper bound
			Hinojosa et al. (2015a)	NE	Multistage series system
Different input/output specifications	#DMUs	#Specifications	Jahangoshai Rezaee et al. (2012a)	NB	Minimum cross-efficiency for each specification as disagreement point
			Jahangoshai Rezaee et al. (2012b)	NB	Minimum cross-efficiency for each specification as disagreement point
			Yang and Morita (2013)	NB	For each specification, minimum efficiency along a given improvement direction as disagreement point

system. Du et al. (2011) consider different ways of setting the disagreement point. One possibility, which the authors do not recommend, is to use the zero as disagreement point. They also try using the efficiency score of the anti ideal operation points of each of the two stages, imposing those efficiencies as lower bounds on the efficiency of the respective stages. Another possibility is to use the centralized efficiency scores or the efficiency scores computed by using a leader-follower approach as disagreement points. They apply the method to two datasets from the literature, involving 30 US commercial banks (Liang et al., 2008) and 24 Taiwanese non-life insurance companies (Kao and Hwang, 2008).

Zhou et al. (2013) also apply NBGT to a two-stage system but they use as disagreement points the minimum efficiency scores of each stage compatible with the overall system efficiency score. They can find the NB solution analytically leading to an efficiency decomposition in which the efficiency score of each stage is the geometric mean of its minimum and maximum efficiency scores. This simple and elegant result occurs because for these two-stage systems the overall system efficiency is the product of the efficiency of the two stages. Since this property is valid for multistage systems, Hinojosa et al. (2015a) have proposed a NE solution to the more general case of multistage systems. This includes as a special case the system studied by Zhou et al. (2013).

Jalali-Naini et al. (2013) also use NBGT but they consider a slightly more complex network topology that includes a first stage followed by two processes in parallel. However, since a leader-follower approach is used, the efficiency of the first stage is computed in a conventional way and it is just for the two follower parallel processes for which the NBGT approach is used. They propose a variant of NB solution in which, instead of using a disagreement point representing lower bounds of the payoffs, the "basic efficiency of each parallel stage" is used as upper bound, and a function that corresponds to the product of the differences between the efficiency score of each process and that upper bound is minimized. They apply their method to two different datasets, one involving 35 Iranian bank branches and the other 20 Iranian power plants.

Finally, a third type of NBGT applications to DEA, apart from the common set of weights and Network DEA problems mentioned above, corresponds to the approaches in Jahangoshai Rezaee et al. (2012a,b) and, Yang and Morita (2013). In the first two papers the idea is to consider two different groups of inputs so that computing the efficiency with each type of inputs leads to a different efficiency score for each DMU. The NB game is played, therefore, for each DMU separately and the two players represent the two types of inputs. The disagreement point used for each DMU is its minimum cross efficiency score, computed using each type of inputs separately. Jahangoshai Rezaee et al. (2012a) apply this method to 54 health centers while Jahangoshai Rezaee et al. (2012b) apply it to 24 Iranian power plants.

The approach in Yang and Morita (2013) is somewhat more complex. They consider different perspectives to assess the DMUs. Each perspective corresponds to a different specification of the inputs and outputs. Depending on the perspective the efficiencies of the DMUs differ. The NB game is played separately for each DMU and the players represent the different perspectives. For each DMU 0, its disagreement point for each perspective is the lowest efficiency score of that perspective allowing for improving that DMU along any given direction. In this way the model, not only computes the efficiency scores for the different perspectives, but also projects the DMUs along the corresponding improvement direction. They apply the method to a dataset of 65 Japanese banks.

9.4 TU Cooperative Game Approaches to DEA

As in the case of NBGT, there are three groups of TU cooperative games (TUCG) approaches to DEA. They are summarized in Table 9.2. The first group of papers comprises those based on the Egoist's dilemma (ED, Nakabayashi and Tone 2006). This is a consensus-building problem that consists in reaching an aggregate score for each player when several scoring criteria exist. The players must choose how to weight the different criterion. The problem appears also when a given amount must be allocated among the players and there are different allocation criteria. Nakabayashi and Tone (2006) studied this problem from a DEA perspective and proposed two TUCG based on it labeled, respectively, *max* DEA game (N, c) and *min* DEA game (N, d). The former is subadditive and leads to a supperadditive zero-normalized and balanced (hence, with non-empty core) TUCG (N, v). The min DEA game is also superadditive and balanced. Moreover, because (N, c) and (N, d) are dual TUCGs, their Shapley value coincide. They also show that this does not necessarily occur with their respective nucleoli. Independently of the solution concept used, a final Linear programming (LP) model is proposed to achieve a common set of weights that gives aggregated scores as close as possible (using a Tchebycheff metric) to the TUCG imputation. They also propose a Benefit-Cost (BC) DEA game. They enumerate several potential applications, such as research grants allocation to applicants by a foundation, burden sharing in UN, NATO and similar organizations, comparison of cities for quality of life, etc.

Jahanshahloo et al. (2006) extend the ED to interval data by applying the approach twice (once for each of the interval limits) and defining the corresponding sum game. In this way the max DEA game, min DEA game and BC DEA game are studied. They apply the method to a dataset of 20 commercial bank branches.

Nakabayashi et al. (2009) study an ED setting with two criteria and show that the Shapley Value and the nucleolus (which always coincide in the case of two players) also coincide with the traditional allocation rule expressed as add them up and divide by two. They show the advantage of ED by incorporating Assurance Region (AR) weight constraints, commonly used in DEA.

Wu et al. (2009b) study the ED problem using the normalized cross efficiency matrix as scoring matrix. The Shapley value of either the max or the min DEA game provides an allocation which is input to a common weights LP model to determine what the authors call ultimate cross-efficiency. Wu et al. (2008) also use ED on the normalized cross efficiency matrix but they propose as ultimate cross efficiency scores the nucleolus of the max DEA game, computed by using a Genetic Algorithm (GA).

Table 9.2 Summary of DEA TUCG approaches

DEA TUCG type	Players	Characteristic function	Reference	Solution concept	Remarks
Egoist's dilemma	Agents	Aggregate score	Nakabayashi and Tone (2006)	Shapley value, Nucleolus	Max DEA game, min DEA game, BC DEA game, closest common weights
			Jahanshahloo et al. (2006)	–	Interval data, ED for lower and upper interval limits, sum TUCG
			Nakabayashi et al. (2009)	Shapley value, Nucleolus	Two players, AR weight restrictions
			Sekine et al. (2014)	Shapley value, Nucleolus	Constant sum TUCG, empty core
	DMUs	Aggregate normalized cross efficiency score	Wu et al. (2008)	Nucleolus	GA for computing nucleolus, ultimate cross efficiency
			Wu et al. (2009b)	Shapley value	Common closest weight, ultimate cross efficiency
Technology sharing and resource pooling	Organizations (owners of DMUs)	Minimum input cost	Lozano (2012)	Shapley value, Nucleolus, τ-value	Sharing information about input/output operating points, balanced TUCG
			Lozano (2013a)	Shapley value	Joint-venture cooperation, multiple facilities, multi-period horizon
		Maximum revenue	Lozano (2013b)	Owen point	DEA production games, totally balanced
		Total output vector	Lozano et al. (2014)	Dominance/ Preference least core	Set-valued DEA production games, two different excess functions
		Maximum fuzzy revenue	Hinojosa et al. (2015b)	Preference least core	Output prices are fuzzy numbers, standard fuzzy order based on α-cuts
Other	Input/output variables	Sum of efficiency change ratios	Li and Liang (2010)	Shapley value	More variables = more discriminant power

Sekine et al. (2014) propose a different TUCG approach to the ED problem. Based on the strategic form DEA game, the authors define a superadditive, constant-sum DEA game. In this type of games the core is empty unless the game is inessential (which occurs if the score of each player is the same for all criteria, a trivial case). They study the Shapley value, which for constant-sum games can be computed with a simpler formula, as well as the nucleolus. They show that both solution concepts coincide in the case of 3 players but that this is not necessarily the case for more than 3 players.

A second group of TUCG approaches to DEA revolve about the idea that if several organizations cooperate and share the data about the feasible input/output operating points they can enlarge their production possibility sets and thus obtain more outputs with the available resources or, alternatively, produce given amount of outputs more cost efficiently. Lozano (2012) was the first to propose TUCG, based on a minimum cost DEA model, to allocate the benefits of input/output information sharing among cooperating organizations. This cost TUCG is subadditive and balanced. Two additional LP models are formulated: one to compute the τ-value and another to check its stability. The approach is applied to a 12 hospitals dataset from the literature (Tone, 2002).

Lozano (2013a) also uses TUCG to measure and allocate the advantages that cooperation brings to DEA. In this case, the problem dealt with is the selection of the best partner to form a joint venture. The synergies of that cooperation depend on the complementarity of their respective technologies, i.e. with which partner(s) will the Production Possibility Set (PPS) of the joint venture be bigger and thus lead to a more cost efficient operation. The Shapley value is proposed as a solution concept to allocate the advantages of the cooperation. The approach can be extended to consider multiple production facilities and a multi-period horizon.

Lozano (2013b) proposes the so-called DEA production games, in which several organizations can cooperate by both sharing their technologies and pooling their resources. A revenue maximization DEA model is used to compute the characteristic function. This TUCG is shown to be superadditive and totally balanced. A simple way of computing a stable solution (based on the Owen point) is proposed. The method is applied to a randomly generated dataset.

For cases in which some of the outputs are not marketable or not all output prices are available, Lozano et al. (2014) study set-valued DEA production games. In this case, the DEA model used does not just maximize revenue but has to consider the whole output Pareto efficient frontier of the total output vectors generated by the coalitions. Two different excess functions measuring the dissatisfaction of the coalitions with respect to a given output allocation are considered, giving rise to two different core concepts: the dominance core and the preference core. While the former is always non-empty the latter, more strict, is very often empty. Models for computing a solution in the corresponding Least Core are proposed.

Hinojosa et al. (2015b) also extend DEA production games but in a different direction. Thus, they consider the situation when the output prices are fuzzy numbers. As in crisp DEA production games, if the organizations cooperate the coalitions can obtain a higher revenue. The problem is how to allocate the fuzzy payoff resulting from the cooperation. By using standard fuzzy orders based on α-cuts, fuzzy DEA production games can be treated as set valued production games and an LP model for computing a preference least core solution can be formulated.

Apart from the two strands of DEA TUCG approaches commented above (namely, ED and DEA production games) there are other possibilities of applying TUCG to DEA. Thus, for example, Li and Liang (2010) present an ingenious approach to measure the importance of variables (inputs and outputs) in a DEA application by looking at the change in the efficiency scores of the whole DMU sample depending on the variables considered. In this game the players are the variables and the characteristic function is the sum, for all DMUs, of the efficiency change ratios due to considering the variables in the coalition with respect to ignoring them. The Shapley Value of such a TUCG gives an estimation of the contribution of each variable to the increase in the discriminant power of the DEA model. This value is thus proposed as a measure of the importance of each variable.

9.5 Further Potential Applications

In this section a number of novel applications of cooperative games to DEA are suggested. These applications involve both Nash bargaining game theory (NBGT) and TU cooperative games (TUCG).

An interesting application of NBGT would be in the context of centralized DEA, in which the players would be the operating units whose production is to be planned and the payoffs would be the monetary value (assuming output prices are known) of the production they would obtain with the allocated inputs. The input allocation is precisely what the players need to agree on. As disagreement point the null vector can be used, i.e. zero payoffs for all players if they do not agree on how to allocate the inputs. The KS solution can be found if the ideal solution is previously computed as the payoff that each operating unit would obtain if it could use all the available inputs. Although the NB solution is harder to obtain, other bargaining solutions (like the egalitarian solution or the cooperative solution) can also be computed and used for comparison.

Another possible application of NBGT is the ED problem mentioned in the previous section. Using the minimum score along the different criteria for each player as its disagreement value and the maximum as its ideal payoff the KS solution can be obtained by solving a simple LP, while the NB solution would require solving a non-linear optimization model subject to the convexity constraints on the allowed weights.

There are other possible applications of NBGT to DEA. For example, the problem of determining a common set of weights to assess the efficiency of the DMUs can be approached using NBGT. The players would be the DMUs and their payoffs can be the efficiency scores that the common set of weights would assign to each DMU. The minimum efficiency score that a DMU can obtain using common set of weights can be the disagreement point. Using the conventional DEA efficiency score (i.e. letting each DMU choose the weights so as to appear under the best possible light) as the ideal point, the KS bargaining solution can be computed by solving a LP optimization model.

As regards TUCG, additional applications to DEA may involve, for example, the ranking of efficient DMUs. The players would be the efficient DMUs and the characteristic function for a given coalition would be the sum, for all inefficient DMUs, of the difference between their efficiency scores assuming the DMUs in the coalition were removed from the sample, and their original efficiency scores computed with the whole sample of DMUs. The idea is that the more important a DMU (or a coalition of DMUs) is, the more affected are the efficiency scores of the inefficient DMUs by its removal. Thus, if an efficient DMU is removed from the sample the efficient frontier shifts backwards and the efficiency scores of the inefficient DMUs increase. The larger the shift of efficient frontier, the higher this increase in the efficiency scores of the inefficient DMUs. By using the Shapley value, a measure of the marginal contribution of each player to this effect can be computed, and this index can be used to rank the efficient DMUs.

To finish this section we present a more detailed description of the ideas behind recent research works by the authors, currently under revision for publication. One of the papers deals with the problem of computing the process efficiency decomposition in a multistage production system. The others correspond to extensions and variants of DEA production games (Fig. 9.4).

9.5.1 *Nash Decomposition for Process Efficiency in Multistage Production Systems*

In this research, DEA models in which each DMU is organized internally as a sequence of processes or stages are considered.

Conventional DEA models consider a DMU as a black box that directly transforms inputs into outputs. There exist, however, a number of DEA applications in

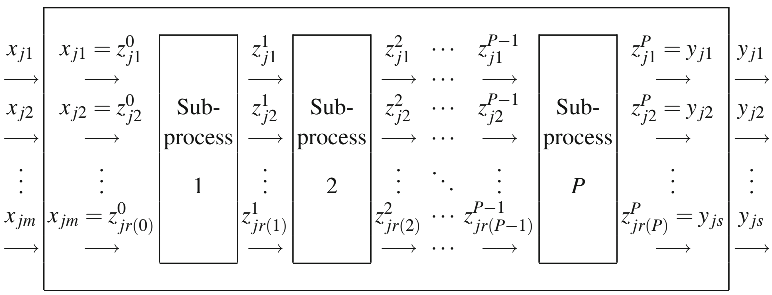


Fig. 9.4 Production process of DMU j ($j = 1, 2 \ldots n$)

which several interrelated stages are distinguished so that intermediate products, internally generated and consumed within the system, are also considered. This more fine-grained approach is generally labelled Network DEA (e.g. Färe and Grosskopf 1996). Many Network DEA models have been proposed in the last years, including, among others, game-theory approaches (Liang et al. 2008), relational Network DEA (Kao and Hwang, 2008, 2010), weighted additive efficiency decomposition (Chen et al., 2009; Cook et al., 2010), Network Slack-Based Measure (NSBM) of efficiency (Tone and Tsutsui 2009), Slacks-Based Inefficiency measure (Fukuyama and Weber 2010), dynamic Network DEA (Tone and Tsutsui 2014), Malmquist index approach (Kao and Hwang 2014), etc. Kao (2014) and Halkos et al. (2014) provide extensive and up-to-date reviews of Network DEA models.

Some of the proposed Network DEA models are based on the so-called multiplier DEA formulation. When solving these models multiple alternative optima may arise and, therefore, alternative efficiency decompositions are possible (see, e.g., Kao and Hwang 2008, Liang et al. 2008). This problem can also affect the decomposition into technical and scale efficiencies (Kao and Hwang 2011) and not only in the case of two-stage systems, but also in the case of the efficiency decomposition of general multistage systems (Kao 2014). In the case of two-stage systems, some authors have proposed different ways of solving the uncertainty about how the processes efficiencies should be computed. One common approach is to compute the best and worst possible efficiency scores of each process, by choosing the best score for one process and the worst for the other process, depending on which process efficiency is the decision maker more concerned with Liang et al. (2008), and Kao and Hwang (2014). Another alternative, proposed also in Liang et al. (2008), is to look for efficiency decompositions which can be regarded as “fair” in relation to some rationality principles. Du et al. (2011) and Zhou et al. (2013) have proposed the use of the Nash bargaining solution for the case of two-stage systems. Recently Wang and Li (2014), deal with an interesting extension. Our research follows the latter path and investigates the decomposition of efficiency in a general multistage system based on the Nash bargaining solution. When the efficiency decomposition problems addressed here are regarded as bargaining problems, a main feature is that they are not convex. Thus, in order to propose a solution for the decomposition and to identify the properties in which it is supported, the so called Nash extension solution is considered. We first prove that for this class of problems, the decomposition generated by the Nash extension solution coincides with that obtained by applying the Nash solution, and subsequently, it is shown that this Nash decomposition can be computed by using a simple and elegant formula. We also provide the interpretation of the properties that the solution fulfills in this context.

9.5.2 DEA Production Games

DEA production games have recently been introduced in a paper by Lozano (2013b). It is an interesting approach to linear production processes showing how a set of recorded observations of the production process can be used to plan future production on the basis of a technology inspired in DEA. In these DEA models the production technology is assumed to be implicit in the input-output data given by the set of recorded observations, and the efficient units are those located on the frontier of the production possibility set defined by the set of observations and the quantities of resources.

Borrero et al. (2016) have recently further investigated these cooperative games. The links between the class of DEA production games and the classes of linear programming games (see Curiel 1997) and linear production games have been established. It is shown that if the agents share both the resources and the technology, then the production can be centralized and taken over by any of the agents in the coalition. Another interesting fact is that if the technology is assumed to exhibit constant returns to scale, then the DEA production game which arises does not depend on the level of cooperation. When the technology exhibits variable returns to scale, the final profit depends on the level of cooperation, and if solely the resources are shared, then in general, optimal production is only achieved when different agents produce.

It is proven that any DEA production game with constant returns to scale can be seen as a linear production game, and vice versa, any linear production game can be written as a DEA production game with constant returns to scale. The transformations needed to establish this equivalence are explicitly provided. In relation to DEA production games with variable returns to scale, a significant subclass is considered: those games for which all coalitions are able to produce with the resources available. In this case, transformations are also provided which enables us to prove that the games in this class are non-negative linear programming games, and hence they can also be considered linear production games.

On the other hand, Owen (1975) used cooperative game theory to study the general class of linear production games and proved that these games have a non-empty core. He also proposed an allocation scheme of the total revenue obtained when all the players cooperate, which yields allocations in the core of the cooperative production game. These core allocations are obtained by assigning to each player the sum of the amount of each resource that this player brings to the production process valued using the dual prices corresponding to the model when all agents cooperate.

The Owen set of DEA production games has been also analyzed in Borrero et al. (2016), and the interpretation of these allocations for different levels of cooperation between agents has been discussed. New insights into the interpretation of these sets of allocations for the various versions of these production games are presented.

DEA production processes have also been modeled as vector-valued DEA production problems (Lozano et al. 2014). This is convenient (even necessary) whenever the price of at least one output is unreliable or unavailable. This can occur because an output is not marketable‘ (e.g. public services or non-tradable emissions), or because its price is uncertain or volatile and cannot be reliably forecasted (e.g. tradable emissions permits). In these cases, it is not possible to aggregate all the outputs in a revenue function, and the output amounts that result from the production process are multi-dimensional. This means that the payoff of each coalition consists of a vector of output quantities that must be allocated among the players in the coalition. Some possible situations in which a vector-valued DEA production problem can be envisaged are: Companies or institutions that may cooperate on their worker training programs; Cooperation of several private or public sector health care providers, each one supplying its own resources and skills; Different associated or independent fitness centers that, through cooperation, may virtually pool their equipment, sports facilities and instructors; Agribusiness companies (e.g. wine producers, see Aparicio et al. 2013) that can benefit from cooperating in the cultivation and harvesting of their respective plots of land; Banks and other financial institutions that may cooperate in some of their backoffice operations; Logistic service providers or other companies that may cooperate in their transportation operations (see Lozano et al. 2013).

Apart from the above mentioned, a different approach has also been conducted by the authors of the present book chapter. In DEA production models, the objective function represents the total revenue obtained from selling certain kinds of products, and the problem is formulated as a linear programming problem in which the revenue is maximized in the production possibility set induced by the set of recorded observations. However, very often, in real world-situations, the assumption of certainty with respect to the nature of the parameters is unrealistic and in many applications, the use of fuzzy logic (Zadeh 1965) has proved to be advantageous to deal with the imprecise nature of the data involved. Particularly, in the analysis of efficiency by using DEA models, imprecision in the data is a main drawback and their representation as fuzzy numbers enables a more realistic assessment of the efficiency of the decision making units (see for instance, Hatami-Marbini et al. 2011 and Emrouznejad et al. 2014).

Hinojosa et al. (2015b) study DEA production games with fuzzy prices. The introduction of uncertainty into the cooperative model raises new and interesting issues, since coalitions can form prior to the resolution of uncertainty and they must discuss divisions of the uncertain revenue by taking into account their potential worths which may also be uncertain. The lack of precision in the parameters of the linear production problem is modeled via fuzzy logic, that is, some of the parameters involved in the objective function and/or in the constraints of the production game are represented by fuzzy numbers.

Several cooperative models involving fuzzyness can be found in the literature. The line initiated in Aubin (1981) studies games with fuzzy coalitions, where the agents may consider different levels of participation in cooperation. Recent work in this line is, for instance, Wu (2012), and Li and Zhang (2009). Our investigation deals with models in which the fuzziness concerns the values that the coalitions can achieve. Nishizaki and Sakawa (2001) have previously addressed the special case of fuzzy cooperative games arising from linear production programming problems with fuzzy parameters for which they proposed an infinite family of cores, each of which consists of a set of non-fuzzy payoff vectors. Recently, in Hinojosa et al. (2013) and in Monroy et al. (2013), a different approach has been presented to analyse the solutions of cooperative games with fuzzy payoffs and applied to the cases of fuzzy linear production games and fuzzy assignment games.

As a first step to analyze DEA production problems in a fuzzy environment, a partial order has to be considered in the set of fuzzy numbers. Hence, the concept of maximization of fuzzy objective functions on a feasible set must be understood as the search for the maximal elements with respect to this partial order. As a consequence, the game arising from the production situation, when the pool of resources is controlled by several agents, is a set-valued game in which each element of the set is a fuzzy number. In this situation, since there is not a total order among the payoffs, the comparisons between the payoffs obtained by the players and by the coalitions are not straightforward as in scalar games and, therefore, classic solution concepts are not applicable.

Previous literature has addressed this difficulty by establishing a utility function in order to induce a scalar game and to obtain allocations of the associated total revenue based on different solution concepts. However, this approach seldom helps towards an accurate analysis of the situation, since the results are non-fuzzy payoffs.

In Hinojosa et al. (2015b) an *ex-ante* analysis of the production situation has been carried out, and a solution for the DEA production game with fuzzy prices which is applicable before the fuzziness is resolved has been proposed, namely the preference least core. In this solution the fuzzy nature of the allocations is preserved, and therefore, the quantity finally assigned to each agent is a fuzzy number. The preference least core has recently been introduced in Lozano et al. (2014) for set-valued DEA production games, and is based on the same idea as the least core in standard TU games. Its main drawback in the fuzzy environment is the difficulty involved in the effective computation of the fuzzy allocations.

Standard fuzzy orders in the set of fuzzy numbers are adopted (see González and Vila 1992, and Ramík and Římánek 1985), and the excess of the coalitions is defined accordingly. For DEA production games, the fuzzy allocations in the preference least core allocate the revenue obtained with one of the efficient production vectors which minimize the excess of the coalitions. The main contribution in the paper is the proposal of a procedure to compute allocations in the preference least core. The procedure requires solving a single linear programming model, which at the same time yields the efficient fuzzy revenue obtained by cooperation and the allocation to the agents of this fuzzy quantity.

The approach is applied in a case study, for which allocations in the preference least core are obtained, both for the case where uncertain prices are represented by triangular fuzzy numbers and trapezoidal fuzzy numbers.

Acknowledgements This research has been partially financed by the Spanish Ministry of Science and Innovation, projects, ECO2011-29801-C02-01 and ECO2011-29801-C02-02, and by the Consejería de Innovación de la Junta de Andalucía, project P11-SEJ-7782 and P10-TEP-6332.

References

Aparicio, J., Borras, F., Pastor, J. T., & Vidal, F. (2013). Accounting for slacks to measure and decompose revenue efficiency in the Spanish designation of origin wines with DEA. *European Journal of Operational Research, 231*, 443–451.

Aubin, J. P. (1981). Cooperative fuzzy games. *Mathematics of Operations Research, 6*, 1–13.

Aumann, R. J., & Maschler, M. (1964). The bargaining set for cooperative games. In M. Dresher, L. S. Shapley, & A. Tucker (Eds.), *Advances in game theory* (Vol. 52, pp. 443–476). Princeton: Princeton University Press.

Bondareva, O. (1963). Some applications of linear programming methods to the theory of cooperative games. *Problemy Kybernetiki, 10*, 119–139.

Borrero, D.V., Hinojosa, M.A., & Mármol, A.M. (2016). DEA production games and Owen allocations. *European Journal of Operational Research, 252*, 921–930.

Chen, Y., Cook, W.D., Li, N., & Zhu, J. (2009). Additive efficiency decomposition in two-stage DEA. *European Journal of Operational Research, 196*, 1170–1176.

Conley, J.P., & Wilkie, S. (1991). Bargaining theory without convexity. *Economics Letters, 36*, 365–369.

Conley, J.P., & Wilkie, S. (1996). An extension of the Nash bargaining solution to non-convex problems. *Games and Economic Behaviour, 13*, 26–38.

Cook, W.D., Liang, L., & Zhu, J. (2010). Measuring performance of two-stage network structures by DEA: A review and future perspective. *Omega, 38*, 423–430.

Curiel, I.J. (1997). *Cooperative game theory and applications*. Dordrecht: Kluwer Academic Publisher.

Du, J., Liang, L., Chen, Y., Cook, W.D., & Zhu, J. (2011). A bargaining game model for measuring performance of two stage network structures. *European Journal of Operational Research, 210*, 390–397.

Emrouznejad, A., Tavana, M., & Hatami-Marbini, A. (2014). The state of the art in fuzzy data envelopment analysis. *Performance Measurement with Fuzzy Data Envelopment Analysis, 309*, 1–45.

Färe, R., & Grosskopf, S. (1996). Productivity and intermediate products: A frontier approach. *Economics Letters, 50*, 65–70.

Fukuyama, H., & Weber, W.L. (2010). A slacks-based inefficiency measure for a two-stage system with bad outputs. *Omega, 38*(5), 398–409.

Gillies, D.B. (1953). Some theorems on n-person games. Ph.D. Thesis, Princeton.

González, A., & Vila, M.A. (1992). Dominance relation on fuzzy numbers. *Information Sciences, 64*, 1–16.

Green, R., Doyle, J., & Cook, W.D. (1996). Preference voting and project ranking using DEA and cross-evaluation. *European Journal of Operational Research, 90*, 461–472.

Halkos, G.E., Tzeremes, N.G., & Kourtzidis, S.A. (2014). A unified classification of two-stage DEA models. *Surveys in Operations Research and Management Science, 19*, 1–16.

Hatami-Marbini, A., Emrouznejad, A., & Tavana, M. (2011). A taxonomy and review of the fuzzy data envelopment analysis literature: Two decades in the making. *European Journal of Operational Research, 214*, 457–472.

Herrero, M.J. (1989) Nash program: non-convex bargaining problems. *Journal of Economic Theory, 49*, 266–277.

Hinojosa, M.A., Lozano, S., & Mármol, A. (2015a). Nash decomposition for process efficiency in multistage production systems. Working paper submitted for publication.

Hinojosa, M.A., Lozano, S., & Mármol, A. (2015b). DEA production games with fuzzy output prices. Working paper submitted for publication.

Hinojosa, M.A., Mármol, A.M., Monroy, L., & Fernández, F.R. (2013). A multi-objective approach to fuzzy linear production games. *International Journal of Information Technology and Decision Making, 12*(5), 927–943.

Jahangoshai Rezaee, M., Moini, A., & Hali-Aji Asgari, F. (2012a). Unified performance evaluation of health centers with integrated model of data envelopment analysis. *Journal of Medical Systems, 36*, 3805–3815.

Jahangoshai Rezaee, M., Moini, A., & Makui, A. (2012b). Operational and non-operational performance evaluation of thermal power plants in Iran: A game theory approach. *Energy, 38*(1), 96–103.

Jahanshahloo, G.R., Hosseinzadeh, L.F., & Sohraiee, S. (2006). Egoist's dilemma with interval data. *Applied Mathematics and Computation, 183*, 94–105.

Jalali-Naini, S.G., Moini, A., & Jahangoshai Rezaee, M. (2013). Nash bargaining game model for two parallel stages process evaluation with shared inputs. *International Journal of Advanced Manufacturing Technology, 67*, 475–484.

Kalai, E., & Smorodinsky, M. (1975). Other solutions to Nash's bargaining problem. *Econometrica, 43*, 513–518.

Kao, C. (2014). Network data envelopment analysis: A review. *European Journal of Operational Research, 239*, 1–16.

Kao, C., & Hwang, S.N. (2008). Efficiency decomposition in two-stage data envelopment analysis: An application to non-life insurance companies in Taiwan. *European Journal of Operational Research, 185*, 418–429.

Kao, C., & Hwang, S.N. (2010). Efficiency measurement for network systems: IT impact on firm performance. *Decision Support Systems, 48(3)*, 437–446.

Kao, C., & Hwang, S.N. (2011). Decomposition of technical and scale efficiencies in two-stage production systems. *European Journal of Operational Research, 211*, 515–559.

Kao, C., & Hwang, S.N. (2014). Multi-period efficiency and Malmquist productivity index in two-stage production systems. *European Journal of Operational Research, 232*, 512–521.

Li, Y., & Liang, L. (2010). A Shapley value index on the importance of variables in DEA models. *Expert Systems with Applications, 37*, 6287–6292.

Li, S., & Zhang, Q. (2009). A simplified expression of the Shapley function for fuzzy games. *European Journal of Operational Research, 196*, 234–245.

Liang, L., Cook, W.D., & Zhu, J. (2008). DEA models for two-stage processes: Game approach and efficiency decomposition. *Naval Research Logistics, 55*, 643–653.

Lozano, S. (2012). Information sharing in DEA: A cooperative game theory approach. *European Journal of Operational Research, 222*, 558–565.

Lozano, S. (2013a). Using DEA to find the best partner for a horizontal cooperation. *Computers & Industrial Engineering, 66*, 286–292.

Lozano, S. (2013b). DEA production games. *European Journal of Operational Research, 231*, 405–413.

Lozano, S., Moreno, P., Adenso-Díaz, B., & Algaba, E. (2013). Cooperative game theory approach to allocating benefits of horizontal cooperation. *European Journal of Operational Research, 229*, 444–452.

Lozano, S., Hinojosa, M.A., & Mármol, A.M. (2014). Set-valued DEA production games. *Omega, 52*, 92–100.

Monroy, L., Hinojosa, M.A., Mármol, A.M., & Fernández, F.R. (2013). Set-valued cooperative games with fuzzy payoffs. The fuzzy assignment game. *European Journal of Operational Research, 225*(1), 85–90.

Nakabayashi, K., & Tone, K. (2006). Egoist's dilemma: A DEA game. *Omega, 34*, 135–148.

Nakabayashi, K., Sahoo, B.K., & Tone, K. (2009). Fair allocation based on two criteria: A DEA game view of "Add them up and divide by two". *Journal of the Operations Research Society of Japan, 52*(2), 131–146.

Nash, J.F. (1950). The bargaining problem. *Econometrica, 28*, 155–162.

Nishizaki, I., & Sakawa, M. (2001). *Fuzzy and multiobjective games for conflict resolution*. Heidelberg: Physica-Verlag

Omrani, H., Gharizadeh Beiragh, R., & Shafiei Kaleibari, S. (2015). Performance assessment of Iranian electricity distribution companies by an integrated cooperative game data envelopment analysis principal component analysis approach. *Electrical Power and Energy Systems, 64*, 617–625.

Owen, G. (1975). On the core of linear production games. *Mathematical Programming, 9*, 358–370.

Ramík, J., & Římánek, J. (1985). Inequality relation between fuzzy numbers and its use in fuzzy optimization. *Fuzzy Sets and Systems, 16*(2), 123–138.

Roth, A.E. (1979). An impossibility result concerning n-person bargaining games. *International Journal of Game Theory, 8*, 129–132.

Schmeidler, D. (1969). The nucleolus of a characteristic function game. *SIAM Journal on Applied Mathematics, 17*(6), 1163–1170.

Sekine, S., Fu, J., & Muto, S. (2014). Game theoretic approaches to weight assignments in data envelopment analysis problems. *Mathematical Problems in Engineering*. doi: 10.1155/2014/434252.

Shapley, L.S. (1953). A value for *n*-person games. In H. Kuhn, & A. Tucker (Eds.), *Contributions to the theory of games II*. Annals of Mathematics Studies. Princeton: Princeton University Press.

Shapley, L.S. (1967). On balanced sets and cores. *Naval Research Logistics Quarterly, 14*, 453–460.

Sugiyama, M., & Sueyoshi, T. (2014). Finding a common weight vector of data envelopment analysis based upon bargaining game. *Studies in Engineering and Technology*. doi:10.11114/set.v1i1.277.

Thomson, W. (1980). Two characterizations of the Raiffa solution. *Economics Letters, 6*, 225–231.

Tijs, S.H. (1981). Bounds for the core and the τ-value. In: O. Moeschlin, & D. Pallaschke (Eds.), *Game theory and mathematical economics* (pp. 123–132). Amsterdam: North-Holland Publishing Company

Tone, K. (2002). A strange case of the cost and allocative efficiencies in DEA. *Journal of the Operational Research Society, 53*, 1225–1231.

Tone, K., & Tsutsui, M. (2009). Network DEA: A slacks-based measure approach. *European Journal of Operational Research, 197*, 243–252.

Tone, K., & Tsutsui, M. (2014). Dynamic DEA with network structure: A slacks-based measure approach. *Omega, 42*, 124–131.

von Neumann, J., & Mongerstern, O. (1944). *The theory of games and economics behaviour*. Princeton: Princeton University Press.

Wang, M., & Li, Y. (2014). Supplier evaluation based on Nash bargaining game model. *Expert Systems with Applications, 41*, 4181–4185.

Weber, R.J. (1988). Probabilistic values for games. In *The Shapley value* (pp. 101–119). Cambridge: Cambridge University Press.

Wu, H.C. (2012), Proper cores and dominance cores of fuzzy games. *Fuzzy Optimization and Decision Making, 11*(1), 47–72.

Wu, J., Liang, L., & Zha, Y.C. (2008). Determination of the weights of ultimate cross efficiency based on the solution of nucleolus in cooperative game. *Systems Engineering-Theory and Practice, 28*(5), 92–97.

Wu, J., Liang, L., Yang, F., & Yan, H. (2009a). Bargaining game model in the evaluation of decision making units. *Expert Systems with Applications, 36*, 4357–4362.

Wu, J., Liang, L., & Yang, F. (2009b). Determination of the weights for the ultimate cross efficiency using Shapley value in cooperative game. *Expert Systems with Applications, 36*, 872–876.

Wu, H., Du, S., Liang, L., & Zhou, Y. (2013). A DEA-based approach for fair reduction and reallocation of emission permits. *Mathematical and Computer Modelling, 58*, 1095–1101.

Yang, X., & Morita, H. (2013) Efficiency improvement from multiple perspectives: An application to Japanese banking industry. *Omega, 41*(3), 501–509.

Zadeh, L.A. (1965). Fuzzy sets. *Information and Control, 8*, 338–353.

Zhou, Z., Sun, L., Yang, W., Liu, W., & Ma, C. (2013). A bargaining game model for efficiency decomposition in the centralized model of two-stage systems. *Computers and Industrial Engineering, 64*, 103–108.

Chapter 10
Measuring Bank Performance: From Static Black Box to Dynamic Network Models

Hirofumi Fukuyama and William L. Weber

Abstract This chapter presents the recently developed dynamic-network bank technology and performance measures of Fukuyama and Weber (Efficiency and productivity growth: Modelling in the financial services industry. Wiley, London, pp. 193–213, 2013; J Product Anal 44(3):249–264, 2015a; Ann Oper Res, in press, 2015b; Japanese bank productivity, 2007-2012: A dynamic network approach. Mimeo, 2016). The method uses DEA to represent the production technology and directional distance functions to measure bank performance. A two stage bank technology where an intermediate product is produced in a first stage and then used to produce final outputs in a second stage is extended over time. The performance measure allows the researcher to compare observed inputs and outputs, including undesirable outputs, with the outputs and inputs that might be produced if a producer were able to optimally choose production plans relative to a dynamic benchmark technology. Although Fukuyama and Weber's studies apply the dynamic network technology to measure the performance of Japanese banks, the method can be applied to banks in other countries and to other types of financial institutions.

Keywords Data envelopment analysis (DEA) • Network DEA model • Dynamic DEA model • Dynamic-network DEA model • Bad outputs • Nonperforming loans • Productivity

H. Fukuyama (✉)
Fukuoka University, 8-19-1 Nanakuma, Jonan-Ku, Fukuoka 814-0180, Japan
e-mail: fukuyama@fukuoka-u.ac.jp

W.L. Weber
Department of Economics and Finance, Southeast Missouri State University, Girardeau, MO, USA
e-mail: wlweber@semo.edu

S.-N. Hwang et al. (eds.), *Handbook of Operations Analytics Using Data Envelopment Analysis*, International Series in Operations Research & Management Science 239, DOI 10.1007/978-1-4899-7705-2_10

10.1 Introduction

In this chapter we present a dynamic-network model of the bank technology which can be used to measure bank performance. The model accounts for exogenous inputs, excess reserves which can be carried over from period to period and final outputs that include desirable outputs and jointly produced undesirable by-products in the form of nonperforming loans. The framework is based on the performance measures of Fukuyama and Weber (2013, 2015a, b, 2016). These models measure bank performance accounting for the following characteristics of the bank technology: (i) banks face a two-stage network technology where deposits and other funds are produced in a first stage and then in a second stage those deposits are used to generate a portfolio of interest-bearing assets (loans) and non-interest bearing assets (securities investments), (ii) banks face credit risk in that the loan production process generates a jointly produced by-product of nonperforming loans, (iii) in the second stage of production bank managers can choose to make loans and securities investments or carry-over some excess reserves for use in a future period, (iv) nonperforming loans produced in one period become an undesirable input to the first stage of production in a future period.

The dynamic aspect of the technology incorporates two outcomes of current period production on future production. First, a bank can choose to either produce final outputs of securities investments and loans, including jointly produced nonperforming loans or they can choose to carry-over some of their raised funds to the second stage of production in a future period. Thus, current production decisions affect future production possibilities. Second, nonperforming loans generated in the current period have a negative effect on the first stage of production in a subsequent period. This is because when nonperforming loans are generated, a bank must raise more financial equity capital or curtail their deposit taking and other fund-raising activities. Thus, static performance indicators that account for only current period outputs and inputs are biased to the extent that bank managers optimize over many periods. Although the lending process links desirable and jointly produced undesirable outputs, securities investments made by the bank are not linked to nonperforming loans.[1] As a consequence we separate the jointly produced linked outputs of performing and nonperforming loans and the unlinked outputs of securities investments following Epure and Lafuente (2015) and Fukuyama and Weber (2016). Since nonperforming loans are an unavoidable by-product of loan production and negatively affect the ability of banks to raise deposits, they are treated as an undesirable input to stage 1 in a subsequent period. On the other hand, carryover assets from a previous period augment future lending and investment opportunities and are treated as a desirable input to stage 2.

[1] Securities investments generally are subject to interest rate risk (Saunders and Cornett 2011) rather than credit risk.

10.2 Selective Literature Review

10.2.1 Network DEA and Dynamic DEA

In the static black-box form of data envelopment analysis (DEA) due to Farrell (1957) and Charnes et al. (1978), inputs and outputs are assumed to be independent across production periods. That is, the inputs and outputs observed in one period have no effect on the production technology in future periods. Färe and Grosskopf (1996) extended the static black box technology and laid the theoretical foundation for network DEA. A two-stage network model where intermediate products are produced in stage 1 and then become the only inputs to stage 2 is commonly used (see Fig. 10.1). The popularity of this model is mainly due to its simple mathematical structure although this network model can be easily extended to more than two stages and to parallel production technologies as well. Sexton and Lewis (2003), Liang et al. (2008), Chen et al. (2009b) and Chen et al. (2010) investigated the static two-stage network method without undesirable outputs. Fukuyama and Weber (2010) presented a static two-stage network model of bank production accounting for the undesirable output of nonperforming loans. Their model used the directional technology distance function to measure performance and accounted for slacks in the constraints that define the DEA technology.

Lewis and Sexton (2004) developed a multi-stage network DEA model which extended the Sexton and Lewis (2003) two-stage network model. Kao and Hwang (2008) provided a two-stage method that simultaneously determined the efficiency of each division and the entire system. Tone and Tsutsui (2009) introduced a network DEA method that compared actual division outputs with potential outputs accounting for slacks in the output and inputs constraints that defined the network technology. Kao (2009) used a multiplier form and introduced a network DEA method by considering the relations among sub-process efficiencies in terms of series, parallel, and mixed methods.

Fukuyama and Mirdehghan (2012) and Mirdehghan and Fukuyama (2016) suggested a two-phase algorithm for identifying the Pareto-Koopmans efficiency status of the decision making unit (DMU). Lozano (2015) presented a network DEA model that compared outputs of all divisions with the outputs of all divisions that

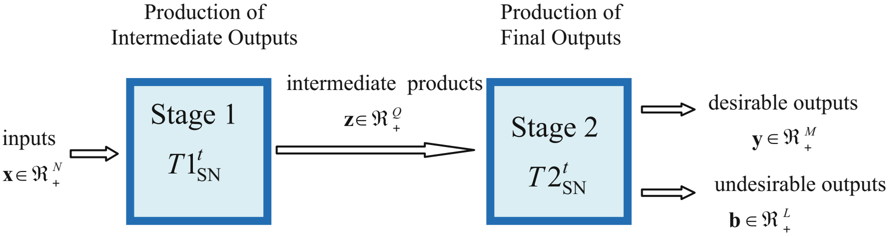


Fig. 10.1 Static two-stage network production with bad outputs. *Legend*: $T1_{SN}^t$: static network stage 1 technology; $T2_{SN}^t$: static network stage 2 technology; SN: static network

could be produced if a central decision-maker allocated inputs to each division or sub-process instead of taking divisional inputs as given. Extending Fukuyama and Weber's (2010) two-stage network SBI (slack-based inefficiency) model, Lozano (2016) generalized Fukuyama and Weber's (2010) two-stage network model by accounting for undesirable outputs. A comprehensive survey of the literature on network DEA is provided by Kao (2014) who categorized network DEA models into nine classes: (i) independent models, (ii) system distance measure models, (iii) process distance models, (iv) factor distance models, (v) slacks-based measure models, (vi) ratio-form system (overall) efficiency models, (vii) ratio-form sub-process efficiency models, (viii) game theoretic models, and (ix) value-based models.

While dynamic DEA studies are sparse compared to those of network DEA, the number of dynamic network DEA studies has been growing. Färe, Grosskopf and Margaritis (2011) extended Shephard and Färe's (1980) basic dynamic production framework by connecting a sequence of single-period technologies and allowing producers to produce either final outputs or carryover some output to augment production in a subsequent period. Färe and Grosskopf (1996) used investment spending as an example of a carryover with producers deciding the mix of GDP to be allocated to the final output of consumer spending and private investment spending that enhances future production possibilities. Sengupta (1994) and Nemoto and Goto (2003) used adjustment cost theory and provided optimal control-theoretic models for assessing the dynamic efficiency of DMUs. To examine the impact of public capital investment on private productivity, Bogetoft et al. (2009) derived a DEA model and calculated the optimal paths and levels of public and private investment spending. Tone and Tsutsui (2014) proposed a slacks-based dynamic-network model that allowed carryover assets to have positive or negative effects on future production. In an examination of Bangladeshi banks Akther et al. (2013) constrained the current-period bank technology on the amount of nonperforming loans generated in a previous period. Fukuyama and Weber (2013, 2015a, b, 2016) extended Akther et al. (2013) to multiple periods and allowed a bank to reduce the current production of loans and securities investments and to save carryovers for use in a future period if future production could be enhanced by more than the loss of current production.

Färe et al. (1992) and Färe et al. (1994) proposed static Malmquist productivity indices which can be decomposed into indexes of efficiency change and technological change. Färe et al. (2011) presented a multiplicative dynamic Malmquist index that allowed DMUs to make inter-temporal decisions on the allocation of scarce resources so as to maximize production over all periods. Similarly, Fukuyama and Weber (2015b, 2016) proposed a dynamic-network Luenberger bank productivity indicator and its additive components of efficiency change and technological change.

De Mateo et al. (2006) and Fallah-Fini et al. (2014) addressed various inter-temporal aspects of production technologies and performance measures. De Mateo et al. (2006) studied various concepts associated with inter-temporal production including the cost, path, and period of adjustment, the appraisal period, profits, and dynamic DEA. Fallah-Fini et al. (2014) determined five main factors that influence the inter-temporal dependence between inputs and outputs: production delays, inventories, quasi-fixed factors, adjustments costs, and disembodied technical change.

10.2.2 Bank Production and Risk

Berger and Humphrey (1997) provide a comprehensive survey of financial institution performance measures. Here we selectively examine studies that estimated bank performance controlling for the effects of bank risk. McAllister and McManus (1993) showed that large US banks had even greater measured scale economies controlling for bank risk. Altunbas et al. (2000) estimated a parametric cost frontier for a sample of Japanese commercial banks to examine the impact of risk and quality factors on bank costs. Altunbas et al. (2000) defined the loan quality as the ratio of nonperforming loans to total loans. For a sample of Japanese banks operating in 1996, Drake and Hall (2003) reported evidence that financial capital had the greatest impact on scale efficiency.

Liu and Tone (2008) used the ratio of credit costs to potential loan losses as an input and found that Japanese banks appeared to be "learning by doing" as efficiency improved during the sample period of 1997–2001. Park and Weber (2006) and Fukuyama and Weber (2003, 2004, 2005) controlled for bank risk by using financial equity capital as an input in their static measures of bank performance. Fukuyama and Weber (2008a, b) estimated the shadow price of nonperforming loans. Drake et al. (2009) documented the importance of accounting for loans and risk in the analysis of Japanese bank efficiency.

Fukuyama and Weber (2010) proposed a two-stage network model for Japanese cooperative Shinkin banks. In stage 1 banks use labor, physical capital and financial equity capital to produce deposits. Then, in stage 2 those deposits serve as an input as banks produce a portfolio of securities investments and loans with some of the loans becoming nonperforming. In a study of Bangladeshi banks Akther, Fukuyama and Weber (2013) advanced this specification by allowing nonperforming loans generated in a preceding period to decrease the production possibility set in a subsequent period. All above-mentioned bank efficiency studies have the view that risk associated to nonperforming loans need to be considered in bank efficiency measurement.

Compared with network DEA, fewer studies have been devoted to the dynamic DEA studies of bank efficiency. Fukuyama and Weber (2013) extended the Färe and Grosskopf (1996) dynamic model to a network setting by specifying a bank technology where a bank can forego current loans and the jointly produced nonperforming loans by expanding carryover assets (excess reserves) for use in a subsequent period when better lending conditions might exist. Although their dynamic model considered only three periods for bank managers to optimize over

Fukuyama and Weber (2015b, 2016) extended the model to more than three periods. Fukuyama and Weber (2015a) added a financial regulatory restraint that constrained the feasible technology by requiring banks to hold a minimum ratio of financial equity capital to assets. Furthermore, Fukuyama and Weber (2015a, b) showed how the primal envelopment form that incorporated the regulatory constraint could be estimated by its dual multiplier form with financial regulatory restraint and how the Luenberger dynamic productivity indicator could be decomposed into productivity gains due to technological progress and productivity gains due to greater efficiency. Epure and Lafuente (2015) accounted for bank risk and distinguished between desirable outputs linked to nonperforming loans and desirable outputs such as securities investments and service fees not linked to jointly produced undesirable outputs.

Table 10.1 presents a summary of the assumptions behind the models of Fukuyama and Weber (2013, 2015a, b, 2016) and the data sets on Japanese banks used in their empirical work. The primary purpose of the current study is to unify these models and provide an extension by incorporating the condition of weak disposability between desirable outputs and jointly produced undesirable outputs when banks operate under variable returns to scale.

10.3 Preliminaries

10.3.1 Black-Box Technology

Let the exogenous inputs that can be employed by a bank in period t be represented by $\mathbf{x}^t = \left(x_1^t, \ldots, x_N^t\right) \in \Re_+^N$ and let total desirable outputs be represented by $\mathbf{y}^t = \left(y_1^t, \ldots, y_M^t\right) \in \Re_+^M$. The static black-box (SBB) technology for period t is defined as

$$T_{\mathrm{SBB}}^t = \left\{(\mathbf{x}^t,\ \mathbf{y}^t) \in \Re_+^N \times \Re_+^M \middle|\ \mathbf{x}^t \text{ can produce } \mathbf{y}^t\right\}. \tag{10.1}$$

The SBB technology considers only one period and neglects the possible effects of past production outcomes such as nonperforming loans and carryover assets that might make a bank appear less efficient than it would otherwise be if those carryover assets were incorporated into the technology. We assume that T_{SBB}^t satisfies strong disposability of exogenous inputs $\mathbf{x}^t$ and desirable outputs $\mathbf{y}^t$ along with other standard properties (Shephard 1970; Färe and Primont 1995). Strong disposability of inputs and outputs means that if $\left(\mathbf{x}_0^t,\ \mathbf{y}_0^t\right) \in T_{\mathrm{SBB}}^t$ then $\left(\mathbf{x}_0^t,\ -\mathbf{y}_0^t\right) \leq \left(\mathbf{x}_1^t,\ -\mathbf{y}_1^t\right)$ implies $\left(\mathbf{x}_1^t,\ \mathbf{y}_1^t\right) \in T_{\mathrm{SBB}}^t$. That is, it is feasible for banks to use more input to produce less output.

Table 10.1 Previous dynamic-network bank efficiency studies with NPLs

	Unique model characteristics	Data used	Common model characteristics
Fukuyama and Weber (2013)	• Envelopment form	Shinkin banks (265 × 8 = 2120)	1. CRS frontiers
	• 3-period directional technology distance function	Period: FY2002–FY2009	2. NPLs as undesirable output
Fukuyama and Weber (2015a)	• Envelopment form and multiplier form	Commercial banks (101 × 5 = 505)	3. Past NPLs affect present production
	• Financial regulatory constraints	Period: FY2006–FY2010	4. Two-stage network structure
	• 3-period directional technology distance function		5. Multi-period dynamic structure
Fukuyama and Weber (2015b)	• Envelopment form	Commercial banks (103 × 7 = 721)	
	• Multi-period directional technology distance function	Shinkin banks (265 × 7 = 1855)	
	• Luenberger indicator	Period: FY2006–FY2012	
Fukuyama and Weber (2016)	• Envelopment Form	Commercial banks (100 × 7 = 700)	
	• Multi-period directional output distance function	Period: FY2006–FY2012	
	• Luenberger indicator		
	• Distinction between linked desirable and undesirable outputs and desirable outputs not linked to undesirable outputs		

FW Fukuyama and Weber, *DN* dynamic-network, *NPLs* nonperforming loans, *FY* fiscal year
Notes: Data size changes due to data availability, differences in research objectives and the functional differences in bank activities. (1) Commercial banks consist of joint-stock city and regional banks. (2) Shinkin banks are credit cooperatives

10.3.2 Network Technology with Bad Outputs

To specify a bank technology and measure DMU performance researchers must determine what inputs are used to produce what outputs. In their examination of Japanese banks Fukuyama and Weber (2003, 2004, 2005) adopted the intermediation approach of Sealey and Lindley (1977) and assumed that Japanese banks employed variable inputs of labor, physical capital, and deposits and a quasi-fixed input in the form of financial equity capital to produce loans and securities investments. Although the SBB technology represented by (10.1) has frequently been used in bank efficiency measurement studies, disagreement exists regarding whether deposits should be treated as an output or as an input. Berger and Humphrey (1997) and Fethi and Pasiouras (2010) provide background on this on-going discussion. To cope with this problem, various authors have used static two-stage network models where deposits are an intermediate output of stage 1 production and an intermediate input of stage 2 production. Let $\mathbf{z}^t = \left(z_1^t, \ldots, z_Q^t\right) \in \Re_+^Q$ represent the vector of intermediate products that are produced in stage 1 and then subsequently used as an input in stage 2. The static stage 1 network (SN) technology is denoted

$$T1_{\mathrm{SN}}^t = \left\{ (\mathbf{x}^t, \mathbf{z}^t) \in \Re_+^N \times \Re_+^Q \middle| \ \mathbf{x}^t \in \Re_+^N \ \text{ can produce } \ \mathbf{z}^t \in \Re_+^Q \right\}. \tag{10.2}$$

In the second stage of production it is commonly assumed that the main activity of a bank is lending to the customers and hence it faces the credit risk associated with lending, i.e., nonperforming loans arise in the stage 2 production process. Let $\mathbf{b}^t = \left(b_1^t, \ldots, b_L^t\right) \in \Re_+^L$ represent the undesirable outputs that are jointly produced as part of the lending process. Accounting for these undesirable outputs the static stage 2 network technology is denoted

$$T2_{\mathrm{SN}}^t = \left\{ (\mathbf{z}^t, \mathbf{y}^t, \mathbf{b}^t) \in \Re_+^N \times \Re_+^M \times \Re_+^L \middle| \ \mathbf{x}^t \ \text{ can produce } \ (\mathbf{y}^t, \mathbf{b}^t) \right\}. \tag{10.3}$$

Nonperforming loans are generally classified by the various stages of delinquency–loans delinquent for less than 3 months, loans delinquent less than 6 months but more than 3 months, etc. Following Wang et al. (1997) and Chen et al. (2009a, b, 2010) the SBB technology can be extended to a static two-stage network technology that accounts for undesirable outputs. This static network technology is denoted

$$T_{\mathrm{SN}}^t = \left\{ (\mathbf{x}^t, \mathbf{z}^t, \mathbf{y}^t) \in \Re_+^N \times \Re_+^M \middle| \ (\mathbf{x}^t, \mathbf{z}^t) \in T1_{\mathrm{SN}}^t, \quad (\mathbf{z}^t, \mathbf{y}^t, \mathbf{b}^t) \in T2_{\mathrm{SN}}^t \right\}. \tag{10.4}$$

Figure 10.1 depicts the two-stage network system with undesirable outputs.

10.3.3 Dynamic Technology with Carryovers

To move from a static to a dynamic technology we follow Färe and Grosskopf (1996) and assume that bank managers have discretion in how to allocate total output produced, $\mathbf{y}^t = \left(\mathbf{y}_1^t, \ldots, \mathbf{y}_M^t\right) \in R_+^M$, between final outputs, $\mathbf{fy}^t = \left(\mathbf{fy}_1^t, \ldots, \mathbf{fy}_M^t\right) \in R_+^M$, and carryover assets, $\mathbf{c}^t = \left(c_1^t, \ldots, c_M^t\right) \in \Re_+^M$ where

$$\mathbf{y}^t = \mathbf{fy}^t + \mathbf{c}^t \in \Re^M_+. \tag{10.5}$$

See also Fukuyama and Weber (2013, 2015a, b). For instance, total outputs can be thought of as the total amount of assets–less required reserves and physical capital assets–that can be allocated to the various divisions that make loans and securities investments. After the managers of those divisions receive their allocation ($\mathbf{y}^t$) they can choose to use their funds to make loans or securities investments ($\mathbf{fy}^t$) or they can save their allocation for use in a future period ($\mathbf{c}^t$) when better lending and investment opportunities might be available because of technological progress or because of a more robust economy (Fig. 10.2).

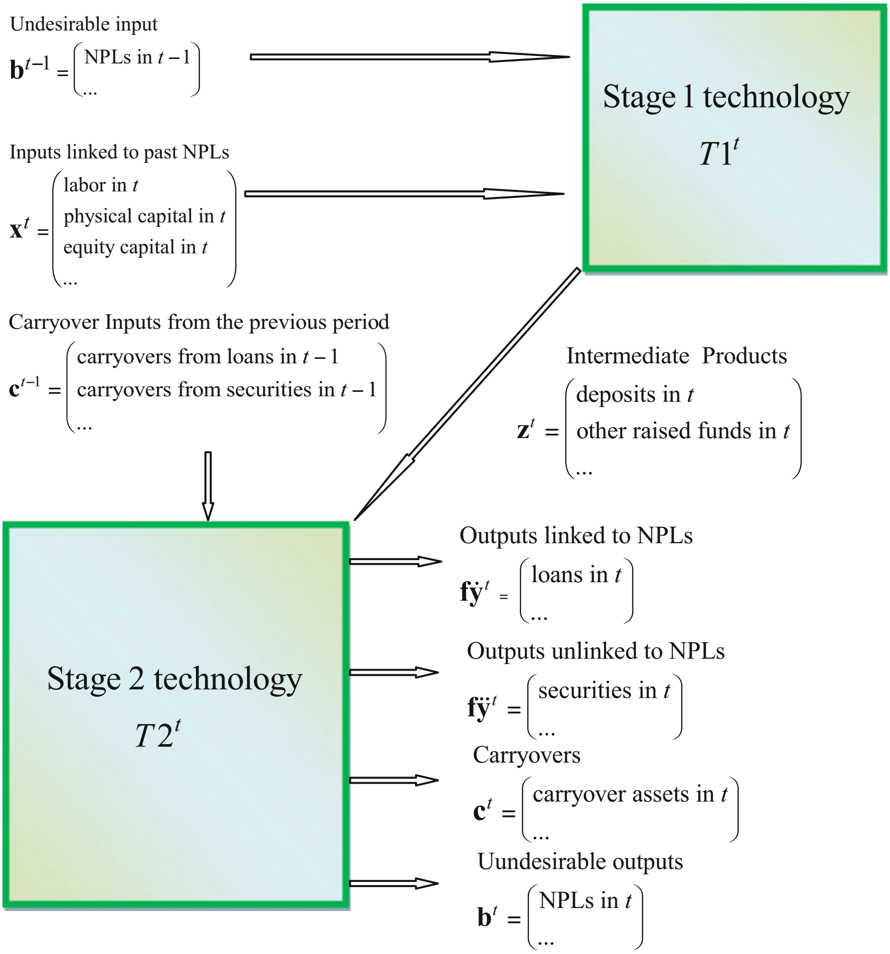


Fig. 10.2 Bank intermediation process with NPLs and carryovers. *Notes*: $T1^t$: dynamic network Stage 1 technology; $T2^t$: dynamic network Stage 2 technology; NPLs: nonperforming loans

Therefore, we define the dynamic black-box (DBB) technology as

$$T_{\mathrm{DBB}} = \left\{ (\mathbf{x}, \mathbf{fy}, \mathbf{c}, \mathbf{b}) \middle| \begin{array}{c} \left(\mathbf{c}^{t-1}, \mathbf{b}^{t-1}, \ \mathbf{x}^t, \mathbf{fy}^t + \mathbf{c}^t, \mathbf{b}^t\right) \text{ is a feasible} \\ \text{production plan for } t = 1, \ldots, T \end{array} \right\} \tag{10.6}$$

where $\mathbf{x} = (\mathbf{x}^1, \mathbf{x}^2, \ldots, \mathbf{x}^T)$, $\mathbf{fy} = (\mathbf{fy}^1, \ \mathbf{fy}^2, \ldots, \ \mathbf{fy}^T)$, $\mathbf{c} = (\mathbf{c}^0, \mathbf{c}^1, \ \ldots, \ \mathbf{c}^T)$ and $\mathbf{b} = \left(\mathbf{b}^0, \mathbf{b}^1, \ldots, \mathbf{b}^T\right)$. Akther et al. (2013) considered lagged nonperforming loans as an undesirable input when studying the efficiency of commercial banks in Bangladesh.

10.3.4 Dynamic-Network Technology

Our final goal is to link the static network technology with the dynamic black box technology. Nonperforming loans generated in a previous period, $\mathbf{b}^{t-1}$, are an undesirable input to stage 1 in period t. Undesirable inputs have the property that if the current level of production is to be maintained, greater use of the undesirable input must be offset by the use of larger amounts of the desirable inputs. For instance, financial equity capital is necessary for banks to engage in fund raising activities. When some of a bank's loans become nonperforming, the ratio of equity capital to total assets falls and bank regulations require banks to either seek additional sources of financial equity capital (the desirable stage 1 input) or reduce fund raising activities. Therefore, we define the dynamic-network stage 1 technology as

$$T1^t = \left\{ \left(\mathbf{b}^{t-1}, \mathbf{x}^t, \mathbf{z}^t\right) \in \Re_+^M \times \Re_+^N \times \Re_+^Q \middle| \left(\mathbf{b}^{t-1}, \mathbf{x}^t\right) \in \Re_+^N \text{ can produce } \ \mathbf{z}^t \in \Re_+^Q \right\}. \tag{10.7}$$

Stage 2 of production also has a dynamic element. Along with the intermediate outputs of raised funds and deposits, carryover assets from a previous period are a desirable input in the production of the portfolio of loans and securities investments in the current period. When lending and other investment opportunities are plentiful in the current period, bank managers might seek to keep carryover assets (excess reserves) to a minimum which reduces production possibilities in a subsequent period. In contrast, bank managers might choose to keep final outputs relatively small and hold a large amount of carryover assets for use in a future, more positive lending environment. Thus, carryover assets, $\mathbf{c}^t = \left(c_1^t, \ldots, c_M^t\right) \in \Re_+^M$ represent the unused assets in period t that are held until period $t+1$, similar to an inventory. When current economic conditions are weakening or when technological progress is expected a bank can delay some portion of the assets for use to a future period at the expense of the current production of outputs. Therefore, we denote the stage 2 technology as

$$T2^t = \left\{ \left(\mathbf{c}^{t-1}, \mathbf{z}^t, \mathbf{fy}^t, \mathbf{b}^t \right) \in \Re_+^N \times \Re_+^M \times \Re_+^L \middle| \; \left(\mathbf{c}^{t-1}, \mathbf{z}^t, \mathbf{fy}^t + \mathbf{c}^t, \mathbf{b}^t \right) \text{ is feasible} \right\}. \tag{10.8}$$

We combine (10.7) and (10.8) to obtain the period t network technology

$$NT^t = \left\{ \left(\mathbf{b}^{t-1}, \mathbf{c}^{t-1}, \ \mathbf{x}^t, \ \mathbf{z}^t, \ \mathbf{fy}^t, \ \mathbf{b}^t, \mathbf{c}^t \right) \;\middle|\; \begin{array}{l} \left(\mathbf{b}^{t-1}, \ \mathbf{x}^t, \ \mathbf{z}^t \right) \in T1^t, \\ \left(\mathbf{c}^{t-1}, \mathbf{z}^t, \mathbf{fy}^t, \mathbf{b}^t \right) \in T2^t \end{array} \right\}. \tag{10.9}$$

The dynamic-network technology (DNT) is formed by extending (10.9) over $t = 1, \ldots, T$ periods:

$$DNT = \left\{ (\mathbf{x}, \ \mathbf{y}, \mathbf{c}, \ \mathbf{b}) \;\middle|\; \begin{array}{l} \left(\mathbf{b}^0, \mathbf{c}^0, \ \mathbf{x}^1, \ \mathbf{z}^1, \ \mathbf{fy}^1, \ \mathbf{b}^1, \mathbf{c}^1 \right) \in NT^1, \\ \qquad \vdots \\ \left(\mathbf{b}^{T-1}, \mathbf{c}^{T-1}, \ \mathbf{x}^T, \ \mathbf{z}^T, \ \mathbf{fy}^T, \ \mathbf{b}^T, \mathbf{c}^T \right) \in NT^T \end{array} \right\}. \tag{10.10}$$

To measure bank performance we use a variant of the directional distance function. Directional distance functions were developed by Chambers et al. (1998) as a functional representation of the production technology, similar to Luenberger's (1992, 1995) benefit function that was used to represent the consumer's choice problem. A single period directional distance function measures the simultaneous expansion in desirable outputs and contraction in undesirable outputs and inputs for the directional scaling vector $\mathbf{g} = \left(\mathbf{g}_{fy}, \mathbf{g}_b, \mathbf{g}_x \right)$. We extend the directional distance function to the dynamic network technology given by (10.10). Let $\Omega_k = \left(\mathbf{b}_k^{t-1}, \ \dot{\mathbf{c}}_k^{t-1}, \ \ddot{\mathbf{c}}_k^{t-1}, \ \mathbf{b}_k^t, \ \mathbf{x}_k^t, \ \mathbf{f}\dot{\mathbf{y}}_k^t, \ \dot{\mathbf{c}}_k^t, \ \mathbf{f}\ddot{\mathbf{y}}_k^t, \ \ddot{\mathbf{c}}_k^t, \ \forall t = 1, \ldots, T \right)$ represent the observed inputs, outputs, and carryovers for bank k in each production period. We define a weighted DN-directional distance function as

$$\begin{aligned} DN\vec{D}(\Omega_k; \mathbf{g}) = \underset{\beta^t, \ \mathbf{z}^t, \ \mathbf{c}^t}{\text{maximize}} & \left\{ \sum_{t=1}^{T} w^t \beta^t \right. \\ & \left| \begin{array}{l} \left(\mathbf{b}^0, \mathbf{c}^0, \ \mathbf{x}^1 - \beta^1 \mathbf{g}_x, \ \mathbf{z}^1, \ \mathbf{fy}^1 + \beta^1 \mathbf{g}_y, \ \mathbf{b}^1 - \beta^1 \mathbf{g}_b, \mathbf{c}^1 \right) \in NT^1, \\ \left(\mathbf{b}^1 - \beta^1 \mathbf{g}_b, \mathbf{c}^1, \ \mathbf{x}^2 - \beta^2 \mathbf{g}_x, \ \mathbf{z}^2, \ \mathbf{fy}^2 + \beta^2 \mathbf{g}_y, \ \mathbf{b}^2 - \beta^2 \mathbf{g}_b, \mathbf{c}^2 \right) \in NT^1, \\ \quad \vdots \\ \left(\mathbf{b}^{T-1} - \beta^{T-1} \mathbf{g}_b, \mathbf{c}^{T-1}, \ \mathbf{x}^T - \beta^T \mathbf{g}_x, \ \mathbf{z}^T, \ \mathbf{fy}^T + \beta^T \mathbf{g}_y, \ \mathbf{b}^T - \beta^T \mathbf{g}_b, \mathbf{c}^T \right) \in NT^T. \end{array} \right\} \end{aligned} \tag{10.11}$$

The weights (w^t) for each period are exogenously chosen. Following Nemoto and Goto (2003), De Mateo et al. (2006) and Fukuyama and Weber (2015a, b, 2016) one might choose the present value factors for the predetermined weights. That is, $w^t = (1 + R)^{t-1}$, where R is the producer's rate of time preference. Inefficient producers have $DN\vec{D}(\Omega_k; \mathbf{g}) > 0$ and in stage 1 of period t they can contract inputs

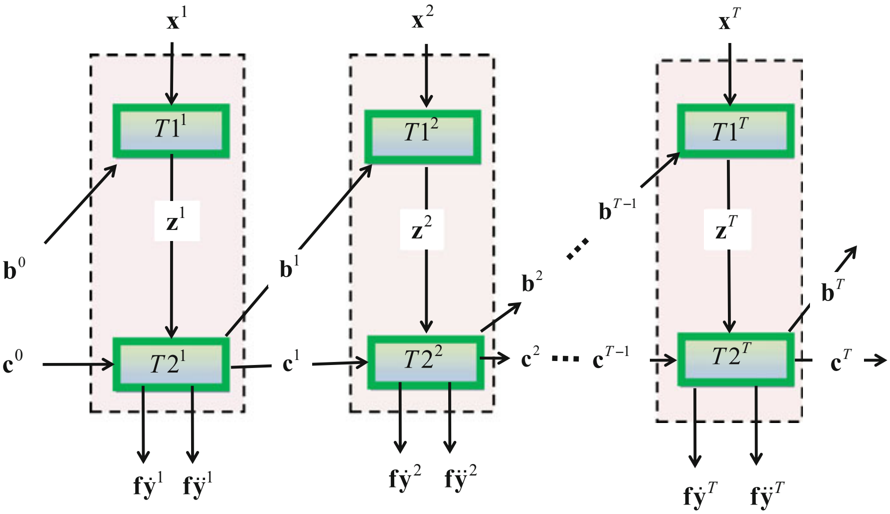


Fig. 10.3 Dynamic network structure of bank production. *Note*: We adapted the multi-period dynamic-network representations given in Fukuyama and Weber (2015b, 2016)

by $\beta^t \mathbf{g}_x$ while in stage 2 of the same period they can simultaneously expand desirable outputs by $\beta^t \mathbf{g}_y$ and contract undesirable outputs by $\beta^t \mathbf{g}_b$. The contraction in stage 2's undesirable outputs means that in the next period the amount of undesirable inputs that enter stage 1 will also decline. A producer who is efficient in a single period has $\beta^t = 0$ and a producer who is efficient in every period has $DN\vec{D}(\Omega_k; \mathbf{g}) = 0$. We note that in the initial period the lagged value of undesirable outputs $(\mathbf{b}^0)$ are fixed. Furthermore, while the producer chooses the amount of carryover assets $(\mathbf{c}^t)$ in each of the $t = 1, \ldots, T$ periods, the lagged value of carryover assets $(\mathbf{c}^0)$ in period $t = 1$ are taken as exogenous. Figure 10.3 depicts the multi-period dynamic-network structure for a bank.

10.4 DEA Implementation

We specify the dynamic network technology and estimate the performance of each DMU using DEA. To incorporate the idea that nonperforming loans from a previous period are an undesirable input to stage 1 of the current period we assume that nonperforming loans $\left(\mathbf{b}^{t-1}\right)$ and other inputs $(\mathbf{x}^t)$ satisfy joint weak input disposability (JWID). The condition of JWID is written as

$$\begin{aligned} \text{JWID}: \quad & \left(\mathbf{b}^{t-1},\ \mathbf{x}^t,\ \mathbf{z}^t\right) \in T1^t \subseteq \Re_+^{L+N+Q} \quad \text{and} \quad \varphi \geq 1 \\ \Rightarrow \quad & \left(\varphi\mathbf{b}^{t-1},\ \varphi\mathbf{x}^t,\ \mathbf{z}^t\right) \in T1^t \end{aligned} \tag{10.12}$$

which indicates that any proportional expansion of undesirable inputs $\mathbf{b}^{t-1}$ and current desirable inputs $\mathbf{x}^t$ can still feasibly produce a fixed level of intermediate products $\mathbf{z}^t$.

Our DEA technology also distinguishes between desirable outputs that are linked to undesirable byproducts and desirable outputs that are not linked to undesirable outputs. To make the distinction we assume that $\dot{M}$ of the desirable outputs are linked to undesirable outputs and that $\ddot{M}$ of the desirable outputs are not linked to undesirable outputs where $M = \dot{M} + \ddot{M}$. That is,

$$\mathbf{y}^t = \mathbf{fy}^t + \mathbf{c}^t = (\mathbf{f\dot{y}}^t + \dot{\mathbf{c}}^t, \mathbf{f\ddot{y}}^t + \ddot{\mathbf{c}}^t). \tag{10.13}$$

In addition to JWID we assume that stage 2 desirable outputs linked to undesirable outputs satisfy joint weak output disposability (JWOD). This condition is written as follows:

$$\begin{aligned} &\text{JWOD}: \ (\mathbf{z}^t,\ \mathbf{f\dot{y}}^t + \dot{\mathbf{c}}^t,\ \mathbf{f\ddot{y}}^t + \ddot{\mathbf{c}}^t,\ \mathbf{b}^t) \in T2^t \subseteq \Re_+^{Q+\dot{M}+\ddot{M}+L} \\ &\text{and } \ 0 \leq \theta^t \leq 1 \qquad \Rightarrow \ (\mathbf{z}^t,\ \theta^t(\mathbf{f\dot{y}}^t + \dot{\mathbf{c}}^t),\ \mathbf{f\ddot{y}}^t + \ddot{\mathbf{c}}^t,\ \theta\mathbf{b}^t) \in T2^t \end{aligned} \tag{10.14}$$

which means that it is feasible to produce proportionally less of the desirable and undesirable outputs with the same amount of input. Weak disposability of outputs is in contrast to the more typical assumption of strong disposability where it is possible to produce less of a single output. Weak disposability means there is an opportunity cost of producing fewer undesirable outputs–fewer desirable outputs must also be produced.

To fully specify the DEA technology we assume that there are $j = 1,\ldots, J$ banks observed in $t = 1, \ldots, T$ periods. Let $\boldsymbol{\lambda}^{1t} = \left(\lambda_1^{1t}, \ldots, \lambda_J^{1t}\right) \in \Re_+^J$ and $\boldsymbol{\lambda}^{2t} = \left(\lambda_1^{2t}, \ldots, \lambda_J^{2t}\right) \in \Re_+^J$ represent the intensity variable vectors that form convex combinations of observed inputs for the stage 1 and stage 2 technologies. Accounting for JWID, the stage 1 DEA technology in period t is

$$\begin{aligned} T1^t &= \left\{ \left(\mathbf{b}^{t-1}, \mathbf{x}^t,\ \mathbf{z}^t\right) \ \middle|\ \left(\mathbf{b}^{t-1},\ \mathbf{x}^t\right) \text{can produce } \mathbf{z}^t \right\} \\ &= \left\{ \left(\mathbf{b}^{t-1},\ \mathbf{x}^t,\ \mathbf{z}^t\right) \in \Re_+^{L+N+Q} \ \middle|\ \begin{array}{l} \mathbf{b}^{t-1} \geq \sum_{j=1}^J \varphi^t \mathbf{b}_j^{t-1} \lambda_j^{1t},\ \ \mathbf{x}^t \geq \sum_{j=1}^J \varphi^t \mathbf{x}_j^t \lambda_j^{1t}, \\ \mathbf{z}^t \leq \sum_{j=1}^J \mathbf{z}_j^t \lambda_j^{1t},\ \boldsymbol{\lambda}^{1t} \geq \mathbf{0},\ \ \varphi^t \geq 1 \end{array} \right\} \end{aligned} \tag{10.15}$$

where "**0**" indicates an appropriate dimensional zero vector. The existence of nonperforming loans from a preceding period requires the bank to raise a greater amount of equity capital and utilize more of the other inputs.

The stage 2 DEA technology for period t is denoted by

$$
\begin{aligned}
T2^t &= \left\{ (\mathbf{z}^t,\ \mathbf{f}\mathbf{y}^t + \mathbf{c}^t,\ \mathbf{b}^t) \in \Re_+^{Q+L+M} \middle|\ (\mathbf{c}^{t-1},\ \mathbf{z}^t)\,\text{can produce}\,(\mathbf{y}^t + \mathbf{c}^t,\ \mathbf{b}^t) \right\} \\
&= \left\{ \begin{pmatrix} \mathbf{z}^t,\ \mathbf{f}\dot{\mathbf{y}}^t + \dot{\mathbf{c}}^t,\ \mathbf{b}^t, \\ \mathbf{f}\ddot{\mathbf{y}}^t + \ddot{\mathbf{c}}^t \end{pmatrix} \in \Re_+^{Q+L+M} \middle| \begin{array}{l} \sum_{j=1}^{J} \mathbf{z}_j^t \lambda_j^t \le \mathbf{z}^{2t},\ \ \mathbf{b}^t \ge \sum_{j=1}^{J} \theta^t \mathbf{b}_j^t \lambda_j^{2t}, \\ \mathbf{f}\dot{\mathbf{y}}^t + \dot{\mathbf{c}}^t \le \sum_{j=1}^{J} \theta^t \dot{\mathbf{y}}_j^t \lambda_j^{2t}, \\ \mathbf{f}\ddot{\mathbf{y}}^t + \ddot{\mathbf{c}}^t \le \sum_{j=1}^{J} \ddot{\mathbf{y}}_j^t \lambda_j^{2t}, \\ \dot{\mathbf{c}}^{t-1} \ge \sum_{j=1}^{J} \dot{\mathbf{c}}_j^{t-1} \lambda_j^{2t},\ \ \ddot{\mathbf{c}}^{t-1} \ge \sum_{j=1}^{J} \ddot{\mathbf{c}}_j^{t-1} \lambda_j^{2t}, \\ \boldsymbol{\lambda}_j^{2t} \ge \mathbf{0},\ \ 0 \le \theta^t \le 1,\ \ \forall t = 1,\ldots,T \end{array} \right\}.
\end{aligned}
\tag{10.16}
$$

Combining (10.15) and (10.16), we obtain the DEA-based technology for t denoted as

$$
NT^t = \left\{ \begin{pmatrix} \mathbf{b}^{t-1},\ \mathbf{x}^t,\ \mathbf{c}^{t-1}, \\ \mathbf{f}\dot{\mathbf{y}}^t + \dot{\mathbf{c}}^t, \\ \mathbf{f}\ddot{\mathbf{y}}^t + \ddot{\mathbf{c}}^t,\ \mathbf{b}^t \end{pmatrix} \in \Re_+^{L+N+M+\dot{M}+\ddot{M}+L} \middle| \begin{array}{l} \mathbf{b}^{t-1} \ge \sum\limits_{j=1}^{J} \varphi^t \mathbf{b}_j^{t-1} \lambda_j^{1t},\ \ \mathbf{x}^t \ge \sum_{j=1}^{J} \varphi^t \mathbf{x}_j^t \lambda_j^{1t}, \\ \sum_{j=1}^{J} \mathbf{z}_j^t \left(\lambda_j^{1t} - \lambda_j^{2t} \right) \ge \mathbf{0},\ \ \mathbf{b}^t \ge \sum_{j=1}^{J} \theta^t \mathbf{b}_j^t \lambda_j^{2t}, \\ \mathbf{f}\dot{\mathbf{y}}^t + \dot{\mathbf{c}}^t \le \sum\limits_{j=1}^{J} \theta^t \dot{\mathbf{y}}_j^t \lambda_j^{2t}, \\ \mathbf{f}\ddot{\mathbf{y}}^t + \ddot{\mathbf{c}}^t \le \sum_{j=1}^{J} \ddot{\mathbf{y}}_j^t \lambda_j^{2t}, \\ \dot{\mathbf{c}}^{t-1} \ge \sum_{j=1}^{J} \dot{\mathbf{c}}_j^{t-1} \lambda_j^{2t},\ \ \ddot{\mathbf{c}}^{t-1} \ge \sum_{j=1}^{J} \ddot{\mathbf{c}}_j^{t-1} \lambda_j^{2t},\ \ \mathbf{z}^t \ge \mathbf{0}, \\ \boldsymbol{\lambda}^{1t} \ge \mathbf{0},\ \ \boldsymbol{\lambda}^{2t} \ge \mathbf{0},\ \ \varphi^t \ge 1,\ \ 0 \le \theta^t \le 1,\ \ \forall t = 1,\ldots,T \end{array} \right\}.
\tag{10.17}
$$

We use $\mathbf{b}^t \ge \sum_{j=1}^{J} \theta^t \mathbf{b}_j^t \lambda_j^{2t}$ rather than $\mathbf{b}^t = \sum_{j=1}^{J} \theta^t \mathbf{b}_j^t \lambda_j^{2t}$ in (10.16) and (10.17), but this treatment does not indicate that undesirable outputs are inputs due to the condition that $0 \le \theta^t \le 1$. In fact, Färe et al. (2016) also replaced the equality "$=$" by the inequality "$\ge$" in a static black-box setting where no distinction was made between desirable outputs linked with jointly produced undesirable outputs and desirable outputs not linked to undesirable outputs. They stated that such a treatment was consistent with treating $\mathbf{b}^t$ as undesirable outputs, rather than inputs.

In (10.17) two sets of constraints link the two stages of production. These constraints are for the intermediate outputs produced in stage 1 which are then used as an input to stage 2. The constraints are

$$\sum_{j=1}^{J} \mathbf{z}_j^t \lambda_j^{1t} \geq \mathbf{z} \quad \text{and} \quad \sum_{j=1}^{J} \mathbf{z}_j^t \lambda_j^{2t} \leq \mathbf{z} \quad \Leftrightarrow \quad \sum_{j=1}^{J} \mathbf{z}_j^t \left(\lambda_j^{1t} - \lambda_j^{2t} \right) \geq \mathbf{0}. \tag{10.18}$$

See Fukuyama and Weber (2010, 2014) and see also Chen et al. (2009b) and Chen et al. (2010). Equation (10.18) allows some of the intermediate outputs produced in stage 1 to be wasted in that not all of the intermediate outputs are needed to produce the final outputs in stage 2. Fukuyama and Weber (2015a) found that Japanese commercial banks produced more deposits in stage 1 than were needed to produce the portfolio of loans and securities investments in stage 2. An et al. (2015) also studied the relation between the degree of centralization and the internal resource waste for a two-stage network DEA problem. Fukuyama and Mirdehghan (2012) and Mirdehghan and Fukuyama (2016) examined (10.18) in a more general network DEA framework from a Pareto-Koopmans efficiency perspective.

The dynamic network directional distance function can be estimated using DEA by substituting the dynamic network DEA technology (10.17) into (10.11). Let $\mathbf{g} = \left(\mathbf{g}_x, \mathbf{g}_{\dot{y}}, \mathbf{g}_{\ddot{y}}, \mathbf{g}_b \right)$ be a predetermined directional vector for exogenous inputs, linked outputs, unlinked outputs and undesirable outputs and let w^t represent the pre-determined weights for each period. The T-period DN-directional technology distance function is estimated using DEA as

$$\begin{aligned}
& DN\vec{D}(\Omega_k;\mathbf{g}) \;=\; \underset{\beta^t,\ \dot{\mathbf{c}}^t,\ \ddot{\mathbf{c}}^t,\ \boldsymbol{\lambda}^{1t},\ \boldsymbol{\lambda}^{2t},\ \varphi^t,\ \theta^t}{\text{maximize}} \quad w^1\beta^1 + w^2\beta^2 + \ldots + w^T\beta^T \\
& \text{subject to:} \\
& \boxed{\text{bank } k \text{ in } t=1 \text{ and technology at } t=1} \\
& \quad \mathbf{b}_k^0 \geq \sum\nolimits_{j=1}^{J} \varphi^1 \mathbf{b}_j^0 \lambda_j^{1,1}, \quad \mathbf{x}_k^1 - \beta^1 \mathbf{g}_x \geq \sum\nolimits_{j=1}^{J} \varphi^1 \mathbf{x}_j^1 \lambda_j^{1,1}, \quad \sum\nolimits_{j=1}^{J} \mathbf{z}_j^1 \left(\lambda_j^{1,1} - \lambda_j^{2,1} \right) \geq \mathbf{0}, \\
& \quad \dot{\mathbf{c}}_k^0 \geq \sum\nolimits_{j=1}^{J} \dot{\mathbf{c}}_j^0 \lambda_j^{2,1}, \quad \ddot{\mathbf{c}}_k^0 \geq \sum\nolimits_{j=1}^{J} \ddot{\mathbf{c}}_j^0 \lambda_j^{2,1}, \quad \mathbf{b}_k^1 - \beta^1 \mathbf{g}_b \geq \sum_{j=1}^{J} \theta^1 \mathbf{b}_j^1 \lambda_j^{2,1}, \\
& \quad \mathbf{f}\dot{\mathbf{y}}_k^1 + \dot{\mathbf{c}}^1 + \beta^1 \mathbf{g}_{\dot{y}} \leq \sum\nolimits_{j=1}^{J} \theta^1 \dot{\mathbf{y}}_j^1 \lambda_j^{2,1}, \quad \mathbf{f}\ddot{\mathbf{y}}_k^1 + \ddot{\mathbf{c}}^1 + \beta^1 \mathbf{g}_{\ddot{y}} \leq \sum\nolimits_{j=1}^{J} \ddot{\mathbf{y}}_j^1 \lambda_j^{2,1}, \\
& \quad \beta^1 \geq 0, \quad \boldsymbol{\lambda}^{1,1} \geq \mathbf{0}, \quad \boldsymbol{\lambda}^{2,1} \geq \mathbf{0}, \quad \dot{\mathbf{c}}^1 \geq \mathbf{0}, \quad \ddot{\mathbf{c}}^1 \geq \mathbf{0}, \quad \varphi^1 \geq 1, \quad 0 \leq \theta^1 \leq 1
\end{aligned} \tag{10.19}$$

$$\boxed{\text{bank } k \text{ in } t=2,\ldots T \text{ and technology at } t=2,\ldots,T}$$
$$\mathbf{b}_k^{t-1}-\beta^{t-1}\mathbf{g_b}\geq\sum\nolimits_{j=1}^{J}\varphi^t\mathbf{b}_j^{t-1}\lambda_j^{1t},\ \mathbf{x}_k^t-\beta^t\mathbf{g}_x\geq\sum\nolimits_{j=1}^{J}\varphi^t\mathbf{x}_j^t\lambda_j^{1t},\ \sum\nolimits_{j=1}^{J}\mathbf{z}_j^t\left(\lambda_j^{1t}-\lambda_j^{2t}\right)\geq\mathbf{0},$$
$$\dot{\mathbf{c}}^{t-1}\geq\sum\nolimits_{j=1}^{J}\dot{\mathbf{c}}_j^{t-1}\lambda_j^{2,t},\ \ddot{\mathbf{c}}^{t-1}\geq\sum\nolimits_{j=1}^{J}\ddot{\mathbf{c}}_j^{t-1}\lambda_j^{2,t},\ \mathbf{b}_k^t-\beta^t\mathbf{g}_b\geq\sum\nolimits_{j=1}^{J}\theta^t\mathbf{b}_j^t\lambda_j^{2t},$$
$$\mathbf{f}\dot{\mathbf{y}}_k^t+\beta^t\mathbf{g}_{\dot{y}}+\dot{\mathbf{c}}^t\leq\sum\nolimits_{j=1}^{J}\theta^t\dot{\mathbf{y}}_j^t\lambda_j^{2t},\ \mathbf{f}\ddot{\mathbf{y}}_k^t+\beta^t\mathbf{g}_{\ddot{y}}+\ddot{\mathbf{c}}^t\leq\sum\nolimits_{j=1}^{J}\ddot{\mathbf{y}}_j^t\lambda_j^{2t},$$
$$\beta^t\geq 0,\ \boldsymbol{\lambda}^{1t}\geq\mathbf{0},\ \boldsymbol{\lambda}^{2t}\geq\mathbf{0},\ \dot{\mathbf{c}}^t\geq\mathbf{0},\ \ddot{\mathbf{c}}^t\geq\mathbf{0},\ \varphi^t\geq 1,\ 0\leq\theta^t\leq 1.$$

The T-period DN-directional technology distance function is a dynamic network version of the directional technology distance function due to Chambers et al. (1996).

Although (10.19) is a nonlinear program it can be transformed into a linear program by transforming the intensity variables using the Kuosmanen's (2005) procedure. Let $\gamma_j^{1t}=\varphi^t\lambda_j^{1t}$ and let $\mu_j^{1t}=(1-\varphi^t)\lambda_j^{1t}$ where $\mu_j^{1t}\quad(j=1,\ldots,J)$ are non-positive. Consequently, the stage 1 intensity variables can be written as $\lambda_j^{1t}=\gamma_j^{1t}+\mu_j^{1t}$. Similarly, let $\gamma_j^{2t}=\theta^t\lambda_j^{2t}$ and let $\mu_j^{2t}=(1-\theta^t)\lambda_j^{2t}$ where $\mu_j^{2t}\quad(j=1,\ldots,J)$ are non-negative. Thus, the stage 2 intensity variables can be written as $\lambda_j^{2t}=\gamma_j^{2t}+\mu_j^{2t}$. Note that γ_j^{1t} and γ_j^{2t} are non-negative. Substituting these transformed variables into (10.19) yields

$$\sum\nolimits_{t=1}^{T}w^t\beta^t[t]\quad=\underset{\beta^t,\ \dot{\mathbf{c}}^t,\ \ddot{\mathbf{c}}^t,\ \gamma_j^{1t},\ \mu_j^{1t},\ \gamma_j^{2t},\ \mu_j^{2t}}{\text{maximize}}\ w^1\beta^1+w^2\beta^2+\ldots+w^T\beta^T$$

subject to :

$$\boxed{\text{bank } k \text{ in } t=1 \text{ and technology in } t=1}$$
$$\begin{aligned}
&\mathbf{b}_k^0\geq\sum\nolimits_{j=1}^{J}\mathbf{b}_j^0\gamma_j^{1,1},\quad \mathbf{x}_k^1-\beta^1\mathbf{g}_x\geq\sum\nolimits_{j=1}^{J}\mathbf{x}_j^1\gamma_j^{1,1},\\
&\sum\nolimits_{j=1}^{J}\mathbf{z}_j^1\left(\left(\gamma_j^{1,1}+\mu_j^{1,1}\right)-\left(\gamma_j^{2,1}+\mu_j^{2,1}\right)\right)\geq\mathbf{0},\\
&\dot{\mathbf{c}}_k^0\geq\sum\nolimits_{j=1}^{J}\dot{\mathbf{c}}_j^0\left(\gamma_j^{2,1}+\mu_j^{2,1}\right),\quad \ddot{\mathbf{c}}_k^0\geq\sum\nolimits_{j=1}^{J}\ddot{\mathbf{c}}_j^0\left(\gamma_j^{2,1}+\mu_j^{2,1}\right),\\
&\mathbf{b}^1-\beta^1\mathbf{g}_b\geq\sum\nolimits_{j=1}^{J}\mathbf{b}_j^1\gamma_j^{2,1},\\
&\mathbf{f}\dot{\mathbf{y}}_k^1+\beta^1\mathbf{g}_{\dot{y}}+\dot{\mathbf{c}}^1\leq\sum\nolimits_{j=1}^{J}\dot{\mathbf{y}}_j^1\gamma_j^{2,1},\\
&\mathbf{f}\ddot{\mathbf{y}}_k^1+\beta^1\mathbf{g}_{\ddot{y}}+\ddot{\mathbf{c}}^1\leq\sum\nolimits_{j=1}^{J}\ddot{\mathbf{y}}_j^1\left(\gamma_j^{2,1}+\mu_j^{2,1}\right),\\
&\beta^1\geq 0,\quad \boldsymbol{\lambda}^{1,1}\geq\mathbf{0},\quad \boldsymbol{\lambda}^{2,1}\geq\mathbf{0},\quad \dot{\mathbf{c}}^1\geq\mathbf{0},\quad \ddot{\mathbf{c}}^1\geq\mathbf{0}\ ,\\
&\gamma_j^{1,1}\geq 0,\quad \mu_j^{1,1}\leq 0,\quad \gamma_j^{2,1}\geq 0,\quad \mu_j^{2,1}\geq 0\quad(j=1,\ldots,J)\\
&\gamma_j^{1,1}+\mu_j^{1,1}\geq 0\quad(j=1,\ldots,J),\ \ \gamma_j^{2,1}+\mu_j^{2,t}\geq 0\quad(j=1,\ldots,J),
\end{aligned}\tag{10.20}$$

$$
\begin{aligned}
&\boxed{\text{bank } k \text{ in } t=2,\ldots T \text{ and technology at } t=2,\ldots,T}\\
&\mathbf{b}_k^{t-1}-\beta^{t-1}\mathbf{g}_b \geq \sum\nolimits_{j=1}^{J}\mathbf{b}_j^{t-1}\gamma_j^{1t},\ \ \mathbf{x}_k^t-\beta^t\mathbf{g}_b \geq \sum\nolimits_{j=1}^{J}\mathbf{x}_j^t\gamma_j^{1t}, \quad ,\\
&\sum\nolimits_{j=1}^{J}\mathbf{z}_j^t\left(\left(\gamma_j^{1t}+\mu_j^{1t}\right)-\left(\gamma_j^{2t}+\mu_j^{2t}\right)\right)\geq \mathbf{0},\\
&\dot{\mathbf{c}}^{t-1}\geq \sum\nolimits_{j=1}^{J}\dot{\mathbf{c}}_j^{t-1}\left(\gamma_j^{2t}+\mu_j^{2t}\right),\ \ \ddot{\mathbf{c}}^{t-1}\geq \sum\nolimits_{j=1}^{J}\ddot{\mathbf{c}}_j^{t-1}\left(\gamma_j^{2t}+\mu_j^{2t}\right),\\
&\mathbf{b}^t-\beta^t\mathbf{g}_b \geq \sum\nolimits_{j=1}^{J}\mathbf{b}_j^t\gamma_j^{2t},\\
&\mathbf{f}\dot{\mathbf{y}}_k^t+\beta^t\mathbf{g}_{\dot{y}}+\dot{\mathbf{c}}^t \leq \sum\nolimits_{j=1}^{J}\dot{\mathbf{y}}_j^t\gamma_j^{2t},\\
&\mathbf{f}\ddot{\mathbf{y}}_k^t+\beta^t\mathbf{g}_{\ddot{y}}+\ddot{\mathbf{c}}^t \leq \sum\nolimits_{j=1}^{J}\ddot{\mathbf{y}}_j^t\left(\gamma_j^{2t}+\mu_j^{2t}\right),\\
&\beta^t\geq 0,\ \ \boldsymbol{\lambda}^{1t}\geq \mathbf{0},\ \ \boldsymbol{\lambda}^{2t}\geq \mathbf{0},\ \ \dot{\mathbf{c}}^t\geq \mathbf{0},\ \ \ddot{\mathbf{c}}^t\geq \mathbf{0}\ ,\\
&\gamma_j^{1t}\geq 0,\ \ \mu_j^{1t}\leq 0,\ \ \gamma_j^{2t}\geq 0,\ \ \mu_j^{2t}\geq 0 \quad (j=1,\ldots,J)\\
&\gamma_j^{1t}+\mu_j^{1t}\geq 0\ \ (j=1,\ldots,J),\quad \gamma_j^{2t}+\mu_j^{2t}\geq 0\ \ (j=1,\ldots,J),\ \ t=2,\ldots,T.
\end{aligned}
$$

where carryover assets, $\dot{\mathbf{c}}^t$, $\ddot{\mathbf{c}}^t$ $(\forall t = 1, \ldots, T)$ are choice variables and the optimized values $\beta^t[t]$ represent bank inefficiency in period t with $DN\vec{D}(\Omega_k;\mathbf{g}) = \sum\nolimits_{t=1}^{T} w^t\beta^t[t]$.

The dynamic-network performance problem (10.20) selects the maximal value of the weighted sum of scaling factors related to exogenous inputs, the undesirable output of nonperforming loans and the desirable outputs of loans and securities investments. The model (10.20) differs from those of Fukuyama and Weber (2013, 2015a, b) by incorporating joint weak input-disposability (JWID) given by (10.12) and joint weak output-disposability (JWOD) given by (10.14). The multi-period dynamic-network directional distance function $DN\vec{D}(\Omega_k;\mathbf{g})$ extends the static black-box directional technology distance function due to Chambers et al. (1998). The optimal values of the intermediate outputs, $\mathbf{z}^t$, can be calculated using the optimal intensity variables with $\sum\nolimits_{j=1}^{J}\mathbf{z}_j^t\left(\gamma_j^{1t}+\mu_j^{1t}\right)$ providing an upper bound estimate for $\mathbf{z}^t$ and $\sum\nolimits_{j=1}^{J}\mathbf{z}_j^t\left(\gamma_j^{2t}+\mu_j^{2t}\right)$ providing a lower bound estimate of $\mathbf{z}^t$. Values of $DN\vec{D}_k = DN\vec{D}(\Omega_k;\mathbf{g}) = 0$ indicate that DMU k is efficient in every period with no ability to simultaneously expand final outputs and contract undesirable outputs given the DEA technology. When $DN\vec{D}_k > 0$, DMU k is inefficient with larger values indicating greater inefficiency.

The objective function of (10.20) is a weighted bank performance score equal to the sum of the product of the weights (w^t) and the period t directional technology distance functions $\beta^t[t]$ over the $t = 1, \ldots, T$ periods. For the estimation of various productivity indicators we need to calculate cross-period directional technology distance functions $\beta^{t+1}[t]$ and $\beta^t[t+1]$, $t = 1, \ldots, T-1$, where $\beta^{t+1}[t]$ measures the distance of the observed banks inputs and outputs in period t relative to the technology in period $t+1$ and $\beta^t[t+1]$ measures the observed bank's inputs and outputs in period $t+1$ relative to the technology in period t. We estimate the cross-period DN-directional technology distance functions by building on Pastor, Asmild, and Lovell's (2011) biennial Malmquist index and Färe et al. (2011) dynamic Malmquist index.

Using the cross-period estimates, the DN-Luenberger productivity indicator $\left(DNL^{t,t+1}\right)$ is obtained as

$$DNL^{t,t+1} = \frac{1}{2}\left[\left(w^t\beta^t[t] - w^t\beta^t[t+1]\right) + \left(w^{t+1}\beta^{t+1}[t] - w^{t+1}\beta^{t+1}[t+1]\right)\right],\\ t = 1, \ldots, T-1. \tag{10.21}$$

A DMU experiences productivity growth (decline) between periods t and $t+1$ if $DNL^{t,t+1}$ is positive (negative). The DN-Luenberger productivity indicator extends Chambers' (2002) static Luenberger productivity indicator for a dynamic network technology. The DN-Luenberger productivity indicator can also be thought of as an additive version of the static Malmquist productivity index of Färe et al. (1994).

To estimate $\beta^t[t+1]$ we solve the following optimization problem:

$$\begin{aligned}
&\sum\nolimits_{t=1}^{T-1} w^t\beta^t[t+1] \quad = \underset{\beta^t,\ \dot{\mathbf{c}}^t,\ \ddot{\mathbf{c}}^t,\ \gamma_j^{1t},\ \mu_j^{1t},\ \gamma_j^{2t},\ \mu_j^{2t}}{\text{maximize}} \quad \sum\nolimits_{t=1}^{T-1} w^t\beta^t\\
&\text{subject to :}\\
&\boxed{\text{bank } k \text{ in } t=2 \text{ and technology in } t=1}\\
&\quad \mathbf{b}_k^1 \geq \sum\nolimits_{j=1}^{J} \mathbf{b}_j^0\gamma_j^{1,1}, \quad \mathbf{x}_k^2 - \beta^1\mathbf{g}_x \geq \sum\nolimits_{j=1}^{J} \mathbf{x}_j^1\gamma_j^{1,1},\\
&\quad \sum\nolimits_{j=1}^{J} \mathbf{z}_j^1\left(\left(\gamma_j^{1,1}+\mu_j^{1,1}\right) - \left(\gamma_j^{2,1}+\mu_j^{2,1}\right)\right) \geq \mathbf{0}, \ \dot{\mathbf{c}}_k^1 \geq \sum\nolimits_{j=1}^{J} \dot{\mathbf{c}}_j^0\left(\gamma_j^{2,1}+\mu_j^{2,1}\right),\\
&\quad \ddot{\mathbf{c}}_k^1 \geq \sum\nolimits_{j=1}^{J} \ddot{\mathbf{c}}_j^0\left(\gamma_j^{2,1}+\mu_j^{2,1}\right), \quad \mathbf{b}^2 - \beta^1\mathbf{g}_b \geq \sum\nolimits_{j=1}^{J} \mathbf{b}_j^1\gamma_j^{2,1},\\
&\quad \mathbf{f}\dot{\mathbf{y}}_k^2 + \beta^1\mathbf{g}_{\dot{y}} + \dot{\mathbf{c}}^2 + \ \dot{\mathbf{s}}^2 = \sum\nolimits_{j=1}^{J} \dot{\mathbf{y}}_j^1\gamma_j^{2,1},\\
&\quad \mathbf{f}\ddot{\mathbf{y}}_k^2 + \beta^1\mathbf{g}_{\ddot{y}} + \ddot{\mathbf{c}}^2 + \ \ddot{\mathbf{s}}^2 = \sum\nolimits_{j=1}^{J} \ddot{\mathbf{y}}_j^1\left(\gamma_j^{2,1}+\mu_j^{2,1}\right),\\
&\quad \beta^1: \ \text{free}, \quad \boldsymbol{\lambda}^{1,1} \geq \mathbf{0}, \quad \boldsymbol{\lambda}^{2,1} \geq \mathbf{0}, \quad \dot{\mathbf{c}}^2 \geq \mathbf{0}, \quad \ddot{\mathbf{c}}^2 \geq \mathbf{0}\ , \quad \dot{\mathbf{s}}^2: \ \text{free}, \quad \ddot{\mathbf{s}}^2: \ \text{free},\\
&\quad \gamma_j^{1,1} \geq 0, \quad \mu_j^{1,1} \leq 0, \quad \gamma_j^{2,1} \geq 0, \quad \mu_j^{2,1} \geq 0 \quad (j=1,\ldots,J)\\
&\quad \gamma_j^{1,1}+\mu_j^{1,1} \geq 0 \quad (j=1,\ldots,J), \quad \gamma_j^{2,1}+\mu_j^{2,1} \geq 0 \quad (j=1,\ldots,J)
\end{aligned} \tag{10.22}$$

$$
\boxed{\text{bank } k \text{ in } t=3,\ldots T \text{ and technology at } t=2,\ldots,T-1}
$$

$$
\begin{aligned}
&\mathbf{b}_k^t-\beta^{t-1}\mathbf{g_b}\geq\sum\nolimits_{j=1}^{J}\mathbf{b}_j^{t-1}\gamma_j^{1t},\ \ \mathbf{x}_k^{t+1}-\beta^t\mathbf{g}_x\geq\sum\nolimits_{j=1}^{J}\mathbf{x}_j^t\gamma_j^{1t},\\
&\sum\nolimits_{j=1}^{J}\mathbf{z}_j^t\Big(\Big(\gamma_j^{1t}+\mu_j^{1t}\Big)-\Big(\gamma_j^{2t}+\mu_j^{2t}\Big)\Big)\geq\mathbf{0},\\
&\dot{\mathbf{c}}^t\geq\sum\nolimits_{j=1}^{J}\dot{\mathbf{c}}_j^{t-1}\Big(\gamma_j^{2t}+\mu_j^{2t}\Big),\ \ \ddot{\mathbf{c}}^t\geq\sum\nolimits_{j=1}^{J}\ddot{\mathbf{c}}_j^{t-1}\Big(\gamma_j^{2t}+\mu_j^{2t}\Big),\\
&\mathbf{b}_k^{t+1}-\beta^t\mathbf{g}_b\geq\sum\nolimits_{j=1}^{J}\mathbf{b}_j^t\gamma_j^{2t},\\
&\mathbf{f}\dot{\mathbf{y}}_k^{t+1}+\beta^t\mathbf{g}_{\dot{y}}+\dot{\mathbf{c}}^{t+1}+\ \dot{\mathbf{s}}^{t+1}=\sum\nolimits_{j=1}^{J}\dot{\mathbf{y}}_j^t\gamma_j^{2t},\\
&\mathbf{f}\ddot{\mathbf{y}}_k^{t+1}+\beta^t\mathbf{g}_{\ddot{y}}+\ddot{\mathbf{c}}^{t+1}+\ \ddot{\mathbf{s}}^{t+1}=\sum\nolimits_{j=1}^{J}\ddot{\mathbf{y}}_j^t\left(\gamma_j^{2t}+\mu_j^{2t}\right),\\
&\beta^t:\ \text{free in sign},\ \ \dot{\mathbf{c}}^t\geq\mathbf{0},\ \ \ddot{\mathbf{c}}^t\geq\mathbf{0}\ ,\\
&\dot{\mathbf{s}}^{t+1}:\ \text{free in sign},\ \ \ddot{\mathbf{s}}^{t+1}:\ \text{free in sign},\\
&\gamma_j^{1t}\geq 0,\ \ \mu_j^{1t}\leq 0,\ \ \gamma_j^{2t}\geq 0,\ \ \mu_j^{2t}\geq 0\ \ (j=1,\ldots,J),\ \ t=2,\ldots,T-1\\
&\gamma_j^{1t}+\mu_j^{1t}\geq 0\ \ (j=1,\ldots,J),\ \ \gamma_j^{2t}+\mu_j^{2t}\geq 0\ \ (j=1,\ldots,J),\ \ t=2,\ldots,T-1.
\end{aligned}
$$

The cross-period distance functions, $\beta^{t+1}[t],\ \ t=1,2,\ldots,T-1$, measure how far a bank's observed inputs and outputs in period t are from the period $t+1$ production frontier. These cross-period distance functions are found by solving the linear programming problem

$$
\begin{aligned}
&\sum\nolimits_{t=1}^{T-1}w^{t+1}\beta^{t+1}[t]=\max_{\beta^t,\ \dot{\mathbf{c}}^t,\ \ddot{\mathbf{c}}^t,\ \gamma_j^{1t},\ \mu_j^{1t},\ \gamma_j^{2t},\ \mu_j^{2t}}\ \sum\nolimits_{t=1}^{T-1}w^{t+1}\beta^{t+1}\\
&\text{subject to}:\\
&\boxed{\text{bank } k \text{ in } t=1 \text{ and technology in } t=2}\\
&\quad\mathbf{b}_k^0\geq\sum\nolimits_{j=1}^{J}\mathbf{b}_j^1\gamma_j^{1,2},\quad \mathbf{x}_k^1-\beta^2\mathbf{g}_x\geq\sum\nolimits_{j=1}^{J}\mathbf{x}_j^2\gamma_j^{1,2},\\
&\quad\sum\nolimits_{j=1}^{J}\mathbf{z}_j^2\Big(\Big(\gamma_j^{1,2}+\mu_j^{1,2}\Big)-\Big(\gamma_j^{2,2}+\mu_j^{2,2}\Big)\Big)\geq\mathbf{0},\\
&\quad\dot{\mathbf{c}}_k^0\geq\sum\nolimits_{j=1}^{J}\dot{\mathbf{c}}_j^1\Big(\gamma_j^{2,2}+\mu_j^{2,2}\Big),\quad \ddot{\mathbf{c}}_k^0\geq\sum\nolimits_{j=1}^{J}\ddot{\mathbf{c}}_j^1\Big(\gamma_j^{2,2}+\mu_j^{2,2}\Big),\\
&\quad\mathbf{b}_k^1-\beta^2\mathbf{g}_b\geq\sum\nolimits_{j=1}^{J}\mathbf{b}_j^1\gamma_j^{2,2},\\
&\quad\mathbf{f}\dot{\mathbf{y}}_k^1+\beta^2\mathbf{g}_{\dot{y}}+\dot{\mathbf{c}}^1+\dot{\mathbf{s}}^1=\sum\nolimits_{j=1}^{J}\dot{\mathbf{y}}_j^2\gamma_j^{2,2},\\
&\quad\mathbf{f}\ddot{\mathbf{y}}_k^1+\beta^2\mathbf{g}_{\ddot{f}}+\ddot{\mathbf{c}}^1+\ddot{\mathbf{s}}^1=\sum\nolimits_{j=1}^{J}\ddot{\mathbf{y}}_j^2\left(\gamma_j^{2,2}+\mu_j^{2,2}\right),\\
&\quad\beta^2:\ \text{free in sign},\quad \dot{\mathbf{c}}^1\geq\mathbf{0},\quad \ddot{\mathbf{c}}^1\geq\mathbf{0},\\
&\quad\dot{\mathbf{s}}^1:\ \ \text{free in sign},\quad \ddot{\mathbf{s}}^1:\ \ \text{free in sign},\\
&\quad\gamma_j^{1,2}\geq 0,\quad \mu_j^{1,2}\leq 0,\quad \gamma_j^{2,2}\geq 0,\quad \mu_j^{2,2}\geq 0\quad (j=1,\ldots,J)\\
&\quad\gamma_j^{1,2}+\mu_j^{1,2}\geq 0\quad (j=1,\ldots,J),\quad \gamma_j^{2,2}+\mu_j^{2,2}\geq 0\quad (j=1,\ldots,J)
\end{aligned}
\tag{10.23}
$$

$$
\begin{array}{l}
\boxed{\text{bank } k \text{ in } t=2,\ldots T-1 \text{ and technology at } t=3,\ldots,T} \\
\mathbf{b}_k^{t-1}-\beta^t\mathbf{g}_b \geq \sum_{j=1}^{J}\mathbf{b}_j^t\gamma_j^{1,t+1}, \quad \mathbf{x}_k^t-\beta^{t+1}\mathbf{g}_x \geq \sum_{j=1}^{J}\mathbf{x}_j^{t+1}\gamma_j^{1,t+1}, \\
\sum_{j=1}^{J}\mathbf{z}_j^{t+1}\left(\left(\gamma_j^{1,t+1}+\mu_j^{1,t+1}\right)-\left(\gamma_j^{2,t+1}+\mu_j^{2,t+1}\right)\right) \geq \mathbf{0}, \\
\dot{\mathbf{c}}^{t-1} \geq \sum_{j=1}^{J}\dot{\mathbf{c}}_j^t\left(\gamma_j^{2,t+1}+\mu_j^{2,t+1}\right), \ \ddot{\mathbf{c}}^{t-1} \geq \sum_{j=1}^{J}\ddot{\mathbf{c}}_j^t\left(\gamma_j^{2,t+1}+\mu_j^{2,t+1}\right), \\
\mathbf{b}_k^t-\beta^{t+1}\mathbf{g}_b \geq \sum_{j=1}^{J}\mathbf{b}_j^t\gamma_j^{2t}, \\
\mathbf{f}\dot{\mathbf{y}}_k^t+\beta^{t+1}\mathbf{g}_{\dot{y}}+\dot{\mathbf{c}}^t+\ \dot{\mathbf{s}}^t=\sum_{j=1}^{J}\dot{\mathbf{y}}_j^{t+1}\gamma_j^{2,t+1}, \\
\mathbf{f}\ddot{\mathbf{y}}_k^t+\beta^{t+1}\mathbf{g}_{\ddot{y}}+\ddot{\mathbf{c}}^t+\ \ddot{\mathbf{s}}^t=\sum_{j=1}^{J}\ddot{\mathbf{y}}_j^{t+1}\left(\gamma_j^{2,t+1}+\mu_j^{2,t+1}\right), \\
\beta^{t+1}: \text{ free in sign}, \quad \dot{\mathbf{c}}^t \geq \mathbf{0}, \quad \ddot{\mathbf{c}}^t \geq \mathbf{0}\ , \\
\dot{\mathbf{s}}^t: \text{ free in sign}, \quad \ddot{\mathbf{s}}^t: \text{ free in sign}, \\
\gamma_j^{1t} \geq 0, \ \ \mu_j^{1t} \leq 0, \ \ \gamma_j^{2t} \geq 0, \ \ \mu_j^{2t} \geq 0 \quad (j=1,\ldots,J), \ \ t=2,\ldots,T-1 \\
\gamma_j^{1,t+1}+\mu_j^{1,t+1} \geq 0 \ \ (j=1,\ldots,J), \ \ \gamma_j^{2,t+1}+\mu_j^{2,t+1} \geq 0 \ \ (j=1,\ldots,J), \ \ t=2,\ldots,T-1.
\end{array}
$$

After solving the linear programming problems (10.20), (10.22), and (10.23), we can obtain the DN-Luenberger productivity indicator, $DNL^{t,t+1}$, given in (10.21). This productivity indicator can be decomposed into a dynamic-network efficiency change indicator, $DNEC^{t,t+1}$, and a dynamic-network technical change indicator, $DNTC^{t,t+1}$:

$$
\begin{aligned}
DNL^{t,t+1} = & \underbrace{w^t\beta^t[t]-w^{t+1}\beta^{t+1}[t+1]}_{DNEC^{t,t+1}} \\
& + \underbrace{\frac{1}{2}\left[\left\{w^{t+1}\beta^{t+1}[t]-w^t\beta^t[t]\right\}+\left\{w^{t+1}\beta^{t+1}[t+1]-w^t\beta^t[t+1]\right\}\right]}_{DNTC^{t,t+1}},
\end{aligned}
\tag{10.24}
$$

A bank exhibits an efficiency gain (loss) if $DNEC^{t,t+1}$ is positive (negative). Similarly, a bank exhibits technological progress (regress) if $DNTC^{t,t+1}$ is positive (negative).

10.5 A Choice of Variables and Regulatory Constraints

10.5.1 Variable Selection: An Example

In this sub-section, we describe the bank inputs and outputs that were used by Fukuyama and Weber (2015a) in their dynamic network model. In their basic model, banks transform labor (x_1), physical capital (x_2) and financial equity capital (x_3) to produce deposits (z_1) and other raised funds (z_2) in stage 1. Then, in stage 2, banks use the intermediate products of stage 1 as inputs in producing loans ($f\dot{y}_1$) and securities investments ($f\ddot{y}_2$) as well as carryover assets ($\mathbf{c}=(\dot{c}_1, \ \ddot{c}_2)$) and an undesirable by-product of nonperforming loans (b_1). Carryover assets are divided into carryover assets that come from loans ($\dot{c}_1$) and carryover assets that come from securities ($\ddot{c}_2$). The total carryover assets ($\dot{c}_1 + \ddot{c}_2$) are derived as:

$$\begin{aligned} \dot{c}_1 + \ddot{c}_2 = \quad & \text{Assets} - \text{Required Reserves} - \text{Physical capital}(x_2) \\ & -\text{Performing loans}(f\dot{y}_1) \quad - \text{Securities investments } (f\ddot{y}_2) \\ & -\text{Nonperforming loans } (b_1). \end{aligned} \tag{10.25}$$

In their study, all carryover assets correspond with securities, i.e., $c_1 = 0$ and $c_2 > 0$. Since all carryovers are from securities investments, the network technology (10.17) exhibits null-jointness, because a proportional reduction in linked desirable and undesirable outputs is technologically feasible given the condition of JWOD. For the calculation of required reserves, see Fukuyama and Weber (2013, 2015a, b).

10.5.2 Imposing Bank Regulatory Constraint

Banks face a variety of financial regulations which constrain their ability to reduce certain kinds of inputs such as financial equity capital or to expand deposits and other raised funds without the use of extra financial equity capital, even if the technology would allow them to do so. In addition, financial regulations also constrain the ability of banks to make certain kinds of risky loans without additional financial equity capital. Fukuyama and Weber (2015a) incorporated these financial regulatory constraints into the DEA technology. Since Japanese domestic banks are required to have qualifying equity capital as a percent of risk-weighted assets (wA_k^t) greater than 4 %, bank k's capital adequacy ratio is expressed as

$$CAR_k^t = \frac{Tier\ \ 1_k^t + Tier\ \ 2_k^t - deduct_k^t}{wA_k^t} \geq 0.04 \tag{10.26}$$

where $Tier\ \ 1_k^t$ is core tier 1 bank capital (primarily shareholders' equity), $Tier\ \ 2_k^t$ is supplementary bank capital and $deduct_k^t$ is a deduction that includes goodwill. Domestically operating banks are required to have a capital adequacy ratio of at least 4 %, whereas the international banks need to have a capital adequacy ratio of at least 8 % (see for example Montgomery and Shimizutani 2009).

Weber and Devaney (1999) and Färe et al. (2004) were the early DEA studies which incorporated risk-based capital constraints in bank efficiency measurement. Let W_{ka}^t be the risk-weight of asset a and let A_{ka}^t be the value of asset a. For assets $a = 1, \ldots, Z$ the weighted sum $wA_k^t = \sum_{a=1}^{Z} W_{ka}^t A_{ka}^t$ represents risk-weighted assets.

The dynamic-network model of Fukuyama and Weber (2015a) assumed that equity capital equals the sum of *Tier* 1 and *Tier* 2 capital, less deductions. Their data source did not report the risk-weights for loans and securities although it did report total risk-weighted assets (wA_k^t). Therefore, they imputed the risk-weights for the two outputs of loans and securities investments from total risk-weighted assets and by using the regulatory risk-weights for various classes of assets. Their imputation procedure is as follows. A bank's total securities consist of central government bonds (Gov_k^t), local and municipal bonds ($Local_k^t$), corporate bonds ($Corp_k^t$), and other securities ($otherSec_k^t$), and securities have risk weights between zero and one with various government bonds having lower risk weights than corporate bonds and other securities. Following Fukuyama and Weber (2015a) the risk-weight for total securities can be computed as $W_{2k}^t = \frac{0 \times Gov_k^t + 0.2 \times Local_k^t + 0.75 \times Corp_k^t + 1 \times otherSec_k^t}{fy_{2k}^t}$. Fukuyama and Weber (2015a) also assumed that cash representing carryover assets has a risk weight of 0 and physical capital has a risk weight of 1; and other assets ($otherA_k^t$) have a risk-weight of 1. Since $wA_k^t = 0 \times Cash_k^t + W_{1k}^t fy_{1k}^t + W_{2k}^t fy_{2k}^t + 1 \times x_{2k}^t + 1 \times otherA_k^t$, the risk-weight for loans is computed as $W_{1k}^t = \frac{wA_k^t - W_{2k}^t \times fy_{2k}^t - 1 \times x_{2k}^t - 1 \times otherA_k^t}{fy_{1k}^t}$. Therefore, the capital adequacy restriction given by (10.26) can be written as $\frac{x_{3k}^t}{W_{1k}^t \times fy_{1k}^t + W_{2k}^t \times fy_{2k}^t + 1 \times x_{2k}^t + 1 \times otherA_k^t} \geq 0.04$ which can be rearranged to yield $\frac{x_{3k}^t}{0.04} \geq \left(W_{1k}^t \times fy_{1k}^t + W_{2k}^t \times fy_{2k}^t + 1 \times x_{2k}^t + 1 \times otherA_k^t \right)$. Thus, taking inefficiency into consideration yields the following regulatory inequality constraint:

$$\frac{x_{3k}^t}{0.04} \geq W_{1k}^t \times fy_{1k}^t + W_{2k}^t \times fy_{2k}^t + x_{2k}^t + otherA_k^t + \beta^t \left(W_{1k}^t \times g_{y1} + W_{2k}^t \times g_{y2} - g_{x2} + \frac{g_{x3}}{.04} \right). \tag{10.27}$$

Therefore, the financial regulatory constraints can be expressed as

$$F_k^t \geq \beta^t G_k^t \quad (t = 1, \ldots, T). \tag{10.28}$$

where $F_k^t = \frac{x_{3k}^t}{0.04} - W_{1k}^t \times fy_{1k}^t - W_{2k}^t \times fy_{2k}^t - x_{2k}^t - otherA_k^t$ and $G_k^t = W_{1k}^t \times g_{y1} + W_{2k}^t \times g_{y2} - g_{x2} + \frac{g_{x3}}{0.04}$. Therefore, the financial regulatory constraint can be imposed by adding (10.28) to (10.19). A consequence of financial regulatory constraints is that measures of inefficiency–the ability to expand desirable outputs and contract inputs–is less than what might be achieved given the technology without the financial regulatory constraint.

Note that the dual multiplier representation of (10.19) with the financial regulatory constraint (10.28) can be developed similar to the method of Fukuyama and Weber (2015a).

10.6 A Summary

Specifying an appropriate technology and measuring financial institution performance has been a fertile area among operations researchers in the past 30 years. Much of the early work relied on a black box technology where inputs entered and outputs emerged from the black box and the performance of a particular financial institution was measured relative to the best-practice producer in a single period. This research was extended to network models that allowed various production divisions within a financial institution to contribute to the production of final outputs. One of the common network models assumed that banks used various exogenous inputs in stage 1 to produce intermediate outputs of deposits and then used those deposits as an input in stage 2 to generate a portfolio of interest bearing assets such as loans and securities investments. These network models were extended to account for the fact that the lending process generates a jointly produced undesirable output in the form of delinquent or nonperforming loans. Furthermore, nonperforming loans generated in one period constrain bank production possibilities in future periods. In addition, instead of immediately making loans as deposits are generated banks can instead choose to carryover some of their deposits if they expect enhanced future production possibilities. Dynamic models extended the black box technology by allowing inter temporal dependence between the input and output decisions of one period on the production possibilities of subsequent periods.

In this chapter, the dynamic network bank technology and performance measures developed by Fukuyama and Weber (2013, 2015a, b) were studied and extended accounting for weak disposability between desirable and undesirable inputs and accounting for weak disposability between desirable and undesirable outputs. Static black box efficiency measures tend to be biased because they ignore inter-temporal dependencies among inputs and outputs. The performance measures that were developed in this chapter help reduce the bias in static black box efficiency measures by comparing observed bank input and output decisions relative to a dynamic best-practice technology that accounts for the effects of input and output decisions of one period on the ability of banks to produce in future periods.

References

Akther, S., Fukuyama, H., & Weber, W. L. (2013). Estimating two-stage network slacks-based inefficiency: An application to Bangladesh banking. *Omega, 41*(1), 88–96.

Altunbas, Y., Liu, M., Molyneux, P., & Seth, R. (2000). Efficiency and risk in Japanese banking. *Journal of Banking and Finance, 58*, 826–839.

An, Q., Yan, H., Wu, J., & Liang, L. (2015). Internal resource waste and centralization degree in two-stage systems: An efficiency analysis. *Omega*. doi:10.1016/j.omega.2015.07.009

Berger, A. N., & Humphrey, D. B. (1997). Efficiency of financial institutions: International survey and directions for future research. *European Journal of Operations Research, 98*, 175–212.

Bogetoft, P., Färe, R., Grosskopf, S., Hayes, K., & Taylor, L. (2009). Dynamic network DEA: An illustration. *Journal of the Operations Research Society of Japan, 52*(2), 147–162.

Chambers, R. G. (2002). Exact nonradial input, ouput, and productivity measurement. *Economic Theory, 20*, 751–765.

Chambers, R. G., Chung, Y., & Färe, R. (1998). Profit, directional distance functions and Nerlovian efficiency. *Journal of Optimization Theory and Applications, 98*(2), 351–364.

Charnes, A., Cooper, W. W., & Rhodes, E. (1978). Measuring the efficiency of decision making units. *European Journal of Operational Research, 2*, 429–444.

Chen, Y., Cook, W. D., Li, N., & Zhu, J. (2009a). Additive efficiency decomposition in two-stage DEA. *European Journal of Operational Research, 196*, 1170–1176.

Chen, Y., Cook, W. D., & Zhu, J. (2010). Deriving the DEA frontier for two-stage processes. *European Journal of Operational Research, 202*, 138–142.

Chen, Y., Liang, L., & Zhu, J. (2009b). Equivalence in two-stage DEA approaches. *European Journal of Operational Research, 193*(2), 600–604.

De Mateo, F., Coelli, T., & O'Donnell, C. (2006). Optimal paths and cost of adjustment in dynamic DEA models: With application to Chilean department stores. *Annals of Operations Research, 145*, 211–227.

Drake, L., Hall, M. J. B., & Simper, R. (2009). Bank modelling methodologies: A comparative non-parametric analysis of efficiency in the Japanese banking sector. *International Journal of Financial Markets, Institutions and Money, 19*, 1–15.

Drake, L., & Hall, M. J. B. (2003). Efficiency in Japanese banking: An empirical analysis. *Journal of Banking & Finance, 27*, 891–917.

Epure, M., & Lafuente, E. (2015). Monitoring bank performance in the presence of risk. *Journal of Productivity Analysis, 44*(3), 265–281.

Fallah-Fini, S., Triantis, K., & Johnson, A. L. (2014). Reviewing the literature on non-parametric dynamic efficiency measurement: State-of-the-art. *Journal of Productivity Analysis, 41*, 51–67.

Färe, R., & Grosskopf, S. (1996). *Intertemporal production frontiers: With dynamic DEA*. Boston, MA: Kluwer Academic.

Färe, R., Grosskopf, S., Lindgren, B., & Roos, P. (1992). Productivity changes in Swedish pharmacies 1980–1989: A non-parametric Malmquist approach. *Journal of Productivity Analysis, 3*, 85–101.

Färe, R., Grosskopf, S., Norris, M., & Zhang, Z. (1994). Productivity growth, technical progress, and efficiency change in industrialized countries. *American Economic Review, 84*(1), 66–83.

Färe, R., Grosskopf, S., & Margaritis, D. (2011). Malmquist productivity indexes and DEA. In W. W. Cooper, L. Seiford, & J. Zhu (Eds.), *Handbook on data envelopment analysis*. New York: Springer.

Färe, R., Grosskopf, S., & Pasurka, C. (2016). Technical change and pollution abatement costs. *European Journal of Operational Research, 248*(2), 715–724.

Färe, R., Grosskopf, S., & Weber, W. L. (2004). The effect of risk-based capital standards on profit efficiency in banking. *Applied Economics, 36*, 1731–1743.

Färe, R., & Primont, D. (1995). *Multi-output and duality: Theory and application*. Boston: Kluwer-Nijhoff Publishing.

Farrell, M. J. (1957). The measurement of production efficiency. *Journal of the Royal Statistical Society Series A (General), 120*(3), 253–281.

Fethi, M. D., & Pasiouras, F. (2010). Assessing bank efficiency and performance with operational research and artificial intelligence techniques: A survey. *European Journal of Operational Research, 204*(2), 189–198.

Fukuyama, H., & Mirdehghan, S. M. (2012). Identifying the efficiency status in network DEA. *European Journal of Operational Research, 220*(1), 85–92.

Fukuyama, H., & Weber, W. L. (2003). Modeling indirect allocative efficiency via distance and quasi-distance functions. *Journal of the Operations Research Society, 46*(3), 264–285.

Fukuyama, H., & Weber, W. L. (2004). Economic inefficiency measurement of input spending when decision making units face different input prices. *Journal of the Operational Research Society, 164*(2), 535–547.
Fukuyama, H., & Weber, W. L. (2005). Estimating output gains by means of Luenberger efficiency measures. *European Journal of Operational Research, 165*(2), 535–547.
Fukuyama, H., & Weber, W. L. (2008a). Japanese banking inefficiency and shadow pricing. *Mathematical and Computer Modelling, 48*(11/12), 1854–1867.
Fukuyama, H., & Weber, W. L. (2008b). Estimating inefficiency, technological change and shadow prices of problem loans for regional banks and Shinkin banks in Japan. *The Open Management Journal, 1*, 1–11.
Fukuyama, H., & Weber, W. L. (2010). A slacks-based inefficiency measure for a two-stage system with bad outputs. *Omega, 38*(5), 239–410.
Fukuyama, H., & Weber, W. L. (2013). A dynamic network DEA model with an application to Japanese Shinkin banks. In F. Pasiouras (Ed.), *Efficiency and productivity growth: Modelling in the financial services industry* (pp. 193–213). London: Wiley. Chapter 9.
Fukuyama, H., & Weber, W. L. (2014). Two-stage network DEA with bad outputs. In W. D. Cook & J. Zhu (Eds.), *Data envelopment analysis: A handbook on the modeling of internal structures and networks* (pp. 451–474). New York: Springer. Chapter 19.
Fukuyama, H., & Weber, W. L. (2015a). Measuring Japanese bank performance: A dynamic network DEA approach. *Journal of Productivity Analysis, 44*(3), 249–264.
Fukuyama, H., & Weber, W. L. (2015b). Measuring bank performance with a dynamic network Luenberger indicator. *Annals of Operations Research,* in press.
Fukuyama, H., & Weber, W. L. (2016). Japanese bank productivity, 2007–2012: A dynamic network approach. Mimeo.
Kao, C. (2009). Efficiency measurement for parallel production systems. *European Journal of Operational Research, 196*, 1107–1112.
Kao, C. (2014). Network data envelopment analysis. *European Journal of Operational Research, 239*, 1–16.
Kao, C., & Hwang, S. N. (2008). Efficiency decomposition in two-stage data envelopment analysis: An application to non-life insurance companies in Taiwan. *European Journal of Operational Research, 185*(1), 418–429.
Kuosmanen, T. (2005). Weak disposability in nonparametric production with undesirable outputs. *American Journal of Agricultural Economics, 87*(4), 1077–1082.
Lewis, H. F., & Sexton, T. R. (2004). Network DEA: Efficiency analysis of organizations with complex internal structure. *Computers and Operations Research, 31*(9), 1365–1410.
Liang, L., Cook, W. D., & Zhu, J. (2008). DEA models for two-stage processes: Game approach and efficiency decomposition. *Naval Research Logistics, 55*, 643–653.
Liu, J., & Tone, K. (2008). A multistage method to measure efficiency and its application to Japanese banking. *Socio-Economic Planning Sciences, 42*(2), 75–91.
Lozano, S. (2015). Alternative SBM model for network DEA. *Computers & Industrial Engineering, 82*, 33–40.
Lozano, S. (2016). Slacks-based inefficiency approach for general networks with bad outputs: An application to the banking sector. *Omega, 60*, 73–84.
Luenberger, D. G. (1992). Benefit functions and duality. *Journal of Mathematical Economics, 21*, 461–481.
Luenberger, D. G. (1995). *Microeconomic theory*. Boston: McGraw-Hill.
McAllister, P. H., & McManus, D. (1993). Resolving the scale efficiency puzzle in banking. *Journal of Banking and Finance, 17*(2–3), 389–405.
Mirdehghan, S. M., & Fukuyama, H. (2016). Pareto-Koopmans efficiency and network DEA. *Omega, 61*, 78–88.
Montgomery, H., & Shimizutani, S. (2009). The effectiveness of bank recapitalization policies in Japan. *Japan and the World Economy*, *21*, 1–25.

Nemoto, J., & Goto, M. (2003). Measurement of dynamic efficiency in production: An application of data envelopment analysis to Japanese electric utilities. *Journal of Productivity Analysis, 19*, 191–210.

Park, K. H., & Weber, W. L. (2006). A note on efficiency and productivity growth in the Korean banking industry, 1992–2002. *Journal of Banking & Finance, 30*, 2371–2386.

Saunders, A., & Cornett, M. (2011). *Financial institutions management: A risk management approach* (7th ed.). New York: McGraw-Hill Irwin.

Sealey, C., & Lindley, J. (1977). Inputs, outputs, and a theory of production and cost at depository financial institutions. *Journal of Finance, 32*, 1251–1266.

Sengupta, J. K. (1994). Evaluating dynamic efficiency by optimal control. *International Journal of Systems Science, 25*(8), 1337–1353.

Sexton, T. R., & Lewis, H. F. (2003). Two-stage DEA: An application to major league baseball. *Journal of Productivity Analysis, 19*, 227–249.

Shephard, R. W. (1970). *Theory of cost and production functions*. New Jersey, Princeton University Press.

Shephard, R. W., & Färe, R. (1980). *Dynamic theory of production correspondences*. Oelgeschlager: Gunn and Hain.

Tone, K., & Tsutsui, M. (2009). Network DEA: A slacks-based measure approach. *Omega, 38*, 145–156.

Tone, K., & Tsutsui, M. (2014). Dynamic DEA with network structure: A slacks-based measure approach. *Omega, 42*(1), 124–131.

Wang, C. H., Gopal, R., & Zionts, S. (1997). Use of data envelopment analysis in assessing information technology impact on firm performance. *Annals of Operations Research, 3*, 191–213.

Weber, W. L., & Devaney, M. (1999). Bank efficiency, risk-based capital and real estate exposure: The credit crunch revisited. *Real Estate Economics, 27*, 1–25.

Chapter 11
Evaluation and Decomposition of Energy and Environmental Productivity Change Using DEA

Ke Wang

Abstract In this chapter, we present an input specific Luenberger energy and environmental productivity indicator. The data envelopment analysis (DEA) approach is utilized to estimate the directional distance function for composing the Luenberger energy and environmental productivity indicator. We further decompose the Luenberger productivity indicator in two ways. Firstly, it can be decomposed into four components that measure the changes of pure efficiency, scale efficiency, pure technology, and scale technology to energy and environmental productivity change. This decomposition helps to identify the effects of catching up to the frontier and the frontier shift, as well as the economy of scale (both from an efficiency perspective and a technical perspective) towards energy and environmental productivity change. Secondly, it can be additionally decomposed into the productivity changes of specific energy input factors and undesirable output (emission) factor. This decomposition enables to examine the contributions of specific input and undesirable output factors toward energy and environmental productivity change. An illustrative empirical application of the Luenberger energy and environmental productivity indicator and its decompositions are also provided in this chapter.

Keywords Carbon productivity • Data envelopment analysis • Input specific productivity indicator

K. Wang (✉)
School of Management and Economics, Beijing Institute of Technology, No. 5 South Zhongguancun Street, Haidian District, Beijing, China

Center for Energy and Environmental Policy Research, Beijing Institute of Technology, No. 5 South Zhongguancun Street, Haidian District, Beijing, China
e-mail: wangkebit@bit.edu.cn

S.-N. Hwang et al. (eds.), *Handbook of Operations Analytics Using Data Envelopment Analysis*, International Series in Operations Research & Management Science 239, DOI 10.1007/978-1-4899-7705-2_11

11.1 Introduction

Many current studies of energy productivity or environmental productivity changes have attempted to identify the sources of observed productivity increase or decrease in major energy consuming and carbon emitting countries. Several hypotheses have been proposed such as environmental regulations or carbon emission control policies impose burdens on economic development since pollutant abatement activities or carbon emission reduction activities are implemented to be in response to environmental policies and these activities are usually costly. Carbon control policies divert part of the energy inputs, which are initially used for producing desirable outputs, to carbon emission abatement activities and thus, it may result in reducing the production of desirable outputs. In other words, the carbon control policies may require a decision maker to employ more energy inputs to produce the same level of the desirable outputs and therefore, it leads to the underestimation of energy productivity growth from the traditional productivity measuring indicators (Färe et al. 2007a). Several researches had provided evidences on this underestimation such as Chung et al. (1997), Färe et al. (2001), and Hailu and Veeman (2001), in which the estimated productivity improvements with the consideration of undesirable (i.e., pollutant emissions) are higher than the traditional productivity improvement measures when undesirable outputs are ignored. Their estimation results support the so called Porter Hypothesis arguing that environmental regulations will lead to enhanced competitiveness (measured by productivity growth) through stimulating innovation on emission control and environmental protection (Porter 1990). However, other studies provide contradicts evidence. For instance, Kumar (2006) indicated that developing countries shown lower productivity growth when the undesirable outputs (e.g., carbon emission) are taken into account and treated as weakly disposable outputs. Similarly, Zhang et al. (2011) pointed out that, in the case of China, the productivity growth was overestimated if undesirable outputs are ignored, and this overestimation indicates that the increase speed of desirable output (production) exceeds the decrease speed (in absolute value) of undesirable output (carbon emission) (Färe et al. 2001). Therefore, the evidence of China does not support the Porter Hypothesis.

The concept of energy and environmental productivity is an extension of ecological productivity which could be defined as ratio between environmental impacts added (e.g., global warming effect caused by carbon emission) and value added from the consumption of energy. Energy and environmental productivity aims to achieve more desirable output productions with the current energy consumptions and pollutant emissions, or to maintain the current goods and service outputs with less energy inputs as well as less carbon emission. Energy and environmental productivity is essentially an indicator of economic performance and resources utilization performance, and thus the measurement of it is important to determine the benchmark of success (best practice of desirable and undesirable output productions), identify the areas for performance improvement (energy consumption and carbon emission reduction potentials), and prioritize actions for keeping economic growth as well as controlling pollutant emissions (Mahlberg and Luptacik 2014).

Malmquist and Luenberger productivity indicators are capable of calculating productivity changes, and under the evaluation framework proposed by Chung et al. (1997), these indicators allow for taking undesirable outputs into account without requiring price information on them. In addition, these indicators credit observations under evaluation for promotions in production or reductions in resources and pollutions, as well as providing a measurement of productivity change. These indicators also provide the information on whether the productivity change is caused by efficiency change (catching up to the best practice frontier) or technical change (best practice frontier shift). In order to provide a more general decomposition framework of productivity change and to avoid so-called scale bias of technical change, Simar and Wilson (1998) and Zofio and Lovell (1998) provides a unifying decomposition of productivity indicator which may deal with a complete characterization of efficiency change and technical change both from a technical and a scale perspective.

The productivity indicators mentioned above are not able to attribute energy and environmental productivity change to changes in consumption of specific energy input factors or in production of specific pollutant undesirable output factors. In order to separately identify the productivity change of each energy input and carbon emission undesirable output, this study, based on the general decomposition framework of Simar and Wilson (1998) and Zofio and Lovell (1998), proposes an input specific Luenberger energy and environmental productivity indicator. This indicator can be further decomposed in the way that enables examining the contribution of specific input and undesirable output factors to energy and environmental productivity change. These decompositions help to identify the catch-up effect, frontier shift effect, and economy of scale towards energy and environmental productivity change, as well as provide the information on which energy inputs and emission output of an observation under evaluation are the driving forces of energy and environmental productivity change. The Luenberger indicators proposed in this study are all computed through the directional distance functions that are estimated using the data envelopment analysis (DEA) approach (Cooper et al. 2011; Zhu 2015).

This chapter is structured as follows. Section 11.2 presents the Luenberger productivity indicator and its decomposition. Section 11.3 presents the DEA approach used for computing the directional distance functions of energy and environmental efficiency measures. An application of the energy and environmental productivity indicator to China's regional energy and environmental productivity change is illustrated in Sect. 11.4. Section 11.5 provides the conclusions.

11.2 Luenberger Productivity Indicator and Its Decomposition

Two commonly used indicators for calculating productivity change and its components (e.g., efficiency change and technical change) are Malmquist productivity index (Färe et al. 1989, 1994) and Luenberger productivity indicator (Chambers et al. 1996). Malmquist index has a ratio structure while Luenberger indicator has an additive structure. Usually, Malmquist index is associated with Russell measure of inefficiency (Färe et al. 1982, 1985; Pastor et al. 1999) which is multiplicative by nature; while Luenberger indicator is associated with slack-based measure of inefficiency through directional distance function or with directional Russell measure of inefficiency (Färe and Grosskopf 2010; Fukuyama and Weber 2009) which is additive by nature.

For decomposing the productivity indicators, Färe et al. (1992) derived an input-oriented Malmquist index for measuring productivity change and decomposed it into indicators measuring changes in efficiency and technology. Then, Färe et al. (1994) proposed an output-oriented Malmquist index and provided an alternative decomposition identifying changes in efficiency, scale and technology. Simar and Wilson (1998) additionally decomposed the Malmquist index into four factors: pure efficiency change, scale efficiency change, pure technology change, and scale of technology change. On the other hand, Chambers et al. (1996) derived a decomposition of non-oriented Luenberger indicator into its efficiency change component and technical change component. This non-oriented Luenberger indicator is constructed through directional distance function that simultaneously adjusts inputs and outputs.

Following, we first introduce the directional distance function and the Luenberger productivity indicators proposed in Chambers et al. (1996), and then propose another decomposition of Luenberger indicator for the evaluation and decomposition of energy and environmental productivity change.

Firstly, the production technology can be described by a set T which defined as:

$$T = \{(x, y) : x \,\text{can produce}\, y\} \tag{11.1}$$

where x is a non-negative vector of inputs and y is a non-negative vector of outputs. Then, the directional distance function denoted by $D_T(x, y; g_x, g_y)$ is defined as:

$$D_T(x,\ y;\ g_x,\ g_y) = \sup\,\{\beta :\ (x - \beta g_x,\ y + \beta g_y) \in T\} \tag{11.2}$$

where (g_x, g_y) is a nonzero directional vector.

The following Fig. 11.1 illustrates the directional distance function assuming constant returns to scale. The input and output vector (x, y) is expanded along a direction, which is given by (g_x, g_y), as much as is feasible, and the maximal expansion is measured as $D_T(x, y; g_x, g_y)$.

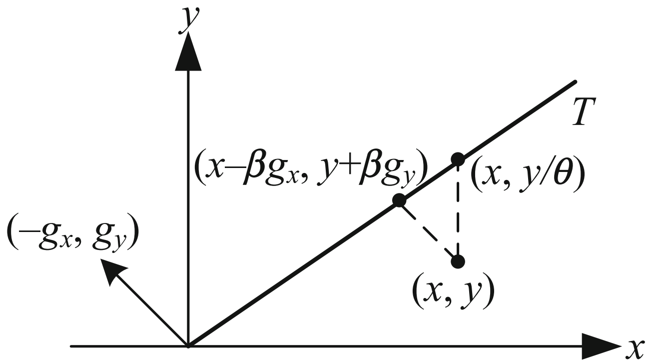


Fig. 11.1 Directional distance function and output distance function

If inputs and outputs are freely disposable, the directional distance function D_T completely characterizes the technology:

$$D_T\left(x,\ y;\ g_x,\ g_y\right) \geq 0 \Leftrightarrow\ (x,\ y) \in T \tag{11.3}$$

And if T exhibits constant returns to scale, then it follows for $\lambda > 0$ that:

$$D_T\left(\lambda x,\ \lambda y;\ g_x,\ g_y\right) = \lambda D_T\left(x,\ y;\ g_x,\ g_y\right) \tag{11.4}$$

If we choose $g_x = 0$ and $g_y = y$, the directional distance function is related to Shephard output distance function as:

$$D_T(x,\ y; 0,\ y) = 1/D_0(x,\ y) - 1 \tag{11.5}$$

where Shephard output distance function is defined as:

$$D_0(x,\ \ y) = \inf\ \{\theta : (x,\ \ y/\theta) \in T\} \tag{11.6}$$

and that

$$D_0(x,\ y) \leq 1\ \ \Leftrightarrow (x, y) \in T \tag{11.7}$$

Equation (11.5) indicates that Shephard output distance function is a special case of the directional distance function.

Next, we introduce Chambers et al. (1996)'s Luenberger productivity indicator. Different from the Malmquist productivity indicator, Luenberger productivity indicator has an additive structure, and it can be formulated as differences rather than ratios of efficiency measures. The Luenberger productivity indicator for periods t and $t + 1$ is defined as:

$$L(x^t, y^t, x^{t+1}, y^{t+1}) = \frac{1}{2}\Big[D_T^{t+1}\left(x^t, y^t; g_x, g_y\right) - D_T^{t+1}\left(x^{t+1}, y^{t+1}; g_x, g_y\right) + D_T^{t}\left(x^t, y^t; g_x, g_y\right) - D_T^{t}\left(x^{t+1}, y^{t+1}; g_x, g_y\right)\Big] \tag{11.8}$$

Positive value of L indicates productivity improvement and negative value of L indicates productivity decline.

Following Chambers et al. (1996), the Luenberger productivity indicator can be further decomposed into two components of efficiency change measure (*LEFCH*) and technical change measure (*LTECH*) as:

$$LEFCH = D_T^{t}\left(x^t, y^t; g_x, g_y\right) - D_T^{t+1}\left(x^{t+1}, y^{t+1}; g_x, g_y\right) \tag{11.9}$$

and

$$LTECH = \frac{1}{2}[D_T^{t+1}(x^{t+1}, y^{t+1}; g_x, g_y) - D_T^{t}(x^{t+1}, y^{t+1}; g_x, g_y) + D_T^{t+1}(x^t, y^t; g_x, g_y) - D_T^{t}(x^t, y^t; g_x, g_y)] \tag{11.10}$$

Then, we have:

$$L\left(x^t, y^t, x^{t+1}, y^{t+1}\right) = LEFCH + LTECH \tag{11.11}$$

The efficiency change measures how close two observations (i.e., one decision making unit in two periods t and $t + 1$) are to the frontiers of two technologies T (i.e., technologies of two periods t and $t + 1$), while the technical change measures the (average) distance between the two technologies.

In the current study, since we plan to measure and decompose energy and environmental productivity change through Luenberger productivity indicator, in which energy inputs, desirable outputs of economic productions, and undesirable outputs of emissions caused by energy consumptions need to be simultaneously measured, we will give another measurement and decomposition of productivity change based on the Luenberger indicator. Before doing this, we first introduce a production technology set that considers a production process of employing a vector of energy inputs (e) and a vector of other resources inputs (x) such as labor and capital to generate a vector of desirable outputs (y) such as gross domestic product, and a vector of undesirable outputs (b), such as CO_2 emission, as byproducts of desirable outputs. This production technology set can be defined as:

$$T' = \{(e,\ x,\ \ y\ , b) : (e,\ \ x) \text{ can produce } (y,\ \ b)\} \tag{11.12}$$

where T' is often assumed to be a closed and bounded set, and in addition, inputs and desirable outputs are supposed to be freely (or strongly) disposable.

For a reasonable modeling of a production technology that includes both desirable and undesirable outputs, additional assumptions need to be imposed. Färe et al. (1989) first proposed to use the weak disposability of undesirable outputs on the production technology set, which implies that the reduction of undesirable outputs is costly and a proportional reduction in both desirable and undesirable outputs is necessary. Moreover, the nulljointness also need to be introduced on the production technology set, which implies that for producing desirable outputs, some undesirable outputs must be generated as well.

The jointly weak disposability of desirable and undesirable outputs assumption is defined as:

$$\text{If } (e,\ x,\ y,\ b) \in T' \text{ and } \theta \in (0, 1], \text{ then } (e,\ x,\ \theta y,\ \theta b) \in T' \tag{11.13}$$

And the nulljointness assumption is defined as:

$$\text{If } (e,\ x,\ y,\ b) \in T' \text{ and } b = 0, \text{ then } y = 0 \tag{11.14}$$

Then, the directional distance function with the consideration of desirable and undesirable outputs denoted by $D(e, x, g, b;\ g_e,\ g_x,\ g_g,\ g_b)$ can be defined as:

$$D(e, x, g,\ b;\ g_e,\ g_x,\ g_y,\ g_b) = \sup\,\{\beta : (e-\beta g_e, x-\beta g_x, y+\beta g_y, b-\beta g_b) \in T'\} \tag{11.15}$$

where (g_e, g_x, g_y, g_b) is a nonzero directional vector.

The following Fig. 11.2 illustrates part of the directional distance function of desirable and undesirable outputs

$(y,\ b)$ which is expanded along a direction denoted by (g_y, g_b) as much as is feasible. The maximal expansion is measured as $D(\mathrm{e},\ x,\ y,\ b;\ g_e,\ g_x,\ g_y,\ g_b)$. Similarly, we have the property of the directional distance function D as:

$$D\left(e,\ x,\ y,\ b;\ g_e,\ g_x,\ g_y,\ g_b\right) \geq 0 \Leftrightarrow (e, x, y, b) \in T'. \tag{11.16}$$

Next, according to the concept of Chambers et al. (1996), we propose the following Luenberger productivity indicator for periods t and $t+1$:

$$\begin{aligned} LP_t^{t+1} &= \frac{1}{2}\Big[D^t\left(e^t, x^t, y^t, b^t; g_e, g_x, g_y, g_b\right) - D^t\left(e^{t+1}, x^{t+1}, y^{t+1}, b^{t+1}; g_e, g_x, g_y, g_b\right) \\ &+ D^{t+1}\left(e^t, x^t, y^t, b^t; g_e, g_x, g_y, g_b\right) - D^{t+1}\left(e^{t+1}, x^{t+1}, y^{t+1}, b^{t+1}; g_e, g_x, g_y, g_b\right)\Big] \end{aligned} \tag{11.17}$$

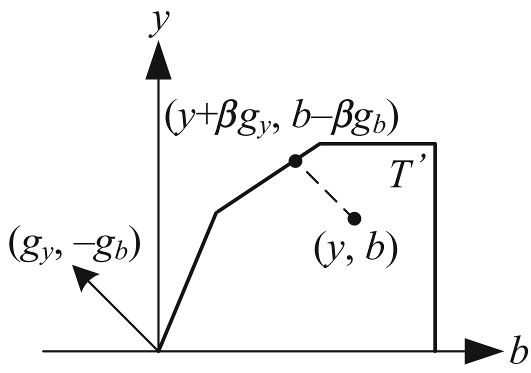


Fig. 11.2 Directional distance function of desirable and undesirable outputs

Positive and negative *LP* values respectively represent productivity increase and decrease. *LP* can also be decomposed into two components, namely efficiency change (*LPEFCH*) and technical change (*LPTECH*) as:

$$LPEFCH_t^{t+1} = D^t\left(e^t, x^t, y^t, b^t; g_e, g_x, g_y, g_b\right) - D^{t+1}\left(e^{t+1}, x^{t+1}, y^{t+1}, b^{t+1}; g_e, g_x, g_y, g_b\right) \quad (11.18)$$

and

$$\begin{aligned} &LPTECH_t^{t+1} \\ &= \frac{1}{2}[D^{t+1}(e^{t+1}, x^{t+1}, y^{t+1}, b^{t+1}; g_e, g_x, g_y, g_b) - D^t(e^{t+1}, x^{t+1}, y^{t+1}, b^{t+1}; g_e, g_x, g_y, g_b) \\ &\quad + D^{t+1}(e^t, x^t, y^t, b^t; g_e, g_x, g_y, g_b) - D^t(e^t, x^t, y^t, b^t; g_e, g_x, g_y, g_b)] \end{aligned} \quad (11.19)$$

The sum of *LPEFCH* and *LPTECH* is equal to *LP*. The efficiency change component measures the catch up effect that indicates the change in distance of an observation to the production frontiers at periods t and $t+1$. The technical change component measures the frontier shift effect that reflects the shift in the production frontiers at periods t and $t+1$.

Figure 11.3 provides an illustration of the *LP*. Suppose that the multiple line segments 0*ABCD* and 0*A'B'C'D'* respectively represent the production technologies of periods t and $t+1$, and assume that points a and d respectively represent one observation, consuming the same amounts of inputs to produce different amounts of desirable and undesirable outputs, at periods t and $t+1$. Thus, the period t desirable and undesirable output vector (y^t, b^t) is denoted by a, and the period $t+1$ desirable and undesirable output vector $\left(y^{t+1},\ b^{t+1}\right)$ is denoted by d. The two technologies at

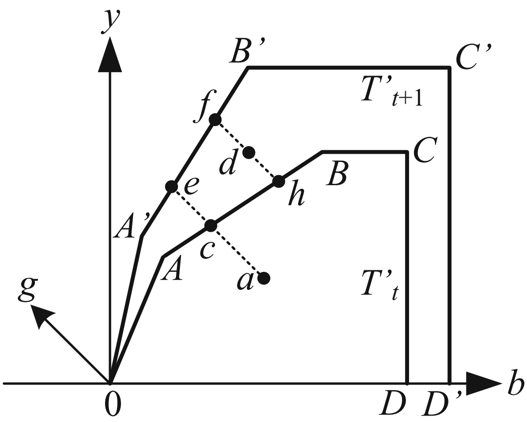


Fig. 11.3 The Luenberger productivity indicator for desirable and undesirable outputs

periods t and $t+1$ are T'_t and T'_{t+1}. Moreover, the direction (g_y, g_b) is denoted by g. Then, we can have:

$$c = a + D^t(a;\ g)\ \cdot\ g \tag{11.20}$$

$$e = a + D^{t+1}(a;\ g)\ \cdot g \tag{11.21}$$

$$f = d + D^{t+1}(d;\ g)\ \cdot\ g \tag{11.22}$$

$$h = d + D^t(d; g)\ \cdot\ g \tag{11.23}$$

and

$$LPEFCH = (c-a)-(f-d) = [D^t(a;\ g)-D^{t+1}(d;\ g)]\ \cdot g \tag{11.24}$$

$$\begin{aligned} LPTECH &= \frac{1}{2}[(f-h) + (e-c)] \\ &= \frac{1}{2}\ [D^{t+1}(d; g)-D^t(d; g) + D^{t+1}(a; g)-D^t(a; g)]\ \cdot g \end{aligned} \tag{11.25}$$

In this study, we apply the Luenberger productivity indicator to measure and decompose energy and environmental productivity change. Inspired by the decomposition of the Malmquist index in Simar and Wilson (1998) and Zofio (2007), we go one step further, than the decomposition introduced above, to decompose Luenberger energy and environmental productivity indicator into four components of pure efficiency change, scale efficiency change, pure technical change, and scale technical change. This decomposition is considered a more comprehensive decomposition that remains generally accepted productivity change definition, as well as efficiency change and technical change definitions from both a technical perspective and a scale perspective, and this decomposition can avoid the scale biases of efficiency change and technical change.

Next, we propose these four components, namely Luenberger pure efficiency change (*LPEC*), Luenberger scale efficiency change (*LSEC*), Luenberger pure technical change (*LPTC*), and Luenberger scale technical change (*LSTC*) as:

$$LPEC_t^{t+1} = D_V^t\left(e^t, x^t, y^t, b^t; g_e, g_x, g_y, g_b\right) - D_V^{t+1}\left(e^{t+1}, x^{t+1}, y^{t+1}, b^{t+1}; g_e, g_x, g_y, g_b\right) \tag{11.26}$$

$$\begin{aligned} LSEC_t^{t+1} = & \left[D_C^t\left(e^t,x^t,y^t,b^t;g_e,g_x,g_y,g_b\right) - D_V^t\left(e^t,x^t,y^t,b^t;g_e,g_x,g_y,g_b\right)\right] \\ & -\left[D_C^{t+1}\left(e^{t+1},x^{t+1},y^{t+1},b^{t+1};g_e,g_x,g_y,g_b\right) - D_V^{t+1}\left(e^{t+1},x^{t+1},y^{t+1},b^{t+1};g_e,g_x,g_y,g_b\right)\right] \end{aligned} \tag{11.27}$$

$$\begin{aligned} LPTC_t^{t+1} = & \frac{1}{2}\Big[D_V^{t+1}\left(e^t,x^t,y^t,b^t;g_e,g_x,g_y,g_b\right) - D_V^t\left(e^t,x^t,y^t,b^t;g_e,g_x,g_y,g_b\right) \\ & +D_V^{t+1}\left(e^{t+1},x^{t+1},y^{t+1},b^{t+1};g_e,g_x,g_y,g_b\right) - D_V^t\left(e^{t+1},x^{t+1},y^{t+1},b^{t+1};g_e,g_x,g_y,g_b\right)\Big] \end{aligned} \tag{11.28}$$

$$\begin{aligned} & LSTC_t^{t+1} \\ & = \frac{1}{2}\Big\{\left[D_C^{t+1}\left(e^t,x^t,y^t,b^t;g_e,g_x,g_y,g_b\right) - D_V^{t+1}\left(e^t,x^t,y^t,b^t;g_e,g_x,g_y,g_b\right)\right] \\ & \quad - \left[D_C^t\left(e^t,x^t,y^t,b^t;g_e,g_x,g_y,g_b\right) - D_V^t\left(e^t,x^t,y^t,b^t;g_e,g_x,g_y,g_b\right)\right] \\ & \quad + \left[D_C^{t+1}\left(e^{t+1},x^{t+1},y^{t+1},b^{t+1};g_e,g_x,g_y,g_b\right) - D_V^{t+1}\left(e^{t+1},x^{t+1},y^{t+1},b^{t+1};g_e,g_x,g_y,g_b\right)\right] \\ & \quad - \left[D_C^t\left(e^{t+1},x^{t+1},y^{t+1},b^{t+1};g_e,g_x,g_y,g_b\right) - D_V^t\left(e^{t+1},x^{t+1},y^{t+1},b^{t+1};g_e,g_x,g_y,g_b\right)\right]\Big\} \end{aligned} \tag{11.29}$$

where D_C indicates directional distance function under constant returns to scale assumption, while D_V indicates directional distance function under variable returns to scale assumption.

LPEC measures contemporaneous pure efficiency change of observation at periods t and $t+1$. Positive and negative *LPEC* values respectively indicate increase and decrease on pure technical efficiency. Note that two directional distance functions used for *LPEC* are all under variable returns to scale.

LSEC measures scale efficiency change resulted from the change in the location of observation in its input and output space between periods t and $t+1$, or change in the shape of the technology, or some combination of these two changes. Positive and negative *LSEC* values respectively indicate increase and decrease on scale efficiency. Note that two sets in the first and the second brackets in (11.27) respectively measure the difference between the two directional distance functions under variable returns to scale and constant returns to scale at periods t and $t+1$.

LPTC measures pure technical change which measures the shift of the technology. It is defined as the arithmetic mean of two shifts respectively relative to observation at periods t and $t+1$. Positive and negative *LPTC* values respectively indicate technical progress (i.e., upward shift of technology) and technical regress (i.e., downward shift of technology). Note that four directional distance functions used for *LPTC* are all under variable returns to scale.

LSTC measures scale technical change which is denoted by the change in returns to scale of technology at two fix position of an observation at periods t and $t+1$. It is also defined as the arithmetic mean of two differences which respectively measure the change in distance between two directional distance functions under variable returns to scale and constant returns to scale. Positive and negative *LSTC* values respectively indicate technology is moving toward constant returns to scale or opposite constant returns to scale.

In the next section, we develop a DEA model for calculating the Luenberger productivity indicator of energy and environmental productivity change measure and decomposition.

11.3 DEA Model for Energy and Environmental Efficiency Measurement

The calculation of Luenberger productivity indicator (*LP*) and its four decompositions: *LPEC*, *LSEC*, *LPTC*, and *LSTC* requires the computation of eight directional distance functions utilized in (11.17) and (11.26) to (11.29). DEA models are applied for estimating these directional distance functions. We assume that at each period $t = 1, \ldots, p$, there are $j = 1, \ldots, n$ observations of energy (e) and other resources (x) inputs, as well as desirable (y) and byproduct undesirable (b) outputs:

$$\left(e_j^t, x_j^t, y_j^t, b_j^t\right) j = 1, \ldots, n, t = 1, \ldots, p \tag{11.30}$$

where e, x, y, and b are all vectors respectively with $h = 1, \ldots, k$; $i = 1, \ldots, m$; $r = 1, \ldots, s$; and $f = 1, \ldots, l$ elements in each vector.

Following Färe et al. (1994), the production technology set derived from the data (11.30) and that meets (i) the jointly weak disposability and nulljointness of desirable and undesirable outputs, (ii) strong disposability of inputs and desirable outputs, and (iii) constant returns to scale is:

$$\begin{aligned} T' = \{(e, x, y, b) : & \textstyle\sum_{j=1}^{n} \lambda_j e_{hj}^t \le e_h^t, h = 1, \ldots, k, \\ & \textstyle\sum_{j=1}^{n} \lambda_j x_{ij}^t \le x_i^t, i = 1, \ldots, m, \\ & \textstyle\sum_{j=1}^{n} \lambda_j y_{rj}^t \ge y_r^t, r = 1, \ldots, s, \\ & \textstyle\sum_{j=1}^{n} \lambda_j b_{fj}^t = b_i^t, f = 1, \ldots, h, \\ & \lambda_j \ge 0, j = 1, \ldots, n\} \end{aligned} \tag{11.31}$$

Then, we adapt an additive featured directional distance function to computing our Luenberger productivity indicator. The computation is conducted through solving the following linear programming problem. For the observation j_0:

$$\begin{aligned} & D_C^t\left(e^t, x^t, y^t, b^t; g_e, g_x, g_y, g_b\right) = \max\beta \\ & s.t. \sum\nolimits_{j=1}^{n} \lambda_j e_{hj}^t \le e_{hj_0}^t - \beta g_e, h = 1, \ldots, k, \\ & \quad \sum\nolimits_{j=1}^{n} \lambda_j x_{ij}^t \le x_{ij_0}^t - \beta g_x, i = 1, \ldots, m, \\ & \quad \sum\nolimits_{j=1}^{n} \lambda_j y_{rj}^t \ge y_{rj_0}^t + \beta g_y, r = 1, \ldots, s, \\ & \quad \sum\nolimits_{j=1}^{n} \lambda_j b_{fj}^t = b_{fj_0}^t - \beta g_b, f = 1, \ldots, l, \\ & \quad \lambda_j \ge 0, j = 1, \ldots, n \end{aligned} \tag{11.32}$$

where β is the inefficiency measure and λ_j is the intensity variable that connects inputs of each observation with its outputs by a convex combination.

Model (11.32) provides the estimation of directional distance function for observation at period t against the technology at the same period. For the estimation at period $t+1$, just change the t period data to $t+1$ period data in Model (11.32). If we need to compute the mixed period directional distance function, then the t period data $(e_j^t, x_j^t, y_j^t, b_j^t)$ is used in the left hand side of the constraints and the $t+1$ period data $\left(e_j^{t+1}, x_j^{t+1}, y_j^{t+1}, b_j^{t+1}\right)$ is used in the right hand side of the constraints in Model (11.32) for computing $D_C^t\left(e^{t+1}, x^{t+1}, y^{t+1}, b^{t+1}; g_e, g_x, g_y, g_b\right)$, or on the contrary, the $t+1$ period data is used in the left hand side while the t period data is used in the right hand side for computing $D_C^{t+1}\left(e^t, x^t, y^t, b^t; g_e, g_x, g_y, g_b\right)$.

Model (11.32) is under constant returns to scale assumption. Next, we propose the following programming for calculating directional distance function under variable returns to scale. Similarly, for the observation j_0:

$$\begin{aligned} & D_V^t\left(e^t, x^t, y^t, b^t; g_e, g_x, g_y, g_b\right) = \max\beta \\ & s.t. \sum\nolimits_{j=1}^{n} \lambda_j e_{hj}^t \le e_{hj_0}^t - \beta g_e, h = 1, \ldots, k, \\ & \quad \sum\nolimits_{j=1}^{n} \lambda_j x_{ij}^t \le x_{ij_0}^t - \beta g_x, i = 1, \ldots, m, \\ & \quad \sum\nolimits_{j=1}^{n} \theta_j \lambda_j y_{rj}^t \ge y_{rj_0}^t + \beta g_y, r = 1, \ldots, s, \\ & \quad \sum\nolimits_{j=1}^{n} \theta_j \lambda_j b_{fj}^t = b_{fj_0}^t - \beta g_b, f = 1, \ldots, l \\ & \quad \sum\nolimits_{j=1}^{n} \lambda_j = 1 \\ & \quad \lambda_j \ge 0, 0 \le \theta_j \le 1, j = 1, \ldots, n \end{aligned} \tag{11.33}$$

where θ_j is an abatement factor to keep the reductions of desirable and undesirable outputs are proportional, i.e., to satisfy weak disposability of undesirable under the variable returns to scale assumption. Model (11.33) is a non-linear programming problem which can be linearized through the following the changes. If we set $\theta_j\lambda_j = \eta_j$ and $(1-\theta_j)\lambda_j = \delta_j$, then $\lambda_j = \eta_j + \delta_j$. Model (11.33) can be translated into:

$$\begin{aligned}
&D_V^t\left(e^t, x^t, y^t, b^t; g_e, g_x, g_y, g_b\right) = \max\beta \\
&s.t. \sum\nolimits_{j=1}^{n} (\eta_j + \delta_j)e_{hj}^t \le e_{hj_0}^t - \beta g_e, h = 1, \ldots, k, \\
&\quad \sum\nolimits_{j=1}^{n} (\eta_j + \delta_j)x_{ij}^t \le x_{ij_0}^t - \beta g_x, i = 1, \ldots, m, \\
&\quad \sum\nolimits_{j=1}^{n} \eta_j y_{rj}^t \ge y_{rj_0}^t + \beta g_y, r = 1, \ldots, s, \\
&\quad \sum\nolimits_{j=1}^{n} \eta_j b_{fj}^t = b_{fj_0}^t - \beta g_b, f = 1, \ldots, l, \\
&\quad \sum\nolimits_{j=1}^{n} (\eta_j + \delta_j) = 1, \\
&\quad \eta_j, \delta_j \ge 0, j = 1, \ldots, n
\end{aligned} \tag{11.34}$$

Model (11.34) computes the directional distance function under variable returns to scale at period t. Similarly, replacing the t period data with the $t+1$ period data, Model (11.34) provides the estimation at period $t+1$. For the calculation of the mixed period, the t period data is used in the left hand side of the constraints and the $t+1$ period data is used in the right hand side of the constraints in Model (11.34) for calculating $D_V^t\left(e^{t+1}, x^{t+1}, y^{t+1}, b^{t+1}; g_e, g_x, g_y, g_b\right)$; the $t+1$ period data is used in the left hand side while the t period data is used in the right hand side for calculating $D_V^{t+1}\left(e^t, x^t, y^t, b^t; g_e, g_x, g_y, g_b\right)$.

Our next task is to determine the directions utilized for computing the eight directional distance functions. From the perspective of energy utilization conservation but keep the generation of outputs unchanged, we could choose $\left(g_e,\ g_x,\ g_y,\ g_b\right) = (1,\ 0,\ 0,\ 0)$; or from the perspective of simultaneously increasing desirable outputs and decreasing undesirable outputs with the consumption of energy inputs unchanged, we could choose $\left(g_e,\ g_x,\ g_y,\ g_b\right) = (0,\ 0,\ 1,\ 1)$. In this study, we plan to minimize the energy consumption and decrease the undesirable carbon emissions as much as possible while keep the desirable production fixed, so as to provide an appropriate evaluation of energy and environmental productivity change, therefore, we should choose the direction vector as $\left(g_e, g_x, g_y, g_b\right) = (1,\ 0,\ 0,\ 1)$.

There is one important feature of the above setting of direction that should be noted. The directional distance function utilized in this study has an additive structure (e.g., e–βg_e, b–βg_b), and as pointed out in Färe et al. (2007b), this additive directional distance function has the advantages that the estimation results are easy to aggregate and it provides a clear connection to production function. However, the estimation results of it could be affected by the data scale, i.e., the estimation results from additive directional distance function is not unit-invariant. To avoid this problem, we chose another direction as follows, which is the observed energy input and undesirable output value of each decision making unit:

$$\left(g_e,\ g_x,\ g_y,\ g_b\right) = (e,\ 0,\ 0, b) \tag{11.35}$$

Then we have the right hand side of the constraints in Models (11.32) to (11.34) as:

$$\left[e^t_{hj_0} - \beta e^t_{hj_0}, x^t_{ij_0}, y^t_{rj_0}, b^t_{fj_0} - \beta b^t_{fj_0}\right]^{\mathrm{T}} = \left[(1-\beta)e^t_{hj_0}, x^t_{ij_0}, y^t_{rj_0}, (1-\beta)b^t_{fj_0}\right]^{\mathrm{T}} \tag{11.36}$$

The choosing of the above direction satisfies the unit-invariance of the estimation of directional distance function.

The energy and environmental inefficiency measure β proposed in Models (11.32) to (11.34) is considered an integrated inefficiency measure that is not able to identify the inefficiency of a specific energy input resource, or to distinguish the different contribution of energy inputs and undesirable emission outputs in the inefficiency measure. In order to provide an insight of the different contribution of a specific energy resource among all input factors, as well as the impact of undesirable outputs in the evaluation of energy and environmental productivity change, we further modify Models (11.32) and (11.34) and propose the following Models in which the singe inefficiency measure β is replaced with a set of energy input specific inefficiency measure $\beta_h,\ \ h = 1, \ldots, k,$ and undesirable output specific inefficiency measure $\beta_f, f = 1, \ldots, l$:

$$\begin{aligned}
&D^t_C\left(e^t, x^t, y^t, b^t; g_e, g_x, g_y, g_b\right) = \max\left(w_e \textstyle\sum_{h=1}^{k} w_h\beta_h + w_b \textstyle\sum_{f=1}^{l} w_f\beta_f\right)\\
&s.t. \textstyle\sum_{j=1}^{n} \lambda_j e^t_{hj} \le e^t_{hj_0} - \beta_h g_e, h = 1, \ldots, k,\\
&\qquad \textstyle\sum_{j=1}^{n} \lambda_j x^t_{ij} \le x^t_{ij_0}, i = 1, \ldots, m,\\
&\qquad \textstyle\sum_{j=1}^{n} \lambda_j y^t_{rj} \ge y^t_{rj_0}, r = 1, \ldots, s,\\
&\qquad \textstyle\sum_{j=1}^{n} \lambda_j b^t_{fj} = b^t_{fj_0} - \beta_f b^t_{fj_0}, f = 1, \ldots, l,\\
&\qquad \lambda_j \ge 0, j = 1, \ldots, n
\end{aligned} \tag{11.37}$$

and

$$
\begin{aligned}
& D_V^t\left(e^t, x^t, y^t, b^t; g_e, g_x, g_y, g_b\right) = \max\left(w_e \sum\nolimits_{h=1}^{k} w_h \beta_h + w_b \sum\nolimits_{f=1}^{l} w_f \beta_f\right) \\
& s.t. \sum\nolimits_{j=1}^{n} \left(\eta_j + \delta_j\right) e_{hj}^t \le e_{hj_0}^t - \beta_h e_{hj_0}^t, h = 1, \ldots, k, \\
& \quad \sum\nolimits_{j=1}^{n} \left(\eta_j + \delta_j\right) x_{ij}^t \le x_{ij_0}^t, i = 1, \ldots, m, \\
& \quad \sum\nolimits_{j=1}^{n} \eta_j y_{rj}^t \ge y_{rj_0}^t, r = 1, \ldots, s, \\
& \quad \sum\nolimits_{j=1}^{n} \eta_j b_{fj}^t = b_{fj_0}^t - \beta_f b_{fj_0}^t, f = 1, \ldots, l, \\
& \quad \sum\nolimits_{j=1}^{n} \left(\eta_j + \delta_j\right) = 1, \\
& \quad \eta_j, \delta_j \ge 0, j = 1, \ldots, n
\end{aligned}
\tag{11.38}
$$

where w_h and w_f are, respectively, the normalized user specified weights associated with energy input and emission output; w_e and w_b are the normalized user specified weights respectively associated with energy inefficiency measure and environmental inefficiency measure.

11.4 Application to China's Regional Energy and Environmental Productivity Change

In this section, we utilize the Luenberger productivity indicator and its decomposition based on DEA directional distance function proposed in Sects. 11.2 and 11.3 to evaluate the energy and environmental productivity of China's regions during the period of 1997–2012 and further decompose the productivity indicator so as to identify the contributions of its change.

Our study period covers approximately four Five-Year Plan (FYP) periods of China, namely the later years of the 9th FYP (1997–2000), the 10th and 11th FYP (2001–2010), and the early years of the 12th FYP (2011–2012) periods. The calculations are based on the data of at the provincial level (China's 30 provinces, autonomous regions and municipalities) and the results are reported at the regional level (China's eight economic-geographic areas) for illustrating convenience.

These eight economic-geographic areas are northeast area, north coast area, east coast area, south coast area, middle Yellow River area, middle Yangtze River area, southwest area, and northwest area. This cluster is according to the characters of the regional economic development and geography of China's 30 provinces (excluding Tibet, Taiwan, Hong Kong, and Macau due to data absence). Figure 11.4 illustrates the distribution of these provinces and areas.

During the study period, China's economic growth mode, energy consumption pattern, energy intensity and carbon emission intensity reduction effort, and government's regulation and policy on national sustainable development changed significantly. Thus the evaluation of China's energy and environmental productivity change over this period is helpful to understand the difference and the trend on

Fig. 11.4 China's eight economic-geographic areas and their energy consumption structures

productivity change in China's different regions, and furthermore, to understand the contributions of different energy input and undesirable output factors toward such productivity change over time. This evaluation helps to improve the performance of energy and environmental policy making and implementation.

11.4.1 Data and Variables

The inputs are total energy consumption (e), labor force (x_1) and capital stock (x_2); the desirable output (y) is GDP (at national level) or GRP (at regional level); the undesirable output (b) is CO_2emissions. Moreover, for computing the different contributions of specific energy resources, the total energy consumption is further decomposed into four final energy consumptions of coal, oil, natural gas and electricity (e_1 to e_4). Input and output data at provincial level are obtained from China Statistical Yearbook and China Energy Statistical Yearbook (for all e, and x_1, y) or calculated ourselves (for x_2 and b) (Wang et al. 2012, 2013).

Table 11.1 summarizes the descriptive statistics of the input and output data for China's eight economic-geographic areas for selected years. Take the total data of 2012 as instances, the consumptions of both the total energy and its composition of coal, oil and electricity in the north coast area were highest and those in the

Table 11.1 Descriptive statistics of inputs and outputs for China's eight economic-geographic areas

Inputs and outputs		Year	Northeast area	North coast area	East coast area	South coast area	Middle Yellow River area	Middle Yangtze River area	Southwest area	Northwest area
Desirable output	GDP (billion RMB)	1997	926.49	1888.34	1951.90	1431.54	1218.89	1336.84	1185.93	352.98
		2012	4653.21	10,122.50	10,322.78	7572.39	6779.07	6788.70	5998.88	1587.48
Non-energy inputs	Labor (thousand person)	1997	49,592.00	92,744.00	72,160.00	57,286.00	93,624.00	116,988.00	129,346.00	23,724.00
		2012	52,212.68	112,641.52	96,292.55	83,897.10	108,249.29	132,545.76	150,457.66	28,996.12
	Capital (billion RMB)	1997	1486.55	3389.38	3616.00	1868.03	1821.86	1763.76	1630.48	697.17
		2012	13,526.73	25,635.26	24,093.53	15,015.99	20,357.23	15,967.41	16,271.45	4913.42
Energy input	Energy (million tce)	1997	140.82	175.98	130.31	78.04	145.00	137.70	134.36	53.10
		2012	348.07	649.46	417.32	292.03	502.05	396.95	425.06	186.90
Specific energy inputs	Coal (million tce)	1997	77.58	104.84	67.58	29.68	105.45	101.76	97.53	30.31
		2012	151.82	339.40	126.49	86.09	286.88	236.05	235.43	82.19
	Oil (million tce)	1997	30.20	35.36	37.31	31.74	14.81	21.48	9.48	11.99
		2012	106.87	130.39	125.89	105.42	70.04	73.22	76.21	27.25
	Natural gas (million tce)	1997	4.38	2.35	0.07	0.77	1.48	0.18	10.95	2.35
		2012	14.34	25.51	17.52	14.70	26.20	10.78	30.68	21.54
	Electricity (million tce)	1997	28.67	33.43	25.34	15.84	23.27	14.29	16.40	8.45
		2012	75.04	154.16	147.42	85.82	118.93	76.89	82.75	55.92
Undesirable outputs	CO_2 (million t)	1997	460.19	540.41	414.86	211.74	495.97	413.25	379.68	152.56
		2012	1018.29	1862.77	1254.15	814.13	2006.22	1169.19	1166.26	631.94

northwest area were lowest. The only exception appeared in the consumption of natural gas that the southwest area was the largest consumer and the middle Yangtze River area had the lowest consumption in China. Different from the regional distribution of energy consumption, the middle Yellow River area was accounted as the largest CO_2 emitter, and the CO_2 emissions from the northwest area were lowest.

During the study period, China's total energy consumption significantly increased by more than two times and in which the most significant increase (more than six times) is in natural gas consumption. Electricity consumption increased by nearly four times. However, the structure of energy consumption rarely changed which is characterized by the feature of "rich in coal, short of oil, and lack of natural gas". Figure 11.5 illustrates the distribution of total energy consumption for four specific energy consumptions in China and the distributions of energy consumption for China's eight areas. In addition, the distribution of total energy consumption for four specific energy consumptions in China's eight areas can be found in Fig. 11.4.

In general, the north coast area was the largest energy consumer, which accounted for about 1/5 of the nation's total energy consumption, followed by the middle Yellow River area (16 %) and the east coast area (14 %). The total energy consumption of the northwest area was the lowest in China (5 %). The relative high deviations of energy consumptions and emissions reveal that the regional inequality on energy consumption evolves differently in regional economic development and geographical distribution.

The deviations can be further identified in Fig. 11.6, in which the ratios of GDP over energy consumptions and CO_2 emissions of China and its eight economic-geographic areas over 1997–2012 are presented. It can be seen that the economic well-developed south and east coast areas enjoyed relatively high desirable output-energy input (GDP-energy) ratios and desirable output-undesirable output (GDP-CO_2) ratios, while the economic less-developed northwest area suffered both the comparatively low GDP-energy ratio and low GDP-CO_2 ratio. In addition, the largest CO_2 emitter (middle Yellow River area) also experienced low GDP-CO_2 ratio. These phenomena reveal that China's coastal areas may have higher energy and environmental productivities over their western and middle counterparts.

Figure 11.6 also shows that, over the entire study period, most of China's eight economic-geographic areas have improvements on their ratios of GDP over energy and CO_2 emissions, which indicates that their GDP outputs grew faster than their energy inputs and carbon emissions. However, the improvements were not continuous, since there were temporal drop downs and fluctuations in the ratios from 2004 to 2006 and several individual years.

Based on these phenomena, we expect overall increases in energy and environmental productivities for most of China's economic-geographic areas over the entire study period, while the productivity changes over different FYP periods may vary.

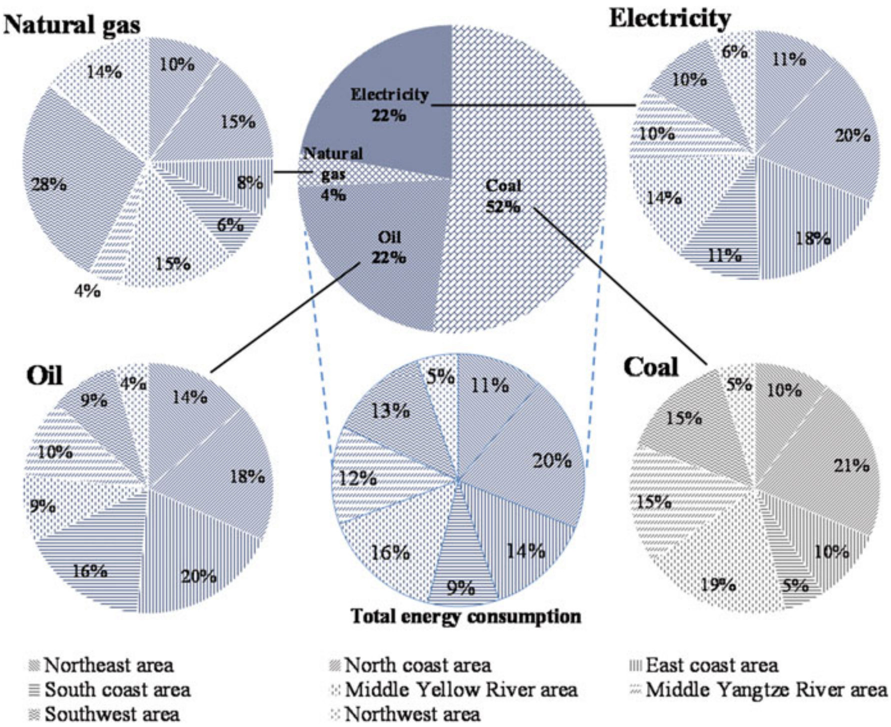


Fig. 11.5 Average energy consumption structure of China (1997–2012)

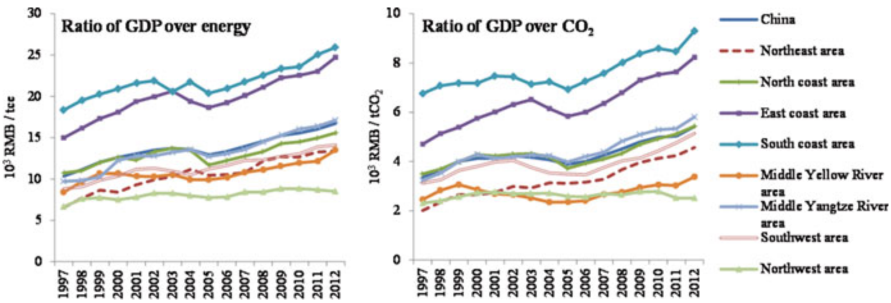


Fig. 11.6 Ratios of GDP over energy and CO_2 of China and its eight areas

11.4.2 Results and Discussions

We compute eight directional distance functions through Models (11.32), (11.34), (11.37) and (11.38), as well as the Luenberger productivity indicator and its decompositions through (11.26) to (11.29) respectively for each China's province at each 2-year period (e.g., 1997/1998, 2011/2012). The energy and environmental productivity indicators (*LP*) are obtained which identifies the energy and environmental productivity change over time. Then the *LP* is further decomposed from two perspectives: (i) The *LP* is decomposed into the measures of pure efficiency change, scale efficiency change, pure technical change, and scale technical change; (ii) The *LP* is decomposed into its specific energy inputs (coal, oil, natural gas and electricity) productivity changes and CO_2 emission undesirable output productivity change. The former decomposition helps to identify the effect of efficiency change or technical change on productivity change, while the latter decomposition helps to identify the contribution of the productivity change of a specific energy resource and carbon emission on the integrated energy and environmental productivity change.

Since the economic development levels and patterns, as well as the natural resources endowments of China's eight economic-geographic areas are quite different, it is expected that the energy and environmental productivity changes of these areas will be quite different, and furthermore, the driving force for their productivity changes and the contributions of different energy inputs towards productivity change for these areas also will be quite different. In this section, we analysis and compare the productivity indicators for China's eight economic-geographic areas during 1997–2012. The evaluation results are reported in Table 11.2.

Note that, in Table 11.2, the last column indicates the energy and environmental productivity change directly computed based on the results of Models (11.32) and (11.34) with the direction of $\left(g_e,\ g_x,\ g_y,\ g_b\right) = (e,\ 0,\ 0,\ b)$. For comparative analysis, we further compute (i) the integrated energy productivity change (reported in column three) based on Models (11.32) and (11.34) with the direction of $\left(g_e,\ g_x,\ g_y,\ g_b\right) = (e,\ 0,\ 0,\ 0)$ and a single inefficiency measure β; and (iii) the aggregated specific energy productivity change (reported in column four) based on Models (11.37) and (11.38) with the direction of $\left(g_e, g_x, g_y, g_b\right) = \left(\left(g_{e_1}, g_{e_2}, g_{e_3}, g_{e_4}\right), g_x, g_y, g_b\right) = ((e_1, e_2, e_3, e_4), 0, 0, 0)$, and four energy input specific inefficiency measure $\beta_h (h = 1, \ldots, 4)$. It can be seen that the effect of energy consumption structure (i.e., contribution of specific energy resource on productivity change) and the effect of undesirable output are not considered in the integrated energy productivity change measures. Moreover, the effect of energy consumption structure is taken in to account, but the effect of undesirable output are omitted in the aggregated specific energy productivity change measures. For the weights, we simply choose $w_e = w_b = 1/2, w_h = 1/4\ (h = 1, \ldots, 4)$, and $w_f = 1\ (f = 1)$, which indicate that the inefficiency measure of energy and carbon emissions have the same importance in the computation of energy and environmental productivity indicator, as well as the inefficiency measure of four specific energy resources have the same importance in the computations of both aggregated specific energy productivity indicator and energy and environmental productivity indicator.

Table 11.2 Productivity indicators of China's eight areas (1997–2012)

Indicator	Area	Integrated energy productivity change	Aggregated specific energy productivity change	Coal specific productivity change	Oil specific productivity change	Natural gas specific productivity change	Electricity specific productivity change	Energy and environmental productivity change
LP	Northeast area	0.1268	0.1288	−0.0287	0.0866	0.9032	0.3227	0.0123
	North coast area	0.0126	0.0368	−0.0717	0.0536	0.5368	0.1402	−0.1350
	East coast area	0.1223	0.0000	0.0000	0.0000	0.0000	0.0000	0.0128
	South coast area	−0.0656	−0.0124	−0.0136	−0.0182	−0.0356	−0.0010	−0.0789
	Middle Yellow River area	0.0291	−0.0805	−0.2274	−0.0442	0.1694	0.1341	−0.0090
	Middle Yangtze River area	0.0511	0.0684	−0.0600	0.1784	0.3797	0.1748	0.0170
	Southwest area	−0.0119	−0.1535	−0.3200	−0.0462	0.3147	0.0000	−0.1298
	Northwest area	−0.0246	−0.1391	−0.2573	0.2865	0.1523	−0.3013	0.0837
LPEC	Northeast area	0.1094	0.2327	0.1852	0.0480	0.8182	0.3582	0.0500
	North coast area	−0.0751	−0.1644	−0.3024	0.0072	0.0736	−0.0733	0.0100
	East coast area	0.0599	0.0000	0.0000	0.0000	0.0000	0.0000	0.0299

(continued)

Table 11.2 (continued)

Indicator	Area	Integrated energy productivity change	Aggregated specific energy productivity change	Coal specific productivity change	Oil specific productivity change	Natural gas specific productivity change	Electricity specific productivity change	Energy and environmental productivity change
	South coast area	−0.0589	0.0000	0.0000	0.0000	0.0000	0.0000	−0.0767
	Middle Yellow River area	0.0126	−0.2202	−0.3882	0.0201	−0.0635	−0.1075	0.1970
	Middle Yangtze River area	−0.0005	0.0552	−0.0042	0.0934	0.3122	0.0922	0.0344
	Southwest area	−0.1062	−0.0488	−0.1249	0.0123	−0.0176	0.0484	0.0047
	Northwest area	−0.0319	0.0569	0.0427	0.2103	0.2332	−0.0728	0.3360
LPTC	Northeast area	0.0122	−0.0466	−0.0906	0.0340	−0.0572	−0.0209	−0.0456
	North coast area	0.0894	0.1209	0.1725	0.0248	0.0673	0.1054	−0.0386
	East coast area	0.1406	0.0000	0.0000	0.0000	0.0000	0.0000	0.1472
	South coast area	−0.0085	−0.0037	−0.0028	−0.0050	−0.0171	−0.0019	−0.0039
	Middle Yellow River area	0.0069	0.0518	0.0759	−0.0281	−0.0337	0.0848	−0.1125
	Middle Yangtze River area	0.0000	−0.0263	−0.0628	0.0222	−0.0383	0.0104	−0.0394

	Southwest area	−0.0110	−0.0784	−0.1141	−0.0287	−0.0107	−0.0605	−0.1600
	Northwest area	−0.0551	−0.0671	−0.0886	0.0375	−0.1269	−0.0957	−0.2646
LSEC	Northeast area	−0.0141	−0.1281	−0.2866	0.0642	0.1736	−0.0254	−0.1104
	North coast area	0.0029	0.1387	0.0872	0.0419	0.6967	0.2075	−0.2085
	East coast area	−0.0831	0.0000	0.0000	0.0000	0.0000	0.0000	−0.1430
	South coast area	−0.0225	−0.0124	−0.0136	−0.0182	−0.0356	−0.0010	−0.0063
	Middle Yellow River area	−0.0099	0.1268	0.1275	−0.0643	0.4184	0.2200	−0.1703
	Middle Yangtze River area	0.0417	0.0579	0.0243	0.0850	0.1002	0.0949	0.0245
	Southwest area	0.0798	−0.1341	−0.2540	−0.0471	0.2731	−0.0453	0.0070
	Northwest area	−0.0508	−0.2313	−0.3734	0.0573	0.0085	−0.2258	−0.1077
LSTC	Northeast area	0.0194	0.0709	0.1633	−0.0597	−0.0314	0.0109	0.1184
	North coast area	−0.0046	−0.0585	−0.0290	−0.0203	−0.3009	−0.0994	0.1020
	East coast area	0.0048	0.0000	0.0000	0.0000	0.0000	0.0000	−0.0214
	South coast area	0.0243	0.0037	0.0028	0.0050	0.0171	0.0019	0.0081

(continued)

Table 11.2 (continued)

Indicator	Area	Integrated energy productivity change	Aggregated specific energy productivity change	Coal specific productivity change	Oil specific productivity change	Natural gas specific productivity change	Electricity specific productivity change	Energy and environmental productivity change
	Middle Yellow River area	0.0195	−0.0390	−0.0425	0.0281	−0.1518	−0.0631	0.0768
	Middle Yangtze River area	0.0099	−0.0185	−0.0173	−0.0222	0.0055	−0.0228	−0.0024
	Southwest area	0.0256	0.1078	0.1730	0.0173	0.0699	0.0574	0.0185
	Northwest area	0.1132	0.1024	0.1620	−0.0186	0.0375	0.0930	0.1200

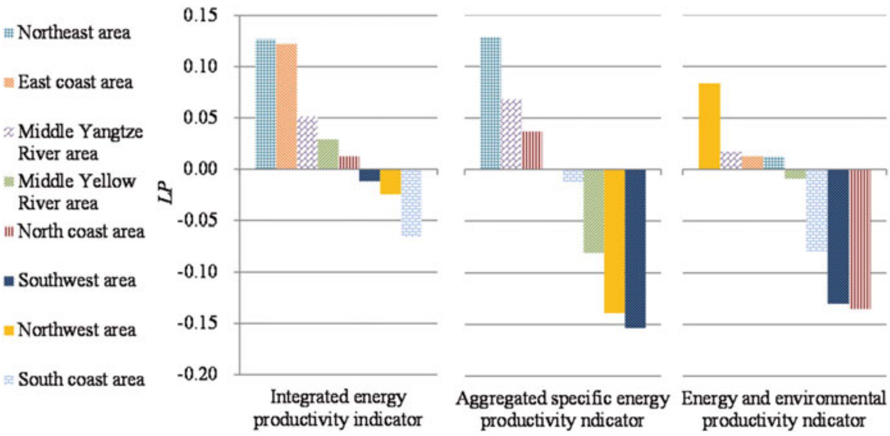


Fig. 11.7 *LP* based comparison of energy and environmental productivity changes of China's eight areas

The scores on *LP* of integrated energy productivity, aggregated specific energy productivity, and energy and environmental productivity of eight Chinese areas over the period of 1997–2012 are illustrated in Fig. 11.7.

With regard to the integrated energy productivity change, east coast area ranked first, followed by northeast area, middle Yangtze River area, middle Yellow River area, and north coast area. Their *LP* scores are all positive indicating integrated energy productivity progress over the study period, in which northeast area shows the highest increase of 12.7 %, and the increase in north coast area is 1.3 %. The remaining three areas all have negative *LP* scores indicating integrated energy productivity regress over the same period. Among which, south coast area experiences the most obvious regress of −6.6 %, followed by northwest area (−2.5 %) and southwest area (−1.2 %).

With respect to the energy and environmental productivity, northwest area has the most significant productivity progress (8.37 %), followed by middle Yangtze River area (1.7 %), east coast area (1.3 %), and northeast area (1.2 %). On the contrary, north coast area has the most obvious productivity regress (−13.5 %), followed by southwest area (−13.0 %), south coast area (−7.9 %), and middle Yellow River area (−0.9 %).

Compared with the integrated energy productivity, it can be found that, when taking the carbon emissions, i.e., environmental effect, into productivity change computation, almost all areas (seven out of eight areas) show productivity indicator reductions. This finding indicates that in general China's energy productivity growth may be overestimated over the period 1997–2012 if ignoring the carbon emission, since for China's most areas, the *LP* scores under energy and environmental productivity indicator are below than those under integrated energy productivity indicator. The only exception happens in northwest area whose energy and environmental productivity indicator is higher than its integrated energy productivity indicator. This indicates that, the measure of energy productivity that ignores undesirable outputs underestimates the "true" energy productivity growth in northwest area. One possible explanation for this underestimation is that environmental regulations, i.e., energy conservation and carbon emission reduction policies, play a positive impact on the energy utilization and production activities in this area over the study period. With the emissions reduction policies, part of energy resources are diverted from desirable output production to pollutant abatement activity, however, integrated energy productivity indicator ignores the positive effect of energy inputs for pollutant abatement activity, and assumes these energy inputs are unproductive for desirable output production. As a matter of fact, these energy resources are utilized for emission reduction and environment promotion through the encouragement of emission reduction policies of adopting pollutant abatement technology and management, or switching from traditional energy consumption process to cleaner production process. Therefore, integrated energy productivity fail to identify northwest area's effort on emission reduction and underestimate it "true" energy productivity growth.

When taking energy utilization structure into evaluation, the aggregated specific energy productivity shows a different distribution among China's eight areas over the period of 1997–2012. Firstly, northeast area (12.9 %), middle Yangtze River area (6.8 %), and north coast area (3.7 %) remain to have positive *LP* scores under aggregated specific energy productivity, which are higher than those under integrated energy productivity. Secondly, *LP* scores of east coast area and middle Yellow River area decline if energy utilization structure is taken into account. East coast area shows no change on aggregated specific energy productivity, and middle Yellow River area even switches to productivity decrease (−8.1 %). Thirdly, the remaining three areas still suffer aggregated specific energy productivity regresses and among which the regresses of northwest area (−13.9 %) and southwest area (−15.4 %) become more obvious. These comparative results reveal that energy consumption structure plays different roles in energy productivity change measures among different areas. It positively affects energy productivity change in China's economic well-developed northern and eastern areas (northeast, north coast, and middle Yangtze River), but negatively affects energy productivity change in China's economic less-developed western area (northwest and southwest). This difference may rise from the different contributions of *LPEC*, *LPTC*, *LSEC*, and *LSTC* to *LP* under the aggregated specific energy productivity among China's eight different areas, and may also cause by the different roles of individual energy inputs to energy productivity changes among China's eight different areas. To answer this question, we further provide a productivity decomposition analysis for different areas in the following part of this section.

Figure 11.8 illustrates the contributions of pure efficiency change, pure technical change, scale efficiency change, and scale technical change to aggregate specific energy productivity of China's eight areas. East coast area is omitted for its zero *LP* score over the study period. It can be seen that, first of all, the largest driving force for the productivity growths of northeast, north coast, and middle Yangtze River areas are different. Energy utilization efficiency growth, i.e., the catch-up effect, is

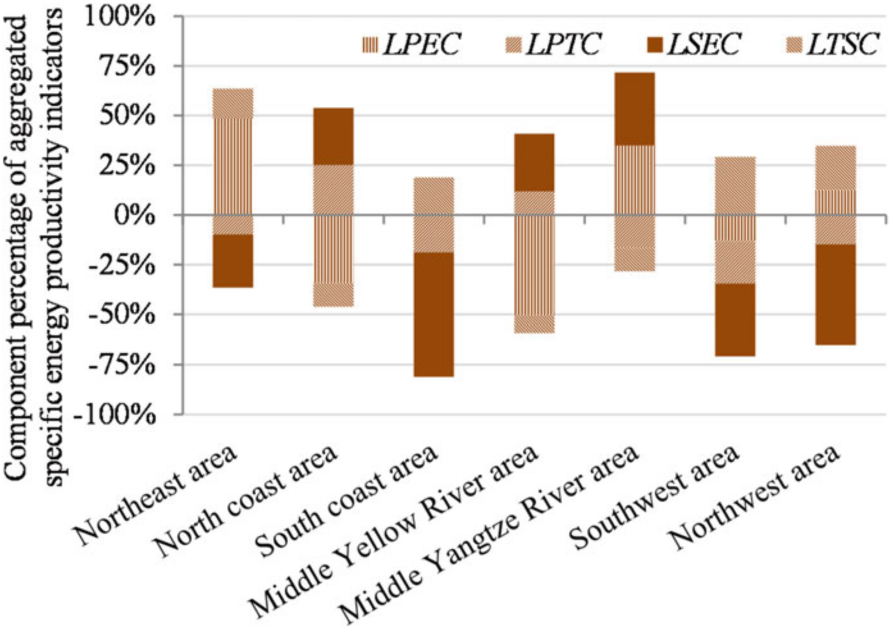


Fig. 11.8 Comparison of energy specific productivity and its decompositions of China's eight areas

the largest contributor in northeast area, which accounts for nearly 50 % of the growth. This finding indicates that in general, provinces in northeast area experienced comparatively higher increases in energy utilization efficiency than other part of China during 1997–2012. Scale efficiency change is the major driving force both in north coast area and middle Yellow River area, which takes approximate 25–35 % of the productivity growth. This result reveals that in general, the movements of approaching the optimal energy utilization and desirable output production scale are more obvious in provinces of these two areas than other parts of China during 1997–2012.

Secondly, scale efficiency change is the largest driving force for the productivity decrease in almost all of the areas shown aggregated specific energy productivity decline, except middle Yellow River area. It can be seen that in south coast, southwest, and northwest areas, the decreases on scale efficiency are obvious, which account for 30–55 % of the productivity decrease. However, the major driving force in Middle Yellow River area is pure efficiency change on energy utilization. These findings reveal that in general, provinces in south coast, southwest, and northwest areas experienced opposite movements against optimal production scale over the observation period, and provinces in middle Yellow River area suffered more significant comparative energy utilization efficiency reduction than other parts of China.

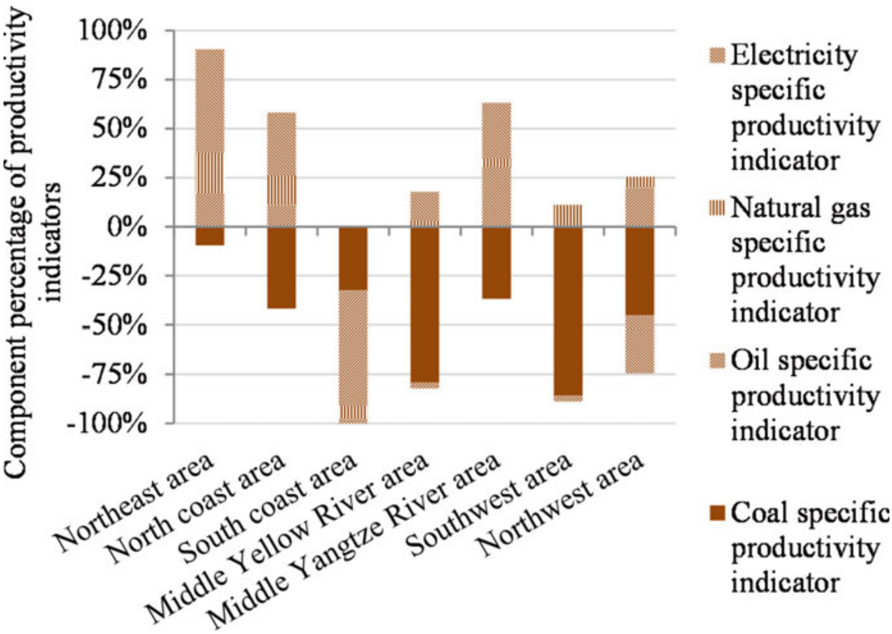


Fig. 11.9 Contribution of specific energy input to energy productivity of China's eight areas

Thirdly, change in returns to scale of the technology plays an obvious positive role on productivity change in China's economic less-developed west areas, although these areas show aggregate specific energy productivity decline during 1997–2012. Figure 11.9 shows the scores on *LSTC* which account for approximate 25 % of the productivity changes in southwest and northwest areas. This indicates that in general, energy utilization technology in these areas is moving towards constant returns to scale.

Figure 11.9 additionally illustrates the contributions of specific energy inputs, i.e., coal, oil, natural gas and electricity, to the energy productivity change during 1997–2012 for China's eight areas. Firstly, it is obvious that coal plays a negative role in energy productivity change in each of these areas, especially in southwest and middle Yellow River areas, the significant regress on coal specific productivity accounts for more than 75 % of energy productivity decrease of these two areas. Secondly, natural gas is measured as a positive contributor for energy productivity growth in almost all areas except south coast and east coast areas. Although the contribution of natural gas is not large compared with coal, oil and electricity, its productivity score is positive in six out of eight areas in China over the entire study period. Thus, increase the consumption percentage of natural gas will have a positive effect in China's energy productivity promotion. Thirdly, the role of oil specific productivity and electricity specific productivity are diversified in different areas. For example, oil specific productivity is the largest driving force for energy productivity decrease in south coast area, but the largest driving force for energy productivity increase in middle Yangtze River area. Similarly, electricity specific productivity is identified as the largest positive contributor of energy productivity change in northeast area, but it is also the second largest negative contributor of energy productivity change in northwest area.

11.5 Conclusions

This chapter proposed an input specific Luenberger energy and environmental productivity indicator for productivity change measurement, and the data envelopment analysis (DEA) approach is utilized to estimate the directional distance functions for composing the Luenberger energy and environmental productivity indicator. Then, the Luenberger productivity indicator is further decomposed from two perspectives. It is decomposed into four components that respectively measure the pure efficiency change, scale efficiency change, pure technical change, and scale technical change which identify the catch-up effect, frontier shift effect, and economy of scale towards energy and environmental productivity change. It is also decomposed into the specific energy inputs productivity changes which examine the different contribution of productivity change of each specific energy resource towards the integrated energy and environmental productivity change.

The Luenberger productivity indicator and its decompositions are applied to measure China's regional energy and environmental productivity change over 1997–2012 and the major results regarding to China's eight economic-geographic areas are as follows.

Firstly, energy consumption structure plays different roles in energy productivity measurement among different areas. It positively affects productivity change in China's economic well-developed areas but negatively affects productivity change in China's economic less-developed areas. Secondly, the overestimations of energy productivity when ignoring emissions are observed in seven out of eight economic-geographic areas in China over the study period, which indicates that the evidence of China does not fully support the Porter Hypothesis. Finally, specific energy input of coal plays a negative role in energy productivity growth in each of the eight areas, and natural gas is identified as the positive contributor for energy productivity growth in almost all eight areas. Thus, the enlarging of consumption percentage of natural gas will continuously help to promote China's energy productivity.

Acknowledgement Ke Wang acknowledges the financial support from the National Natural Science Foundation of China (No. 71471018 and No. 71101011).

References

Chambers, R. G., Färe, R., & Grosskopf, S. (1996). Productivity growth in APEC countries. *Pacific Economic Review, 1*(3), 181–190.

Chung, Y. H., Färe, R., & Grosskopf, S. (1997). Productivity and undesirable outputs: A directional distance function approach. *Journal of Environmental Management, 51*, 229–240.

Cooper, W. W., Seiford, L. M., & Zhu, J. (2011). *Handbook on data envelopment analysis* (Vol. 164). New York: Springer.

Färe, R., & Grosskopf, S. (2010). Directional distance functions and slacks-based measures of efficiency. *European Journal of Operational Research, 200*(1), 320–322.

Färe, R., Grosskopf, S., Lindgren, B., & Roos, P. (1989). *Productivity development in Swedish hospitals: A Malmquist output index approach*. Working Paper, Southern Illinois University.

Färe, R., Grosskopf, S., Lindgren, B., & Roos, P. (1992). Productivity changes in Swedish pharmacies 1980–1989: A non-parametric approach. *Journal of Productivity Analysis, 3* (1–2), 85–101.

Färe, R., Grosskopf, S., Lindgren, B., & Roos, P. (1994). Productivity developments in Swedish hospitals: A Malmquist output index approach. In A. Charnes, W. Cooper, A. Lewin, & L. Seiford (Eds.), *Data envelopment analysis: Theory, methodology, and applications*. The Netherlands: Springer.

Färe, R., Grosskopf, S., & Lovell, C. K. (1985). *The measurement of efficiency of production* (Vol. 6). New York: Springer.

Färe, R., Grosskopf, S., & Pasurka, C. A., Jr. (2001). Accounting for air pollution emissions in measures of state manufacturing productivity growth. *Journal of Regional Science, 41*, 381–409.

Färe, R., Grosskopf, S., & Pasurka, C. A. (2007a). Pollution abatement activities and traditional productivity. *Ecological Economics, 62*, 673–682.

Färe, R., Grosskopf, S., & Pasurka, C. A. (2007b). Environmental production functions and environmental directional distance functions. *Energy, 32*(7), 1055–1066.

Färe, R., Lovell, C. A. K., & Zieschang, K. (1982). Measuring the technical efficiency of multiple output technologies. In W. Eichhorn et al. (Eds.), *Quantitative studies in production and prices*. Wurzburg: Physica-Verlag.

Fukuyama, H., & Weber, W. L. (2009). A directional slacks-based measure of technical inefficiency. *Socio-Economic Planning Sciences, 43*, 274–287.

Hailu, A., & Veeman, T. S. (2001). Non-parametric productivity analysis with undesirable outputs: An application to the Canadian pulp and paper industry. *American Journal of Agriculture Economics, 83*, 605–616.

Kumar, S. (2006). Environmentally sensitive productivity growth: A global analysis using Malmquist-Luenberger index. *Ecological Economics, 56*, 280–293.

Mahlberg, B., & Luptacik, M. (2014). Eco-efficiency and eco-productivity change over time in a multisectoral economic system. *European Journal of Operational Research, 234*(3), 885–897.

Pastor, J. T., Ruiz, J. L., & Sirvent, I. (1999). An enhanced DEA Russell graph efficiency measure. *European Journal of Operational Research, 115*(3), 596–607.

Porter, M. E. (1990). The competitive advantage of nations. *Harvard Business Review, 68*, 73–93.

Simar, L., & Wilson, P. W. (1998). *Productivity growth in industrialized countries*. Discussion Paper \#9810, Institut de Statistique, Universit\'e Catholique de Louvain, Louvain-la-Neuve, Belgium.

Wang, K., Wei, Y. M., & Zhang, X. (2012). A comparative analysis of China's regional energy and emission performance: Which is the better way to deal with undesirable outputs? *Energy Policy, 46*, 574–584.

Wang, K., Wei, Y. M., & Zhang, X. (2013). Energy and emissions efficiency patterns of Chinese regions: A multi-directional efficiency analysis. *Applied Energy, 104*, 105–116.

Zhang, C., Liu, H., Bressers, H., & Buchanan, K. S. (2011). Productivity growth and environmental regulations—Accounting for undesirable outputs: Analysis of China's thirty provincial regions using the Malmquist-Luenberger index. *Ecological Economics, 70*, 2369–2379.

Zofio, J. (2007). Malmquist productivity index decompositions: A unifying framework. *Applied Economics, 39*(18), 2371–2387.

Zofio, J., & Lovell, C. A. K. (1998). *Yet another Malmquist productivity index decomposition*, Mimeo. Department of Economics, Universidad Autónoma de Madrid, Madrid, University of Georgia, Athens, Georgia.

Zhu, J. (2015). *Data envelopment analysis. A handbook of models and methods* (International series in operations research & management science, Vol. 221). New York: Springer.

Chapter 12
Identifying the Global Reference Set in DEA: An Application to the Determination of Returns to Scale

Mahmood Mehdiloozad and Biresh K. Sahoo

Abstract In data envelopment analysis (DEA), any set of observed decision making units (DMUs) that produce a projection point of an inefficient DMU is called a *reference set* of this DMU. Based on this definition, however, the concept of reference set is not mathematically well defined in the non-radial DEA setting. This is because a given projection point may be generated by multiple *unary reference sets*, and different projection points may result in multiple *maximal reference sets*. In this chapter, first, we address this issue by differentiating between the uniquely found reference set, called the *global reference set (GRS)*, and the unary and maximal types of the reference set for which the multiplicity issue may occur. Second, to identify the GRS, we propose a general linear programming based approach that is computationally more efficient than its alternatives. Third, we define the returns to scale (RTS) of an inefficient DMU at its projection point that is produced by *all*—but not some—of the units in its GRS. By this definition, the notion of RTS is unambiguous, since the GRS is unique and the projection points generated by all the possible reference units all exhibit the same type of RTS. Fourth, using a non-radial DEA model, we develop two precise multiplier- and envelopment-based methods to determine RTS possibilities of the DMUs. To demonstrate the ready applicability of our approach, we finally conduct an empirical analysis based on a real-life data set.

Keywords Data envelopment analysis • Linear programming • Global reference set • Minimum face • Returns to scale

M. Mehdiloozad
Department of Mathematics, College of Sciences, Shiraz University, Shiraz 71454, Iran
e-mail: m.mehdiloozad@gmail.com

B.K. Sahoo (✉)
Xavier Institute of Management, Xavier University, Bhubaneswar 751013, India
e-mail: biresh@ximb.ac.in

S.-N. Hwang et al. (eds.), *Handbook of Operations Analytics Using Data Envelopment Analysis*, International Series in Operations Research & Management Science 239, DOI 10.1007/978-1-4899-7705-2_12

12.1 Introduction

Data envelopment analysis (DEA), introduced by Charnes et al. (1978, 1979), is a linear programming (LP) based method for measuring the relative efficiency of a homogeneous group of decision making units (DMUs) with multiple inputs and multiple outputs. Based on observed data and a set of postulates, DEA defines a reference technology set relative to which a DMU can be classified as *efficient* or *inefficient*. To evaluate the performance of an inefficient DMU, each standard DEA model projects it—in a unique or multiple way(s)—onto efficiency frontier of the technology set. As the benchmark-units of the inefficient DMU, it also identifies a set of observed efficient DMUs that produce its projection point. This identified set is referred to as a *reference (or peer) set* of the inefficient DMU and each of its elements is called a *reference DMU*.

The identification of *all* the possible reference DMUs of an inefficient unit is an important issue in DEA. This issue has received significant attention in the literature due to its wide range of potential applications in ranking (Jahanshahloo et al. 2007), benchmarking and target setting (Bergendahl 1998; Camanho and Dyson 1999), and measuring returns to scale (RTS) (Tone 1996, 2005; Tone and Sahoo 2006; Cooper et al. 2007; Sueyoshi and Sekitani 2007a, b; Krivonozhko et al. 2014; Mehdiloozad et al. 2015; Mehdiloozad and Sahoo 2015).

From a managerial perspective, identification of all the reference DMUs is specifically important for two reasons. First, to improve the performance of an inefficient DMU, it may not be practical to introduce an unobserved (virtual) projection as its benchmark. In such a situation, however, the identification offers the possibility to derive practical guidelines from benchmarking against the reference DMUs. Second, when some—but not all—reference DMUs are identified for an inefficient DMU, the decision maker may be still of the opinion that the identified reference DMUs are not appropriate benchmarks and may wish to have more options in choosing benchmarks. In such a case, the identification allows him/her to incorporate the preference information into analysis in order to yield a projection with the most preferred (i) closeness (Tone 2010), (ii) values of inputs and outputs, and (iii) shares of reference units in its formation.

The pioneering attempt to find out all the reference DMUs via non-radial DEA models was made by Sueyoshi and Sekitani (2007b). Based on the strong complementary slackness conditions[1] of linear programming, they proposed a primal–dual based method by means of the *range-adjusted measure* (RAM) model of Cooper et al. (1999). The proposed method in their impressive study is very interesting as a theoretical idea. However, as Krivonozhko et al. (2012b) argued, not only the computational burden of Sueyoshi and Sekitani's (2007b) approach is high, but it also seems that the basic matrices defined in their approach are likely to be ill-conditioned, which may lead to erroneous and unacceptable results even for

[1] For more details about these conditions, the interested readers may refer to Mehdiloozad et al. (2016).

medium-size problems. Furthermore, the economic interpretation of some of the constraints of their proposed model does not make much sense. In a conscious attempt to overcome these difficulties, Krivonozhko et al. (2014) later proposed another primal–dual based procedure. Although their proposed procedure outperforms the approach of Sueyoshi and Sekitani (2007b), it requires solving several LP problems.

The studies by Sueyoshi and Sekitani (2007b) and Krivonozhko et al. (2014) correctly found all the observed DMUs on *minimum face*—a face of minimum dimension on which all the projection points are located—as a *unique* reference set of the DMU under evaluation. At the same time, they pointed out that the occurrence of *multiple reference sets* (we call this as the *multiplicity* issue) was likely. However, none of these studies explicitly made a clear distinction between the uniquely found reference set and other types of reference set for which the issue of multiplicity might occur. This lack of discrimination creates an ambiguity about the uniqueness, and consequently, about the mathematical well-definedness of the concept of reference set.

In a more recent study, Mehdiloozad et al. (2015) effectively eliminated this ambiguity by defining three types of the reference set in a mathematically well-defined manner. To identify all the reference units of an inefficient DMU, they then proposed an LP model. In this regard, Mehdiloozad (2016) subsequently formulated a mixed 0–1 LP model and demonstrated that the LP model of Mehdiloozad et al. (2015) can be alternatively derived from his model. As an important application for finding all the reference units, Mehdiloozad et al. (2015) and Mehdiloozad and Sahoo (2015) developed two RTS measurement methods, one by applying the BCC model (Banker et al. 1984) and the other by the CCR (Charnes et al. 1978) model.

In this chapter, we discuss and present the results established by Mehdiloozad et al. (2015), Mehdiloozad (2016), and Mehdiloozad and Sahoo (2015) in two parts, with Part I dealing with the identification of all the reference DMUs, and Part II with the measurement of RTS.

Part I: On Identification of the Global Reference Set

First, we identify potential sources of the origin of the multiplicity issue and define three types of the reference set. The first source of the origin is that a given projection may be generated by multiple convex combinations of the observed DMUs. In other words, the presence of alternative optimal intensity vectors for the given projection (hereafter referred to as problem Type I) is the first source of the origin. We call the reference sets identified by such vectors as the *unary reference sets (URSs)*. That is, we define a URS as the set of efficient DMUs that are active in a specific convex combination producing the given projection. To deal with problem Type I, we define the notion of *maximal reference set (MRS)* as the union of all the URSs associated with the given projection. The second source of the origin in the RAM model is the occurrence of multiple projections (hereafter referred to as problem Type II). To deal with problem Type II, we further define the union of all the MRSs as the unique *global reference set (GRS)* of the evaluated DMU. Note that

- the three introduced concepts are all mathematically well defined.
- the URS and MRS help demonstrate the occurrence of the multiplicity issue associated with a single and multiple projection(s), respectively.
- while both URS and MRS may face with the issue of multiplicity, the GRS presents a unique reference set that contains all the possible reference DMUs.

Then, as to the linkage between the GRS and the minimum face, we prove that the convex hull of the GRS is equal to the minimum face. As to our formulation of the set of all possible optimal intensity vectors of the RAM model, we show that the GRS can be identified by finding a *maximal element* of this set—an element with the maximum number of positive components. Based on this finding, we formulate a mixed 0–1 LP model for identifying the GRS and then, transform it into an equivalent LP model using the LP relaxation method.

The proposed approach has several important features. First, it can deal effectively with the simultaneous occurrence of problems Type I and II. Second, this approach is computationally more efficient than its two primal–dual based counterparts. This is because it involves solving a single LP problem, and it is based on the primal (envelopment) form that is computationally more efficient than the dual (multiplier) form (Cooper et al. 2007). Third, the computational efficiency can be enhanced by using the simplex algorithm[2] adopted for solving the LP problems with upper-bounded variables. Fourth, the developed approach can be used readily in the 'additive model' (Charnes et al. 1985), the 'BAM model' (Pastor 1994; Pastor and Ruiz 2007; Cooper et al. 2011), the 'RAM/BCC model' (Aida et al. 1998), the 'DSBM model' (Jahanshahloo et al. 2012), the 'GMDDF model' (Mehdiloozad et al. 2014), and any radial DEA model like the 'BCC model'. Fifth, the proposed approach is free from the restricting assumption that the input–output data must be non-negative, so it can deal effectively with negative data. Finally, this approach that is developed based on the assumption of variable RTS can be successfully adapted to the case of constant RTS, just by removing the convexity constraint.

[2] The simplex algorithm for bounded variables was published by Dantzig (1955) and was independently developed by Charnes and Lemke (1954). This algorithm is much more efficient than the ordinary simplex algorithm for solving the LP problem with upper-bounded variables (Winston 2003).

Part II: On Determination of the RTS

As is well known in the literature,[3] the concept of RTS is meaningful only when the relevant DMU lies on efficiency frontier of the technology set. With regard to this, the RTS of an inefficient DMU is determined at its projection point on the efficiency frontier. Obviously, if the used projection point is unique, the RTS of this inefficient DMU is determined without any ambiguity. However, under the occurrence of problem Type II, determining the RTS uniquely and, consequently, achieving the mathematical precision of this definition are not guaranteed, and these may lead to erroneous inferences as to RTS possibilities of the inefficient DMUs. This is because multiple projections may reveal different types of RTS for an inefficient DMU. Without consideration of this fact, any RTS determination method may yield conflicting inferences on RTS possibilities for the inefficient DMUs facing with multiple projections.

To resolve the above-mentioned issue, the RTS must be defined over a subset of the *projection set*—the set of all the possible projection points—that its elements all exhibit the same RTS possibility. In this regard, Krivonozhko et al. (2012c) have shown that all relative interior points of the minimum face operate under the same type of RTS. This interesting finding reveals that the definition of RTS for an inefficient DMU can be made unambiguous by requiring its projection point to be in the relative interior of the associated minimum face. Hence, following Krivonozhko et al. (2014), Mehdiloozad et al. (2015) and Mehdiloozad and Sahoo (2015), we define the RTS of an inefficient DMU over the intersection of the projection set with the relative interior of the minimum face.

Based on this definition, we develop two precise RTS determination algorithms by using the LP model proposed for finding the GRS. The first one is a three-stage algorithm that uses the multiplier form of the BCC model, whereas the second one is a two-stage algorithm that applies the envelopment form of the CCR model. On a comparison between these two algorithms, the second one is computationally more efficient than the first one.

The remainder of this chapter unfolds as follows. Section 12.2 deals with description of the technology followed by a brief review of the RAM model. Section 12.3, first, presents the three notions URS, MRS, and GRS; second, investigates the properties of the GRS; third, proposes an approach for the identification of the GRS; fourth, discusses the properties of this approach; and finally, elaborates on the proposed approach with a numerical example. Section 12.4, first, defines the RTS of the inefficient DMUs; second, develops two RTS determination algorithms; and finally, presents illustrations of these algorithms. Section 12.5

[3] See, e.g., Banker et al. (1996), Førsund (1996), Sahoo et al. (1999), Fukuyama (2000, 2001, 2003), Tone and Sahoo (2003, 2004, 2005), Sengupta and Sahoo (2006), Sahoo (2008), Podinovski et al. (2009), Podinovski and Førsund (2010), Sahoo et al. (2012), Sahoo and Tone (2013, 2015), Sahoo and Sengupta (2014), Sahoo et al. (2014a, b), among others.

provides an illustrative empirical application. Section 12.6 presents summary of the chapter with some concluding remarks.

12.2 Background

As far as notations are concerned, let $\mathbb{R}_+^d$ be the non-negative Euclidean d-orthant. We denote vectors and matrices in bold letters, vectors in lower case and matrices in upper case. All vectors are column vectors. We denote the transpose of vectors and matrices by a superscript T. We use $\mathbf{0}_d$ and $\mathbf{1}_d$ to show d-dimensional vectors with the values of 0 and 1 in every entry, respectively. The identity matrix of order d is also symbolized by $\mathbf{I}_d$.

Throughout this chapter, we consider a set of n observed DMUs (with the index set J), where each uses m inputs to produce s outputs. We denote, respectively, the input and output vectors of each DMU$_j$ ($j \in J$) by $\mathbf{x}_j = \left(x_{1j}, \ldots, x_{mj}\right)^T \in \mathbb{R}_+^m$ and $\mathbf{y}_j = \left(y_{1j}, \ldots, y_{sj}\right)^T \in \mathbb{R}_+^s$, and the input and output matrices by $\mathbf{X} = [\mathbf{x}_1 \ \ldots \ \mathbf{x}_n]$ and $\mathbf{Y} = [\mathbf{y}_1 \ \ldots \ \mathbf{y}_n]$. We also consider DMU$_o$ ($o \in J$) to be the DMU under evaluation.

12.2.1 Technology Set

The technology set, T, is defined as the set of all feasible input–output combinations, i.e.,

$$T = \left\{(\mathbf{x}, \mathbf{y}) \in \mathbb{R}_+^m \times \mathbb{R}_+^s \,|\, \mathbf{x} \text{ can produce } \mathbf{y}\right\}. \tag{12.1}$$

Under the variable returns to scale (VRS) framework, the nonparametric DEA representation of T can be set up as follows (Banker et al. 1984):

$$T_{VRS}^{DEA} = \left\{(\mathbf{x}, \mathbf{y}) \in \mathbb{R}_+^m \times \mathbb{R}_+^s \,\middle|\, \mathbf{X}\boldsymbol{\lambda} \leq \mathbf{x} \ , \ \mathbf{Y}\boldsymbol{\lambda} \geq \mathbf{y}, \ \mathbf{1}_n^T\boldsymbol{\lambda} = 1, \ \boldsymbol{\lambda} \geq \mathbf{0}_n\right\}. \tag{12.2}$$

In the following definition, we introduce two basic notions. For more details, see Rockafellar (1970) and Davtalab-Olyaie et al. (2015).

Definition 1 Let $H = \{(\mathbf{x}, \mathbf{y}) |\, \mathbf{u}^T(\mathbf{y} - \overline{\mathbf{y}}) - \mathbf{v}^T(\mathbf{x} - \overline{\mathbf{x}}) = 0\}$ be a supporting hyperplane of T_{VRS}^{DEA} at $(\overline{\mathbf{x}}, \overline{\mathbf{y}})$. Then, H and its corresponding face, i.e., $F = H \cap T_{VRS}^{DEA}$, are called *strong* if and only if components of the coefficient vectors $\mathbf{u}$ and $\mathbf{v}$ are all positive.

12.2.2 The RAM Model

In reference to T_{VRS}^{DEA}, the RAM model of Cooper et al. (1999) is set up as

$$\begin{array}{ll} \rho_o = \min & 1 - \dfrac{1}{m+s}\left(\mathbf{w}^{-T}\mathbf{s}^- + \mathbf{w}^{+T}\mathbf{s}^+\right) \\ \text{subject to} & \\ \mathbf{X}\boldsymbol{\lambda} + \mathbf{s}^- = \mathbf{x}_o, & \\ \mathbf{Y}\boldsymbol{\lambda} - \mathbf{s}^+ = \mathbf{y}_o, & \\ \mathbf{1}_n^T\boldsymbol{\lambda} = 1, & \\ \boldsymbol{\lambda} \geq \mathbf{0}_n,\ \mathbf{s}^- \geq \mathbf{0}_m,\ \mathbf{s}^+ \geq \mathbf{0}_s, & \end{array} \tag{12.3}$$

where $\mathbf{s}^-$ and $\mathbf{s}^+$ represent, respectively, the input and output slack vectors; and, $\mathbf{w}^-$ and $\mathbf{w}^+$ are defined, respectively, as

$$\begin{array}{ll} \dfrac{1}{w_i^-} = \max\limits_{j\in J}\left\{x_{ij}\right\} - \min\limits_{j\in J}\left\{x_{ij}\right\}, & i = 1, \ldots, m; \\ \dfrac{1}{w_r^+} = \max\limits_{j\in J}\left\{y_{rj}\right\} - \min\limits_{j\in J}\left\{y_{rj}\right\}, & r = 1, \ldots, s. \end{array} \tag{12.4}$$

Let $\left(\boldsymbol{\lambda}^*, \mathbf{s}^{-*}, \mathbf{s}^{+*}\right)$ be an optimal solution to model (12.3). Then, the *efficiency* and *improvement* of DMU$_o$ are defined as follows.

Definition 2 (RAM-efficiency) DMU$_o$ is said to be *efficient* if and only if $\rho_o = 1$, i.e., $\mathbf{s}^{-*} = \mathbf{0}_m$ and $\mathbf{s}^{+*} = \mathbf{0}_s$. Otherwise, it is called inefficient.

Definition 3 (RAM-improvement) For an inefficient DMU$_o$, a *projection* is defined by

$$P := \left(\mathbf{x}_o^{\mathrm{RAM}}, \mathbf{y}_o^{\mathrm{RAM}}\right) = \left(\mathbf{X}\boldsymbol{\lambda}^*, \mathbf{Y}\boldsymbol{\lambda}^*\right) = \left(\mathbf{x}_o - \mathbf{s}^{-*}, \mathbf{y}_o + \mathbf{s}^{+*}\right). \tag{12.5}$$

The set of all the possible projection points, denoted by Λ_o, is called the *projection set*. It can be easily verified that $\Lambda_o \subseteq E$, where E represents the set of all the efficient DMUs. Henceforth, $\mathbf{X}_E$ and $\mathbf{Y}_E$ denote the input and output matrices of the efficient DMUs, respectively; and, e denotes the cardinality of E. For sake of convenience and without loss of generality, we assume that $E = \{(\mathbf{x}_1, \mathbf{y}_1), \ldots, (\mathbf{x}_e, \mathbf{y}_e)\}$.

12.3 Identifying the Global Reference Set (GRS)

12.3.1 Definition of the GRS

Here, we present some key definitions, concepts and results, which are all essential in the sequel.

Definition 4 Let $\boldsymbol{\lambda}^*$ be a partial optimal solution to model (12.3) that is associated with a given projection P. We define the set of DMUs with positive λ_j^* as the *unary reference set (URS)* for DMU$_o$ and denote it by R_{oP}^U as

$$R_{oP}^U = \left\{ \left(\mathbf{x}_j, \mathbf{y}_j\right) \middle| \lambda_j^* > 0 \right\}. \tag{12.6}$$

We refer to each member of R_{oP}^U as a *reference DMU* of DMU$_o$. The reference units of DMU$_o$ are all efficient, and are all located on the supporting hyperplane (s) of T_{VRS}^{DEA} at P.

Since the projection P may be expressed as multiple convex combinations of the observed DMUs, multiple optimal values may take place for the intensity vector $\boldsymbol{\lambda}$ (problem Type I), leading to the occurrence of multiple URSs.[4] To deal with problem Type I, we define a reference set containing all the URSs.

Definition 5 We define the union of *all* the URSs associated with a given projection P as the *maximal reference set (MRS)* for DMU$_o$ and denote it by R_{oP}^M as

$$\begin{aligned} R_{oP}^M &= \cup R_{oP}^U \\ &= \left\{ \left(\mathbf{x}_j, \mathbf{y}_j\right) \middle| \lambda_j^* > 0 \text{ in some optimal solution of model (12.3) associated with } P \right\}. \end{aligned} \tag{12.7}$$

Because the RAM model is non-radial in nature, it may produce multiple projections (problem Type II) for DMU$_o$, resulting thus in the occurrence of multiple MRSs.[5] To deal with problem Type II, we use the concept of *minimum face*[6] that was discussed in detail by Sueyoshi and Sekitani (2007b) and Krivonozhko et al. (2014) from different perspectives. Toward this, first we define Ω_o as the set of intensity variables that are associated with all the optimal solutions of model (12.3), that is,

$$\Omega_o : = \left\{ \boldsymbol{\mu} \middle| \begin{bmatrix} \mathbf{X}_E & \mathbf{I}_m & \mathbf{0}_{m\times s} \\ \mathbf{Y}_E & \mathbf{0}_{s\times m} & -\mathbf{I}_s \\ \mathbf{1}_e^T & \mathbf{0}_m^T & \mathbf{0}_s^T \\ \mathbf{0}_e^T & \mathbf{w}^{-T} & \mathbf{w}^{+T} \end{bmatrix} \begin{bmatrix} \boldsymbol{\mu} \\ \mathbf{s}^- \\ \mathbf{s}^+ \end{bmatrix} = \begin{bmatrix} \mathbf{x}_o \\ \mathbf{y}_o \\ 1 \\ (m+s)(1-\rho_o) \end{bmatrix}, \begin{bmatrix} \boldsymbol{\mu} \\ \mathbf{s}^- \\ \mathbf{s}^+ \end{bmatrix} \geq \mathbf{0}_{\mathrm{e+m+s}} \right\}. \tag{12.8}$$

[4] Under such an occurrence, the determination of RTS via Tone's (1996, 2005) method may be problematic. For a detailed discussion on this issue, interested readers may refer to the illustrative Figs. 1 and 2 in Krivonozhko et al. (2012c).

[5] The occurrence of multiple MRSs is illustrated via an example in Sect. 3 in Sueyoshi and Sekitani (2007b).

[6] For a graphical illustration of the minimum face, see Fig. 4 in Sueyoshi and Sekitani (2007b).

Note that each $\boldsymbol{\mu}$ in Ω_o is an e-dimensional intensity vector whose components are associated with the efficient DMUs. If $\boldsymbol{\mu} \in \Omega_o$, the n-dimensional vector $\boldsymbol{\lambda}^*_{\boldsymbol{\mu}} = \begin{pmatrix} \boldsymbol{\mu} \\ \mathbf{0}_{n-e} \end{pmatrix}$ is an optimal intensity vector for model (12.3), and vice versa.

From (12.8), the projection set Λ_o can be expressed as follows:

$$\Lambda_o = \{(\mathbf{X}_E\boldsymbol{\mu}, \mathbf{Y}_E\boldsymbol{\mu}) |\ \boldsymbol{\mu} \in \Omega_o\}. \tag{12.9}$$

As demonstrated by Krivonozhko et al. (2014), there exists a face of minimum dimension, $\Gamma_o^{\min}$, which contains Λ_o. This face is referred to as the minimum face and is, indeed, the intersection of all the faces of T_{VRS}^{DEA} that contain Λ_o, i.e.,

$$\Gamma_o^{\min} = \bigcap_{\substack{F \text{ is a face of } T_{VRS}^{DEA} \\ \text{and } \Lambda_o \subseteq F}} F. \tag{12.10}$$

Now, we provide the following definition that takes the occurrence of multiple MRSs associated with multiple projections into consideration.

Definition 6 We define the union of *all* the MRSs of DMU$_o$ as its *global reference set (GRS)* and denote it by R_o^G as

$$\begin{aligned} R_o^G &= \bigcup_{P \in \Lambda_o} R_{oP}^M \\ &= \left\{ \left(\mathbf{x}_j, \mathbf{y}_j\right) \middle|\ \lambda_j^* > 0 \text{ in some optimal solution of model (12.3)} \right\}. \end{aligned} \tag{12.11}$$

12.3.2 Properties of the GRS

In this section, we investigate some important properties of the GRS. First, we provide the following lemma to geometrically characterize the GRS.

Lemma 1 The convex hull of the GRS, $conv(R_o^G)$, is a strong face of T_{VRS}^{DEA}.

Proof It is easy to verify that Λ_o is a convex set. Let $(\bar{\mathbf{x}}, \bar{\mathbf{y}})$ be a relative interior point of this set. Since $(\bar{\mathbf{x}}, \bar{\mathbf{y}})$ is efficient, the strong complementary slackness conditions of linear programming imply the existence of a strong supporting hyperplane of T_{VRS}^{DEA} at this point. Without loss of generality, let H^S be such a hyperplane whose associated strong face $F^S := H^S \cap T_{VRS}^{DEA}$ is of minimum dimension. By Theorem 6.4 in Rockafellar (1970), the convexity of Λ_o implies that H^S is binding on all the DMUs in R_o^G. Therefore, $conv\left(R_o^G\right) \subseteq H^S$, indicating that $conv\left(R_o^G\right) \subseteq F^S$.

To prove the equality, first note that F^S is a polytope (bounded polyhedral set) as per Theorem 2 in Davtalab-Olyaie et al. (2014). Hence, the equality holds if $\Lambda_o \cap ri(F^S) \neq \emptyset$, where $ri(F^S)$ denotes the relative interior of F^S. This is because this relation implies that all the observed DMUs on F^S belong to R_o^G. Assume on the contrary, that $\Lambda_o \cap ri(F^S) = \emptyset$ or, equivalently, that $\Lambda_o \subseteq \partial(F^S)$. Then, there exists a unique face of F^S of minimum dimension, namely K^S, for which $\Lambda_o \subseteq K^S \subseteq \partial(F^S) \subsetneq F^S$. Following Rockafellar (1970), K^S is a strong face of T_{VRS}^{DEA} that contains Λ_o. Since the dimension of K^S is less than that of F^S, we have a contradiction. Thus, the proof is complete by Definition 1. ∎

Applying the above lemma, the following theorem establishes a linkage between the GRS and the minimum face. Precisely, it shows that the GRS spans the minimum face.

Theorem 1 The minimum face is equal to the convex hull of the GRS, i.e., $\Gamma_o^{\min} = conv(R_o^G)$.

Proof As proved in Lemma 1, $conv(R_o^G)$ is a strong face of T_{VRS}^{DEA} that contains Λ_o. Thus, according to (12.10), it will suffice to show that $conv(R_o^G) \subseteq \Gamma_o^{\min}$. By the definition of a face, there exists a supporting hyperplane, namely $H^{\min}$, such that $\Gamma_o^{\min} = H^{\min} \cap T_{VRS}^{DEA}$. Since $H^{\min}$ is binding at each projection, it passes through each DMU in R_o^G. Thus, by the convexity of $H^{\min}$, we have that $conv(R_o^G) \subseteq H^{\min}$, which completes the proof. ∎

As an immediate corollary of Theorem 1, the minimum face can be expressed in terms of the units in the GRS.

Corollary 1 The minimum face is a polytope that can be explicitly represented as

$$\Gamma_o^{\min} = \left\{ (\mathbf{x}, \mathbf{y}) \;\middle|\; \mathbf{x} = \sum_{j \in J_o^G} \delta_j \mathbf{x}_j,\ \mathbf{y} = \sum_{j \in J_o^G} \delta_j \mathbf{y}_j,\ \sum_{j \in J_o^G} \delta_j = 1,\ \delta_j \geq 0,\ \forall j \in J_o^G \right\}, \tag{12.12}$$

where J_o^G is the index set of the units in R_o^G.

We now turn to present a theorem that plays an important role in developing an approach to finding the GRS in the immediately subsequent section. Toward this, we first provide the following lemma as a straightforward consequence of Definitions 5 and 6 and (12.9).

Lemma 2 DMU_j lies in R_o^G if and only if $\mu_j > 0$ in some $\boldsymbol{\mu} \in \Omega_o$. That is,

$$R_o^G = \bigcup_{\boldsymbol{\mu} \in \Omega_o} \left\{ (\mathbf{x}_j, \mathbf{y}_j) \,\middle|\, \mu_j > 0 \right\}. \tag{12.13}$$

Theorem 2 Let $\boldsymbol{\mu}_o^{\max}$ be a *maximal element* of Ω_o—an element with the maximum number of positive components. Then,

$$R_o^G = \left\{ \left(\mathbf{x}_j, \mathbf{y}_j \right) \middle| \, \mu_{oj}^{\max} > 0 \right\}. \tag{12.14}$$

Proof By assumption, we have $\boldsymbol{\mu}_o^{\max} \in \Omega_o$. Hence, from equation (12.13), we need only to prove that $R_o^G \subseteq \left\{ \left(\mathbf{x}_j, \mathbf{y}_j \right) \middle| \, \mu_{oj}^{\max} > 0 \right\}$. Again from (12.13), this is equivalent to demonstrating that $\boldsymbol{\mu}_o^{\max}$ takes positive values in any positive component of any $\boldsymbol{\mu} \in \Omega_o$.

By contradiction, assume that there exists an element $\overline{\boldsymbol{\mu}} \in \Omega_o$ and an index $j_0 \in \left\{ j \middle| \, \overline{\mu}_j > 0 \right\}$ for which $\mu_{oj_0}^{\max} = 0$. Further, let $\hat{\boldsymbol{\mu}}$ be a strict convex combination of $\boldsymbol{\mu}_o^{\max}$ and $\overline{\boldsymbol{\mu}}$. Then, $\hat{\boldsymbol{\mu}} \in \Omega_o$ since Ω_o is a convex set. Moreover, $\left\{ j \middle| \, \hat{\mu}_j > 0 \right\} = \left\{ j \middle| \, \mu_{oj}^{\max} > 0 \right\} \cup \left\{ j \middle| \, \overline{\mu}_j > 0 \right\}$ and $\left\{ j \middle| \, \mu_{oj}^{\max} > 0 \right\} \subsetneqq \left\{ j \middle| \, \hat{\mu}_j > 0 \right\}$, accordingly. This contradicts the maximality of $\boldsymbol{\mu}_o^{\max}$ and, thus, the proof is complete. ∎

Theorem 2 shows that the problem of identifying the GRS is the same as the problem of finding a maximal element of Ω_o.

12.3.3 Identification of the GRS

As shown in the previous section, identifying the GRS is equivalent to finding a maximal element of Ω_o. To find such an element, we develop the following simple algorithm:

Stage 1 (*Initialization*) Let $J_1 = E$ and set $t = 1$.
Stage 2 Solve the following LP problem:

$$\begin{array}{ll} [\mathrm{LP}_t] & \max \displaystyle\sum_{j \in J_t} \mu_j \\ & \text{subject to} \\ & \boldsymbol{\mu} \in \Omega_o. \end{array}$$

Stage 3 Let $\boldsymbol{\mu}^t$ be an optimal solution to LP_t and set $J_{t+1} := J_t - \left\{ j \middle| \, \mu_j^t > 0 \right\}$.
Stage 4 If $\displaystyle\sum_{j \in J_t} \mu_j^t > 0$, set $t \leftarrow t + 1$ and return to Stage 2; otherwise, the algorithm is terminated.

If the algorithm terminates at iteration T, then it returns $\boldsymbol{\mu}_o^{\max}$ as $\boldsymbol{\mu}_o^{\max} = \frac{1}{T}\sum_{t=1}^{T}\boldsymbol{\mu}^t$. While the implementation of the above algorithm is straightforward, it may require solving many problems when n is considerably large. This motivates us to propose a new approach that requires the execution of a single LP problem. Toward this end, we set up the following mixed 0–1 LP problem:

$$\begin{aligned}
&\max \;\; \mathbf{1}^T\boldsymbol{\alpha} + \sigma \\
&\text{subject to} \\
&\begin{bmatrix} \mathbf{X}_E & \mathbf{I}_m & \mathbf{0}_{m\times s} \\ \mathbf{Y}_E & \mathbf{0}_{s\times m} & -\mathbf{I}_s \\ \mathbf{1}_e^T & \mathbf{0}_m^T & \mathbf{0}_s^T \\ \mathbf{0}_e^T & \mathbf{w}^{-T} & \mathbf{w}^{+T} \end{bmatrix} \begin{bmatrix} \boldsymbol{\mu} \\ \mathbf{s}^- \\ \mathbf{s}^+ \end{bmatrix} - \begin{bmatrix} \mathbf{x}_o \\ \mathbf{y}_o \\ 1 \\ (m+s)(1-\rho_o) \end{bmatrix} \delta = \mathbf{0}_{m+s+2}, \\
&\boldsymbol{\alpha} \le \boldsymbol{\mu},\; \sigma \le \delta, \\
&\boldsymbol{\alpha} : \text{binary},\; \sigma : \text{binary}, \\
&\boldsymbol{\mu} \ge \mathbf{0}_e,\; \mathbf{s}^- \ge \mathbf{0}_m,\; \mathbf{s}^+ \ge \mathbf{0}_s,\; \delta \ge 0.
\end{aligned} \tag{12.15}$$

The idea behind developing model (12.15) originates from two points. First, $\alpha_j > 0$ ($\sigma > 0$) implies that $\mu_j > 0$ ($\delta > 0$). Second, since $\boldsymbol{\alpha}$ and σ are both binary, maximizing $\mathbf{1}^T\boldsymbol{\alpha} + \sigma$ results in the identification of $\boldsymbol{\mu}_o^{\max}$, which is formally demonstrated below.

Lemma 3 For any $\boldsymbol{\mu} \in \Omega_o$, there exists a feasible solution $\left(\boldsymbol{\mu}', \mathbf{s}^{-'}, \mathbf{s}^{+'}, \delta', \boldsymbol{\alpha}', \sigma'\right)$ with $\sigma' = 1$ to model (12.15) such that $\mathbf{1}^T\boldsymbol{\alpha}' = n^+(\boldsymbol{\mu})$, where $n^+(\cdot)$ denotes the number of positive components of a vector.

Proof of Lemma 3 Let $\boldsymbol{\mu} \in \Omega_o$ and define l as

$$l := \min\{\mu_j | \mu_j > 0\}, \;\; k := \begin{cases} 1 & \text{if } \; l \ge 1, \\ \frac{1}{l} & \text{if } \; l < 1. \end{cases}$$

Then, from the definition of Ω_o in (12.8), the solution $\left(\boldsymbol{\mu}', \mathbf{s}^{-'}, \mathbf{s}^{+'}, \delta', \boldsymbol{\alpha}', \sigma'\right)$ defined by

$$98\%\boldsymbol{\mu}' := k\boldsymbol{\mu},\; \mathbf{s}^{-'} := k\mathbf{s}^-,\; \mathbf{s}^{+'} := k\mathbf{s}^+,\; \delta' := k,\; \alpha_j' := \begin{cases} 1 & \mu_j' > 0, \\ 0 & \mu_j' = 0, \end{cases} \;\; \sigma' := 1 \tag{12.16}$$

is feasible to model (12.15) and $\mathbf{1}_e^T\boldsymbol{\alpha}' = n^+(\boldsymbol{\mu})$. ∎

By using Lemma 3, the following theorem shows that $\boldsymbol{\mu}_o^{\max}$ can be found by virtue of an optimal solution of model (12.15).

Theorem 3 Let $\left(\boldsymbol{\lambda}^*, \mathbf{s}^{-*}, \mathbf{s}^{+*}, \delta^*, \boldsymbol{\alpha}^*, \sigma^*\right)$ be an optimal solution to model (12.15). Then, $\boldsymbol{\mu}_o^{\max} = \frac{1}{\delta^*}\boldsymbol{\mu}^*$.

Proof of Theorem 3 Let $(\boldsymbol{\mu}^*, \mathbf{s}^{-*}, \mathbf{s}^{+*}, \delta^*, \boldsymbol{\alpha}^*, \sigma^*)$ be an optimal solution to model (12.8). Since the first $m+s+2$ constraints of model (12.8) constitute a homogeneous system of linear equalities, it can be proved (by the way of contradiction) that $\mu_j^* \geq 1$ for any j that $\mu_j^* > 0$. Since model (12.8) is a maximization LP problem, we therefore have $\alpha_j^* = 1$ for any j that $\mu_j^* > 0$. This indicates that $n^+(\boldsymbol{\mu}^*) = \mathbf{1}_e^T\boldsymbol{\alpha}^*$.

We claim that $\delta^* > 0$. To prove our claim, assume on the contrary that $\delta^* = 0$. Then, the constraints of model (12.8) imply that $\sigma^* = 0$. Now, consider an arbitrary element $\boldsymbol{\mu}$ of Ω_o. Then, the solution $\left(\boldsymbol{\mu}', \mathbf{s}^{-'}, \mathbf{s}^{+'}, \delta', \boldsymbol{\alpha}', \sigma'\right)$ defined by (12.16) is feasible to model (12.8). Consequently, the solution $\left(\hat{\boldsymbol{\mu}}, \hat{\mathbf{s}}^-, \hat{\mathbf{s}}^+, \hat{\delta}, \hat{\boldsymbol{\alpha}}, \hat{\sigma}\right)$ defined by

$$\left(\hat{\boldsymbol{\mu}}, \hat{\mathbf{s}}^-, \hat{\mathbf{s}}^+, \hat{\delta}, \hat{\boldsymbol{\alpha}}, \hat{\sigma}\right) := \left(\boldsymbol{\mu}^* + \boldsymbol{\mu}', \hat{\mathbf{s}}^{-*} + \mathbf{s}^{-'}, \hat{\mathbf{s}}^{+*} + \mathbf{s}^{+'}, \delta', \boldsymbol{\alpha}^*, 1\right) \tag{11}$$

is feasible to model (12.8) and its corresponding objective function value is strictly greater than $\mathbf{1}_e^T\boldsymbol{\alpha}^*$. This contradicts the optimality of $(\boldsymbol{\mu}^*, \mathbf{s}^{-*}, \mathbf{s}^{+*}, \delta^*, \boldsymbol{\alpha}^*, \sigma^*)$ and hence, proves our claim.

By dividing both sides of the first $m+s+2$ constraints of model (12.8) at optimality by δ^*, we have $\frac{1}{\delta^*}\boldsymbol{\mu}^* \in \Omega_o$, implying $n^+\left(\boldsymbol{\mu}^*\right) \leq n^+\left(\boldsymbol{\mu}_o^{\max}\right)$. Therefore, the equality holds immediately by Lemma 3. ∎

Theorem 3 follows that the GRS can be found with the help of model (12.15). However, as is known, this method is not computationally efficient when the size of the model is considerably large. In what follows, we deal effectively with this issue by demonstrating that the LP relaxation of model (12.15) provides an equivalent LP model, which is computationally more efficient and, hence, practically more relevant.

Theorem 4 Model (12.15) is equivalent to the following LP model:

$$\begin{aligned}
&\max \;\; \mathbf{1}^T\boldsymbol{\alpha} + \sigma \\
&\text{subject to} \\
&\begin{bmatrix} \mathbf{X}_E & \mathbf{I}_m & \mathbf{0}_{m\times s} \\ \mathbf{Y}_E & \mathbf{0}_{s\times m} & -\mathbf{I}_s \\ \mathbf{1}_e^T & \mathbf{0}_m^T & \mathbf{0}_s^T \\ \mathbf{0}_e^T & \mathbf{w}^{-T} & \mathbf{w}^{+T} \end{bmatrix} \begin{bmatrix} \boldsymbol{\mu} \\ \mathbf{s}^- \\ \mathbf{s}^+ \end{bmatrix} - \begin{bmatrix} \mathbf{x}_o \\ \mathbf{y}_o \\ 1 \\ (m+s)(1-\rho_o) \end{bmatrix} \delta = \mathbf{0}_{m+s+2}, \\
&\boldsymbol{\alpha} \leq \boldsymbol{\mu}, \; \sigma \leq \delta, \\
&\mathbf{0}_e \leq \boldsymbol{\alpha} \leq \mathbf{1}_e, \; 0 \leq \sigma \leq 1, \\
&\boldsymbol{\mu} \geq \mathbf{0}_e, \; \mathbf{s}^- \geq \mathbf{0}_m, \; \mathbf{s}^+ \geq \mathbf{0}_s, \; \delta \geq 0.
\end{aligned} \tag{12.17}$$

Proof Since model (12.17) is the LP relaxation of model (12.15), the optimal objective function value of model (12.17) is an upper bound to that of model (12.15). Hence, by letting $\left(\boldsymbol{\mu}^*, \mathbf{s}^{-*}, \mathbf{s}^{+*}, \delta^*, \boldsymbol{\alpha}^*, \sigma^*\right)$ be an optimal solution to model (12.17), it will suffice to show that $\sigma^* = 1$ and $\alpha_j^* = 1$ for any j that $\alpha_j^* > 0$.

Since Ω_o is a non-empty convex set and model (12.17) is a maximization LP problem, it can be easily verified—by the way of contradiction—that $\sigma^* > 0$. We claim that $\sigma^* = 1$. To prove our claim, assume by contradiction that $\sigma^* < 1$. Then, dividing both sides of the constraints of model (12.17) at optimality by σ^* yields that the vector $\left(\boldsymbol{\mu}', \mathbf{s}^{-\prime}, \mathbf{s}^{+\prime}, \delta', \alpha', \sigma'\right)$, defined by

$$\boldsymbol{\mu}' := \frac{1}{\sigma^*}\boldsymbol{\mu}^*, \ \mathbf{s}^{-\prime} := \frac{1}{\sigma^*}\mathbf{s}^{-*}, \ \mathbf{s}^{+\prime} := \frac{1}{\sigma^*}\mathbf{s}^{+*}, \ \delta' := \frac{\delta^*}{\sigma^*}, \ \alpha_j' := \min\left\{1, \frac{\alpha_j^*}{\sigma^*}\right\}, \ \sigma' := 1, \tag{12.18}$$

is a feasible solution to model (12.17). Since $\alpha_j' \geq \alpha_j^*$ for any $j = 1, \ldots, e$ and $\sigma' > \sigma^*$, the objective function value associated with this solution is strictly greater than $\mathbf{1}^T\boldsymbol{\alpha}^* + \sigma^*$. This contradicts the optimality of $\left(\boldsymbol{\lambda}^*, \mathbf{s}^{-*}, \mathbf{s}^{+*}, \delta^*, \boldsymbol{\alpha}^*, \sigma^*\right)$ and proves our claim. In a similar way, it can also be proved that $\alpha_j^* = 1$ for any j that $\alpha_j^* > 0$ and so the proof is complete. ∎

Notice that, as per Theorems 3 and 4, $\boldsymbol{\mu}_o^{\max}$ can be identified via the LP relaxation model (12.17). To reduce the number of constraints, we now make the substitutions $\boldsymbol{\beta} := \boldsymbol{\mu} - \boldsymbol{\alpha}$ and $\varphi := \delta - \sigma$ in model (12.17) that transform it to the following upper-bounded LP model:

$$\begin{aligned}
&\max \ \mathbf{1}^T\boldsymbol{\alpha} + \sigma \\
&\text{subject to} \\
&\begin{bmatrix} \mathbf{X}_E & \mathbf{I}_m & \mathbf{0}_{m\times s} \\ \mathbf{Y}_E & \mathbf{0}_{s\times m} & -\mathbf{I}_s \\ \mathbf{1}_e^T & \mathbf{0}_m^T & \mathbf{0}_s^T \\ \mathbf{0}_e^T & \mathbf{w}^{-T} & \mathbf{w}^{+T} \end{bmatrix} \begin{bmatrix} \boldsymbol{\alpha} + \boldsymbol{\beta} \\ \mathbf{s}^- \\ \mathbf{s}^+ \end{bmatrix} - \begin{bmatrix} \mathbf{x}_o \\ \mathbf{y}_o \\ 1 \\ (m+s)(1-\rho_o) \end{bmatrix} (\sigma + \varphi) = \mathbf{0}_{m+s+2}, \\
&\mathbf{0}_e \leq \boldsymbol{\alpha} \leq \mathbf{1}_e, \ 0 \leq \sigma \leq 1, \\
&\boldsymbol{\beta} \geq \mathbf{0}_e, \ \mathbf{s}^- \geq \mathbf{0}_m, \ \mathbf{s}^+ \geq \mathbf{0}_s, \ \varphi \geq 0.
\end{aligned} \tag{12.19}$$

Let $\left(\boldsymbol{\alpha}^*, \boldsymbol{\beta}^*, \mathbf{s}^{-*}, \mathbf{s}^{+*}, \sigma^*, \varphi^*\right)$ be an optimal solution to model (12.19). From Theorems 3 and 4, we then have $\boldsymbol{\mu}_o^{\max} = \frac{1}{\sigma^* + \varphi^*}\left(\boldsymbol{\alpha}^* + \boldsymbol{\beta}^*\right)$. The projection associated with $\boldsymbol{\mu}_o^{\max}$ is given by

$$P_o^{\max} = \left(\mathbf{x}_o^{\max}, \mathbf{y}_o^{\max}\right) = \left(\mathbf{X}_E\boldsymbol{\mu}_o^{\max}, \mathbf{Y}_E\boldsymbol{\mu}_o^{\max}\right), \tag{12.20}$$

which lies in the relative interior of $\Gamma_o^{\min}$ by Corollary 1.

12.3.4 Properties of the Proposed Approach

Some useful properties of the proposed approach are presented below.

- *Computational efficiency*

  In any practical application that involves determination of the GRS, the proposed approach is computationally more efficient than the ones by Sueyoshi and Sekitani (2007b) and Krivonozhko et al. (2014) due to two reasons. First, it involves the execution of a single LP problem. Second, unlike the previous primal–dual based methods, it is based only on the primal (envelopment) form that is computationally more efficient than the dual (multiplier) form (Cooper et al. 2007).

  Note that since model (12.19) contains several upper-bounded variables, its computational efficiency can be enhanced by using the simplex algorithm adopted for solving the LP problems with upper-bounded variables. This is because considering model (12.19) as an LP problem with upper-bounded variables leads to a further reduction in its size. More precisely, the size of the basic matrices during the solution process becomes $(m + s + 2) \times (m + s + 2)$, which is one times greater than that of the size of the basic matrices in model (12.3).
- *Extension to other DEA models*

  The proposed approach can readily be used without any change in the 'additive model' (Charnes et al. 1985) and the 'BAM model' (Pastor 1994; Pastor and Ruiz 2007; Cooper et al. 2011). This is because the difference between each of these two models and the RAM model lies only in the objective-function weights associated with the input and output slacks. With some minor changes, it can also be adopted for the 'RAM/BCC model' of Aida et al. (1998), the 'DSBM model' of Jahanshahloo et al. (2012) and the 'GMDDF model' of Mehdiloozad et al. (2014). Furthermore, it can be implemented in any radial DEA model, as has been used in Mehdiloozad and Sahoo (2015).[7]
- *Extension to constant returns to scale case*

  The assumption of VRS is maintained in the proposed approach. This is because when a data set contains some negative values, one may not be able to define an efficient frontier, passing through the origin, as is assumed under constant returns to scale (CRS). Therefore, as argued by Silva Portela and Thanassoulis (2010), the assumption of CRS is untenable with negative data. It is, however, worth noting that while the minimum face is a polytope in the VRS-based technology, it is an unbounded polyhedral cone in the CRS-based technology that is generated by the reference units in the GRS. Despite this structural difference between the two technologies,[8] the presented results can

[7] To the best of our knowledge, the other studies on identification of all the possible reference DMUs using the BCC model include Sueyoshi and Sekitani (2007a), Jahanshahloo et al. (2008), Krivonozhko et al. (2012a), and Roshdi et al. (2014).

[8] For more details about the facial structure of the CRS- and VRS-based technologies, see, e.g., Davtalab-Olyaie et al. (2014a; b) and Jahanshahloo et al. (2013).

still be successfully adapted to the case of CRS by removing the convexity constraint, i.e., $\mathbf{1}_n^T\boldsymbol{\lambda} = 1$. This is because the proposed approach is primarily based on finding a maximal element of a convex set, and is independent of the existence of the convexity constraint, accordingly.

- *Dealing with negative input–output data*

  Being independent of the data sets used, the proposed approach is free from the restricting assumption that the input–output data must be non-negative, which makes identification of the GRS possible in the presence of negative data. From a practical point of view, this can be very beneficial since negative inputs or outputs may appear in many real-life applications.[9]

12.3.5 Numerical example

We consider a data set from Mehdiloozad et al. (2015) that is exhibited in Table 12.1. It consists of eight hypothetical DMUs with one input and one output. Based on these data, Fig. 12.1 depicts the frontier spanned in the two-dimensional input–output space.

To illustrate the application of the approach proposed in Sect. 12.3.3, we first evaluate each DMU using the RAM model. Table 12.2 exhibits the efficiency score and the projection point for each DMU. The results reveal that DMU_1, DMU_2, DMU_3, and DMU_4 form the efficient frontier, and are, hence, efficient. Amongst the inefficient DMUs (DMU_5, DMU_6, DMU_7, and DMU_8), DMU_8 has the minimum efficiency score of $\rho_8 = 0.643$.

To comprehend the results better, we first illustrate the geometric properties of the GRS and the minimum face. As far as DMU_5 and DMU_6 are concerned, their corresponding projections are unique (i.e., DMU_4 for DMU_5 and DMU_2 for DMU_6). For units such as DMU_7 and DMU_8, DMU_3 is found to be one of their projections. Moreover, since none of these units is uniquely projected, the projection sets of DMU_7 and DMU_8 are, respectively, the line segment connecting DMU_2 and DMU_3 and the line segment connecting DMU_2 and DMU_4 ($w_1^- = 1/7$ and $w_1^+ = 1/7$). Therefore,

Table 12.1 Input and output data for Example 12.3.5

	DMU_1	DMU_2	DMU_3	DMU_4	DMU_5	DMU_6	DMU_7	DMU_8
Input	1	2	3	5	8	2	3	6
Output	2	5	6	8	8	1	3	4

[9] While dealing with the estimation of a piecewise log-linear technology, one may encounter negative data since the log transformation of values less than 1 are always negative (Mehdiloozad et al. 2014). One may also refer to, e.g., Pastor and Ruiz (2007), Sahoo and Tone (2009) and Sahoo et al. (2012), among others, for several examples of applications with negative data.

Fig. 12.1 The production frontier

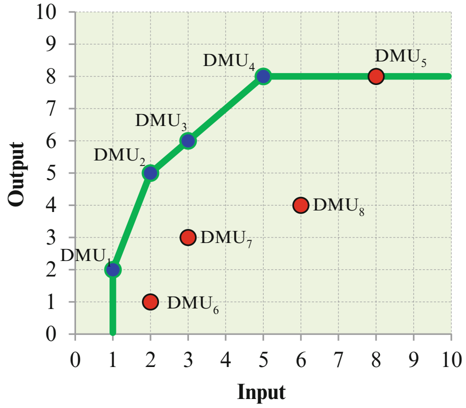


Table 12.2 The results obtained from the RAM model

	DMU_1	DMU_2	DMU_3	DMU_4	DMU_5	DMU_6	DMU_7	DMU_8
ρ_o	1	1	1	1	0.786	0.714	0.786	0.643
$(\boldsymbol{x}_o^{\mathrm{RAM}}, \boldsymbol{y}_o^{\mathrm{RAM}})$	DMU_1	DMU_2	DMU_3	DMU_4	DMU_4	DMU_2	DMU_3	DMU_4

Table 12.3 The GRSs of the inefficient DMUs

		DMU_5	DMU_6	DMU_7	DMU_8
	$(\boldsymbol{x}_o^{\max}, \boldsymbol{y}_o^{\max})$	DMU_4	DMU_2	DMU_3	(3.333,6.333)
Reference weights	$\lambda_1^{\max}$				
	$\lambda_2^{\max}$		1	0.500	0.333
	$\lambda_3^{\max}$			0.250	0.333
	$\lambda_4^{\max}$	1		0.250	0.333

the minimum face associated with each of these units is the line segment connecting DMU_2 and DMU_4. This indicates that some units with different projection sets may have the same minimum face.

Having obtained the efficiency score for each DMU, we use model (12.19) to identify its GRS and determine a relative interior point of its corresponding minimum face. The results are all presented in Table 12.3.

Since DMU_5 and DMU_6 have unique, efficient projection points (i.e., DMU_4 for DMU_5 and DMU_2 for DMU_6), the GRS for each of these units is the same as its unique projection point. Formally, $J_5^G = \{4\}$ and $J_6^G = \{2\}$.

Now, consider the case of DMU_7 suffering from the simultaneous occurrence of problems Type I and II:

- Type I: The sets $\{DMU_3\}$, $\{DMU_2, DMU_4\}$ and $\{DMU_2, DMU_3, DMU_4\}$ are the three URSs for DMU_7 associated with the projection DMU_3. So, the MRS of DMU_7 associated with DMU_3 is $\{DMU_2, DMU_3, DMU_4\}$.
- Type II: Λ_7 is the line segment connecting DMU_2 and DMU_3.

As can be seen in Table 12.2, the GRS of DMU_7 consists of DMU_2, DMU_3 and DMU_4 with the respective weights of 0.5, 0.25 and 0.25, i.e., $J_7^G = \{2, 3, 4\}$. This finding confirms to our Theorem 1 and its corollary, i.e., the convex hull of DMU_2, DMU_3 and DMU_4 is $\Gamma_7^{\min}$. Moreover, DMU_3 is determined as a relative interior point of $\Gamma_7^{\min}$.

DMU_2, DMU_3 and DMU_4 with the respective weights of 0.5, 0.25 and 0.25 also constitute the GRS of DMU_8, i.e., $J_8^G = \{2, 3, 4\}$.

12.4 Determination of Returns to Scale (RTS)

12.4.1 Definition of RTS for an Inefficient DMU

As is well known, the concept of RTS is meaningful only if the relevant DMU is efficient. Based on this, the standard approach followed in the DEA literature for evaluating the RTS of an inefficient DMU is first to project it onto the efficiency frontier and then to determine its RTS at its projection point $(\mathbf{x}_o^{\text{RAM}}, \mathbf{y}_o^{\text{RAM}})$ as defined in (12.5). However, determining the RTS uniquely and, consequently, achieving the mathematical preciseness of this standard definition cannot be always guaranteed since multiple projections may reveal different types of RTS for the DMU under evaluation.

Therefore, in order to resolve this issue, the RTS must be defined over a subset of Λ_o that its elements all exhibit the same RTS possibility. To accomplish the task, we resort to the concept of minimum face. As demonstrated by Krivonozhko et al. (2012c), all relative interior points of the minimum face operate under the same type of RTS. Thus, following Krivonozhko et al. (2014), Mehdiloozad et al. (2015), and Mehdiloozad and Sahoo (2015), the RTS of an inefficient DMU is well defined over the intersection of Λ_o with the relative interior of the minimum face. Based on this fact, we present the following precise definition of RTS for the inefficient DMUs.

Definition 7 The RTS of an inefficient DMU is defined at $P_o^{\max} = \left(\mathbf{x}_o^{\max}, \mathbf{y}_o^{\max}\right)$, as given in (12.20).

12.4.2 Determination of RTS Via the BCC Model

The multiplier form of the BCC model (Banker et al. 1984) for DMU_o is set up as:

$$\begin{array}{l} \max \quad \mathbf{u}^T\mathbf{y}_o - u_0 \\ \text{subject to} \\ \mathbf{v}^T\mathbf{x}_o = 1, \\ \mathbf{u}^T\mathbf{Y} - \mathbf{v}^T\mathbf{X} - \mathbf{u}_0\mathbf{1}_n \leq \mathbf{0}_n, \\ \mathbf{u} \geq \mathbf{0}_s, \ \mathbf{v} \geq \mathbf{0}_m, \ u_0 : \text{free in sign.} \end{array} \tag{12.21}$$

To determine the RTS possibility of DMU_o, one needs to find the lower and upper bounds of u_0 in the set of all the optimal solutions of model (12.21). Let $\underline{u}_0$ and $\bar{u}_0$ be, respectively, the min and max of u_0, which can be obtained from the following models:

$$\begin{aligned}
&\underline{u}_0(\overline{u}_0) = \min(\max) \quad u_0 \\
&\text{subject to} \\
&\mathbf{u}^T\mathbf{y}_o - u_0 = 1, \\
&\mathbf{v}^T\mathbf{x}_o = 1, \\
&\mathbf{u}^T\mathbf{Y}_E - \mathbf{v}^T\mathbf{X}_E - \mathbf{u}_0\mathbf{1}_e^T \le \mathbf{0}_e^T, \\
&\mathbf{u} \ge \mathbf{0}_s, \ \mathbf{v} \ge \mathbf{0}_m, \ u_0 : \text{free in sign.}
\end{aligned} \tag{12.22}$$

Then, the following theorem identifies RTS based on the signs of $\underline{u}_0$ and $\bar{u}_0$.

Theorem 5 (Banker and Thrall 1992; Banker et al. 2004) Let DMU_o be efficient. Then,

(i) increasing RTS prevail at DMU_o if and only if $\overline{u}_0 < 0$.
(ii) constant RTS prevail at DMU_o if and only if $\underline{u}_0 \le 0 \le \overline{u}_0$.
(iii) decreasing RTS prevail at DMU_o if and only if $\underline{u}_0 > 0$.

In this theorem, DMU_o is assumed to be efficient; otherwise, it is replaced by its projection point defined in (12.5). Based on Definition 5 and Theorem 5, we design the following three-stage algorithm to determine the RTS possibility of DMU_o:

Algorithm I
Stage 1: Solve the RAM model for $(\mathbf{x}_o, \mathbf{y}_o)$.

- If $(\mathbf{x}_o, \mathbf{y}_o)$ is efficient, go to Stage 2.
- Else, solve model (12.19) and obtain $(\mathbf{x}_o^{\max}, \mathbf{y}_o^{\max})$ from (12.20). Then, replace $(\mathbf{x}_o, \mathbf{y}_o)$ with $(\mathbf{x}_o^{\max}, \mathbf{y}_o^{\max})$ and go to Stage 2.

Stage 2: Solve the minimization form of model (12.22) for $(\mathbf{x}_o, \mathbf{y}_o)$.

- If $\underline{u}_0 = 0$, then constant RTS prevail.
- Else if $\underline{u}_0 > 0$, decreasing RTS prevail.
- Else, go to Step 3.

Stage 3: Solve the maximization form of model (12.22) for $(\mathbf{x}_o, \mathbf{y}_o)$.

- If $\overline{u}_0 < 0$, increasing RTS prevail.
- Else, constant RTS prevail.

Algorithm I is based on a multiplier-based DEA model and determines the RTS of a DMU by examining the intercept(s) of the supporting hyperplane(s) at, the given DMU if it is efficient or, its projection point if it is inefficient. Obviously, Algorithm I encounters no problem as long as there is a unique supporting hyperplane. Moreover, as we show below, it precisely determines RTS under the occurrence of multiple supporting hyperplanes (problem Type III).

First, let DMU_o be efficient. Then, the face of minimum dimension containing DMU_o, denoted by $F_{(\mathbf{x}_o,\mathbf{y}_o)}$, is the intersection of T_{VRS}^{DEA} with all the supporting hyperplanes at $(\mathbf{x}_o, \mathbf{y}_o)$. Hence, if the dimension of $F_{(\mathbf{x}_o,\mathbf{y}_o)}$ is $m + s - 1$—i.e., it is a 'Full Dimensional Efficient Facet' (Olesen and Petersen 1996, 2003), then there is a unique supporting hyperplane at $(\mathbf{x}_o, \mathbf{y}_o)$. Otherwise, problem Type III arises due to the non-full dimensionality of $F_{(\mathbf{x}_o,\mathbf{y}_o)}$ as the unique source of its origin. According to Theorem 5, Algorithm I deals with the difficulty arising from this source in its second and third stages.

For an inefficient DMU, problem Type III originates from two sources. The first one is the non-full dimensionality of $F_{P_o^{\max}} = \Gamma_o^{\min}$, which is circumvented in the second and third stages of Algorithm I. The second one is Problem Type II in which case, multiple projections are associated with multiple MRSs, and each MRS characterizes at least one supporting hyperplane at the corresponding projection. The effective way to address the difficulty arising from Problem Type II is to determine RTS using the supporting hyperplanes that are characterized by all of the MRSs, i.e., the GRS, but not by any specific MRS. This is done in the first stage of Algorithm I by applying the projection point $P_o^{\max}$.

12.4.3 Determination of RTS Via the CCR Model

Let us consider the envelopment form of the CCR model (Charnes et al. 1978), as given in the following form:

$$\begin{aligned} \theta_o^{\mathrm{CCR}} = \min \quad & \theta \\ & \text{subject to} \\ & \mathbf{X\lambda} \geq \theta \mathbf{x}_o, \\ & \mathbf{Y\lambda} \leq \mathbf{y}_o, \\ & \boldsymbol{\lambda} \geq \mathbf{0}_n. \end{aligned} \tag{12.23}$$

We refer to θ_o^{CCR} as the CCR-efficiency score. The following theorem enables us to determine the RTS of DMU_o by looking at the optimal solution of model (12.23).

Theorem 6 (Zarepisheh et al. 2006) Let DMU_o be efficient. Then,

(i) constant RTS prevail at DMU_o if and only if $\theta_o^{\mathrm{CCR}} = 1$.
(ii) decreasing RTS prevail at DMU_o if and only if $\theta_o^{\mathrm{CCR}} < 1$ and $\mathbf{1}_n^T\boldsymbol{\lambda}^* > 1$ in any optimal solution of model (12.23).
(iii) increasing RTS prevail at DMU_o if and only if $\theta_o^{\mathrm{CCR}} < 1$ and $\mathbf{1}_n^T\boldsymbol{\lambda}^* < 1$ in any optimal solution of model (12.23).

To apply this theorem for an inefficient DMU, it is replaced by its projection point defined in (12.5). Based on Definition 5 and Theorem 6, we propose the following two-stage algorithm to determine the RTS possibility of DMU_o:

Algorithm II

Stage 1: Solve the RAM model for $(\mathbf{x}_o, \mathbf{y}_o)$.

- If $(\mathbf{x}_o, \mathbf{y}_o)$ is efficient, go to Stage 2.
- Else, solve model (12.19) and obtain $(\mathbf{x}_o^{\max}, \mathbf{y}_o^{\max})$ from (12.20). Then, replace $(\mathbf{x}_o, \mathbf{y}_o)$ with $(\mathbf{x}_o^{\max}, \mathbf{y}_o^{\max})$ and go to Stage 2.

Stage 2: Solve the CCR model for $(\mathbf{x}_o, \mathbf{y}_o)$.

- If $\theta_o^{\text{CCR}} = 1$, then constant RTS prevail.
- Else if $\mathbf{1}_n^T \boldsymbol{\lambda}^* > 1$, decreasing RTS prevail.
- Else, increasing RTS prevail.

Algorithm II is based on an envelopment-based DEA model and determines RTS based on the CCR-efficiency score and the optimal sum of the intensity variables. The main advantage of this algorithm lies in its computational efficiency, which is due to two reasons. First, to find out the projection point of an inefficient DMU, the approach by Mehdiloozad et al. (2015) is computationally more efficient than its alternatives. Second, to determine RTS, only the envelopment form of the CCR model is required to be solved.

12.4.4 Numerical Example

Here, we illustrate Algorithms I and II by applying them to the data set exhibited in Table 12.1, which was also used in Example 12.3.5. Using each algorithm, we estimate RTS of the observed DMUs by executing the following three main steps:

Step 1 We first evaluate all the DMUs via the RAM model to determine their efficiency statuses.

Step 2 We determine RTS statuses of the efficient DMUs.

Step 3 We determine RTS statuses of the inefficient DMUs.

Remark 1 Note that after determining the RTS possibilities of all the efficient DMUs in Step 2, the method of Tone (1996, 2005) can be applied in Step 3. This is sensible from a computational perspective because it avoids the requirement of solving any additional model.

Table 12.4 The RTS statuses of eight DMUs determined via Algorithm I

	DMU_1	DMU_2	DMU_3	DMU_4	DMU_5	DMU_6	DMU_7	DMU_8
$\underline{u}_0$	-1	-0.167	1	0.6	–	–	–	–
$\bar{u}_0$	-0.333	1.5	–	–	–	–	–	–
RTS	I	C	D	D	D	C	D	D

Note: C, D, and I stand for constant RTS, decreasing RTS, and increasing RTS, respectively

12.4.4.1 Determining RTS Statuses of the DMUs Using Algorithm I

As can be seen in Table 12.2, out of eight units, only four units (DMU_1, DMU_2, DMU_3, and DMU_4) are efficient. First, we determine the RTS status of each efficient unit by using Algorithm I and report the results in Table 12.4.

For each of these units, we directly proceed to Stage 2 and solve the minimization form of model (12.22). Since $\underline{u}_{03} = 1$ and $\underline{u}_{04} = 0.6$, DMU_3 and DMU_4 exhibit decreasing RTS. Since $\underline{u}_{01} = -1 < 0$ and $\underline{u}_{02} = -0.167 < 0$, we go to Step 3 and solve the maximization form of model (12.22) for DMU_1 and DMU_2. The results reveal that DMU_1 and DMU_2 operate under increasing RTS and constant RTS, respectively.

We now turn to determine the RTS statuses of the remaining inefficient units. For each of these inefficient units, first, we solve model (12.19) to obtain a projection point in the relative interior of its corresponding minimum face from (12.20). As can be seen in Table 12.3, model (12.19) projects DMU_5, DMU_6, and DMU_7 onto the observed efficient units DMU_4, DMU_2, and DMU_3, respectively. Hence, without solving any additional models, the RTS statuses of these units are determined at their corresponding projection points. However, by applying model (12.19) to DMU_8, we obtain the unobserved activity $(3.333, 6.666)$ as its projection point and identify DMU_2, DMU_3, and DMU_4 as its reference units. Since these reference units exhibit either constant RTS or decreasing RTS, the RTS status of DMU_8 is found to be decreasing as per Remark 1.

In summary, Algorithm I requires solving of six LP models for determining the RTS statuses of all the eight DMUs.

12.4.4.2 Determining RTS Statuses of the DMUs Using Algorithm II

Here, we illustrate the use of Algorithm II, as developed in Sect. 12.4.3, for determining the RTS possibilities of the DMUs. First, we determine the RTS statuses of DMU_1, DMU_2, DMU_3, and DMU_4 using Algorithm II. Since these units are all efficient, we directly evaluate them via the CCR model. For each DMU, the second and third rows of Table 12.5 present the CCR efficiency score and the optimal sum of the intensity variables, respectively.

Table 12.5 The RTS statuses of eight DMUs determined via Algorithm II

	DMU_1	DMU_2	DMU_3	DMU_4	DMU_5	DMU_6	DMU_7	DMU_8
θ_o^{CCR}	0.8	1	0.8	0.64	–	–	–	–
$\mathbf{1}_n^T\boldsymbol{\lambda}*$	0.4	–	1.2	1.6	–	–	–	–
RTS	I	C	D	D	D	C	D	D

Note: C, D, and I stand for constant RTS, decreasing RTS, and increasing RTS, respectively

Since the CCR efficiency score for DMU_2 is equal to one, this unit operates under constant RTS. Since for DMU_1, . . ., we have $\theta_1^{CCR} < 1$ and $\mathbf{1}_n^T\boldsymbol{\lambda}^* < 1, \ldots,$ Theorem 6 implies that this unit exhibits increasing RTS. Similarly, Theorem 6 implies that DMU_3 and DMU_4 operate under decreasing RTS, since $\theta_o^{CCR} < 1$ and $\mathbf{1}_n^T\boldsymbol{\lambda}^* > 1$ hold for these units. As discussed in Sect. 12.4.4.1, the RTS possibilities of the inefficient units can be determined at their projection points obtained by model (12.19), without solving any additional model. Hence, Algorithm II enables us to estimates the RTS statuses of all the eight DMUs by solving the CCR model four times.

12.5 Empirical Application

To demonstrate the ready applicability of our proposed approach, we conduct an illustrative empirical analysis based on a real-life data set of 70 public schools in the United States, which was taken from Charnes et al. (1981).

The data consists of five inputs and three outputs. The inputs of schools are the education level of mother as measured in terms of percentage of high school graduates among female parents (x_1), the highest occupation of a family member according to a pre-arranged rating scale (x_2), the parental visit index representing the number of visits to the school site (x_3), the parent counseling index calculated from data on the time spent with child on school-related topics such as reading together, etc. (x_4), and the number of teachers at a given site (x_5). The outputs are the Total Reading Score as measured by the Metropolitan Achievement Test (y_1), the Total Mathematics Score as measured by the Metropolitan Achievement Test (y_2), and the Coopersmith Self-Esteem Inventory, intended as a measure of self-esteem (y_3).

Table 12.6 Descriptive statistics of efficiency scores and projection points obtained by the RAM model for the inefficient schools

	ρ_o	x_1^{RAM}	x_2^{RAM}	x_3^{RAM}	x_4^{RAM}	x_5^{RAM}	y_1^{RAM}	y_2^{RAM}	y_3^{RAM}
Min	0.844	4.379	2.233	7.958	8.170	2.274	8.817	9.369	6.350
Max	0.985	39.969	14.650	51.902	52.132	8.314	54.530	63.557	39.100
Mean	0.938	15.634	8.396	28.938	29.847	4.947	30.319	36.529	22.367

12.5.1 Evaluation of Schools via the RAM Model

We first make efficiency assessment of the schools using the RAM model. The results show that 26 (37 %) schools are efficient. For the remaining 44 (63 %) inefficient schools, Table 12.6 provides summary statistics of the efficiency scores and the inputs and outputs of the projection points. It can be observed that the values of ρ_o range from 0.844 to 0.965 with the mean efficiency score of 0.938.

12.5.2 Determining RTS Statuses of the Efficient Schools

We apply our Algorithm I and II for the 26 efficient schools in order to determine their RTS possibilities. The results are all provided in Table 12.7. Out of these efficient schools, schools S5, S32, S38, and S45 are found operating under increasing RTS, schools S11 and S12 under decreasing RTS and the remaining ones under constant RTS.

12.5.3 Determining RTS Statuses of the Inefficient Schools

Now, we proceed to determine RTS statuses of the inefficient schools. First, we apply model (12.19) to each inefficient school, to identify its reference schools as its benchmarks, and to obtain a projection point in the relative interior point of its corresponding minimum face. For the 44 inefficient schools, the results on the GRSs together with their associated intensity vectors are all reported in the first six columns of Table 12.8.

For example, the efficient schools S44, S58, and S59 with the respective weights of 0.052, 0.915, and 0.033 appear in the GRS of the most inefficient school S66. This means that the target inputs and outputs of school S66 are a convex combination of the inputs and outputs of schools S44, S58, and S59. Thus, in order for school S66 to become efficient, it must adjust its inputs and outputs so that it produces the output vector $0.052 \times \mathbf{y}_{S44} + 0.915 \times \mathbf{y}_{S58} + 0.033 \times \mathbf{y}_{S59}$ by consuming the input vector $0.052 \times \mathbf{x}_{S44} + 0.915 \times \mathbf{x}_{S58} + 0.033 \times \mathbf{x}_{S59}$.

Table 12.7 The RTS statuses of the 26 efficient schools

DMU	Algorithm I		Algorithm II		RTS
	$\underline{u}_0$	$\bar{u}_0$	θ_o^{CCR}	$\mathbf{1}_n^T\boldsymbol{\lambda}^*$	
S5	−1	−0.26	0.929	0.564	I
S11	0.026	0.686	0.976	1.919	D
S12	0.072	0.668	0.973	1.615	D
S15	−0.518	0.157	1	–	C
S17	−1	0.18	1	–	C
S18	−0.011	0.243	1	–	C
S20	−0.165	0.88	1	–	C
S21	−0.023	0.405	1	–	C
S22	−0.017	0.054	1	–	C
S24	−0.318	0.46	1	–	C
S27	−0.167	0.439	1	–	C
S32	−1	−0.606	0.895	0.703	I
S35	−0.006	0.368	1	–	C
S38	−1	−0.249	0.873	0.394	I
S44	−0.051	1.477	1	–	C
S45	−1	−0.325	0.88	0.418	I
S47	−0.166	0.455	1	–	C
S48	−1	0.003	1	–	C
S49	−0.083	0.125	1	–	C
S52	−0.052	0.617	1	–	C
S54	−0.03	0.929	1	–	C
S56	−0.263	0.082	1	–	C
S58	−0.407	0.279	1	–	C
S59	−0.29	0	1	–	C
S62	−1	0.099	1	–	C
S69	−1	0.374	1	–	C

Note: C, D, and I stand for constant RTS, decreasing RTS, and increasing RTS, respectively

Once the GRS and the intensity vector $\boldsymbol{\lambda}_o^{\max}$ are identified for each school, its corresponding projection $P_o^{\max}$ is obtained from (12.20). Table 12.9 gives the statistics of the obtained projection points. By applying Algorithms I and II to the obtained projection points, the RTS statuses of the inefficient schools are then classified. The results are reported in the last five columns of Table 12.8. As the results show, out of 44 inefficient schools, 3 (7 %) schools exhibit decreasing RTS, 1 (2 %) school exhibits increasing RTS, and the remaining 40 (91 %) schools exhibit constant RTS.

As an interesting point, note that the RTS statuses for schools such as S1, S10, S28, and S50 cannot be determined directly from their GRSs. This is because the reference units of each of these schools all exhibit constant RTS.

Table 12.8 The GRSs and the RTS statuses of the inefficient schools

DMU	Global reference set					Algorithm I		Algorithm II		RTS
						$\underline{u}_0$	$\bar{u}_0$	θ_o^{CCR}	$\mathbf{1}_n^T\boldsymbol{\lambda}^*$	
S1	S52(0.652)	S58(0.171)	S59(0.177)			0.003	–	0.997	2.108	D
S2	S44(0.250)	S58(0.750)				−0.091	0.073	1	–	C
S3	S44(0.201)	S52(0.275)	S58(0.525)			−0.049	0.065	1	–	C
S4	S44(0.036)	S58(0.954)	S59(0.010)			−0.134	0.11	1	–	C
S6	S58(0.862)	S62(0.138)				−0.428	0.053	1	–	C
S7	S58(0.944)	S59(0.056)				−0.225	0.211	1	–	C
S8	S44(0.003)	S58(0.739)	S59(0.258)			−0.071	0.052	1	–	C
S9	S58(0.831)	S59(0.169)				−0.171	0.141	1	–	C
S10	S44(0.159)	S52(0.555)	S58(0.236)	S59(0.049)		0.003	–	0.999	1.311	D
S13	S58(0.828)	S59(0.172)				−0.17	0.14	1	–	C
S14	S58(0.104)	S62(0.624)	S69(0.272)			−0.525	0.071	1	–	C
S16	S44(0.337)	S58(0.558)	S59(0.105)			−0.064	0.049	1	–	C
S19	S44(0.397)	S58(0.560)	S59(0.043)			−0.067	0.052	1	–	C
S23	S44(0.478)	S58(0.500)	S59(0.022)			−0.064	0.05	1	–	C
S25	S44(0.234)	S58(0.299)	S62(0.467)			−0.122	0.032	1	–	C
S26	S44(0.201)	S58(0.698)	S59(0.101)			−0.077	0.059	1	–	C
S28	S17(0.396)	S20(0.047)	S27(0.369)	S47(0.084)	S62(0.104)	−0.181	−0.061	0.998	0.946	I
S29	S62(0.627)	S69(0.373)				−1	0.081	1	–	C
S30	S58(0.973)	S59(0.027)				−0.245	0.241	1	–	C
S31	S58(0.990)	S59(0.010)				−0.258	0.263	1	–	C
S33	S44(0.288)	S58(0.563)	S59(0.150)			−0.063	0.047	1	–	C
S34	S58(0.768)	S59(0.232)				−0.151	0.119	1	–	C
S36	S58(0.997)	S59(0.003)				−0.264	0.273	1	–	C
S37	S58(0.978)	S59(0.022)				−0.249	0.247	1	–	C

S39	S20(0.396)	S27(0.461)	S44(0.143)			−0.089	0.354	1	–	C
S40	S52(0.239)	S58(0.666)	S69(0.095)			−0.092	0.092	1	–	C
S41	S44(0.229)	S58(0.767)	S59(0.004)			−0.093	0.075	1	–	C
S42	S17(0.482)	S20(0.482)	S58(0.036)			−0.209	0.062	1	–	C
S43	S44(0.114)	S58(0.776)	S59(0.110)			−0.085	0.065	1	–	C
S46	S44(0.032)	S58(0.710)	S59(0.259)			−0.069	0.05	1	–	C
S50	S52(0.545)	S58(0.399)	S59(0.056)			0.004	0.139	0.999	1.345	D
S51	S58(0.274)	S62(0.726)				−0.556	0.078	1	–	C
S53	S58(0.948)	S59(0.052)				−0.227	0.214	1	–	C
S55	S15(0.010)	S44(0.117)	S52(0.076)	S58(0.554)	S69(0.243)	−0.062	0.024	1	–	C
S57	S52(0.431)	S58(0.499)	S69(0.070)			−0.077	0.077	1	–	C
S60	S44(0.091)	S58(0.143)	S62(0.766)			−0.187	0.054	1	–	C
S61	S58(0.009)	S62(0.447)	S69(0.544)			−0.57	0.086	1	–	C
S63	S44(0.017)	S52(0.012)	S58(0.437)	S62(0.058)	S69(0.476)	−0.107	0.05	1	–	C
S64	S44(0.032)	S58(0.863)	S59(0.105)			−0.099	0.076	1	–	C
S65	S44(0.006)	S52(0.075)	S58(0.219)	S62(0.224)	S69(0.476)	−0.111	0.053	1	–	C
S66	S44(0.052)	S58(0.915)	S59(0.033)			−0.119	0.095	1	–	C
S67	S44(0.063)	S58(0.834)	S59(0.103)			−0.094	0.073	1	–	C
S68	S44(0.147)	S58(0.249)	S59(0.134)	S62(0.471)		−0.085	0.025	1	–	C
S70	S44(0.027)	S58(0.640)	S62(0.333)			−0.188	0.054	1	–	C

Note: C, D, and I stand for constant RTS, decreasing RTS, and increasing RTS, respectively

Table 12.9 Descriptive statistics of relative interior points of the minimum faces

DMU	x_1^{max}	x_2^{max}	x_3^{max}	x_4^{max}	x_5^{max}	y_1^{max}	y_2^{max}	y_3^{max}
Min	4.379	2.233	7.958	8.170	2.274	8.817	9.369	6.350
Max	39.969	14.650	51.902	52.132	8.314	54.530	63.557	39.100
Mean	15.634	8.396	28.938	29.847	4.947	30.319	36.529	22.367

12.6 Summary and Concluding Remarks

The crucial issue concerning identification of the reference set of an inefficient DMU is the occurrence of multiple reference sets, i.e., the issue of multiplicity. To deal effectively with this issue, this chapter proposes a general LP-based approach for identifying all the possible reference units of an inefficient DMU, and then applies it for determining RTS. Toward these two objectives, first, two potential sources of the origin of the multiplicity issue are identified. The first one is the presence of alternative optimal intensity vectors for a given projection point (problem Type I), and the second one is the occurrence of multiple projection points (problem Type II).

Obviously, the reference set of an inefficient DMU is not well defined under the issue of multiplicity, and under either problem Types I or Type II or both, accordingly. To overcome the problem of the multiplicity issue, the uniquely found reference set containing all the possible reference units must be discriminated from the two other types of reference set for which the multiplicity issue occurs due to problems Types I and II. In this chapter, this discrimination is made by introducing three types of the reference set—i.e., URS, MRS and GRS. Corresponding to a given projection point of an inefficient DMU, the URS is defined as the set of efficient DMUs that are active in a specific convex combination generating this point. The union of all the URSs associated with the given projection is defined as its associated MRS. The union of the MRSs associated with all the projection points is also defined as the GRS of the evaluated DMU.

With the help of the introduced notions, it is then demonstrated that the convex hull of the GRS is equal to the minimum face, from which it is immediately concluded that the minimum face is a polytope. It is also proved that the GRS can be accurately identified by finding a maximal element of the set of all intensity vectors at optimality of the RAM model. To find out such an element, first, a mixed 0–1 LP model is proposed in the envelopment form and then, it is transformed into an equivalent upper-bounded LP model by using the LP relaxation method. The proposed approach has several advantages as outlined below:

- It is computationally more efficient than its alternatives, since it requires the execution of a single LP model.
- Its computational efficiency is higher than those of its primal–dual based alternatives, as it is primal based.
- Its computational efficiency can be substantially improved by using the simplex algorithm adopted for solving the LP problems with upper-bounded variables.

- It can be easily applied to both radial and non-radial DEA models.
- It is independent of the imposed RTS assumption.

As an important application of the proposed approach, two precise methods are developed to determine RTS statuses of the DMUs. In this regard, first, the RTS of an inefficient DMU is defined at its projection point that lies in the relative interior of the minimum face. This definition is precise since the minimum face is spanned by the GRS, and all the relative interior points of the minimum face exhibit the same type of RTS. Then, based on this definition, two RTS determination algorithms are proposed by extending those of Banker et al. (2004) and Zarepisheh et al. (2006). The first one is a three-stage algorithm that uses the multiplier form of the BCC model and determines RTS by examining the intercept(s) of the supporting hyperplane(s). This algorithm deals effectively with the occurrence of multiple supporting hyperplanes that arises either from problem Type II or from the non-full dimensionality of the minimum face. The second one is a two-stage algorithm that applies the envelopment form of the CCR model and determines RTS by looking at the sum of the optimal intensity variables. From computational perspective, the second algorithm is superior to the first one.

References

Aida, K., Cooper, W. W., Pastor, J. T., & Sueyoshi, T. (1998). Evaluating water supply services in Japan with Ram: A range-adjusted measure of inefficiency. *Omega, 26*, 207–232.

Banker, R. D., Bardhan, I., & Cooper, W. W. (1996). A note on returns to scale in DEA. *European Journal of Operational Research, 88*, 583–585.

Banker, R. D., Charnes, A., & Cooper, W. W. (1984). Some models for the estimation of technical and scale inefficiencies in data envelopment analysis. *Management Science, 30*, 1078–1092.

Banker, R. D., Cooper, W. W., Seiford, L. M., Thrall, R. M., & Zhu, J. (2004). Returns to scale in different DEA models. *European Journal of Operational Research, 154*, 345–362.

Banker, R. D., & Thrall, R. M. (1992). Estimation of returns to scale using data envelopment analysis. *European Journal of Operational Research, 62*, 74–84.

Bergendahl, G. (1998). DEA and benchmarks an application to Nordic banks. *Annals of Operations Research, 82*, 233–250.

Camanho, A. S., & Dyson, R. G. (1999). Efficiency, size, benchmarks and targets for bank branches: An application of data envelopment analysis. *Journal of the Operational Research Society, 50*, 903–915.

Charnes, A., Cooper, W. W., Golany, B., Seiford, L., & Stutz, J. (1985). Foundations of data envelopment analysis for Pareto-Koopmans efficient empirical productions functions. *Journal of Econometrics, 30*, 91–107.

Charnes, A., Cooper, W. W., & Rhodes, E. (1978). Measuring the efficiency of decision making units. *European Journal of Operational Research, 2*, 429–441.

Charnes, A., Cooper, W. W., & Rhodes, E. (1979). Short communication: Measuring the efficiency of decision making units. *European Journal of Operational Research, 3*, 339.

Charnes, A., Cooper, W. W., & Rhodes, E. (1981). Evaluating program and managerial efficiency: An application of data envelopment analysis to program follow through. *Management Science, 27*, 668–697.

Charnes, A., & Lemke, C. E. (1954). *Computational theory of linear programming; I: The bounded variables problem*. United States, Office of Naval Research, O.N.R. research memorandum. Graduate School of Industrial Administration, Carnegie Institute of Technology.

Cooper, W. W., Park, K. S., & Pastor, J. T. (1999). RAM: A range adjusted measure of inefficiency for use with additive models and relations to other models and measures in DEA. *Journal of Productivity Analysis, 11*, 5–42.

Cooper, W. W., Pastor, J. T., Borras, F., Aparicio, J., & Pastor, D. (2011). BAM: A bounded adjusted measure of efficiency for use with bounded additive models. *Journal of Productivity Analysis, 35*, 85–94.

Cooper, W. W., Seiford, L. M., & Tone, K. (2007). *Data envelopment analysis: A comprehensive text with models, applications, references and DEA-solver software*. Boston: Kluwer Academic.

Dantzig, G. B. (1955). Upper bounds, secondary constraints, and block triangularity in linear programming. *Econometrica, 23*, 174–183.

Davtalab-Olyaie, M., Roshdi, I., Jahanshahloo, G. R., & Asgharian, M. (2014). Characterizing and finding full dimensional efficient facets in DEA: A variable returns to scale. *Journal of the Operational Research Society, 65*, 1453–1464.

Davtalab-Olyaie, M., Roshdi, I., Partovi Nia, V., & Asgharian, M. (2015). On characterizing full dimensional weak facets in DEA with variable returns to scale technology. *Optimization: A Journal of Mathematical Programming and Operations Research, 64*(11), 2455–2476.

Førsund, F. R. (1996). On the calculation of scale elasticities in DEA models. *Journal of Productivity Analysis, 7*, 283–302.

Fukuyama, H. (2000). Returns to scale and scale elasticity in data envelopment analysis. *European Journal of Operational Research, 125*, 93–112.

Fukuyama, H. (2001). Returns to scale and scale elasticity in Farrell, Russell and additive models. *Journal of Productivity Analysis, 16*, 225–239.

Fukuyama, H. (2003). Scale characterizations in a DEA directional technology distance function framework. *European Journal of Operational Research, 144*, 108–127.

Jahanshahloo, G. R., Hosseinzadeh Lotfi, F., Mehdiloozad, M., & Roshdi, I. (2012). Connected directional slack-based measure of efficiency in DEA. *Applied Mathematical Sciences, 6*, 237–246.

Jahanshahloo, G. R., Junior, H. V., Hosseinzadeh Lotfi, F., & Akbarian, D. (2007). A new DEA ranking system based on changing the reference set. *European Journal of Operational Research, 181*, 331–337.

Jahanshahloo, G. R., Roshdi, I., & Davtalab-Olyaie, M. (2013). Characterizing and finding full dimensional efficient facets of PPS with constant returns to scale technology. *International Journal of Industrial Mathematics, 5*, 149–159.

Jahanshahloo, G. R., Shirzadi, A., & Mirdehghan, S. M. (2008). Finding the reference set of a decision making unit. *Asia-Pacific Journal of Operational Research, 25*, 563–573.

Krivonozhko, V. E., Førsund, F. R., & Lychev, A. V. (2012a). Methods for determination of multiple reference sets in the DEA models. *Mathematics and Statistics, 85*, 134–138.

Krivonozhko, V. E., Førsund, F. R., & Lychev, A. V. (2012b). A note on imposing strong complementary slackness conditions in DEA. *European Journal of Operational Research, 220*, 716–721.

Krivonozhko, V. E., Førsund, F. R., & Lychev, A. V. (2012c). Returns-to-scale properties in DEA models: The fundamental role of interior points. *Journal of Productivity Analysis, 38*, 121–130.

Krivonozhko, V. E., Førsund, F. R., & Lychev, A. V. (2014). Measurement of returns to scale using non-radial DEA models. *European Journal of Operational Research, 232*, 664–670.

Mehdiloozad, M. (2016). Identifying the global reference set in DEA: A mixed 0-1 LP formulation with an equivalent LP relaxation. *Operational Research*. doi:10.1007/s12351-015-0222-9

Mehdiloozad, M., Mirdehghan, S. M., Sahoo, B. K., & Roshdi, I. (2015). On the identification of the global reference set in data envelopment analysis. *European Journal of Operational Research, 245*, 779–788.

Mehdiloozad, M., & Sahoo, B. K. (2015). Measurement of returns to scale in DEA using the CCR model. Retrieved August 7, 2015, from http://arxiv.org/pdf/1508.01736v1

Mehdiloozad, M., Sahoo, B. K., & Roshdi, I. (2014). A generalized multiplicative directional distance function for efficiency measurement in DEA. *European Journal of Operational Research, 214*, 679–688.

Mehdiloozad, M., Tone, K., Askarpour, R., & Ahmadi, M. B. (2016). Finding a maximal element of a convex set through its characteristic cone: An application to finding a strictly complementary solution. *Computational and Applied Mathematics*. doi:10.1007/s40314-016-0324-x

Olesen, O. B., & Petersen, N. C. (1996). Indicators of ill-conditioned data sets and model misspecification in data envelopment analysis: An extended facet approach. *Management Science, 42*, 205–219.

Olesen, O. B., & Petersen, N. C. (2003). Identification and use of efficient faces and facets in DEA. *Journal of productivity Analysis, 20*, 323–360.

Pastor, J. T. (1994). *New additive models for handling zero and negative data*. Working paper: Universidad de Alicante, Departamento de Estadí'stica e Investigación Operativa, Spain.

Pastor, J. T., & Ruiz, J. L. (2007). Variables with negative values in DEA. In W. D. Cook & J. Zhu (Eds.), *Modeling data irregularities and structural complexities in data envelopment analysis* (pp. 63–84). New York: Springer.

Podinovski, V. V., & Førsund, F. R. (2010). Differential characteristics of efficient frontiers in data envelopment analysis. *Operations Research, 58*, 1743–1754.

Podinovski, V. V., Førsund, F. R., & Krivonozhko, V. E. (2009). A simple derivation of scale elasticity in data envelopment analysis. *European Journal of Operational Research, 197*, 149–153.

Rockafellar, R. T. (1970). *Convex analysis*. Princeton, NJ: Princeton University Press.

Roshdi, I., Van de Woestyne, I., & Davtalab-Olyaie, M. (2014). Determining maximal reference set in data envelopment analysis. Retrieved July 9, 2014, from http://arxiv.org/pdf/1407.2593v1

Sahoo, B. K. (2008). A non-parametric approach to measuring short-run expansion path. *Journal of Quantitative Economics, 6*, 137–150.

Sahoo, B. K., Kerstens, K., & Tone, K. (2012). Returns to growth in a non-parametric DEA approach. *International Transactions in Operational Research, 19*, 463–486.

Sahoo, B. K., Mohapatra, P. K. J., & Trivedi, M. L. (1999). A comparative application of data envelopment analysis and frontier translog production function for estimating returns to scale and efficiencies. *International Journal of Systems Science, 30*, 379–394.

Sahoo, B. K., & Sengupta, J. K. (2014). Neoclassical characterization of returns to scale in nonparametric production analysis. *Journal of Quantitative Economics, 12*, 78–86.

Sahoo, B. K., & Tone, K. (2009). Radial and non-radial decompositions of profit change: With an application to Indian banking. *European Journal of Operational Research, 196*, 1130–1146.

Sahoo, B. K., & Tone, K. (2013). Non-parametric measurement of economies of scale and scope in non-competitive environment with price uncertainty. *Omega, 41*, 97–111.

Sahoo, B. K., & Tone, K. (2015). Scale elasticity in non-parametric DEA approach. In J. Zhu (Ed.), *Data envelopment analysis—A handbook of models and methods* (pp. 269–290). New York: Springer.

Sahoo, B. K., Zhu, J., Tone, K., & Klemen, B. M. (2014a). Decomposing technical efficiency and scale elasticity in two-stage network DEA. *European Journal of Operational Research, 233*, 584–594.

Sahoo, B. K., Zhu, J., & Tone, K. (2014b). Decomposing efficiency and returns to scale in two-stage network systems. In W. D. Cook & J. Zhu (Eds.), *Data envelopment analysis: A handbook of modeling internal structure and network* (pp. 137–164). New York: Springer.

Sengupta, J. K., & Sahoo, B. K. (2006). *Efficiency models in data envelopment analysis: Techniques of evaluation of productivity of firms in a growing economy*. London: Palgrave Macmillan.

Silva Portela, M. C. A., & Thanassoulis, E. (2010). Malmquist-type indices in the presence of negative data: An application to bank branches. *Journal of Banking & Finance, 34*, 1472–1483.

Sueyoshi, T., & Sekitani, K. (2007a). The measurement of returns to scale under a simultaneous occurrence of multiple solutions in a reference set and a supporting hyperplane. *European Journal of Operational Research, 181*, 549–570.

Sueyoshi, T., & Sekitani, K. (2007b). Measurement of returns to scale using a non-radial DEA model: A range-adjusted measure approach. *European Journal of Operational Research, 176*, 1918–1946.

Tone, K. (1996). A simple characterization of return to scale in DEA. *Journal of Operations Research Society of Japan, 39*, 604–613.

Tone, K. (2005). Errata: A simple characterization of returns to scale in DEA. *Journal of Operations Research Society of Japan, 48*, 172.

Tone, K. (2010). Variations on the theme of slacks-based measure of efficiency in DEA. *European Journal of Operational Research, 200*, 901–907.

Tone, K., & Sahoo, B. K. (2003). Scale, indivisibilities and production function in data envelopment analysis. *International Journal of Production Economics, 84*, 165–192.

Tone, K., & Sahoo, B. K. (2004). Degree of scale economies and congestion: A unified DEA approach. *European Journal of Operational Research, 158*, 755–772.

Tone, K., & Sahoo, B. K. (2005). Evaluating cost efficiency and returns to scale in the life insurance corporation of India using data envelopment analysis. *Socio-Economic Planning Sciences, 39*, 261–285.

Tone, K., & Sahoo, B. K. (2006). Re-examining scale elasticity in DEA. *Annals of Operations Research, 145*, 69–87.

Winston, W. L. (2003). *Operations research: Applications and algorithms*. Boston: Duxbury Press.

Zarepisheh, M., Soleimani-damaneh, M., & Pourkarimi, L. (2006). Determination of returns to scale by CCR formulation without chasing down alternative optimal solutions. *Applied Mathematics Letters, 19*, 964–967.

Chapter 13
Technometrics Study Using DEA on Hybrid Electric Vehicles (HEVs)

Dong-Joon Lim, Shabnam R. Jahromi, Timothy R. Anderson, and Anca-Alexandra Tudorie

Abstract The Toyota Prius was first introduced in 1997 and since then over 150 hybrid electric vehicles (HEVs) have been brought to the automobile market around the world. This was spurred by a major interest in the future of vehicles using 'alternative fuel' for addressing environmental and fuel dependency concerns. This study evaluates and compares the technological advancement observed in different HEV market segments over the past 15 years. The results indicate that the introduction of a wide range of midsize HEVs is posing a threat to the two-seaters and compact HEV segments while an SUV segment shows a fast adoption with a significant performance improvement. The rates of change for each segment are also provided to give insights into the estimation of the future performance levels for new product development target setting purposes.

Keywords Hybrid electric vehicle • Technological forecasting • Data envelopment analysis • Market segment • Rate of change

---

Part of this chapter is based upon Lim, D.-J., Jahromi, S.R., Anderson, T.R., Tudorie, A.-A., Comparing technological advancement of hybrid electric vehicles (HEV) in different market segments. Technological Forecasting and Social Change, Vol. 97 (2015), 140–153. (Lim et al. 2015b) with permissions from Elsevier.

D.-J. Lim • S.R. Jahromi • T.R. Anderson (✉)
Department of Engineering and Technology Management, Portland State University, Portland, OR 97201, USA
e-mail: dongjoon@pdx.edu; rshabnam@pdx.edu; tim.anderson@pdx.edu

Anca-Alexandra Tudorie
Department of Policy Analysis, Delft University of Technology, Delft 2628 BX, The Netherlands
e-mail: a.tudorie@googlemail.com

S.-N. Hwang et al. (eds.), *Handbook of Operations Analytics Using Data Envelopment Analysis*, International Series in Operations Research & Management Science 239, DOI 10.1007/978-1-4899-7705-2_13

13.1 Introduction

Increasing fuel prices, government regulation, and a general desire to reduce environmental concerns have resulted in increased sales for fuel efficient vehicles. The Toyota Prius, introduced in 1997, was the first major hybrid electric vehicle (HEV) and since then most other manufacturers have introduced HEVs with varying success. While popular, the Prius and other vehicles were small and did not satisfy the needs of many other market segments. Over the following years, manufacturers have developed HEVs to serve other segments.

Electric vehicles can be broadly categorized as 'pure-electric' (i.e., using only a battery and an electric motor for propulsion without tailpipe) or 'hybrid-electric' (i.e., combining the conventional internal combustion engine with an electric motor and battery). As the electric vehicle market grows, related technologies are progressing every year especially in terms of driving range and fuel economy. In particular, the anxiety on the travel range of pure electric vehicles has been reduced by the advent of HEV. Besides, the fuel economy of the HEV has been greatly improved in plug-in HEV that can be recharged from an external grid.

Building on prior works (Jahromi et al. 2013a; Tudorie 2012), this study proposes a Technometrics model using data envelopment analysis (DEA) and applies it to the HEV industry so that technological advancement patterns in different market segments can be investigated. In addition, the rates of change for each segment are provided to give insights into the estimation of the future performance levels for new product development target setting purposes.

13.2 Methodology

Technometrics is a discipline aiming for the measurement of scientific or technological changes (Sahal 1985). As in traditional statistical literature, technometrics models can be conveniently divided into two groups: parametric and nonparametric approaches. The former approach constitutes the technology frontier by fitting it to a predefined functional form such as hyper-plane (Alexander and Nelson 1973), ellipsoid (Dodson 1985), generalized convex curve (Martino 1985), or iso-time surface derived from multiple S-curves (Danner 2006). Parametric models therefore tend to be robust to noise by filtering them with a predefined 'general' pattern. The latter approach, in contrast, purely adapts the technology frontier to data without being shaped *a priori*, which renders resulting frontier to be a piecewise linear combination rather than a curved surface (Lim 2015a).

Technology forecasting using DEA (TFDEA), which may be classified as one of the nonparametric RAND[1] techniques, iterates a technology frontier formation

[1] The term RAND was originated from the RAND Corporation. It is a generic term for technometrics model using time as a dependent variable whereas price is used in the hedonic approach.

through accumulating technologies over time to capture the rate of technological change (Inman 2004) (Fig. 13.1).

As previously noted, its nonparametric nature makes it possible to identify multiple facets constructing the technology frontier in which tradeoffs of corresponding design process can be explicitly considered. This feature is best exemplified by the segmented rate of change which varies along the frontier and therefore it becomes possible to obtain a relevant rate of technological change considering each technology's characteristics (Fig. 13.2). Lim and Anderson's study showed that capturing local rates of change from identified frontier facets and utilizing them for individual forecasting targets improve the forecasting accuracy in general (Lim et al. 2015a; Lim and Anderson 2014).

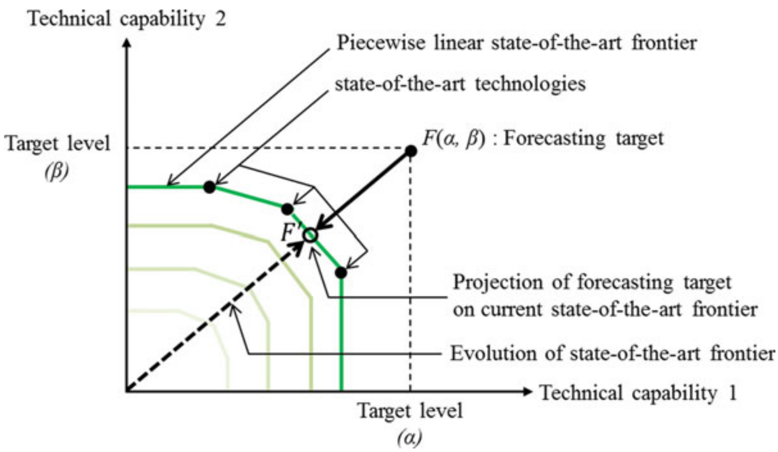


Fig. 13.1 Two-dimensional illustration of technology forecasting using DEA

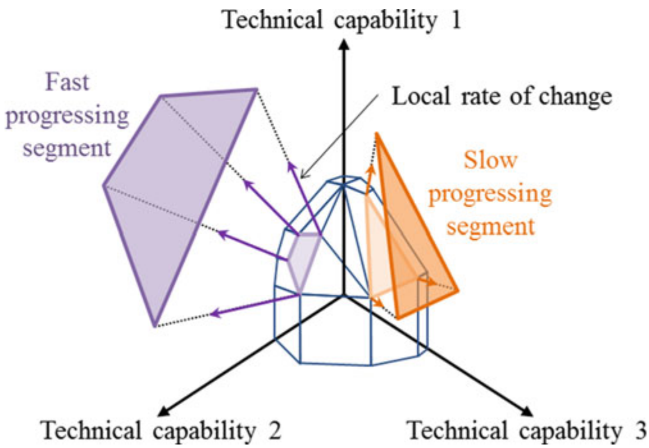


Fig. 13.2 Illustration of segmented rate of change

To formulate the TFDEA model, suppose there are n decision making units (DMUs), i.e., technologies, and let $x = (x_1, \ldots, x_m) \in \Re_+^M$ denote an input vector, $y = (y_1, \ldots, y_s) \in \Re_+^S$ denote an output vector. Following the minimum extrapolation principle (Banker et al. 1984), the production possibility set (PPS) for each DMU k can be constructed as (13.1). Note that variable returns to scale (VRS) is assumed and the frontier separation is imposed to deal with the external nondiscretionary factor. This restricts the reference set for each technology being evaluated to technologies presenting only same or more disadvantageous conditions in terms of the categorical index (Ruggiero 1996; Banker and Morey 1986). Therefore, this requires the categorical variables to be arranged in a rank order according to the favorable condition. We introduce a categorical variable for this study in the following section to account for the nondiscretionary factor.

$$PPS_k = \left\{ (x, y) \left| \begin{array}{lll} \sum_{j=1}^{n} \lambda_{jk} x_{ij} \geq x_{ik}, & \sum_{j=1}^{n} \lambda_{jk} y_{rj} \leq y_{rk}, & \forall i, r \\ \sum_{j=1}^{n} \lambda_{jk} = 1, & \lambda_{jk} \geq 0, & \forall j \\ \lambda_{jk} = 0, & \forall j \mid y_{rj} < \ y_{rk}\, for\ \ r \in ND & \end{array} \right. \right\} \tag{13.1}$$

Having specified the PPS, efficiency measurement can take a various forms. In this study, we employ an output-oriented model as can be denoted in (13.2).

$$\phi_k{}^* = max \left\{ \ \phi_k : (x, \phi_k y) \in PPS_k \ \right\} \tag{13.2}$$

The evolution of technology frontier is then captured by the efficiency changes of past DMUs. To quantify this, let $\phi_k^t *$ be an obtained efficiency score of DMU k from PPS_k including DMUs up to time t, t_k be the release date of DMU k, and T be the vantage point from which the rate of change is being captured. Then $\phi_k^{t_k} * = 1$ and $\phi_k^T * > 1$ indicates that DMU k was located on the technology frontier at the time of release but later superseded by the newly created technology frontier at T. Combining this information with the time gap between technology frontiers, the rate of change observed by each DMU can be obtained as formulated in (13.3).[2]

$$\gamma_k^T = \left(\phi_k^T * \right)^{\frac{1}{\frac{\sum_{j=1}^{n} \lambda_{jk}^{T^*} t_j}{\sum_{j=1}^{n} \lambda_{jk}^{T^*}} - t_k}}, \ \forall k \mid \phi_k^{t_k} * = 1, \quad \phi_k^T * \rangle 1 \tag{13.3}$$

[2] Note that (13.3) may suffer from the issue of alternative optimal solutions and in such case the secondary objective can be applied as described in (Lim et al. 2014).

Next, the local rate of change is computed for DMU(s) located on the technology frontier at T, that is, for peer DMU j of surpassed past DMU k. Each local rate of change therefore represents a growth pattern of adjacent frontier facets based on the technological advancement observed from related past technologies (Lim 2015b). Consequently, this enables an identification of how much frontier expansion has been made by each benchmark technology among others. This is denoted as below.

$$\delta_j^T = \frac{\sum_{k=1}^{n} \lambda_{jk}^{T^*} \gamma_k^T}{\sum_{k=1,\gamma_k^T>0}^{n} \lambda_{jk}^{T^*}}, \qquad \forall j \mid \phi_j^T * = 1 \tag{13.4}$$

13.3 Research Model and Dataset

13.3.1 TFDEA Parameters

13.3.1.1 Input Variable

MSRP: Manufacturer's suggested retail price can be considered as a reasonable proxy for manufacturing cost due to a high presumed correlation. The vehicles in the dataset were from different countries and released in different years therefore the actual MSRP for each vehicle was converted into 2013 U.S. dollar value through the following steps:

1. The vehicle's MSRP in the year of release was found through the manufacturers' website or car review websites.
2. If the MSRP was in currency other than U.S. dollars, the value was converted to the equivalent amount in U.S. dollars using the exchange rate of the year of release. This study used the historical exchange rates provided by OANDA Corporation for the conversions (OANDA Corporation 2013). Equation (13.5) shows the formula to convert the MSRPs in the original currency to U.S. dollar equivalent:

$$MSRP_{U.S.\ dollar\ equivalent} = Exchange\ rate_{year\ of\ release} {}^*MSRP_{in\ original\ currency} \tag{13.5}$$

3. To inflate a past dollar value into present value, (13.6) was used by applying the historical consumer price index (CPI) and the CPI of the year 2013. The CPI

values were obtained from the Bureau of Labor Statistics and the formula can be found as below (Boskin et al. 1998):

$$MSRP_{2013\ equivalent} = MSRP_{year\ of\ release} * (2013\ CPI)\ /(Year\ of\ release\ CPI) \tag{13.6}$$

13.3.1.2 Output Variables

Acceleration rate: This value determines the time (in seconds) it takes for a vehicle to go from 0 to 100 km (or 60 miles). Equation (13.7) shows the formula to calculate the acceleration rate:

$$Acceleration\ rate\left(\frac{km}{hour} per\ second\right) = \frac{speed\ range\left(\frac{km}{h}\right)}{time(second)} \tag{13.7}$$

Fuel economy: Fuel economy shows the distance a vehicle can travel in one unit of fuel. The Environmental Protection Agency (EPA) provides information on fuel economy for the vehicles available in the U.S. market (Environmental Protection Agency (EPA) 2013). This study used the fuel economy value for combined city and highway driving cycles that was officially announced by the EPA.

Note that the fuel economy estimation is complicated for plug-in HEVs as they can drive in pure electric mode from having been charged with the grid. Therefore the fuel economy of plug-in HEV was modified so that it takes account of hybrid mode only. To consider the additional dimension of plug-in HEV's performance, i.e., pure electric mode, another output of fuel economy is needed to be incorporated in the model as discussed below.

Max of MPG and MPG equivalent: The EPA developed a mile per gallon equivalent (MPGe) for plug-in HEVs to take all-electric range into account. This value is based on the gasoline-equivalent energy of electricity (Hybrid Vehicle Research and Development, Demonstration Program 2000). Specifically, 1 gal of gasoline can be approximated to 33.7 kW/h of electric energy. For vehicles that were not introduced in the U.S. market, the value of MPGe was calculated using (13.8):

$$MPG\ equivalent = \frac{33.7 * driving\ range}{battery\ capacity} \tag{13.8}$$

Since this parameter takes the maximum of MPG and MPGe, conventional HEVs have the same value as their fuel economy. Consequently, adding this parameter can address the additional feature of plug-in HEV without penalizing conventional hybrid cars in TFDEA model.

13.3.1.3 Categorical Parameter

Vehicle class: Unlike the earlier work by Jahromi et al. (Jahromi et al. 2013b) that included seating capacity as one of the output parameters to take capacity of the vehicle into account, this study used vehicle class as a categorical parameter. This is because seating capacity is more of design characteristics suitably determined for the target market than performance characteristic that manufacturers want to increase. Furthermore, vehicle class can be used to classify the different types of vehicle more precisely than seating capacity. For example, Prius C is a compact vehicle and Prius V is a midsize vehicle while they have the same seating capacity of five.

The EPA defines vehicle classes based on interior passenger and cargo volumes as well as design purposes (Environmental Protection Agency (EPA) 2013). This study adopted the EPA's criteria and grouped HEVs into seven classes: two-seaters (TS), compact (C), midsize (M), large (L), sport utility vehicle (SUV), minivan (MV), and pickup truck (PT). By using the above order of vehicle classes[3] as categorical indices, HEVs can only be compared to HEVs in the same or following classes. For example, HEVs in the last class (i.e., pickup truck) are only compared with HEVs in the same class, but HEVs in category M are compared with HEVs in the same and/or following classes (i.e., M, L, SUV, MV, and PT) in terms of per price performances. Intuitively, the category M vehicle will not be compared against any vehicles from preceding classes (i.e., TS and C). Consequently, this enables to reflect a great deal of information contained in each HEV market segment that would be lost in any point-comparison without consideration on environmental factors (Ruggiero 1996).

13.3.2 Dataset

The dataset has been updated to cover total 154 HEVs including 11 plug-in HEVs from 1997 to 2013 (see Table 13.1). The EPA database was the main source to collect the required information of technical attributes. Other sources were cross referenced especially for the vehicles released outside the U.S. and, in such a case, information was prioritized in order of technical report, product manual, benchmarking journals, and review sites. The whole dataset is available in (Lim et al. 2015b).

[3] This should be understood as the order of difficulty to achieve per price performances due to structural requirements for each market segment rather than mere vehicle sizes. For example, while pickup trucks have a range of sizes, the industrial loads that need to be carried in pickup trucks may cause design demands beyond that of minivans that are typically reflected in lower fuel economy. Also, note that EPA only applies volume criteria for cars (TS, C, M, and L) and weight criteria for trucks (SUV, MV, and PT).

Table 13.1 Dataset summary

Vehicle class		Two-seaters	Compact	Midsize	Large	SUV	Minivan	Pickup truck
Number of vehicles		9	32	56	8	37	4	8
First introduction (Years)		2000	1997	2004	2009	2004	2003	2004
MSRP (2013 equivalent)	Max	$21,435	$49,650	$118,544	$104,300	$97,238	$38,085	$57,095
	Average	$19,521	$27,908	$37,335	$85,251	$47,495	$29,616	$39,819
	Min	$18,936	$14,072	$11,849	$25,200	$17,045	$16,394	$30,090
Acceleration (km/h/s)	Max	12.20	14.93	19.61	20.41	18.52	9.26	12.35
	Average	9.99	9.84	12.63	15.97	12.99	7.85	11.12
	Min	9.24	7.04	7.14	12.35	8.33	6.29	9.09
MPG	Max	60.69	68.21	72.92	43.00	33.64	58.80	22.35
	Average	50.08	43.54	35.82	26.06	26.22	49.28	19.89
	Min	37.00	28.00	20.00	21.00	18.82	40.46	17.00
Max of MPG and MPGe	Max	60.69	98.00	100.00	43.00	38.00	58.80	22.35
	Average	50.08	50.95	41.42	26.06	26.45	49.28	19.89
	Min	37.00	28.00	20.00	21.00	18.82	40.46	17.00

13.4 Analysis of the Technological Advancement Patterns

The model was implemented using the software[4] developed by Lim and Anderson (2012). Figure 13.3 provides a sketch of what segment has been dominating the market in terms of technological superiority by showing how the state-of-the-art frontier of hybrid electric vehicles over time has been made up of vehicles from different segments. That is, the percentage indicates the amount of which each HEV segment stakes out the state-of-the-art frontier that any particular HEV is aiming for. In 1997 for example, the state-of-the-art frontier was constructed by a sole compact HEV, the first generation of Prius, without a competition therefore the dark blue region (i.e., compact segment) filled up the entire frontier space. As other types of HEVs began to be released in the market over time, the state-of-the-art frontier has been made up of a wide variety HEVs.

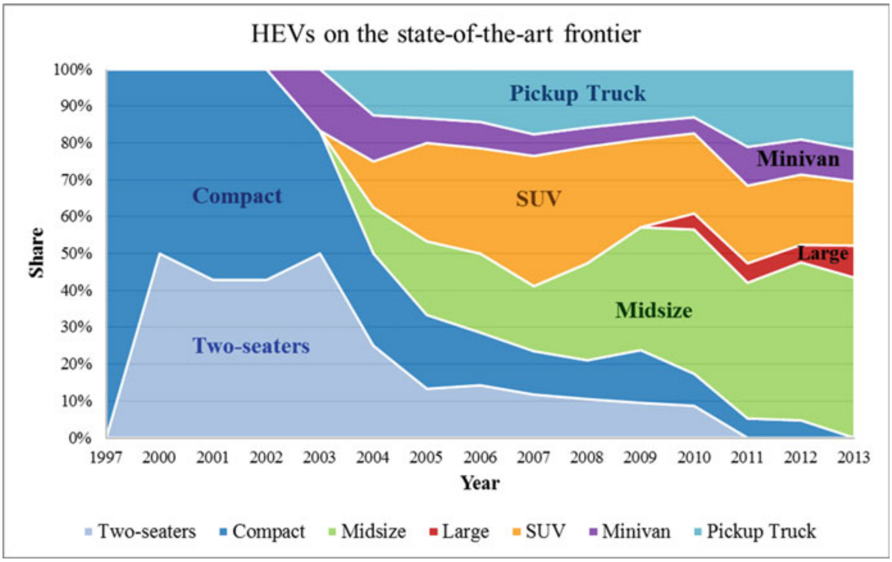


Fig. 13.3 State-of-the-art HEV distribution

[4] R package for a standard TFDEA is available at http://cran.r-project.org/web/packages/TFDEA/index.html

A web-based version and Excel add-in are also available at http://tfdea.com and in (Lim and Anderson 2012) respectively.

13.4.1 Two-Seaters and Compact Segments: "Stagnated"

Until 2003, all HEVs in our dataset were either two-seaters or compact automobiles. This resulted in these two segments dominating the HEV market but the introduction of vehicles in other segments resulted in an erosion of this dominance. Despite consecutive introductions of successful lineups such as Honda Insight and Toyota Prius C, the technological dominance has been shrinking as the other types of HEVs' market advance.

Note that there were no two-seaters or compact HEVs on the state-of-the-art frontier in 2013. This indicates that two-seaters and compact HEVs are no longer competitive with vehicles in other segments, though they presumably have a light weight advantage. This is particularly attributed to the encroachment of the midsize HEVs that is extending its target market with a fast technological advancement recently. One can verify this by the list of benchmarks of two-seaters and compact HEVs in 2013. Table 13.2 contains this information. The combination of benchmark and dominated set can be understood as a competitor group in terms of their product spec where the former is found to be outperforming the latter. For example, Prius (first generation), indicated as vehicle number of 1, has become obsolete since its introduction in 1997 and it was superseded by its benchmarks: Accord Hybrid (21), Prius alpha (V) (80), and Fit Shuttle Hybrid (82).

Except for the Fit Shuttle Hybrid (82), benchmarks of all two-seaters and compact HEVs were found to be midsize HEVs. This suggests that midsize HEVs are outperforming HEVs from those two segments with similar technical characteristics. That is, midsize HEVs are penetrating the market niche that has been dominated by two-seaters and compact HEVs. In fact, the bar for energy efficiency is constantly being raised as more competitors including bigger vehicles have come into the market place with innovative features such as plug-in

Table 13.2 Benchmarks of two-seaters and compact HEVs

Benchmarks (class)	Dominated set[a]	
	Two-seaters	Compact
21 (M)	4, 6, 7, 9, 12, 18, 55, 90, 136	1, 3, 5, 10, 16, 19, 43, 47, 63, 66, 71, 77, 78, 79, 88, 97, 102, 111, 112, 113, 117, 138, 140, 141
40 (M)		10
56 (M)		99
67 (M)		2, 46, 81
80 (M)	4, 6, 7, 9, 12, 18, 55, 90, 136	1, 2, 3, 5, 16, 19, 43, 46, 47, 54, 63, 66, 71, 77, 78, 79, 81, 88, 97, 102, 111, 112, 113, 117, 138, 140, 141
82 (MV)	4, 6, 7, 9, 12, 18, 55, 90, 136	1, 3, 5, 10, 16, 19, 43, 47, 63, 66, 77, 88, 97, 102, 111, 117
145 (M)		46, 71, 78, 79, 112, 113, 138, 140, 141
152 (M)		68, 99, 154
153 (M)		103

[a]List of HEVs who cited the corresponding state-of-the-art HEV as a benchmark

technology. Hence, high fuel economy is not entirely the domain of smaller vehicles any more (Sanchez 2013). This instigated makers of small HEVs to engage in more ingenious designs and development improvement (e.g., Toyota's new global architecture project) (Toyota 2012).

13.4.2 Midsize Segment: "Flourishing"

Continuing the previous discussion, it is noteworthy that midsize segment has shown a fast adoption rate with a superior technological performance recently. Indeed, hybrid technology has gained substantial popularity not only in fuel prices but also in reliability and longevity of power train that almost every auto manufacturers began to add hybrid version of their conventional midsize models to their brochures (CarsDirect 2013).

Figure 13.4 further explains the market penetration of midsize HEVs into the compact segment. Although the average price of midsize HEVs is still slightly higher than compact HEVs, not only the acceleration of midsize HEVs outperforms compact HEVs but also the gap of average fuel economy between compact and midsize HEVs is getting narrower. Especially, recent midsize plug-in HEVs such as Ford C-Max Energi (152) and Fusion Energi (153) have surpassed the fuel economy of any other compact HEVs as shown in the bottom right figure. This would attract customers who pine for a sportier vehicle in addition to roomier interior and safety features to the midsize segment with a variety of purchase options.

Almost by definitions, benchmarks (i.e., state-of-the-art HEVs) targeting a niche market won't have a big dominated set who cited them as a benchmark (Doyle and Green 1991). In contrast, state-of-the-art HEVs with a broad scope must have been cited as a benchmark by many other competitors. Consequently, it would be possible to reveal whether an HEV on the 2013 state-of-the-art frontier is the niche or the broad player if the information on which and how many HEVs were compared with them was available. This has been done in Table 13.3.

In the midsize segment, three dominant players can be identified: Honda Accord Hybrid (21), Toyota Prius alpha (V) (80), and Infiniti M35h (145). One can further classify them such that Accord Hybrid as a low-end, Prius alpha (V) a middle-end, and M35h a high-end benchmark based on their MSRPs and performance characteristics.

Local rates of change of state-of-the-art HEVs indicate how much technological advancement has been observed from their dominated sets. Using the foregoing classifications, middle-end midsize HEVs have shown the fastest rate of change, i.e., 3.66 % of annual improvement for acceleration and fuel economies, whereas low-end and high-end midsize HEVs' progresses were relatively moderate, 1.56 % and 1.96 % respectively.

It is also interesting to note that BYD F3DM (56) and Ford C-Max Energi (152) were found to be state-of-the-art plug-in HEVs that have been competed against other plug-in HEVs listed in their dominated sets. However the technological advancement of plug-in HEV in midsize segment appeared to be modest so far

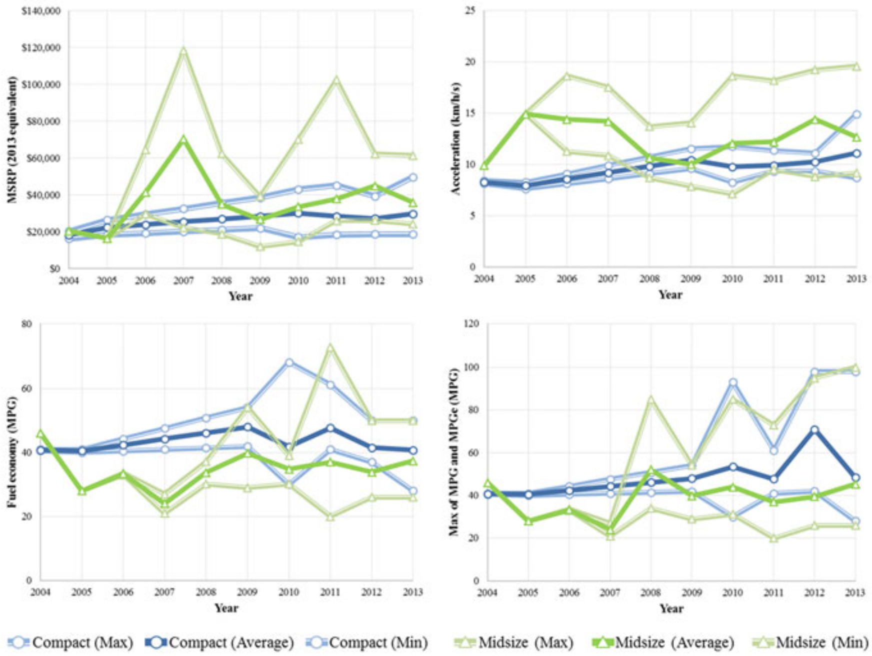


Fig. 13.4 Encroachment of midsize segment into the compact segment

possibly due to the fact that the current battery technology has been struggling with technical challenges along with cost and complexity coming from dual power trains (U.S. Department of Energy 2006; Shao et al. 2009).

13.4.3 Large Segment: "Emerging"

Two large HEVs are on the 2013 state-of-the-art frontier: the BMW ActiveHybrid 7 Series (61) and the Ford C-Max Hybrid FWD (116). One may notice that these HEVs are representing two very different regions within a large HEV segment. Indeed, the BMW ActiveHybrid 7 Series, which has a 2013 equivalent MSRP of $104,300, constitutes the most expensive HEV market segment. This is a noteworthy segment in that it is penetrating a niche of luxury market with a powerful engine and electric motor combination while still getting satisfactory MPG. In fact, the high-end automakers have finally begun to push green cars, e.g., Mercedes' S hybrid series or Porsche's Panamera S series, right after Tesla proved that there is a sufficient number of upscale customers in the electric vehicle market (Garthwaite 2013).

In contrast, the Ford C-Max Hybrid FWD, which has a 2013 equivalent MSRP of $25,200, stakes out the other end of the large segment. This unique vehicle is, in fact, targeting the niche between midsize and minivan segments to satisfy

Table 13.3 Benchmarks and local rates of change observed from 2013 state-of-the-art HEVs

Class	State-of-the-art HEV	Dominated set[a]	Local rate of change
Midsize	21	1, 3, 4, 5, 6, 7, 9, 10, 12, 13, 16, 18, 19, 21, 24, 27, 33, 38, 39, 43, 45, 47, 50, 55, 60, 62, 63, 64, 66, 70, 71, 77, 78, 79, 83, 84, 86, 88, 89, 90, 91, 92, 97, 102, 104, 105, 106, 107, 111, 112, 113, 114, 115, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 136, 137, 138, 139, 140, 141	1.01562
	40	10, 40, 60	1.00422
	56	38, 56, 99	1.00083
	67	2, 25, 36, 46, 49, 67, 81, 100, 101, 108, 144, 146, 147	1.00867
	80	1, 2, 3, 4, 5, 6, 7, 9, 12, 13, 16, 18, 19, 24, 25, 27, 33, 36, 39, 43, 45, 46, 47, 49, 50, 54, 55, 62, 63, 64, 66, 70, 71, 77, 78, 79, 80, 81, 83, 84, 86, 88, 89, 90, 91, 92, 97, 101, 102, 104, 105, 106, 107, 111, 112, 113, 114, 115, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 136, 137, 138, 139, 140, 141, 144, 146, 147	1.03664
	145	24, 25, 27, 30, 33, 39, 45, 46, 49, 50, 62, 64, 70, 71, 78, 79, 83, 84, 86, 89, 91, 92, 96, 100, 101, 104, 105, 106, 107, 108, 112, 113, 115, 118, 119, 120, 121, 122, 123, 124, 125, 126, 137, 138, 139, 140, 141, 144, 145, 146, 147	1.01961
	152	38, 68, 99, 152, 153, 154	1.01367
Large	61	30, 44, 58, 61, 76, 96, 103, 109, 148, 149, 150	N/A
	116	116, 148	N/A
SUV	51	17, 20, 28, 31, 34, 51, 52, 69, 72, 87, 93, 95, 110, 128, 131	1.04067
	58	11, 17, 23, 28, 29, 31, 32, 41, 53, 58, 69, 72, 73, 74, 87, 93, 110, 127, 128, 129, 131, 133, 134	1.03854
	59	20, 34, 59	1.05082
	94	11, 15, 23, 28, 29, 32, 37, 41, 42, 48, 52, 53, 57, 65, 73, 74, 94, 95, 110, 127, 128, 129, 130, 131, 132, 133, 134, 135, 148	1.03080
Minivan	26	8, 26	1.03721
	82	1, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 23, 29, 31, 32, 34, 37, 41, 42, 43, 44, 47, 53, 55, 57, 60, 63, 65, 66, 69, 72, 73, 74, 76, 77, 82, 85, 87, 88, 90, 93, 97, 102, 109, 111, 114, 117, 127, 129, 130, 132, 133, 134, 135, 136, 149, 150	1.01971
Pickup truck	142	8, 142	1.03721

Kruskal-Wallis test has been conducted to verify the non-parametric significance of differences between groups. It was shown that identified segments are non-identical populations with respect to the local rates of change (chi-squared = 9.8938, df = 3, p-value = 0.02964)

[a]List of HEVs who cited the corresponding state-of-the-art HEV as a benchmark

customers craving for stylish and spacious HEV but not as big as minivans (Voelcker 2013). Besides, the kinetic design deliberately shrinking the cargo space enables to deliver MPG of 43 which is the highest fuel economy in the large segment.

The local rates of change for the large segment could not be calculated due to their recent debut on the state-of-the-art frontier. That is, successive introductions of large HEVs could show two notable sub-segments within the frontier but the evolution of corresponding frontier facets hasn't occurred yet. Nevertheless, this emerging large HEV segment may be signaling one of the disruptive paths of future HEV development such as the recent adoption of diesel hybrid sheds light on an attempt to get a substantial boost in MPG and meet the stringent CO_2 regulations at the same time (Hazeldine et al. 2009; Borras 2013).

13.4.4 SUV Segment: "Forging Ahead"

Many industry reports point out that the SUV market is declining mostly due to the growing crossover segments as well as a low fuel economy (Siu 2013). However, at the same time, SUV is still recognized as a pure utility of a 'go anywhere' spirit that no other segment can replace in today's auto market. This motivated manufacturers to incorporate hybrid technology, especially plug-in, into the SUV market so that the hybrid SUV segment can address a market demand with the improved fuel economy (Duvall 2002; Greene et al. 2004).

The fast rates of change observed by all four state-of-the-art SUVs, Saturn Vue Hybrid (51), Audi Q5 (58), Jeep Patriot EV (59), and Porsche Cayenne S Hybrid (94), are supporting the previous argument. In particular, a relatively inexpensive SUV niche represented by Jeep Patriot EV and its dominated set show the fastest local rate of change of 5.08 % across all HEV segments. Furthermore, the dominant vehicles of medium and large SUVs: Audi Q5 and Cayenne S Hybrid, show local rates of change of 3.85 % and 3.08 % respectively. One may find it interesting to see how these cheap plug-in SUV and full-size luxury SUV segments would leverage the SUV market with current rate of technological advancement as opposed to the other crossover vehicles.

13.4.5 Minivan Segment: "Crossover"

As previously discussed, the cardinality of dominated set may imply the state-of-the-art HEV's positioning in the market. According to this, the Fit Shuttle Hybrid (82) can be regarded as a good all-round performer. Specifically, its dominated set includes all types of HEVs, which indicates that this vehicle would be one of the most representative designs across all HEV segments. However, the local rate of change of this cheap and economic minivan was found to be 1.97 %. This is slower than the larger minivan segment's, represented by Estima Hybrid (26), 3.72 %.

It should be noted here that minivans have been successful in Asia and Europe but have yet to be produced for the U.S. market. It is often pointed out that minivan's signature feature of three rows for seven (or eight) passenger capacity would face a difficulty in the U.S. market without ensuring sufficient cargo and legroom space (Young 2013). In addition, carmakers claim that minivans wouldn't get much fuel economy improvement due to their big and boxy structure. Furthermore, minivan customers want to have not only high fuel efficiency but also long cruising ranges, which requires the optimal placement of hybrid battery packs to keep them from using up valuable space. Therefore one may have to keep in mind that current minivan segment represented by Fit Shuttle Hybrid might be valid in a specific market that values economic design, hence, not be applicable to the U.S. market nor for the expected rate of technological advancement.

13.4.6 Pickup Truck Segment: "Steady"

There is actually only one hybrid pickup truck model (under two different brand names: Chevrolet Silverado and GMC Sierra both from General Motors) and therefore this segment reflects how much performance of this product line has advanced throughout the generations. Not surprisingly, the most recent model, Silverado 15 Hybrid 2WD (142), was found to be a state-of-the-art truck with annualized performance improvement of 3.72 %.

However, the hybrid pickup truck segment requires a cautious view on its future. The state-of-the-art hybrid truck today has fuel economy of 21 MPG and acceleration of 12.35 km/h/s with MSRP of $41,135. One may find it unclear if this hybrid truck is more appealing than its solid gasoline version, i.e., Silverado C15 2WD with 17 MPG, acceleration of 13.70 km/h/s, and $23,590 price tag. Assuming $5 a gallon gasoline and 20,000 miles per year, the payback period would be over 15 years. Although hybrid technology may be a good choice for other reasons, current efficiency-cost analysis suggests that the premium upfront cost for hybrid trucks is not likely offset by fuel savings. This indicates a faster rate of change through additional innovation may be needed for hybrid pickup trucks to become more prominent in the future HEV market.

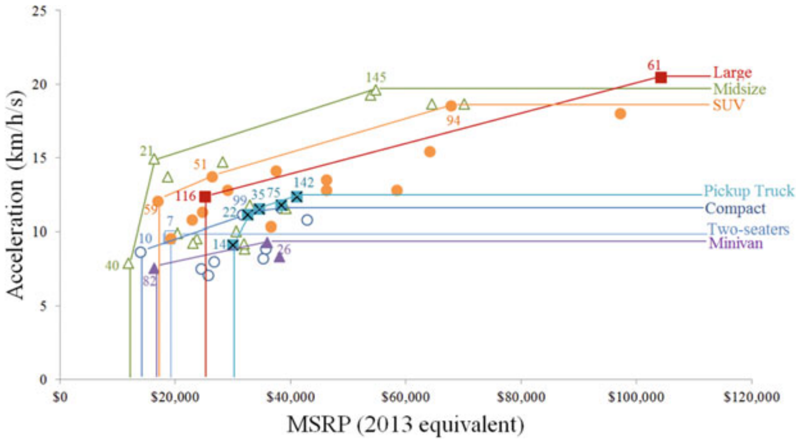
13.5 Conclusion

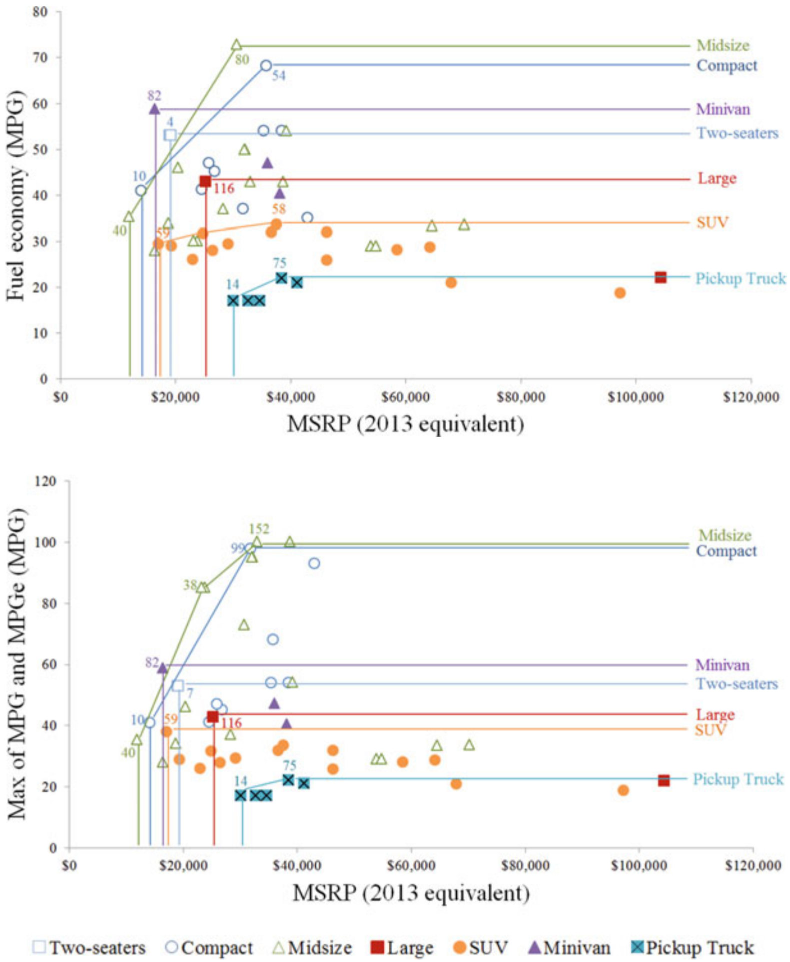
This study evaluates and compares the technological advancement observed in different HEV market segments over the past 15 years. The results indicate that three sub-segments exist in midsize HEVs and middle class represented by Prius alpha (V) showed the faster technological progress than other two. The performance growth as well as diversification of midsize HEVs seems to be posing a threat to two-seaters and compact segments. The overall rate of the SUV segment's technological advancement, from low price plug-in to full-size, was shown to be the fastest across the all HEV segments. The large HEVs are targeting a luxury market niche whereas minivans are showing the universal design characteristics in non U.S. markets. Finally, hybrid pickup trucks showed a steady performance upgrade however they are competing against their own solid gasoline versions to prove the utility of hybrid technologies.

In addition, the rate of technological advancement identified in each market (sub) segment was found to give an insight into the target setting practice for a new product development planning. Therefore, manufacturers may position their products within the current state-of-the-art frontier and utilize the corresponding rate of change to see whether their design targets would locate on the estimated future frontiers.

As a future work, trade-offs between technological characteristics need to be examined so that various future technological possibilities can be estimated based on identified rate of changes. Technological forecasting for Battery Electric Vehicles (BEV) using a similar approach could suggest another future work with the growing interest in pure electric vehicles.

Appendix: 2013 State-of-the-Art Frontiers of Different HEV Segments





References

Alexander, A. J., & Nelson, J. R. (1973). Measuring technological change: Aircraft turbine engines. *Technological Forecasting and Social Change, 5*, 189–203.

Banker, R. D., Charnes, A., & Cooper, W. W. (1984). Some models for estimating technical and scale inefficiencies in data envelopment analysis. *Management, 30*, 1078–1092.

Banker, R. D., & Morey, R. C. (1986). Use of categorical variables in data envelopment analysis. *Management Science, 32*, 1613–1627.

Borras, J. (2013). LA 2013: Volkswagen Announces 256 MPG Diesel Hybrid, GAS2.

Boskin, M. J., Dulberger, E. R., Gordon, R. J., Griliches, Z., & Jorgenson, D. W. (1998). Consumer prices, the consumer price index, and the cost of living. *Journal of Economic Perspectives, 12*, 3–26.

CarsDirect. (2013). Hybrid Car Buying Guide.

Danner, T. W. (2006). *A formulation of multidimensional growth models for the assessment and forecast of technology attributes*. Atlanta, GA: Georgia Institute of Technology.
Dodson, E. N. (1985). Measurement of state of the art and technological advance. *Technological Forecasting and Social Change, 27*, 129–146.
Doyle, J. R., & Green, R. H. (1991). Comparing products using data envelopment analysis. *Omega, 19*, 631–638.
Duvall, M. (2002). *Comparing the benefits and impacts of hybrid electric vehicle options for compact sedan and sport utility vehicles*. Palo Alto, CA: EPRI.
Environmental Protection Agency (EPA). (2013). The official U.S. government source for fuel economy information.
Garthwaite, J. (2013). Tesla motors' success gives electric car market a charge. *National Geographic Magazine*.
Greene, D. L., Duleep, K. G., & McManus, W. S. (2004). *Future potential of hybrid and diesel powertrains in the US light-duty vehicle market*. Oak Ridge National Laboratory, pp. 1–77.
Hazeldine, T., Kollamthodi, S., Brannigan, C., Morris, M., & Deller, L. (2009). *Market outlook to 2022 for battery electric and plug-in hybrid electric vehicles*. Oxfordshire: AEA.
Electric and Hybrid Vehicle Research, Development, and Demonstration Program. (2000). Petroleum-equivalent fuel economy calculation.
Inman, O. L. (2004). *Technology forecasting using data envelopment analysis*. Portland, OR: Portland State University.
Jahromi, S., Anderson, T., & Tudori, A. (2013). Forecasting hybrid electric vehicles using TFDEA. in *PICMET-Technology Management IT-Driven Services*, PICMET, San Jose, pp. 2098–2107.
Jahromi, S. R., Tudori, A. A., & Anderson, T. R. (2013). Forecasting hybrid electric vehicles using TFDEA. in *Technology Management in the IT-Driven Services*, PICMET, San Jose, pp. 2098–2107.
Lim, D.-J. (2015a). *Technological forecasting based on segmented rate of change*. Portland, OR: Portland State University.
Lim, D.-J. (2015b). Technology forecasting using DEA in the presence of infeasibility. *International Transactions in Operational Research* doi:10.1111/itor.12198
Lim, D.-J., & Anderson, T. R. (2012). An introduction to technology forecasting with a TFDEA excel add-in. in *Technology Management for Emerging Technologies (PICMET)*, *2012 Proceedings PICMET'12*.
Lim, D.-J., & Anderson, T. R. (2014). Improving forecast accuracy by a segmented rate of change in technology forecasting using data envelopment analysis (TFDEA). in *PICMET-Infrastructure and Service Integration*, PICMET, Kanazawa, pp. 2903–2907.
Lim, D.-J., Anderson, T. R., & Inman, O. L. (2014). Choosing effective dates from multiple optima in Technology Forecasting using Data Envelopment Analysis (TFDEA). *Technological Forecasting and Social Change, 88*, 91–97.
Lim, D.-J., Anderson, T. R., & Shott, T. (2015a). Technological forecasting of supercomputer development: The march to Exascale computing. *Omega, 51*, 128–135.
Lim, D.-J., Jahromi, S. R., Anderson, T. R., & Tudorie, A. A. (2015b). Comparing technological advancement of hybrid electric vehicles (HEV) in different market segments. *Technological Forecasting and Social Change, 97*, 140–153.
Martino, J. P. (1985). Measurement of technology using tradeoff surfaces. *Technological Forecasting and Social Change, 27*, 147–160.
OANDA Corporation. (2013). OANDA Currency Converter.
Ruggiero, J. (1996). On the measurement of technical efficiency in the public sector. *European Journal of Operational Research, 90*, 553–565.
Sahal, D. (1985). Foundations of technometrics. *Technological Forecasting and Social Change, 27*, 1–37.
Sanchez, E. A. (2013). Toyota targeting higher efficiency with next generation prius. *Automotive News Reports*.

Shao, S. S. S., Pipattanasomporn, M., & Rahman, S. (2009). Challenges of PHEV penetration to the residential distribution network. in *2009 I.E. Power Energy Society General Meeting*.

Siu, J. (2013). Top 10 Cheapest SUVs, AutoGuide.

Toyota. (2012). *Toyota's medium to long term growth initiatives*. Toyota Annual Report.

Tudorie, A. A. (2012). *Technology forecasting of electric vehicles using data envelopment analysis*. Delft, The Netherlands: Delft University of Technology.

U.S. Department of Energy. (2006). *Discussion meeting on plug-in hybrid electric vehicles*. Washington, DC.

Voelcker, J. (2013). Ford C-Max—Review, Car Connect.

Young, J. (2013). Getting small with hybrid mini vans, 1st Hybrid.

Chapter 14
A Radial Framework for Estimating the Efficiency and Returns to Scale of a Multi-component Production System in DEA

Jingjing Ding, Chenpeng Feng, and Huaqing Wu

Abstract This chapter provides radial measurements of efficiency for the production process possessing multi-components under different production technologies. Our approach is based on the construction of various empirical production possibility sets. Then we propose a procedure that is unaffected affected by multiple optima for estimating returns to scale. The theoretical connections between the traditional black box and the proposed multi-component approach are established, which ascertains consistency in estimating the efficiency and returns to scale. Moreover, we introduce two homogeneity conditions, which clarify the difference between our approach and the existing one, and are important for evaluating performance in multi-component setting. Finally, an empirical study of the pollution treatment processes in China is presented, and compared to the results from black-box approach. Many insightful findings related to the operations of the pollution treatment processes in China are secured.

Keywords Data envelopment analysis • Efficiency • Returns to scale • Multi-component • Pollution treatment process

J. Ding (✉) • C. Feng
School of Management, Hefei University of Technology, No. 193 Tunxi Road, Hefei, Anhui Province 230009, P.R. China
e-mail: jingding@hfut.edu.cn; cpfeng@hfut.edu.cn

H. Wu
School of Economics, Hefei University of Technology, No. 193 Tunxi Road, Hefei, Anhui Province 230009, P.R. China
e-mail: wuhuaqing@hfut.edu.cn

S.-N. Hwang et al. (eds.), *Handbook of Operations Analytics Using Data Envelopment Analysis*, International Series in Operations Research & Management Science 239, DOI 10.1007/978-1-4899-7705-2_14

14.1 Introduction

We consider the estimation of the efficiency and returns to scale (RTS) for a production system which can be modeled as having multi-components based on data envelopment analysis (DEA). There are many production systems bearing this situation. For example, Beasley (1995) studied the performances of universities, each of which had two components: research and teaching. Cook et al. (2000) modeled a banking production system as having two components: service and sales. We are mainly concerned with radial measurements, and the theoretical connection with the existing black-box approach.

DEA is a nonparametric technique for measuring the relative efficiencies of a set of peer decision-making units (DMUs) involving multiple inputs and outputs. Charnes et al. (1978) first introduced it. In this pioneer paper, the authors constructed a nonlinear programming model to evaluate the efficiency of activity conducted by a non-profit organization. The model is known as the CCR model in the literature. As is known, the CCR model captures both technical and scale inefficiencies. Banker et al. (1984) proposed a new model (BCC model) which extended the CCR model by separating technical efficiency and scale efficiency. Recently, DEA has been extended to many areas in management science and operational research field.

At the early stage of development, DEA treats a DMU under evaluation as a black box. Thus, it cannot provide users with specific information concerning the sources of inefficiency within an organization. Färe and Grosskopf (2000) introduced a network DEA technique, which opened the black box, and explicitly modeled the internal mechanism of a DMU. Lewis and Sexton (2004) also published a research paper in this direction. Färe and Grosskopf (2000) and Lewis and Sexton (2004) proposed radial measurements of efficiency in network DEA literature. By contrast, Tone and Tsutsui (2009) extended radial measurements in network DEA to non-radial measurements of efficiency by introducing slack-based network DEA model. Kao and Hwang (2008) and Kao (2009a, b) proposed models for evaluating DMUs with serial network structure, parallel network structure and the mixture of the above two structures. DMUs with a two-stage production process have been extensively studied both from a theoretical and from a practical perspective. Included among these studies are Liang et al. (2008) and Chen et al. (2006, 2009a, b, 2010). We refer the reader to review papers, such as Cook et al. (2010) and Castelli et al. (2010) for more references.

The value of returns to scale (RTS) measures the percentage change in output from a given percentage change in inputs in economic theory. Unlike main researches in economic literature, which are concerned about production processes with a single output, extensions to the situations of multiple outputs are spurred by Banker et al. (1984). Since then, RTS has been studied extensively. Banker et al. (2004) published an excellent review on different methods used to handle RTS. According to the paper, there are two approaches followed in the literature to study RTS. The first approach is proposed by Färe et al. (1985, 1994) and the other one is devised by Banker et al. (1984). In this paper, we follow the first approach, which has the advantage of being unaffected by the possible existence of multiple optima.

The existing papers concerning RTS are mainly based on the black-box assumption. However, very few of these papers deal with RTS, when the black-box assumption is dropped. Research papers with RTS consideration include Chen et al. (2009a), Tsai and Molinero (2002). Those two papers both follow the framework proposed by Banker et al. (1984), and could suffer from the existence of multiple optima.

Our current paper studies a production process with a multi-component structure. Before moving on, we firstly differentiate two cases of production processes having a multi-component according to data availability. The first case has the data on how the shared inputs and shared outputs are split among sub-decision making units (SDMUs). The second case does not have data on how the shared inputs/outputs are split among SDMUs. Beasley (1995) and Cook et al. (2000) investigated models for evaluating performance in the second case, but did not study the RTS of the productions. In addition, how to extend their models to treat RTS is not clear. The difficulties are twofold in multi-component setting: (1) the nonlinearity of the proposed models and (2) the impact of potential multiple optima on testing RTS by following Banker's approach. Our work focuses on production processes with multi-components of (1). In doing so, we avoid the problem of nonlinearity, to center on investigating RTS.

The contributions of our work mainly lie in three aspects. Firstly, we propose radial measurements for efficiency evaluation and a procedure to determine the RTS of a DMU that is unaffected by possible multiple optima. Secondly, we establish theoretical connection between the black-box approach by Färe et al. (1985, 1994) and our multi-component approach, which helps to connect the black-box approach with the network approach, and ensures consistency between both approaches in dealing with RTS. In addition, two homogeneity conditions are proposed and are important for evaluating performance in multi-component setting. They are not pointed out before in the literature. Thirdly, in this work, we use the proposed method to study the efficiency and RTS of pollution treatment processes in China based on real data. We model the processes as having two components, which is different from the traditional approach, and secure various insightful findings related to the operations of the pollution treatment processes in China.

The paper unfolds as follows: Section 14.2 proposes a radial evaluation model under variable returns to scale assumption (14.2.1), and establishes the theoretical connection of the proposed model to the black-box model (14.2.2). Section 14.3 provides a procedure for determining the RTS of a DMU. Section 14.4 establishes the theoretical connection of the proposed approach for estimating RTS to Färe et al. (1985, 1994). In Sect. 14.5, we apply the prospective method to study the performance of pollution treatment processes in China. Section 14.6 concludes the paper.

14.2 Radial Performance Measurement for a Multi-component System

A production unit (denoted as a DMU) with multi-component structure studied in this paper is depicted in Fig. 14.1. The DMU consists of two sub-decision-making units (SDMUs) without loss of generality. It is assumed that some inputs of DMU are shared by $SDMU_1$ and $SDMU_2$, and some outputs are the results of $SDMU_1$ and $SDMU_2$. In addition to shared inputs and outputs, there are inputs and outputs of the DMU dedicated to, or are the results of, $SDMU_1$ or $SDMU_2$ exclusively. We assume to deal with n DMUs in this paper. In the sequel, when referring to a specific DMU, we denote it by a subscript j, that is, DMU_j, $SDMU_{1j}$, and $SDMU_{2j}$ ($j = 1, \ldots, n$).

The variables in Fig. 14.1 are defined as follows: $X_1 = \left(x_1^1, \ldots, x_m^1\right)$ indicates m inputs dedicated to $SDMU_1$; $X_2 = \left(x_1^2, \ldots, x_h^2\right)$ indicates h inputs dedicated to $SDMU_2$; $X_s = \left(x_1^s, \ldots, x_l^s\right)$ indicates l inputs shared by $SDMU_1$ and $SDMU_2$; $Y_1 = \left(y_1^1, \ldots, y_s^1\right)$ indicates s outputs produced exclusively by $SDMU_1$; $Y_2 = \left(y_1^2, \ldots, y_q^2\right)$ indicates q outputs produced exclusively by $SDMU_2$; $Y_s = \left(y_1^s, \ldots, y_u^s\right)$ indicates u outputs produced together by $SDMU_1$ and $SDMU_2$. When referring to the specific data of DMU_j, we shall use a secondary index j. For instance, the m inputs dedicated to $SDMU_{1j}$, the $SDMU_1$ of DMU_j, are denoted as $X_{1j} = \left(x_{1j}^1, \ldots, x_{mj}^1\right)$.

We differentiate two cases of production processes with multi-component structure according to data availability. In the first case, the data on how the shared inputs and shared outputs are split between $SDMU_1$ and $SDMU_2$ are available. In this case, we use $X_{s1} = \left(x_1^{s1}, \ldots, x_l^{s1}\right)$, $X_{s2} = \left(x_1^{s2}, \ldots, x_l^{s2}\right)$, and $Y_{s1} = \left(y_1^{s1}, \ldots, y_u^{s1}\right)$, $Y_{s2} = \left(y_1^{s2}, \ldots, y_u^{s2}\right)$ to denote the observational data fulfilling $X_s = X_{s1} + X_{s2}$ and $Y_s = Y_{s1} + Y_{s2}$. Note that these are component wise additions indicating $X_s(i) = X_{s1}(i) + X_{s2}(i)$, $i, \ldots, l$, and $Y_s(j) = Y_{s1}(j) + Y_{s2}(j)$, $j = 1, \ldots, u$. In the second case, it is not known how the shared inputs/outputs are split. *We deal with the former case in this paper.*

To be specific, we take pollution treatment processes in China as an example. If we are going to investigate the performances of pollution treatment processes in all provinces, provinces are naturally modeled as DMUs. When the black box of a

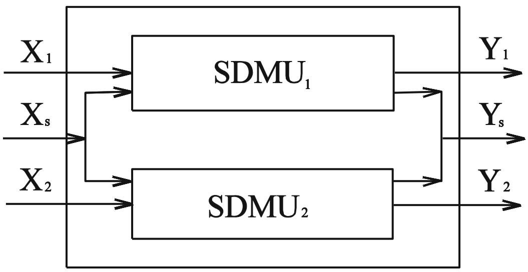


Fig. 14.1 Structure of multi-component system

DMU is opened, it can be found that cities can be further classified into two SDMUs: capital city and non-capital cities. The capital city is the political, economic and cultural center of a province. Thus, the environment beyond the control of the management of the pollution treatment process in capital city and non-capital cities is arguably different. This makes sense: For example, a capital city often consumes more inputs such as capital inputs: pollution treatment facilities. As will be shown in this paper, the average capital city consumes approximately more than one fifth of the total inputs, but produces less than one fifth of the total outputs. In this case, we might reasonably claim that the capital city consumes more inputs as compared with noncapital cities.

14.2.1 Basic Model

Let us begin with the construction of production possibility set (PPS) of each SDMU. Based on the PPS of SDMUs, the PPS of a DMU is derived. We assume first variable returns of scale for all SDMUs. Note that the PPS considered is similar to that in Tsai and Molinero (2002).

The PPS of SDMU_1:

$$T_1^{VRS} = \left\{ \left(X^1, Y^1\right) \left| \begin{array}{l} \sum_{j=1}^{n} \lambda_j^1 x_{ij}^1 \le x_i^1, i = 1, \ldots, m, \sum_{j=1}^{n} \lambda_j^1 y_{rj}^1 \ge y_r^1, r = 1, \ldots, s \\ \sum_{j=1}^{n} \lambda_j^1 x_{ij}^{s1} \le x_i^{s1}, i = 1, \ldots, l, \sum_{j=1}^{n} \lambda_j^1 y_{rj}^{s1} \ge y_r^{s1}, r = 1, \ldots, u \\ \sum_{j=1}^{n} \lambda_j^1 = 1, \lambda_j^1 \ge 0 \end{array} \right. \right\} \tag{14.1}$$

where $\left(X^1, Y^1\right) = \left(x_1^1, \ldots, x_m^1, x_1^{s1}, \ldots, x_l^{s1}, y_1^1, \ldots, y_s^1, y_1^{s1}, \ldots, y_u^{s1}\right)$.

The PPS of SDMU_2:

$$T_2^{VRS} = \left\{ \left(X^2, Y^2\right) \left| \begin{array}{l} \sum_{j=1}^{n} \lambda_j^2 x_{ij}^2 \le x_i^2, i = 1, \ldots, h, \sum_{j=1}^{n} \lambda_j^2 y_{rj}^2 \ge y_r^2, r = 1, \ldots, q \\ \sum_{j=1}^{n} \lambda_j^2 x_{ij}^{s2} \le x_i^{s2}, i = 1, \ldots, l, \sum_{j=1}^{n} \lambda_j^2 y_{rj}^{s2} \ge y_r^{s2}, r = 1, \ldots, u \\ \sum_{j=1}^{n} \lambda_j^2 = 1, \lambda_j^2 \ge 0 \end{array} \right. \right\} \tag{14.2}$$

where $\left(X^2, Y^2\right) = \left(x_1^2, \ldots, x_h^2, x_1^{s2}, \ldots, x_l^{s2}, y_1^2, \ldots, y_q^2, y_1^{s2}, \ldots, y_u^{s2}\right)$.

The PPS of DMU:

$$T^{VRS}=\left\{(X,Y)\left|\begin{array}{l}\sum_{j=1}^{n}\lambda_j^1 x_{ij}^1\le x_i^1, i=1,\ldots,m, \sum_{j=1}^{n}\lambda_j^1 x_{ij}^{s1}+\sum_{j=1}^{n}\lambda_j^2 x_{ij}^{s2}\le x_i^s, i=1,\ldots,l\\ \sum_{j=1}^{n}\lambda_j^2 x_{ij}^2\le x_i^2, i=1,\ldots,h, \sum_{j=1}^{n}\lambda_j^1 y_{rj}^1\ge y_r^1, r=1\ldots s\\ \sum_{j=1}^{n}\lambda_j^1 y_{rj}^{s1}+\sum_{j=1}^{n}\lambda_j^2 y_{rj}^{s2}\ge y_r^s, r=1,\ldots,u, \sum_{j=1}^{n}\lambda_j^2 y_{rj}^2\ge y_r^2, r=1,\ldots,q\\ \sum_{j=1}^{n}\lambda_j^1=1, \sum_{j=1}^{n}\lambda_j^2=1, \lambda_j^1,\lambda_j^2\ge 0\end{array}\right.\right\} \tag{14.3}$$

where $(X,Y)=\left(x_1^1,\ldots,x_m^1,x_1^s,\ldots,x_l^s,x_1^2,\ldots,x_h^2,y_1^1,\ldots,y_s^1,y_1^s,\ldots,y_u^s,y_1^2,\ldots,y_q^2\right)$.

It should be noted that the PPS of DMU is the addition of the PPS's of $SDMU_1$ and $SDMU_2$. We assume that if $SDMU_1$ (X^1, Y^1) and $SDMU_2$ (X^2, Y^2) are possible, then one can set up a DMU consisting of a $SDMU_1$ and a $SDMU_2$. Most importantly, the two SDMUs do not interfere with each other and carry out (X^1, Y^1) and (X^2, Y^2) independently. The result is then that DMU built in this way consumes (X^1+X^2), and produces (Y^1+Y^2).

The performance of a DMU can be measured under two different situations: first, price information is given, and second, prices are not available. In the latter situation, Shephard's input distance function is a frequently used measurement (Shephard's 1970). Suppose $L(Y)$ is the input requirement set derived from T^{VRS}. Shephard's input distance function is given below.

$$D(X,Y)=\max\{\lambda : X/\lambda\in L(Y), \lambda\in R\} \tag{14.4}$$

Clearly, $D(X,Y)$ is greater than or equal to 1, if $X\in L(Y)$, with $D(X,Y)=1$, if and only if it is impossible to improve input vector X proportionately without worsening the output vector. Let $\theta=1/\lambda$. It follows that

$$[D(X,Y)]^{-1}=\min\{\theta : \theta X\in L(Y)\} \tag{14.5}$$

According to (14.3) and (14.5), the performance of DMU_0 with multi-components can be estimated by the following linear programming model.

$$
\begin{aligned}
&\theta_T^* = \min\ \theta \\
&\text{s.t.}\ \sum_{k=1}^{2}\sum_{j=1}^{n}\lambda_j^k x_{ij}^{sk} \leq \theta x_{io}^{s} && i = 1, \ldots, l\ \text{(shared inputs)} \\
&\sum_{j=1}^{n}\lambda_j^1 x_{ij}^1 \leq \theta x_{io}^1 && i = 1, \ldots, m\ (\text{inputs dedicated to SDMU}_1) \\
&\sum_{j=1}^{n}\lambda_j^2 x_{ij}^2 \leq \theta x_{io}^2 && i = 1, \ldots, h\ (\text{inputs dedicated to SDMU}_2) \\
&\sum_{k=1}^{2}\sum_{j=1}^{n}\lambda_j^k y_{rj}^{sk} \geq y_{ro}^{s} && r = 1, \ldots, u\ \text{(shared outputs)} \\
&\sum_{j=1}^{n}\lambda_j^1 y_{rj}^1 \geq y_{ro}^1 && r = 1, \ldots, s\ (\text{outputs produced by SDMU}_1) \\
&\sum_{j=1}^{n}\lambda_j^2 y_{rj}^2 \geq y_{ro}^2 && r = 1, \ldots, q\ (\text{outputs produced by SDMU}_2) \\
&\sum_{j=1}^{n}\lambda_j^k = 1 && k = 1, 2 \\
&\lambda_j^k \geq 0, k = 1, 2, && j = 1, \ldots, n.
\end{aligned}
\tag{14.6}
$$

where the decision variables are $\lambda_j^k\ (j = 1, \ldots, n;\ k = 1, 2)$ and θ. It should be noted that x_{ij}^k, x_{ij}^{sk}, y_{rj}^k and y_{rj}^{sk} are observational data that correspond to the types of inputs and outputs labeled in (14.6).

14.2.2 Theoretical Connection with Black-Box Approach

In this section, we formally derive the black-box equivalent PPS that corresponds to T^{VRS}, which can give an insight into model (14.6). Before moving on, we assume that the structure depicted in Fig. 14.1 consumes all inputs shared by SDMU_1 and SDMU_2, and all the outputs of DMU are the results of SDMU_1 and SDMU_2. We adopt the convention that DMU consumes m inputs $X_j = \left(x_{1j}, \ldots, x_{mj}\right)$ and produces s outputs $Y_j = \left(y_{1j}, \ldots, y_{sj}\right)$. Thus, based on the notations provided above for DMUs with multi-component structure, the assumption here implies that $X_j^{sk} = \left(x_{1j}^{sk}, \ldots, x_{mj}^{sk}\right)$ and $Y_j^{sk} = \left(y_{1j}^{sk}, \ldots, y_{sj}^{sk}\right)$ with $X_j^{s1} + X_j^{s2} = X_j^s = X_j$, and $Y_j^{s1} + Y_j^{s2} = Y_j^s = Y_j$. Later in the paper, the s in the superscript is deleted for

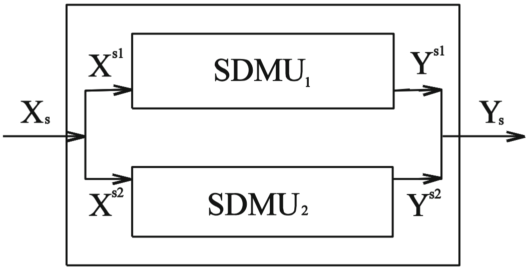


Fig. 14.2 Structure of DMU with all shared inputs and outputs

simplicity. In cases where inputs or outputs are not entirely shared by SDMU_1 and SDMU_2 (See Fig. 14.1), the values of those inputs/outputs dedicated to SDMU_1 (SDMU_2) are zeros for SDMU_2 (SDMU_1). Therefore, the structure of the DMU in Fig. 14.1 reduces to structure provided in Fig. 14.2.

In light of the structure depicted in Fig. 14.2, T_1^{VRS}, T_2^{VRS} and T^{VRS} in the previous section are rewritten as follows:

$$T_1^{VRS} = \left\{ \left(X^1, Y^1\right) \middle| \sum_{j=1}^{n} \lambda_j^1 x_{ij}^1 \le x_i^1, i = 1, \ldots, m, \right. \\ \left. \sum_{j=1}^{n} \lambda_j^1 y_{rj}^1 \ge y_r^1, r = 1, \ldots, s, \sum_{j=1}^{n} \lambda_j^1 = 1, \lambda_j^1 \ge 0 \right\} \tag{14.7}$$

where $\left(X^1, Y^1\right) = \left(x_1^1, \ldots, x_m^1, y_1^1, \ldots, y_s^1\right)$.

$$T_2^{VRS} = \left\{ \left(X^2, Y^2\right) \middle| \sum_{j=1}^{n} \lambda_j^2 x_{ij}^2 \le x_i^2, i = 1, \ldots, m, \right. \\ \left. \sum_{j=1}^{n} \lambda_j^2 y_{rj}^2 \ge y_r^2, r = 1, \ldots, s, \sum_{j=1}^{n} \lambda_j^2 = 1, \lambda_j^2 \ge 0 \right\} \tag{14.8}$$

where $\left(X^2, Y^2\right) = \left(x_1^2, \ldots, x_m^2, y_1^2, \ldots, y_s^2\right)$.

$$T^{VRS} = \left\{ (X, Y) \middle| \begin{array}{l} \sum_{k=1}^{2} \sum_{j=1}^{n} \lambda_j^k x_{ij}^k \le x_i, i = 1, \ldots, m, \sum_{k=1}^{2} \sum_{j=1}^{n} \lambda_j^k y_{rj}^k \ge y_r, r = 1, \ldots, s \\ \sum_{j=1}^{n} \lambda_j^k = 1, \lambda_j^k \ge 0, k = 1, 2 \end{array} \right\} \tag{14.9}$$

where $(X, Y) = (x_1, \ldots, x_m, y_1, \ldots, y_s)$.

We proceed to give a result on the convexity of T^{VRS} that is necessary for the exposition of this paper.

Property 1 T^{VRS} is convex set.

Proof Suppose (X_1, Y_1) and (X_2, Y_2) belong to T^{VRS}. By definition, there are sets of nonnegative multipliers $\lambda_j^{k1*}, \lambda_j^{k2*}$ with $\sum_{j=1}^{n} \lambda_j^{k1*} = 1$ and $\sum_{j=1}^{n} \lambda_j^{k2*} = 1$ such that

$$\sum_{k=1}^{2}\sum_{j=1}^{n}\lambda_j^{k1*} y_{rj}^k \geq y_r^1, r = 1, \ldots, s, \sum_{k=1}^{2}\sum_{j=1}^{n}\lambda_j^{k1*} x_{ij}^k \leq x_i^1, i = 1, \ldots, m,$$

$$\sum_{k=1}^{2}\sum_{j=1}^{n}\lambda_j^{k2*} y_{rj}^k \geq y_r^2, r = 1, \ldots, s, \sum_{k=1}^{2}\sum_{j=1}^{n}\lambda_j^{k2*} x_{ij}^k \leq x_i^2, i = 1, \ldots, m.$$

For any convex pair α, β, we have $\sum_{k=1}^{2}\sum_{j=1}^{n}\left(\alpha\lambda_j^{k1*} + \beta\lambda_j^{k2*}\right) y_{rj}^k \geq \alpha y_r^1 + \beta y_r^2, r = 1, \ldots, s,$ $\sum_{k=1}^{2}\sum_{j=1}^{n}\left(\alpha\lambda_j^{k1*} + \beta\lambda_j^{k2*}\right) x_{ij}^k \leq \alpha x_i^1 + \beta x_i^2, i = 1, \ldots, m,$ and $\sum_{j=1}^{n}\left(\alpha\lambda_j^{k1*} + \beta\lambda_j^{k2*}\right) = 1.$ This ensures that $\alpha(X_1, Y_1) + \beta(X_2, Y_2) = (\alpha X_1 + \beta X_2, \alpha Y_1 + \beta Y_2) \in T^{VRS}$. □

Assumption 1 Assume there are n DMUs, each of which consists of two production units $\text{SDMU}_{1\text{j}}$, $\text{SDMU}_{2\text{j}}$, $j = 1,\ldots,n$ using the production technology characterized by T_1^{VRS} and T_2^{VRS} respectively. Let there be an extended data set (**EDS**) of n^2 distinct DMUs, each of which comprises $\text{SDMU}_{1\text{j}}$ and $\text{SDMU}_{2\text{k}}$ with $j, k \in \{1, \ldots, n\}$.

Let (x_{ij}, y_{rj}) denote the input and output bundle of DMU_j in **EDS**. Define T_b^{VRS}, T_b^{CRS}, and T_b^{NIRS} as below, where the superscripts CRS and NIRS, respectively, stand for constant returns to scale and non-increasing returns to scale:

$$T_b^{VRS} = \left\{ (X, Y) \middle| \sum_{j=1}^{n^2} \lambda_j x_{ij} \leq x_i, i = 1, \ldots, m, \sum_{k=1}^{n^2} \lambda_j y_{rj} \geq y_r, r = 1, \ldots, s, \sum_{j=1}^{n^2} \lambda_j = 1, \lambda_j \geq 0 \right\}$$

$$T_b^{CRS} = \left\{ (X, Y) \middle| \sum_{j=1}^{n^2} \lambda_j x_{ij} \leq x_i, i = 1, \ldots, m, \sum_{k=1}^{n^2} \lambda_j y_{rj} \geq y_r, r = 1, \ldots, s, \lambda_j \geq 0 \right\}$$

$$T_b^{NIRS} = \left\{ (X,Y) \middle| \sum_{j=1}^{n^2} \lambda_j x_{ij} \leq x_i, i = 1, \ldots, m, \sum_{k=1}^{n^2} \lambda_j y_{rj} \geq y_r, r = 1, \ldots, s, \right.$$
$$\left. \sum_{j=1}^{n^2} \lambda_j \leq 1, \lambda_j \geq 0 \right\}$$

where $(X,Y) = (x_1, \ldots, x_m, y_1, \ldots, y_s)$.

We now establish that the PPS of the general multi-component system with two different SDMUs can be recovered by DMUs in **EDS** through the black-box approach. The connections between the multi-component PPS's and the above mentioned black-box PPS's are summarized in Theorem 1.

Theorem 1 *$T_b^{VRS} = T^{VRS}$, $T_b^{CRS} = T^{CRS}$, and $T_b^{NIRS} = T^{NIRS}$, where*

$$T^{CRS} = \left\{ (X,Y) \middle| \begin{array}{l} \sum_{k=1}^{2}\sum_{j=1}^{n} \lambda_j^k x_{ij}^k \leq x_i, i = 1, \ldots, m, \sum_{k=1}^{2}\sum_{j=1}^{n} \lambda_j^k y_{rj}^k \geq y_r, r = 1, \ldots, s \\ \sum_{j=1}^{n} \lambda_j^1 = \sum_{j=1}^{n} \lambda_j^2, \lambda_j^k \geq 0, k = 1, 2 \end{array} \right\}$$

and

$$T^{NIRS} = \left\{ (X,Y) \middle| \begin{array}{l} \sum_{k=1}^{2}\sum_{j=1}^{n} \lambda_j^k x_{ij}^k \leq x_i, i = 1, \ldots, m, \sum_{k=1}^{2}\sum_{j=1}^{n} \lambda_j^k y_{rj}^k \geq y_r, r = 1, \ldots, s \\ \sum_{j=1}^{n} \lambda_j^1 = \sum_{j=1}^{n} \lambda_j^2 \leq 1, \lambda_j^k \geq 0, k = 1, 2 \end{array} \right\}$$

Proof See Appendix. □

Let us close this section by pointing out the difference between T^{CRS} and $\overline{T}^{CRS}$, which is defined by

$$\overline{T}^{CRS} = \left\{ (X,Y) \middle| \sum_{k=1}^{2}\sum_{j=1}^{n} \lambda_j^k x_{ij}^k \leq x_i, i = 1, \ldots, m, \sum_{k=1}^{2}\sum_{j=1}^{n} \lambda_j^k y_{rj}^k \geq y_r, r = 1, \ldots, s, \right.$$
$$\left. \lambda_j^k \geq 0, k = 1, 2 \right\}.$$

Researchers in the literature tend to define $\overline{T}^{CRS}$ as the CRS PPS for the production system in Fig. 14.1. Tsai and Molinero (2002) is a case in point. Obviously, the production frontier determined by T^{CRS} is dominated by the one defined by $\overline{T}^{CRS}$. In Fig. 14.3, we use a set of two DMUs with one input and one output for illustration.

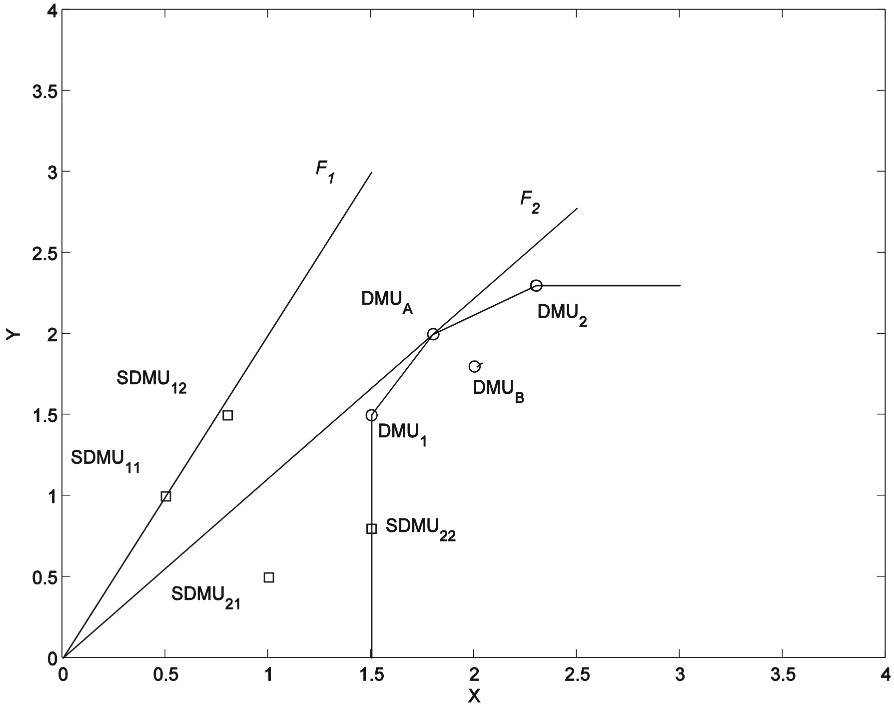


Fig. 14.3 Graphical illustration of T^{CRS} and $\overline{T}^{CRS}$

Here, DMU_1 and DMU_2 comprise of (SDMU_{11}, SDMU_{21}) and (SDMU_{12}, SDMU_{22}) respectively. DMU_A and DMU_B are generated by combining respectively SDMU_{11} and SDMU_{22}, SDMU_{21} and SDMU_{22}. In light of Theorem 1, T^{CRS} is the conic hull constructed by DMU_1, DMU_2, DMU_A and $\mathrm{DMU}_\mathrm{B.}$ This is the region to the right of frontier F_2. PPS provided by $\overline{T}^{CRS}$ is the region to the right of frontier F_1.

Figure 14.3 shows that the production frontier of $\overline{T}^{CRS}$ is determined by SDMU_{11}. Apparently, the production process of SDMU is arguably different from that of DMU. Therefore, the use of SDMU as a benchmarking point for DMU is not appropriate. To highlight the difference between SDMU and DMU, criteria for homogeneity are essential. The homogeneity in this context refers to the characteristic of the efficient frontier that a benchmarking point on the frontier constructed for evaluating the performance of a DMU should be comparable to the DMU in terms of the internal production process. Two homogeneity conditions for the construction of a virtual DMU, i.e., weak condition and strong condition, are introduced below:

(1) Weak homogeneity condition: $t_1 = 0$ if and only if $t_2 = 0$.
(2) Strong homogeneity condition: $t_1 = t_2$.

Clearly, if a virtual DMU built by SDMU_1 and SDMU_2 satisfies the strong homogeneity condition, the weak homogeneity condition is automatically satisfied. However, the opposite is not true. Comparing the definition of T^{CRS} with that of $\overline{T}^{CRS}$, the difference is the distinct requirements of the sum of the levels of elementary activities involved (i.e., $t_1 = \sum_{j=1}^{n} \lambda_j^1, t_2 = \sum_{j=1}^{n} \lambda_j^2$). Specifically, T^{CRS} requires $t_1 = t_2$, while $\overline{T}^{CRS}$ does not. $\overline{T}^{CRS}$ is claimed to violate the strong homogeneity condition.

This small example shows that SDMU_{11} is chosen as a benchmarking point, as can be seen from Fig. 14.3 where F_1 is completely specified by SDMU_{11}. If we do not set conditions for choosing a benchmarking point, the frontier is arguably too ideal. The main consequence is the potential under estimation of the efficiency of a DMU, since an improper benchmarking point is chosen. This specification of conditions is comparable to the modeling consideration in the evaluation considering environment constraints. One might expect that the performance of a DMU be evaluated by comparing it to the DMUs possessing similar environment characteristics (See, for example, Ruggiero (1998)).

14.3 Procedure for Estimating the Returns to Scale

In economic theory, the value of RTS measures the percentage change in output from a given percentage change in inputs. Let $y = f(x)$ denote a production function for a single-output technology. The production function is said to have IRS if $f(ax) > af(x)$, for any $a > 1$. The production function exhibits DRS if $f(ax) > af(x)$, for any $a \in [0, 1)$. If $f(ax) = af(x)$ for all scalars $a \geq 0$, the production function exhibits CRS. Banker et al. (1984), who introduced the concept of Most Productive Scale Size (MPSS) into the DEA literature, spurred extensions to the situations of multiple inputs and outputs. For a technically efficient DMU_0 with input and output bundle (X_0, Y_0) to be MPSS, the following optimization model should achieve a value of one. Note that the subscript 0 is usually used to indicate the DMU under evaluation in the literature. In the sequel, we shall frequently refer to DMU_0 when a specific DMU is discussed.

$$\begin{aligned} \max\ & \frac{\beta}{\alpha} \\ \text{s.t.}\ & (\alpha X_0, \beta Y_0) \in T \\ & \alpha, \beta \geq 0. \end{aligned} \tag{14.10}$$

where T is the empirical production possibility set. If the optimal value is larger than 1, it means that either the current input level can be reduced with a less percentage of losses in outputs, or it can be increased with a larger percentage of gains in outputs. Therefore, DMU_0 can benefit from the adjustment of input levels.

By analogy, the following model is proposed for testing whether DMU_0 with multi-component structure is MPSS, where T in (14.10) is substituted by T^{VRS}.

$$
\begin{aligned}
&\max \frac{\phi}{\theta} \\
&\text{s.t.} \sum_{k=1}^{2}\sum_{j=1}^{n}\lambda_j^k x_{ij}^{sk} + s^{s-} = \theta x_{io}^s \qquad i = 1, \ldots, l \text{ (shared inputs)} \\
&\sum_{j=1}^{n}\lambda_j^1 x_{ij}^1 + s_i^{1-} = \theta x_{io}^1 \qquad i = 1, \ldots, m \text{ (inputs dedicated to SDMU}_1) \\
&\sum_{j=1}^{n}\lambda_j^2 x_{ij}^2 + s_i^{2-} = \theta x_{io}^2 \qquad i = 1, \ldots, h \text{ (inputs dedicated to SDMU}_2) \\
&\sum_{k=1}^{2}\sum_{j=1}^{n}\lambda_j^k y_{rj}^{sk} - s_r^{s+} = \phi y_{ro}^s \qquad r = 1, \ldots, u \text{ (shared outputs)} \\
&\sum_{j=1}^{n}\lambda_j^1 y_{rj}^1 - s_r^{1+} = \phi y_{ro}^1 \qquad r = 1, \ldots, s \text{ (outputs produced by SDMU}_1) \\
&\sum_{j=1}^{n}\lambda_j^2 y_{rj}^2 - s_r^{2+} = \phi y_{ro}^2 \qquad r = 1, \ldots, q \text{ (outputs produced by SDMU}_2) \\
&\sum_{j=1}^{n}\lambda_j^k = 1 \qquad k = 1, 2, \\
&\lambda_j^k \geq 0, k = 1, 2, j = 1, \ldots, n.
\end{aligned}
\tag{14.11}
$$

Cooper et al. (1996) proposed an approach to transform the above non-linear model to an equivalent linear model. Firstly, let us divide both sides of the constraints by ϕ. The resulting model is given in (14.12). Secondly, by letting $\theta/\phi = t$, $s_r^{s+}/\phi = \bar{s}_r^{s+}$, $s_r^{1+}/\phi = \bar{s}_r^{1+}$, $s_r^{2+}/\phi = \bar{s}_r^{2+}$, $s_i^{s-}/\phi = \bar{s}_i^{s-}$, $s_i^{1-}/\phi = \bar{s}_i^{1-}$, $s_i^{2-}/\phi = \bar{s}_i^{2-}$ and $\lambda_j^k/\phi = \bar{\lambda}_j^k$, we can obtain model (14.13). Since ϕ in (14.13) is a free variable, it is safe to delete it. Finally, model (14.13) can be further reduced to an equivalent model (14.14). Note that we call two optimization problems equivalent if from a solution of one, a solution of the other is readily found, and vice versa.

$$
\begin{aligned}
&\max \frac{\phi}{\theta} \\
&\text{s.t.} \ \sum_{k=1}^{2}\sum_{j=1}^{n}\frac{\lambda_j^k}{\phi}x_{ij}^{sk}+\frac{s_i^{s-}}{\phi}=\frac{\theta}{\phi}x_{io}^{s}\ i=1,\ldots,l\,(\text{shared inputs}) \\
&\quad \sum_{j=1}^{n}\frac{\lambda_j^1}{\phi}x_{ij}^1+\frac{s_i^{1-}}{\phi}=\frac{\theta}{\phi}x_{io}^1 \quad i=1,\ldots,m\,(\text{inputs dedicated to SDMU}_1) \\
&\quad \sum_{j=1}^{n}\frac{\lambda_j^2}{\phi}x_{ij}^2+\frac{s_i^{2-}}{\phi}=\frac{\theta}{\phi}x_{io}^2 \quad i=1,\ldots,h\,(\text{inputs dedicated to SDMU}_2) \\
&\quad \sum_{k=1}^{2}\sum_{j=1}^{n}\frac{\lambda_j^k}{\phi}y_{rj}^{sk}-\frac{s_r^{s+}}{\phi}=y_{ro}^{s} \quad r=1,\ldots,u\,(\text{shared outputs}) \\
&\quad \sum_{j=1}^{n}\frac{\lambda_j^1}{\phi}y_{rj}^1-\frac{s_r^{1+}}{\phi}=y_{ro}^1 \quad r=1,\ldots,s\,(\text{outputs produced by SDMU}_1) \\
&\quad \sum_{j=1}^{n}\frac{\lambda_j^2}{\phi}y_{rj}^2-\frac{s_r^{2+}}{\phi}=y_{ro}^2 \quad r=1,\ldots,q\,(\text{outputs produced by SDMU}_2) \\
&\quad \sum_{j=1}^{n}\frac{\lambda_j^k}{\phi}=\frac{1}{\phi} \quad k=1,2 \\
&\quad \lambda_j^k,\phi\geq 0,k=1,2,j=1,\ldots,n.
\end{aligned}
\tag{14.12}
$$

$$
\begin{aligned}
&\max \frac{1}{t} \\
&\text{s.t.} \ \sum_{k=1}^{2}\sum_{j=1}^{n}\overline{\lambda}_j^k x_{ij}^{sk}+\overline{s}_i^{s-}=tx_{io}^{s} \quad i=1,\ldots,l\,(\text{shared inputs}) \\
&\quad \sum_{j=1}^{n}\overline{\lambda}_j^1 x_{ij}^1+\overline{s}_i^{1-}=tx_{io}^1 \quad i=1,\ldots,m\,(\text{inputs dedicated to SDMU}_1) \\
&\quad \sum_{j=1}^{n}\overline{\lambda}_j^2 x_{ij}^2+\overline{s}_i^{2-}=tx_{io}^2 \quad i=1,\ldots,h\,(\text{inputs dedicated to SDMU}_2) \\
&\quad \sum_{k=1}^{2}\sum_{j=1}^{n}\overline{\lambda}_j^k y_{rj}^{sk}-\overline{s}_r^{s+}=y_{ro}^{s} \quad r=1,\ldots,u\,(\text{shared outputs}) \\
&\quad \sum_{j=1}^{n}\overline{\lambda}_j^1 y_{rj}^1-\overline{s}_r^{1+}=y_{ro}^1 \quad r=1,\ldots,s\,(\text{outputs produced by SDMU}_1) \\
&\quad \sum_{j=1}^{n}\overline{\lambda}_j^2 y_{rj}^2-\overline{s}_r^{2+}=y_{ro}^2 \quad r=1,\ldots,q\,(\text{outputs produced by SDMU}_2) \\
&\quad \sum_{j=1}^{n}\overline{\lambda}_j^1=\sum_{j=1}^{n}\overline{\lambda}_j^2=\frac{1}{\phi} \\
&\quad \lambda_j^k\geq 0,k=1,2,j=1,\ldots,n.
\end{aligned}
\tag{14.13}
$$

$$
\begin{aligned}
& t^* = \min\ t \\
\text{s.t. } & \sum_{k=1}^{2}\sum_{j=1}^{n}\overline{\lambda}_j^k x_{ij}^{sk} + \overline{s}_i^{s-} = t x_{io}^s \qquad i = 1, \ldots, l \text{ (shared inputs)} \\
& \sum_{j=1}^{n}\overline{\lambda}_j^1 x_{ij}^1 + \overline{s}_i^{1-} = t x_{io}^1 \qquad i = 1, \ldots, m \text{ (inputs dedicated to SDMU}_1\text{)} \\
& \sum_{j=1}^{n}\overline{\lambda}_j^2 x_{ij}^2 + \overline{s}_i^{2-} = t x_{io}^2 \qquad i = 1, \ldots, h \text{ (inputs dedicated to SDMU}_2\text{)} \\
& \sum_{k=1}^{2}\sum_{j=1}^{n}\overline{\lambda}_j^k y_{rj}^{sk} - \overline{s}_r^{s+} = y_{ro}^s \qquad r = 1, \ldots, u \text{ (shared outputs)} \\
& \sum_{j=1}^{n}\overline{\lambda}_j^1 y_{rj}^1 - \overline{s}_r^{1+} = y_{ro}^1 \qquad r = 1, \ldots, s \text{ (outputs produced by SDMU}_1\text{)} \\
& \sum_{j=1}^{n}\overline{\lambda}_j^2 y_{rj}^2 - \overline{s}_r^{2+} = y_{ro}^2 \qquad r = 1, \ldots, q \text{ (outputs produced by SDMU}_2\text{)} \\
& \sum_{j=1}^{n}\overline{\lambda}_j^1 = \sum_{j=1}^{n}\overline{\lambda}_j^2 \\
& \lambda_j^k \geq 0, k = 1, 2, j = 1, \ldots, n.
\end{aligned}
\tag{14.14}
$$

Assume that $t^*, \overline{\lambda}_j^{k*}$ are the optimal solution to model (14.14). It follows that $\phi^* = 1/\sum_{j=1}^{n}\lambda_j^{2*}$ and $\theta^* = t^*\phi^* = t^*/\sum_{j=1}^{n}\overline{\lambda}_j^{1*} = t^*/\sum_{j=1}^{n}\overline{\lambda}_j^{2*}$. Apparently, Proposition 1 holds.

Proposition 1 *If* $t^* = 1$, *then DMU is MPSS, and constant returns to scale prevails at DMU; Otherwise, the unit is not MPSS.*

RTS generally has an unambiguous meaning only if DMU_0 is on the efficiency frontier. For any inefficient DMU0 to become efficient, based on the optimal solutions of model (14.6), it can be projected onto the efficient frontier by formulas as follows:

(1) $\overline{y}_{ro}^s = y_{ro}^s + s_r^{+s*}, \overline{y}_{ro}^1 = y_{ro}^1 + s_r^{+1*}, \overline{y}_{ro}^2 = y_{ro}^2 + s_r^{+2*}$.
(2) $\overline{x}_{io}^s = t^* x_{io}^s - s_i^{-s*}, \overline{x}_{io}^1 = t^* x_{io}^1 - s_i^{-1*}, \overline{x}_{io}^2 = t^* x_{io}^2 - s_i^{-2*}$.

For those who are interested in the projection operation and the concept of efficient frontier, we recommend Cooper et al. (2004). A full treatment of the topics is beyond the scope of this paper. Before proceeding to discuss how to determine RTS of a DMU, we now introduce the scale efficiency of a production unit in Definition 1.

Definition 1 Scale efficiency: $\theta_S^* = t^*/\theta_T^*$.

Scale efficiency reflects the RTS characteristic of DMU_0. It should be noted that if DMU_0 is not an efficient unit, the scale efficiency actually reflects the RTS characteristic of the corresponding projection on the efficient frontier by formulas (14.1) and (14.2). Let us denote it as DMU_o^* for the convenience of reference.

Obviously, it can be seen that $\theta_S^* \leq 1$, since the feasible set of model (14.6) is a subset of the feasible set of model (14.14). If $\theta_S^* = 1$, DMU_o^* should achieve an efficiency rating of 1 by model (14.14). If not, it contradicts that $\theta_S^* = 1$, i.e., $t^* = \theta_T^*$. Therefore, by Proposition 1, DMU_0^* is MPSS. In other words, DMU_0 exhibits or is projected onto a region of the efficient frontier exhibits constant returns to scale.

If $\theta_S^* < 1$, or equivalently, the optimal objective function (ϕ/θ) of model (14.11) is larger than 1, the current input–output data of DMU_0^* can be improved in productivity by adjusting the scale of it. This is because the percentage by which the outputs gain equiproportionate increase due to the adjustment of the scale will outweigh the percentage by which the inputs increase equiproportionate, or the input equiproportionate reduction will outweigh the output equiproportionate reduction. To sum up, if $\theta_S^* < 1$, DMU_0 is currently not located in CRS region of the frontier or not projected onto a region of the frontier that exhibits CRS.

Below we provide Proposition 2 to shed light on how to determine whether IRS or DRS prevail at DMU_0 with the aid of model (14.15).

Proposition 2. (Conditions for the Determination of RTS (Multi-component))

(1) *If* $\theta_S^* = 1$, *then DMU0 exhibits or is projected onto a region of the efficient frontier exhibits constant returns to scale.*
(2) *If* $\theta_S^* < 1$ *and the optimal values of models (14.14) and (14.15) below coincide, then* DMU_0 *exhibits or is projected onto a region of the efficient frontier that exhibits increasing returns to scale.*
(3) *If* $\theta_S^* < 1$ *and the optimal values of models (14.6) and (14.15) below coincide,* DMU_0 *exhibits or is projected onto a region of the efficient frontier that exhibits decreasing returns to scale.*

A short proof of the proposition is in order. We consider the condition (2): $\theta_S^* < 1$ and the optimal values of models (14.14) and (14.15) coincide. The condition (3) can be established similarly.

Let $\bar{\lambda}_j^{1*}$ and $\bar{\lambda}_j^{2*}$ be the optimal solutions of models (14.14) and (14.15). It is clear that $\sum_{j=1}^{n} \bar{\lambda}_j^{1*} = \sum_{j=1}^{n} \bar{\lambda}_j^{2*} < 1$. DMU_0^* can make improvement through output augmentation since $\phi^* = 1/\sum_{j=1}^{n} \bar{\lambda}_j^{1*} > 1$. As DMU_0^* is technically efficient, the only way that it can increase the output level is by increasing the level of inputs. As the percentage by which the outputs increase outweighs the percentage by which the inputs increase, DMU_0 is currently located in the region that shows increasing returns to scale.

We have to show now it is impossible to lower its output level, and at the same time improve the productivity, i.e., achieve MPSS, since we have not checked if model (14.15) can achieve an value less than that of model (14.6) (i.e., θ_T^*) if $\sum_{j=1}^n \lambda_j^1 = \sum_{j=1}^n \lambda_j^2 \leq 1$ is replaced by $\sum_{j=1}^n \lambda_j^1 = \sum_{j=1}^n \lambda_j^2 \geq 1$. It should be noted that an optimal value less than θ_T^* in this context indicates DMU_0^* can gain benefits by lowering its input level. If this were true, the RTS of DMU_0^* will have an ambiguous meaning, since it can gain positive change in productivity by either lowering or augmenting its input level.

We claim impossibility by contradiction. Suppose $\overline{\lambda}_{1j}^{1*}, \overline{\lambda}_{1j}^{2*}, t_1^*$ and $\overline{\lambda}_{2j}^{1*}, \overline{\lambda}_{2j}^{2*}, t_2^*$ are the respective optimal solutions of model (14.15) and the model similar to model (14.15) except that $\sum_{j=1}^n \lambda_{1j}^{1*} = \sum_{j=1}^n \lambda_{1j}^{2*} < 1$ is replaced by $\sum_{j=1}^n \lambda_{2j}^{1*} = \sum_{j=1}^n \lambda_{2j}^{2*} > 1$. In addition, $t_1^* = t^* \leq t_2^* < \theta_T^*$ (i.e., $\theta_S^* < 1$). Thus, there exists a convex combination of the two solutions with $t^* = at_1^* + (1-a)t_2^* < \theta_T^*$, and $\sum_{j=1}^n \left(a\lambda_{1j}^{1*} + (1-a)\lambda_{2j}^{1*}\right) = \sum_{j=1}^n \left(a\lambda_{1j}^{2*} + (1-a)\lambda_{2j}^{2*}\right) = 1$, which contradicts the premise that θ_T^* is the optimal value of model (14.6). Thus, impossibility holds and condition (2) has an unambiguous meaning.

$$
\begin{aligned}
& t_{nirs}^* = \min\ t \\
& \text{s.t.}\ \sum_{k=1}^{2}\sum_{j=1}^{n}\overline{\lambda}_j^k x_{ij}^{sk} + \overline{s}_i^{s-} = tx_{io}^s \qquad i = 1, \ldots, l \text{ (shared inputs)} \\
& \quad \sum_{j=1}^{n}\overline{\lambda}_j^1 x_{ij}^1 + \overline{s}_i^{1-} = tx_{io}^1 \qquad i = 1, \ldots, m \text{ (inputs dedicated to } \mathrm{SDMU}_1\text{)} \\
& \quad \sum_{j=1}^{n}\overline{\lambda}_j^2 x_{ij}^2 + \overline{s}_i^{2-} = tx_{io}^2 \qquad i = 1, \ldots, h \text{ (inputs dedicated to } \mathrm{SDMU}_2\text{)} \\
& \quad \sum_{k=1}^{2}\sum_{j=1}^{n}\overline{\lambda}_j^k y_{rj}^{sk} - \overline{s}_r^{s+} = y_{ro}^s \qquad r = 1, \ldots, u \text{ (shared outputs)} \\
& \quad \sum_{j=1}^{n}\overline{\lambda}_j^1 y_{rj}^1 - \overline{s}_r^{1+} = y_{ro}^1 \qquad r = 1, \ldots, s \text{ (outputs produced by } \mathrm{SDMU}_1\text{)} \\
& \quad \sum_{j=1}^{n}\overline{\lambda}_j^2 y_{rj}^2 - \overline{s}_r^{2+} = y_{ro}^2 \qquad r = 1, \ldots, q \text{ (outputs produced by } \mathrm{SDMU}_2\text{)} \\
& \quad \sum_{j=1}^{n}\overline{\lambda}_j^1 = \sum_{j=1}^{n}\overline{\lambda}_j^2 \leq 1 \\
& \quad \lambda_j^k \geq 0, k = 1, 2, j = 1, \ldots, n.
\end{aligned}
\tag{14.15}
$$

14.4 Theoretical Connection Between Black Box Approach and Multi-component Approach

In this section, we establish the equivalence between the method proposed in the previous section and the traditional black approach provided by Färe et al. (1985, 1994). This further ensures consistency in transition from black box to multi-component setting.

The efficiency measurements based on CRS, VRS, and NIRS respectively are provided as follows:

1. Efficiency index based on CRS;

$$\begin{aligned} \theta_b^{crs} &= \min\ \theta \\ \text{s.t.}\ &\sum_{j=1}^{n^2} \lambda_j y_{rj} \geq y_{ro} \qquad r = 1, \ldots, s. \\ &\sum_{j=1}^{n^2} \lambda_j x_{ij} \leq \theta x_{io} \qquad i = 1, \ldots, m. \\ &\lambda_j \geq 0, j = 1, \ldots, n^2. \end{aligned} \tag{14.16}$$

2. Efficiency index based on VRS;

$$\begin{aligned} \theta_b^{vrs} &= \min\ \theta \\ \text{s.t.}\ &\sum_{j=1}^{n^2} \lambda_j y_{rj} \geq y_{ro} \qquad r = 1, \ldots, s. \\ &\sum_{j=1}^{n^2} \lambda_j x_{ij} \leq \theta x_{io} \qquad i = 1, \ldots, m. \\ &\sum_{j=1}^{n^2} \lambda_j = 1 \\ &\lambda_j \geq 0, j = 1, \ldots, n^2. \end{aligned} \tag{14.17}$$

3. Efficiency index based on NIRS;

$$\begin{aligned} \theta_b^{nirs} &= \min\ \theta \\ \text{s.t.}\ &\sum_{j=1}^{n^2} \lambda_j y_{rj} \geq y_{ro} \qquad r = 1, \ldots, s. \\ &\sum_{j=1}^{n^2} \lambda_j x_{ij} \leq \theta x_{io} \quad i = 1, \ldots, m. \\ &\sum_{j=1}^{n^2} \lambda_j \leq 1 \\ &\lambda_j \geq 0, j = 1, \ldots, n^2. \end{aligned} \tag{14.18}$$

Färe et al. (1985, 1994) provided the following proposition for determining RTS.

Proposition 3 (Conditions for Determination of RTS (Black Box))

(1) *DMU_0 exhibits or is projected onto a region of the efficient frontier that exhibits constant returns to scale, if* $\theta_b^{crs} = \theta_b^{vrs} = \theta_b^{nirs}$.
(2) *DMU_0 exhibits or is projected onto a region of the efficient frontier that exhibits increasing returns to scale, if* $\theta_b^{crs} = \theta_b^{nirs} < \theta_b^{vrs}$.
(3) *DMU_0 exhibits or is projected onto a region of the efficient frontier that exhibits decreasing returns to scale, if* $\theta_b^{crs} < \theta_b^{nirs} = \theta_b^{vrs}$.

Formally, the following theorem holds.

Theorem 2 *Proposition 2 is equivalent to Proposition 3.*

Proof In light of Theorem 1, we can derive that $\theta_b^{vrs} = \theta_T^*, \theta_b^{crs} = t^*$ and $\theta_b^{crs} = t_{nirs}^*$, since the corresponding PPS's are equal. Since $\theta_s^* = 1$ indicates $t^* = \theta_T^* = t_{nirs}^*$, it follows that the first condition of Proposition 3 is equivalent to the first condition of Proposition 2. By the same reasoning, condition 2 of the propositions is equivalent as well as their conditions 3. Thus, Proposition 3 is equivalent to Proposition 2. □

14.5 Application

In this section, data extracted from *Environmental Statistics 2009* are used for illustration. We analyze the performances (efficiency and RTS) of the pollution treatment processes for waste water and waste air in China. Provinces are deemed as DMUs, each of which consists of two SDMUs, namely, capital city and non-capital cities. The pollution treatment process is depicted in Fig. 14.4.

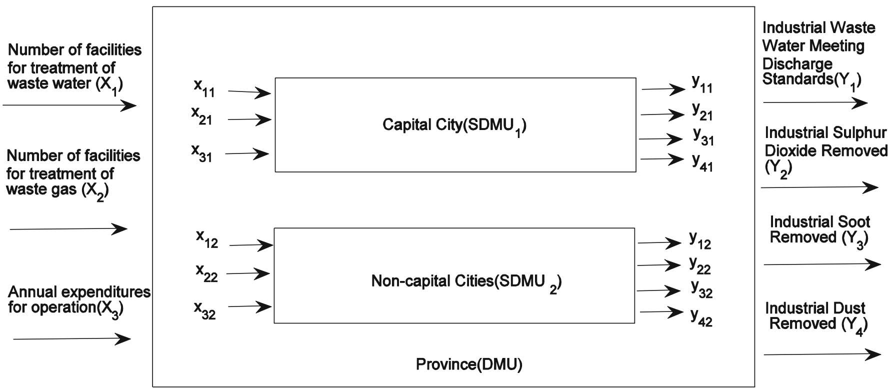


Fig. 14.4 Treatment process for wastewater and gas

The inputs involved in this application are three indicators: (1) number of facilities for treatment of wastewater in set (X_1); (2) number of facilities for treatment of waste gas in set (X_2); (3) annual expenditures in 10,000 Yuan (X_3). The outputs include (1) the industrial wastewater meeting discharge standards in 10,000 t (Y_1), (2) industrial sulphur dioxide removed in 10,000 t (Y_2), (3) industrial soot removed in 10,000 t (Y_3), and (4) industrial dust removed in 10,000 t (Y_4).

The inputs (X_1, X_2, X_3) are shared by capital city (SDMU_1) and non-capital cities (SDMU_2), and the outputs are the results of SDMU_1 and SDMU_2 fulfilling $X_i = \sum_{k=1}^{2} x_{ik}$ and $Y_r = \sum_{k=1}^{2} y_{rk}$.

Table 14.1 provides the input/output data by DMU (province), and Table 14.2 provides data on inputs/outputs by SDMU_1 (capital city). Table 14.3 presents the descriptive statistics of the data on inputs/outputs. In light of Table 14.3, capital city consumes relative more inputs and produces comparatively less outputs. An average capital city consumes inputs 19 %, 21 % and 26 % of the means of X_1, X_2 and X_3 respectively. However, the amounts produced account for 20 %, 17 %, 17 % and 14 % respectively of the means of Y_1, Y_2, Y_3 and Y_4 by an average capital city. Thus, roughly speaking, the average capital city consumes approximately more than one fifth of the total inputs, but produces less than one fifth of the total outputs. In this case, we might reasonably claim that the capital city consumes more inputs as compared with the noncapital cities. In the sequel, we will present the computational results associated with efficiency and returns to scale.

14.5.1 Efficiency

The efficiencies of DMUs by using the black-box approach and the proposed multi-component approach are presented in Table 14.4. From the black-box approach, the results of $\overline{\theta}_o^{crs}$ (CCR model), $\overline{\theta}_o^{nirs}$ and $\overline{\theta}_o^{vrs}$ (BCC model) are reported in columns 2–4. Column 5 presents results by Kao's parallel model which, in fact, are based on the $\overline{T}^{CRS}$ (see Kao (2009b)). Using the multi-component approach the results of $t^*, \theta^*_{nirs}, \theta^*_T$ by models (14.14), (14.15) and (14.6) are presented in columns 6–8.

Now we focus on the results of $\overline{\theta}_o^{vrs}$ and θ^*_T, both of which are based on the VRS assumption. Note that $\overline{\theta}_o^{vrs}$ is the result of the black-box approach without considering the internal mechanism of a DMU, and θ^*_T is the result of multi-component approach. The difference between the two efficiency indexes can be attributed to the level of information requirements. Obviously, if more information is available, we are able to refine the results from the black-box approach. Overall, notice from Table 14.4 that the mean of θ^*_T is approximately 87.6 % of the mean of $\overline{\theta}_o^{vrs}$, with a standard deviation of 0.103. Their distributions are provided in Fig. 14.5. The distribution of θ^*_T is more bell-shaped, while the distribution of $\overline{\theta}_o^{vrs}$ is obviously skewed to the left.

Table 14.1 Number of facilities, annual expenditures for operation and treatment of wastewater and waste gas by province

Province	X_1 (set)	X_2 (set)	X_3 (10,000 Yuan)	Y_1 (10,000 t)	Y_2 (10,000 t)	Y_3 (10,000 t)	Y_4 (10,000 t)
Beijing	514.00	2547.00	115,631.00	8221.00	11.20	199.90	108.40
Tianjin	875.00	3052.00	230,766.70	20,413.00	22.10	307.90	119.30
Hebei	5822.00	12,998.00	837,126.40	115,699.00	92.80	2158.30	741.80
Shanxi	2797.00	9216.00	588,709.40	35,229.00	108.70	2102.50	306.70
Inner Mongolia	836.00	4406.00	343,880.80	24,092.00	141.10	2358.20	236.20
Liaoning	1822.00	10,247.00	427,415.30	73,544.00	89.10	1520.40	675.30
Jilin	629.00	2983.00	118,217.20	33,443.00	8.90	885.50	293.50
Heilongjiang	990.00	4396.00	320,333.30	33,768.00	4.10	1133.30	107.00
Shanghai	1790.00	3839.00	394,044.20	41,364.00	24.00	524.40	103.40
Jiangsu	6469.00	11,365.00	994,868.20	253,940.00	177.20	2117.30	295.30
Zhejiang	7630.00	13,600.00	683,111.40	182,094.00	128.60	1297.20	475.70
Anhui	1795.00	4669.00	307,930.90	64,438.00	133.40	1097.20	321.00
Fujian	4196.00	6833.00	244,448.40	137,825.00	29.70	485.80	226.90
Jiangxi	1767.00	3786.00	202,398.00	63,863.00	130.60	1058.90	497.50
Shandong	4590.00	11,453.00	1,071,702.00	174,953.00	222.20	2683.20	614.00
Henan	3211.00	8225.00	1,113,638.00	126,308.00	134.60	2382.30	585.00
Hubei	2050.00	4921.00	378,840.30	87,753.00	71.80	781.30	362.10
Hunan	3149.00	4884.00	270,813.90	85,057.00	70.40	816.10	295.90
Guangdong	9968.00	13,876.00	1,031,626.00	191,413.00	108.80	1108.60	609.30
Guangxi	2552.00	5687.00	257,770.60	176,290.00	56.80	503.90	248.60
Hainan	293.00	401.00	53,475.40	5674.00	4.00	68.60	9.60
Chongqing	1550.00	2978.00	389,378.20	62,648.00	65.30	279.80	41.70
Sichuan	4757.00	6811.00	373,030.70	103,191.00	61.70	1006.90	239.30
Guizhou	1798.00	2708.00	185,327.00	8386.00	80.20	1015.10	143.90
Yunnan	2032.00	5267.00	283,327.80	30,574.00	114.40	817.60	373.70

(continued)

Table 14.1 (continued)

Province	X_1 (set)	X_2 (set)	X_3 (10,000 Yuan)	Y_1 (10,000 t)	Y_2 (10,000 t)	Y_3 (10,000 t)	Y_4 (10,000 t)
Tibet	13.00	39.00	563.50	274.00	0.00	0.10	0.00
Shananxi	2780.00	4036.00	203,039.20	47,132.00	36.40	440.60	190.70
Gansu	747.00	2734.00	604,078.40	9670.00	139.10	327.50	84.00
Qinghai	148.00	771.00	40,161.50	3767.00	1.10	139.10	66.80
Ningxia	380.00	1259.00	91,000.20	17,884.00	13.30	495.70	37.70
Xinjiang	775.00	4177.00	106,242.50	15,078.00	4.60	429.80	61.10

Table 14.2 Number of facilities, annual expenditures for operation and treatment of wastewater and waste gas by capital city

Capital city	X_1 (set)	X_2 (set)	X_3 (10,000 Yuan)	Y_1 (10,000 t)	Y_2 (10,000 t)	Y_3 (10,000 t)	Y_4 (10,000 t)
Beijing	514.00	2547.00	115,631.00	8221.00	11.20	199.90	108.40
Tianjin	875.00	3052.00	230,766.70	20,413.00	22.10	307.90	119.30
Shijiazhuang	547.00	1434.00	80,319.80	20,795.00	19.60	278.20	29.30
Taiyuan	310.00	1234.00	133,666.90	1994.00	19.00	529.20	87.40
Hohhot	74.00	550.00	42,033.10	2802.00	14.40	320.30	8.60
Shenyang	352.00	2181.00	25,760.90	6706.00	3.00	91.10	1.60
Changchun	103.00	671.00	16,906.40	5181.00	0.60	153.90	85.00
Harbin	156.00	863.00	32,357.70	3367.00	2.10	180.40	16.60
Shanghai	1790.00	3839.00	394,044.20	41,364.00	24.00	524.40	103.40
Nanjing	701.00	1123.00	152,446.10	36,606.00	42.00	271.80	78.00
Hangzhou	1051.00	1672.00	144,510.20	63,889.00	8.60	120.20	78.80
Hefei	170.00	389.00	14,842.70	2011.00	0.50	89.10	13.10
Fuzhou	367.00	537.00	58,322.90	5429.00	6.30	129.20	1.60
Nanchang	198.00	499.00	28,708.40	9471.00	2.30	47.90	27.80
Jinan	208.00	765.00	125,283.30	4693.00	12.60	157.60	68.60
Zhengzhou	388.00	1408.00	66,162.40	12,696.00	6.60	295.80	55.70
Wuhan	248.00	551.00	69,037.50	23,603.00	10.00	249.90	35.20
Changsha	289.00	315.00	8248.70	3679.00	2.60	50.10	20.10
Guangzhou	1161.00	2069.00	176,626.00	33,045.00	15.60	209.10	5.90
Nanning	367.00	840.00	28,386.60	12,696.00	2.00	38.60	34.10
Haikou	35.00	19.00	2984.30	481.00	0.00	0.00	0.00
Chongqing	1550.00	2978.00	389,378.20	62,648.00	65.30	279.80	41.70
Chengdu	1759.00	1545.00	67,992.50	20,421.00	5.60	216.70	13.50
Guiyang	385.00	753.00	51,893.60	2244.00	24.10	95.50	32.20
Kunming	546.00	1414.00	102,620.40	4405.00	63.20	125.20	41.00

(continued)

Table 14.2 (continued)

Capital city	X_1 (set)	X_2 (set)	X_3 (10,000 Yuan)	Y_1 (10,000 t)	Y_2 (10,000 t)	Y_3 (10,000 t)	Y_4 (10,000 t)
Lhasa	11.00	11.00	355.00	267.00	0.00	0.00	0.00
Xi'an	414.00	920.00	18,480.40	17,862.00	2.90	65.90	14.60
Lanzhou	73.00	512.00	493,456.40	2311.00	2.00	103.70	11.10
Xinning	123.00	420.00	32,555.10	3443.00	1.10	115.80	38.40
Yinchuan	106.00	242.00	18,244.00	4923.00	1.40	12.70	6.90
Urumqi	103.00	474.00	27,511.10	4564.00	0.40	39.80	10.40

Table 14.3 Descriptive statistics on input and output variables

Variables	Mean (province)	Std. dev. (province)	Mean (capital city)	Std. dev. (capital city)	Mean (non-capital city)	Std. dev. (non-capital city)
X_1 (set)	2539.5	2368.8	483.03	494.6	2056.5	2188.7
X_2 (set)	5618.2	3920.8	1155.7	949.74	4462.5	3801.1
X_3 (10,000 Yuan)	395,580	319,360	101,600	122,600	293,980	302,330
Y_1 (10,000 t)	72,064	67,491	14,265	17,043	57,799	61,115
Y_2 (10,000 t)	73.748	59.378	12.616	16.88	61.132	57.118
Y_3 (10,000 t)	985.26	758.18	170.96	134.95	814.3	711.99
Y_4 (10,000 t)	273.27	210.9	38.332	35.447	234.94	212.8

Furthermore, according to Fig. 14.5, 15 provinces are classified as efficient by the BCC. It can be seen the discrimination power of BCC model in this application is too weak. By contrast, 12 of them are degraded in efficiencies by the multi-component approach. They are Hebei, Liaoning, Zhejiang, Jiangxi, Shandong, Henan, Guangdong, Guangxi, Tibet, Gansu, Qinghai, and Ningxia. Seven of them are given efficiency scores lower than 0.9.

Finally, we point out that the efficiency scores based on T^{CRS} are almost the same as those based on $\overline{T}^{CRS}$. Though the differences of θ^*_{Kao} and t^* are negligible, we can find that the efficiencies of some DMUs such as Zhejiang and Hunan are adjusted slightly.

14.5.2 Returns to Scale

The RTS of provinces can be determined by Proposition 3 (black box), and Proposition 2 (multi-component). The results are presented in Table 14.5.

Table 14.5 shows that approximately half of the provinces which are classified by the black-box approach as CRS and IRS are reclassified as DRS or CRS by the multi-component approach. Those classified as DRS by the black-box approach remain the same by the both approaches. We concentrate here on the results of the multi-component approach. In summary, six provinces show IRS, five provinces show CRS and the rest show DRS. Among those that show CRS, Inner Mongolia and Jilin have the MPSS because the optimal value in Model (14.10) that corresponds to t^* in Table 14.4 equals one. We proceed to rearrange the results by the multi-component approach according to the administrative regions of China. The results are provided in Table 14.6.

From Table 14.6, the developed provinces are more likely to show DRS. In particular, East China shows DRS entirely. Another obvious finding is that the provinces that show IRS are mainly located in the west of China, which is less developed area of China.

Table 14.4 Results of various models

Provinces	θ_b^{crs}	θ_b^{nirs}	θ_b^{vrs}	θ_{Kao}	t^*	θ_{nirs}^*	θ_T^*
Beijing	0.4944	0.4944	0.5062	0.373	0.373	0.373	0.373
Tianjin	0.4681	0.4681	0.4748	0.3937	0.3937	0.4003	0.4003
Hebei	0.5428	1	1	0.4718	0.4718	0.8754	0.8754
Shanxi	0.5161	0.5654	0.5654	0.4952	0.4952	0.536	0.536
Inner Mongolia	1	1	1	1	1	1	1
Liaoning	0.902	1	1	0.7609	0.7609	0.8761	0.8761
Jilin	1	1	1	1	1	1	1
Heilongjiang	0.7037	0.7037	0.7049	0.5528	0.5528	0.6043	0.6043
Shanghai	0.5177	0.5177	0.519	0.3241	0.3241	0.3756	0.3756
Jiangsu	0.9221	1	1	0.6711	0.6711	1	1
Zhejiang	0.5352	1	1	0.5034	0.5036	0.7536	0.7536
Anhui	0.9099	0.9405	0.9405	0.7872	0.7872	0.7941	0.7941
Fujian	0.8612	0.8612	0.8614	0.7703	0.7703	0.8023	0.8023
Jiangxi	1	1	1	0.9605	0.9618	0.9709	0.9709
Shandong	0.8142	1	1	0.6254	0.6254	0.9473	0.9473
Henan	0.9133	1	1	0.7209	0.7209	0.9412	0.9412
Hubei	0.8199	0.8199	0.8201	0.6874	0.6874	0.7555	0.7555
Hunan	0.774	0.774	0.7742	0.7004	0.7006	0.7006	0.7006
Guangdong	0.5486	1	1	0.4723	0.4723	0.7266	0.7266
Guangxi	1	1	1	0.9612	0.9614	0.9708	0.9708
Hainan	0.6608	0.6608	0.7323	0.4066	0.4066	0.4066	0.4243
Chongqing	0.9163	0.9163	0.9203	0.6495	0.6495	0.7123	0.7123
Sichuan	0.6593	0.6593	0.6596	0.615	0.6154	0.6179	0.6179
Guizhou	0.8909	0.8909	0.8961	0.8397	0.8397	0.8407	0.8407
Yunnan	0.6877	0.6877	0.6882	0.5962	0.5965	0.5965	0.597
Tibet	0.711	0.711	1	0.5031	0.5031	0.5031	0.8519
Shananxi	0.5248	0.5248	0.5274	0.4946	0.495	0.495	0.4953
Gansu	1	1	1	0.9309	0.9346	0.9346	0.9351
Qinghai	0.9673	0.9673	1	0.6992	0.6992	0.6992	0.706
Ningxia	1	1	1	0.889	0.8903	0.8903	0.8903
Xinjiang	0.5407	0.5407	0.5434	0.4591	0.4591	0.5198	0.5198

14.6 Summary and Conclusion

This paper studies the efficiency evaluation and RTS estimation in the situation where a DMU has multi-component structure. Radial measurements for efficiency evaluation and a procedure to determine the RTS of a DMU that is unaffected by possible multiple optima are provided. In doing so, we emphasize the theoretical connections between the black-box approach, which has been extensively studied in the literature, and the proposed methods. The strong relationship as is given by theorem 1 ensures a consistent transition from the black-box approach to the multi-component approach.

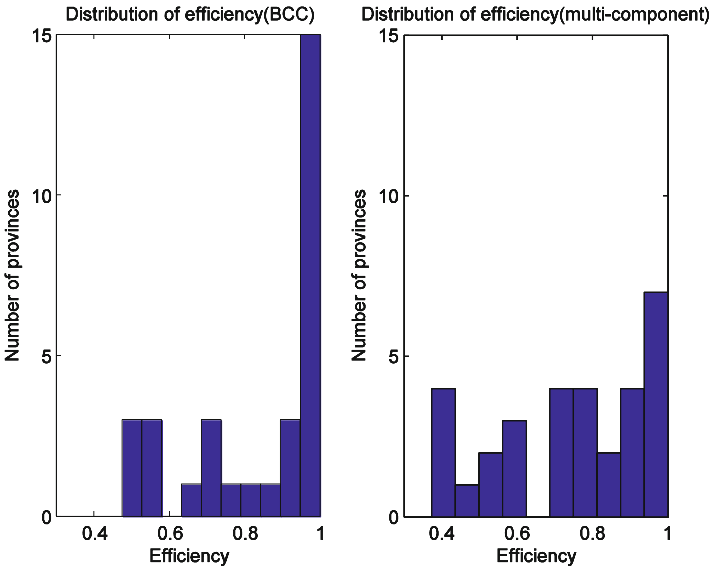


Fig. 14.5 Distribution of efficiency scores

In the application section, we use the proposed method to study the efficiencies and RTS of pollution treatment processes in China. The results show that the multi-component approach has strong discrimination power: the efficiency scores obtained are distributed in a bell-shaped manner, contrast this to the weak discrimination power as evidenced by the black-box approach with the distribution of efficiency scores skewed to the left. It is also found that six provinces show IRS, five provinces show CRS, and the rest show DRS. Among those that show CRS, Inner Mongolia and Jilin have the MPSS. Furthermore, the developed provinces are more likely to show DRS. In particular, East China shows DRS entirely. In contrast, the provinces that show IRS are mainly located in the west, which is a less developed area of China.

Acknowledgments The research is supported by National Natural Science Funds of China (Nos. 71301155; 71471053) and the Fundamental Research Funds for the Central Universities (No. JZ2015HGBZ0481).

Appendix

Proof of Theorem 1 Before we prove theorem 1, we establish Lemma 1.

Table 14.5 Results of various models

Provinces	RTS (black box)	RTS (multi-component)
Beijing	IRS	CRS
Tianjin	IRS	DRS
Hebei	DRS	DRS
Shanxi	DRS	DRS
Inner Mongolia	CRS	CRS
Liaoning	DRS	DRS
Jilin	CRS	CRS
Heilongjiang	IRS	DRS
Shanghai	IRS	DRS
Jiangsu	DRS	DRS
Zhejiang	DRS	DRS
Anhui	DRS	DRS
Fujian	IRS	DRS
Jiangxi	CRS	DRS
Shandong	DRS	DRS
Henan	DRS	DRS
Hubei	IRS	DRS
Hunan	IRS	CRS
Guangdong	DRS	DRS
Guangxi	CRS	DRS
Hainan	IRS	IRS
Chongqing	IRS	DRS
Sichuan	IRS	DRS
Guizhou	IRS	DRS
Yunnan	IRS	IRS
Tibet	IRS	IRS
Shananxi	IRS	IRS
Gansu	CRS	IRS
Qinghai	IRS	IRS
Ningxia	CRS	CRS
Xinjiang	IRS	DRS

Lemma A1 *Define $\hat{T}_b^{VRS}, \hat{T}^{VRS}$ as follows:*

$$\hat{T}_b^{VRS} = \left\{ (X, Y) \middle| \sum_{j=1}^{n^2} \lambda_j x_{ij} = x_i, i = 1, \ldots, m, \right.$$

$$\left. \sum_{j=1}^{n^2} \lambda_j y_{rj} = y_r, r = 1, \ldots, s, \sum_{j=1}^{n^2} \lambda_j = 1, \lambda_j \geq 0 \right\}$$

and

Table 14.6 RTS by administrative regions

Region	Provinces		
	IRS	CRS	DRS
North China		Beijing, Inner Mongolia	Tianjin, Hebei, Shanxi
Northeast		Jilin	Liaoning, Heilongjiang
East China			Shanghai, Jiangsu, Zhejiang, Anhui, Fujian, Jiangxi, Shandong
South-central China	Hainan	Hunan	Henan, Hubei, Guangdong, Guangxi
Southwest	Yunnan, Tibet		Chongqing, Sichuan, Guizhou
Northwest	Shaanxi, Gansu, Qinghai	Ningxia	Xinjiang
Total	6	5	20

$$\hat{T}^{VRS} = \left\{ (X, Y) \middle| \sum_{k=1}^{2} \sum_{j=1}^{n} \lambda_j^k x_{ij}^k = x_i, i = 1, \ldots, m, \sum_{k=1}^{2} \sum_{j=1}^{n} \lambda_j^k y_{rj}^k = y_r, r = 1, \ldots, s, \right.$$

$$\left. \sum_{j=1}^{n} \lambda_j^k = 1, \lambda_j^k \geq 0 \right\}. \textit{ Then } \hat{T}_b^{VRS} = \hat{T}^{VRS}.$$

Proof (1) $\hat{T}_b^{VRS} \subseteq \hat{T}^{VRS}$;

Let DMU_j be some DMU in **EDS**, and $(x_{1j}, \ldots, x_{mj}, y_{1j}, \ldots, y_{rj})$ be its input–output bundle. Suppose it is made of $\text{SDMU}_{1\text{k}}$, and $\text{SDMU}_{2\text{m}}$, where $k, m \in \{1, \ldots, n\}$. Obviously, $\left(x_{1j}, \ldots, x_{mj}, y_{1j}, \ldots, y_{rj}\right) \in \hat{T}_b^{VRS}$, since it can be decomposed into input–output bundle of $\text{SDMU}_{1\text{k}}$, and that of $\text{SDMU}_{2\text{m}}$. To put it another way, if we set a multiplier corresponding to $\text{SDMU}_{1\text{k}}$ and $\text{SDMU}_{2\text{m}}$ equal to 1 and other multipliers equal to zero, we can see that $(x_{1j}, \ldots, x_{mj}, y_{1j}, \ldots, y_{rj})$ satisfies the condition to be an element of $\hat{T}^{VRS}$. Therefore $\hat{T}_b^{VRS} \subseteq \hat{T}^{VRS}$ holds.

(2) $\hat{T}_b^{VRS} \supseteq \hat{T}^{VRS}$;

For any $(X, Y) \in \hat{T}^{VRS}$, there exist two sets of convex multipliers $(\lambda_1^1, \ldots, \lambda_n^1)$ and $\left(\lambda_1^2, \ldots, \lambda_n^2\right) \left(\lambda_j^1, \lambda_j^2 \geq 0, \sum_{j=1}^{n} \lambda_j^1 = 1, \sum_{j=1}^{n} \lambda_j^2 = 1 \right)$ such that

$$\begin{aligned} x_i &= \sum_{j=1}^{n} \lambda_j^1 x_{ij}^1 + \sum_{j=1}^{n} \lambda_j^2 x_{ij}^2 (i = 1, \ldots, m), \\ y_r &= \sum_{j=1}^{n} \lambda_j^1 y_{rj}^1 + \sum_{j=1}^{n} \lambda_j^2 y_{rj}^2 (r = 1, \ldots, s). \end{aligned} \tag{14.19}$$

We need to show that there always exists a convex multiplier $\sum_{j=1}^{n^2}\lambda_j = 1, \lambda_j \geq 0$, such that $x_i = \sum_{j=1}^{n^2}\lambda_j x_{ij}, y_r = \sum_{j=1}^{n^2}\lambda_j y_{rj}$, where $(x_{1j}, \ldots, x_{mj}, y_{1j}, \ldots, y_{rj})$ is the input–output bundle of $\mathrm{DMU_j}$ in **EDS**. In other words, there is a convex multiplier such that the following equations hold:

$$
\begin{aligned}
x_i &= \sum_{j=1}^{n}\lambda_j\left(x_{i1}^1 + x_{ij}^2\right) + \sum_{j=n+1}^{2n}\lambda_j\left(x_{i2}^1 + x_{i(j-n)}^2\right) + ,\ldots, + \sum_{j=n^2-n+1}^{n^2}\lambda_j\left(x_{in}^1 + x_{i(j-n^2-n)}^2\right)\\
y_r &= \sum_{j=1}^{n}\lambda_j\left(y_{r1}^1 + y_{rj}^2\right) + \sum_{j=n+1}^{2n}\lambda_j\left(y_{r2}^1 + y_{r(j-n)}^2\right) + ,\ldots, + \sum_{j=n^2-n+1}^{n^2}\lambda_j\left(y_{rn}^1 + y_{r(j-n^2-n)}^2\right)
\end{aligned}
\tag{14.20}
$$

where $(x_{1j}^1, \ldots, x_{mj}^1, y_{1j}^1, \ldots, y_{sj}^1)$ and $(x_{1j}^2, \ldots, x_{mj}^2, y_{1j}^2, \ldots, y_{sj}^2)$, $j = 1, \ldots, n$, are the respective input bundle and output bundle of $\mathrm{SDMU_{1j}}$, and $\mathrm{SDMU_{2j}}$. That is to say, $\sum_{j=1}^{n^2}\lambda_j = 1, \lambda_j \geq 0$ must satisfy the following conditions:

$$
\lambda_j^1 = \sum_{k=(j-1)n+1}^{(j-1)n+n}\lambda_k,\ \lambda_j^2 = \sum_{k=1}^{n}\lambda_{n(j-1)+k},\ j = 1, \ldots, n \tag{14.21}
$$

To facilitate understanding, we organize the conditions as matrix products.

$$
\begin{bmatrix} \lambda_1 & \lambda_{n+1} & \ldots & \lambda_{n^2-n+1} \\ \lambda_2 & \lambda_{n+2} & \ldots & \lambda_{n^2-n} \\ \ldots & \ldots & \ldots & \ldots \\ \lambda_n & \lambda_{n+n} & \ldots & \lambda_{n^2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ \ldots \\ 1 \end{bmatrix} = \begin{bmatrix} \lambda_1^2 \\ \lambda_2^2 \\ \ldots \\ \lambda_n^2 \end{bmatrix} \tag{14.22}
$$

$$
\begin{bmatrix} \lambda_1 & \lambda_{n+1} & \ldots & \lambda_{n^2-n+1} \\ \lambda_2 & \lambda_{n+2} & \ldots & \lambda_{n^2-n} \\ \ldots & \ldots & \ldots & \ldots \\ \lambda_n & \lambda_{n+n} & \ldots & \lambda_{n^2} \end{bmatrix}^T \begin{bmatrix} 1 \\ 1 \\ \ldots \\ 1 \end{bmatrix} = \begin{bmatrix} \lambda_1^1 \\ \lambda_2^1 \\ \ldots \\ \lambda_n^1 \end{bmatrix} \tag{14.23}
$$

The above illustration indicates that the row j of the matrix is summed to λ_j^2, and the column j the matrix is summed to λ_j^1. Let us now combine (14.22) and (14.23) into the following equations where A is $2n$ by n^2.

$$\mathbf{A}\lambda = \begin{bmatrix} \overbrace{11,\ldots,1}^{n} & \overbrace{00,\ldots,0}^{n} & \overbrace{00,\ldots,0}^{n} & \ldots & \overbrace{00,\ldots,0}^{n} \\ 00,\ldots,0 & 11,\ldots,1 & 00,\ldots,0 & \ldots & 00,\ldots,0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 00,\ldots,0 & 00,\ldots,0 & 00,\ldots,0 & \ldots & 11,\ldots,1 \\ 10,\ldots,0 & 10,\ldots,0 & 10,\ldots,0 & \ldots & 10,\ldots,0 \\ 01,\ldots,0 & 01,\ldots,0 & 01,\ldots,0 & \ldots & 01,\ldots,0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 00,\ldots,1 & 00,\ldots,1 & 00,\ldots,1 & \ldots & 00,\ldots,1 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \cdots \\ \lambda_{n^2} \end{bmatrix} = \begin{bmatrix} \lambda_1^1 \\ \lambda_2^1 \\ \cdots \\ \lambda_n^1 \\ \lambda_1^2 \\ \lambda_2^2 \\ \cdots \\ \lambda_n^2 \end{bmatrix} = \mathbf{\Gamma} \tag{14.24}$$

We are going to prove (14.24) always has a nonnegative solution $\lambda_1^*, \ldots, \lambda_{n^2}^*$. Note that $\sum_{j=1}^{n^2} \lambda_j^* = 1$ automatically holds provided $\sum_{j=1}^{n} \lambda_j^1 = 1$ and $\sum_{j=1}^{n} \lambda_j^2 = 1$. Our problem reduces to the existence of nonnegative solution to (14.24). We claim the nonnegative solution always exists, by way of contradiction. Before moving on, we reduce (14.24) to (14.25).

$$\overline{\mathbf{A}}\lambda = \begin{bmatrix} \overbrace{00,\ldots,0}^{n} & \overbrace{11,\ldots,1}^{n} & \overbrace{00,\ldots,0}^{n} & \ldots & \overbrace{00,\ldots,0}^{n} \\ 00,\ldots,0 & 00,\ldots,0 & 11,\ldots,1 & \ldots & 00,\ldots,0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 00,\ldots,0 & 00,\ldots,0 & 00,\ldots,0 & \ldots & 11,\ldots,1 \\ 10,\ldots,0 & 10,\ldots,0 & 10,\ldots,0 & \ldots & 10,\ldots,0 \\ 01,\ldots,0 & 01,\ldots,0 & 01,\ldots,0 & \ldots & 01,\ldots,0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 00,\ldots,1 & 00,\ldots,1 & 00,\ldots,1 & \ldots & 00,\ldots,1 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \cdots \\ \lambda_{n^2} \end{bmatrix} = \begin{bmatrix} \lambda_2^1 \\ \lambda_3^1 \\ \cdots \\ \lambda_n^1 \\ \lambda_1^2 \\ \lambda_2^2 \\ \cdots \\ \lambda_n^2 \end{bmatrix} = \overline{\mathbf{\Gamma}} \tag{14.25}$$

Note that we have eliminated the first row of $\mathbf{A}$ and the first element of $\mathbf{\Gamma}$ by elementary row operation. Assume, now, that $\overline{A}\lambda = \overline{\Gamma}$ doesn't have a nonnegative solution, i.e., $\overline{\mathbf{\Gamma}}$ doesn't belong to the conic hull constructed by the column vectors of $\bar{A}$. By Farkas lemma, there exists $\mathbf{x} \in R^{2n-1}$, such that

(1) $\mathbf{x}^{\mathbf{T}}\overline{\Gamma} > 0$;
(2) $\mathbf{x}^{\mathbf{T}}\overline{\mathbf{A}}(i) \le 0, \overline{\mathbf{A}}(i)$ denotes the i th column of $\bar{\mathbf{A}}$, $i = 1, \ldots, n^2$.

By (2), it follows that

(1) $\mathbf{x}(i) \le 0,\ i = n, \ldots, 2n-1,\ \big(\mathbf{x}(i)$ denotes the ith component of vector $\mathbf{x}\big)$;
(2) For any $k = 1, \ldots, n-1$, we have $x(k) + x(i) \le 0,\ i = n, \ldots, 2n-1$, i.e., $x(k) \le \min_{j=n,\ldots 2n-1} -x(j)$.

Combining the previous two conditions, we obtain

$$
\begin{aligned}
x^T\overline{\Gamma} &= \sum_{k=1}^{n-1} x(k)\lambda_{k+1}^1 + \sum_{j=n}^{2n-1} x(j)\lambda_j^2 \le \left(\min_{j=n,\ldots,2n-1} -x(j)\right)\sum_{k=1}^{n-1}\lambda_{k+1}^1 + \sum_{j=n}^{2n-1} x(j)\lambda_j^2 \\
&= \left(-\max_{j=n,\ldots,2n-1} x(j)\right)\sum_{k=1}^{n-1}\lambda_{k+1}^1 + \sum_{j=n}^{2n-1} x(j)\lambda_j^2 \\
&\le \left(-\max_{j=n,\ldots,2n-1} x(j)\right)\sum_{k=1}^{n-1}\lambda_{k+1}^1 + \max_{j=n,\ldots,2n-1} x(j) \\
&= \left(\max_{j=n,\ldots,2n-1} x(j)\right)\left(1 - \sum_{k=1}^{n-1}\lambda_{k+1}^1\right) \le 0
\end{aligned}
\tag{14.26}
$$

To see why the last relation holds, note that $\sum_{j=1}^{n}\lambda_j^1 = 1$ and $\mathbf{x}(i) \le 0$, $i = n, \ldots, 2n-1$. So it follows that $1 - \sum_{k=1}^{n-1}\lambda_{k+1}^1 = \lambda_1^1 \ge 0$, and $\max_{j=n,\ldots,2n-1} x(j) \le 0$. Therefore, the product of the two parts is less than or equal to zero.

This contradicts $\mathbf{x}^T\overline{\boldsymbol{\Gamma}} > 0$. Therefore, $\overline{\boldsymbol{\Gamma}}$ belongs to the conic hull constructed by the column vectors of $\bar{\mathbf{A}}$, i.e., there is $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_{n^2}) \ge 0$ such that $\overline{\mathbf{A}}\lambda = \overline{\boldsymbol{\Gamma}}$, which also means that $\mathbf{A}\lambda = \boldsymbol{\Gamma}$. By our construction, we know that there exists $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_{n^2}) \ge 0$ such that (14.22) and (14.23) hold. In turn, this establishes that $(X, Y) \in \hat{T}_b^{VRS}$. □

Proof of Theorem 1 Let $(x_{1j}, \ldots, x_{mj}, y_{1j}, \ldots, y_{rj})$ be an arbitrary point in T_b^{VRS}. We first prove that $T_b^{VRS} \subseteq T^{VRS}$. By definition, there exists one point $\left(\bar{x}_{1j}, \ldots, \bar{x}_{mj}, \bar{y}_{1j}, \ldots, \bar{y}_{rj}\right)$ in $\hat{T}_b^{VRS}$ such that $x_{ij} \ge \bar{x}_{ij}$ and $y_{rj} \le \bar{y}_{rj}$. In light of Lemma 1, $\left(\bar{x}_{1j}, \ldots, \bar{x}_{mj}, \bar{y}_{1j}, \ldots, \bar{y}_{rj}\right)$ also belongs to $\hat{T}^{VRS}$. Therefore $\left(x_{1j}, \ldots, x_{mj}, y_{1j}, \ldots, y_{rj}\right) \in T^{VRS}$, since there is a point in T^{VRS} such that $x_{ij} \ge \bar{x}_{ij}$ and $y_{rj} \le \bar{y}_{rj}$ hold. By analogy, we can prove $T_b^{VRS} \supseteq T^{VRS}$. Therefore, $T_b^{VRS} = T^{VRS}$ holds.

By substituting the convex condition in the definition of T^{VRS} and T_b^{VRS} for $\sum_{j=1}^{n}\lambda_j^k = t\ (k = 1, 2)$ and $\sum_{j=1}^{n^2}\lambda_j = t\ (t \ge 0)$ respectively, it follows that $T^{VRS}(t) = T_b^{VRS}(t)$, since they are obtained by scaling up or down T^{VRS} and T_b^{VRS} by the same factor t. Given the fact that $T_b^{CRS} = \bigcup_{t\in[0,\infty)} T_b^{VRS}(t)$, $T_b^{NIRS} = \bigcup_{t\in[0,1]} T_b^{VRS}(t)$, and $T^{CRS} = \bigcup_{t\in[0,\infty)} T^{VRS}(t)$, $T^{NIRS} = \bigcup_{t\in[0,1]} T^{VRS}(t)$, it follows $T_b^{CRS} = T^{CRS}$ and $T_b^{NIRS} = T^{NIRS}$. □

References

Banker, R. D., Charnes, A., & Cooper, W. W. (1984). Some models for the estimation of technical and scale inefficiencies in data envelopment analysis. *Management Science, 30*(9), 1078–1092.

Banker, R. D., Cooper, W. D., Seiford, L. M., Thrall, R. M., & Zhu, J. (2004). Returns to scale in different DEA models. *European Journal of Operational Research, 154*(2), 345–362.

Beasley, J. E. (1995). Determining teaching and research efficiencies. *Journal of the Operational Society, 46*(4), 441–452.

Castelli, L., Pesenti, R., & Ukovich, W. (2010). A classification of DEA models when the internal structure of the decision making units is considered. *Annals of Operations Research, 173*(1), 207–235.

Charnes, A., Cooper, W. W., & Rhodes, E. (1978). Measuring the efficiency of decision making units. *European Journal of Operational Research, 2*(6), 429–444.

Chen, Y., Cook, W. D., Li, N., & Zhu, J. (2009a). Additive efficiency decomposition in two-stage DEA. *European Journal of Operational Research, 196*(3), 1170–1176.

Chen, Y., Cook, W. D., & Zhu, J. (2010). Deriving the DEA frontier for two-stage processes. *European Journal of Operational Research, 202*(1), 138–142.

Chen, Y., Liang, L., Yang, F., & Zhu, J. (2006). Evaluation of information technology investment: A data envelopment analysis approach. *Computers & Operations Research, 33*(5), 1368–1379.

Chen, Y., Liang, L., & Zhu, J. (2009b). Equivalence in two-stage DEA approaches. *European Journal of Operational Research, 193*(2), 600–604.

Cook, W. D., Hababou, M., & Tuenter, H. J. H. (2000). Multicomponent efficiency measurement and shared inputs in data envelopment analysis: An application to sales and service performance in bank branches. *Journal of Productivity Analysis, 14*(3), 209–224.

Cook, W. D., Liang, L., & Zhu, J. (2010). Measuring performance of two-stage network structures by DEA: A review and future perspective. *Omega, 38*(6), 423–430.

Cooper, W. W., Seiford, L. M., & Zhu, J. (2004). *Handbook on data envelopment analysis*. Boston: Kluwer Academic.

Cooper, W. W., Thompson, R. G., & Thrall, R. M. (1996). Extensions and new developments in DEA. *Annals of Operations Research, 66*(2), 1–45.

Färe, R., & Grosskopf, S. (2000). Network DEA. *Socio-Economic Planning Sciences, 34*(1), 25–49.

Färe, R., Grosskopf, S., & Lovell, C. A. K. (1985). *The measurement of efficiency of production*. Boston: Kluwer Nijhoff.

Färe, R., Grosskopf, S., & Lovell, C. A. K. (1994). *Production frontiers*. Cambridge: Cambridge University Press.

Kao, C. (2009a). Efficiency decomposition in network data envelopment analysis: A relational model. *European Journal of Operational Research, 192*(3), 949–962.

Kao, C. (2009b). Efficiency measurement for parallel production systems. *European Journal of Operational Research, 196*(3), 1107–1112.

Kao, C., & Hwang, S.-N. (2008). Efficiency decomposition in two-stage data envelopment analysis: An application to non-life insurance companies in Taiwan. *European Journal of Operational Research, 185*(1), 418–429.

Lewis, H. F., & Sexton, T. R. (2004). Network DEA: Efficiency analysis of organizations with complex internal structure. *Computers & Operations Research, 31*(9), 1365–1410.

Liang, L., Cook, W. W., & Zhu, J. (2008). DEA models for two-stage processes: Game approach and efficiency decomposition. *Naval Research Logistics, 55*(7), 643–653.

Ruggiero, J. (1998). Non-discretionary inputs in data envelopment analysis. *European Journal of Operational Research, 111*(3), 461–469.

Shephard, R. W. (1970). The theory of cost and production functions. Princeton: Princeton University Press.

Tone, K., & Tsutsui, M. (2009). Network DEA: A slacks-based measure approach. *European Journal of Operational Research, 197*(1), 243–252.

Tsai, P. F., & Molinero, C. M. (2002). A variable returns to scale data envelopment analysis model for the joint determination of efficiencies with an example of the UK health service. *European Journal of Operational Research, 141*(1), 21–38.

Chapter 15
DEA and Accounting Performance Measurement

Julie Harrison and Paul Rouse

Abstract This chapter considers the use of accounting information in DEA. We examine some of the advantages and pitfalls of using this type of information in DEA models. We also discuss some typical accounting measures used in DEA and suggest three models using publicly available accounting information. The chapter also examines how DEA can be used in conjunction with accounting approaches to measurement, including the balanced scorecard and activity-based costing. We demonstrate, using case studies, how the combined use of these methods can improve the insights obtained. The chapter concludes by discussing contingency theory and how it can be used to inform DEA research on the relationship between performance and environmental factors.

Keywords Data envelopment analysis • Accounting performance measurement • GAAP • Inflation adjusted inputs and outputs • Activity-based costing • Balanced scorecard • Indexing • Ratio analysis • Contingency theory

15.1 Introduction

In this chapter we discuss the potentially valuable complementary relationships between Data Envelopment Analysis (DEA) and Accounting Performance Measurement (APM). We illustrate these relationships and discuss how they can be best exploited. Also, we examine potential pitfalls that can arise when accounting information is used for productivity analysis.

DEA can be based on a productivity model usually comprising specification of inputs to produce a set of outputs, the construction of an efficient frontier, and use of distance measures to provide efficiency scores for units of interest, called decision making units (DMUs). However, as Cook et al. (2014: 1) point out "it must be remembered that ultimately DEA is intended as a method for performance

J. Harrison (✉) • P. Rouse
Department of Accounting and Finance, The University of Auckland, Private Bag 92019, Auckland 1142, New Zealand
e-mail: j.harrison@auckland.ac.nz

S.-N. Hwang et al. (eds.), *Handbook of Operations Analytics Using Data Envelopment Analysis*, International Series in Operations Research & Management Science 239, DOI 10.1007/978-1-4899-7705-2_15

evaluation and benchmarking against best-practice." Contextual variables about the DMUs' environments can also be used to explore reasons for variability in efficiency or performance. In many cases accounting information is used in DEA models either to provide proxies for physical measures, to facilitate the identification of allocative efficiencies, or to model financial processes within businesses.

Accounting performance measurement (APM) is a broader topic that ranges from an examination of specific ratios within common frameworks, such as the Balanced Scorecard, to the preparation of data needed for financial accounts prepared in compliance with regulatory reporting requirements. There are also links to costing systems, especially Activity Based Costing, that often inform performance and productivity ratios concerning processes and resource utilization. Furthermore, there is a large area of management accounting research that uses contingency theory that examines the influence of contextual variables on aspects of organisational structure, operation, and/or performance. This theory provides a valuable theoretical complement to research into contextual variables in DEA.

This chapter therefore focuses on the nexus between DEA and APM and aims to fill a gap in understanding how APM can be used in DEA and where care needs to be taken in using APM in productivity models. We start by discussing accounting information generally, its advantages and disadvantages and our view on how it can be best used in supplying information for productivity modelling.

15.2 Accounting Information

Practically all accounting systems in the Western world produce similar types of information. These are typically reported in Income Statements, Balance Sheets and Statements of Cash Flow (or names similar to these). Regulatory reporting requirements in most countries ensure that a large proportion of firms and often public sector organisations, report this information publicly. Over the past two decades, these reports are increasingly available in electronic form in databases such as Compustat, often in XBRL[1] format and available from Government websites (e.g. SEC Financial Statement Data Sets[2]).

In addition to the ubiquity of financial accounting information in readily available forms, there are certain advantages that this information provides to researchers. First, the accounting information system is the primary data system in every organisation from large to small. Naturally large and even medium-size organisations have other data systems, but the accounting system remains the base essential for any organisation. Second, financial reports for firms that are listed on a stock exchange are required to be prepared in accordance with local Generally

[1] eXtensible Business Reporting Language (XBRL) is an open source digital standard for reporting financial data, see www.xbrl.org (accessed 24 September 2015).

[2] http://www.sec.gov/dera/data/financial-statement-data-sets.html (accessed 7 September 2015).

Accepted Accounting Practice (GAAP), meaning that comparability of financial data across such entities within a country is high. Third, external reporting requirements in almost all cases require that this information is audited independently by an external party so the quality of the data is exceptionally high. Fourth, International Financial Reporting Standards[3] (IFRS) have been adopted in many countries reducing the variability of accounting choices available resulting in increasing comparability of financial statements across countries, as well as increasing the amount of information required to be reported. Lastly, as noted this information is available in electronic form and forms an important part of the trend to “big data” resource availability for researchers.

Conversely, there are also disadvantages with financial reporting data that researchers need to be aware of. First, there are limits to what is disclosed and the type of information disclosed will be affected by operational decisions, such as whether to lease or buy an asset. If the lease falls under the category of a finance lease, it must be capitalised and depreciated with an equivalent liability recognised in the balance sheet. An operating lease can still be expensed in the income statement but recent work by the International Accounting Standards Board and the Financial Accounting Standards Board may change this. Organisations report only the information legally required,[4] which may not include separate disclosure of items such as salary and wages, marketing expenses and other administrative items. Of particular concern, are organisations that use subcontractors instead of employees, as even if information on salaries and wages is disclosed, information on contractors is unlikely to be separately disclosed. This will create problems when selecting financial variables that adequately capture the factors of production used by companies being compared (e.g. labour).

The Balance Sheet generally contains most items of interest, but care must be taken with using this data as it brings us to our second disadvantage: mixed measurement bases. Historic cost has traditionally been the main measurement principle for fixed assets (also called non-current assets) and some items of inventory. Recent changes towards “fair value accounting” have diluted the historic cost convention, but it is likely that the value for most fixed assets (especially goodwill and other intangible assets) will differ from the realisable value or replacement cost. On the liability side, debt may be reported using a variety of valuation methods depending on the terms of the debt and materiality. Thus the assets and liabilities in the balance sheet can be a mix of current and historic values.

Third, there may not always be a “good” relationship or matching between inputs and outputs. For example, Smith (1990) notes that research and development is often not recorded in the same accounting period as any benefits it might provide. Nonetheless, expenses are meant to be recognised in the income statement on the

[3] Previously International Accounting Standards (IAS).

[4] This is a grey area. For example, IAS1 states that “Material Omissions or misstatements of items are material if they could, individually or collectively, influence the economic decisions that users make on the basis of the financial statements.”

basis of a direct association between the costs incurred and the earning of specific items of income. This notion of "matching" is incorporated within the accruals concept and provides some assurance that there is a good relationship between expenses and revenues.

Fourth, while GAAP generally limits the choices available to organisations in the recognition and measurement of financial items, there are still many alternatives available. Furthermore, organisations are not restricted to their initial choices and changes in accounting policies can significantly impact the treatment and value of individual items. However, an entity can only change their accounting policy if it is required by an applicable accounting standard or the change results in the income statement providing more reliable and relevant information. For example, although organisations generally use straight line depreciation for fixed assets, there can still be considerable variability in the estimation of useful lives and future salvage values, both of which significantly affect the annual depreciation expense which affects both the income statement and balance sheet. Firms can choose their own estimates of useful life and salvage value, which can lead to major issues in assuming comparability among organisations even in the same industry. Morell (2013) described how Singapore Airlines changed its depreciation policy for its aircraft from 10 years useful life and salvage value of 20 % to a useful life of 15 years and salvage value of 10 %. The impact on its 2001/2002 profits was expected to be an increase of US$160 million with a significant reduction in depreciation expense as well as an increase in the balance sheet value for its aircraft. Any panel data that contains these types of changes can result in a much distorted time series of productivity results. For other items such as inventory, organisations can use different measurement bases such as FIFO (first in, first out) or weighted average. These can produce significant differences in cost of sales (Income Statement) and inventory valuations (Balance Sheet).

Nonetheless, financial reporting data can be extremely useful for the advantages listed above but care should be taken when using it. For the purposes of calculating allocative efficiency, splitting revenue and expense flows into quantities and prices can be problematic depending on the organisation type and market. For example, in the case of banks the price of debt can be estimated using interest expense in relation to the debt amount; similarly with interest income. However, this can be more difficult for manufacturing firms where individual prices for each firm may not be known for their sales and materials; however, if there are market prices then sector wide prices could be used for this purpose. One area that is particularly challenging concerns estimating prices for fixed assets. We do not believe that using depreciation or repairs and maintenance or combinations of the two provides a good estimate of prices since depreciation is merely an allocation of cost with almost non-existent consideration of market prices; while repairs and maintenance are the costs of ensuring the asset operates as required.

15.3 Accounting Ratios for Performance Measurement

Ratios are one of the most common methods managers use when appraising performance. One of the classic accounting ratio models is the DuPont model (or DuPont Analysis), developed by the DuPont company in the 1920s. This model provides a decomposition of an important financial ratio, *return on investment* (variations examine *return on equity* and *return on assets*). The purpose of the model is to break down the overall return into the component parts to identify the source of increases and/or decreases in firm performance. Figure 15.1 provides an illustration of the Du Pont decomposition.

Reading from left to right, it shows how changes in underlying financial items affect overall performance. In Fig. 15.1 investment is defined as Assets and the first decomposition is into Profit Margin (Net Income divided by Sales) divided by Asset Turnover (Sales divided by Total Assets). These in turn can be decomposed into their respective subcomponents.

There are other decompositions used in accounting. The key similarity is their focus on ratios and shared objective of isolating the causes of changes in performance. More recent models often include non-financial measures as well as financial data in recognition of the limitations of performance assessments based solely on financial data. While most decompositions focus on businesses, similar models

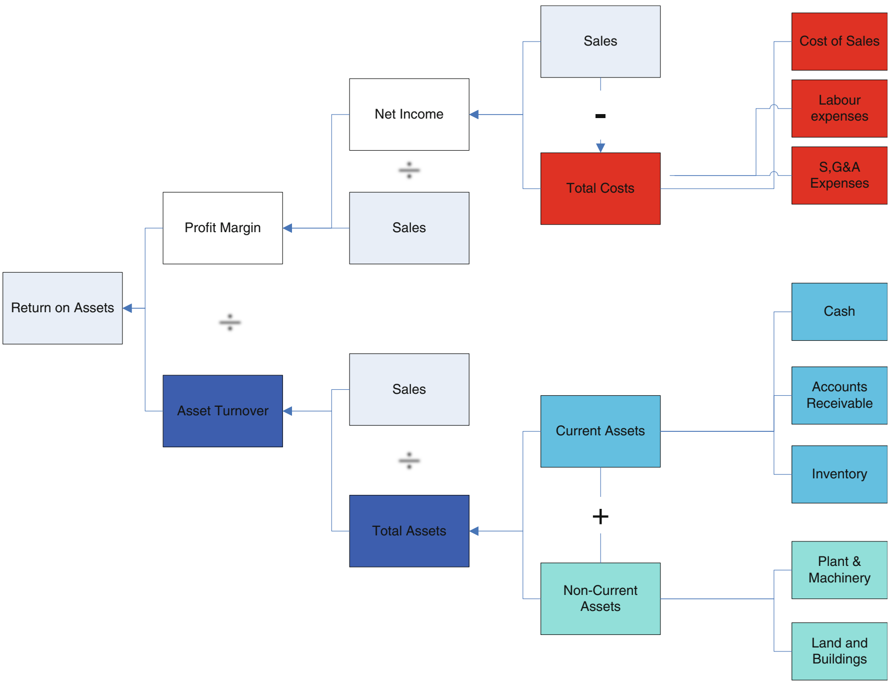


Fig. 15.1 The DuPont return on investment decomposition

have been developed to capture important ratios for public sector organisations. One of the more useful models is that provided by Ramanathan (1985) for the benefit cost ratio (15.1):

$$\frac{B}{C} = \frac{B}{OC} \times \frac{OC}{O} \times \frac{O}{I} \times \frac{I}{C} \tag{15.1}$$

Where the financial data are (B) the financial measures of the benefits and (C) costs, and the non-financial measures are outcomes (OC), outputs (O), and inputs (I). These can be further interpreted as measures of effectiveness (OC/O), efficiency (O/I) and economy (I/C). Other variations are possible such as cost effectiveness using the ratio (OC/C) or cost efficiency (O/C). Rouse and Chiu (2009) used these classifications to determine the optimal mix of road maintenance activities for New Zealand highway maintenance by identifying best practice in terms of both efficiency and effectiveness.

Accounting performance measures often use partial productivity ratios with the numerator in the form of an output and denominator as an input e.g. sales per employee, net income per branch. Bragg (2012) provided a comprehensive list of around 250 ratios for general and more specialised use in support departments. There are also ratios that combine profitability with capacity in the area of revenue management. These take the form of revenue per unit of available capacity. For example, a measure used for restaurants, RevPASH, represents the amount of revenue per available seat hour thus combining the revenue obtained from diners with restaurant seating capacity for a period of time. Many of these are described by Sheryl Kimes at Cornell, as illustrated by Kimes and Singh (2009).

As noted above, managers are very familiar with ratios and tend to understand ratios better than a model comprising multiple inputs and multiple outputs. Accordingly, using ratios in a DEA model is often more understandable and acceptable to managers. One of the first studies to incorporate ratios into DEA was Greenberg and Nunamaker (1987) in their "DEA-based multiple control model" which used five ratios to measure the performance of 16 small (under 100 beds) hospitals. The ratios were quite diverse (i.e. of a formative nature) and comprised percent occupancy, total inpatient revenue per inpatient day, accounts receivable turnover, total cost per routine patient day and average length of stay. As stated by the authors, these are familiar measures in practice and are easily understandable by managers. The form of the DEA model was different from most DEA models at that time since the ratios for each hospital were maximised (requiring the last two ratios to be inverted), there was only one input, which is set equal to one for each DMU, and the model was constant returns to scale (CRS).

Smith (1990) applied DEA to financial statements using inputs and outputs based upon the ratio of return on capital employed. The inputs used were average equity and average debt (components of capital employed), and outputs were earnings available for shareholders, interest payments and tax payments with the latter two treated as "bad" outputs. Thanassoulis (1995) examined the performance of police forces using DEA with ratios constructed using the inputs and outputs from a DEA model. A noteworthy observation from this paper is the use of performance indicators (PIs) to better communicate the DEA results. For example, the ratios for DMUs from the efficient reference set were reported to enable forces to compare their own (familiar) ratios with these. Thanassoulis et al. (1996) compared ratio analysis and DEA for district health authorities in England finding significant differences in rankings between the two approaches. Notwithstanding they comment insightfully that for health authorities that had similar results "it can be seen that PIs can be thought of as a subset of possible DEA efficiency measures, and indeed any unit which has the maximum value on a PI will appear with 100 % or almost 100 % efficient under DEA... The units with maximum PI values will thus also appear as subsets of DEA efficient or almost efficient units" (Thanassoulis et al. 1996: 238).

A similar comparison was made by Bowlin (1999) who developed a DEA model using only accounting measures such as operating profit, assets, operating expenses as the outputs and inputs. He also used these measures to perform traditional ratio analysis and compared the results of the two approaches. To our knowledge Bowlin (1999) and Smith (1990) were the first studies to use solely financial accounting numbers in the DEA analysis. Oral and Yalalan (1990) used non-monetary items for part of their analysis of banks and financial statement data for the second part. This was followed by Feroz et al. (2003) who employed the DuPont model in their selection of inputs (sales,[5] total assets and common equity) and output (net income).

Bradbury and Rouse (2002) used risk factors associated with auditing the branches of a large pharmaceutical company to construct a frontier of high and low risk branches in order to better allocate audit resources. Although previous studies had described themselves as "audits", this was the first study to measure audit risk in the proper sense. Risk factors from a previous study by Miltz et al. (1991) were used and included quality of internal control, size, internal pressure on management and activities of the branch. Risk measures ranged from one (low risk) to five (high risk) so a branch that had high scores on all six risk factors would lie on the high risk frontier. A low risk frontier could be easily constructed by reversing the measurement order so that a score of 1 denoted high risk and 5 low risk. Applying DEA to the data provided a ranking of branches in terms of high risk or low risk. A refinement to the model was the inclusion of weight restrictions to reflect experts' opinion from the Miltz et al. (1991) study on appropriate upper and lower bounds for each risk factor.

Notwithstanding the attraction of using ratios that are well understood in practice, Hollingsworth and Smith (2003: 735) pointed out a problem with using ratios with different denominators and recommended the use of variable returns to scale to ensure that "all comparison between units is by interpolation only". This avoids unrealistic target values for inefficient DMUs exceeding the maximum range.

When convexity is assumed, Emrouznejad and Amin (2009) show that this can lead to inappropriate convex combinations of efficient units on the production

[5] The treatment of sales as an input is unusual although one can see their logic. Usually, sales is treated as an output.

possibility frontier. They propose two models to overcome this problem which involve including the numerator and denominator as separate output and input constraints.

In summary, ratios have the advantage of being readily understood by managers and are often readily available. Managers are familiar with ratios calculated using both financial and non-financial measures, but often rely on partial productivity measures. Non-accounting ratios are frequently used in DEA research on compiling indices in topics such as human development (Despotis 2005), and best city (Zhu 2001). While there are some issues, both technical and conceptual, around the use of ratios, their widespread use and familiarity suggest that they could be very useful in "selling" DEA to practitioners as a useful tool.

We turn next to using accounting information as the basis for inputs and outputs in DEA models.

15.4 Accounting Information and Its Interpretation in Productivity Measurement

There are a number of studies that use accounting information in productivity measurement. Since the focus of this book is on DEA, we confine our discussion to DEA models. As noted, accounting information has a number of advantages but care needs to be taken in its use and selection of appropriate accounting items. Christensen and Hemmer (2007) examined the relationship between production functions and cost systems and noted that where production is constant returns to scale (and assuming a Leontief structure), a two-(cost) pool procedure can replicate the marginal cost for all products simultaneously; under variable returns to scale,[6] bias can arise. "The economics of production and the interaction among products are determinant of the efficiency of the accounting system" (Christensen and Hemmer, 2007: 562). This needs to be considered when using both product level cost data as well as firm-level aggregated data.

The classic economics model typically assumes production is a function of labour and capital (and land). According to Reddy and Saraswathi (2007), raw materials are omitted since they have a constant relation to output at all levels of production. "This constancy of input-output relations leaves the methods of production unaffected" (Reddy and Saraswathi, 2007: 162). Typically labour is regarded as a variable input and capital as fixed in the short run. Most DEA models of technical efficiency use quantities of inputs and outputs which may be in physical quantities, but occasionally can include monetary measures. On the other hand there are some models that use entirely monetary measures for inputs and outputs e.g. the intermediation models in banking studies. The combination of price and quantities into a single measure calls into question whether it is technical, allocative

[6] Plus economies of scope.

or a combination of the two which is represented by the resulting efficiency score. Indeed Banker et al. (2007) describe an approach to decomposing an aggregated efficiency score into its technical and allocative components.

To illustrate the range of inputs and outputs used, Table 15.1 details a number of DEA studies that have used only financial data. This shows the variation in the data

Table 15.1 Financial inputs and outputs from the literature

Study	Inputs	Outputs	Comments
Smith (1990)	Equity	EBIT	Interest and tax are treated as bad outputs although it is unclear how they were treated
	Debt	Interest expense	
		Tax	
Oral and Yalalan (1990)	Personnel expenses	Interests on loans	
	Admin expenses	Non-interest income	
	Depreciation		
	Interests on deposits		
Bowlin (1999)	Operating expenses	Operating profit	Assets appear to be fixed assets
	Identifiable assets (equipment, facilities etc.)	Operating cash flows	
		Sales	
Feroz et al. (2003)	Sales	Net income	The classification of sales as an input is unusual
	Total assets		
	Common equity		
Rodriguez-Perez et al. (2011)	Total expenses	Total revenue	
	Land and buildings		
	Financial investments in associated and group companies		
	Other financial investments		
	Other assets		
Demerjian et al. (2012)	Cost of goods sold	Sales	The close relationship between cost of sales and sales suggests the need for a weight constraint explicitly linking these
	Selling and administration expenses		
	Net property, plant and equipment		
	Net operating leases		
	Net research and development		
	Purchased goodwill		
	Other intangibles assets		

used and in their treatment as either inputs, outputs, or "bad" outputs. There appears to be no consensus on the items to include in the analysis and there is generally little discussion on the rationale for including/excluding different items. Another concern is the large numbers of inputs and outputs used in some models, which would require a reasonable number of DMUs to satisfy the dimensionality requirements.[7] There are also issues around whether some of the items should have weight restrictions as per ARII of Allen et al. (1997). There is also some question whether some inputs used such as purchased goodwill, research and development expenditure and other intangible assets either relate to the generation of sales or profit or match to the same period of sales occurrence e.g. research and development expenditure in a current year may not eventuate in sales until later years.

An essential question when using accounting data is the purpose of the model. Are the accounting numbers proxies for some underlying economic measure or are the data being used to produce an accounting production model? For example, are total assets a proxy for capital and sales a proxy for produced output?

We provide some models below that provide guidance for the appropriate selection of accounting data. Note that these are confined to items found in publicly available financial statement data; other data may be available from other sources such as numbers of employees, physical volumes of output or input, specific capital equipment utilisation such as machine hours. Also, note that these models may require modification to reflect industry differences that significantly affect the recognition, measurement and disclosure of financial items e.g. banking and insurance industries.

15.4.1 Model 1: Production Process

Purpose: To use financial data to model a firm's technical efficiency.

Inputs: Property, plant and equipment (PPE), operating expense (with depreciation added back if included).

Outputs: Cost of production (equal to cost of sales (COS) plus closing inventory minus opening inventory). If this is not available, then use sales as an alternative proxy.

This is a fairly specific model that focuses on PPE as the capital input. Operating expense is used instead of labour cost as typically this is not disclosed separately, consequently, it will include non-manufacturing labour. As PPE is already included as an input, depreciation should be added back if it has been included in operating expenses. If selling, general and administrative expenses are disclosed, there is an argument for excluding these from operating expense to focus on the manufacturing or production side.

[7] See footnote 10 for a rule of thumb. However, for benchmarking purposes the size of the sample or number of DMUs may be irrelevant (Cook et al. 2014).

We suggest COS with inventory adjustments is the best proxy for production volume as it is not affected by pricing margins, which are included in sales (although, we note that it may be affected by changes in input prices if these change during the period). A possible complication is in manufacturing situations where the cost of production labour is already contained in the cost of production and thus also in COS and inventory. Given that this information is not conventionally available, we argue that COS with inventory adjustments provides the best proxy.

It can be argued this model provides a mix of technical and allocative efficiency depending on how the accounting items are envisaged. If as proxies, then the DEA score can be argued to be a measure of technical efficiency. If as production items in their own right, then the scores could be viewed as aggregate efficiency comprising technical and allocative efficiency.

15.4.2 Model 2: Firm Financial Efficiency Model

Purpose: To use financial data to model a firm's financial efficiency.

Inputs: Total assets, operating expense (with depreciation added back if already included).

Outputs: Sales, net income.

There are several options for the measurement of total assets: non-current assets plus current assets or non-current assets plus working capital. For non-current assets, items that could be excluded include intangibles such as goodwill, adjustments relating to mark-to-market of assets or liabilities, deferred tax (although this is normally a liability), and some long-term investments that are not directly connected to a firm's trading strategy. As noted above, operating expense will include non-manufacturing expenses, which in the case of firm efficiency, are legitimate for inclusion.

Outputs in this model include sales to represent the output activity of the firm and net income as a measure of the quality of sales. Efficiency, therefore, relates to how well a firm generates revenue for the assets invested.

15.4.3 Model 3: Funding Efficiency Model

Purpose: To use financial data to model a firm's funding efficiency.

Inputs: Equity, debt.

Outputs: Earnings before Interest and Tax (EBIT).

There are several options for the measurement of Equity. This could refer to ordinary shareholders funds excluding any preference capital. While there might be some reserves included under shareholders' funds that are unusual, it is probably best to leave them in the measure to ensure consistency in treatment across firms. Debt should capture longer-term financing and, therefore, should consist of non-current liabilities, excluding any provisions such as deferred tax. The basic principle is to identify the long-term providers of finance for the firm. Some might argue that bank overdrafts should be included in this and that could be a valid argument in some circumstances.

The output is EBIT which encapsulates the overall return to the long term providers of funds. Efficiency, therefore, relates to how well a firm generates profits from its different sources of capital, that is, how well it maximises its return on investment.

15.5 Indexing Dollar Values and Translation of Foreign Currencies

While inflation rates are low currently, they can still impact on DEA modelling especially where panel data are used and a pooled analysis is undertaken i.e. combining several years within the one analysis for a set of DMUs. To provide the intuition for this, consider a firm with two employees and production output of 200 units which is sold for a price of $10 per unit. The sales per employee is thus $1000. Assuming inflation of 10 % and no change in employees or output, the following year the selling price is $11, sales of output are 200 units, but the ratio increases to $1100 per employee purely because of inflation and without any change in productivity. If inflation is ignored, we would run the risk of assuming that productivity has improved by 10 %. This is easy to see in a simple example where one item is financial and the other is non-financial. It is tempting to think that when all items are in financial terms that somehow or other it all cancels out since the inputs and outputs have the same "value" because they are in the same year and are not affected by other years.

To illustrate the potential problems of ignoring inflation in pooled datasets consider the financial data provided in Table 15.2, which contains input and output data in dollars for six DMUs for two time periods: Year 1 and Year 3, with no restatement for inflation.

The left hand graph of Fig. 15.2 plots the data for year 1 and it can be seen that all DMUs lie on a variable returns to scale (VRS) efficiency frontier with DMUs C1 and D1 comprising the constant returns to scale (CRS) section of the frontier. The right hand graph of Fig. 15.2 plots the DMUs for the pooled sample of Years 1 and

Table 15.2 Input and output data in dollars for years 1 and 3

DMU year 1	Input	Output	DMU year 3	Input	Output
A1	1.20	1.00	A3	1.45	1.21
B1	1.50	2.30	B3	1.82	2.78
C1	2.00	3.50	C3	2.42	4.24
D1	3.00	5.25	D3	3.63	6.35
E1	4.00	6.00	E3	4.84	7.26
F1	5.00	6.50	F3	6.05	7.87

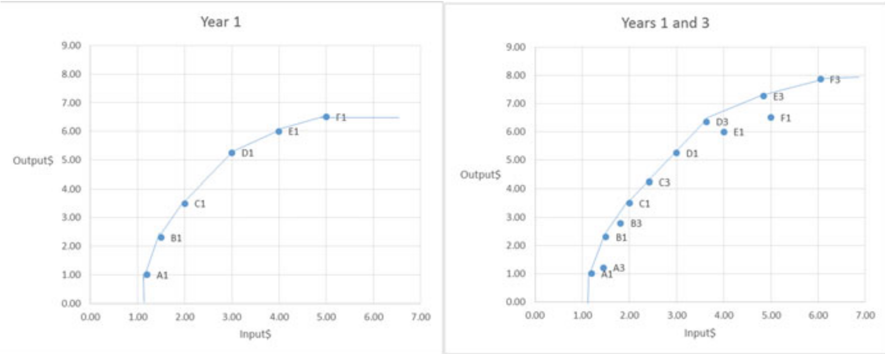


Fig. 15.2 DMU input and output data for year 1 and years 1 and 3

3. DMUs lying on the frontier are A1, B1, C1, C3, D1, D3, E3 and F3 with all others lying below the frontier. Are these other DMUs inefficient?

Assume that inflation over the period was 10 % per annum. Restating the year 1 amounts for 2 years compounded inflation results in exactly the same numbers as the Year 3 input and output columns. In other words, productivity has not changed at all but inflation has increased both inputs and outputs from Year 1 to Year 3. In fact all DMUs are still efficient. The effect of inflation in this case is to leave the DMUs that were increasing returns to scale (IRS) efficient in Year 1 unchanged while the Year 3 IRS DMUs appear to be inefficient; DMUs on the decreasing returns to scale (DRS) part of the frontier now dominate their Year 1 counterparts who have now become inefficient. Note that the DMUs on the CRS segment are efficient for both years.

Thus, for any pooled analysis when dealing with inputs and/or outputs measured in financial terms, the amounts must be restated to take inflation into account. Failure to do so can result in incorrect efficiency scores unless the technology assumes CRS. Even relatively low rates of inflation can lead to distortions in the analysis particularly when several time periods are involved. This problem becomes more complicated when inputs and outputs are subject to different rates of price inflation.

A final complication can arise with the analysis of panel financial data from several countries. Based on the example above, one should restate the financial amounts to take inflation into account. But if the currency exchange rates have fluctuated over time, then should one translate the currency into a common currency (e.g. $US) and then reflate using a US price index or restate the local currency using the local price index and then translate into $US?[8] We favour the second option, restate-translate, for several reasons. First, it recognises the local price changes and

[8] These alternatives, known as the translate-restate versus restate-translate options, were discussed thoroughly in the Financial Accounting Standards Board Discussion Document in the early 1970s concerning FASB 8 *Accounting for the Translation of Foreign Currency Transactions and Foreign Currency Financial Statements*.

makes more sense than restating using another country's price index. Second, although the exchange rate is likely to behave in the long run according to purchasing power parity theory, this may not happen in the short to medium term. Third, the translate-restate method results in a mix of exchange rate and price changes making it difficult to interpret any trend in the dollar amounts. The restate-translate method provides a much clearer notion of trends and has the advantage of using only one exchange rate.

We next describe the similarity between DEA and activity-based costing (ABC) with an application to child immunisation in New Zealand medical General Practices.

15.6 Activity-Based Costing and DEA: Congenial Twins

The foundations of Activity-Based Costing (ABC) probably lie in activity analysis (Koopmans 1951) and were first introduced into the accounting literature by Staubus (1971), which was largely ignored until the 1980s when Cooper and Kaplan successfully promulgated its adoption through a series of articles in the Harvard Business Review e.g. Kaplan and Cooper (1988). The essence of ABC is a focus on activities in a two-stage model where the first stage measures the consumption of resources by activities to construct activity cost pools which feed the second stage which measures the consumption of activities (cost pools) by cost objects (e.g. products or services). It is thus a consumption model and, as noted by Christensen and Demski (1995), is linear with separable and additive activities. An important aspect to note is the use of resource drivers, which are used to trace consumption of resources by activities (e.g. square footage, number of personnel) and activity drivers, which are used to trace consumption of activities (or their cost pools) by cost objects (e.g. machine hours, labour hours) as shown in Fig. 15.3. These drivers are usually non-financial and often correspond to the inputs and outputs used in DEA.

Compare ABC with its focus on an activity consuming resources to produce some product or service with DEA. DEA focuses on a process that consumes inputs to produce outputs. The similarity is obvious, but there are differences. Generally DEA uses fairly aggregated inputs and outputs and the process is often at firm or branch level i.e. the process contains activities. ABC deals with specific activities, outputs and resources, and the activities can be at fairly micro levels.

Figure 15.3 illustrates the interactions between ABC and DEA. The top part of Fig. 15.3 shows a conventional DEA model where the process can be influenced by environmental or contextual factors. The lower part of the figure shows the ABC model (horizontal) plus a vertical section labelled Activity Based Management (ABM). ABM is the action part of ABC systems where management use the ABC information to find ways to improve the process by improving activities. This involves better understanding activities through identifying cost drivers: those things that explain why an activity exists and the level at which it operates, see, for example, Turney (1992). Measuring the performance of activities provides

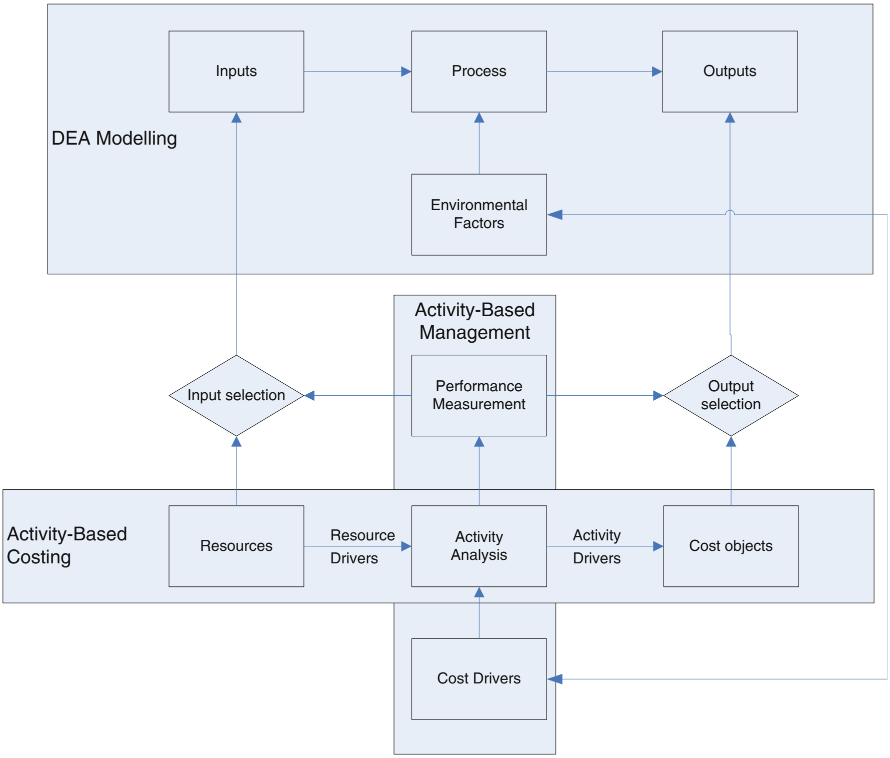


Fig. 15.3 DEA and ABC interactions

insights into whether improvement is possible and in Fig. 15.3 is the interface between ABC and DEA.

The arrows connecting input selection with resources and inputs determine which and how resources are to be aggregated or translated into inputs for the DEA model. Similarly the arrows connecting output selection with cost objects and outputs determine which and how cost objects are to be aggregated or translated into outputs for the DEA model. Finally cost drivers and environmental factors are linked as both are attempting to explain behaviour either at the activity level or process level.

We illustrate the interactions between the models through a study of immunisation activities of New Zealand (NZ) general practices (GPs) reported in Rouse et al. (2010). Twenty-four general practices agreed to participate in detailed time keeping and provision of information to identify the cost of immunisations provided. Data collected included total practice overheads and specific costs relating to immunisation and the total hours worked by all practice staff, total immunisation time involvement over an average week, and a daily log of the tasks involved in immunisation service delivery. In addition, questionnaires covering the less common, monthly events were completed by all staff involved in immunisation tasks.

Table 15.3 DEA results for the 24 GP practices

	Constant returns to scale	Variable returns to scale		Scale efficiency	
DMU	CRS	Input	Output	Input	Output
Mean	60 %	69 %	75 %	87 %	80 %
Median	47 %	68 %	76 %	94 %	74 %
Stdev	24 %	24 %	21 %	14 %	17 %
# efficient	3	5	5	4	3

An Activity Based Costing (ABC) model was developed in which the main immunisation related activities were identified and traced to vaccination events via measurement of both resources and activities. These activities were checking registration, vaccine preparation, obtaining informed consent, administering the vaccine, documentation, checking and routine follow-up. Resources consumed were mainly nurse time and the cost object was the child immunisation. Other activities were at the batch level (activities required to manage the service) such as waste removal costs specifically for sharps (needles), printing, postage and stationery incurred in making appointments and other correspondence concerning immunisations, vaccine ordering, audit procedures, generating routine immunisation appointments and reminders, and late immunisations. There were other costs at the product sustaining level (activities to provide "resource capability") e.g. initial vaccination training, annual staff training and updates, and at the facility level (practice level costs/activities required to meet the infrastructure or organisation requirements) e.g. administration (rent, utilities, subscriptions, insurance, depreciation), the cost of support staff, including receptionists and practice managers.

The ABC model was used to provide more aggregated inputs for the DEA model as shown in Fig. 15.3. The DEA model had 24 DMUs and 2 inputs (the main immunisation activities aggregated by time and aggregated batch costs) and a single output (number of vaccinations). Results are reported in Table 15.3.

Efficiency scores improved considerably under the assumption of VRS as opposed to CRS and there was a higher level of efficiency when adopting an output as opposed to an input orientation. Approximately one-fifth (5 out of 24) of the practices were fully-efficient under VRS and over half were 76 % efficient or more. Scale efficiency scores show that there was considerable differences in scale effects (87 % for input and 80 % for output orientations). This suggests that the practices varied in terms of size (reflected in the total vaccinations for each practice which ranged from 65 to 4949) and that the VRS model was probably a better fit than the CRS model.

As noted, the DEA inputs aggregated primary activities time and batch level costs from the ABC model. The detailed information from ABC in terms of resource drivers and disaggregated activities can be used to analyse the DEA results to identify environmental factors that influenced efficiency scores, which in turn can provide insights into cost drivers.

The details of each aggregated input are shown in Table 15.4 organised by quartile efficiency scores. Thus in the top panel, which reports the VRS output

Table 15.4 DEA scores by quartiles across unit and batch level activities

VRS output oriented	*Ave eff*	Reg check	Vaccine prep	Informed consent	Giving vacc	Documentation	Checking	Routine follow up	Total minutes	Reg patients	Reg patients <5 years	Depriv	UR_POP	
DMUs in quarters														
1	48 %	3.6	5.7	5.3	6.6	3.5	5.1	1.0	30.8	3420	278	5	117	
2	66 %	2.0	4.1	4.9	4.8	5.4	1.9	3.4	26.4	7745	770	6	110	
3	86 %	2.2	2.2	4.3	3.6	3.8	2.4	2.4	20.9	7895	543	7	88	
4	100 %	3.1	2.2	3.4	2.7	2.9	1.5	1.4	17.1	8700	680	8	66	
			Y	**Y**	**Y**	**Y**	**Y**			**Y**	**Y**	**Y**	**Y**	
DMUs in quarters		Claim and opport cost	GP time	Removal	PPS	Vacc ordering STD	Audit STD	Gen appoints STD	Gen P rems STD	Late imms STD Q5	PMS STD Q5	Org ORS STDM Q5	AES STD adverse events	Late imms sub-total
CRS	*Ave eff*													
1	37 %	$504	$2469	$66	$363	$23	$3	$66	$25	$108	$63	$7	$7	$182
2	43 %	$860	$3768	$150	$768	$39	$8	$15	$22	$106	$55	$12	$9	$180
3	62 %	$440	$1071	$70	$1065	$12	$7	$92	$4	$101	$26	$12	$8	$128
4	97 %	$214	$329	$162	$803	$19	$7	$25	$4	$7	$11	$7	$4	$15
		Y	**Y**						**Y**	**Y**	**Y**		**Y**	**Y**

orientation results, there are four quartiles of GP practices with mean efficiency scores of 48 %, 66 %, 86 % and 100 %, respectively. The corresponding mean times for each activity at unit level are shown across the columns for each row.

A "Y" was placed in the row below columns where there appears to be a trend.[9] For example, higher levels of efficiency are associated with lower times for vaccine preparation, informed consent, giving the vaccination etc. Column 11 (number of registered patients) reveals increasing efficiency with larger practices with a weaker trend in column 12 (registered patients under 5 years old). There was a counter-intuitive trend in the deprivation index (higher scores indicate higher deprivation) where more inefficient practices were associated with lower levels of deprivation. The explanation for this was that practices in affluent areas took more time with clients, who were more likely to seek in-depth information about immunisation benefits and risks. In deprived areas clients asked fewer questions, were more trusting of the health professional, so informed consent was often obtained in less time. Furthermore, in more deprived areas there was the possibility of multiple vaccinations if larger families were all immunised in the same visit.

The lower panel of Table 15.4 shows batch level costs by activity. The CRS results are reported since batch level costs are not volume driven but are more time related. Using the CRS results, it can be seen that DEA inefficiency was affected by activities pertaining to claiming from the health authorities, GP time, vaccine ordering and audit requirements, reminding clients of appointments and chasing up no-shows i.e., late immunisations. Late immunisations were regarded as a major nuisance by most practices and the rightmost column shows a clear trend for the CRS results with the most efficient practices having the lowest late immunisation costs.

15.7 DEA and the Balanced Scorecard: A New Approach to an Old Problem

Variable selection is always an issue with performance measurement. The old adage that "what you measure is what you get" should always be foremost in the minds of those selecting measures and constructing models. In DEA, the problem is, in theory, mitigated by the use of a productivity model. This imposes the requirement that selected measures relate to production inputs and outputs. However, in practice this does not eliminate all choice as most production processes have many inputs and outputs that could be included. One option would be to

[9] Regression results were not significant due to, we believe, the small number of GP practices in our sample. While our interpretation of trends is not supported statistically, it proved of interest to clinicians involved in the study and health specialists in this area.

include all possible inputs and outputs; however, the greater the number of measures used in a DEA model the higher the average efficiency scores[10] for the DMUs being compared and the lower the discriminatory power of the model. Accordingly, a well-designed DEA model should be parsimonious in the inclusion of inputs and outputs, restricting the model to the key production inputs and outputs.

Golany and Roll (1989: 239) set out some guidelines for selecting DMUs and measures where the "initial list of factors to be considered for assessing DMU performance should be as wide as possible". The next step is to refine this list using expert judgement, non-DEA quantitative analysis and DEA based analysis. The latter two include correlations and regressions between inputs and outputs, aggregation possibilities, and examination of the DEA weights for inputs and outputs used in the DEA analysis (a close to zero weight across DMUs for a measure suggests it is a candidate to be dropped).

While this is a worthwhile process, Rouse et al. (1997) proposed an alternative approach where measures should be located within a holistic framework comprising a linked structure performance pyramid based upon the Balanced Scorecard of Kaplan and Norton (1992):

> "The pyramid represents a comprehensive, fully integrated performance measurement system that captures multiple perspectives, ensures that measures reflect strategic directions and provides explanation and choice of actions through identification of underlying drivers. . . . The DEA analysis provides information for productivity and performance measurement, benchmarking and comparisons of actual versus target measures which can be fed back into the performance measurement system." Rouse et al. (1997: 131)

Measures can be located within the pyramid and their appropriateness and validity can be established in terms of their links to strategy and underlying process drivers. Only those measures that can be so located are included in the DEA model. Furthermore, the DEA results can be interpreted and explained by reference to the pyramid and traced to these process drivers for deciding upon actions to be taken. Examples of such actions were described in Rouse et al. (2002) where an airline maintenance division combined the linked structure performance pyramid with DEA for performance management purposes.

Further studies have examined integrating DEA with the BSC by using measures from the four perspectives to construct linked DEA models. Eilat et al. (2008) used a BSC tailored to research and development projects adding a fifth perspective, uncertainty. Twenty four measures were identified and grouped under the five perspectives (or "cards" in their description). Preferences (referred to as "balance") among the cards were effected through bounds on the proportions of the card scores with increasing discrimination among projects as more balancing constraints were

---

[10] In the absence of weight restrictions on the inputs and outputs in a DEA model, an increase in the number of measures will, ceteris paribus, result in higher average efficiency scores as individual DMUs are more likely to be able to find a combination of inputs and outputs that compares favourably to other DMUs. To mitigate this problem a rule of thumb of twice the number of inputs multiplied by the number of outputs is suggested as a minimum sample size (Dyson et al. 2001).

included. Their model thus focused on managerial preferences among the five perspectives to obtain greater discriminatory power.

García-Valderrama et al. (2009) refined the integration process further by constructing DEA models linking pairs of five BSC perspectives, the normal four perspectives plus an innovation perspective. This is in line with the spirit of the linked perspectives of the strategy maps proposed by Kaplan and Norton (2001). For example, the first DEA model took the financial perspective measures as outputs and the customer perspective measures as inputs; the second DEA model took the customer measures as outputs and the internal perspective measures as inputs and so on until they had five DEA models. This enabled them to examine differences in efficiencies in different parts of the organisation as modelled by the BSC. Parts of their work suggest the possibility of using network DEA, an approach subsequently applied by Amado et al. (2012).[11]

Amado et al. (2012) focused on strategy maps and developed strategic objectives with associated critical success factors which in turn informed the identification of performance indicators. Similar to García-Valderrama et al. (2009), their DEA models were "interconnected following the cause and effect relationships hypothesised in the BSC literature" (Amado et al., 2012: 395). Thus the outputs of the perspectives below the financial perspective were the inputs for the perspective above. While describing their approach as "network DEA", it appears that the models were run independently as opposed to a network model with intermediate outputs becoming inputs to successive perspectives.

Notwithstanding, this selection of research into combining DEA and the BSC reveals some potentially powerful insights from this more incisive and informed analysis with areas for future research using network DEA and possibly hyperbolic or directional distance functions. As mentioned at the start of this section, the BSC provides a good model of the organisation since the measures contained are those considered to be important by management thus enabling the DEA modeller to be better informed as to the validity or appropriateness of the measures selected for DEA study.

The earlier discussion around cost drivers and environmental or contextual variables leads us to the final section concerning contextual performance and contingency theory.

15.8 Understanding Contextual Performance to "Do Better"

The importance of contextual variables has long been recognised in accounting and, in particular, the study and design of management control systems, including performance measurement systems. Contingency theorists have considered the impact on the effectiveness of management controls of a variety of exogenous

[11] This study contains a good literature review of research into DEA and BSC.

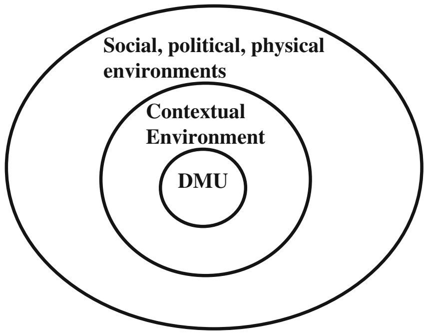


Fig. 15.4 The DMU and its contingent environments

factors including the environment, technology, strategy and culture of different organisations (Chenhall 2003). Emanating from general systems theory, contingency theory is based on the assumption that "organisation variables are in a complex interrelationship with one another and with conditions in the environment" (Lawrence and Lorsch 1967: 157). In contrast to a systems theory view of an organisation's form being determined independently, contingency theory argues that the form is determined by external pressures which vary depending upon the size of the organisation, its strategy for operation and the external environment itself (Donaldson 2001). Thus, an organisation matches its form and strategy to the contingency pressures it faces. While the environment impacts upon the organisation the latter is also involved in a process of "...mutual influence and interdependence" (Burrell and Morgan 1979: 168). In other words, organisations both shape and are shaped by their environment.

In short, contingency theory posits that the behaviour of a system is contingent or depends on its environment. More specifically, we could say that the performance of a unit of interest is affected by factors that comprise its immediate contextual and broader environments as depicted in Fig. 15.4.

If we accept that performance is contingent on such contextual and environmental factors, then any attempt to measure performance requires a broader exploration of these. Defining performance measurement as the "comparison of actual against expectations with the implied objective of learning to do better" (Rouse and Putterill 2003: 795) poses two tasks: first, how to gauge the comparative expectations and, second, how to learn to do better. Frontier based methods such as Stochastic Frontier Analysis (SFA) and Data Envelopment Analysis (DEA) can provide answers to the first; using this information to do better is the subject of contingency theory and various DEA models suggested in the literature.

Consistent with contingency theory, an early distinction between discretionary and non-discretionary resources recognised the existence of inputs or outputs beyond the control of DMU managers (Charnes et al. 1980). These could encompass physical environmental circumstances as well as constraints arising from organisational and managerial policies i.e. contextual variables. Define X as inputs and Y as outputs split into subsets for controllable inputs (Xc) and outputs (Yc) and noncontrollable inputs (Xnc) and outputs (Ync); then z for the exogenous contextual input (Zx) and output (Zy) variables. Early approaches (Bessent and Bessent 1980; Bessent et al. 1982; Jesson et al. 1987) using single-stage models treated both controllable and environmental factors as discretionary inputs or outputs as shown in Fig. 15.5a.[12] Note that environmental factors could be treated as either inputs (Zx) or outputs (Zy) or even both.

These were refined by modified DEA models that separated controllable from uncontrollable (including environmental factors) where the former are used to evaluate efficiency with the latter determining the reference peers for evaluation purposes (Banker and Morey 1986a).

Figure 15.5b shows this "all-inclusive" approach where controllable inputs and outputs and non-controllable/environmental factors were incorporated directly into the DEA model, but the latter are not subject to direct evaluation for efficiency purposes.

A further refinement known as the "categorical model" was made by Banker and Morey (1986b) in the situation where some of the non-controllable or environmental factors can be used to rank the DMUs using an ordinal scale from least to most favourable in terms of the influence of these factors. The ranking is then used to separate DMUs into groups which are evaluated from the lowest to highest ranking whereby lower ranked groups retain their efficiency score from their respective peer ranking evaluation.

Figure 15.5c shows how the categorical variable (in this figure depicted by an input or output environmental factor) lies outside of the input and output set (which could include non-controllable items[13]). Several non-controllable or environmental factors can be reduced to a single categorical variable using a weighted "index" according to the results of a regression analysis (Ruggiero 1998).

Finally, a multi-stage approach can be followed where the first stage DEA model uses only controllable inputs and outputs followed by a second stage regression where the dependent variable can be the efficiency score or the amount of radial (possibly including nonradial) slack (see for example, Ray 1988; Fried et al. 1999, 2002). The results of the regression can be used to adjust the controllable inputs and outputs which are then used in a third stage DEA model.

Figure 15.5d illustrates this third approach which only includes controllable factors in the first DEA model and regresses either the efficiency score or slack on the environmental factors (note that these could also include the non-controllable factors). The results are then used to remove the impact of both non-controllable and exogenous variables from the calculation of managerial inefficiency i.e. this last approach is the only one to isolate the portion of inefficiency solely due to the

[12] Given the controllability element, we prefer to call these controllable and non-controllable items.

[13] Which raises the possibility of a mixed non-discretionary and categorical model.

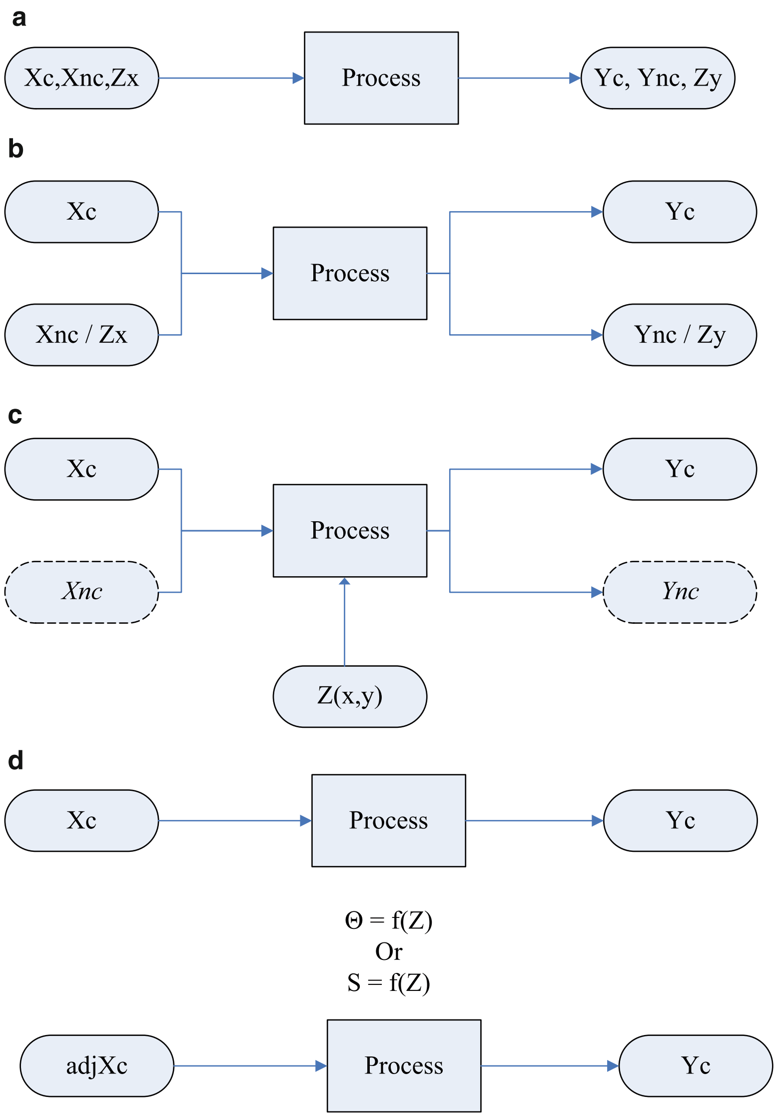


Fig. 15.5 (**a**) Single stage with all inputs and outputs treated as discretionary. (**b**) Single stage separating controllable from non-controllable/environmental. (**c**) Categorical model using environmental factor(s) to evaluate DMUs by ranking. (**d**) Three stage DEA approach showing controllable inputs adjusted in the third stage

actions of the DMU managers. By using the stochastic frontier analysis method in the second stage, Fried et al. (2002) are able to decompose stage one inefficiency into that attributable to the environment and that attributable to managerial inefficiency.

Table 15.5 Distinctions between different contextual variables

	Endogenously determined		Exogenously determined
	Inputs (X)	Outputs (Y)	Environmental (Z)
Controllable	Material	Services	Market conditions
	Labour	Units of production	
	Plant		
Non-controllable	Land	Level of service provision (e.g. traffic volume, contracted output)	Climate
	Buildings		Geography (e.g. topology)
	Assets with high specificity		

There are many more variations on the above reported in the literature e.g. Harrison et al. (2012), Fried et al. (2008). It is obvious from the discussion above, that the terminology between controllable (discretionary), non-controllable (non-discretionary) and environmental is fairly loose. We attempt to tighten this terminology next by reference to the management accounting literature that has been influenced by contingency theory notions.

Endogenous inputs and *outputs* are those resources and objects that are traceable to a process (albeit not necessarily traceable to a specific object from a product costing perspective). These may be controllable or non-controllable depending on the time period (e.g. short versus long term), availability (e.g. labour/capital markets), level of management being evaluated (e.g. branch versus division) and management or legislative policy (e.g. management policy on overtime, trading hours).

Environmental variables, in contrast, are exogenously determined and tend to be not explicitly traceable to a process even though they may be recognised as affecting such process. Table 15.5 provides some examples of these distinctions.

Although usually noncontrollable in the short term, some environmental variables may be controllable partially in a longer term time horizon e.g. stabilisation of terrain for road maintenance, obtaining of licences to control market competition. In addition, whether a variable is controllable or non-controllable can also reflect the management level being evaluated e.g., factors, such as policy, that are non-controllable to operations-level managers, may be controllable by senior managers.

In summary, contingency theory provides a theoretical basis for this analysis of contextual variables in DEA models. Furthermore, it provides a framework for the identification of potential variables and consideration of whether they are endogenous inputs and outputs that are only non-controllable at certain levels of management or in the short term versus those environmental variables that are exogenously determined.

15.9 Summary

This chapter has focused on the use of accounting information in DEA and provided some caveats regarding its use. We have also mentioned its advantages and discussed some typical accounting measures in the form of ratios. Ratios should not be overlooked in DEA research given not only that the original (Charnes et al. 1978) model started with a ratio, but also their familiarity to managers and ease of communication. We also examined the types of accounting measures commonly used in DEA models and suggest three models with publicly available accounting information that could be useful in DEA research.

We discussed some common problems in DEA and how accounting can inform the approaches used. When using financial information over time it is essential to adjust for price changes if any pooled analysis over years is to be performed. We show how failure to do this can distort DEA results. We then extended our discussion to the use of activity-based costing models and show their correspondence and links to DEA together with a case study of child immunisation. Next, we discussed research using both DEA and balanced scorecards and their mutually supporting roles in the selection of variables for DEA models as well as the potential to integrate DEA into a balanced scorecard. We concluded with a discussion of contingency theory and its relationship to contextual and environmental factors. Most DEA studies of contextual or environmental factors ignore theory so we believe that awareness of an underpinning theory can add to this literature's rigour.

Acknowledgement We would like to thank our colleague, Chris Clarke, for her comments on financial reporting requirements.

References

Allen, R., Athanassopoulos, A., Dyson, R. G., & Thanassoulis, E. (1997). Weights restrictions and value judgements in Data Envelopment Analysis: Evolution, development and future directions. *Annals of Operations Research, 73*, 13–34.

Amado, C., Santos, S., & Marques, P. (2012). Integrating the Data Envelopment Analysis and the Balanced Scorecard approaches for enhancing performance assessment. *Omega International Journal of Management Science, 40*, 390–403.

Banker, R. D., Chang, H., & Natarajan, R. (2007). Estimating DEA technical and allocative inefficiency using aggregate cost or revenue data. *Journal of Productivity Analysis, 27*, 115–121.

Banker, R. D., & Morey, R. C. (1986a). Efficiency analysis for exogenously fixed inputs and outputs. *Operations Research, 34*, 513–521.

Banker, R. D., & Morey, R. C. (1986b). The use of categorical variables in Data Envelopment Analysis. *Management Science, 32*, 1613–1627.

Bessent, A. M., & Bessent, E. W. (1980). Determining the comparative efficiency of schools through Data Envelopment Analysis. *Educational Administration Quarterly, 16*, 57–75.

Bessent, A. M., Bessent, E. W., Kennington, J., & Regan, B. (1982). An Application of Mathematical Programming to assess productivity in the Houston Independent School District. *Management Science, 28*, 1355–1367.
Bowlin, W. F. (1999). An analysis of the financial performance of defense business segments using Data Envelopment Analysis. *Journal of Accounting and Public Policy, 18*, 287–301.
Bradbury, M., & Rouse, P. (2002). An application of Data Envelopment Analysis to the evaluation of audit risk. *Abacus, 38*, 263–280.
Bragg, S. M. (2012). *Business ratios and formulas: A comprehensive guide*. New Jersey: Wiley.
Burrell, G., & Morgan, G. (1979). *Sociological paradigms and organisational analysis*. London: Heinemann.
Charnes, A., Cooper, W. W., Lewin, A., Morey, R. C., & Rousseau, J. (1980). *Efficiency analysis with non-discretionary resources*. Austin, TX: The University of Texas at Austin, Center for Cybernetic Studies.
Charnes, A., Cooper, W. W., & Rhodes, E. (1978). Measuring the efficiency of decision making units. *European Journal of Operational Research, 2*, 429–444.
Chenhall, R. (2003). Management control systems design within its organizational context: Findings from contingency-based research and directions for the future. *Accounting, Organizations and Society, 28*, 127–168.
Christensen, J., & Demski, J. S. (1995). The classical foundations of modern costing. *Management Accounting Research, 6*, 13–32.
Christensen, J., & Hemmer, T. (2007). Analytical modeling of cost in Management Accounting Research. In C. S. Chapman, A. G. Hopwood, & M. D. Shields (Eds.), *Handbook of Management Accounting Research*. Oxford: Elsevier.
Cook, W. D., Tone, K., & Zhu, J. (2014). Data Envelopment Analysis: Prior to choosing a model. *Omega International Journal of Management Science, 44*, 1–4.
Demerjian, P., Lev, B., & Mcvay, S. (2012). Quantifying managerial ability: A new measure and validity tests. *Management Science, 58*(7), 1229–1248.
Despotis, D. K. (2005). A reassessment of the human development index via Data Envelopment Analysis. *Journal of the Operational Research Society, 56*, 969–980.
Donaldson, L. (2001). *The contingency theory of organizations*. London: Sage.
Dyson, R. G., Allen, R., Camanho, A. S., Podinovshi, V. V., Sarrico, C. S., & Shale, E. A. (2001). Pitfalls and protocols in DEA. *European Journal of Operational Research, 132*, 245–259.
Eilat, H., Golany, B., & Shtub, A. (2008). R&D project evaluation: An integrated DEA and balanced scorecard approach. *Omega International Journal of Management Science, 36*, 895–912.
Emrouznejad, A., & Amin, G. R. (2009). DEA Models for ratio data: Convexity consideration. *Applied Mathematical Modelling, 33*, 486–498.
Feroz, E. H., Kim, S., & Raab, R. L. (2003). Financial statement analysis: A Data Envelopment Analysis approach. *Journal of the Operational Research Society, 54*, 48–58.
Fried, H. O., Lovell, C. A. K., Schmidt, S. S., & Yaisawarng, S. (2002). Accounting for environmental effects and statistical noise in Data Envelopment Analysis. *Journal of Productivity Analysis, 17*, 157–174.
Fried, H. O., Lovell, C. A. K., & Schmidt, S. S. (Eds.). (2008). *The measurement of productive efficiency and productivity change*. New York: Oxford University Press.
Fried, H. O., Schmidt, S. S., & Yaisawarng, S. (1999). Incorporating the operating environment into a nonparametric measure of technical efficiency. *Journal of Productivity Analysis, 12*, 249–267.
García-Valderrama, T., Mulero-Mendigorri, E., & Revuelta-Bordoy, D. (2009). Relating the perspectives of the balanced scorecard for R&D by means of DEA. *European Journal of Operational Research, 196*, 1177–1189.
Golany, B., & Roll, Y. (1989). An application procedure for DEA. *Omega, 17*, 237–250.
Greenberg, R., & Nunamaker, T. (1987). A generalized multiple criteria model for control and evaluation of nonprofit organizations. *Financial Accountability & Management, 3*, 331–342.

Harrison, J., Rouse, P., & Armstrong, J. (2012). Categorical and continuous non-discretionary variables in Data Envelopment Analysis: A comparison of two single-stage models. *Journal of Productivity Analysis, 37*, 261–276.
Hollingsworth, B., & Smith, P. (2003). Use of ratios in Data Envelopment Analysis. *Applied Economics Letters, 11*, 733–735.
Jesson, D., Mayston, D., & Smith, P. (1987). Performance assessment in the education sector: Educational and economic perspectives. *Oxford Review of Education, 13*, 249–266.
Kaplan, R. S., & Cooper, R. (1988). Make costs right: Make the right decisions. *Harvard Business Review, 66*, 96–103.
Kaplan, R. S., & Norton, D. P. (1992). The balanced scorecard—Measures that drive performance. *Harvard Business Review, 70*(1), 71–79.
Kaplan, R. S., & Norton, D. P. (2001). Transforming the Balanced Scorecard from performance measurement to strategic management: Part I. *Accounting Horizons, 15*, 87–104.
Kimes, S., & Singh, S. (2009). Spa Revenue Management. *Cornell Hospitality Quarterly, 50*, 82–95.
Koopmans, T. C. (1951). An analysis of production as an efficient combination of activities. In Koopmans, T. C. (Ed.), *Activity analysis of production and allocation, cowles commission for research in economics, monograph no. 13*. New York: Wiley.
Lawrence, P. R., & Lorsch, J. W. (1967). *Organization and environment: Managing differentiation and integration*. Boston: Graduate School of Business, Harvard University.
Miltz, D., Calomme, G. J., & Willekens, M. (1991). A risk-based allocation of internal audit time: a case study. *Auditing: A Journal of Practice and Theory, 10*(2), 49–61.
Morell, P. S. (2013). *Airline finance*. Farnham: Ashegate.
Oral, M., & Yalalan, R. (1990). An empirical study on measuring operating efficiency and profitability of bank branches. *European Journal of Operational Research, 46*, 282–294.
Ramanathan, K. V. (1985). A proposed framework for designing management control systems in not-for-profit organizations. *Financial Accountability & Management in Governments, Public Services, and Charities, 1*, 75–92.
Ray, S. C. (1988). Data Envelopment Analysis, nondiscretionary inputs and efficiency: An alternative interpretation. *Socio-Economic Planning Sciences, 22*, 167–176.
Reddy, M. K., & Saraswathi, S. (2007). *Managerial economics and financial accounting*. New Delhi: Prentice-Hall.
Rodriguez-Perez, G., Slof, J., Sola, M., Torrent, M., & Vilardell, I. (2011). Assessing the impact of fair-value accounting on financial statement analysis: A Data Envelopment Analysis approach. *Abacus, 47*, 61–84.
Rouse, P., & Chiu, T. (2009). Towards Optimal Life Cycle Management in a road maintenance setting using DEA. *European Journal of Operational Research, 196*, 672–681.
Rouse, P., Harrison, J., & Turner, N. (2010). Cost and performance: Complements for improvement. *Journal of Medical Systems, 35*, 1063–1074.
Rouse, P., & Putterill, M. (2003). An integral framework for performance measurement. *Management Decision, 41*, 791–805.
Rouse, A. P., Putterill, M., & Ryan, D. (1997). Towards a general managerial framework for performance measurement: A comprehensive highway maintenance application. *Journal of Productivity Analysis, 8*, 127–149.
Rouse, P., Putterill, M., & Ryan, D. (2002). Integrated performance measurement design: Insights from an application in aircraft maintenance. *Management Accounting Research, 13*, 229–248.
Ruggiero, J. (1998). Non-discretionary inputs in Data Envelopment Analysis. *European Journal of Operational Research, 111*, 461–469.
Smith, P. (1990). Data Envelopment Analysis applied to financial statements. *Omega International Journal of Management Science, 18*, 131–139.
Staubus, G. J. (1971). *Activity costing and input-output accounting*. Homewood, IL: Richard D. Irwin.

Thanassoulis, E. (1995). Assessing police forces in England and Wales using Data Envelopment Analysis. *European Journal of Operational Research, 87*, 641–657.

Thanassoulis, E., Boussofiane, A., & Dyson, R. (1996). A comparison of Data Envelopment Analysis and ratio analysis as tools for performance assessment. *Omega International Journal of Management Science, 24*, 229–244.

Turney, P. B. (1992). Activity-based management. *Management Accounting, 73*, 20–25.

Zhu, J. (2001). Multidimensional quality-of-life measure with an application to Fortune's best cities. *Socio-Economic Planning Sciences, 35*, 263–284.

Chapter 16
DEA Environmental Assessment (I): Concepts and Methodologies

Toshiyuki Sueyoshi

Abstract This study consists of two chapters (I and II, in Chaps. 16 and 17, respectively). One of the two chapters discusses an empirical study in which this research explains how to use DEA environmental assessment to establish corporate sustainability. The other chapter summarizes previous works on the research area. The first chapter (I) discusses that environmental assessment and protection are important concerns in modern business. Consumers are interested in environmental protection and they avoid purchasing products from dirty-imaged companies even if their prices are much less than the ones produced by green-imaged companies. A green image (often, not reality) of corporations is recently becoming very important for corporate survivability in a global market. By extending previous works on environment assessment and corporate sustainability, where companies need to consider both economic prosperity and pollution prevention in their business operations, this study discusses a use of Data Envelopment Analysis (DEA) for environmental assessment by utilizing the radial measurement. The proposed approach analytically incorporates different combinations of disposability concepts into the proposed radial models. It is easily envisioned that the proposed radial measurement for environmental assessment can guide corporate leaders and managers in identifying how to invest for eco-technology innovation on the abatement of undesirable outputs (e.g., industrial pollution). To document the practicality, this study applies the proposed approach to 153 observations on S&P 500 corporations in 2012 and 2013. The empirical investigation confirms that investors pay more serious attention on company's green image for corporate sustainability in a long horizon than profitability in a short horizon.

Keywords Environment • Radial measurement • Technology innovation • Corporate sustainability

T. Sueyoshi (✉)
Department of Management, New Mexico Institute of Mining and Technology,
801, Leroy Place, Socorro, NM 87801, USA
e-mail: toshi@nmt.edu

S.-N. Hwang et al. (eds.), *Handbook of Operations Analytics Using Data Envelopment Analysis*, International Series in Operations Research & Management Science 239, DOI 10.1007/978-1-4899-7705-2_16

16.1 Introduction

This study consists of two chapters. This chapter discusses an empirical study in which this research explains how to use DEA environmental assessment[1] to establish corporate sustainability within U.S. industrial sectors where DEA stands for Data Envelopment Analysis and U.S. stands for the United States of America. The other section summarizes previous works on the research area.

The Intergovernmental Panel on Climate Change (IPCC),[2] established by United Nations environmental program, has recently reported a policy suggestion in April (2014) that it is necessary for us to reduce an amount of Greenhouse Gas (GHG) emissions, in particular CO_2, by 40–70 % (compared with 2010) until 2050 and to reduce it at the level of almost zero by the end of this twenty-first century via shifting our current systems to energy-efficient ones. Otherwise, the global warming and climate change will destroy our natural and socio-economic systems.

Although it is almost impossible to establish our societies that do not produce any GHG emissions, the global warming and climate change have been influencing corporate behaviors and operations because all firms need to change their business strategies in order to adapt various regulation changes for preventing industrial pollutions at the level of U.S. federal and local governments. More importantly, consumers do not purchase any products and services from a dirty-imaged company even if the price is much less than that of a green-imaged company. The conventional business logic and practice (e.g., less expensive price and high quality) do not function anymore in modern corporations because they are now belonging to part of a world-wide trend toward a sustainable society.

The benefits from adapting GHG technologies range from intangible ones, such as improved public images as good (green) corporate citizen, to measurable ones such as their lower direct and indirect emission levels. Unfortunately, acknowledging the importance of reducing GHG emissions, many companies often misunderstand a business linkage between the cost of GHG technologies and their overall performances and business opportunities. It may be true in a myopic horizon that

---

[1] Glover and Sueyoshi (2009) discussed the history of DEA from the contributions of Professor William W. Cooper who first invented DEA from the linkage of L1 regression proposed in eighteenth century. Both DEA and L1 regression have a close linkage in these developments. See also Ijiri and Sueyoshi (2010) that discussed the contributions of Professor Cooper from the perspective of "social economics" and "social accounting", both have provided DEA development with a conceptual backbone. A contribution of the previous DEA efforts for environmental assessment was that they found the importance of separating outputs into desirable and undesirable outputs. That was a contribution, indeed. Previous DEA research efforts in the past decades, including Boccard (2014), Chitkara (1999), Cooper et al. (1996), Korhonen and Luptacik (2004), Mou (2014), Sarica and Or (2007), Shrivastava et al. (2012), Sueyoshi and Goto (2011), Sueyoshi and Yuan (2015a, 2015b), Zhang et al. (2013), Zhou et al. (2013) and many other articles. An important feature of these previous DEA studies was that they mainly used radial models for DEA environmental assessment.

[2] See IPCC's webpage (http://ipcc.ch/index.htm).

environmental protection requires a large amount of investment for GHG reduction and the investment does not produce any direct benefit to them.

However, such a business concern is different in a long term horizon. As discussed by Porter and van der Linde (1995), “environmental regulation does not jeopardize corporate performance, but rather providing firms with an opportunity to improve efficiency and competitiveness through environmental innovations in processes and products”. In modern business reality, some companies clearly understand the trade-off between their investments for low GHG emissions, including low-carbon technologies, and enhancement in operational performance and its related profit. The companies with a green image become more competitive and strategic in today’s environmentally conscious markets. This clearly indicates that modern corporations in all the sectors need to consider their technology investments on environmental protection and corporate performance enhancement from the perspective of corporate sustainability in short and long term horizons.

A business difficulty, associated with attaining such corporate sustainability, is that business leaders and academia do not have a practical methodology for assessing the performance of firms in terms of their operational and environmental achievements. Furthermore, is there any methodology that can guide their investment strategies for attaining the corporate stainability?

In replying such important inquiries, this study proposes a holistic methodology, or DEA, to evaluate the performance of firms from their levels of corporate sustainability. The proposed use of DEA, referred to as “DEA environmental assessment”, has four research concerns to be explored in this study. First, it incorporates two disposability concepts such as natural disposability and managerial disposability, where operational performance is the first priority and environmental performance is the second priority in the natural disposability. An opposite priority order is found in the managerial disposability. Outputs and inputs, characterizing their operational and environmental performance, are separated under disposability combinations. Second, this study investigates the concept of congestion on undesirable outputs, referred to as “Desirable Congestion (DC)”, in order to identify effective investment for preventing industrial pollutions. The conventional concept of congestion is “Undesirable Congestion (UC)”, which is applied to desirable outputs. Third, as an empirical study, this research applies the proposed approach, originated from different disposability combinations, for the performance evaluation of S&P 500 companies. It is necessary for us to examine different disposability concepts and methodologies to obtain useful policy and business suggestions for guiding a large policy issue such as the global warming and climate change. See Wang et al. (2014), Sueyoshi and Wang (2014a, b) and Sueyoshi and Yuan (2015b). Finally, this study describes business implications obtained from the proposed DEA application.

The remainder of this study is organized as follows. Section 16.2 provides a brief literature review on DEA environmental assessment. See Chap. 17 of this study provides a detailed literature study on DEA environmental assessment. Section 16.3 discusses underlying concepts incorporated into the proposed approach. Section 16.4 describes radial models under different disposability concepts.

Section 16.5 summarizes investment strategy. Section 16.6 applies the proposed approach to evaluate the unified (environmental and operational) performance of S&P 500 companies and summarizes empirical results obtained from the application. Section 16.7 concludes this research along with future extensions.

16.2 Literature Review

First of all, see Chap. 17 of this study that lists 407 previous studies. Therefore, it is important for us to note only the position of this research. That is, a limited number of previous studies on applied energy have discussed corporate sustainability and investment strategy by using DEA environmental assessment. Exceptions can be found in Wang et al. (2014), Sueyoshi and Wang (2014a, b) and Sueyoshi and Yuan (2015b). Such a business concern is very important for environmental assessment for all industrial sectors in not only the U.S. but also other industrial nations. This research will explore the issue as an empirical study. That is the purpose of this study.

16.3 Underlying Concepts for DEA Environmental Assessment

16.3.1 Abbreviations and nomenclatures

All abbreviations and nomenclatures used in this study (Chaps. 16 and 17) are summarized as follows.

DC: Desirable Congestion,
DMU: Decision Making Unit,
DEA: Data Envelopment Analysis,
DTS: Damages to Scale,
DTR: Damages to Return,
EPA: Environmental Protection Agency
GHG: Greenhouse Gas
IPCC: Intergovernmental Panel on Climate Change
OPEC: Organization of the Petroleum Exporting Counties
RTS: Returns to Scale,
UC: Undesirable Congestion,
URS: Unrestricted,
UE: Unified Efficiency,
UEN: Unified Efficiency under Natural disposability,
UEM: Unified Efficiency under Managerial disposability,
UENM: Unified Efficiency under Natural & Managerial disposability,

X: A column vector of m inputs,
G: A column vector of s desirable outputs,
B: A column vector of h undesirable outputs,
d_i^x: An unknown slack variable of the ith input,
d_r^g: An unknown slack variable of the rth desirable output,
d_f^b: An unknown slack variable of the fth undesirable output,
λ: An unknown column vector of intensity (or structural) variables,
R_i^x: A data range related to the ith input,
R_r^g: A data range related to the rth desirable output,
R_f^b: A data range related to the fth undesirable output,
v_i: A dual variable of the ith input,
u_r: A dual variable of the rth desirable output,
w_f: A dual variable of the fth undesirable output and
σ: A dual variable to indicate the intercept of a supporting hyperplane on a production and pollution possibility set.

16.3.2 Natural and Managerial Disposability

Let us consider $X \in R_+^m$ as an input vector, $G \in R_+^s$ as a desirable output vector and $B \in R_+^h$ as an undesirable output vector. These vectors are referred to as "production factors" in this study. In addition to the vectors, the subscript (j) is used to stand for the jth DMU (Decision Making Unit: corresponding to an organization in private and public sectors) and λ_j indicates the jth intensity variable ($j = 1, \ldots, n$) which is used for connecting production factors.

Using an axiomatic expression, unified (operational and environmental) production and pollution possibility sets to express natural and managerial disposability are specified by the following two types of output vectors and an input vector, respectively:

$$\begin{aligned} P^N(X) &= \left\{ (G,B) : G \le \sum_{j=1}^n G_j\lambda_j,\, B \ge \sum_{j=1}^n B_j\lambda_j,\, X \ge \sum_{j=1}^n X_j\lambda_j,\, \sum_{j=1}^n \lambda_j = 1,\, \lambda_j \ge 0 \ \ (j = 1, \ldots, n) \right\} \ \& \\ P^M(X) &= \left\{ (G,B) : G \le \sum_{j=1}^n G_j\lambda_j,\, B \ge \sum_{j=1}^n B_j\lambda_j,\, X \le \sum_{j=1}^n X_j\lambda_j,\, \sum_{j=1}^n \lambda_j = 1,\, \lambda_j \ge 0 \ \ (j = 1, \ldots, n) \right\}. \end{aligned} \tag{16.1}$$

The difference between the two concepts on disposability is that production technology under natural disposability, or $P^N(X)$, has $X \ge \sum_{j=1}^n X_j\lambda_j$. Meanwhile, the managerial disposability, or $P^M(X)$, has $X \le \sum_{j=1}^n X_j\lambda_j$. The two disposability concepts intuitively appeal to us because an efficiency frontier for desirable outputs locates above or on all observations, while that of undesirable outputs locates below or on the observations. See Porter and van der Linde (1995) on a description on the use of managerial disposability from corporate strategy.

It is important to note that the operational performance is the first priority and the environmental performance is the second one under natural disposability in assessing the unified efficiency. In contrast, the managerial disposability has an opposite priority order in the assessment. This study considers the disposability concepts as two different criteria for environmental assessment.

In the previous research efforts by DEA environmental assessment, an input vector is usually assumed to project toward a decreasing direction. This assumption is often inconsistent with the reality of environmental protection in a private sector. For example, let us consider a manufacturing firm that can increase the input vector if its marginal (or average) cost is less than the marginal (or average) sale because the business condition produces profit to the firm. Thus, the conventional use of DEA is often unacceptable in a private sector because the previous DEA studies have implicitly assumed the minimization on total production cost. The cost concept may be acceptable for the performance analysis of many organizations in a public sector, but not for a private sector. Thus, it can be easily imagined that DEA environmental assessment in the private sector, as discussed in this study, is different from that of the public sector. The cost concept for guiding organizations in the private sector is marginal cost or average cost, not the total cost. Furthermore, the opportunity cost, originated from business risk due to industrial pollutions and the other types of various problems (e.g., the disaster of Fukushima Daiichi nuclear power plant), has a major role in modern business. As mentioned previously, no consumer buys products from dirty-imaged companies even if their prices are much less than those of green-imaged companies. Such opportunity cost is very important in managing modern business. See, for example, the corporate sandal of Volkswagen, found in 2015, that has been long cheating on CO_2 emission produced by its cars. It will take a long time for the car company to recover the trust from consumers.

16.3.3 Unification Between Natural and Managerial Disposability

Figure 16.1, adapted from Sueyoshi and Yuan (2015b), depicts a unification process for combining desirable and undesirable outputs, which is separated into the three stages (from I to III). For our visual convenience, Fig. 16.1 depicts the case of a single component of the three production factors. It is easily extendable to the case of multiple components in the proposed DEA formulation.

First stage (I) has two components (A) and (B). The first component (A) of the stage (I) indicates the production relationship between an input (x) and a desirable output (g) under the assumption that all DMUs produce a same amount of undesirable output (b). The production possible set (PrPS) is listed below a convex curve

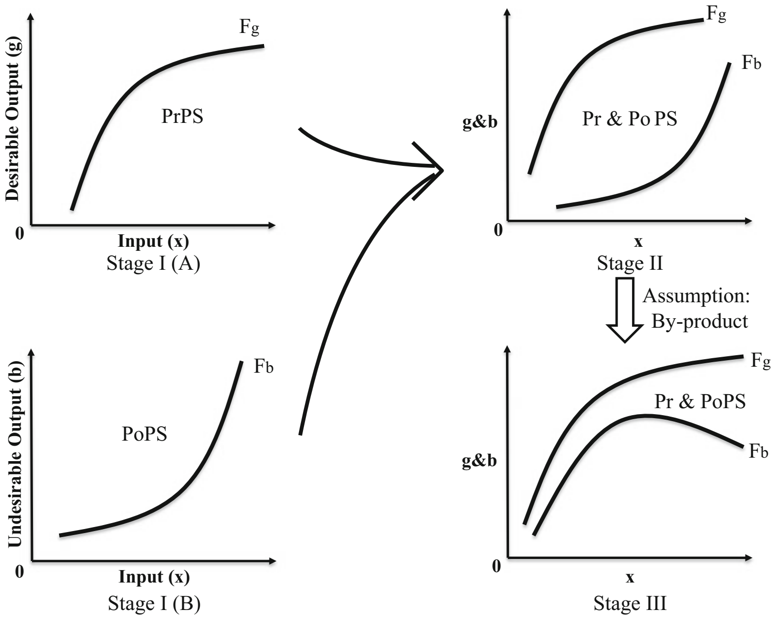


Fig. 16.1 Unification between natural and managerial disposability concepts

(F_g) in the x-g space. The set, indicating the location of all DMUs under the convex curve, is structured by the concept of natural disposability.

Here, it is important to note that, as summarized in Table 17.1, most of previous studies on DEA environmental assessment belong to Stage I(A) in their conceptual frameworks.

The Stage I has the other component (B) which is structured by the concept of managerial disposability. A pollution possibility set (PoPS), locating above the concave curve of a pollution function (F_b), indicates the location of all DMUs in the x-b space under the assumption that they produce a same amount of a desirable output (g).

The second stage (II) unifies the two components of Stage I. The horizontal and vertical coordinates for Stage II indicate x and $g\&b$, respectively. The unification makes it possible to identify the production and pollution possibility sets (Pr&PoPS) between the convex (F_g) and concave (F_b) curves. All DMUs, locating within the Pr&PoPS, is shaped by an intersection between the production and pollution possibility sets.

Assumption for Output Unification: The third stage (III) incorporates the assumption that "undesirable outputs are by-products of desirable outputs". The assumption seems trivial to us, but it drastically changes the structure of DEA environmental assessment. For example, the assumption changes the two curves (F_g and F_b) to be shaped by a convex form, as depicted in the bottom-right hand side of Fig. 16.1. Here, it is important to note that the production curve (F_g) should have an increasing trend along with an input enhancement. However, the pollution curve (F_b) should have an increase tread due to the assumption, and then it should have a decrease trend because of eco-technology innovation or other types of environmental efforts for pollution reduction (a fuel mix strategy or a use of inputs with less CO_2 emission). Consequently, both curves should have a convex form, which is structurally different from the two (I and II) stages of Fig. 16.1. Thus, Fig. 16.1 visually describes a rationale regarding why DEA environmental assessment is more complicated and more difficult than a conventional use of DEA, as mentioned previously. Thus, the existence of undesirable outputs makes the assessment very difficult from the conventional use of DEA, as depicted in Fig. 16.1.

16.3.4 Desirable Congestion (DC)

Figure 16.2 exhibits a desirable output (g) on the horizontal axis and an undesirable output (b) on the vertical axis. The negative slope of a supporting hyperplane indicates an occurrence of DC, or eco-technology to reduce an amount of an undesirable output. The occurrence of DC implies that an enlarged input (x) increases a desirable output (g) and decreases an undesirable output vector (b). This study is interested in an occurrence of DC because we look for corporate sustainability that indicates economic prosperity and environmental protection by eco-technology innovation. Equality constraints should be assigned to desirable outputs (G) in the case.

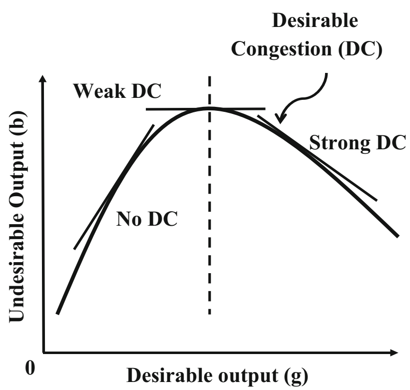


Fig. 16.2 Desirable congestion (DC)

It is important to note the following two concerns on Fig. 16.2.

(a) The convex curve needs an assumption that undesirable outputs are by-products of desirable output. It becomes a concave curve without the assumption.
(b) An occurrence of desirable congestion (DC) implies that we can measure eco-technology innovation to reduce the amount of undesirable outputs.

16.4 Unified Efficiency

16.4.1 Unified Efficiency (UE)

The unified (operational and environmental) performance, or often referred to as Debrew[3]-Farrell[4] measure, of DMUs (Decision Making Unites: organizations to be

[3] Gérard Debreu (July 4, 1921–December 31, 2004) was a French economist and mathematician, who came to have United States citizenship. Best known as a professor of economics at the University of California, Berkeley, where he began working in 1962. Gerard Debreu's contributions are in general equilibrium theory—highly abstract theory about whether and how each market reaches equilibrium. In a famous paper, coauthored with Kenneth Arrow and published in 1954, Debreu proved that under fairly unrestrictive assumptions, prices may exist for bring markets into equilibrium. In his 1959 book, The Theory of Value, Debreu introduced more general equilibrium theory, using complex analytic tools from mathematics—set theory and topology—to prove his theorems. In 1983 Debreu was awarded the Nobel Prize for having incorporated new analytical methods into economic theory and for his rigorous reformulation of the theory of general equilibrium. See http://books.google.co.jp/books?id=Z6Oy4L6LSwC&pg=PA140&lpg=PA140&dq=debreu+farrell&source=bl&ots=aLkVeuwk9u&sig=SYkaHtL56JXvZjUW0jJHg33cw0o&hl=ja&sa=X&ei=QZ03VPP1CtXc8AWAyoCQDA&ved=0CEoQ6AEwBg#v=onepage&q=debreu%20farrell&f=false

[4] His name was Michael James Farrell who was an applied economist at University of Cambridge, UK. Unfortunately, his study had a difficulty in finding his personal information on his birth and death dates. Since his contribution had been long supported by many production economists, this study needs to review his contributions from the perspective of DEA. Our review discussion is based upon the three articles (Farrell 1954; Farrell 1957; Farrell and Fieldhouse 1962). The first article (Farrell 1954: An application of activity analysis to the theory on the firm) was prepared when he visited Yale University (USA) where he could meet T.C. Koopmans and J. Tobin. In the article (1954, p. 292), he discussed "activity analysis", proposed by Koopmans, which could explore the corporate behavior of a firm by an application of "linear programming". In his article, the production relationship between production factors could be expressed by a static model in multiple periods. As a result, linear programming could be applicable to the assessment of corporate behavior. The second article (Farrell 1957: The measurement of productive efficiency) was innovative and it was closely related to the classical DEA development by providing the methodology with a conceptual basis. The article discussed an efficient production function, inspired by the activity analysis of linear programming (1957, p. 11) and started discussing an efficiency measure, referred to as "technical efficiency", which was first discussed in Debreu's "coefficient of resource utilization" (Debreu 1951). In addition to the concept of technical efficiency, According to his article (1957, p. 255), "an efficient production function might be

measured) is characterized by their production activities that utilize inputs to yield desirable and undesirable outputs. This study considers n DMUs, or organizations to be evaluated by DEA. An important feature of DEA environmental assessment is that the achievement of each DMU is relatively compared with those of the remaining others. The performance level is referred to as "an efficiency measure".

The proposed approach uses the following data ranges related to inputs, desirable and undesirable outputs:

$$R_i^x = (m+s+h)^{-1}\left(max\left\{x_{ij}\middle|j=1,\ldots,n\right\} - min\left\{x_{ij}\middle|j=1,\ldots,n\right\}\right)^{-1} \text{ for } i=1,\ldots,m,$$
$$R_r^g = (m+s+h)^{-1}(max\{g_{rj}|j=1,\ldots,n\} - min\{g_{rj}|j=1,\ldots,n\}^{-1} \text{ for } r=1,\ldots,s \text{ and}$$
$$R_f^b = (m+s+h)^{-1}\left(max\left\{b_{fj}\middle|j=1,\ldots,n\right\} - min\left\{b_{fj}\middle|j=1,\ldots,n\right\}\right)^{-1} \text{ for} f=1,\ldots,h,$$

respectively. All the three data ranges are identified from an observed data set so that they are given to us before computing the proposed approach. Later, the inputs are further separated into two categories by the two disposability concepts. However, it is not necessary to change anything on the input ranges because they are determined by observations on each input.

---

expressed by a theoretical function specified by "engineers". However, such an engineering-based empirical function was complicated and practically impossible to measure the theoretical efficiency function from the perspective of production economics. This study pays attention to the fact that Farrell (1957) has used the term "technical efficiency" because of his awareness on the engineering perspective, following Debreu (1951). Here, we may have simple questions such as "what engineering was" and "what type of technology was" in his economics context. It is very clear to us that the production technology in the middle of the twentieth century is by far different from the current one in the beginning of the twenty-first century. Fully acknowledging his contribution in production economics, this study does not use the term "technical efficiency" to avoid our confusion with "technology innovation" on industrial pollution that is the gist of this chapter. The second article (1957, p. 255 and p. 260) also discussed "price efficiency" and "overall efficiency" under increasing and diminishing RTS. These economic concepts have long provided us with a conceptual basis on DEA. No wonder why many studies have discussed his contribution as a staring study of DEA even if he did not mention anything on DEA. Finally, the third article (1962, Farrell and Fieldhouse: Estimating efficient production function under increasing returns to scale) extended Farrell's study (1967) by discussing a linear programming structure that was solved by the simplex method of linear programming (1967, pp. 265–266). Their study documented two interesting concerns from our perspective. One of the interesting concerns was that they knew an occurrence of degeneracy, or multiple solutions. The other concern was that they discussed the importance of a dual formulation, not discussed by production economists even nowadays. As discussed by Glover and Sueyoshi (2009), it is easily imagined that their appendix on the method of computation (1967, pp. 264–267) was guided by Alan Hoffman, as a reviewer of their manuscript, who was an operations researcher. Consequently, their description on computation is still useful in modern DEA algorithmic development. It may be true that many DEA researchers have been long discussing the concept of technical efficiency, due to Farrell's engineering concern, but not paying serious attention its dual formulation, as discussed by their works (1967). As documented in their study (1967), the collaboration between production economics and operations research/management science is essential in extending new research dimensions on DEA.

The research efforts by Sueyoshi and Goto (2012b, c, 2013a, b, 2014a, b) have proposed the following radial model for DEA environmental assessment:

$$
\begin{aligned}
&Maximize \quad && \xi + \varepsilon_s \left[\sum_{i=1}^{m} R_i^x \left(d_i^{x+} + d_i^{x-} \right) + \sum_{r=1}^{s} R_r^g d_r^g + \sum_{f=1}^{h} R_f^b d_f^b \right] \\
&s.t. && \sum_{j=1}^{n} x_{ij}\lambda_j - d_i^{x+} + d_i^{x-} = x_{ik} \quad (i = 1, \ldots, m), \\
& && \sum_{j=1}^{n} g_{rj}\lambda_j - d_r^g - \xi g_{rk} = g_{rk} \quad (r = 1, \ldots, s), \\
& && \sum_{j=1}^{n} b_{fj}\lambda_j + d_f^b + \ \xi b_{fk} = b_{fk} \quad (f = 1, \ldots, h), \\
& && \sum_{j=1}^{n} \lambda_j = 1, \\
& && \lambda_j \geq 0 \quad (j = 1, \ldots, n), \ \xi : URS, \\
& && d_i^{x+} \geq 0 \quad (i = 1, \ldots, m), d_i^{x-} \geq 0 \quad (i = 1, \ldots, m), \\
& && d_r^g \geq 0 \quad (r = 1, \ldots, s), \ \ and \ \ d_f^b \geq 0 \quad (f = 1, \ldots, h),
\end{aligned}
\tag{16.2}
$$

where ξ is an inefficiency score, indicating a distance between an efficiency frontier and an observed vector of desirable and undesirable outputs. This study sets ε_s as 0.0001 for our computation convenience to reduce an influence of slacks. A subjective decision may occur on the selection of ε_s. Historically, it was considered that ε_s was a Non-Archimedean small number in DEA. However, none knows what it is in reality. In avoiding such a specification difficulty, it is possible for us to use $\varepsilon_s = 0$ in Model (16.2). However, in the case, dual variables may become zero on some production factors so that information on production factors in a data set is not fully utilized in Model (16.2). This is problematic and unacceptable as a computational result of DEA performance assessment.

The two slacks related to the ith input are mathematically defined as $d_i^{x+} = \left(\left| d_i^x \right| + d_i^x \right)/2$ and $d_i^{x-} = \left(\left| d_i^x \right| - d_i^x \right)/2$. They are mutually exclusive so that a simultaneous occurrence of both $d_i^{x+} > 0$ and $d_i^{x-} > 0$ ($i = 1, \ldots, m$) should be excluded from the optimal solution of Model (16.2). When the simultaneous occurrence occurs on Model (16.2), a computer code usually produces "an unbounded solution" because of violating the nonlinear conditions.

To make Model (16.2) satisfy the nonlinear conditions, the previous studies (e.g., Sueyoshi and Goto 2012a) have suggested the following two computational alternatives:

(a) One of the two alternatives is that Model (16.2) incorporates the nonlinear conditions into Model (16.2) as side constraints and then we solve Model (16.2) with $d_i^{x+} d_i^{x-} = 0$ ($i = 1, \ldots, m$) as a nonlinear programming problem.
(b) The other alternative is that Model (16.2) incorporate the following side constraints: $d_i^{x+} \leq M z_i^+$, $d_i^{x-} \leq M z_i^-$, $z_i^+ + z_i^- \leq 1$, z_i^+ and z_i^-: binary (i = 1, ... m) and solve Model (16.2) with the side constraints as a mixed integer

programming problem. Here, M stands for a very large number that we need to prescribe before our computational operation.

After solving Model (16.2) with the nonlinear conditions, the level of unified efficiency (UE) of the kth DMU is determined by

$$UE = 1 - \left[\xi^* + \varepsilon_s\left(\sum_{i=1}^{m} R_i^x\left(d_i^{x+*} + d_i^{x-*}\right) + \sum_{r=1}^{s} R_r^g d_r^{g*} + \sum_{f=1}^{h} R_f^b d_f^{b*}\right)\right]. \quad (16.3)$$

Here, the inefficiency score and slacks within the parentheses are obtained from the optimality of Model (16.2).

16.4.2 Unified Efficiency under Natural Disposability (UEN)

Formulation for Stage I (A): The research efforts of Sueyoshi and Goto (2012b, c, 2013a, b, 2014a, b) and Sueyoshi and Yuan (2015a, b) have proposed the following radial model to measure the unified efficiency of the kth DMU under natural disposabilit)y:

$$\begin{aligned}
Maximize \quad & \xi + \varepsilon_s\left[\sum_{i=1}^{m} R_i^x d_i^{x-} + \sum_{r=1}^{s} R_r^g d_r^g + \sum_{f=1}^{h} R_f^b d_f^b\right] \\
s.t. \quad & \sum_{j=1}^{n} x_{ij}\lambda_j + d_i^{x-} = x_{ik} \qquad (i = 1, \ldots, m), \\
& \sum_{j=1}^{n} g_{rj}\lambda_j - d_r^g - \xi g_{rk} = g_{rk} \quad (r = 1, \ldots, s), \\
& \sum_{j=1}^{n} b_{fj}\lambda_j + d_f^b + \xi b_{fk} = b_{fk} \quad (f = 1, \ldots, h), \\
& \sum_{j=1}^{n} \lambda_j = 1, \\
& \lambda_j \geq 0 \quad (j = 1, \ldots, n),\ \xi : URS,\ d_i^{x-} \geq 0 \quad (i = 1, \ldots, m), \\
& d_r^g \geq 0 \quad (r = 1, \ldots, s), \quad and \quad d_f^b \geq 0 \quad (f = 1, \ldots, h).
\end{aligned} \quad (16.4)$$

A unified efficiency score (UEN))under natural disposability on the kth DMU is measured by

$$UEN = 1 - \left[\xi^* + \varepsilon_s\left(\sum_{i=1}^{m} R_i^x d_i^{x-*} + \sum_{r=1}^{s} R_r^g d_r^{g*} + \sum_{f=1}^{h} R_f^b d_f^{b*}\right)\right], \quad (16.5)$$

where the inefficiency score and all slack variables are determined on the optimality of Model (16.4). The equation within the parenthesis, obtained from the optimality of Model (16.4), indicates the level of unified inefficiency under natural disposability. The unified efficiency is obtained by subtracting the level of inefficiency from unity.

16.4.3 Unified Efficiency under Managerial Disposability (UEM)

Formulation for Stage I (B): Shifting our research interest from natural disposability to managerial disposability, where the first priority is environmental performance and the second priority is operational performance, this study utilizes the following radial model that measures the unified efficiency of the kth DMU (Sueyoshi and Goto 2012b, c, 2013a, b, 2014a, b; Sueyoshi and Yuan 2015a, b):

$$\begin{aligned}
&\textit{Maximize} \quad && \xi + \varepsilon_s \left[\sum_{i=1}^{m} R_i^x d_i^{x+} + \sum_{r=1}^{s} R_r^g d_r^g + \sum_{f=1}^{h} R_f^b d_f^b \right] \\
&s.t. && \sum_{j=1}^{n} x_{ij}\lambda_j - d_i^{x+} = x_{ik} \qquad (i = 1, \ldots, m), \\
& && \sum_{j=1}^{n} g_{rj}\lambda_j - d_r^g - \xi g_{rk} = g_{rk} \quad (r = 1, \ldots, s), \\
& && \sum_{j=1}^{n} b_{fj}\lambda_j + d_f^b + \xi b_{fk} = b_{fk} \quad (f = 1, \ldots, h), \\
& && \sum_{j=1}^{n} \lambda_j = 1, \\
& && \lambda_j \geq 0 \quad (j = 1, \ldots, n), \quad \xi : URS,\ d_i^{x+} \geq 0 \quad (i = 1, \ldots, m), \\
& && d_r^g \geq 0 \quad (r = 1, \ldots, s), \quad and \quad d_f^b \geq 0 \quad (f = 1, \ldots, h).
\end{aligned} \tag{16.6}$$

An important feature of Model (16.6) is that it changes $+d_i^{x-}$ of Model (16.4) to $-d_i^{x+}$ in order to attain the status of managerial disposability. No other change is found in Model (16.6).

A unified efficiency score (UEM) on the kth DMU under managerial disposability is measured by

$$UEM = 1 - \left[\xi^* + \varepsilon_s \left(\sum_{i=1}^{m} R_i^x d_i^{x+*} + \sum_{r=1}^{s} R_r^g d_r^{g*}, + \sum_{f=1}^{h} R_f^b d_f^{b*} \right) \right], \tag{16.7}$$

where the inefficiency score and all slack variables are determined on the optimality of Model (16.6). The equation within the parenthesis, obtained from the optimality of Model (16.6), indicates the level of unified inefficiency under managerial disposability. The unified efficiency is obtained by subtracting the level of inefficiency from unity.

16.4.4 Unified Efficiency under Natural and Managerial Disposability (UENM)

Formulation for Stage II: A possible unification between Models (16.4) and (16.6) is that it combines the two models along with the separation on inputs into two categories under natural and managerial disposability. Consequently, inputs and outputs are classified into four categories (2 input groups × 2 output groups) for the measurement of *UENM*. This study proposes the following radial model measures the level of UENM (Goto et al. 2014):

$$
\begin{aligned}
&\textit{Maximize} \quad && \xi + \varepsilon_s \left[\sum_{i=1}^{m^-} R_i^x d_i^{x-} + \sum_{q=1}^{m^+} R_q^x d_q^{x+} + \sum_{r=1}^{s} R_r^g d_r^g + \sum_{f=1}^{h} R_f^b d_f^b \right] \\
&\textit{s.t.} && \sum_{j=1}^{n} x_{ij}^- \lambda_j + d_i^{x-} = x_{ik}^- \qquad (i = 1, \ldots, m^-), \\
& && \sum_{j=1}^{n} x_{qj}^+ \lambda_j - d_q^{x+} = x_{qk}^+ \qquad (q = 1, \ldots, m^+), \\
& && \sum_{j=1}^{n} g_{rj} \lambda_j - d_r^g - \xi g_{rk} = g_{rk} \qquad (r = 1, \ldots, s), \\
& && \sum_{j=1}^{n} b_{fj} \lambda_j + d_f^b + \xi b_{fk} = b_{fk} \qquad (f = 1, \ldots, h), \\
& && \sum_{j=1}^{n} \lambda_j = 1, \\
& && \lambda_j \geq 0 \quad (j = 1, \ldots, n), \quad \xi : URS, \\
& && d_i^{x-} \geq 0 \quad (i = 1, \ldots, m^-), \quad d_q^{x+} \geq 0 \quad (q = 1, \ldots, m^+), \\
& && d_r^g \geq 0 \quad (r = 1, \ldots, s) \quad and \quad d_f^b \geq 0 \quad (f = 1, \ldots, h).
\end{aligned}
\tag{16.8}
$$

Here, the number of original m inputs are newly separated into m^- (under natural disposability) and m^+ (under managerial disposability), respectively, in Model (16.8). The model maintains $m = m^- + m^+$. One of the two input categories uses inputs $\left(x_{ij}^-\right)$ whose slacks $\left(d_i^{x-}\right)$ for $i = 1, \ldots, m^-$ are formulated under natural disposability. For example, the number of employees belongs to the input category. Meanwhile, the other category contains inputs $\left(x_{qj}^+\right)$ whose slacks $\left(d_q^{x+}\right)$ for $q = 1, \ldots, m^+$ are formulated under managerial disposability. For example, the amount of capital investment for eco-technology innovation belongs to the input category. As formulated in Model (16.8), the input vector of the *j*th DMU is separated into the two groups under natural and managerial disposability.

The level of unified efficiency (*UENM*) under natural and managerial disposability is measured by

$$UENM = 1 - \left[\xi^* + \varepsilon_s\left(\sum_{i=1}^{m^-} R_i^x d_i^{x-*} + \sum_{q=1}^{m^+} R_q^x d_q^{x+*} + \sum_{r=1}^{s} R_r^g d_r^{g*} + \sum_{f=1}^{h} R_f^b d_f^{b*}\right)\right], \tag{16.9}$$

The dual formulation of Model (16.8) becomes as follows:

$$\begin{aligned}
&\text{Minimize} \sum_{i=1}^{m^-} v_i x_{ik}^- - \sum_{q=1}^{m^+} z_q x_{qk}^+ - \sum_{r=1}^{s} u_r g_{rk} + \sum_{f=1}^{h} w_f b_{fk} + \sigma \\
&s.t. \quad \sum_{i=1}^{m^-} v_i x_{ij}^- - \sum_{q=1}^{m^+} z_q x_{qj}^+ - \sum_{r=1}^{s} u_r g_{rj} + \sum_{f=1}^{h} w_f b_{fj} + \sigma \geq 0 && (j = 1, \ldots, n), \\
&\qquad \sum_{r=1}^{s} u_r g_{rk} + \sum_{f=1}^{h} w_f b_{fk} = 1 \\
&\qquad v_i \geq \varepsilon_s R_i^x && (i = 1, \ldots, m^-), \\
&\qquad z_q \geq \varepsilon_s R_q^x && (q = 1, \ldots m^+) \\
&\qquad u_r \geq \varepsilon_s R_r^g && (r = 1, \ldots, s), \\
&\qquad w_f \geq \varepsilon_s R_f^b && (f = 1, \ldots, h), \\
&\qquad \sigma : URS,
\end{aligned} \tag{16.10}$$

where v_i $(i = 1, \ldots, m^-)$, z_q $(q = 1, \ldots, m^+)$, u_r $(r = 1, \ldots, s)$ and w_f $(f = 1, \ldots, h)$ are all dual variables related to the first, second, third and fourth groups of constraints in Model (16.8). The dual variable (σ), which is unrestricted, is obtained from the last equation of Model (16.8). The objective value of Model (16.8) equals that of Model (16.10) on optimality.

A contribution of *UENM*, measured by Models (16.8) and (16.10), is that these models combine the two disposability concepts into a single criterion where they are equally treated in environmental assessment. A drawback of *UENM* is that it does not incorporate an occurrence of DC, or eco-green technology innovation on undesirable outputs. See Stage II of Fig. 16.1 that visually describes the methodological difficulty.

To intuitively describe a rationale on why Models (16.8) and (16.10) have a difficulty in measuring eco-technology innovation, this study returns to Model (16.10) by which the supporting hyperplane is expressed by $vx^- - zx^+ - ug + wb + \sigma = 0$, or

$wb = -vx^- + zx^+ + ug - \sigma$, in the case where all production factors have a single component. Since w is positive in its sign, the supporting hyperplane is unacceptable because an increase in the input under natural disposability (x^-) decreases the undesirable output. Such an observation should be reversely applicable to the input under managerial disposability (x^+). The relationship is unacceptable for this study so that Models (16.8) and (16.10) need to be reorganized as in the next section.

16.4.5 Unified Efficiency under Natural and Managerial Disposability: UENM(DC) with a Possible Occurrence of Desirable Congestion (Eco-technology Innovation)

Formulation for Stage III: To identify a possible occurrence DC, or eco-technology innovation, this study reorganizes the hyperplane like $vx^- - zx^+ + ug - wb + \sigma = 0$. The corresponding dual formulation to satisfy the requirement in the case of multiple production factors becomes as follows:

$$
\begin{aligned}
&\text{Minimize} \sum_{i=1}^{m^-} v_i x_{ik}^- - \sum_{q=1}^{m^+} z_q x_{qk}^+ + \sum_{r=1}^{s} u_r g_{rk} - \sum_{f=1}^{h} w_f b_{fk} + \sigma \\
&\quad s.t. \quad \sum_{i=1}^{m^-} v_i x_{ij}^- - \sum_{q=1}^{m^+} z_q x_{qj}^+ + \sum_{r=1}^{s} u_r g_{rj} - \sum_{f=1}^{h} w_f b_{fj} + \sigma \geq 0 && (j = 1, \ldots, n), \\
&\qquad \sum_{r=1}^{s} u_r g_{rk} = 1 \\
&\qquad v_i \geq \varepsilon_s R_i^x && (i = 1, \ldots, m^-), \\
&\qquad z_q \geq \varepsilon_s R_q^x && (q = 1, \ldots, m^+), \\
&\qquad u_r : URS && (r = 1, \ldots, s), \\
&\qquad w_f \geq \varepsilon_s R_f^b && (f = 1, \ldots, h), \\
&\qquad \sigma : URS,
\end{aligned}
\tag{16.11}
$$

The primal formulation of Model (16.11) can be specified as follows:

$$
\begin{aligned}
&\textit{Maximize} \quad \xi + \varepsilon_s \left[\sum_{i=1}^{m^-} R_i^x d_i^{x^-} + \sum_{q=1}^{m^+} R_q^x d_q^{x^+} + \sum_{f=1}^{h} R_f^b d_f^b \right] \\
&s.t. \quad \sum_{j=1}^{n} x_{ij}^- \lambda_j + d_i^{x^-} = x_{ik}^- \qquad (i = 1, \ldots, m^-), \\
&\qquad \sum_{j=1}^{n} x_{qj}^+ \lambda_j - d_q^{x^+} = x_{qk}^+ \qquad (q = 1, \ldots, m^+), \\
&\qquad \sum_{j=1}^{n} g_{rj} \lambda_j + \xi g_{rk} = g_{rk} \qquad (r = 1, \ldots, s), \\
&\qquad \sum_{j=1}^{n} b_{fj} \lambda_j - d_f^b = b_{fk} \qquad (f = 1, \ldots, h), \\
&\qquad \sum_{j=1}^{n} \lambda_j = 1, \\
&\qquad \lambda_j \geq 0 \quad (j = 1, \ldots, n), \quad \xi : URS, d_i^{x^-} \geq 0 \quad (i = 1, \ldots, m^-), \\
&\qquad d_q^{x^+} \geq 0 \ (q = 1, .., m^+) \ \textit{and} \ d_f^b \geq 0 \ (f = 1, .., h).
\end{aligned}
\tag{16.12}
$$

The unified efficiency score, or *UENM(DC)*, is measured by

$$
UENM(DC) = 1 - \left[\xi^* + \varepsilon_s \left(\sum_{i=1}^{m^-} R_i^x d_i^{x^-*} + \sum_{q=1}^{m^+} R_q^x d_q^{x^+*}, + \sum_{f=1}^{h} R_f^b d_f^{b*} \right) \right], \tag{16.13}
$$

where the inefficiency score and slacks are determined on the optimality of Model (16.12).

16.5 Investment Strategy

After solving Model (16.12), this study can identify an occurrence of DC, or green technology innovation for pollution mitigation, by the following rule along with the assumption on a unique optimal solution (Sueyoshi and Goto 2014b):

(a) if $u_r^{+*} = 0$ for some (at least one) r, then "weak DC" occurs on the kth DMU,
(b) if $u_r^{+*} < 0$ for some (at least one) r, then "strong DC" occurs on the kth DMU and
(c) if $u_r^{+*} > 0$ for all r, then "no DC" occurs on the kth DMU.

Note that if $u_r^{+*} < 0$ for some r and $u_{r'}^{+*} = 0$ for the other r', then this study considers that the strong DC occurs on the kth DMU. It is indeed true that $u_r^{+*} < 0$ for all r is the best case because an increase in any desirable output always decreases an amount of undesirable outputs. Meanwhile, if $u_r^{+*} < 0$ is identified for some r, then it indicates that there is a chance to reduce an amount of undesirable output(s). Therefore, this study also considers the second case as an investment opportunity because we want to reduce an amount of industrial pollution as much as possible.

Under an occurrence of strong DC (i.e., $u_r^{+*} < 0$ for at least one r), the effect of investment on undesirable outputs is determined by the following rule:

(a) if $z_q^* > \varepsilon_s R_q^x$ for q in Model (16.12), then the qth input for investment under managerial disposability can effectively decrease an amount of undesirable outputs and
(b) if $z_q^* = \varepsilon_s R_q^x$ for q in Model (16.12), then the qth input for investment has a limited effect on decreasing an amount of undesirable outputs.

The investment on inputs under managerial disposability is not recommended in the other two cases (i.e., no and weak DC). Furthermore, this study uses "a limited effect" in the second case. The term implies that if this study drops the data range on the qth input in Model (16.12), then there is a high likelihood that z_q^* may become zero. Moreover, $z_q^* > \varepsilon R_q^x$ are required for some q, but not necessary for all q.

Finally, it is important to note that the proposed investment classification needs at least two desirable outputs because unrestricted u in Model (16.12) cannot produce a negative value on the dual variable, so being unable to identify an investment opportunity, in the case of a single desirable output. Thus, the investment rule discussed in this study needs multiple desirable outputs.

16.6 Empirical Study

This study obtains a data set from Wang et al. (2014) and Sueyoshi and Wang (2014a) whose data source is the Carbon Disclosure Project (CDP) and COMPUSTAT. The CDP builds the world's largest database regarding corporate performance and climate change by collecting data sets via annual online questionnaire sent out to major firms across the world. This study utilizes the data on S&P 500 companies for 2012 and 2013, including the companies' direct and indirect GHG emission, the investment in carbon mitigation and the corresponding total estimated GHG saving.

It is important to note the two concerns on the data set. One of the two concerns is that among the S&P 500 companies responding to the CDP survey, some companies choose not to provide detailed information of their climate change strategies. This study excludes all of such companies that have refused to disclose information in any of the above data fields. The other concern is that the usage of survey data depends upon the accuracy and trustworthiness of the self-reported information. The CDP data indicates whether a company's emission has been verified by a third-party institution. To address the second concern about data accuracy, this study restricts the data sample to companies that have obtained third-party verification of their GHG emissions. Eventually, this study has obtained a panel of 153 observations from S&P 500 companies over the annual periods 2012–2013. The selected companies include consumer discretionary companies such as General Motors, consumer staple companies such as PepsiCo, energy companies such as Chevron, health care companies such as Pfizer, industrial companies such as Boeing, information technology like Google and Intel, material companies such as Alcoa. This study has confirmed matching between the CDP data set and the operational characteristics and financial performance of firms obtained from COMPUSTAT.

The data set consists of the following operational, environmental and financial factors:

(a) *Estimated CO_2 Saving*: This indicates the annual CO_2 saving from a company's current emission level after the investment in abatement technologies. The variable can be regarded as a measure of a company's technology capacity.
(b) *Return on Assets*: This is defined as the ratio between net income and total assets. It is incorporated as a measure of firm profitability.
(c) *Direct CO_2 Emission*: This measures an amount of emissions from sources owned by a company. The cost of adapting pollution prevention practices and the effectiveness of pollution prevention as a strategy for reducing emissions may vary with a scale of current emission.
(d) *Indirect CO_2 Emission*: This measures an amount of emissions from generation of electricity, steam, heating and cooling purchased by a company offsite.
(e) *Number of* Employees: This is regarded as a proxy for a firm size. Larger firms may have more resources to adapt CO_2 mitigation practices.
(f) *Working Capital*: This is included to indicate the operating liquidity of a firm. Firms with higher working capital may invest more in CO_2 mitigation.
(g) *R&D Expense*: This is another measure of a firm's technology capacity. It is expected that firms with higher R&D expense is more likely to acquire and implement efficient emission control technology.
(h) *Total Assets*: This includes current assets, property, plant and equipment, all of which are used as another proxy for a corporate size.
(i) *Investment in CO_2 Abatement*: This gives a total amount of investment that a company is required to make to achieve the estimated annual CO_2 saving. Profit maximizing firms are expected to choose technology according to their cost performance and effectiveness in mitigating the amount of CO_2 emissions.

In summary, this study utilizes two desirable outputs (i.e., estimated annual CO_2 saving and return on assets), two undesirable outputs (i.e., direct and indirect CO_2 emissions), three inputs under natural disposability (i.e., number of employees, working capital and total assets), two inputs under managerial disposability (i.e., investment in CO_2 abatement and R&D expense).

Table 16.1 documents descriptive statistics on the data set used in this study in which Avg., S.D. Min. and Max. indicate average, standard deviation, minimum

Table 16.1 Descriptive statistics

Outputs and inputs		Desirable outputs		Undesirable outputs		Inputs				
Variables		Estimated CO2 savings	Return on assets	Direct CO2 emission	Indirect CO2 emission	Employees	Working capital	R&D expense	Total assets	Investment in CO2 abatement
Unit		Metric tonnes	Index	1000 tons	1000 tons	Thousand	Million $	Million $	Million $	Thousand $
Consumer discretionary	Avg.	21,041.1	0.0604	452.3511	876.6882	44.4199	2400.4422	738.1463	17,918.1577	6414.48
	S.D.	54,807.2	0.0338	765.4351	1531.2482	56.3165	4237.8357	1995.8894	39,857.5618	11,402.27
	Min.	20.0	0.0125	5.9850	17.4220	5.5000	98.9320	41.0000	837.4000	122.00
	Max.	199,907.0	0.1012	2454.7550	5531.3800	213.0000	16,004.0000	7368.0000	149,422.0000	37,035.00
Consumer staples	Avg.	42,759.4	0.1053	576.2264	500.8998	44.2445	917.5033	126.9574	13,969.0686	35,101.54
	S.D.	115,410.8	0.0648	1104.6655	517.0599	78.8626	468.8002	168.1833	20,382.8222	107,782.81
	Min.	767.6	0.0238	58.3440	119.9750	4.8500	103.0000	14.4000	3258.2000	350.00
	Max.	390,000.0	0.1911	3854.7840	1928.4900	278.0000	1805.6000	552.0000	74,638.0000	360,000.00
Energy	Avg.	157,412.7	0.0654	10,144.5206	940.0859	36.8375	5275.6919	290.1308	57,027.8392	101,641.31
	S.D.	289,914.2	0.0289	15,911.8134	1059.5143	41.3664	6300.9056	419.1814	59,514.6891	331,992.70
	Min.	100.0	0.0275	485.0000	61.6300	1.8760	5.0000	0.1000	12,670.9090	600.00
	Max.	1,000,000.0	0.1144	58,559.2200	3849.3190	118.0000	21,508.0000	1168.0000	232,982.0000	1,205,600.00
Health care	Avg.	20,291.3	0.0916	309.8158	430.4136	41.7609	10,704.3921	3520.7566	50,562.2260	12,200.18
	S.D.	42,912.0	0.0411	408.9799	428.6841	38.3091	10,375.7373	3029.5218	52,192.6653	19,879.12
	Min.	21.0	0.0179	7.2320	18.4200	4.4600	390.6540	132.6390	3901.7620	7.00
	Max.	206,193.0	0.1953	1402.5280	1256.6640	127.6000	32,796.0000	9112.0000	188,002.0000	88,788.98
Industrials	Avg.	23,992.9	0.0731	522.3643	543.3526	80.9725	3293.9876	953.1261	29,580.8274	9162.75
	S.D.	27,081.9	0.0231	1243.7752	430.4336	65.0745	3212.8384	1167.7591	28,960.1830	14,048.50
	Min.	120.5	0.0423	8.6820	25.4220	3.1210	45.1300	15.4000	688.0910	22.07
	Max.	107,875.0	0.1311	5532.8440	1756.2750	218.3000	12,327.0000	3918.0000	89,409.0000	63,409.80

Information Technology	Avg.	11,058.6	0.0904	75.1610	342.9409	30.7352	6733.2885	1617.9462	21,148.3242	2705.57
	S.D.	34,572.1	0.0493	203.1143	519.2890	39.9831	13,212.1084	2498.4823	29,831.1172	8635.49
	Min.	2.0	0.0131	0.0339	11.0490	2.3800	54.3860	25.0340	1172.1660	8.05
	Max.	169,787.0	0.2307	897.7590	2331.0480	147.6000	52,396.0000	10,148.0000	121,271.0000	59,000.00
Materials	Avg.	183,627.9	0.0600	7027.4129	4601.3531	34.0613	2246.4306	351.0438	19,313.3611	17,723.61
	S.D.	236,157.1	0.0371	9052.2963	5530.0427	21.9178	2225.4220	621.7890	14,383.7926	40,253.65
	Min.	172.0	0.0048	53.9270	88.5250	5.7000	72.0000	13.0000	3249.6000	1.24
	Max.	638,000.0	0.1393	30,628.1040	16,659.7360	70.0000	7642.0000	2067.0000	49,736.0000	177,000.00
Total	Avg.	54,670.6	0.0805	2095.4156	1103.0311	42.0503	5451.6729	1394.2375	29,380.3075	18,392.98
	S.D.	143,424.1	0.0441	6579.1242	2599.8684	48.4773	9414.8361	2302.9263	38,871.3393	102,637.18
	Min.	2.0	0.0048	0.0339	11.0490	1.8760	5.0000	0.1000	688.0910	1.24
	Max.	1,000,000.0	0.2307	58,559.2200	16,659.7360	278.0000	52,396.0000	10,148.0000	232,982.0000	1,205,600.00

and maximum, respectively. To control for heterogeneity across sectors, this study further calculates industry-adjusted index to be used in DEA models for all the variables. The index of a variable is the ratio of its actual value to the industry average of that variable.

Table 16.2 summarizes five unified efficiency scores, or *UE, UEN, UEM, UENM* and *UENM(DC)*, of firms in the IT industry as an illustrative purpose. As summarized at the bottom of Table 16.2, the five unified efficiency measures are 0.3587 in *UE*, 0.4853 in *UEN*, 0.5800 in *UEM*, 0.6295 in *UENM* and 0.7425 in *UENM(DC),* respectively, on average. *UE* is a unified efficiency measure for operational and environmental performance, *UEN* is a measure which has the first priority on operational performance and the second one on environmental performance. *UNM* indicates an opposite case. *UENM* combines the two disposability concepts. *UENM(DC)* incorporates a possible occurrence of eco-technology innovation to reduce the amount of undesirable outputs. For example, Applied Materials Inc. (2012) exhibited the status of efficiency in the five efficiency measures. The other firms have some level of inefficiency in these measures.

To explain the implications of the five efficiency measures, let us pay attention to Yahoo! Inc. The firm had 0.0354 in *UE*, 0.0622 in *UEN*, 0.5679 in *UEM*, 0.3346 in *UENM* and 1.0000 in *UENM(DC)* during 2012. These measures indicated the status of inefficiency in the first four performance measures. However, the efficiency (1.0000) in *UENM(DC)* indicated that the firm had a very high level of investment opportunity in 2012. Such an investment changed the status of the other four efficiency measures from inefficiency to efficiency in 2013. As a result of investment on technology innovation in 2012, the Yahoo! did not need the investment for technology innovation anymore so that the measure of *UENM(DC)* dropped to 0.1317 in 2013.

Table 16.3 allows us to compare the performance of the seven main industrial sectors and their industrial subgroups. In the table, the materials sector exhibited the best performance (0.501), the energy sector was the second (0.3709) and the IT sector was the third (0.3587) in their *UE* measures. Meanwhile, the energy sector was the best (0.7290), the IT sector was the second (0.6295) and the material sector was the third (0.5919) in terms of their *UENM* measures. The major difference between *UE* and *UENM* is that the latter has the input classification under the two disposability concepts, but the former does not have such a classification. This study used both R&D expenditure and investment in CO_2 abatement as the inputs under managerial disposability. The computational result of Table 16.3 indicated that the energy sector had the most promising area for investment on technology innovation among the seven industrial sectors.

Table 16.4 lists dual variables, the type of DC (S: Strong DC and No: No DC) and the type of investment effect (E: effective and L: limited) on the IT sector. A blank space indicates that the type is no DC so that it is not necessary for us to consider an opportunity for investment. Table 16.5 summarizes the effective and limited investment opportunities on the seven industrial sectors. On overall average, 46 observations (30.07 %) were rated as efficient observations and 2 firms

Table 16.2 Unified efficiency measures of IT Industry

Company name	UE	UEN	UEM	UENM	UENM (DC)
Adobe Systems, Inc. (2012)	0.4975	0.5340	0.7685	0.6200	0.1850
Adobe Systems, Inc. (2013)	0.3680	0.4187	0.5834	0.4265	0.2607
Automatic Data Processing, Inc. (2012)	0.0980	0.0756	0.7548	0.0760	0.4584
Automatic Data Processing, Inc. (2013)	0.0715	0.0715	0.5911	0.0747	0.3684
Akamai Technologies Inc. (2012)	0.5275	1.0000	0.7847	1.0000	0.6043
Akamai Technologies Inc. (2012)	0.1480	1.0000	0.1738	1.0000	0.9378
Altera Corp. (2012)	1.0000	1.0000	1.0000	1.0000	0.4993
Altera Corp. (2013)	0.6842	0.8539	1.0000	1.0000	1.0000
Applied Materials Inc. (2012)	1.0000	1.0000	1.0000	1.0000	1.0000
Broadcom Corporation (2012)	0.2210	0.2239	0.6159	0.6638	1.0000
Broadcom Corporation (2013)	0.2427	0.2581	1.0000	1.0000	1.0000
CA Technologies (2012)	0.1491	0.6130	0.2172	1.0000	0.6477
CA Technologies (2013)	0.1570	0.3810	0.2247	0.6239	0.3332
Spansion Inc. (2013)	0.0329	0.0841	0.0329	0.0809	1.0000
Compuware Corp. (2012)	0.3702	1.0000	0.3890	1.0000	1.0000
EMC Corporation (2012)	0.0291	0.0934	0.3527	1.0000	1.0000
EMC Corporation (2013)	0.0352	0.0462	0.4039	1.0000	1.0000
Fairchild Semiconductor (2013)	0.0636	0.0643	0.0248	0.0630	1.0000
Google Inc. (2012)	0.3542	0.2806	1.0000	0.5454	1.0000
Google Inc. (2013)	0.2537	0.0966	1.0000	0.4660	1.0000
Intel Corporation (2013)	1.0000	0.9893	1.0000	1.0000	1.0000
Jabil Circuit, Inc. (2012)	0.1510	1.0000	0.6985	0.3490	0.7048
Jabil Circuit, Inc. (2013)	0.1915	1.0000	1.0000	0.2007	0.8015
JDS Uniphase Corp. (2012)	1.0000	0.3034	0.2235	0.3223	0.7630
Juniper Networks, Inc. (2013)	0.1943	0.1304	0.3533	0.2901	1.0000
KLA-Tencor Corporation (2013)	0.8238	0.8928	0.8579	0.8722	0.2707
LSI Corporation (2012)	0.4624	0.8477	0.5238	1.0000	1.0000
LSI Corporation (2013)	0.2625	0.4659	0.2981	1.0000	1.0000
Lexmark International, Inc. (2013)	0.0989	0.1286	0.1134	0.2311	1.0000
Microchip Technology (2012)	0.0803	0.4493	0.1026	0.4015	0.1773
Microchip Technology (2013)	0.0488	0.0635	0.0401	0.0620	0.4456
Marvell Technology Group Ltd. (2012)	0.2966	0.4010	0.5209	0.6436	1.0000
Marvell Technology Group Ltd. (2013)	0.3975	0.4563	1.0000	1.0000	1.0000
Microsoft Corporation (2012)	0.9168	0.9168	1.0000	1.0000	1.0000
Microsoft Corporation (2013)	0.3524	0.3524	1.0000	1.0000	1.0000
NetApp Inc. (2013)	0.0705	0.0757	0.1953	0.1077	0.5535
NVIDIA Corporation (2012)	0.2959	0.3046	0.6037	0.6131	0.3787
Oracle Corporation (2013)	0.3450	0.3450	1.0000	0.7059	0.4189
SanDisk Corporation (2012)	0.3967	0.6041	0.6451	0.4624	0.3433
Symantec Corporation (2012)	0.0759	1.0000	0.3840	1.0000	1.0000
Symantec Corporation (2013)	0.0666	0.1348	0.2669	0.4339	1.0000
Teradyne Inc. (2012)	0.6609	1.0000	0.6697	1.0000	0.3371

(continued)

Table 16.2 (continued)

Company name	UE	UEN	UEM	UENM	UENM (DC)
Teradyne Inc. (2013)	0.3803	0.7042	0.4599	0.7226	0.5906
Texas Instruments Incorporated (2012)	0.2582	0.3589	0.2985	0.4565	1.0000
Texas Instruments Incorporated (2013)	0.0518	0.1325	0.1005	0.2741	1.0000
Xerox Corporation (2013)	1.0000	0.0813	1.0000	0.0915	0.4302
Yahoo! Inc. (2012)	0.0354	0.0622	0.5679	0.3346	1.0000
Yahoo! Inc. (2013)	1.0000	1.0000	1.0000	1.0000	0.1317
Avg.	0.3587	0.4853	0.5800	0.6295	0.7425
S.D.	0.3195	0.3653	0.3403	0.3534	0.3074

(1.31 %) among 153 total observations were rated as limited investments in terms of developing green technology innovation for corporate sustainability.

In Table 16.5, the energy sector had the highest fraction (46.15 %) of effective investments, marked by E, among the seven industrial sectors, along with limited investment effect (7.69 %). This result indicated that investment for technology innovation in the energy section was the most effective in developing corporate sustainability, compared with the other six industrial sectors. In other words, the energy sector produces a large amount of CO_2 emission. Therefore, it is important for the United States to start controlling the amount of CO_2 emission by paying attention to the investment to firms in the energy sector.

It is important to note that the examination of firms with E (effective investment) provides us with a guidance on which firms have proper technology to enhance corporate sustainability. Such a firm selection can reduce the number of technically advanced firms and makes it possible that we can identify the type of technology to be used for a specific industry although different industries have distinct technology structures and developments on production and environmental protection.

Finally, as documented in these tables, DEA environmental assessment may provide corporate leaders, investors and other individuals who are interested in corporate sustainability with a guideline on which firm(s) they should invest for enhancing the corporate sustainability.

16.7 Conclusion and Future Extensions

Environmental assessment and corporate sustainability have recently become a very important business concern because consumers are interested in environmental protection. A green image is recently essential for corporate survivability in a global market where companies must compete with each other in domestic and international markets.

As a new type of methodology for assessing the corporate sustainability, this study proposed a use of DEA radial measurement for environmental assessment. By shifting DEA models from the non-radial measurement (Sueyoshi and Goto 2012a)

Table 16.3 Unified efficiency measures of seven industry sectors

Sector	Company type	UE	UEN	UEM	UENM	UENM (DC)	# of DMUs
Consumer Discretionary	Automobiles & Components	0.0797	0.2949	0.3558	0.3136	0.8560	6
	Consumer Durables & Apparel	0.5972	0.8369	0.6003	0.8314	0.8759	5
	Retailing	0.1440	0.8089	0.1792	0.4483	0.5593	2
	Overall	0.2886	0.5824	0.4227	0.5335	0.8180	13
Consumer Staples	Food, Beverage & Tobacco	0.1399	0.1720	0.5395	0.2386	0.6378	8
	Household & Personal Products	0.7649	0.8435	0.8450	0.9032	0.5490	3
	Overall	0.3103	0.3551	0.6228	0.4198	0.6136	11
Energy	Energy Equipment & Services	0.3627	0.3686	1.0000	0.9903	0.7288	4
	Oil & Gas	0.3746	0.9881	0.5089	0.6128	0.5955	9
	Overall	0.3709	0.7975	0.6600	0.7290	0.6365	13
Healthcare	Health Care Equipment & Services	0.0967	0.1462	0.4709	0.2100	0.3348	2
	Pharmaceuticals & Biotechnology	0.3422	0.3747	0.3902	0.4898	0.6457	25
	Overall	0.3240	0.3578	0.3962	0.4690	0.6227	27
Industrials	Capital Goods	0.1903	0.2991	0.3757	0.3100	0.4169	17
	Commercial & Professional Services	0.5688	1.0000	0.5788	0.7176	0.6159	2
	Overall	0.2302	0.3729	0.3971	0.3529	0.4379	19
Information Technology	Semiconductors	0.3823	0.5032	0.5342	0.6765	0.7736	19
	Software & Services	0.3215	0.5157	0.6503	0.6837	0.6859	18
	Technology Hardware & Equipment	0.3788	0.4048	0.5441	0.4595	0.7815	11
	Overall	0.3587	0.4853	0.5800	0.6295	0.7425	48
Materials	Chemicals	0.5738	0.7932	0.8226	0.8268	0.7021	13
	Containers & Packaging	0.4948	0.4251	0.4296	0.3163	0.3565	3
	Metals & Mining	0.0684	0.6356	0.1834	0.0474	1.0000	2
	Paper & Forest Products	0.4875	0.6420	0.4353	0.3073	0.5008	4
	Overall	0.5014	0.7012	0.6405	0.5919	0.6455	22

Source: Sueyoshi and Wang (2014a)

to the radial measurement, this study discussed the new use of environmental assessment to determine the five unified efficiency measures that could serve as an empirical basis for developing corporate sustainability. Furthermore, in discussing a use of DEA environmental assessment, this study considered both R&D expenditure and investment in CO_2 abatement as inputs for managerial

Table 16.4 Dual variables, desirable congestion and investment strategy on IT Industry

Company Name	Dual variables									DC	R&D expense	Investment in CO_2	Overall
	Estimated CO_2 savings	Return on assets	Direct CO_2 emission	Indirect CO_2 emission	Employees	Working capital	R&D exp'ense	Total assets	Investment in CO_2 abatememt				
Adobe Systems, Inc. (2012)	0.1775	0.8232	0.0000	0.0000	0.0000	0.0032	0.1496	0.0000	0.0005	No			
Adobe Systems, Inc. (2013)	2.5256	0.7617	0.0000	0.0000	0.3226	0.1274	0.5576	0.0000	0.0000	No			
Automatic Data Processing, Inc. (2012)	1.6154	1.9692	0.0000	0.0000	0.0000	0.0472	0.5827	0.0000	0.0000	No			
Automatic Data Processing, Inc. (2013)	0.5238	1.7194	0.0000	0.0000	0.0000	0.0399	0.3747	0.0000	0.0000	No			
Akamai Technologies Inc. (2012)	3.1128	0.6854	0.0000	0.0000	0.59703	0.0000	0.0000	0.0000	0.1797	No			
Akamai Technologies Inc. (2013)	11.2380	0.9281	0.0000	0.7775	7.4811	0.0000	0.0000	0.0000	0.0073	No			
Altera Corp. (2012)	6.5270	0.4526	0.0000	0.0000	3.5031	0.0000	0.6775	0.0000	0.0654	No			
Altera Corp. (2013)	69.1044	0.2388	5.0984	0.4914	102.2038	1.5171	5.1060	5.1691	21.6652	No			
Applied Materials Inc. (2012)	−9.3064	91.4528	0.3288	1.2799	36.4636	1.0847	7.6938	15.5084	4.8507	S	E	E	E
Broadcom Corporation (2012)	141.1422	−0.1839	1.1853	2.9311	55.2952	47.5982	43.7637	1.2001	0.7590	S	E	E	E
Broadcom Corporation (2013)	8.9608	−0.9052	0.8760	4.2767	33.4818	95.3201	74.6531	4.5927	1.1793	S	E	E	E
CA Technologies (2012)	0.4432	1.0421	0.2489	0.0000	0.0000	10.5398	1.2515	0.0000	0.1075	No			
CA Technologies (2013)	2.2309	0.8847	0.0000	0.0000	0.0993	1.3345	0.9046	0.0000	0.0732	No			
Spansion Inc. (2013)	6.1119	1.8150	12.2755	3.6390	70.6178	17.2968	13.7915	62.2360	0.7083	No			

Compuware Corp. (2012)	21.9705	1.3912	0.8758	1.2568	13.5444	103.0976	11.5776	4.8133	0.7342	No			
EMC Corporation (2012)	37.2408	−4.3929	2.2003	6.8622	0.9999	127.8919	72.7823	0.7791	2.9424	S	E	E	E
EMC Corporation (2013)	12.2348	−3.6796	1.5785	7.6816	0.8923	111.2428	72.6548	0.6347	0.9246	S	E	E	E
Fairchild Semiconductor (2013)	29.4322	6.3777	16.8759	4.2283	5.9974	20.6440	5.3168	71.5521	1.7258	No			
Google Inc. (2012)	22.4605	0.5681	1.1646	37.3245	21.3493	1.3118	12.3654	9.0472	5.4525	No			
Google Inc. (2013)	138.4366	−2.6500	9.3241	6.1131	42.3860	0.2699	38.6730	0.3209	1.4426	S	E	E	E
Intel Corporation (2013)	−0.0144	0.7898	0.2206	6.8127	4.0358	3.3037	7.2107	7.9754	3.1437	S	E	E	E
Jabil Circuit, Inc. (2012)	−0.1608	1.9945	0.0000	0.5951	0.0000	1.6438	0.0000	0.7099	0.1335	S	E	L	E
Jabil Circuit, Inc. (2013)	−0.2349	2.3675	0.0000	0.7331	0.0000	1.6473	0.0000	1.0295	0.0000	S	L	L	L
JDS Uniphase Corp. (2012)	−1.2719	3.0752	0.0000	0.0000	0.0000	0.0000	10.5668	48.0976	1.6758	S	E	E	E
Juniper Networks, Inc. (2013)	−0.5365	4.8671	0.0353	0.2250	6.8151	5.5666	4.2026	0.1009	0.2666	S	E	E	E
KLA-Tencor Corporation (2013)	4.2112	0.5125	0.0000	0.0000	3.3435	0.0000	0.8421	0.0000	0.0245	No			
LSI Corporation (2012)	35.8088	−1.4953	0.9589	11.9479	37.6283	44.6531	58.2309	116.7256	2.7827	S	E	E	E
LSI Corporation (2013)	1.0402	0.8090	1.1038	5.2296	39.7990	59.3700	62.6907	102.9850	2.0307	No			
Lexmark International, Inc. (2013)	−0.1090	2.8626	0.0831	2.7775	0.0260	50.6814	16.8418	22.9877	1.3594	S	E	E	E
Microchip Technology (2012)	2.0726	0.4276	1.2925	0.0000	6.6525	0.0000	0.0000	0.0000	0.5838	No			
Microchip Technology (2013)	0.0187	2.5267	0.0000	0.0704	1.2290	0.0000	0.2619	0.0000	0.0000	No			

(continued)

Table 16.4 (continued)

Company Name	Dual variables									DC	R&D expense	Investment in CO_2	Overall
	Estimated CO_2 savings	Return on assets	Direct CO_2 emission	Indirect CO_2 emission	Employees	Working capital	R&D exp'ense	Total assets	Investment in CO_2 abatememt				
Marvell Technology Group, Ltd. (2012)	220.4684	0.5376	0.9823	1.7127	22.6917	38.5207	26.2614	0.4451	3.6961	No			
Marvell Technology Group, Ltd. (2013)	−0.5632	1.7263	0.0348	0.0307	6.2616	30.5193	35.3471	65.4028	3.2435	S	E	E	E
Microsoft Corporation (2012)	32.2560	−0.8047	1.0125	30.9364	3.9330	10.1730	25.0133	2.9205	5.7558	S	E	E	E
Microsoft Corporation (2013)	22.7528	−1.9337	1.1099	17.5582	3.4089	1.1596	17.6268	2.0939	5.5452	S	E	E	E
NetApp Inc. (2013)	5.2241	1.5755	0.0000	0.0000	0.6673	0.2636	1.1533	0.0000	0.0000	No			
NVIDIA Corporation (2012)	3.3077	0.4948	0.0000	0.0379	3.3217	0.1490	0.9000	0.0000	0.0172	No			
Oracle Corporation (2013)	0.2871	0.4434	0.0000	0.0037	0.0000	0.0000	0.1134	0.0000	0.0000	No			
SanDisk Corporation (2012)	0.3036	0.7188	0.0000	0.0000	5.8008	0.0000	1.1901	0.0000	0.0803	No			
Symantec Corporation (2012)	42.6041	0.1704	1.0364	9.8108	14.2522	152.4651	58.9106	4.7293	2.4060	No			
Symantec Corporation (2013)	252.7384	−1.2723	2.0519	6.8658	0.4074	32.3343	20.7193	0.3593	2.3438	S	E	E	E
Teradyne Inc. (2012)	2.5566	0.4170	0.0000	0.0000	2.7705	0.1278	0.7446	0.0000	0.0144	No			
Teradyne Inc. (2013)	4.5379	0.7369	0.0000	0.0000	4.9145	0.1255	1.2007	0.0000	0.0289	No			
Texas Instruments Incorporated (2012)	4.8749	−14.5837	10.1190	8.6390	8.3258	5.6786	5.4799	12.4549	2.3151	S	E	E	E

Texas Instruments Incorporated (2013)	0.8628	−2.6931	5.5824	23.2012	11.9651	4.1897	6.5988	29.1600	1.6439	S	E	E	E
Xerox Corporation (2013)	0.5681	1.8382	0.1675	0.0000	0.0000	0.0784	0.4199	0.0000	0.0854	No			
Yahoo! Inc. (2012)	107.9684	−1.1052	0.0006	2.2199	1.3699	4.1354	3.7229	0.0010	0.7937	S	E	E	E
Yahoo! Inc. (2013)	0.4495	0.3138	0.0000	0.1148	0.3024	0.0790	0.2464	0.0000	0.0000	No			

Table 16.5 Investment strategy on seven industrial sectors

Sector	# of effective investments	Percentage (%)	# of limited investments	Percentage (%)
Consumer Discretionary	6	46.15	0	0.00
Consumer Staples	1	9.09	0	0.00
Energy	6	46.15	1	7.69
Healthcare	4	14.81	0	0.00
Industrials	2	10.53	0	0.00
Information Technology	19	39.58	1	2.08
Materials	8	36.36	0	0.00
Overall	46	30.07	2	1.31

Source: Sueyoshi and Wang (2014a)

disposability. This type of application had never been explored in any previous DEA studies on environmental assessment. It is easily envisioned that the proposed DEA approach will provide corporate leaders with guidance on environmental strategy and investment on technology selection. Such selection, identified by examining firms with strong DC, is useful in establishing corporate sustainability.

To demonstrate the practicality of the proposed approach, this study applied it to 153 observations of all S&P 500 corporations in 2012 and 2013. The empirical investigation suggested that it is necessary for investors to pay more serious attention to company's green image, so enhancing sustainability, than profitability in a short term horizon. A contribution of this study was that corporate leaders and investors could evaluate and plan the development of their corporate sustainability by utilizing information generated by the proposed approach.

It is true that the proposed environmental assessment is not yet perfect. There are four research issues as future extensions of this study. First, the technology innovation needs a time lag until it can fully exert its effect. Thus, the proposed approach needs to incorporate a time horizon in the computational process. For the research purpose, it is necessary for us to combine the proposed approach with the time series measurement proposed by the research efforts (Sueyoshi and Goto 2014c, Sueyoshi and Wang 2014b). Second, it is also important to make a theoretical linkage between the proposed approach and investment behavior in portfolio analysis. Third, technology innovation and selection may depend upon the type of industry. Different industries need different technology structures. See Sueyoshi and Yuan (2015b). Hence, the technology selection needs to consider a combination among different technology structures. This study did not explore the important aspect on technology. Finally, this study assumes that the proposed DEA approach produces a unique solution. However, DEA often suffers from an occurrence of multiple solutions. It is important to incorporate SCSCs (Strong Complementary Slackness Conditions) into the proposed computational framework. Such research tasks will be important future extensions of this study.

In conclusion, it is hoped that this study makes a contribution in the development of corporate sustainability. We look forward to seeing research extension, as discussed in this study.

References

Boccard, N. (2014). The cost of nuclear electricity: France after Fukushima. *Energy Policy, 66*, 450–461.

Chitkara, P. (1999). A data envelopment analysis approach to evaluation of operational inefficiencies in power generating units: A case study of Indian power plants. *IEEE Transactions on Power Systems, 14*, 419–425.

Cooper, W. W., Huang, Z., Li, S., Lelas, V., & Sullivan, D. W. (1996). Survey of mathematical programming models in air pollution management. *European Journal of Operational Research, 96*, 1–35.

Debreu, G. (1951). The Coefficient of Resource Utilization, *Econometrica, 19*, 273–292.

Debreu, G. (1959). *Theory of Value*, Colonial Press, Inc, Massachusetts, USA.

Farrell, M.J. (1954). An application of activity analysis to the theory of the firm. *Econometrica, 22*, 291–302.

Farrell, M.J. (1957). The measurement of productive efficiency. *Journal of the Royal Statistical Society*, Series A, 120, Part *3*, 253–290.

Farrell, M.J., & Fieldhouse, M. (1962). Production functions under increasing returns to scale. Journal of the Royal Statistical Society, Series A (General) *125*, 252–267

Glover, F., & Sueyoshi, T. (2009). Contributions of Professor William W. Cooper in operations research and management science. *European Journal of Operational Research, 197*, 1–16.

Goto, M., Ootsuka, A., & Sueyoshi, T. (2014). DEA (data envelopment analysis) assessment of operational and environmental efficiencies on Japanese regional industries. *Energy, 66*, 535–549.

Ijiri, Y., & Sueyoshi, T. (2010). Accounting essays by Professor William W. Cooper: Revisiting in commemoration of his 95th birthday. *ABACUS: A Journal of Accounting, Finance and Business Studies, 46*, 464–505.

Korhonen, P. J., & Luptacik, M. (2004). Eco-efficiency analysis of power plants: An extension of data envelopment analysis. *European Journal of Operational Research, 2004*, 437–446.

Mou, D. (2014). Understanding China's electricity market reform from the perspective of the coal-fired power disparity. *Energy Policy, 74*, 224–234.

Porter, M. E., & van der Linde, C. (1995). Toward a new conception of the environment competitiveness relationship. *Journal of Economic Perspectives, 9*, 97–118.

Sarica, K., & Or, I. (2007). Efficiency assessment of Turkish power plants using data envelopment analysis. *Energy, 32*, 1484–1499.

Shrivastava, N., Sharma, S., & Chauhan, K. (2012). Efficiency assessment and benchmarking of thermal power plants in India. *Energy Policy, 40*, 159–176.

Sueyoshi, T., & Goto, M. (2011). DEA approach for unified efficiency measurement: Assessment of Japanese fossil fuel power generation. *Energy Economics, 33*, 195–208.

Sueyoshi, T., & Goto, M. (2012a). Data envelopment analysis for environmental assessment: Comparison between public and private ownership in petroleum industry. *European Journal of Operational Research, 216*, 668–678.

Sueyoshi, T., & Goto, M. (2012b). DEA radial measurement for environmental assessment and planning: Desirable procedures to evaluate fossil fuel power plants. *Energy Policy, 41*, 422–432.

Sueyoshi, T., & Goto, M. (2012c). Environmental assessment by DEA radial measurement: U.S. coal-fired power plants in ISO (independent system operator) and RTO (regional transmission organization). *Energy Economics, 34*, 663–676.

Sueyoshi, T., & Goto, M. (2013a). A comparative study among fossil fuel power plants in PJM and California ISO by DEA environmental assessment. *Energy Economics, 40*, 130–145.

Sueyoshi, T., & Goto, M. (2013b). DEA environmental assessment in a time horizon: Malmquist index on fuel mix, electricity and CO_2 of industrial nations. *Energy Economics, 40*, 370–382.

Sueyoshi, T., & Goto, M. (2014a). DEA radial measurement for environmental assessment: A comparative study between Japanese chemical and pharmaceutical firms. *Applied Energy, 115*, 502–513.

Sueyoshi, T., & Goto, M. (2014b). Investment strategy for sustainable society by development of regional economies and prevention of industrial pollutions in Japanese manufacturing sectors. *Energy Economics, 42*, 299–312.

Sueyoshi, T., & Goto, M. (2014c). Environmental assessment for corporate sustainability by resource utilization and technology innovation: DEA radial measurement on Japanese industrial sectors. *Energy Economics, 46*, 295–307.

Sueyoshi, T., & Wang, D. (2014a). Radial and non-radial approaches for environmental assessment by data envelopment analysis: Corporate sustainability and effective investment for technology innovation. *Energy Economics, 45*, 537–551.

Sueyoshi, T., & Wang, D. (2014b). Sustainability development for supply chain management in U.S. petroleum industry by DEA environmental assessment. *Energy Economics, 46*, 360–374.

Sueyoshi, T., & Yuan, Y. (2015a). China's regional sustainability and diversity: DEA environmental assessment on economic development and air pollution. *Energy Economics, 49*, 239–256.

Sueyoshi, T., & Yuan, Y. (2015b). Comparison among U.S. industrial sectors by DEA environmental assessment: Equipped with analytical capability to handle zero and negative values in production factors. *Energy Economics* 52(Part A), 69–86.

Wang, D., Li, S., & Sueyoshi, T. (2014). DEA environmental assessment on U.S. industrial sectors: Investment for improvement in operational and environmental performance for corporate sustainability. *Energy Economics, 45*, 254–267.

Zhang, N., Zhou, P., & Choi, Y. (2013). Energy efficiency, CO2 emission performance and technology gaps in fossil fuel electricity generation in Korea: A meta-frontier non-radial directional distance function analysis. *Energy Policy, 56*, 653–662.

Zhou, Y., Xing, X., Fang, K., Liang, D., & Xu, C. (2013). Environmental efficiency analysis of power industry in China based on an entropy SBM model. *Energy Policy, 57*, 68–75.

Chapter 17
DEA Environmental Assessment (II): A Literature Study

Toshiyuki Sueyoshi and Yan Yuan

Abstract This chapter systematically summarizes previous research efforts, including concepts and methodologies, on DEA environmental assessment applied to energy in the past three decades. Industrial developments are very important for all nations in terms of their economic prosperities. The problem is that the development produces various pollutions on air, water and others types of contaminations, all of which are usually associated with our health problems and climate changes. Thus, it is necessary for us to think how to make a balance between economic success and pollution mitigation in order to maintain a high level of social and corporate sustainability in the world. It is widely considered among researchers and practitioners that DEA is one of methodologies to examine the level of sustainability. The purpose of this chapter is to describe the importance of DEA in assessing unified (operational and environmental) performance of various entities in public and private sectors by summarizing previous research efforts on environmental assessment. The literature survey in this chapter covers 407 articles on DEA applications in energy and environment. It is true that DEA is not a perfect methodology, rather being an approximation methodology for performance assessment. The methodology has strengths and drawbacks in applications. Therefore, it is very important for us to carefully use DEA for guiding large policy and business strategies regarding the global warming and climate change. An underlying premise of this study is that technology innovation in engineering may solve the pollution and climate problem by linking it with economic and business concerns. The DEA provides such a linkage between engineering and social science, so enhancing the practicality in mitigating environmental pollutions. It is envisioned that the literature study, along with a summary on conceptual and methodological developments discussed in Chap. 16, provides researchers and individuals who are interested in social and corporate sustainability with analytical and methodological guidelines for their future research works on DEA environmental assessment.

T. Sueyoshi (✉) • Y. Yuan
Department of Management, New Mexico Institute of Mining and Technology,
801 Leroy Place, Socorro, NM 87801, USA
e-mail: toshi@nmt.edu; yyuan@nmt.edu

S.-N. Hwang et al. (eds.), *Handbook of Operations Analytics Using Data Envelopment Analysis*, International Series in Operations Research & Management Science 239, DOI 10.1007/978-1-4899-7705-2_17

Keywords Energy • Sustainability • Disposability • Environment

17.1 Introduction

As discussed in Chap. 16, it is necessary for us to reduce an amount of Greenhouse Gases (GHG) emissions, in particular CO_2, by 40–70 % (compared with 2010) until 2050 and to the level of almost zero by the end of the twenty-first century via shifting our current systems to energy efficient ones. Otherwise, we will have to bear life threatening consequences, such as strong hurricanes, heat waves, droughts, floods, food crisis as well as other damages to human health, social and economic systems. For example, a heat wave in 2003 caused about 50,000 deaths in Europe and another one in 1995 caused 600 deaths in Chicago. From Environmental Protection Agency (EPA) report, heat waves cause more deaths in the United States every year than hurricanes, tornadoes, floods, and earthquakes combined. Not necessary to mention that air pollution can make asthma and other lung conditions worse and climate change might allow some infectious diseases to spread. Our challenge on the climate change makes conventional profit-driven business inappropriate on logic and practice and incompatible with a world-wide trend toward a sustainable society.

To overcome the practical difficulty, many researchers in business and economics have proposed a use of DEA environmental assessment, as discussed in Chap. 16. Such importance of DEA[1] has been well known in assessing unified (operational and environmental) performance of many different entities in public and private sectors.[2] In order to achieve social and corporate sustainability, especially on environmental protection, DEA is a useful approach to overcome the difficulty. The purpose of this chapter is to summarize previous works on DEA environmental assessment, covering 407 articles, by paying attention to conceptual and methodological developments. It is important to note that this study covers only energy-related articles because of a page limit of this chapter. We clearly understand that there are many articles on DEA applications for sustainability developments.

Here, it is necessary to mention that DEA is not a perfect methodology because it is an approximation methodology for performance assessment. It has methodological strengths and drawbacks in applications. Therefore, it is very important for us to carefully use DEA in guiding large policy and business issues such as the global warming and climate change in the world. This chapter considers that DEA is just one of useful mathematical approaches for assessing the status of social and

---

[1] Professor William W. Cooper developed DEA with Professor A, Charnes. His historical contributions are summarized in the two articles. See Glover, F., Sueyoshi, T., 2009. Contributions of Professor William W. Cooper in operations research and management science. European Journal of Operational Research 197, 1–16 and Ijiri, Y., Sueyoshi, T., 2010. Accounting essays by Professor William W. Cooper: revisiting in commemoration of his 95th birthday. ABACUS: A Journal of Accounting, Finance and Business Studies 46, 464–505.

[2] See the third and fourth footnotes of Chap. 16, which discusses why this study uses “operational efficiency”, not a conventional term “technical efficiency”.

corporate sustainability (i.e., economic success and environmental protection). An underlying concept of this study is that eco-technology innovation in engineering may solve various pollution problems, but such efforts need to be linked to economic and business concerns. Otherwise, the engineering capability does not produce expected results on energy efficiency and sustainability. The rule of DEA is to make a methodological linkage between engineering and social sciences, so enhancing a level of sustainability by mitigating various environmental pollutions. It is hoped that this literature survey will provide many researchers and practitioners with a research guideline for their future works concerning environmental protection and sustainability.

The remaining structure of this study is structured as follows: Sect. 17.2 discusses a rationale regrading why we need DEA for environmental assessment. Section 17.3 describes conceptual developments on weak and strong disposability as well as natural and managerial disposability. The section also discusses these disposability concepts from their analytical capabilities. Section 17.4 summarizes previous research efforts on electric power industry by using DEA environmental assessment. Section 17.5 summarizes previous research efforts on petroleum and coal industries. Section 17.6 discusses DEA applications on pollution prevention efforts in agriculture, fishery, manufacturing, transportation and other industries, respectively. Section 17.7 summarizes the previous research work on economic development and corporate strategy. Section 17.8 discusses the methodology development of DEA environmental assessment. Section 17.9 concludes this chapter along with future research directions.

At the end of this chapter, it is important to note that a new book prepared by Sueyoshi and Goto (2017). Environmental Assessment on Energy and Sustainability by Data Envelopment Analysis will be published by John Wiley & Sons, London, UK. The new book contains 28 chapters on DEA environmental assessment and 693 previous research efforts in the area. Their literature classifications are different from the ones in this chapter. Readers, who are interested in DEA environment assessment, may refer to the new book with updated information on DEA environmental assessment.

17.2 DEA Environmental Assessment

Following their description prepared by Sueyoshi and Goto (2015b), Fig. 17.1 visually describes the importance of DEA as a social intelligence process for both reducing the amount of various environmental pollutions and enhancing the status of sustainability. Each organization in public and private sectors needs all of such intelligence capabilities to mitigate the amount of industrial pollution. A methodology selection (e.g., DEA) is determined as part of such an intelligence process, where “intelligence” implies a social capability of an organization (e.g., an energy company in a private sector or United Nations in a public sector) for holistic adjustment to various changes (e.g., a regulation change on industrial pollution),

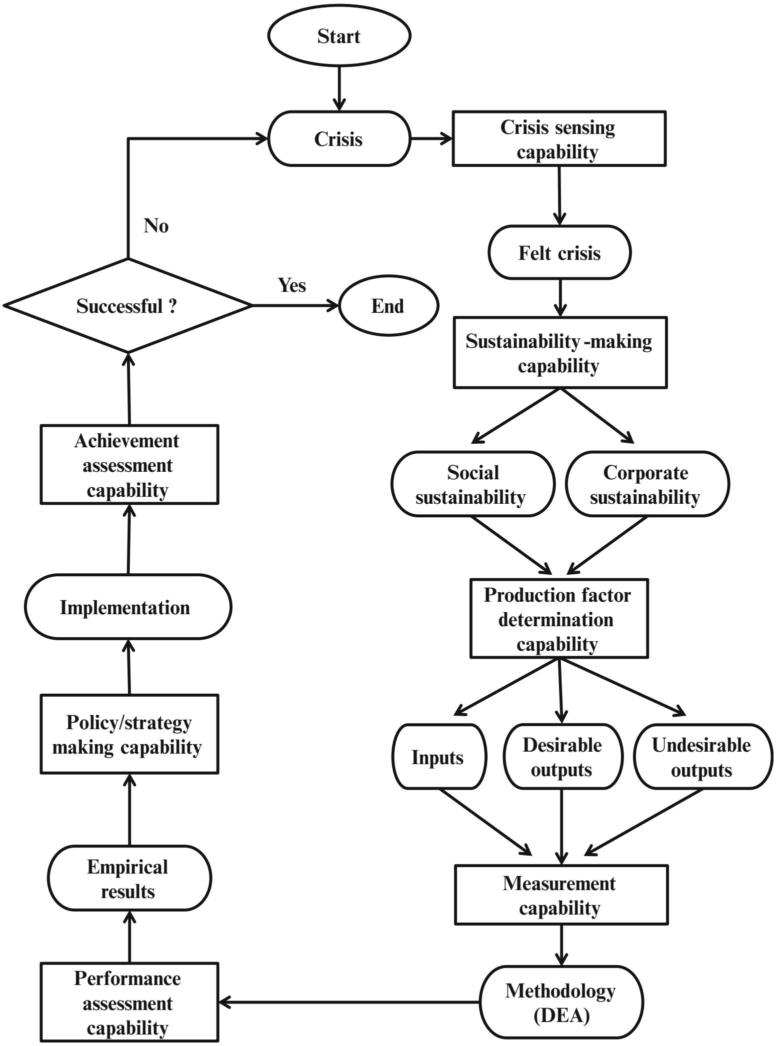


Fig. 17.1 DEA environmental assessment as social I intelligence system. *Source*: Sueyoshi and Goto (2015b)

so not directly linking to any conventional engineering implication (e.g. artificial intelligence in computer science). Of course, such a social capability depends upon the utilization of a data processing ability (i.e., on a modern personal computer) that can simultaneously deal with many big data sets related to environmental protection.

As an initial step, an organization (e.g., a petroleum firm in this study) needs to identify that it faces a serious crisis due to industrial pollution (e.g., air and/or water pollution). As a result of pollution, consumers do not purchase products from a dirty-imaged company. They purchase products from a green company even if the prices are much higher than those of the dirty-imaged company. More seriously, a very large opportunity cost often occurs along with industrial pollutions that make a serious damage on each organization. For example, the disaster of Fukushima Daiichi's nuclear power plant, occurred on March 11, 2011, was such an example of the opportunity cost. This type of opportunity cost is much larger than any cost components in modern business. It is necessary for the organization to have a "sensing capability" by which it can identify the existence of a serious business crisis due to industrial pollutions. Such a sensing capability may be often found in an individual member from the top to the bottom or a group of members within the organization. A result of the sensing capability, incorporated in the organization, is referred to as "felt crisis" in this study.

The felt crisis is extended for serving as a necessity of "social sustainability" in a public sector or "corporate sustainability" in a private sector. The concept of "social sustainability", supported by many international organizations such as United Nations, is related to global social responsibility and social welfare by preventing the climate change and global warming that have been gradually changing our ecological and economic systems. The concept of social sustainability contains world-wide implications in a long-term horizon. In contrast, "corporate sustainability" has another type of implication, so being different from the social sustainability, is related to modern business perspectives. That is, the corporate sustainability is closely related to its survivability via a public image as a good corporate citizen. Moreover, the opportunity cost (e.g., damage due to industrial pollutions), which is much larger than any production-related cost items, may terminate its existence as a going concern. Thus, it is important for us to clearly distinguish between "social" and "corporate" sustainability concepts in discussing between world-wide policy issues (e.g., the global warming and climate change) and modem corporate behaviors for preventing industrial pollutions.

After identifying the organizational objective (social or corporate sustainability), each organization needs to determine what consists of production factors (i.e., inputs, desirable and undesirable outputs) for environmental assessment. This type of social intelligence is referred to as "production factor determination capability" in this study. The DEA, as a holistic approach, is used in this stage as one of many methodological alternatives for assessing the level of social or corporate sustainability. Some member(s) should have the analytical capability to apply DEA for environmental assessment. This type of social intelligence is referred to as "measurement capability".

After applying DEA for performance assessment, the organization can obtain empirical results, indicating the level of sustainability that is measured by relatively comparing its performance with others. This is referred to as "performance assessment capability". Based upon the empirical results, the organization needs to prepare policy/strategy for pollution prevention. The type of intelligence is referred to as "policy/strategy making capability". The organization implements the policy or strategy for pollution reduction. The type of intelligence is referred to as "achievement assessment capability". If the implementation is unsuccessful, the result brings the necessity of restarting the crisis feeling within the organization. A gradual improvement process is necessary in repeating the whole social intelligence process, as depicted in Fig. 17.1, until it can eliminate the crisis feeling due to environmental pollutions. Therefore, the pollution reduction effort needs to be explored in a time horizon, as discussed in this study. Finally, note that if one part of such intelligence capacities does not exist in the organization, then it will be unable to survive in our modern society and business. Such is the destination of dirty-imaged companies.

17.3 Disposability Concepts

Färe et al. (1989) did the ground-breaking research work that separated the concept of disposability into weak and strong categories. Unfortunately, no research clearly mentioned a rationale regarding why they used the two disposability concepts. Therefore, it is important for us to describe them more clearly in this section (II).

An underlying premise for separating between the two disposability concepts is an occurrence of congestion, defined by a conventional economic context. To describe the concept, this study follows Färe et al. (1989, pp. 91–92). Considering both undesirable and desirable output vectors, their study (1989, p. 92) reorganized an output vector into desirable and undesirable output vectors (G, B). Then, the weak disposability was defined by the following vector notation on the two output vectors:

$$P^w(X) = \left\{ (G,B) : G \leq \sum_{j=1}^{n} G_j\lambda_j,\ B = \sum_{j=1}^{n} B_j\lambda_j,\ X \geq \sum_{j=1}^{n} X_j\lambda_j, \sum_{j=1}^{n} \lambda_j = 1 \,\&\, \lambda_j \geq 0 \quad (j = 1,..,n) \right\},$$

where the subscript (j) stands for the jth DMU and λ_j indicates the jth structural or intensity variable ($j = 1, \ldots, n$). The production technology is listed by P and the superscript (w) indicates the weak disposability.

Meanwhile, the strong disposability is defined by the following vector notation on the two output vectors:

$$P^s(X) = \left\{ (G,B) : G \leq \sum_{j=1}^{n} G_j\lambda_j,\ B \leq \sum_{j=1}^{n} B_j\lambda_j,\ X \geq \sum_{j=1}^{n} X_j\lambda_j, \sum_{j=1}^{n} \lambda_j = 1 \,\&\, \lambda_j \geq 0 \quad (j = 1,..,n) \right\},$$

where the inequality constraints $\left(B \leq \sum_{j=1}^{n} B_j\lambda_j \right)$ make it possible to change from the weak disposability to the strong disposability. The superscript (s) indicates the strong disposability. Here, $G \leq \sum_{j=1}^{n} G_j\lambda_j$ and $B \leq \sum_{j=1}^{n} B_j\lambda_j$ are structured by the assumption that “undesirable outputs are by-products of desirable outputs”.

Thus, the two types of outputs have $\leq$ in their vector notations. As easily imagined, $B \geq \sum_{j=1}^{n} B_j\lambda_j$ should be used to express the undesirable output vector. However, Färe et al. (1989) and many other studies assume that undesirable outputs are by-products of desirable outputs. As a result, $B \leq \sum_{j=1}^{n} B_j\lambda_j$ as found in the strong disposability. See Stage III of Fig. 16.1 that visually describes the shapes of efficiency frontiers on a production and pollution possibility set.

Table 17.1 summarizes part of previous conceptual developments on DEA environmental assessment. The first part included the studies with the weak and strong disposability concepts, including a straight-forward use of conventional DEA (so, only strong disposability), which has followed a pioneering work of Färe et al. (1989). The second part included the new concepts with natural and managerial disposability concepts discussed in Chap. 16. As summarized in Table 17.1, the concept of weak and strong disposability has served a conceptual basis for DEA environmental assessment.

To describe analytical implications of the weak and strong disposability concepts, this study needs to first introduce the concept of "congestion", which is classified into Undesirable Congestion (UC) under weak and strong disposability concepts, and Desirable Congestion (DC) under natural and managerial disposability concepts discussed in Chap. 16. Figure 17.2 visually describes a possible occurrence of UC or DC.

First, the left hand side of Fig. 17.2 exhibits an undesirable output (b) on the horizontal axis and a desirable output (g) on the vertical axis. For our visual convenience, this chapter considers the case of a single component of production factors. The convex curve indicates the relationship between the two production factors (g and b). In the figure, a supporting hyperplane visually specifies the shape of a production curve that indicates an occurrence of UC. For example, a negative slope of the supporting hyperplane indicates an occurrence of UC. In contrast, a positive slope implies an opposite case (i.e., no occurrence of UC). Inequality constrains on B, as specified by the strong disposability, exclude the occurrence of UC. In contrast, equality constraints on B, as specified by the weak disposability, allow the occurrence of UC.

An occurrence of inefficiency, discussed in DEA applications, is identified in such a manner that a reduction in an input(s) results in an increase in a maximum possible desirable output(s) without worsening other inputs and desirable outputs. Conversely, an occurrence of UC implies that an increase in an input(s) leads to a decrease in a desirable output(s) without worsening other production factors. This type of inefficiency is different from the concept of "operational inefficiency", that indicates an existence of an excess amount of input(s) and/or an existence of a shortfall of a desirable output(s). A typical example can be found in a line limit of transmission in the electric power industry.

Table 17.1 Classification by disposability concepts

Concept	Related papers
Weak/strong disposability	Azadeh et al. (2009a, b), Azadi et al. (2015), Bi et al. (2014), Bian and Yang (2010), Bretholt and Pan (2013), Burnett and Hansen (2008), Çelen (2013), Chang and Yang (2011), Chang et al. (2013a, b), Chen et al. (2015a, b), Chiu et al. (2011), Dai and Kuosmanen (2014), Fallahi et al. (2011), Fang et al. (2009, 2013), Feng et al. (2015), Giannakis et al. (2005), Gómez-Calvet et al. (2014), Guo et al. (2011), Hernández-Sancho et al. (2000, 2001), Honma and Hu (2008, 2009, 2014a, b), Hu and Lee (2008), Huang et al. (2014), Iglesias et al. (2010), Jamasb et al. (2004), Jaraite and Di Maria (2012), Kashani (2005a, b), Khoshnevisan et al. (2013a, b), Khoshroo et al. (2013), Kumar Mandal (2010), Kumar (2006), Li et al. (2013), Liu et al. (2010), Liu (2015), Madlener et al. (2009), Mandal (2010), Menegaki (2013), Meng et al. (2013), Molinos-Senante et al. (2014), Mousavi-Avval et al. (2012), Oggioni et al. (2011), Olatubi and Dismukes (2000), Oude Lansink and Bezlepkin (2003), Pacudan and de Guzman (2002), Pan et al. (2013), Pasurka (2006), Picazo-Tadeo et al. (2005), Rao et al. (2012), Riccardi et al. (2012), Sala-Garrido et al. (2012), Sarkis and Cordeiro (2012), Scheel (2001), Shi et al. (2010), Song and Wang (2014), Sözen et al. (2012), Taghavifar et al. (2014), Tao and Zhang (2013), Tone and Tsutsui (2007), Tsolas (2011), Vaninsky (2006), Vasco Correa (2012), Wang and Wei (2014), Wang et al. (2007, 2012a, b, c, 2013a, b, c, d, e, f, 2014a, b, 2015), Wei et al. (2011), Wu et al. (2012, 2013a, b, c, d), Yang and Pollitt (2009, 2010), Yeh et al. (2010), Zaim and Taskin (2000a, b), Zhang and Choi (2013a, 2014), Zhang et al. (2011, 2013a, b), Zhou and Ang (2008a, b), Zhou et al. (2008a, b, 2010, 2012a, b, 2013, 2014a, b)
Natural/managerial disposability and RTS/DTS/DTR	Goto et al. (2014), Sueyoshi and Goto (2011a, b, 2012a, c, g, h, i, j, k, 2013d, 2014a, 2015a, b, c), Sueyoshi et al. (2010, 2013a), Sueyoshi and Yuan (2015a, b), Goto et al. (2014), Wang et al. (2014b)

(a) See Chap. 16 of this study that describes the concept of natural and managerial disposability. (b) The weak disposability can be used to identify an occurrence of UC. The natural disposability with equality on B (undesirable outputs) is also used to identify the occurrence of DC. Meanwhile, the managerial disposability with equality on G (desirable outputs) is used to identify an occurrence of DC, or eco-technology innovation. See Fig. 17.2 on a visual difference between UC and DC

Acknowledging the importance of previous contributions summarized in Table 17.2, this chapter needs to mention the other group of studies that shifts the conceptual framework from weak and strong disposability to natural and managerial disposability. As discussed in Chap. 16, the concept of "natural disposability," indicates that a firm decreases a vector of inputs to decrease a vector of undesirable outputs. Given a reduced vector of inputs, the firm increases a vector of desirable outputs as much as possible. For example, let us consider a coal-fired power plant

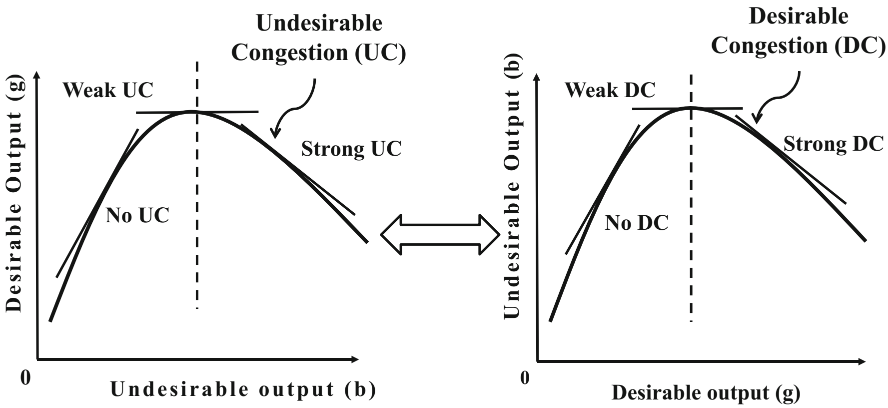


Fig. 17.2 Undesirable and desirable congestion concepts. (a) The left hand of the figure indicates a possible occurrence of Undesirable Congestion (UC) under natural disposability. The occurrence of UC is classified into three categories: no, weak and strong. This type of congestion has been long discussed in the DEA community. In DEA environmental assessment, the strong UC indicates that an enhanced component(s) of an input vector increases some component(s) of an undesirable output vector, but decreases some component(s) of a desirable output vector. This type of congestion indicates a capacity limit on generation and/or a line limit on transmission in the electric power industry. No UC implies no congestion on the whole grid system. The weak UC is between weak and strong UC. For identifying a possible occurrence of strong UC, the equality constraints (so, no slack) are assigned to undesirable outputs in DEA environmental assessment. The weak disposability can identify an occurrence of UC, as well. Thus, this occurrence of UC is measured under natural disposability with no slack (i.e., equality) on undesirable outputs. (b) The right hand side of figure indicates a possible occurrence of Desirable Congestion (DC), or eco-technology innovation for pollution prevention under managerial disposability. The DC is classified into three categories: no, weak and strong. In DEA environmental assessment, the occurrence of DC indicates that an enhanced component(s) of the input vector increases some component(s) of a desirable output vector, but decreases some component(s) of an undesirable output vector. This chapter hopes that the eco-technology innovation can solve various pollution issues. For identifying a possible occurrence of DC, the equality constraints (so, no slack) are assigned to desirable outputs in the proposed formulations under managerial disposability. (c) The convex function at the right hand depends upon the assumption that undesirable outputs are "by-products" of desirable outputs. It should be a concave function without such an assumption. The assumption is acceptable when we consider the relationship between desirable and undesirable outputs. That is, an amount of undesirable outputs may proportionally increase along with an amount of desirable outputs without eco-technology. However, the eco-technology may change the situation so that it should be shaped by a convex function with a declining trend. (d) *Source*: The figure at the right hand side is obtained from Fig. 16.2

where CO_2 emission is produced by coal combustion. The coal is used as an input for the operation of a coal-fired power plant. Consequently, if the coal-fired power plant reduces the amount of coal combustion, the reduction immediately decreases the amount of CO_2 emission at the level that it can satisfy the amount determined by governmental regulation. Given the amount of coal combustion, the coal-fired power plant maximizes the amount of electricity generation. That is the natural disposability. In this case, the coal-fired power plant may attain the reduction of

Table 17.2 Electric power industry

Fuel	Related papers
Coal	Boyd and Pang (2000), Byrnes et al. (1988), Chitkara (1999), Fallahi et al. (2011), Fang et al. (2009), Färe et al. (1983, 1985, 1986, 1990, 1996, Färe et al. 2005), Fleishman et al. (2009), Galindo et al. (2012), Golany et al. (1994), Goto and Tsutsui (1998), Hampf and Rødseth (2015), Kulshreshtha and Parikh (2002), Mou (2014), Nag (2006), Olatubi and Dismukes (2000), Park and Lesourd (2000), Pasurka (2006), Sarkis and Cordeiro (2012), Sheng et al. (2015), Song et al. (2015a, b), Sueyoshi and Goto (2012f, 2013a), Tao and Zhang (2013), Thompson et al. (1995), Tyteca (1997, 1998), Wei et al. (2015), Yang and Pollitt (2009, 2010), Zhang et al. (2013a, b), Zhang and Choi (2013a), Zhou et al. (2012a, b)
Oil/gas	Azadeh et al. (2013), Barros and Assaf (2009), Barros and Managi (2009), Chitkara (1999), Erbetta and Rappuoli (2008), Ertürk and Türüt-Aşık (2011), Fleishman et al. (2009), Ghiasi and Mohammadi (2014), Hawdon (2003), Kashani (2005a, b), Lee et al. (2010, 2013), Li et al. (2013a, b, c, d), Price and Weyman-Jones (1996), Sheng et al. (2015), Song et al. (2015a, b), Thompson et al. (1992, 1996), Zhang et al. (2013a, b)
Renewable	Apergis et al. (2015), Barros (2008), Barros and Peypoch (2008), Bi et al. (2014), Blokhuis et al. (2012), Boubaker (2012a, b), Criswell and Thompson (1996), Duan et al. (2016), Grigoroudis et al. (2014), Jebaraj and Iniyan (2006), Kim et al. (2015), Lam and Shiu (2001), Lins et al. (2012), Liu et al. (2010, 2015), Løken (2007), Mallikarjun and Lewis (2014), Menegaki (2013), Ramanathan (2001), Sarica and Or (2007), Sözen et al. (2010, 2012), Stallard et al. (2008), Sueyoshi and Goto (2014d), Tone and Tsutsui (2007), Woo et al. (2015), Yunos and Hawdon (1997); Zaneslla et al. (2015)
Nuclear	Pollitt (1996), Vaninsky (2006)
Others	Abbott (2005, 2006), Agrell and Bogetoft (2005), Arabi et al. (2014), Arocena (2008), Athanassopoulos et al. (1999), Azadeh et al. (2009a, b), Azadeh and Mousavi Ahranjani (2014), Bagdadioglu et al. (1996), Burnett and Hansen (2008), Cambini et al. (2014), Çelen (2013), Çelen and Yalçın (2012), Chang and Yang (2011), Cherchye et al. (2015), Claggett and Ferrier (1998), Cook and Green (2005), Dai and Kuosmanen (2014), Giannakis et al. (2005), Goto and Sueyoshi (2009a, b), Gouveia et al. (2015), Han et al. (2015), Hattori et al. (2005), Huang et al. (1995), Jamasb et al. (2004), Jamasb and Pollitt (2000, 2003), Jaraite and Di Maria (2012), Korhonen and Syrjänen (2003), Korhonen and Luptacik (2004), Kuosmanen et al. (2013), Leme et al. (2014), Lo et al. (2001), Ma and Zhao (2015), Miliotis (1992), Omrani et al. (2015), Pacudan and de Guzman (2002), Pahwa et al. (2003), Pombo and Taborda (2006), Resende (2002), Sadjadi and Omrani (2008), Sueyoshi (1999), Sueyoshi and Goto (2001, 2011a, 2014d), Thakur et al. (2006), Vaninsky (2006), Vazhayil and Balasubramanian (2013), Wang et al. (2007), Weyman-Jones (1991), Yadav et al. (2010, 2011, 2013), Yaisawarng and Klein (1994), Yeh et al. (2010), Zhang and Bartels (1998)

CO_2 emission with a limited corporate effort at the level that is required by governmental regulation.

The other concept, referred to as "managerial disposability", indicates an opposite case of the natural disposability. In the disposability concept, a firm increases a vector of inputs to decrease a vector of undesirable outputs by utilizing technology innovation on undesirable outputs. Given the increased input vector, the firm

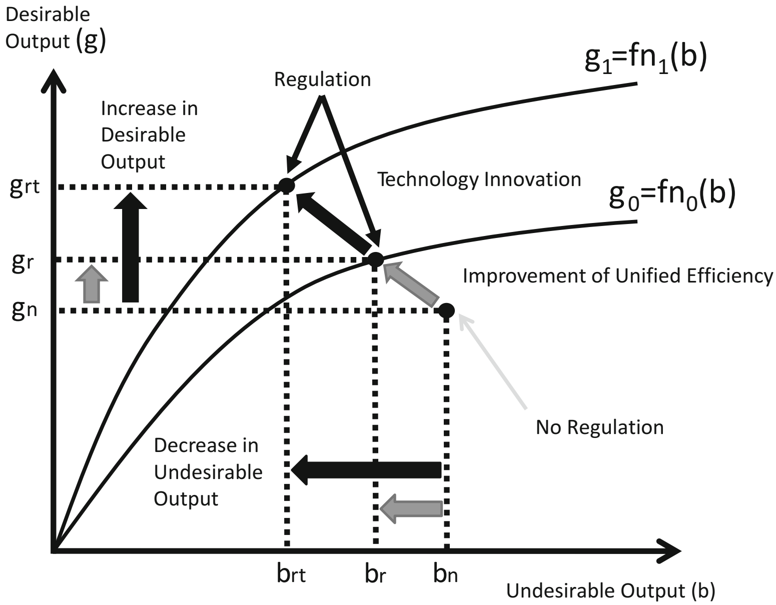


Fig. 17.3 Natural and managerial disposability by technology innovation. (a) *Source*: Sueyoshi et al. (2013b). This study obtains the figure from their research effort. (b) The amount of an input is not listed in the figure for our descriptive convenience

increases a vector of desirable outputs as much as possible. For example, a coal-fired power plant increases the amount of coal combustion so that it can increase the amount of electricity generation. Here, even if the power plant increases the amount of coal combustion, the increase can reduce the amount of CO_2 emission by a managerial effort by using high quality coal with less CO_2 emission and/or an engineering effort to utilize new generation technology (e.g., clean coal technology). Management of the power plant considers a change on environmental regulation as a business opportunity. The challenge of the power plant is a corporate effort for environmental protection, so implying the status of managerial disposability. See Chap. 16 on the axiomatic description on both natural and managerial disposability concepts.

Figure 17.3 visually describes strategies related to the two disposability concepts from production economics and regulation. The conventional strategy to adapt a regulation change can be considered in such a manner that a DMU decreases the amount of an input to decrease an undesirable output so that it can satisfy the regulation change. The amount of an undesirable output is shifted from b_n to b_r. However, it can simultaneously increase the amount of the desirable output from g_n to g_r by its corporate effort so that the DMU can attain an efficiency frontier, or a convex curve shaped by $g_0 = fn_0(b)$ in Fig. 17.3. The adaptive strategy indicates concept of natural disposability.

The second strategy for the adaptation is considered to utilize technology innovation, usually along with a managerial change, that shifts the functional form from g_0 to $g_1 = fn_1(b)$. The operation of the DMU is improved by its engineering capability. In Fig. 17.3, the adaptive strategy makes it possible for the DMU to increase an amount of the input to attain a more desirable combination (g_{rt}, b_{rt}) by both increasing the desirable output and decreasing the undesirable output from the previous combination (g_r, b_r). The second adaptive strategy is the concept of managerial disposability, indicating an opposite case of the natural disposability.

It is important to note that although this study does not address the production cost minimization in an explicit manner, eco-technological innovation and environmental protection are associated with long-term cost saving through increased competitiveness and successful adaptation to a regulation change.

Finally, a possible occurrence of UC and DC can be incorporated into the natural and managerial disposability concepts after slightly modifying output vectors and an input vector, respectively, as follows:

$$P_{UC}^{N}(X) = \left\{ (G,B) : G \leq \sum_{j=1}^{n} G_j\lambda_j,\ B = \sum_{j=1}^{n} B_j\lambda_j,\ X \geq \sum_{j=1}^{n} X_j\lambda_j,\ \sum_{j=1}^{n} \lambda_j = 1\ \&\ \lambda_j \geq 0,\ j = 1, .., n \right\} \text{ and}$$

$$P_{DC}^{M}(X) = \left\{ (G,B) : G = \sum_{j=1}^{n} G_j\lambda_j,\ B \geq \sum_{j=1}^{n} B_j\lambda_j,\ X \leq \sum_{j=1}^{n} X_j\lambda_j,\ \sum_{j=1}^{n} \lambda_j = 1\ \&\ \lambda_j \geq 0, j = 1, .., n \right\}.$$

The first equation incorporates a possible occurrence of UC under natural disposability. The equation is the same as the weak disposability. The other equation incorporates a possible occurrence of DC under managerial disposability. The difference between the two equations can be found on the allocation of the equality sign on desirable or undesirable outputs, as depicted in Fig. 17.1. See Sueyoshi and Yuan (2015a, b) for a detailed description on UC and DC.

17.4 Electric Power Industry

Table 17.2 classifies part of previous research efforts on DEA applications to the electric power industry. The classification is based upon coal, oil and gas, renewable, nuclear generations and other types of business function. The business function of the electric power industry is separated into generation, transmission, distribution and retailing. The previous research efforts focus on discussing generation efficiency because DEA is used to measure the efficiency and productivity performance of their supply capabilities in electric power companies. Therefore, Table 17.2 summarizes the previous studies based upon the type of fuels or a fuel mix used in their generation capabilities. Many generation facilities use different fuel mixes (e.g., coal and gas) so that the classification is not a strength-forward matter. Hence, this study classifies them based upon main fuel used for a generation facility. The category of others includes the research about electricity distribution (i.e. distribution system, network regulation, maintenance and outage repair), the impact of GHG control, the impact of government regulation, the comparison of private versus government ownership, the consequent impact on efficiency levels and so on.

17.5 Petroleum and Coal Industries

Table 17.3 summarizes part of previous studies on petroleum (i.e., oil and gas) and coal industries. They are functionally classified into upstream and downstream. The business components of upstream in petroleum industry include technology and signal processing, geo-tech analysis, exploration/evaluation delineation, technical and market evaluation, facility development and production. After production, the function could be gas processing, crude upgrading and gas-to-liquids, or transport marine and pipeline. From upstream, gas and oil could be produced. Therefore, in downstream, gas could be directly sold or used to generate power, which in turn will be sold in market. On the other hand, after oil is refined, it could be made to primary chemicals, special chemicals or upgraded environmental fuels. After distribution, they could be sold on markets. Hence, the function of downstream includes further production, distribution, marketing and sales.

As summarized in Table 17.3, the previous works discussed the performance of upstream in many petroleum and coal firms because DEA was used to measure their supply side capabilities. An interesting concern on the industry was whether a supply chain capability could enhance the performance of firms because the industries were considered as a huge network system from production places to end users. Another research concern was interested in whether firms (e.g., oil firms in the United States) under private ownership outperformed ones under public ownership because many oil producers (e.g., OPEC nations) were partly owned by governments where OPEC stands for Organization of the Petroleum Exporting Counties. Thus, the supply chain and the ownership issue are important business concerns in the petroleum and coal industries.

Table 17.3 Petroleum and coal industries

Function	Related papers
Upstream	Barros and Assaf (2009), Kashani (2005a, b), Lee et al. (2010, 2013), Li et al. (2013a, b, c, d), Shakouri et al. (2014), Song et al. (2015a, b), Sueyoshi and Goto (2012a, 2015b), Sueyoshi and Wang (2014b), Thompson et al. (1996)
Downstream	Al-Salem (2015), Bevilacqua and Braglia (2002), Erbetta and Rappuoli (2008), Ertürk and Türüt-Aşık (2011), Hawdon (2003), Price and Weyman-Jones (1996), Sueyoshi (2000)

17.6 Agriculture, Fishery, Manufacturing and Transportation Industries

Table 17.4 summarizes part of previous studies on DEA applications in agriculture, fishery, manufacturing, transportation industries. In the manufacturing sector, it includes many industries such as cement industry, iron and steel industry and chemical industry and so on. Others include some related research work such as water usage and wastewater decontamination, service sectors. The previous works were concerned with how energy was efficiently used to enhance the performance

Table 17.4 Applications in agriculture, fishery, manufacturing and transportation

Industry type	Related papers
Agriculture and fishery	Banaeian et al. (2011), Banaeian and Zangeneh (2011), Blancard and Martin (2014), Bozoğlu and Ceyhan (2009), Ebrahimi and Salehi (2015), Ederer (2015), Houshyar et al. (2012), Iglesias et al. (2010), Iribarren et al. (2013, 2014), Khoshnevisan et al. (2013a, b, c), Khoshroo et al. (2013), Madlener et al. (2009), Mobtaker et al. (2012), Mohammadi et al. (2013), Mousavi-Avval et al. (2011a, b, 2012), Nabavi-Pelesaraei et al. (2014), Nassiri and Singh (2007), Nouri et al. (2013), Odeck (2009), Omid et al. (2011), Oude Lansink and Bezlepkin (2003), Oude Lansink and Silva (2003), Pahlavan et al. (2011), Reinhard et al. (2000), Ren et al. (2014), Serra et al. (2014), Serra and Poli (2015), Skevas et al. (2014), Vlontzos et al. (2014), Wu et al. (2013a, b, c, d)
Manufacture	Aparicio et al. (2015), Azadeh et al. (2007, 2008), Azadi et al. (2015), Boyd and McClelland (1999), Boyd et al. (2002), Chauhan et al. (2006), Färe et al. (2001), Ghulam and Jaffry (2015), Han et al. (2015), Hernández-Sancho et al. (2000), Lee and Zhang (2012), Li and Lin (2015b), Long et al. (2015), Lozano et al. (2009), Mohan et al. (2009), Mukherjee (2008), Munksgaard et al. (2005), Oh and Shin (2015), Picazo-Tadeo et al. (2005), Ramli and Munisamy (2015), Triantis and Otis (2004), Voltes-Dorta et al. (2013), Zaim (2004), Zofío and Prieto (2001)
Transportation	Azadi et al. (2014), Chang et al. (2013a, b), Chiu et al. (2011), Cui and Li (2014, 2015a, b), González et al. (2015), Ramanathan (2000, 2005b), Wei et al. (2012), Zhou et al. (2013, 2014a, b)
Others	Bian et al. (2014), Bian and Yang (2010), Blomberg et al. (2012), Brännlund et al. (1998), Chien et al. (2003), Chung (2011), Chung et al. (1997), Diaz-Balteiro et al. (2006), Dyckhoff and Allen (2001), Fang et al. (2013), Färe et al. (1989, 1994a, b), Ferrier and Hirschberg (1992), He et al. (2013), Hernández-Sancho et al. (2011, 2009), Herrala et al. (2012), Hu et al. (2013), Huang and Li (2013), Kumar Mandal and Madheswaran (2010), Lampe and Hilgers (2015), Lee et al. (2011a, b), Lee and Kung (2011), Liu et al. (2013), Lv et al. (2013), Mandal (2010), Molinos-Senante et al. (2014), Oggioni et al. (2011), Önüt and Soner (2006), Raczka (2001), Riccardi et al. (2012), Rogge and De Jaeger (2012), Sala-Garrido et al. (2012), Sanhueza et al. (2004), Sarkis and Weinrach (2001), Tsolas (2011), Tyteca (1996), Vasco Correa (2012), Wei et al. (2007), Wu et al. (2013a, b, c, d), Ylvinger (2003), Yu and Chan (2012), Zhang and Choi (2014), Zhou et al. (2006a)

of organizations in these industries under various governmental regulations. For example, vehicles (e.g., cars and tracks) for the transportation sector produce a large amount of Greenhouse Gas (GHG) emission. In the United States, federal and state governments (e.g., the State of California) assign strict regulations to control the amount of GHG emission. Thus, the gas efficiency became very important for the transportation industry. The business concern on the transportation industry is also applicable to the other industries listed in Table 17.4.

17.7 Economic Development and Corporate Strategy

Table 17.5 summarizes part of previous studies on DEA applications on economic development and corporate strategy. DEA was originally developed for the performance assessment of various entities in public sectors. For example, many studies were interested in the performance of regional sectors in their operational and/or environmental efficiencies. The previous DEA studies found that there were regional

Table 17.5 Economic development and corporate strategy

Development	Related papers
Economy	Arcelus and Arocena (2005), Bampatsou et al. (2013), Barla and Perelman (2005), Bian et al. (2013), Bretholt and Pan (2013), Callens and Tyteca (1999), Chang (2014, 2015), Chen et al. (2015a, b), Chien and Hu (2007), Chiu et al. (2012), Choi et al. (2012), Cui et al. (2014), Färe et al. (2004), Feng et al. (2015), Guo et al. (2011), Hang et al. (2015), He (2015), Honma and Hu (2008, 2009, 2014a, b), Hu and Lee (2008), Hu and Kao (2007), Hu et al. (2011), Hu and Wang (2006), Huang et al. (2014), Jin et al. (2014), Keirstead (2013), Kim and Kim (2012), Kounetas (2015), Kumar (2006), Lee et al. (2011a, b), Lei et al. (2013), Li et al. (2013a, b, c, d), Li and Hu (2012), Li and Lin (2015a), Liang et al. (2004), Lin and Du (2015a, b), Lin and Liu (2012), Liu (2015), Lozano and Gutiérrez (2008), Meng et al. (2013), Murillo-Zamorano (2005), Pan et al. (2013, 2015), Ramanathan (2005a), Rashidi and Farzipoor Saen (2015), Rao et al. (2012), Scheel (2001), Shi et al. (2010), Song et al. (2013a, b), Song and Wang (2014), Sueyoshi and Goto (2010b), Sueyoshi and Goto (2015a, b), Sueyoshi and Yuan (2015a), Taghavifar et al. (2014), Taskin and Zaim (2001), Wang (2013, 2015), Wang and Wei (2014), Wang et al. (2012a, b, c, 2013a, b, c, d, e, f, 2014a, b, 2015), Wang and Feng (2015), Wei et al. (2011), Wu et al. (2012, 2013a, b, c, d, 2015), Yang et al. (2013, 2015), Yang and Wang (2013), Yao et al. (2015), Zaim and Taskin (2000a, b), Zhang and Choi (2013b), Zhang et al. (2008, 2011, 2015), Zhao et al. (2014), Zhou and Ang (2008a, b), Zhou et al. (2006b, 2007a, b, 2008b, 2010, 2012a, b, 2014a, b), Zou et al. (2013)
Corporate strategy	Barla and Perelman (2005), de Castro Camioto et al. (2014), Chang et al. (2013a, b), Goto et al. (2014), Olanrewaju et al. (2012, 2013), Sözen and Alp (2009), Sueyoshi and Goto (2009a, b, 2010a, 2014b, c), Sueyoshi et al. (2009), Sueyoshi and Wang (2014a), Wang et al. (2014b), Zhou et al. (2007a, b, 2015)

differences, leading to social imbalance and often producing income imbalance among them. Such differences among regions usually produced social imbalance, which produces a serious chaos in their social systems. Thus, DEA was considered as a methodology to measure the social imbalance in the previous studies.

It is important to note that some recent studies started to measure corporate sustainability by using DEA as a methodology. Through industry comparison, the investment direction could be determined for the firms. Furthermore, the corporate performance could be evaluated at different levels so that the corporate planning in the future could be adjusted according to the department performance and potential development ability.

17.8 Methodology Developments

Table 17.6 summarizes part of previous methodical efforts on DEA environmental assessment. The environmental assessment in energy needed to consider a time lag until it can identify the effect of energy policy and business strategy. Therefore, it was necessary for them to discuss the performance assessment in a time horizon. Many previous studies incorporated the Malmquist index into DEA environmental assessment to examine an occurrence of a frontier shift among multiple periods. The window analysis, often used in previous DEA studies, was also applied to examine a time shift in efficiency scores. The time shift of efficiency frontiers among different periods was considered as a main concern of this previous research group.

At the end of this section, it is important to describe that DEA is generally classified into radial and non-radial approaches in discussing the measurement

Table 17.6 Methodology developments

Data	Related papers
Cross sectional/ pooled data	Sueyoshi and Goto (2011c), 2012b, d, e), Zhang and Ye (2015)
Time series	Abbott (2006), Banaeian and Zangeneh (2011), Barros et al. (2009b), Barros and Peypoch (2008), Boyd and Pang (2000), Boyd et al. (2002), Chung et al. (1997), Dai and Kuosmanen (2014), Fleishman et al. (2009), Giannakis et al. (2005), Goto and Tsutsui (1998), Hattori et al. (2005), Kashani (2005a, b), Li et al. (2013a, b, c, d), Lin and Liu (2012), Liu (2015), Murillo-Zamorano (2005), Olanrewaju et al. (2012), Price and Weyman-Jones (1996), Skevas et al. (2014), Sueyoshi and Goto (2013c, 2015b), Sueyoshi et al. (2013b), Tone and Tsutsui (2007), Vaninsky (2006), Wang et al. (2013a, b, c, d, e, f), Wang et al. (2007), Zhao et al. (2014), Yaisawarng and Klein (1994), Yunos and Hawdon (1997), Zaim and Taskin (2000a, b), Zhang et al. (2013a, b) Zhou et al. (2014a, b)
Others	Sueyoshi and Goto (2013b)

criterion on efficiency. In a similar manner, DEA environmental assessment is classified into the two categories. The radial approach is based upon the effort of Debreu[3] and Farrell[4] while the non-radial approach is based upon Pareto[5] and Koopmans.[6] Most of the previous works belong to the radial approach. The difference between the two groups of measurement is that the radial approach uses an efficiency score or an inefficiency score to determine the level of efficiency. Meanwhile, the non-radial approach uses slacks, not an efficiency or inefficiency score, to determine the level of efficiency. It is difficult for us to discuss which approach is better in terms of efficiency measurement. However, it is desirable for us to use both approaches so that they do not produce any major differences. This type of problem, referred to as "a methodological bias", should be carefully discussed in any empirical study.

---

[3] See the third footnote of Chap. 16.

[4] See the fourth footnote of Chap. 16.

[5] Vilfredo Federico Damaso Pareto (15 July 1848–19 August 1923) was an Italian engineer, sociologist, economist, political scientist, and philosopher. Pareto is best known for two concepts that are named after him. The first and most familiar is the concept of Pareto optimality. A Pareto-optimal allocation of resources is achieved when it is not possible to make anyone better off without making someone else worse off. The second is Pareto's law of income distribution. This law, which Pareto derived from British data on income, showed a linear relationship between each income level and the number of people who received more than that income. Pareto found similar results for Prussia, Saxony, Paris, and some Italian cities. Although Pareto thought that his law should be "provisionally accepted as universal," he realized that exceptions were possible. As it turns out, many have been found. See http://books.google.co.jp/books?id=Z6Oy4L6LSwC&pg=PA140&lpg=PA140&dq=debreu+farrell&source=bl&ots=aLkVeuwk9u&sig=SYkaHtL56JXvZjUW0jJHg33cw0o&hl=ja&sa=X&ei=QZ03VPP1CtXc8AWAyoCQDA&ved=0CEoQ6AEwBg#v=onepage&q=debreu%20farrell&f=false.

[6] Tjalling Charles Koopmans (August 28, 1910–February 26, 1985) was a Dutch American mathematician and economist. He began his university education at the Utrecht University at seventeen, specializing in mathematics. Three years later, in 1930, he switched to theoretical physics. In 1933, he met Jan Tinbergen, the 1969 Bank of Sweden prize winner, and moved to Amsterdam to study mathematical economics under him. Koopmans shared the 1975 Nobel Prize with Leonid Kantorovich "for their contributions to the theory of optimum allocation of resources." In 1938, he succeeded Jan Tinbergen at the League of Nations in Geneva and then left in 1940 when Hitler invaded the Netherlands. In the United States, Koopmans became a statistician with the Combined Shipping Adjustment Board in Washington where he tried to solve the practical problem of how to reorganize shipping to minimize transportation costs. The problem was complex: the variables included thousands of merchant ships, millions of tons of cargo, and hundreds of ports. He solved it. The technique he developed to do so was called "activity analysis" and is now called linear programming. His first write-up of the analysis is in a 1942 memorandum. His techniques were very similar to those used by Kantorovich, whose work he discovered only much later. See http://books.google.co.jp/books?id=Z6Oy4L6LSwC&pg=PA140&lpg=PA140&dq=debreu+farrell&source=bl&ots=aLkVeuwk9u&sig=SYkaHtL56JXvZjUW0jJHg33cw0o&hl=ja&sa=X&ei=QZ03VPP1CtXc8AWAyoCQDA&ved=0CEoQ6AEwBg#v=onepage&q=debreu%20farrell&f=false.

17.9 Conclusion

This chapter summarized and classified 407 articles in past three decades on previous research works by using DEA environmental assessment. First of all, the conceptual development of disposability was discussed from the pioneering work of Färe et al. (1989). Since then, DEA environmental assessment was often applied to energy and social sustainability. It is indeed true that industrial developments are very important for all nations in terms of their economic prosperities. Then, Sections from 17.3 to 17.6 discussed the application to electric power industry, petroleum industry, Agriculture, fishery, manufacturing, transportation and others industries. However, a problem associated with the developments was that their developments usually produced various pollutions on air, water and others types of contaminations, all of which were usually associated with our health problems and serious social issues such as the global warming and climate change. It is necessary for us to consider how to make a balance between economic success and pollution mitigation in order to attain a high level of social and corporate sustainability. Therefore, Sect. 17.7 discussed about economic development and corporate strategy. Finally, the methodology developments of DEA environmental assessment in assessing unified (operational and environmental) performance of organization were discussed in Sect. 17.7, which could be useful to analyze the social and corporate sustainability in energy and environmental protection in public and private sectors.

The DEA discussed in this chapter is one of the most important methodologies to examine the level of such sustainability. However, it is true that the methodology has strengths and drawbacks in applications. It is very important for us to carefully use the DEA approach for guiding large energy policy and business issues in nations and corporations related to energy and industrial pollutions. Technology innovation in engineering can solve various energy and climate change issues by linking it with political, economic and business concerns. The DEA method can incorporate such technology innovation into environmental assessment so that it can provide us with such a methodological linkage between engineering and social sciences to enhance the level of economic success and to mitigate the amount of environmental pollution. Consequently, the enhanced economic success and environmental protection by utilizing DEA may improve the social sustainability in public sectors and corporate sustainability in private sectors.

At the end of this concluding section, this chapter needs to mention that many research efforts on DEA environmental assessment belong to multiple categories. This study selects one of such multiple categories. Unfortunately, this chapter does not cover all the research efforts. It is true that there are many other works, not listed in the references of this chapter. That is clearly a drawback of this study.

In conclusion, it is hoped that this literature study on DEA environmental assessment, along with a discussion on conceptual and methodological developments in Chap. 16, may provide researchers and individuals who are interested in energy and sustainability with analytical and methodological guidelines for their future research works in the area of energy, business and economics. We look forward to seeing future research extensions as suggested Chapters 16 and 17.

References

Abbott, M. (2005). Determining levels of productivity and efficiency in the electricity industry. *The Electricity Journal, 18*, 62–72.

Abbott, M. (2006). The productivity and efficiency of the Australian electricity supply industry. *Energy Economics, 28*, 444–454.

Agrell, P. J., & Bogetoft, P. (2005). Economic and environmental efficiency of district heating plants. *Energy Policy, 33*, 1351–1362.

Al-Salem, S. M. (2015). Carbon dioxide (CO2) emission sources in Kuwait from the downstream industry: Critical analysis with a current and futuristic view. *Energy, 81*, 575–587.

Aparicio, J., Pastor, J. T., & Zofio, J. L. (2015). How to properly decompose economic efficiency using technical and allocative criteria with non-homothetic DEA technologies. *European Journal of Operational Research, 240*, 882–891.

Apergis, N., Aye, G. C., Barros, C. P., Gupta, R., & Wanke, P. (2015). Energy efficiency of selected OECD countries: A slacks based model with undesirable outputs. *Energy Economics, 51*, 45–53.

Arabi, B., Munisamy, S., Emrouznejad, A., & Shadman, F. (2014). Power industry restructuring and eco-efficiency changes: A new slacks-based model in Malmquist-Luenberger Index measurement. *Energy Policy, 68*, 132–145.

Arcelus, F. J., & Arocena, P. (2005). Productivity differences across OECD countries in the presence of environmental constraints. *Journal of Operational Research Society, 56*, 1352.

Arocena, P. (2008). Cost and quality gains from diversification and vertical integration in the electricity industry: A DEA approach. *Energy Economics, 30*, 39–58.

Athanassopoulos, A. D., Lambroukos, N., & Seiford, L. (1999). Data envelopment scenario analysis for setting targets to electricity generating plants. *European Journal of Operational Research, 115*, 413–428.

Azadeh, A., Amalnick, M. S., Ghaderi, S. F., & Asadzadeh, S. M. (2007). An integrated DEA PCA numerical taxonomy approach for energy efficiency assessment and consumption optimization in energy intensive manufacturing sectors. *Energy Policy, 35*, 3792–3806.

Azadeh, A., Amalnick, M. S., Ghaderi, S. F., & Asadzadeh, S. M. (2008). Corrigendum to "An integrated DEA PCA numerical taxonomy approach for energy efficiency assessment and consumption optimization in energy intensive manufacturing sectors?" [Energy Policy 35 (2007) 3792-3806]. *Energy Policy, 36*, 2770.

Azadeh, A., Ghaderi, S. F., & Omrani, H. (2009a). A deterministic approach for performance assessment and optimization of power distribution units in Iran. *Energy Policy, 37*, 274–280.

Azadeh, A., Ghaderi, S. F., Omrani, H., & Eivazy, H. (2009b). An integrated DEA-COLS-SFA algorithm for optimization and policy making of electricity distribution units. *Energy Policy, 37*, 2605–2618.

Azadeh, A., & Mousavi Ahranjani, P. (2014). The impact of job security, satisfaction and stress on performance assessment and optimization of generation companies. *Journal of Loss Prevention in the Process Industries, 32*, 343–348.

Azadeh, A., Saberi, M., Asadzadeh, S. M., Hussain, O. K., & Saberi, Z. (2013). A neuro-fuzzy-multivariate algorithm for accurate gas consumption estimation in South America with noisy inputs. *International Journal of Electrical Power & Energy Systems, 46*, 315–325.

Azadi, M., Jafarian, M., Farzipoor Saen, R., & Mirhedayatian, S. M. (2015). A new fuzzy DEA model for evaluation of efficiency and effectiveness of suppliers in sustainable supply chain management context. *Computers & Operations Research, 54*, 274–285.

Azadi, M., Shabani, A., Khodakarami, M., & Farzipoor Saen, R. (2014). Planning in feasible region by two-stage target-setting DEA methods: An application in green supply chain management of public transportation service providers. *Transportation Research Part E: Logistics and Transportation Review, 70*, 324–338.

Bagdadioglu, N., Waddams Price, C. M., & Weyman-Jones, T. G. (1996). Efficiency and ownership in electricity distribution: A non-parametric model of the Turkish experience. *Energy Economics, 18*, 1–23.

Bampatsou, C., Papadopoulos, S., & Zervas, E. (2013). Technical efficiency of economic systems of EU-15 countries based on energy consumption. *Energy Policy, 55*, 426–434.

Banaeian, N., Omid, M., & Ahmadi, H. (2011). Energy and economic analysis of greenhouse strawberry production in Tehran province of Iran. *Energy Conversion and Management, 52*, 1020–1025.

Banaeian, N., & Zangeneh, M. (2011). Study on energy efficiency in corn production of Iran. *Energy, 36*, 5394–5402.

Barla, P., & Perelman, S. (2005). Sulphur emissions and productivity growth in industrialized countries. *Annals of Public and Cooperative Economics, 76*, 275–300.

Barros, C. P. (2008). Efficiency analysis of hydroelectric generating plants: A case study for Portugal. *Energy Economics, 30*, 59–75.

Barros, C. P., & Assaf, A. (2009). Bootstrapped efficiency measures of oil blocks in Angola. *Energy Policy, 37*, 4098–4103.

Barros, C. P., & Managi, S. (2009). Productivity assessment of Angola's oil blocks. *Energy, 34*, 2009–2015.

Barros, C. P., & Peypoch, N. (2008). Technical efficiency of thermoelectric power plants. *Energy Economics, 30*, 3118–3127.

Bevilacqua, M., & Braglia, M. (2002). Environmental efficiency analysis for ENI oil refineries. *Journal of Cleaner Production, 10*, 85–92.

Bi, G.-B., Song, W., Zhou, P., & Liang, L. (2014). Does environmental regulation affect energy efficiency in China's thermal power generation? Empirical evidence from a slacks-based DEA model. *Energy Policy, 66*, 537–546.

Bian, Y., He, P., & Xu, H. (2013). Estimation of potential energy saving and carbon dioxide emission reduction in China based on an extended non-radial DEA approach. *Energy Policy, 63*, 962–971.

Bian, Y., Yan, S., & Xu, H. (2014). Efficiency evaluation for regional urban water use and wastewater decontamination systems in China: A DEA approach. *Resources, Conservation and Recycling, 83*, 15–23.

Bian, Y., & Yang, F. (2010). Resource and environment efficiency analysis of provinces in China: A DEA approach based on Shannon's entropy. *Energy Policy, 38*, 1909–1917.

Blancard, S., & Martin, E. (2014). Energy efficiency measurement in agriculture with imprecise energy content information. *Energy Policy, 66*, 198–208.

Blokhuis, E., Advokaat, B., & Schaefer, W. (2012). Assessing the performance of Dutch local energy companies. *Energy Policy, 45*, 680–690.

Blomberg, J., Henriksson, E., & Lundmark, R. (2012). Energy efficiency and policy in Swedish pulp and paper mills: A data envelopment analysis approach. *Energy Policy, 42*, 569–579.

Boubaker, K. (2012a). Renewable energy in upper North Africa: Present versus 202: Horizon perspectives optimization using a data envelopment analysis (DEA) framework. *Renewable Energy, 43*, 364–369.

Boubaker, K. (2012b). A review on renewable energy conceptual perspectives in North Africa using a polynomial optimization scheme. *Renewable and Sustainable Energy Reviews, 16*, 4298–4302.

Boyd, G. A., & McClelland, J. D. (1999). The Impact of environmental constraints on productivity improvement in integrated paper plants. *Journal of Environmental Economics and Management, 38*, 121–142.

Boyd, G. A., & Pang, J. X. (2000). Estimating the linkage between energy efficiency and productivity. *Energy Policy, 28*, 289–296.

Boyd, G. A., Tolley, G., & Pang, J. (2002). Plant level productivity, efficiency, and environmental performance of the container glass industry. *Environmental and Resource Economics, 23*, 29–43.

Bozoğlu, M., & Ceyhan, V. (2009). Energy conversion efficiency of trout and sea bass production in the Black Sea, Turkey. *Energy, 34*, 199–204.

Brännlund, R., Chung, Y., Färe, R., & Grosskopf, S. (1998). Emissions trading and profitability: The Swedish pulp and paper industry. *Environmental and Resource Economics, 12*, 345–356.

Bretholt, A., & Pan, J.-N. (2013). Evolving the latent variable model as an environmental DEA technology. *Omega, 41*, 315–325.

Burnett, R. D., & Hansen, D. R. (2008). Ecoefficiency: Defining a role for environmental cost management. *Accounting, Organizations and Society, 33*, 551–581.

Byrnes, P., Färe, R., Grosskopf, S., & Knox Lovell, C. A. (1988). The effect of unions on productivity: U.S. surface mining of Coal. *Management Science, 34*, 1037–1053.

Callens, I., & Tyteca, D. (1999). Towards indicators of sustainable development for firms: A productive efficiency perspective. *Ecological Economics, 28*, 41–53.

Cambini, C., Croce, A., & Fumagalli, E. (2014). Output-based incentive regulation in electricity distribution: Evidence from Italy. *Energy Economics, 45*, 205–216.

Çelen, A. (2013). Efficiency and productivity (TFP) of the Turkish electricity distribution companies: An application of two-stage (DEA & amp; Tobit) analysis. *Energy Policy, 63*, 300–310.

Çelen, A., & Yalçın, N. (2012). Performance assessment of Turkish electricity distribution utilities: An application of combined FAHP/TOPSIS/DEA methodology to incorporate quality of service. *Utilities Policy, 23*, 59–71.

Chang, M.-C. (2014). Energy intensity, target level of energy intensity, and room for improvement in energy intensity: An application to the study of regions in the EU. *Energy Policy, 67*, 648–655.

Chang, M.-C. (2015). Room for improvement in low carbon economies of G7 and BRICS countries based on the analysis of energy efficiency and environmental Kuznets curves. *Journal of Cleaner Production, 99*, 140–151.

Chang, D.-S., Kuo, L.-C. R., & Chen, Y.-T. (2013a). Industrial changes in corporate sustainability performance: An empirical overview using data envelopment analysis. *Journal of Cleaner Production, 56*, 147–155.

Chang, D.-S., & Yang, F.-C. (2011). Assessing the power generation, pollution control, and overall efficiencies of municipal solid waste incinerators in Taiwan. *Energy Policy, 39*, 651–663.

Chang, Y.-T., Zhang, N., Danao, D., & Zhang, N. (2013b). Environmental efficiency analysis of transportation system in China: A non-radial DEA approach. *Energy Policy, 58*, 277–283.

Chauhan, N. S., Mohapatra, P. K. J., & Pandey, K. P. (2006). Improving energy productivity in paddy production through benchmarking? An application of data envelopment analysis. *Energy Conversion and Management, 47*, 1063–1085.

Chen, J., Song, M., & Xu, L. (2015a). Evaluation of environmental efficiency in China using data envelopment analysis. *Ecological Indicators, 52*, 577–583.

Chen, P.-C., Yu, M.-M., Chang, C.-C., Hsu, S.-H., & Managi, S. (2015b). The enhanced Russell-based directional distance measure with undesirable outputs: Numerical example considering CO_2 emissions. *Omega, 53*, 30–40.

Cherchye, L., De Rock, B., & Walheer, B. (2015). Multi-output efficiency with good and bad outputs. *European Journal of Operational Research, 240*, 872–881.

Chien, T., & Hu, J.-L. (2007). Renewable energy and macroeconomic efficiency of OECD and non-OECD economies. *Energy Policy, 35*, 3606–3615.

Chien, C.-F., Lo, F.-Y., & Lin, J. T. (2003). Using DEA to measure the relative efficiency of the service center and improve operation efficiency through reorganization. *IEEE Transactions on Power Systems, 18*, 366–373.

Chitkara, P. (1999). A data envelopment analysis approach to evaluation of operational inefficiencies in power generating units: A case study of Indian power plants. *IEEE Transactions on Power Systems, 14*, 419–425.

Chiu, Y.-H., Huang, C.-W., & Ma, C.-M. (2011). Assessment of China transit and economic efficiencies in a modified value-chains DEA model. *European Journal of Operational Research, 209*, 95–103.

Chiu, Y.-H., Lee, J.-H., Lu, C.-C., Shyu, M.-K., & Luo, Z. (2012). The technology gap and efficiency measure in WEC countries: Application of the hybrid meta frontier model. *Energy Policy, 51*, 349–357.

Choi, Y., Zhang, N., & Zhou, P. (2012). Efficiency and abatement costs of energy-related CO_2 emissions in China: A slacks-based efficiency measure. *Applied Energy, 98*, 198–208.

Chung, W. (2011). Review of building energy-use performance benchmarking methodologies. *Applied Energy, 88*, 1470–1479.

Chung, Y. H., Färe, R., & Grosskopf, S. (1997). Productivity and undesirable outputs: A directional distance function approach. *Journal of Environmental Management, 51*, 229–240.

Claggett, E. T., & Ferrier, G. D. (1998). The efficiency of TVA power distributors. *Managerial and Decision Economics, 19*, 365–376.

Cook, W. D., & Green, R. H. (2005). Evaluating power plant efficiency: A hierarchical model. *Computers & Operations Research, 32*, 813–823.

Criswell, D. R., & Thompson, R. G. (1996). Data envelopment analysis of space and terrestrially-based large scale commercial power systems for earth: A prototype analysis of their relative economic advantages. *Solar Energy, 56*, 119–131.

Cui, Q., Kuang, H.-B., Wu, C.-Y., & Li, Y. (2014). The changing trend and influencing factors of energy efficiency: The case of nine countries. *Energy, 64*, 1026–1034.

Cui, Q., & Li, Y. (2014). The evaluation of transportation energy efficiency: An application of three-stage virtual frontier DEA. *Transportation Research Part D: Transport and Environment, 29*, 1–11.

Cui, Q., & Li, Y. (2015a). An empirical study on the influencing factors of transportation carbon efficiency: Evidences from fifteen countries. *Applied Energy, 141*, 209–217.

Cui, Q., & Li, Y. (2015b). Evaluating energy efficiency for airlines: An application of VFB-DEA. *Journal of Air Transport Management, 44–45*, 34–41.

Dai, X., & Kuosmanen, T. (2014). Best-practice benchmarking using clustering methods: Application to energy regulation. *Omega, 42*, 179–188.

de Castro Camioto, F., Mariano, E. B., & do Nascimento Rebelatto, D. A. (2014). Efficiency in Brazil's industrial sectors in terms of energy and sustainable development. *Environmental Science & Policy, 37*, 50–60.

Diaz-Balteiro, L., Casimiro Herruzo, A., Martinez, M., & González-Pachón, J. (2006). An analysis of productive efficiency and innovation activity using DEA: An application to Spain's wood-based industry. *Forest Policy and Economics, 8*, 762–773.

Duan, N., Guo, J.-P., & Xie, B.-C. (2016). Is there a difference between the energy and CO_2 emission performance for China's thermal power industry? A bootstrapped directional distance function approach. *Applied Energy, 162*, 1552–1563.

Dyckhoff, H., & Allen, K. (2001). Measuring ecological efficiency with data envelopment analysis (DEA). *European Journal of Operational Research, 132*, 312–325.

Ebrahimi, R., & Salehi, M. (2015). Investigation of CO_2 emission reduction and improving energy use efficiency of button mushroom production using data envelopment analysis. *Journal of Cleaner Production, 103*, 112–119.

Ederer, N. (2015). Evaluating capital and operating cost efficiency of offshore wind farms: A DEA approach. *Renewable and Sustainable Energy Reviews, 42*, 1034–1046.

Erbetta, F., & Rappuoli, L. (2008). Optimal scale in the Italian gas distribution industry using data envelopment analysis. *Omega, 36*, 325–336.

Ertürk, M., & Türüt-Aşık, S. (2011). Efficiency analysis of Turkish natural gas distribution companies by using data envelopment analysis method. *Energy Policy, 39*, 1426–1438.

Fallahi, A., Ebrahimi, R., & Ghaderi, S. F. (2011). Measuring efficiency and productivity change in power electric generation management companies by using data envelopment analysis: A case study. *Energy, 36*, 6398–6405.

Fang, C.-Y., Hu, J.-L., & Lou, T.-K. (2013). Environment-adjusted total-factor energy efficiency of Taiwan's service sectors. *Energy Policy, 63*, 1160–1168.

Fang, H., Wu, J., & Zeng, C. (2009). Comparative study on efficiency performance of listed coal mining companies in China and the US. *Energy Policy, 37*, 5140–5148.

Färe, R., & Grosskopf, S. (2004). Modeling undesirable factors in efficiency evaluation: Comment. *European Journal of Operational Research, 157*, 242–352.

Färe, R., Grosskopf, S., Lindgren, B., & Roos, P. (1994a). Productivity developments in Swedish Hospitals: A Malmquist output index approach. Data envelopment analysis: Theory, methodology, and applications (pp. 253–272). Amsterdam: Springer, ISBN: 978-0-7923-9480-8.

Färe, R., Grosskopf, S., Lovell, C. A. K., & Pasurka, C. (1989). Multilateral productivity comparisons when some outputs are undesirable: A nonparametric approach. *The Review of Economics and Statistics, 71*, 90–98.

Färe, R., Grosskopf, S., Noh, D.-W., & Weber, W. (2005). Characteristics of a polluting technology: Theory and practice. *Journal of Econometrics, 126*, 469–492.

Färe, R., Grosskopf, S., & Hernandez-Sancho, F. (2004). Environmental performance: An index number approach. *Resource and Energy Economics, 26*, 343–352.

Färe, R., Grosskopf, S., & Logan, J. (1983). The relative efficiency of Illinois electric utilities. *Resources and Energy, 5*, 349–367.

Färe, R., Grosskopf, S., & Logan, J. (1985). The relative performance of publicly-owned and privately-owned electric utilities. *Journal of Public Economics, 26*, 89–106.

Färe, R., Grosskopf, S., & Lovell, C. A. K. (1994b). *Production frontiers*. Cambridge: Cambridge University Press.

Färe, R., Grosskopf, S., & Pasurka, C. (1986). Effects on relative efficiency in electric power generation due to environmental controls. *Resources and Energy, 8*, 167–184.

Färe, R., Grosskopf, S., & Pasurka, J. C. A. (2001). Accounting for air pollution emissions in measures of state manufacturing productivity growth. *Journal of Regional Science, 41*, 381–409.

Färe, R., Grosskopf, S., & Tyteca, D. (1996). An activity analysis model of the environmental performance of firms-application to fossil-fuel-fired electric utilities. *Ecological Economics, 18*, 161–175.

Färe, R., Grosskopf, S., Yaisawarng, S., Li, S. K., & Wang, Z. (1990). Productivity growth in Illinois electric utilities. *Resources and Energy, 12*, 383–398.

Feng, C., Chu, F., Ding, J., Bi, G., & Liang, L. (2015). Carbon emissions abatement (CEA) allocation and compensation schemes based on DEA. *Omega, 53*, 78–89.

Ferrier, G. D., & Hirschberg, J. G. (1992). Climate control efficiency. *The Energy Journal, 13*, 37–54.

Fleishman, R., Alexander, R., Bretschneider, S., & Popp, D. (2009). Does regulation stimulate productivity? The effect of air quality policies on the efficiency of US power plants. *Energy Policy, 37*, 4574–4582.

Galindo, P., Schäffer, A., Brechtel, K., Unterberger, S., & Scheffknecht, G. (2012). Experimental research on the performance of CO_2-loaded solutions of MEA and DEA at regeneration conditions. *Fuel, 101*, 2–8.

Ghiasi, M. M., & Mohammadi, A. H. (2014). Rigorous modeling of CO_2 equilibrium absorption in MEA, DEA, and TEA aqueous solutions. *Journal of Natural Gas Science and Engineering, 18*, 39–46.

Ghulam, Y., & Jaffry, S. (2015). Efficiency and productivity of the cement industry: Pakistani experience of deregulation and privatisation. *Omega, 54*, 101–115.

Giannakis, D., Jamasb, T., & Pollitt, M. (2005). Benchmarking and incentive regulation of quality of service: An application to the UK electricity distribution networks. *Energy Policy, 33*, 2256–2271.

Golany, B., Roll, Y., & Rybak, D. (1994). Measuring efficiency of power plants in Israel by data envelopment analysis. *IEEE Transactions on Engineering Management, 41*, 29–301.

Gómez-Calvet, R., Conesa, D., Gómez-Calvet, A. R., & Tortosa-Ausina, E. (2014). Energy efficiency in the European Union: What can be learned from the joint application of directional distance functions and slacks-based measures? *Applied Energy, 132*, 137–154.

González, E., Cárcaba, A., & Ventura, J. (2015). How car dealers adjust prices to reach the product efficiency frontier in the Spanish automobile market. *Omega, 51*, 38–48.
Goto, M., Otsuka, A., & Sueyoshi, T. (2014). DEA (Data Envelopment Analysis) assessment of operational and environmental efficiencies on Japanese regional industries. *Energy, 66*, 535–549.
Goto, M., & Sueyoshi, T. (2009a). *Financial ratio analysis: An application to US Energy Industry* (Productivity, efficiency, and economic growth in the Asia-Pacific Region). Berlin: Physica-Verlag HD.
Goto, M., & Sueyoshi, T. (2009b). Productivity growth and deregulation of Japanese electricity distribution. *Energy Policy, 37*, 3130–3138.
Goto, M., & Tsutsui, M. (1998). Comparison of productive and cost efficiencies among Japanese and US electric utilities. *Omega, 26*, 177–194.
Gouveia, M. C., Dias, L. C., Antunes, C. H., Boucinha, J., & Inácio, C. F. (2015). Benchmarking of maintenance and outage repair in an electricity distribution company using the value-based DEA method. *Omega, 53*, 104–114.
Grigoroudis, E., Petridis, K., & Arabatzis, G. (2014). RDEA: A recursive DEA based algorithm for the optimal design of biomass supply chain networks. *Renewable Energy, 71*, 113–122.
Guo, X.-D., Zhu, L., Fan, Y., & Xie, B.-C. (2011). Evaluation of potential reductions in carbon emissions in Chinese provinces based on environmental DEA. *Energy Policy, 39*, 2352–2360.
Hampf, B., & Rødseth, K. L. (2015). Carbon dioxide emission standards for U.S. power plants: An efficiency analysis perspective. *Energy Economics, 50*, 140–153.
Han, Y., Geng, Z., Zhu, Q., & Qu, Y. (2015). Energy efficiency analysis method based on fuzzy DEA cross-model for ethylene production systems in chemical industry. *Energy, 83*, 685–695.
Hang, Y., Sun, J., Wang, Q., Zhao, Z., & Wang, Y. (2015). Measuring energy inefficiency with undesirable outputs and technology heterogeneity in Chinese cities. *Economic Modeling, 49*, 46–52.
Hattori, T., Jamasb, T., & Pollitt, M. (2005). Electricity distribution in the UK and Japan: A comparative efficiency analysis 1985-1998. *The Energy Journal, 26*, 23–47.
Hawdon, D. (2003). Efficiency, performance and regulation of the international gas industry—A bootstrap DEA approach. *Energy Policy, 31*, 1167–1178.
He, X. (2015). Regional differences in China's CO_2 abatement cost. *Energy Policy, 80*, 145–152.
He, F., Zhang, Q., Lei, J., Fu, W., & Xu, X. (2013). Energy efficiency and productivity change of China's iron and steel industry: Accounting for undesirable outputs. *Energy Policy, 54*, 204–213.
Hernández-Sancho, F., Molinos-Senante, M., & Sala-Garrido, R. (2011). Energy efficiency in Spanish wastewater treatment plants: A non-radial DEA approach. *Science of the Total Environment, 409*, 2693–2699.
Hernández-Sancho, F., Picazo-Tadeo, A., & Reig-Martinez, E. (2000). Efficiency and environmental regulation. *Environmental and Resource Economics, 15*, 365–378.
Hernández-Sancho, F., & Sala-Garrido, R. (2009). Technical efficiency and cost analysis in wastewater treatment processes: A DEA approach. *Desalination, 249*, 230–234.
Herrala, M. E., Huotari, H., & Haapasalo, H. J. O. (2012). Governance of Finnish waterworks—A DEA comparison of selected models. *Utilities Policy, 20*, 64–70.
Honma, S., & Hu, J.-L. (2008). Total-factor energy efficiency of regions in Japan. *Energy Policy, 36*, 821–833.
Honma, S., & Hu, J.-L. (2009). Total-factor energy productivity growth of regions in Japan. *Energy Policy, 37*, 3941–3950.
Honma, S., & Hu, J.-L. (2014a). Industry-level total-factor energy efficiency in developed countries: A Japan-centered analysis. *Applied Energy, 119*, 67–78.
Honma, S., & Hu, J.-L. (2014b). A panel data parametric frontier technique for measuring total-factor energy efficiency: An application to Japanese regions. *Energy, 78*, 732–739.

Houshyar, E., Azadi, H., Almassi, M., Sheikh Davoodi, M. J., & Witlox, F. (2012). Sustainable and efficient energy consumption of corn production in Southwest Iran: Combination of multi-fuzzy and DEA modeling. *Energy, 44*, 672–681.
Hu, J.-L., & Kao, C.-H. (2007). Efficient energy-saving targets for APEC economies. *Energy Policy, 35*, 373–382.
Hu, J. L., & Lee, Y. C. (2008). Efficient three industrial waste abatements for regions in China. *International Journal of Sustainable Development and World Ecology, 15*, 132–144.
Hu, J.-L., Lio, M.-C., Yeh, F.-Y., & Lin, C.-H. (2011). Environment-adjusted regional energy efficiency in Taiwan. *Applied Energy, 88*, 2893–2899.
Hu, J., Liu, J., Yu, J., & Dai, G. (2013). CO_2 absorption into highly concentrated DEA solution flowing over a vertical plate with rectangular windows. *International Journal of Greenhouse Gas Control, 19*, 13–18.
Hu, J.-L., & Wang, S.-C. (2006). Total-factor energy efficiency of regions in China. *Energy Policy, 34*, 3206–3217.
Huang, C.-W., Chiu, Y.-H., Fang, W.-T., & Shen, N. (2014). Assessing the performance of Taiwan's environmental protection system with a non-radial network DEA approach. *Energy Policy, 74*, 547–556.
Huang, R., & Li, Y. (2013). Undesirable input-output two-phase DEA model in an environmental performance audit. *Mathematical and Computer Modelling, 58*, 971–979.
Huang, J. P., Poh, K. L., & Ang, B. W. (1995). Decision analysis in energy and environmental modeling. *Energy, 20*, 843–855.
Iglesias, G., Castellanos, P., & Seijas, A. (2010). Measurement of productive efficiency with frontier methods: A case study for wind farms. *Energy Economics, 32*, 1199–1208.
Iribarren, D., Martín-Gamboa, M., & Dufour, J. (2013). Environmental benchmarking of wind farms according to their operational performance. *Energy, 61*, 589–597.
Iribarren, D., Vázquez-Rowe, I., Rugani, B., & Benetto, E. (2014). On the feasibility of using energy analysis as a source of benchmarking criteria through data envelopment analysis: A case study for wind energy. *Energy, 67*, 527–537.
Jamasb, T., Nillesen, P., & Pollitt, M. (2004). Strategic behavior under regulatory benchmarking. *Energy Economics, 26*, 825–843.
Jamasb, T., & Pollitt, M. (2000). Benchmarking and regulation: International electricity experience. *Utilities Policy, 9*, 107–130.
Jamasb, T., & Pollitt, M. (2003). International benchmarking and regulation: An application to European electricity distribution utilities. *Energy Policy, 31*, 1609–1622.
Jaraite, J., & Di Maria, C. (2012). Efficiency, productivity and environmental policy: A case study of power generation in the EU. *Energy Economics, 34*, 1557–1568.
Jebaraj, S., & Iniyan, S. (2006). A review of energy models. *Renewable and Sustainable Energy Reviews, 10*, 281–311.
Jin, J., Zhou, D., & Zhou, P. (2014). Measuring environmental performance with stochastic environmental DEA: The case of APEC economies. *Economic Modelling, 38*, 80–86.
Kashani, H. A. (2005a). Regulation and efficiency: An empirical analysis of the United Kingdom continental shelf petroleum industry. *Energy Policy, 33*, 915–925.
Kashani, H. A. (2005b). State intervention causing inefficiency: An empirical analysis of the Norwegian Continental Shelf. *Energy Policy, 33*, 1998–2009.
Keirstead, J. (2013). Benchmarking urban energy efficiency in the UK. *Energy Policy, 63*, 575–587.
Khoshnevisan, B., Rafiee, S., Omid, M., & Mousazadeh, H. (2013a). Applying data envelopment analysis approach to improve energy efficiency and reduce GHG (greenhouse gas) emission of wheat production. *Energy, 58*, 588–593.
Khoshnevisan, B., Rafiee, S., Omid, M., & Mousazadeh, H. (2013b). Reduction of CO_2 emission by improving energy use efficiency of greenhouse cucumber production using DEA approach. *Energy, 55*, 676–682.

Khoshnevisan, B., Rafiee, S., Omid, M., Yousefi, M., & Movahedi, M. (2013c). Modeling of energy consumption and GHG (greenhouse gas) emissions in wheat production in Esfahan province of Iran using artificial neural networks. *Energy, 52*, 333–338.

Khoshroo, A., Mulwa, R., Emrouznejad, A., & Arabi, B. (2013). A non-parametric data envelopment analysis approach for improving energy efficiency of grape production. *Energy, 63*, 189–194.

Kim, K., & Kim, Y. (2012). International comparison of industrial CO_2 emission trends and the energy efficiency paradox utilizing production-based decomposition. *Energy Economics, 34*, 1724–1741.

Kim, K.-T., Lee, D. J., Park, S.-J., Zhang, Y., & Sultanov, A. (2015). Measuring the efficiency of the investment for renewable energy in Korea using data envelopment analysis. *Renewable and Sustainable Energy Reviews, 47*, 694–702.

Korhonen, P. J., & Luptacik, M. (2004). Eco-efficiency analysis of power plants: An extension of data envelopment analysis. *European Journal of Operational Research, 154*, 437–446.

Korhonen, P., & Syrjänen, M. (2003). Evaluation of cost efficiency in Finnish electricity distribution. *Annals of Operations Research, 121*, 105–122.

Kounetas, K. (2015). Heterogeneous technologies, strategic groups and environmental efficiency technology gaps for European countries. *Energy Policy, 83*, 277–287.

Kulshreshtha, M., & Parikh, J. K. (2002). Study of efficiency and productivity growth in opencast and underground coal mining in India: A DEA analysis. *Energy Economics, 24*, 439–453.

Kumar, S. (2006). Environmentally sensitive productivity growth: A global analysis using Malmquist-Luenberger index. *Ecological Economics, 56*, 280–293.

Kumar Mandal, S., & Madheswaran, S. (2010). Environmental efficiency of the Indian cement industry: An interstate analysis. *Energy Policy, 38*, 1108–1118.

Kuosmanen, T., Saastamoinen, A., & Sipiläinen, T. (2013). What is the best practice for benchmark regulation of electricity distribution? Comparison of DEA, SFA and StoNED methods. *Energy Policy, 61*, 740–750.

Lam, P.-L., & Shiu, A. (2001). A data envelopment analysis of the efficiency of China's thermal power generation. *Utilities Policy, 10*, 75–83.

Lampe, H. W., & Hilgers, D. (2015). Trajectories of efficiency measurement: A bibliometric analysis of DEA and SFA. *European Journal of Operational Research, 240*, 1–21.

Lee, Y.-C., Hu, J.-L., & Kao, C.-H. (2011a). Efficient saving targets of electricity and energy for regions in China. *International Journal of Electrical Power & Energy Systems, 33*, 1211–1219.

Lee, W.-S., & Kung, C.-K. (2011b). Using climate classification to evaluate building energy performance. *Energy, 36*, 1797–1801.

Lee, S. K., Mogi, G., & Hui, K. S. (2013). A fuzzy analytic hierarchy process (AHP)/data envelopment analysis (DEA) hybrid model for efficiently allocating energy R&D resources: In the case of energy technologies against high oil prices. *Renewable and Sustainable Energy Reviews, 21*, 347–355.

Lee, S. K., Mogi, G., Lee, S. K., Hui, K. S., & Kim, J. W. (2010). Econometric analysis of the R&D performance in the national hydrogen energy technology development for measuring relative efficiency: The fuzzy AHP/DEA integrated model approach. *International Journal of Hydrogen Energy, 35*, 2236–2246.

Lee, S. K., Mogi, G., Li, Z., Hui, K. S., Lee, S. K., Hui, K. N., et al. (2011b). Measuring the relative efficiency of hydrogen energy technologies for implementing the hydrogen economy: An integrated fuzzy AHP/DEA approach. *International Journal of Hydrogen Energy, 36*, 12655–12663.

Lee, M., & Zhang, N. (2012). Technical efficiency, shadow price of carbon dioxide emissions, and substitutability for energy in the Chinese manufacturing industries. *Energy Economics, 34*, 1492–1497.

Lei, M., Zhao, X., Deng, H., & Tan, K.-C. (2013). DEA analysis of FDI attractiveness for sustainable development: Evidence from Chinese provinces. *Decision Support Systems, 56*, 406–418.

Leme, R. C., Paiva, A. P., Steele Santos, P. E., Balestrassi, P. P., & de Lima Galvão, L. (2014). Design of experiments applied to environmental variables analysis in electricity utilities efficiency: The Brazilian case. *Energy Economics, 45*, 111–119.

Li, L.-B., & Hu, J.-L. (2012). Ecological total-factor energy efficiency of regions in China. *Energy Policy, 46*, 216–224.

Li, L., Li, M., & Wu, C. (2013a). Production efficiency evaluation of energy companies based on the improved super-efficiency data envelopment analysis considering undesirable outputs. *Mathematical and Computer Modelling, 58*, 1057–1066.

Li, K., & Lin, B. (2015a). Metafroniter energy efficiency with CO2 emissions and its convergence analysis for China. *Energy Economics, 48*, 230–241.

Li, K., & Lin, B. (2015b). The efficiency improvement potential for coal, oil and electricity in China's manufacturing sectors. *Energy, 86*, 403–413.

Li, Y., Sun, L., Feng, T., & Zhu, C. (2013b). How to reduce energy intensity in China: A regional comparison perspective. *Energy Policy, 61*, 513–522.

Li, H., Yang, W., Zhou, Z., & Huang, C. (2013c). Resource allocation models' construction for the reduction of undesirable outputs based on DEA methods. *Mathematical and Computer Modelling, 58*, 913–926.

Li, X.-G., Yang, J., & Liu, X.-J. (2013d). Analysis of Beijing's environmental efficiency and related factors using a DEA model that considers undesirable outputs. *Mathematical and Computer Modelling, 58*, 956–960.

Liang, L., Wu, D., & Hua, Z. (2004). MES-DEA modelling for analyzing anti-industrial pollution efficiency and its application in Anhui province of China. *International Journal of Global Energy Issues, 22*, 88–98.

Lin, B., & Du, K. (2015a). Energy and CO_2 emissions performance in China's regional economies: Do market-oriented reforms matter? *Energy Policy, 78*, 113–124.

Lin, B., & Du, K. (2015b). Modeling the dynamics of carbon emission performance in China: A parametric Malmquist index approach. *Energy Economics, 49*, 550–557.

Lin, B., & Liu, X. (2012). Dilemma between economic development and energy conservation: Energy rebound effect in China. *Energy, 45*, 867–873.

Lins, M. E., Oliveira, L. B., da Silva, A. C. M., Rosa, L. P., & Pereira, A. O., Jr. (2012). Performance assessment of alternative energy resources in Brazilian power sector using data envelopment analysis. *Renewable and Sustainable Energy Reviews, 16*, 898–903.

Liu, W. (2015). Measuring efficiency and scope economies of cogeneration enterprises in Northeast China under the background of energy saving and emission reduction. *Ecological Indicators, 51*, 173–179.

Liu, C. H., Lin, S. J., & Lewis, C. (2010). Evaluation of thermal power plant operational performance in Taiwan by data envelopment analysis. *Energy Policy, 38*, 1049–1058.

Liu, J. S., Lu, L. Y. Y., Lu, W.-M., & Lin, B. J. Y. (2013). A survey of DEA applications. *Omega, 41*, 893–902.

Liu, Y., Ren, L., Li, Y., & Zhao, X.-g. (2015). The industrial performance of wind power industry in China. *Renewable and Sustainable Energy Reviews, 43*, 644–655.

Lo, F.-Y., Chien, C.-F., & Lin, J. T. (2001). A DEA study to evaluate the relative efficiency and investigate the district reorganization of the Taiwan power company. *IEEE Transactions on Power Systems, 16*, 170–178.

Løken, E. (2007). Use of multicriteria decision analysis methods for energy planning problems. *Renewable and Sustainable Energy Reviews, 11*, 1584–1595.

Long, X., Zhao, X., & Cheng, F. (2015). The comparison analysis of total factor productivity and eco-efficiency in China's cement manufactures. *Energy Policy, 81*, 61–66.

Lozano, S., & Gutiérrez, E. (2008). Non-parametric frontier approach to modelling the relationships among population, GDP, energy consumption and CO_2 emissions. *Ecological Economics, 66*, 687–699.

Lozano, S., Villa, G., & Brännlund, R. (2009). Centralised reallocation of emission permits using DEA. *European Journal of Operational Research, 193*, 752–760.

Lv, W., Zhou, Z., & Huang, H. (2013). The measurement of undesirable output based-on DEA in E&E: Models development and empirical analysis. *Mathematical and Computer Modelling, 58*, 907–912.
Ma, C., & Zhao, X. (2015). China's electricity market restructuring and technology mandates: Plant-level evidence for changing operational efficiency. *Energy Economics, 47*, 227–237.
Madlener, R., Antunes, C. H., & Dias, L. C. (2009). Assessing the performance of biogas plants with multi-criteria and data envelopment analysis. *European Journal of Operational Research, 197*, 1084–1094.
Mallikarjun, S., & Lewis, H. F. (2014). Energy technology allocation for distributed energy resources: A strategic technology-policy framework. *Energy, 72*, 783–799.
Mandal, S. K. (2010). Do undesirable output and environmental regulation matter in energy efficiency analysis? Evidence from Indian Cement Industry. *Energy Policy, 38*, 6076–6083.
Menegaki, A. N. (2013). Growth and renewable energy in Europe: Benchmarking with data envelopment analysis. *Renewable Energy, 60*, 363–369.
Meng, F. Y., Fan, L. W., Zhou, P., & Zhou, D. Q. (2013). Measuring environmental performance in China's industrial sectors with non-radial DEA. *Mathematical and Computer Modelling, 58*, 1047–1056.
Miliotis, P. A. (1992). Data envelopment analysis applied to electricity distribution districts. *Journal of the Operational Research Society, 43*, 549–555.
Mobtaker, H. G., Akram, A., Keyhani, A., & Mohammadi, A. (2012). Optimization of energy required for alfalfa production using data envelopment analysis approach. *Energy for Sustainable Development, 16*, 242–248.
Mohammadi, A., Rafiee, S., Jafari, A., Dalgaard, T., Knudsen, M. T., Keyhani, A., et al. (2013). Potential greenhouse gas emission reductions in soybean farming: A combined use of life cycle assessment and data envelopment analysis. *Journal of Cleaner Production, 54*, 89–100.
Mohan, S. V., Raghavulu, S. V., Mohanakrishna, G., Srikanth, S., & Sarma, P. N. (2009). Optimization and evaluation of fermentative hydrogen production and wastewater treatment processes using data enveloping analysis (DEA) and Taguchi design of experimental (DOE) methodology. *International Journal of Hydrogen Energy, 34*, 216–226.
Molinos-Senante, M., Hernández-Sancho, F., Mocholí-Arce, M., & Sala-Garrido, R. (2014). Economic and environmental performance of wastewater treatment plants: Potential reductions in greenhouse gases emissions. *Resource and Energy Economics, 38*, 125–140.
Mou, D. (2014). Understanding China's electricity market reform from the perspective of the coal-fired power disparity. *Energy Policy, 74*, 224–234.
Mousavi-Avval, S. H., Mohammadi, A., Rafiee, S., & Tabatabaeefar, A. (2012). Assessing the technical efficiency of energy use in different barberry production systems. *Journal of Cleaner Production, 27*, 126–132.
Mousavi-Avval, S. H., Rafiee, S., Jafari, A., & Mohammadi, A. (2011a). Improving energy use efficiency of canola production using data envelopment analysis (DEA) approach. *Energy, 36*, 2765–2772.
Mousavi-Avval, S. H., Rafiee, S., Jafari, A., & Mohammadi, A. (2011b). Optimization of energy consumption for soybean production using data envelopment analysis (DEA) approach. *Applied Energy, 88*, 3765–3772.
Mukherjee, K. (2008). Energy use efficiency in the Indian manufacturing sector: An interstate analysis. *Energy Policy, 36*, 662–672.
Munksgaard, J., Pade, L.-L., & Fristrup, P. (2005). Efficiency gains in Danish district heating. Is there anything to learn from benchmarking? *Energy Policy, 33*, 1986–1997.
Murillo-Zamorano, L. R. (2005). The role of energy in productivity growth: A controversial issue? *The Energy Journal, 26*, 69–88.
Nabavi-Pelesaraei, A., Abdi, R., Rafiee, S., & Mobtaker, H. G. (2014). Optimization of energy required and greenhouse gas emissions analysis for orange producers using data envelopment analysis approach. *Journal of Cleaner Production, 65*, 311–317.

Nag, B. (2006). Estimation of carbon baselines for power generation in India: The supply side approach. *Energy Policy, 34*, 1399–1410.
Nassiri, S., & Singh, S. (2007). Study on energy use efficiency for paddy crop using data envelopment analysis (DEA) technique. *Applied Energy, 86*, 1320–1325.
Nouri, J., Hosseinzadeh Lotfi, F., Borgheipour, H., Atabi, F., Sadeghzadeh, S. M., & Moghaddas, Z. (2013). An analysis of the implementation of energy efficiency measures in the vegetable oil industry of Iran: A data envelopment analysis approach. *Journal of Cleaner Production, 52*, 84–93.
Odeck, J. (2009). Statistical precision of DEA and Malmquist indices: A bootstrap application to Norwegian grain producers. *Omega, 37*, 1007–1017.
Oggioni, G., Riccardi, R., & Toninelli, R. (2011). Eco-efficiency of the world cements industry: A data envelopment analysis. *Energy Policy, 39*, 2842–2854.
Oh, S.-C., & Shin, J. (2015). The impact of mismeasurement in performance benchmarking: A Monte Carlo comparison of SFA and DEA with different multi-period budgeting strategies. *European Journal of Operational Research, 240*, 518–227.
Olanrewaju, O. A., Jimoh, A. A., & Kholopane, P. A. (2012). Integrated IDA-ANN-DEA for assessment and optimization of energy consumption in industrial sectors. *Energy, 46*, 629–635.
Olanrewaju, O. A., Jimoh, A. A., & Kholopane, P. A. (2013). Assessing the energy potential in the South African industry: A combined IDA-ANN-DEA (Index Decomposition Analysis-Artificial Neural Network-Data Envelopment Analysis) model. *Energy, 63*, 225–232.
Olatubi, W. O., & Dismukes, D. E. (2000). A data envelopment analysis of the levels and determinants of coal-fired electric power generation performance. *Utilities Policy, 9*, 47–59.
Omid, M., Ghojabeige, F., Delshad, M., & Ahmadi, H. (2011). Energy use pattern and benchmarking of selected greenhouses in Iran using. *Energy Conversion and Management, 52*, 153–162.
Omrani, H., Gharizadeh Beiragh, R., & Shafiei Kaleibari, S. (2015). Performance assessment of Iranian electricity distribution companies by an integrated cooperative game data envelopment analysis principal component analysis approach. *International Journal of Electrical Power & Energy Systems, 64*, 617–625.
Önüt, S., & Soner, S. (2006). Energy efficiency assessment for the Antalya region hotels in Turkey. *Energy and Buildings, 38*, 964–971.
Oude Lansink, A., & Bezlepkin, I. (2003). The effect of heating technologies on CO_2 and energy efficiency of Dutch greenhouse firms. *Journal of Environmental Management, 68*, 73–82.
Oude Lansink, A., & Silva, E. (2003). CO_2 and energy efficiency of different heating technologies in the Dutch glasshouse industry. *Environmental and Resource Economics, 24*, 395–407.
Pacudan, R., & de Guzman, E. (2002). Impact of energy efficiency policy to productive efficiency of electricity distribution industry in the Philippines. *Energy Economics, 24*, 41–54.
Pahlavan, R., Omid, M., & Akram, A. (2011). Energy use efficiency in greenhouse tomato production in Iran. *Energy, 36*, 6714–6719.
Pahwa, A., Xiaoming, F., & Lubkeman, D. (2003). Performance evaluation of electric distribution utilities based on data envelopment analysis. *IEEE Transactions on Power Systems, 18*, 400–405.
Pan, H., Haiyun, Z., & Xulu, Z. (2013). China's provincial industrial energy efficiency and its determinants. *Mathematical and Computer Modelling, 58*, 1032–1039.
Pan, X., Liu, Q., & Peng, X. (2015). Spatial club convergence of regional energy efficiency in China. *Ecological Indicators, 51*, 25–30.
Park, S.-U., & Lesourd, J.-B. (2000). The efficiency of conventional fuel power plants in South Korea: A comparison of parametric and non-parametric approaches. *International Journal of Production Economics, 63*, 59–67.
Pasurka, C. A. (2006). Decomposing electric power plant emissions within a joint production framework. *Energy Economics, 28*, 26–43.
Picazo-Tadeo, A. J., Reig-Martínez, E., & Hernández-Sancho, F. (2005). Directional distance functions and environmental regulation. *Resource and Energy Economics, 27*, 131–142.

Pollitt, M. G. (1996). Ownership and efficiency in nuclear power production. *Oxford Economic Papers, 48*, 342–360.
Pombo, C., & Taborda, R. (2006). Performance and efficiency in Colombia's power distribution system: Effects of the 1994 reform. *Energy Economics, 28*, 339–369.
Price, C. W., & Weyman-Jones, T. (1996). Malmquist indices of productivity change in the UK gas industry before and after privatization. *Applied Economics, 28*, 29–39.
Raczka, J. (2001). Explaining the performance of heat plants in Poland. *Energy Economics, 23*, 355–370.
Ramanathan, R. (2000). A holistic approach to compare energy efficiencies of different transport modes. *Energy Policy, 28*, 743–747.
Ramanathan, R. (2001). Comparative risk assessment of energy supply technologies: A data envelopment analysis approach. *Energy, 26*, 197–203.
Ramanathan, R. (2005a). An analysis of energy consumption and carbon dioxide emissions in countries of the Middle East and North Africa. *Energy, 30*, 2831–2842.
Ramanathan, R. (2005b). Estimating energy consumption of transport modes in India using DEA and application to energy and environmental policy. *Journal of the Operational Research Society, 56*, 732–737.
Ramli, N. A., & Munisamy, S. (2015). Eco-efficiency in greenhouse emissions among manufacturing industries: A range adjusted measure. *Economic Modelling, 47*, 219–227.
Rao, X., Wu, J., Zhang, Z., & Liu, B. (2012). Energy efficiency and energy saving potential in China: An analysis based on slacks-based measure model. *Computers & Industrial Engineering, 63*, 578–584.
Rashidi, K., & Farzipoor Saen, R. (2015). Measuring eco-efficiency based on green indicators and potentials in energy saving and undesirable output abatement. *Energy Economics, 50*, 18–26.
Reinhard, S., Knox Lovell, C. A., & Thijssen, G. J. (2000). Environmental efficiency with multiple environmentally detrimental variables; estimated with SFA and DEA. *European Journal of Operational Research, 121*, 287–303.
Ren, J., Tan, S., Dong, L., Mazzi, A., Scipioni, A., & Sovacool, B. K. (2014). Determining the life cycle energy efficiency of six biofuel systems in China: A data envelopment analysis. *Bioresource Technology, 162*, 1–7.
Resende, M. (2002). Relative efficiency measurement and prospects for yardstick competition in Brazilian electricity distribution. *Energy Policy, 30*, 637–647.
Riccardi, R., Oggioni, G., & Toninelli, R. (2012). Efficiency analysis of world cement industry in presence of undesirable output: Application of data envelopment analysis and directional distance function. *Energy Policy, 44*, 140–152.
Rogge, N., & De Jaeger, S. (2012). Evaluating the efficiency of municipalities in collecting and processing municipal solid waste: A shared input DEA-model. *Waste Management, 32*, 1968–1978.
Sadjadi, S. J., & Omrani, H. (2008). Data envelopment analysis with uncertain data: An application for Iranian electricity distribution companies. *Energy Policy, 36*, 4247–4254.
Sala-Garrido, R., Hernández-Sancho, F., & Molinos-Senante, M. (2012). Assessing the efficiency of wastewater treatment plants in an uncertain context: A DEA with tolerances approach. *Environmental Science & Policy, 18*, 34–44.
Sanhueza, R., Rudnick, H., & Lagunas, H. (2004). DEA efficiency for the determination of the electric power distribution added value. *IEEE Transactions on Power Systems, 19*, 919–925.
Sarica, K., & Or, I. (2007). Efficiency assessment of Turkish power plants using DEA. *Energy, 32*, 1484–1499.
Sarkis, J., & Cordeiro, J. J. (2012). Ecological modernization in the electrical utility industry: An application of a bads-goods DEA model of ecological and technical efficiency. *European Journal of Operational Research, 219*, 386–395.
Sarkis, J., & Weinrach, J. (2001). Using data envelopment analysis to evaluate environmentally conscious waste treatment technology. *Journal of Cleaner Production, 9*, 417–427.

Scheel, H. (2001). Undesirable outputs in efficiency valuations. *European Journal of Operational Research, 132*, 400–410.

Serra, T., Chambers, R. G., & Oude Lansink, A. (2014). Measuring technical and environmental efficiency in a state-contingent technology. *European Journal of Operational Research, 236*, 706–717.

Serra, T., & Poli, E. (2015). Shadow prices of social capital in rural India, a nonparametric approach. *European Journal of Operational Research, 240*, 892–903.

Shakouri, G. H., Nabaee, M., & Aliakbarisani, S. (2014). A quantitative discussion on the assessment of power supply technologies: DEA (data envelopment analysis) and SAW (simple additive weighting) as complementary methods for the "Grammar". *Energy, 64*, 640–647.

Sheng, Y., Wu, Y., Shi, X., & Zhang, D. (2015). Energy trade efficiency and its determinants: A Malmquist index approach. *Energy Economics, 50*, 306–314.

Shi, G.-M., Bi, J., & Wang, J.-N. (2010). Chinese regional industrial energy efficiency evaluation based on a DEA model of fixing non-energy inputs. *Energy Policy, 38*, 6172–6179.

Skevas, T., Stefanou, S. E., & Oude Lansink, A. (2014). Pesticide use, environmental spillovers and efficiency: A DEA risk-adjusted efficiency approach applied to Dutch arable farming. *European Journal of Operational Research, 237*, 658–664.

Song, C., Li, M., Zhang, F., He, Y.-L., & Tao, W.-Q. (2015a). A data envelopment analysis for energy efficiency of coal-fired power units in China. *Energy Conversion and Management, 102*, 121–130.

Song, M. L., & Wang, S. H. (2014). DEA decomposition of China's environmental efficiency based on search algorithm. *Applied Mathematics and Computation, 247*, 562–572.

Song, M., Yang, L., Wu, J., & Lv, W. (2013a). Energy saving in China: Analysis on the energy efficiency via bootstrap-DEA approach. *Energy Policy, 57*, 1–6.

Song, M.-L., Zhang, L.-L., Liu, W., & Fisher, R. (2013b). Bootstrap-DEA analysis of BRICS' energy efficiency based on small sample data. *Applied Energy, 112*, 1049–1055.

Song, M., Zhang, J., & Wang, S. (2015b). Review of the network environmental efficiencies of listed petroleum enterprises in China. *Renewable and Sustainable Energy Reviews, 43*, 65–71.

Sözen, A., & Alp, I. (2009). Comparison of Turkey's performance of greenhouse gas emissions and local/regional pollutants with EU countries. *Energy Policy, 37*, 5007–5018.

Sözen, A., Alp, I., & Kilinc, C. (2012). Efficiency assessment of the hydro-power plants in Turkey by using data envelopment analysis. *Renewable Energy, 46*, 192–202.

Sözen, A., Alp, I., & Özdemir, A. (2010). Assessment of operational and environmental performance of the thermal power plants in Turkey by using data envelopment analysis. *Energy Policy, 38*, 6194–6203.

Stallard, T., Rothschild, R., & Aggidis, G. A. (2008). A comparative approach to the economic modelling of a large-scale wave power scheme. *European Journal of Operational Research, 185*, 884–898.

Sueyoshi, T. (1999). Tariff structure of Japanese electric power companies: An empirical analysis using DEA. *European Journal of Operational Research, 118*, 350–374.

Sueyoshi, T. (2000). Stochastic DEA for restructure strategy: An application to a Japanese petroleum company. *Omega, 28*, 385–398.

Sueyoshi, T., & Goto, M. (2001). Slack-adjusted DEA for time series analysis: Performance measurement of Japanese electric power generation industry in 1984-1993. *European Journal of Operational Research, 133*, 232–259.

Sueyoshi, T., & Goto, M. (2009a). Can environmental investment and expenditure enhance financial performance of US electric utility firms under the clean air act amendment of 1990? *Energy Policy, 37*, 4819–4826.

Sueyoshi, T., & Goto, M. (2009b). Can R&D expenditure avoid corporate bankruptcy? Comparison between Japanese machinery and electric equipment industries using DEA-discriminant analysis. *European Journal of Operational Research, 196*, 289–311.

Sueyoshi, T., & Goto, M. (2010a). Measurement of a linkage among environmental, operational, and financial performance in Japanese manufacturing firms: A use of data envelopment

analysis with strong complementary slackness condition. *European Journal of Operational Research, 207*, 1742–1753.
Sueyoshi, T., & Goto, M. (2010b). Should the US clean air act include CO_2 emission control?: Examination by data envelopment analysis. *Energy Policy, 38*, 5902–5911.
Sueyoshi, T., & Goto, M. (2011a). DEA approach for unified efficiency measurement: Assessment of Japanese fossil fuel power generation. *Energy Economics, 33*, 292–303.
Sueyoshi, T., & Goto, M. (2011b). Measurement of returns to scale and damages to scale for DEA-based operational and environmental assessment: How to manage desirable (good) and undesirable (bad) outputs? *European Journal of Operational Research, 211*, 76–89.
Sueyoshi, T., & Goto, M. (2011c). Methodological comparison between two unified (operational and environmental) efficiency measurements for environmental assessment. *European Journal of Operational Research, 210*, 684–693.
Sueyoshi, T., & Goto, M. (2012a). Data envelopment analysis for environmental assessment: Comparison between public and private ownership in petroleum industry. *European Journal of Operational Research, 216*, 668–678.
Sueyoshi, T., & Goto, M. (2012b). DEA environmental assessment of coal fired power plants: Methodological comparison between radial and non-radial models. *Energy Economics, 34*, 1854–1863.
Sueyoshi, T., & Goto, M. (2012c). DEA radial and non-radial models for unified efficiency under natural and managerial disposability: Theoretical extension by strong complementary slackness conditions. *Energy Economics, 34*, 700–713.
Sueyoshi, T., & Goto, M. (2012d). DEA radial measurement for environmental assessment and planning: Desirable procedures to evaluate fossil fuel power plants. *Energy Policy, 41*, 422–432.
Sueyoshi, T., & Goto, M. (2012e). Efficiency-based rank assessment for electric power industry: A combined use of data envelopment analysis (DEA) and DEA-discriminant analysis (DA). *Energy Economics, 34*, 634–644.
Sueyoshi, T., & Goto, M. (2012f). Environmental assessment by DEA radial measurement: U.S. coal-fired power plants in ISO (independent system operator) and RTO (regional transmission organization). *Energy Economics, 34*, 663–676.
Sueyoshi, T., & Goto, M. (2012g). Returns to scale and damages to scale on U.S. fossil fuel power plants: Radial and non-radial approaches for DEA environmental assessment. *Energy Economics, 34*, 2240–2259.
Sueyoshi, T., & Goto, M. (2012h). Returns to scale and damages to scale under natural and managerial disposability: Strategy, efficiency and competitiveness of petroleum firms. *Energy Economics, 34*, 645–662.
Sueyoshi, T., & Goto, M. (2012i). Returns to scale and damages to scale with strong complementary slackness conditions in DEA assessment: Japanese corporate effort on environment protection. *Energy Economics, 34*, 1422–1434.
Sueyoshi, T., & Goto, M. (2012j). Returns to scale, damages to scale, marginal rate of transformation and rate of substitution in DEA environmental assessment. *Energy Economics, 34*, 905–917.
Sueyoshi, T., & Goto, M. (2012k). Weak and strong disposability vs. natural and managerial disposability in DEA environmental assessment: Comparison between Japanese electric power industry and manufacturing industries. *Energy Economics, 34*, 686–699.
Sueyoshi, T., & Goto, M. (2013a). A comparative study among fossil fuel power plants in PJM and California ISO by DEA environmental assessment. *Energy Economics, 40*, 130–145.
Sueyoshi, T., & Goto, M. (2013b). Data envelopment analysis as a data mining methodology for environmental assessment. *Journal of Data and Information Processing, 1*, 19–27.
Sueyoshi, T., & Goto, M. (2013c). DEA environmental assessment in a time horizon: Malmquist index on fuel mix, electricity and CO_2 of industrial nations. *Energy Economics, 40*, 370–382.
Sueyoshi, T., & Goto, M. (2013d). Returns to scale vs. damages to scale in data envelopment analysis: An impact of U.S. clean air act on coal-fired power plants. *Omega, 41*, 164–175.

Sueyoshi, T., & Goto, M. (2014a). DEA radial measurement for environmental assessment: A comparative study between Japanese chemical and pharmaceutical firms. *Applied Energy, 115*, 502–513.

Sueyoshi, T., & Goto, M. (2014b). Environmental assessment for corporate sustainability by resource utilization and technology innovation: DEA radial measurement on Japanese industrial sectors. *Energy Economics, 46*, 295–307.

Sueyoshi, T., & Goto, M. (2014c). Investment strategy for sustainable society by development of regional economies and prevention of industrial pollutions in Japanese manufacturing sectors. *Energy Economics, 42*, 299–312.

Sueyoshi, T., & Goto, M. (2014d). Photovoltaic power stations in Germany and the United States: A comparative study by data envelopment analysis. *Energy Economics, 42*, 271–288.

Sueyoshi, T., & Goto, M. (2015a). DEA environmental assessment on coal-fired power plants in U.S. north-east region by DEA non-radial measurement. *Energy Economics, 50*, 125–139.

Sueyoshi, T., & Goto, M. (2015b). DEA environmental assessment in time horizon: Radial approach for Malmquist index measurement on petroleum companies. *Energy Economics, 51*, 329–345.

Sueyoshi, T., & Goto, M. (2015c). Japanese fuel mix strategy after disaster of Fukushima Daiichi's nuclear power plant: Lessons from international comparison among industrial nations measured by DEA environmental assessment in time horizon. *Energy Economics, 52*, 87–103.

Sueyoshi, T., Goto, M., & Shang, J. (2009). Core business concentration vs. corporate diversification in the US electric utility industry: Synergy and deregulation effects. *Energy Policy, 37*, 4583–4594.

Sueyoshi, T., Goto, M., & Snell, M. A. (2013a). DEA environmental assessment: Measurement of damages to scale with unified efficiency under managerial disposability or environmental efficiency. *Applied Mathematical Modelling, 37*, 7300–7314.

Sueyoshi, T., Goto, M., & Sugiyama, M. (2013b). DEA window analysis for environmental assessment in a dynamic time shift: Performance assessment of U.S. coal-fired power plants. *Energy Economics, 40*, 845–857.

Sueyoshi, T., Goto, M., & Ueno, T. (2010). Performance analysis of US coal-fired power plants by measuring three DEA efficiencies. *Energy Policy, 38*, 1675–1688.

Sueyoshi, T., & Wang, D. (2014a). Radial and non-radial approaches for environmental assessment by data envelopment analysis: Corporate sustainability and effective investment for technology innovation. *Energy Economics, 45*, 537–551.

Sueyoshi, T., & Wang, D. (2014b). Sustainability development for supply chain management in U.S. petroleum industry by DEA environmental assessment. *Energy Economics, 46*, 360–374.

Sueyoshi, T., & Yuan, Y. (2015a). China's regional sustainability and diversity: DEA environmental assessment on economic development and air pollution. *Energy Economics, 49*, 239–256.

Sueyoshi, T., & Yuan, Y. (2015b). Comparison among U.S. industrial sectors by DEA environmental assessment: Equipped with analytical capability to handle zero and negative values in Production factors. *Energy Economics, 52*(Part A), 69–86.

Taghavifar, H., Mardani, A., & Karim-Maslak, H. (2014). Multi-criteria optimization model to investigate the energy waste of off-road vehicles utilizing soil bin facility. *Energy, 73*, 762–770.

Tao, Y., & Zhang, S. (2013). Environmental efficiency of electric power industry in the Yangtze river delta. *Mathematical and Computer Modelling, 58*, 927–935.

Taskin, F., & Zaim, O. (2001). The role of international trade on environmental efficiency: A DEA approach. *Economic Modelling, 18*, 1–17.

Thakur, T., Deshmukh, S. G., & Kaushik, S. C. (2006). Efficiency evaluation of the state owned electric utilities in India. *Energy Policy, 34*, 2788–2804.

Thompson, R. G., Dharmapala, P. S., Rothenberg, L. J., & Thrall, R. M. (1996). DEA/AR efficiency and profitability of 14 major oil companies in U.S. exploration and production. *Computers & Operations Research, 23*, 357–373.

Thompson, R. G., Dharmapala, P. S., & Thrall, R. M. (1995). Linked-cone DEA profit ratios and technical efficiency with application to Illinois coal mines. *International Journal of Production Economics, 39*, 99–115.

Thompson, R. G., Lee, E., & Thrall, R. M. (1992). DEA/AR-efficiency of U.S. independent oil/gas producers over time. *Computers & Operations Research, 19*, 377–391.

Tone, K., & Tsutsui, M. (2007). Decomposition of cost efficiency and its application to Japanese-US electric utility comparisons. *Socio-Economic Planning Sciences, 41*, 91–106.

Triantis, K., & Otis, P. (2004). Dominance-based measurement of productive and environmental performance for manufacturing. *European Journal of Operational Research, 154*, 447–464.

Tsolas, I. E. (2011). Performance assessment of mining operations using nonparametric production analysis: A bootstrapping approach in DEA. *Resources Policy, 36*, 159–167.

Tyteca, D. (1996). On the measurement of the environmental performance of firms: A literature review and a productive efficiency perspective. *Journal of Environmental Management, 46*, 281–308.

Tyteca, D. (1997). Linear programming models for the measurement of environmental performance of firms: Concepts and empirical results. *Journal of Productivity Analysis, 8*, 183–197.

Tyteca, D. (1998). Sustainability indicators at the firm level. *Journal of Industrial Ecology, 2*, 61–77.

Vaninsky, A. (2006). Efficiency of electric power generation in the United States: Analysis and forecast based on data envelopment analysis. *Energy Economics, 28*, 326–338.

Vasco Correa, C. A. (2012). Economic evaluation of current conditions of competition and efficiency of automotive and rail systems in Colombia. *Energy Policy, 46*, 78–87.

Vazhayil, J. P., & Balasubramanian, R. (2013). Optimization of India's power sector strategies using weight-restricted stochastic data envelopment analysis. *Energy Policy, 56*, 456–465.

Vlontzos, G., Niavis, S., & Manos, B. (2014). A DEA approach for estimating the agricultural energy and environmental efficiency of EU countries. *Renewable and Sustainable Energy Reviews, 40*, 91–96.

Voltes-Dorta, A., Perdiguero, J., & Jiménez, J. L. (2013). Are car manufacturers on the way to reduce CO_2 emissions?: A DEA approach. *Energy Economics, 38*, 77–86.

Wang, C. (2013). Changing energy intensity of economies in the world and its decomposition. *Energy Economics, 40*, 637–644.

Wang, H. (2015). A generalized MCDA-DEA (multi-criterion decision analysis-data envelopment analysis) approach to construct slacks-based composite indicator. *Energy, 80*, 114–122.

Wang, Z., & Feng, C. (2015). Sources of production inefficiency and productivity growth in China: A global data envelopment analysis. *Energy Economics, 49*, 380–389.

Wang, Z., Feng, C., & Zhang, B. (2014a). An empirical analysis of China's energy efficiency from both static and dynamic perspectives. *Energy, 74*, 322–330.

Wang, D., Li, S., & Sueyoshi, T. (2014b). DEA environmental assessment on U.S. Industrial sectors: Investment for improvement in operational and environmental performance to attain corporate sustainability. *Energy Economics, 45*, 254–267.

Wang, K., Lu, B., & Wei, Y.-M. (2013a). China's regional energy and environmental efficiency: A range-adjusted measure based analysis. *Applied Energy, 112*, 1403–1415.

Wang, J. H., Ngan, H. W., Engriwan, W., & Lo, K. L. (2007). Performance based regulation of the electricity supply industry in Hong Kong: An empirical efficiency analysis approach. *Energy Policy, 35*, 609–615.

Wang, K., & Wei, Y.-M. (2014). China's regional industrial energy efficiency and carbon emissions abatement costs. *Applied Energy, 130*, 617–631.

Wang, K., Wei, Y.-M., & Zhang, X. (2012a). A comparative analysis of China's regional energy and emission performance: Which is the better way to deal with undesirable outputs? *Energy Policy, 46*, 574–584.

Wang, K., Wei, Y.-M., & Zhang, X. (2013b). Energy and emissions efficiency patterns of Chinese regions: A multi-directional efficiency analysis. *Applied Energy, 104*, 105–116.

Wang, K., Yu, S., & Zhang, W. (2013c). China's regional energy and environmental efficiency: A DEA window analysis based dynamic evaluation. *Mathematical and Computer Modelling, 58*, 1117–1127.

Wang, Z., Zeng, H., Wei, Y.-M., & Zhang, Y. (2012b). Regional total factor energy efficiency: An empirical analysis of industrial sector in China. *Applied Energy, 97*, 115–123.

Wang, K., Zhang, X., Wei, Y.-M., & Yu, S. (2013d). Regional allocation of CO_2 emissions allowance over provinces in China by 2020. *Energy Policy, 54*, 214–229.

Wang, Q., Zhao, Z., Shen, N., & Liu, T. (2015). Have Chinese cities achieved the win-win between environmental protection and economic development? From the perspective of environmental efficiency. *Ecological Indicators, 51*, 151–158.

Wang, Q., Zhao, Z., Zhou, P., & Zhou, D. (2013e). Energy efficiency and production technology heterogeneity in China: A meta-frontier DEA approach. *Economic Modelling, 35*, 283–289.

Wang, Q., Zhou, P., & Zhou, D. (2012c). Efficiency measurement with carbon dioxide emissions: The case of China. *Applied Energy, 90*, 161–166.

Wang, H., Zhou, P., & Zhou, D. Q. (2013f). Scenario-based energy efficiency and productivity in China: A non-radial directional distance function analysis. *Energy Economics, 40*, 795–803.

Wei, Y.-M., Liao, H., & Fan, Y. (2007). An empirical analysis of energy efficiency in China's iron and steel sector. *Energy, 32*, 2262–2270.

Wei, C., Löschel, A., & Liu, B. (2015). Energy-saving and emission-abatement potential of Chinese coal-fired power enterprise: A non-parametric analysis. *Energy Economics, 49*, 33–43.

Wei, C., Ni, J., & Sheng, M. (2011). China's energy inefficiency: A cross-country comparison. *The Social Science Journal, 48*, 478–488.

Wei, Z., Zhao, P., & Ai, S. (2012). Efficiency evaluation of Beijing intelligent traffic management system based on super-DEA. *Journal of Transportation Systems Engineering and Information Technology, 12*, 19–23.

Weyman-Jones, T. G. (1991). Productive efficiency in a regulated industry: The area electricity boards of England and Wales. *Energy Economics, 13*, 116–122.

Woo, C., Chung, Y., Chun, D., Seo, H., & Hong, S. (2015). The static and dynamic environmental efficiency of renewable energy: A Malmquist analysis of OECD countries. *Renewable and Sustainable Energy Reviews, 47*, 367–376.

Wu, J., An, Q., Ali, S., & Liang, L. (2013a). DEA based resource allocation considering environmental factors. *Mathematical and Computer Modelling, 58*, 1128–1137.

Wu, J., An, Q., Xiong, B., & Chen, Y. (2013b). Congestion measurement for regional industries in China: A data envelopment analysis approach with undesirable outputs. *Energy Policy, 57*, 7–13.

Wu, H., Du, S., Liang, L., & Zhou, Y. (2013c). A DEA-based approach for fair reduction and reallocation of emission permits. *Mathematical and Computer Modelling, 58*, 1095–1101.

Wu, F., Fan, L. W., Zhou, P., & Zhou, D. Q. (2012). Industrial energy efficiency with CO_2 emissions in China: A nonparametric analysis. *Energy Policy, 49*, 164–172.

Wu, C., Li, Y., Liu, Q., & Wang, K. (2013d). A stochastic DEA model considering undesirable outputs with weak disposability. *Mathematical and Computer Modelling, 58*, 980–989.

Wu, J., Lv, L., Sun, J., & Ji, X. (2015). A comprehensive analysis of China's regional energy saving and emission reduction efficiency: From production and treatment perspectives. *Energy Policy, 84*, 166–176.

Yadav, V. K., Chauhan, Y. K., Padhy, N. P., & Gupta, H. O. (2013). A novel power sector restructuring model based on data envelopment analysis (DEA). *International Journal of Electrical Power & Energy Systems, 44*, 629–637.

Yadav, V. K., Padhy, N. P., & Gupta, H. O. (2010). A micro level study of an Indian electric utility for efficiency enhancement. *Energy, 35*, 4053–4063.

Yadav, V. K., Padhy, N. P., & Gupta, H. O. (2011). Performance evaluation and improvement directions for an Indian electric utility. *Energy Policy, 39*, 7112–7120.

Yaisawarng, S., & Klein, J. D. (1994). The effects of sulfur dioxide controls on productivity change in the U.S. electric power industry. *The Review of Economics and Statistics, 76*, 447–460.

Yang, L., Ouyang, H., Fang, K., Ye, L., & Zhang, J. (2015). Evaluation of regional environmental efficiencies in China based on super-efficiency-DEA. *Ecological Indicators, 51*, 13–19.

Yang, H., & Pollitt, M. (2009). Incorporating both undesirable outputs and uncontrollable variables into DEA: The performance of Chinese coal-fired power plants. *European Journal of Operational Research, 197*, 1095–1105.

Yang, H., & Pollitt, M. (2010). The necessity of distinguishing weak and strong disposability among undesirable outputs in DEA: Environmental performance of Chinese coal-fired power plants. *Energy Policy, 38*, 4440–4444.

Yang, L., & Wang, K.-L. (2013). Regional differences of environmental efficiency of China's energy utilization and environmental regulation cost based on provincial panel data and DEA method. *Mathematical and Computer Modelling, 58*, 1074–1083.

Yang, F., Yang, M., & Nie, H. (2013). Productivity trends of Chinese regions: A perspective from energy saving and environmental regulations. *Applied Energy, 110*, 82–89.

Yao, X., Zhou, H., Zhang, A., & Li, A. (2015). Regional energy efficiency, carbon emission performance and technology gaps in China: A meta-frontier non-radial directional distance function analysis. *Energy Policy, 84*, 142–154.

Yeh, T.-l., Chen, T.-y., & Lai, P.-y. (2010). A comparative study of energy utilization efficiency between Taiwan and China. *Energy Policy, 38*, 2386–2394.

Ylvinger, S. (2003). Light-duty vehicles and external impacts: Product- and policy-performance assessment. *European Journal of Operational Research, 144*, 194–208.

Yu, F. W., & Chan, K. T. (2012). Improved energy management of chiller systems by multivariate and data envelopment analyses. *Applied Energy, 92*, 168–174.

Yunos, J. M., & Hawdon, D. (1997). The efficiency of the national electricity board in Malaysia: An intercountry comparison using DEA. *Energy Economics, 19*, 255–269.

Zaim, O. (2004). Measuring environmental performance of state manufacturing through changes in pollution intensities: A DEA framework. *Ecological Economics, 48*, 37–47.

Zaim, O., & Taskin, F. (2000a). A Kuznets curve in environmental efficiency: An application on OECD countries. *Environmental and Resource Economics, 17*, 21–36.

Zaim, O., & Taskin, F. (2000b). Environmental efficiency in carbon dioxide emissions in the OECD: A non-parametric approach. *Journal of Environmental Management, 58*, 95–107.

Zaneslla, A., Camanho, A. S., & Dias, T. G. (2015). Undesirable outputs and weighting schemes in composite indicators based on data envelopment analysis. *European Journal of Operational Research, 245*, 517–530.

Zhang, Y., & Bartels, R. (1998). The effect of sample size on the mean efficiency in DEA with an application to electricity distribution in Australia, Sweden and New Zealand. *Journal of Productivity Analysis, 9*, 187–204.

Zhang, B., Bi, J., Fan, Z., Yuan, Z., & Ge, J. (2008). Eco-efficiency analysis of industrial system in China: A data envelopment analysis approach. *Ecological Economics, 68*, 306–316.

Zhang, X.-P., Cheng, X.-M., Yuan, J.-H., & Gao, X.-J. (2011). Total-factor energy efficiency in developing countries. *Energy Policy, 39*, 644–650.

Zhang, N., & Choi, Y. (2013a). A comparative study of dynamic changes in CO_2 emission performance of fossil fuel power plants in China and Korea. *Energy Policy, 62*, 324–332.

Zhang, N., & Choi, Y. (2013b). Environmental energy efficiency of China's regional economies: A non-oriented slacks-based measure analysis. *The Social Science Journal, 50*, 225–234.

Zhang, N., & Choi, Y. (2014). A note on the evolution of directional distance function and its development in energy and environmental studies 1997-2013. *Renewable and Sustainable Energy Reviews, 33*, 50–59.

Zhang, H.-Y., Ji, Q., & Fan, Y. (2013a). An evaluation framework for oil import security based on the supply chain with a case study focused on China. *Energy Economics, 38*, 87–95.

Zhang, N., Kong, F., & Yu, Y. (2015). Measuring ecological total-factor energy efficiency incorporating regional heterogeneities in China. *Ecological Indicators, 51*, 165–172.
Zhang, Z., & Ye, J. (2015). Decomposition of environmental total factor productivity growth using hyperbolic distance functions: A panel data analysis for China. *Energy Economics, 47*, 87–97.
Zhang, N., Zhou, P., & Choi, Y. (2013b). Energy efficiency, CO_2 emission performance and technology gaps in fossil fuel electricity generation in Korea: A meta-frontier non-radial directional distance function analysis. *Energy Policy, 56*, 653–662.
Zhao, X., Yang, R., & Ma, Q. (2014). China's total factor energy efficiency of provincial industrial sectors. *Energy, 65*, 52–61.
Zhou, P., & Ang, B. W. (2008a). Decomposition of aggregate CO_2 emissions: A production-theoretical approach. *Energy Economics, 30*, 1054–1067.
Zhou, P., & Ang, B. W. (2008b). Linear programming models for measuring economy-wide energy efficiency performance. *Energy Policy, 36*, 2911–2916.
Zhou, P., Ang, B. W., & Han, J. Y. (2010). Total factor carbon emission performance: A Malmquist index analysis. *Energy Economics, 32*, 194–201.
Zhou, P., Ang, B. W., & Poh, K. L. (2006a). Decision analysis in energy and environmental modeling: An update. *Energy, 31*, 2604–2622.
Zhou, P., Ang, B. W., & Poh, K. L. (2006b). Slacks-based efficiency measures for modeling environmental performance. *Ecological Economics, 60*, 111–118.
Zhou, P., Ang, B. W., & Poh, K. L. (2007a). A mathematical programming approach to constructing composite indicators. *Ecological Economics, 62*, 291–297.
Zhou, P., Ang, B. W., & Poh, K. L. (2008a). A survey of data envelopment analysis in energy and environmental studies. *European Journal of Operational Research, 189*, 1–18.
Zhou, P., Ang, B. W., & Poh, K. L. (2008b). Measuring environmental performance under different environmental DEA technologies. *Energy Economics, 30*, 1–14.
Zhou, P., Ang, B. W., & Wang, H. (2012a). Energy and CO_2 emission performance in electricity generation: A non-radial directional distance function approach. *European Journal of Operational Research, 221*, 625–635.
Zhou, P., Ang, B. W., & Zhou, D. Q. (2012b). Measuring economy-wide energy efficiency performance: A parametric frontier approach. *Applied Energy, 90*, 196–200.
Zhou, G., Chung, W., & Zhang, X. (2013). A study of carbon dioxide emissions performance of China's transport sector. *Energy, 50*, 302–314.
Zhou, G., Chung, W., & Zhang, Y. (2014a). Measuring energy efficiency performance of China's transport sector: A data envelopment analysis approach. *Expert Systems with Applications, 41*, 709–772.
Zhou, X., Fan, L. W., & Zhou, P. (2015). Marginal CO2 abatement costs: Findings from alternative shadow price estimates for Shanghai industrial sectors. *Energy Policy, 77*, 109–117.
Zhou, P., Poh, K. L., & Ang, B. W. (2007b). A non-radial DEA approach to measuring environmental performance. *European Journal of Operational Research, 178*, 1–9.
Zhou, P., Sun, Z. R., & Zhou, D. Q. (2014b). Optimal path for controlling CO_2 emissions in China: A perspective of efficiency analysis. *Energy Economics, 45*, 99–110.
Zofío, J. L., & Prieto, A. M. (2001). Environmental efficiency and regulatory standards: The case of CO_2 emissions from OECD industries. *Resource and Energy Economics, 23*, 63–83.
Zou, G., Chen, L., Liu, W., Hong, X., Zhang, G., & Zhang, Z. (2013). Measurement and evaluation of Chinese regional energy efficiency based on provincial panel data. *Mathematical and Computer Modelling, 58*, 1000–1009.

Chapter 18
Corporate Environmental Sustainability and DEA

Joseph Sarkis

Abstract Data envelopment analysis (DEA) is a flexible management tool and methodology that can be utilized in a variety of ways. This flexibility is evident in the applications of DEA for investigating corporate environmental sustainability and management. In this chapter an overview of DEA and how it can be utilized alone and with other techniques to investigate corporate environmental sustainability questions is presented. Discussion on how DEA has been used for environmental sustainability theory development and testing using empirical information makes up a core aspect of some of the major contributions DEA has provided in this field. DEA is also used as a management decision support tool, which includes benchmarking and multiple criteria decision making. Some details on how each was used with exemplary references are included. Some future DEA directions that could be used for research and application in corporate environmental sustainability is also defined.

Keywords Data envelopment analysis • Greening • Environmental • Business • Benchmarking • Decision making

18.1 Introduction

Data envelopment analysis (DEA) has seen many years of application on issues related to organizational environmental sustainability in general and environmental performance in particular. Although emerging from the economics literature as a production frontier methodology with the traditional economic efficiency of outputs generated from inputs, DEA has expanded in perspective and application. The use of DEA has expanded as a descriptive analysis tool, to generate data for statistical analysis and inferencing, and as a prescriptive decision support tool for organizational decision making.

J. Sarkis (✉)
Foisie School of Business, Worcester Polytechnic Institute, 100 Institute Road, Worcester, MA 01609, USA
e-mail: jsarkis@wpi.edu

S.-N. Hwang et al. (eds.), *Handbook of Operations Analytics Using Data Envelopment Analysis*, International Series in Operations Research & Management Science 239, DOI 10.1007/978-1-4899-7705-2_18

In this chapter we present how some literature investigates corporate environmental sustainability. Much of the work reviewed here focuses on DEA based developments and research by the author of this chapter and some of the lessons learned. A summary of these investigations and future research is also presented. The discussion will focus on the application and usefulness of the DEA tools. Mathematical modeling of the DEA models, many of which are covered elsewhere in this book, will not be detailed. Only general presentation of the DEA-based models is discussed. Individual DEA models and joint application with supporting synergistic tools are also mentioned without delving too deeply in to the mathematical notation and development. The original papers provide a better and detailed exposition of the reviewed models and papers.

Thus, a descriptive and applied perspective will be the methodological approach used in this chapter.

18.2 Corporate Environmental Sustainability

Concerns about industrial and commerce related environmental issues have increased over the years. The reasons for this concern are manifold and range from social, scientific, and technological developments over the years to various specific regional and global pressures faced by these organizations. Social media, instant communication, advances in science, evolving regulatory have all contributed to this increased knowledge and pressures by citizens, communities, regulators, competitors, and consumers.

The science around some of today's environmental problems has been critical in convincing society and organizations that the concerns are real and require some form of alteration on practices. Two areas where this is especially true are in the depletion of natural resources, necessary for continual production, and climate change. Socially, there is greater awareness throughout the world, especially in emerging nations such as China, India, and Brazil, that we must do more to protect ourselves and our environment. As the world continues to develop economically, environmental burdens and their impact on quality of life have raised social awareness. Communications technology, such as the internet and social media, have made environmental information and social communication easier to access than at any other time in history. New regulations that are flexible and voluntary, sometimes supported with market mechanisms or incentives are becoming more evident and putting organizations in unique positions to more carefully respond to them. Finally, industrial self-regulation for corporate social responsibility through certifications, eco-labels, and industrial best practices are also becoming more evident.

The response by industry has not only been from a risk and liability reduction perspective with only a focus on minimizing negative regulatory exposure, but also from more competitive and business case reasons. One of the major reasons that organizations seek to implement practices and technology to reduce ecological footprints is because it can save costs. This win-win opportunity of lowering costs arises when waste is eliminated from a system. This joint gain arises from building organizational 'eco-efficiency'. Eco-efficiency is closely aligned with general efficiency in seeking to minimize inputs and bad outputs, while maximizing good outputs (Dyckhoff and Allen 2001; Egilmez and Park 2014). Usually waste is considered a bad output and seeking to be minimized. Waste can be solid, gaseous, or liquid. Clearly, measuring and evaluating efficiency, eco-efficiency or otherwise is a DEA goal.

In addition organizations may wish to make investments associated with improving environmental sustainability performance. DEA can be effectively be used in this situation as a multiple criteria decision tool (Sarkis 2000a; Gonzalez et al. 2015). Thus, if an organization seeks to make a selection decision it would consider multiple dimensions including environmental, economic, and business dimensions.

Given that DEA is valuable for performance measurement. It can simplify multiple dimensions to a smaller set of performance metrics. Corporate environmental sustainability and green supply chains are both examples of situations where performance measurement has gained in interest and importance.

From a research perspective DEA can be used to evaluate large sources of data for empirical relationships and statistical inferencing. The research questions may be directly related to DEA outputs to determine if there are differences in efficiency scores or as dependent or independent variables of standard econometric approaches.

Each of these DEA-based applications with example situations and references are now overviewed. Additional resources for application of DEA for corporate environmental sustainability topics do exist (e.g. see Sarkis and Talluri 2004a, for additional examples).

18.3 Theory Testing and Statistical Inferencing with DEA: An Environmental Perspective

DEA results may be statistically evaluated and this approach is valuable for broader theoretical or econometric evaluation. The major difficulty with DEA data is that it does not necessarily fall within some of the distribution requirements assumed by various statistical inferential tools. Thus, there is a reliance on non-parametric statistical inferential techniques.

Results of DEA are typically relative efficiencies for organizations or units within organizations. These efficiency scores can then be used to evaluate theory using non-parametric statistical techniques. The major non-parametric tools that have been identified by the literature are based on ranking statistics. The two models that are specifically recommended by Sarkis (2000a, b) are the Mann-Whitney U-test and the Kruskal-Wallis rank tests.

Some of the theory testing from a corporate sustainability perspective has included evaluating waste management location analysis as a comparative analysis with other multiple criteria decision tool to determine if rank orders were similar (Sarkis 2000a). In this situation, Kendall's Tau-b test was utilized to evaluating of the rankings by outranking and DEA approaches were statistically similar. The methodology utilized a weight constrained DEA approach and found that the more constrained the DEA model, based on relative importance considerations, the better the match to the outranking approaches.

18.3.1 Financial and Environmental Performance Relationship

In the above approaches a direct ranking relationship and determining whether significant differences were completed using univariate tests. For more advanced econometric testing the utilization of multivariate techniques would be more appropriate. In this situation a number of variations using DEA were utilized.

A direct approach of using DEA to determine environmental efficiency with the U.S. environmental protection agency's toxics releases inventory was used as an independent variable for the following relationship test (Sarkis and Cordeiro 2001):

$$\begin{aligned}&\text{Firm short-run financial performance}\\&\quad = \mathrm{f}(\text{Environmental efficiency; firm size; firm leverage; error})\end{aligned}$$

In this situation the statistical and theoretical examination focused on whether a relationship existed between firm environmental efficiency and short-run financial performance. This relationship is probably one of the most studied and focuses on whether 'doing good', on environmental performance is related to 'doing well' economically. The selection of the input and output variables in this case were focused on altering the DEA model where one model focused on pollution prevention efficiency, and the other model focused on end-of-pipe efficiency. The theoretical proposition was that stronger relationships would exist with pollution prevention.

The efficiency measures utilize a time difference approach. Where previous year's data was compared to current year data to show improvements in performance.

18.3.2 Ecological Efficiency and Technological Disposition Relationship

An ecological modernization perspective (Sarkis and Cordeiro 2009, 2012) argues that technology can help countries and organizations separate economic growth from environmental degradation. Ecological modernization theory began as a broad-based national policy instrument, but has been discussed as an opportunity for industry and individual organizations. This theory sets the foundation for investigating the relationship between technological choices and ecological and/or economic efficiency. For example, the empirical evaluation can be completed using the following empirical relationship for efficiency at the electrical plant level.

$$\begin{aligned} \mathit{Plant\,Efficiency} = b0 &+ b1(\mathit{Average\,Generator\,age}) \\ &+ b2\,(\mathit{Utility\,Ownership}) + b3\,(\mathit{Plant\,operates\,Scrubber}) \\ &+ b4\,\left(\mathit{Plant\,usesGas\,Fuel}\right) + b5\,(\mathit{Plant\,Uses\,Coal\,Fuel}) \\ &+ b6\,\left(\mathit{Plant\,Used\,FGD\,Scrubber\,Technology\,to\,Comply}, \text{“}\mathit{End\,of\,Pipe}\text{”}\right) \\ &+ b7\,(\mathit{Plant\,Purchased\,Credits\,to\,Comply}) \\ &+ b8\,(\mathit{Plant\,Changed\,Fuels\,or\,Fuel\,Blend}, \text{“}\mathit{In\text{-}Process}\text{”}) \\ &+ b9\text{-}18\,(\mathit{Location}, \mathit{Regions}\,1\text{-}9, 11) + \mathit{error} \end{aligned}$$

The inputs and outputs for these models can be altered to identify ecological versus technical (business) efficiency, which is the dependent variable in this empirical relationship.

The methodologies for each paper, although the data and theory were similar, had variations in the types of DEA models used and multivariate regression analysis.

A variety of DEA models can be used to evaluate the efficiency. One characteristic that I typically choose are DEA models that may have a broader variation in efficiency scores. That is, efficiency scores that are not truncated at either 1 or at 0. One such model is the Tchebycheff radius DEA model (Rousseau and Semple 1995). The efficiency scores in this situation can take on a continuous positive and negative number values. In these situations, undesireable outputs (e.g. pollution effluents) were just treated as inputs into the system, where lessening of undesireable outputs was a goal similar to inputs when seeking to create greater efficiency. Another DEA model used to help reduce the issue of truncation was the superefficiency slack based model (Cooper et al. 2007).

The resulting conclusions of these studies were that the inclusion and consideration of ecological factors into performance evaluation by organizations can significantly change how organizations view operational and investment decisions. A complete analysis of goods and bads from a technical and ecological efficiency perspective rather than from the perspective of technical efficiency alone may alter management's perspectives on their operational decisions.

Another version of DEA models that is very popular in environmental efficiency modeling is the use of bad inputs and outputs that may be separable or inseparable (Cooper et al. 2007). This was the model used in the second in the series on ecological modernization (Sarkis and Cordeiro 2012). The theory was also focused on whether more proactive measure would be better suited for overall and technical efficiency. The separable-non-separable factors in inputs and outputs were meant to determine which factors were more closely and directly aligned. For example boiler heat was directly related to emissions, but boiler capacity may be more separable and not as directly correlated with emissions. This modeling allows bad outputs to remain as outputs.

The multivariate regression analysis utilized Tobit regression when the Tchebycheff radius methodology and slack-based superefficiency models were used (Sarkis and Cordeiro 2012). This usage occurred even though the only truncation of data occurred in the slack-based model. It is acknowledged that the use of second-stage explanatory regression models in DEA, while frequently employed, continues to be viewed by some as controversial. If suitable alternatives are non-existent for second stage multiple regression analysis of DEA results (e.g. including all variables in a DEA model), the use of truncated multiple regression approaches may be the only alternative. Other techniques to overcome some of the correlation issues have been recommended (Simar and Wilson 2007).

18.3.3 Environmental Practices, Performance and Risk Management

Many organizations seek to adopt environmental practices to help reduce risks. This risk management perspective requires that organizations consider how to minimize risks by reducing hazardous waste materials, liability exposure, and improved human health. In fact, much of the U.S. Environmental Protection Agency programs have to do with limiting human health exposure with respect to environmental issues.

In one study (Sarkis 2006) that considered these relationships a series of questions relating to when and what was adopted in risk management and environmental practices were compared to environmental performance. The research questions were general and included:

1. Are earlier adopting organizations better environmental performers, and do they adopt more environmental and risk management programs and practices?
2. Is there a positive relationship between better current environmental performers and adoption of environmental practices?
3. Is there a positive relationship between organizations that improve their environmental performance over time and the amount of environmental and risk management programs adopted?

4. Should organizations adopt more environmental and risk management programs?

To test various hypotheses that seek to answer the above questions, this paper provided two different modeling approaches. First, it varied the input and output measures to determine the specific type of environmental performance that was to be evaluated. Also, variations in the type of DEA model were also used.

Unlike the undesireable outputs approaches described in the previous sections (e.g. making the outputs inputs, or having a different negative valuation in slack-based approaches) this study rescales the undesireable outputs. The rescaling was completed by taking the largest value for each of the outputs and subtracting the value for each facility from this large value. Thus, with this rescaling a larger value is considered to be better, as is the requirement for output data.

The Mann-Whitney U non-parametric independent samples test was used to evaluate a number of hypotheses. This is unlike the use of multivariate regression models. To be able to complete this analysis two groups were formed those that had various factor above and below average valuations and then the inference test was utilized based on efficiency scores and whether statistically significant differences occurred.

18.4 Benchmarking and Key Performance Indicators with DEA

In practice and application, DEA can be used to help organizations complete benchmarking and performance evaluation. DEA as a benchmarking tool can help identify organizational environmental performance and eco-efficiency weaknesses and to address those issues. Alternatively, it can help organizations identify best practices that can be diffused throughout the organizations. Benchmarking and performance measurement are ways that managers can continuously improve their operations. Using DEA as a performance measurement and benchmarking tool has become commonplace (Zhu 2014).

External benchmarking using DEA has typically been on financial or marketing performance and measures, for example with the banking industry. Internal benchmarking has also been developed for internal process improvement. Benchmarking using DEA has been used with respect to the envelopment side of the ratio based linear programming formulation. That is, the units that have a positive efficiency score form the facet set and are regarded as the benchmark DMUs. In other words, it is these DMUs that should be benchmark partners for the organization that wishes to improve its operations.

Benchmarking using DEA may not just be focused on using the facet sets from DEA based models. Another approach that may be useful is through identification of weights used for identifying efficient units in the objective function. Unfortunately, this is not a guaranteed approach since DEA models can generate alternative weight sets for the optimal efficiency score. But, if weights are to be used, various clustering approaches of weights can identify benchmark partners and groups (e.g. Sarkis and Talluri 2004b).

Using fossil fuel electricity generating utilities benchmarking across plants was completed using DEA and clustering approaches (Sarkis 2004). This study evaluated the eco-efficiencies of the top 100 major U.S. fossil-fueled electricity generating plants from 1998 data. The efficiency scores were treated by a clustering method in identifying benchmarks for improving poorly performing plants. Efficiency measures were based on three resource input measures including boiler generating capacity, total fuel heat used, and total generator capacity, and four output measures including actual energy generated, SO2 (sulfur dioxide), NOx (nitrous oxide), and CO2 (carbon dioxide) emissions. The benchmarking was completed to show some characteristics of the benchmarked plants and groups. These characteristics may or may not be in control of management but could provide insights into what may contribute to various performance characteristics of DMUs (plants). Cross efficiency approaches can also be applied in these circumstances to help identify averaged solutions (Talluri and Sarkis 1997, 2002).

The organizational supply chain is an important and emergent area of benchmarking for organizational environmental sustainability (Yakovleva et al. 2012). Although much of the current focus on supply chain sustainability is on the dyadic relationship, extensions to multiple tiers of the supply chain and identifying critical success factors is a recent area of research (Grimm et al. 2014). Benchmarking individual dyads or multiple tiers from sustainable supply chain perspective, in itself, is a complex issue. Not only can their multiple dimensions of business and activities in the supply chain.

The supply chain operations reference (SCOR) model is an example of the complexity and variety of measures that can be used for benchmarking supply chains in general. The SCOR model categorizes the processes of five supply chain stages: plan, source, make, deliver and return. Within each of these stages there SCOR categorizes performance measures on cost, time, quality, flexibility, and innovation dimensions (Bai et al. 2012). Adding these dimensions environmental factors to economic and business factors only adds to the complexity. The literature has accepted the multidimensional and complex relationships of supply chain sustainability performance evaluation (Varsei et al. 2014). Given the potentially large data sets and the need to capture all this data, finding the best, key performance, metrics for sustainable supply chains requires significant development and thought. DEA alone, or with other tools, can provide some important answers. Identifying the most pertinent data in terms of additional information provided by the data may be a way of limiting the complexity.

Along this track, the use of rough set theory, an information set theory methodology for data mining, along with DEA can provide a tool for helping to filter and identify key performance indicators (Bai and Sarkis 2014). The results show that key performance indicators can be determined using neighborhood rough set by reducing overlapping and closely related performance metrics. DEA performance results provide insight into relative performance, benchmarking, of suppliers.

The supply chain sustainability performance results from both the neighborhood rough set and DEA can be quite sensitive to the parameters selected and sustainability key performance indicator sets that were determined. Thus, careful monitoring of using these joint tools may still be required. Although, the use of rough set and DEA can greatly reduce the number of metrics and measures that are needed in this complex environment.

Advances in DEA for supply chain management may include network based techniques to complete benchmarking and decision making associated with sustainability and supply chains (e.g. Chen and Yan 2011; Aviles-Sacoto et al. 2015).

18.5 Multiple Criteria Decision Making with DEA

DEA as a singular approach or jointly with other approaches can be effectively applied to decision making contexts. Decisions facing corporate sustainability and environmental contexts, as mentioned in the previous section are complex. DEA is a tool that can help with data mining and simplifying complex and multiple dimensions to a single or smaller subset of dimensions can prove helpful for decision making.

DEA can be used effectively as a multiple criteria decision making tool (Cook et al. 2014; Doyle and Green 1993; Sarkis 1997, 1999; Sarkis and Talluri 1999). The evaluation of environmental projects or programs is one application of the various DEA models. Since environmental technology and programs are typically strategic, the use of multiple factors and complex factors is standard practice. These multiple factors may include tangible and intangible characteristics. DEA is suitable for this mixture of criteria and factors. With the DEA ranking approaches available, the decision making for these programs become clear. Managerial information can be integrated with these approaches by introducing weight limitation constraints, also defined as cone ratios and assurance regions (Sarkis 1999). This flexibility in DEA allows for a number of ways that ranking and multiple criteria techniques can be used. Clearly, one of the limitations of this set of models is that only deterministic and discrete alternatives can initially be considered since the decision objects and alternatives are typically the DMUs.

In this context DEA can be used as a valuable managerial decision tool. For example DMU's can be various environmental technologies that an organization needs to investigate for potential adoption, or selection of suppliers based on environmental sustainability criterion (Mahdiloo et al. 2015). The criteria may be represented as inputs and outputs. Typically, in a multiple criteria decision making environment, criteria that improve as their value decreases (e.g. cost, emissions) may be considered inputs. For criteria that improve as their values increase (e.g. energy delivered) these would be considered outputs in a DEA model. The results can then be analyzed from a ranking perspective, assuming that there is ample discrimination amongst the efficiency scores of the DEA methodologies.

DEA techniques that are good discriminators are more valuable for selection decisions. Thus, the use of a number of approaches could be considered as more preferable techniques. Although care should be taken in the selection of the technique, since in many cases the final rankings are not always similar. A portfolio selection approach (where sets of environmental decisions are to be made) may also give different groupings of best choices depending on the technique. To overcome these discrepancies additional decision tools or other factors can be considered in final evaluations, or some form of portfolio score can be determined. These are issues that require additional investigations.

18.5.1 Justifying and Choosing Environmental Technologies

One very important multiple criteria decision making analysis approach is the justification, selection and management of environmental technologies. Environmental technologies and innovations can be defined broadly. One set of technologies can include standard hard, tangible technologies, such as scrubbers for end-of-pipe solutions in the utility industry or purchase of solar panels for renewable energy generation (Sarkis and Tamarkin 2005; Sarkis and Cordeiro 2009, 2012). There are softer environmental technologies such as green information systems and software to help in planning and design of environmentally sound products (Bai and Sarkis 2013). Examples would include life cycle analysis tools and computer aided design systems for ecological design of products. Another innovation or technology category may include control technologies that help monitor and address environmental sustainability issues (Sarkis and Weinrach, 2001). These tools and technologies can be software or hardware oriented as well, but help to manage processes by limiting emissions or quality and scrap of materials during processing. They may also provide information to help manage environmental sustainability such as with smart grids and energy reporting (Bai and Sarkis 2013; Sarkis et al. 2013). One additional set of organizational technologies are organizational process innovations. For example environmental management systems and standards such as ISO 14000 may be considered organizational technological innovations. Inter-organizational innovations would be various green supply chain practices (Zhu et al. 2012).

A couple of ways to help filter the decisions and incorporate managerial preference is through the integration of DEA with other decision tools such as the analytical hierarchy process (AHP) or analytical network process (ANP) (Saaty 1996) and multiattribute utility theory (Keeney and Raiffa 1976), may help management filter to a better solution. As mentioned earlier managerial preferences for criteria may help restrict the weights (or relative weights) that are given to each of the criteria. One approach of completing this step is by adding assurance regions (AR).

The concept of AR is described in detail by Thompson et al. (1990). AR requires a definition of upper and lower bounds for each input and output weight. The upper and lower bounds for each weight can help define constraints that relate the weight values, and potentially managerial importance values, of various factors. A simple example for defining the AR constraints for two input weights v1 and v2 is initiated by setting lower (LB) and upper bounds (UB) on each weight. These LB and UB may be ranges for preference weights for each of the criteria from the decision makers. The AR constraints relate the weights and their bounds to each other.

These constraints can be added to various DEA models directly. If the upper and lower bounds of the weights for all the factors are known with certainty, or have exact agreement among managers, and do not need a range to define them, the AR constraints would be equalities. From a computational perspective additional constraints may slow the procedure down. For examples of this application using AHP to limit the weights see Sarkis (1997, 1999).

Various factors for decision making in this environment may be mixed and incorporate business and environmental sustainability dimensions. For example, Cost, Quality, Recyclability, Process Waste Reduction, Packaging Waste Reduction, and Regulatory Compliance may all be decision factors that influence the selection of an environmentally significant technology (Bai and Sarkis, 2012; Sarkis and Dijkshoorn, 2007). The first two factors, Cost and Quality, would be considered standard business performance measures that may be used to evaluate any program or project within an organization. The remaining measures are those that focus primarily on the environmental operations and manufacturing characteristics. These environmentally based factors cover a spectrum from reactive environmental measures (e.g. Regulatory compliance) to proactive measures (e.g. process waste reduction). There may be many more factors that could be considered, as evidenced in the benchmarking discussion. Some filtration process, e.g. using information theoretic approaches, may allow for some initial evaluation of the factors that eliminates less important ones and considers these factors as the primary ones that should be used to evaluate these programs.

Others have applied multiattribute utility theory (MAUT) approaches and ANP as data generators for DEA data. Using these approaches to help generate data can overcome some of the difficulties of qualitative data while incorporating managerial preferences. Thus, the ordering of methodologies, DEA/ANP/MAUT can work interchangeably in terms of developing and implementing various measures for multiple criteria decision making.

18.6 Future Research Directions

Given the hundreds of variations and developments in DEA there remains ample opportunity for utilizing these tools for further investigations. Multi-tier and network DEA can be utilized to investigate sustainable supply chain issues that allow for consideration of multiple levels of supply chain tiers. The field of multi-tier sustainable supply chains is very much in its infancy and even the most basic models can make a contribution to the body of knowledge in corporate environmental sustainability. The difficulty arises in finding practical data and examples. The integration of DEA with life cycle analysis data may be the most appropriate approach to link the two techniques. Although some modeling has been completed to link DEA and life cycle assessment, the analysis is still for a single stage in the supply chain.

Many applications of DEA and corporate eco-efficiency and environmental sustainability research has focused on traditionally polluting and industrial organizations. Service organizations can also have significant environmental sustainability implications. DEA has been effectively applied for service organization performance (Sherman and Zhu 2013). Identification of various environmental measures in this context is important and may include various energy and materials waste aspects. For example, information technology plays a big role in service organizations and greening information technology investigations using DEA with empirical, benchmarking and decision making approaches is a fertile area for research and application.

Bootstrapping of information and data with application to DEA may be helpful when data is not available to make a complete analysis or requiring some form of discrimination amongst DMUs. Bootstrapping methodologies to help randomly generate data based on current data availability and characteristics is an important aspect of DEA that has significant room for application in environmental sustainability management within organizations. Although Lothgren (1998)) describes and evaluates alternative DEA-based bootstrapping estimation, which can be used in these studies, other techniques do exist. For example, the use and application of Bayesian analysis based simulation procedures to generate data and their impact on DEA is a potential bootstrapping approach which is a fruitful direction for future research. Currently, some modeling using Bayesian for sustainable supplier selection has been applied (Sarkis and Dhavale 2015), expanding these valuations with an integration of DEA as a benchmarking or MCDM tool could be a multiple methodological extension that can address some of the limitations of data generation and analysis.

Comparing and contrasting DEA with other productivity analysis approaches in environmental programs provides another opportunity for research. Evaluation using stochastic frontier analysis (SFA) and DEA can be investigated from an environmental perspective. One study (Cordeiro et al. 2012) found that results may be relatively similar, which bodes well in validating some of the DEA techniques. This methodological finding provides confidence in the pattern of results, since the approach and assumptions utilized in SFA complement those of the DEA approach. The technique was applied for environmentally oriented dimensions, comparing and contrasting to a variety of DEA under various experiments can provide insights into limitations and sensitivity of DEA in these contexts.

Multiple stages of evaluation, as in supply chain management stages with multiple organizational levels, can also be considered for temporal factors. Time based panel data to test the evolution of environmental sustainability across industries and time is a fertile area for model development, applications, and theoretical study. Basic questions that can be answered include whether organizations can and do become more efficient in their environmental sustainability efforts over time and also whether different industries make more rapid gains in environmental sustainability over time. Moreover the role played by salient environmental practices such as environmental auditing, waste monitoring, environmental policies and other support practices for improving sustainability in organizations and across supply chains is well worth investigating (Bai and Sarkis, 2012).

The further integration of DEA with a broad variety of methodologies can still be completed. Multiple criteria tools such as TOPSIS, VIKOR, and Outranking, along with various data mining tools related to entropy and information theory, can also be avenues for multi-methodological integration. An important application is to be able to aid in decision making and management of the extant environmental sustainability performance measures that exist.

The use of DEA for quantitatively oriented corporate environmental sustainability information has been well developed. Extending DEA and research to incorporate less tangible measures, such as reputation and image outcomes or programmatic characteristics, may require some adjustment or development of categorical DEA methodologies. Extensions of research to include social sustainability in organizational and supply chain activities are another important direction for research (Brandenburg et al. 2014). Broader organizational and supply chain sustainability investigation that incorporate social sustainability performance measures may also benefit from models that incorporate intangible characteristics. Social sustainability is more likely to incorporate intangible dimensions such as equity, child labor, and diversity issues that are difficult to measure.

18.7 Conclusion

DEA has had a substantial history as a tool to investigate organizational environmental sustainability. It has proven valuable for the understanding and advancement of practice in this field from the perspective of theory evaluation and development, managerial decision making, and organizational benchmarking. Whether as a stand-alone tool or with other methodologies insights gained have been very valuable. DEA is especially beneficial because the complexities involved in understanding and managing sustainability can be effectively addressed.

Further investigations are warranted and DEA as a tool can help us make our organizations more thoughtful, efficient, and sustainable, not only for this generation but for future generations. Hopefully this chapter helps provide insights to old and new researchers to further advance study in this critically important study.

References

Aviles-Sacoto, S., Cook, W. D., Imanirad, R., & Zhu, J. (2015). Two-stage network DEA: When intermediate measures can be treated as outputs from the second stage. *Journal of the Operational Research Society, 66*(11), 1868–1877.

Bai, C., & Sarkis, J. (2012). Performance measurement and evaluation for sustainable supply chains using rough set and data envelopment analysis. In T. Boone, V. Jayaraman, & R. Ganeshan (Eds.), *Sustainable supply chains* (pp. 223–241). New York: Springer.

Bai, C., & Sarkis, J. (2013). Green information technology strategic justification and evaluation. *Information Systems Frontiers, 15*(5), 831–847.

Bai, C., & Sarkis, J. (2014). Determining and applying sustainable supplier key performance indicators. *Supply Chain Management: An International Journal, 19*(3), 275–291.

Bai, C., Sarkis, J., Wei, X., & Koh, L. (2012). Evaluating ecological sustainable performance measures for supply chain management. *Supply Chain Management: An International Journal, 17*(1), 78–92.

Brandenburg, M., Govindan, K., Sarkis, J., & Seuring, S. (2014). Quantitative models for sustainable supply chain management: Developments and directions. *European Journal of Operational Research, 233*(2), 299–312.

Chen, C., & Yan, H. (2011). Network DEA model for supply chain performance evaluation. *European Journal of Operational Research, 213*(1), 147–155.

Cook, W. D., Tone, K., & Zhu, J. (2014). Data envelopment analysis: Prior to choosing a model. *Omega, 44*, 1–4.

Cooper, W. W., Seiford, L. M., & Tone, K. (2007). *Data envelopment analysis: A comprehensive text with models, applications, references and DEA-solver software*. New York: Springer.

Cordeiro, J. J., Sarkis, J., Vazquez-Brust, D., Frater, L., & Dijkshoorn, J. (2012). An evaluation of technical efficiency and managerial correlates of solid waste management by Welsh SMEs using parametric and non-parametric techniques. *Journal of the Operational Research Society, 63*(5), 653–664.

Doyle, J., & Green, R. (1993). Data envelopment analysis and multiple criteria decision making. *Omega, 21*(6), 713–715.

Dyckhoff, H., & Allen, K. (2001). Measuring ecological efficiency with data envelopment analysis (DEA). *European Journal of Operational Research, 132*(2), 312–325.

Egilmez, G., & Park, Y. S. (2014). Transportation related carbon, energy and water footprint analysis of US manufacturing: An eco-efficiency assessment. *Transportation Research Part D: Transport and Environment, 32*, 143–159.

Gonzalez, E. D., Sarkis, J., Huisingh, D., Huatuco, L. H., Maculan, N., Montoya, J., et al. (2015). Making real progress toward more sustainable societies using decision support models and tools: Introduction to the special volume. *Journal of Cleaner Production, 105*, 1–13.

Grimm, J. H., Hofstetter, J. S., & Sarkis, J. (2014). Critical factors for sub-supplier management: A sustainable food supply chains perspective. *International Journal of Production Economics, 152*, 159–173.

Lothgren, M. (1998). *How to bootstrap DEA estimators: A Monte Carlo comparison* (WP in Economics and Finance, 223).

Mahdiloo, M., Saen, R. F., & Lee, K. H. (2015). Technical, environmental and eco-efficiency measurement for supplier selection: An extension and application of data envelopment analysis. *International Journal of Production Economics, 168*, 279–289.

Keeney R. L. & Raiffa H. (1976). Decisions with Multiple Objectives. J. Wiley & Sons, New York.

Rousseau, J. J., & Semple, J. H. (1995). Radii of classification preservation in data envelopment analysis: A case study of 'Program Follow-Through'. *Journal of the Operational Research Society, 46*, 943–957.

Saaty, T. L. (1996). Analytical network process. Pittsburgh: RWS Publications.

Sarkis, J. (1997). Evaluating flexible manufacturing systems alternatives using data envelopment analysis. *The Engineering Economist, 43*(1), 25–47.
Sarkis, J. (1999). A methodological framework for evaluating environmentally conscious manufacturing programs. *Computers & Industrial Engineering, 36*(4), 793–810.
Sarkis, J. (2000a). A comparative analysis of DEA as a discrete alternative multiple criteria decision tool. *European Journal of Operational Research, 123*(3), 543–557.
Sarkis, J. (2000b). An analysis of the operational efficiency of major airports in the United States. *Journal of Operations Management, 18*(3), 335–351.
Sarkis, J. (2004). Environmental benchmarking of the largest fossil-fueled electricity generating plants in the US. In *Photonics Technologies for Robotics, Automation, and Manufacturing* (pp. 182–192). Massachusetts: International Society for Optics and Photonics.
Sarkis, J. (2006). The adoption of environmental and risk management practices: Relationships to environmental performance. *Annals of Operations Research, 145*(1), 367–381.
Sarkis, J., & Cordeiro, J. J. (2001). An empirical evaluation of environmental efficiencies and firm performance: Pollution prevention versus end-of-pipe practice. *European Journal of Operational Research, 135*(1), 102–113.
Sarkis, J., & Cordeiro, J. J. (2009). Investigating technical and ecological efficiencies in the electricity generation industry: Are there win-win opportunities? *Journal of the Operational Research Society, 60*(9), 1160–1172.
Sarkis, J., & Cordeiro, J. J. (2012). Ecological modernization in the electrical utility industry: An application of a bads–goods DEA model of ecological and technical efficiency. *European Journal of Operational Research, 219*(2), 386–395.
Sarkis, J., & Dhavale, D. G. (2015). Supplier selection for sustainable operations: A triple-bottom-line approach using a Bayesian framework. *International Journal of Production Economics, 166*, 177–191.
Sarkis, J., & Dijkshoorn, J. (2007). Relationships between solid waste management performance and environmental practice adoption in Welsh small and medium-sized enterprises (SMEs). *International Journal of Production Research, 45*(21), 4989–5015.
Sarkis, J., Koo, C., & Watson, R. T. (2013). Green information systems & technologies—This generation and beyond: Introduction to the special issue. *Information Systems Frontiers, 15*(5), 695–704.
Sarkis, J., & Talluri, S. (1999). A decision model for evaluation of flexible manufacturing systems in the presence of both cardinal and ordinal factors. *International Journal of Production Research, 37*(13), 2927–2938.
Sarkis, J., & Talluri, S. (2004a). Ecoefficiency measurement using data envelopment analysis: Research and practitioner issues. *Journal of Environmental Assessment Policy and Management, 6*(1), 91–123.
Sarkis, J., & Talluri, S. (2004b). Performance based clustering for benchmarking of US airports. *Transportation Research Part A: Policy and Practice, 38*(5), 329–346.
Sarkis, J., & Tamarkin, M. (2005). Real options analysis for "green trading": The case of greenhouse gases. *The Engineering Economist, 50*(3), 273–294.
Sarkis, J., & Weinrach, J. (2001). Using data envelopment analysis to evaluate environmentally conscious waste treatment technology. *Journal of Cleaner Production, 9*(5), 417–427.
Sherman, H. D., & Zhu, J. (2013). Analyzing performance in service organizations. *Sloan Management Review, 54*(4), 36–42.
Simar, L., & Wilson, P. W. (2007). Estimation and inference in two-stage, semi-parametric models of production processes. *Journal of Econometrics, 136*(1), 31–64.
Talluri, S., & Sarkis, J. (1997). Extensions in efficiency measurement of alternate machine component grouping solutions via data envelopment analysis. *IEEE Transactions on Engineering Management, 44*(3), 299–304.
Talluri, S., & Sarkis, J. (2002). A model for performance monitoring of suppliers. *International Journal of Production Research, 40*(16), 4257–4269.

Thompson, R. G., Langemeier, L. N., Lee, E., & Thrall, R. M. (1990). DEA sensitivity analysis of efficiency measures with an application to Kansas farming. *Journal of Econometrics*, *46*(1), 2.

Varsei, M., Soosay, C., Fahimnia, B., & Sarkis, J. (2014). Framing sustainability performance of supply chains with multidimensional indicators. *Supply Chain Management: An International Journal, 19*(3), 242–257.

Yakovleva, N., Sarkis, J., & Sloan, T. (2012). Sustainable benchmarking of supply chains: The case of the food industry. *International Journal of Production Research, 50*(5), 1297–1317.

Zhu, J. (2014). *Quantitative models for performance evaluation and benchmarking: Data envelopment analysis with spreadsheets* (Vol. 213). Basel: Springer.

Zhu, Q., Sarkis, J., & Lai, K. H. (2012). Green supply chain management innovation diffusion and its relationship to organizational improvement: An ecological modernization perspective. *Journal of Engineering and Technology Management, 29*(1), 168–185.

Index

A
AB123, 59, 62, 63, 67, 70, 74, 80–82
AB13, 61–63, 69–71, 74, 80–82
Academic Ranking of World Universities (ARWU), 2
Acceleration rate, 336
Accounting performance measurement (APM)
  accounting information, 392, 393
  balance sheets, 386, 387
  DEA model, 391
  disadvantages, 387
  DuPont model, 389
  income statements, 386
  non-accounting ratios, 392
  regulatory reporting requirements, 386
  risk measures, 391
  Statements of Cash Flow, 386
Achievement assessment capability, 449
Activity-based costing (ABC) model, 398–400, 402
Activity based management (ABM), 398
ad hoc manner, 52
Advanced manufacturing technology (AMT), 2
Aggregation, 39
American Banks (case study)
  data set
    deposits, 56–59
    inputs and outputs, 61
    interest and non-interest income, 59
    labor, 56
    loans, 59–67
    physical capital, 56
American banks efficiency, 53
Analytical hierarchy process (AHP), 492
Analytical network process (ANP), 492

B
Balanced scorecard, 402–404
BAM model, 313
Bangladeshi banks, 244
Bank technology
  bad outputs, 247–248
  directional distance function, 251
  DMU performance researchers, 247
  dynamic technology, 248–250
  production and risk, 245–246
  regulatory constraint, 261–262
Battery electric vehicles (BEV), 346
Benchmarking using DEA, 489, 490
Benevolent/aggressive models, 7
Black-box approach
  benchmarking point, 362
  DMUs, 357, 358, 360
  EDS, 359
  vs. multi-component approach, 368–369
  PPS, 360
  returns to scale, 359
  SDMU *vs.* DMU, 361
  theorem, 360
  T^{VRS}, 359
  weak and strong homogeneity condition, 361, 362
Black-box technology, 246
Bootstrapped efficiency scores, 207, 209

S.-N. Hwang et al. (eds.), *Handbook of Operations Analytics Using Data Envelopment Analysis*, International Series in Operations Research & Management Science 239, DOI 10.1007/978-1-4899-7705-2

Bootstrapping
  algorithm, 196, 198
  Brazilian container and bulk terminals, 212
  confidence intervals, 188, 189
  convexity assumptions, 188
  corroborate empirical evidences, 212
  DEA analysis, 188
  empirical application, 189
  estimation, 194–196
  Gaussian kernel density function, 196
  input/output-orientation projection, 188
  Maple 12, 211
  Monte-Carlo experiments, 196
  nonparametric efficiency, 187, 188
  productive unit, 188
  pseudo datasets, 197, 198
  returns to scale, 189
  sampling variation, 188
  statistical inference, 188
Borda–Kendall (BK) method, 6
Brazil, 198
Brazilian port terminals
  ANTAQ, 199, 200
  BRICs, 198
  convexity assumption, 201
  DEA and FDH, 199
  decision-makers, 199
  DMUs, 199
  public policies, 211
  statistics, 200, 201
  stressing, 199
  terminals researched, 200
  Transportation Ministry, 199
Brown field approach, 116

C
Carbon Disclosure Project (CDP), 430
Carbon productivity
  ecological productivity, 268
  energy productivity growth, 268
  environmental regulations, 268
Categorical parameter, 337
CCR DEA model, 4
CCR-efficiency score, 318
CCR model, 4, 123
Child immunisation, 409
China Banking Regulatory Commission (CBRC), 116
China Development Bank, 119
China Exim Bank, 119
China's Regional Energy
  data and variables, 282, 284
  integrated energy productivity change, 291
Chinese banks, 116, 117
Chung Shan Institute of Science and Technology (CSIST), 170
Citicorp (CIH), 67
Clean coal technology, 455
Coal-fired power plant, 453
Cobb-Douglas function, 135
Common set of weights (CSW), 3
COMPUSTAT, 430
Congestion, 451
Constant returns-to-scale (CRS), 135, 190, 313
Consumer price index (CPI), 335
Contingency theory, 405, 408
Convex curve, 451
Cooperative game theory
  Kalai-Smorodinsky solution, 219–220
  Nash bargaining solution, 217–219
  TU game (*see* TU cooperative games (TUCG))
Core production process, 211
Corporate environmental sustainability
  communications technology, 484
  DEA-based applications, 485
  depletion of natural resources, 484
  ecological footprints, 484
  ecological modernization, 487
  environmental practices, performance and risk managemen, 488–489
  financial and environmental performance relationship, 486
  and green supply chains, 485
  industrial and commerce related environmental issues, 484
  multiple regression approaches, 488
  multivariate regression analysis, 488
  performance measurement, 485
  risk and liability reduction, 484
  technological disposition relationship, 487–488
  waste management location analysis, 486
Corporate sustainability, 449
Cross-efficiency evaluation
  aggregation, 16–20
  ARWU, 2
  CSW, 3
  DEA models, 2
  DEA weights, 6
  MADM, 1
  principal advantages, 5
  ranking methods, 3–4
  ranking ranges, 11–13
  TOPSIS, 6

Cross-period distance functions, 259
CYN, 57, 61, 63, 66, 67, 70, 82

D

Data envelopment analysis (DEA), 32, 52–55, 61, 62, 67, 73, 80, 81, 83, 84, 147–148, 243
 agriculture, fishery, manufacturing and transportation industries, 458–459
 applications, 32, 483, 494
 applications to electric power industry, 456
 BCC–DEA model, 33
 benchmarking and performance evaluation, 489
 bootstrapped efficiency scores, 207–209
 CCR and BCC, 128
 China's eight economic-geographic areas, 282
 Chinese Banking Sector, 115–117
 CIs, 208, 209
 computer aided design systems, 492
 container *vs*. bulk terminals, 205, 206
 contextual variables, 118–119, 404–406, 408
 and contingent environments, 405
 convexity assumption, 207–209
 data and method, 119–127
 data and selection of indicators, 149–152
 data generation and analysis, 494
 definition, 122
 descriptive statistics, 119, 283
 directional distance functions, 269, 277–279
 DMUs, 33, 385
 economic development and corporate strategy, 459
 economic efficiency, 483
 EEI Analysis, 40
 efficient, 300
 electric power industry, 454
 energy consumption structure, 295
 energy input specific inefficiency measure, 280
 energy specific productivity and decompositions, 293
 energy utilization conservation, 279
 energy utilization efficiency growth, 292
 environmental assessment, 447–449, 451, 453, 462
 environmental efficiency index, 35–37
 environmental operations and manufacturing characteristics, 493
 environmental performance measurement, 31
 environmental productivity index, 37–38
 environmental technologies, 492
 evaluation stages, 494
 financial inputs and outputs, 393
 GDP over energy, 285
 graphical comparison, 34
 HEI, 152–153
 identification, 204
 importance of, 446
 inefficient DMU, 300
 initial estimates, 201–204
 inputs and outputs, 206–207
 life cycle analysis tools, 492
 linear programming problem, 278
 log-ratio plot *vs*. number of deleted terminals, 207, 208
 LP based comparison of energy and environmental productivity, 291
 LP based method, 300
 and MDS methodologies, 154–159
 method, 147–148
 methodology developments, 460
 model specification, 205
 modeling performance, 117
 MRSs, 302
 multidimensional scaling method, 148–149
 multiple criteria decision, 491–493
 non-linear programming problem, 278
 non-parametric statistical techniques, 485
 non-radial DEA models, 300
 optimal intensity vectors, 301
 optimization process, 122
 outliers, 207
 for performance assessment, 449
 petroleum and coal industries, 457
 points representing variables, 159
 preliminary insights, 154–159
 productivity indicators, 287, 289, 290
 by quartiles across unit and batch level activities, 401
 RAM model, 305
 ranking statistics, 485
 RTS characterizations, 209–210
 scale efficiency change, 293
 selling liabilities, 117
 and SFA, 114, 115
 specification searches using multivariate methods, 67–72
 specifications, 153
 standardised variables, 157
 technology sets, 205, 304

Data envelopment analysis (DEA) (*cont.*)
variables, 151
VRS, 34
Dataset summary, 338
DEA-based multiple control model, 390
DEA games
applications of cooperative games
multistage production systems, 231–232
Nash Decomposition, process efficiency, 231–232
production games, 233–235
game theoretical approaches, 215
DEA and MDS Methodologies, 147–149
DEA models, 2, 22–23, 54
DEA radial measurement, 436
Decision making units (DMUs), 2, 33, 122, 146, 169, 189, 243, 352, 450, 455, 456
applications, 314, 327
descriptive statistics, efficiency scores and projection points, 322
efficiency score, 315
inefficient, GRSs, 315
input and output data, 314
minimum face, 303
multiplicity issue, 326
RAM model, 315
two-dimensional input–output space, 314
Desirable congestion (DC), 415
Directional distance functions, 22
Disable congestion (DC), 420, 421
Discordant behavior, 61
Disposability concepts, 450–456
Distribution Free Approach (DFA), 52, 55
DN-Luenberger productivity indicator, 258
Dominance relations, 15
DSBM model, 313
DuPont model (DuPont analysis), 389
DuPont return, 389
Dynamic DEA, 243–245
Dynamic Malmquist index, 244
Dynamic-network performance problem, 257
Dynamic-network technology, 250–252

E
Econometrics, 52
Economic development and corporate strategy and DEA, 459
Eco-technology innovation, 428, 447
EEI, 40
Efficiency, 51, 63, 78, 461
ECM, 130–132
score, 82–83
spillovers, 115, 116
Efficiency contribution measures (ECM), 130–132
Electric power industry and DEA, 454, 456
Electric vehicles, 332
Emerging countries/BRICs, 198
Empirical analysis, 321
Envelopment-based DEA model, 319
Environmental assessment and DEA, 446–448, 451, 453, 460–462
business implications, 415
climate change, 414, 415
CO_2 abatement, 431
corporate performance enhancement, 415
corporate sustainability, 416, 436
disposability concepts, 434
efficiency measures, 434
empirical investigation, 442
environmental protection, 436
estimated CO_2 saving, 431
firm profitability, 431
GHG emissions, 414
global warming, 414, 415
indirect CO_2 emission, 431
industry-adjusted index, 434
investment effect, 434
investment strategy, 416
low-carbon technologies, 415
managerial disposability, 415
natural disposability, 415
nomenclatures, 416, 417
pollution prevention, 431
R&D expenditure, 434
R&D expense, 431
radial models, 415
technology innovation and selection, 442
technology innovation investment, 434
Environmental efficiency index (EEI), 35–37, 39
Environmental efficiency modeling, 488
Environmental productivity index, 37–38
Environmental Protection Agency (EPA), 336, 446
EPI analysis, 45
Extended data set (EDS), 359
Extended model, 129

F
FDEA, 24
Felt crisis, 449
Financial Accounting Standards Board, 387

Financial Institutions, efficiency modeling, 53, 54
Firm financial efficiency model, 395
Fixed cost and resource allocation, 21
Flourishing, 341–342
Foreign currencies translation, 396–398
Fossil fuel electricity, 490
Free Disposal Hull (FDH), 52, 55, 190
Fuel economy, 336
Fukushima Daiichi nuclear power plant, 418, 449
Full time equivalent (FTE), 150
Funding efficiency model, 395, 396
Fuzzy cross-efficiency evaluation, 24
Fuzzy efficiency scores, 102
Fuzzy multi-period efficiency, 91, 95–99

G
Game cross efficiency, 25
Gaussian kernel density functions, 201
Generally accepted accounting practice (GAAP)
  advantages, 388
  cost of sales (Income Statement), 388
  financial reporting data, 388
  inventory valuations (Balance Sheet), 388
Global efficiency score, 69
Global reference set (GRS)
  CCR model, 303
  computational efficiency, 313
  definitions, 305–307
  identification, 309–312
  negative input–output data, 314
  primal–dual based counterparts, 302
  properties, 307–309
  RAM model, 303
  RTS determination algorithms, 303
  type I and II, 302
  types, 303
Global warming and climate change, 446, 449
GMDDF model, 313
Green field investment, 116
Greenhouse gases (GHG) emissions, 446
Gross domestic production (GDP), 39

H
Hierarchical cluster analysis (HCA), 54, 78, 149
Higher education (HE) systems, 146
Hybrid electric vehicle (HEV, 332

I
IFC, 57, 61, 64, 66, 67, 70, 78, 82
IFC bank, 78
Index numbers (IN), 52
Indexing Dollar Values, 396–398
Induced ordered weighted averaging (IOWA), 17
Industrial pollution, 448
Inner and outer programs, 96
Input specific productivity indicators, 269
International Accounting Standards Board, 387
Investment strategy, 429, 430, 438–442
IOWA operators, 19

J
Japanese banks, 246

K
Kolmogorov-Smirnov test (K-S test), 138
Kruskal-Wallis rank tests, 486
Kruskal-Wallis test, 207

L
Large segment, 342–344
Linear programming (LP) model, 189, 356
Liquidity coverage ratio (LCR), 140
LP relaxation method, 302
Luenberger productivity indicator
  definition, 272
  directional distance function, 273
  economic productions, 272
  efficiency and technical changes, 275
  LEFCH, 272
  LPEFCH, 274
  LPTECH, 274
  LTECH, 272
  nulljointness assumption, 273
Luenberger pure efficiency change (*LPEC*), 276
Luenberger pure technical change (*LPTC*), 276
Luenberger scale efficiency change (*LSEC*), 276
Luenberger scale technical change (*LSTC*), 276, 277

M
Malmquist productivity index, 37, 270, 271
Managerial disposability, 454, 455

Mann-Whitney U non-parametric independent samples test, 489
Mann-Whitney U-test, 486
Maximal reference set (MRS), 301, 306
MDS configuration, 156
Measurement capability, 449
Measuring environmental performance, 35–39
Metric/classical scaling, 148
Metropolitan achievement test, 321
Midsize segment, 341–342
Minivan segment, 344–345
Mixed optimal strategy (MOS), 52
Most productive scale size (MPSS), 193, 362
MSRP, 335
Multi-attribute decision making (MADM), 1
Multi-attribute utility theory (MAUT) approaches, 493
Multi-component production system
  black-box approach, 352, 353
  capital city, 370, 373
  CCR model, 352
  DEA, 352
  discrimination power, 377
  DMUs, 352
  efficiency, 370–375, 377
  indicators, 370
  input and output variables, 370, 375
  pollution treatment processes, 353, 354
  province, 370, 371
  radial measurements, 353
  RTS, 352, 353, 375
  SDMUs, 353, 354
  shared inputs/outputs, 354
  variables, 354
  wastewater and gas treatment process, 369
Multi-criteria decision-making methods, 4
Multidimensional scaling (MDS) method, 146, 148–149
Multi-period efficiency measurement
  characteristics, 90
  description, 90
  manufacturing system, 91
  weight restrictions, 91–94
Multi-period system, 94
Multiplier-based DEA model, 317
Multiplier form DEA, 124
Multiplier models, 190

N
Nash bargaining game theory (NBGT), 217–219
  multistage production systems, 231–232
  Nash Decomposition, process efficiency, 231–232
  network topology, 226
  production games, 233–235
  target aggression model, 224
  types, probelms, 223, 225
Natural disposability, 452, 455
Natural and managerial disposability, 417–420, 432, 433
Network DEA, 175, 243–245
Nonperforming loans, 242, 248

O
OLS regression, 124
OPEC. *See* Organization of the Petroleum Exporting Counties (OPEC)
Operational inefficiency, 451
Ordered weighted averaged (OWA) operators, 17
Ordinary least squares (OLS), 182
Organization for Economic Cooperation and Development (OECD), 45, 168
Organization of the Petroleum Exporting Counties (OPEC), 457

P
Paired weights, 106
Panel data regression, 182
PCA results, 69
Peer-restricted, 10
Petroleum and coal industries and DEA, 457
Pickup truck segment, 345
Piecewise log-linear technology, 314
Policy/strategy making capability, 449
Pollutant abatement technology and management, 292
Pollution, 448
Pollution possibility set (PoPS), 419
Pollution treatment process, 369
Port Modernization Law, 198
Primal–dual based procedure, 301
Principal component analysis (PCA), 54, 55, 146
Printing Technology Research Institute, 170
Production possibility set (PPS), 355, 356
Property fitting (ProFit), 55, 68, 72–79, 81, 82, 155, 158

R
RAM/BCC model, 313
RAM–DEA model, 38
Range-adjusted measure (RAM) model, 38, 300
Ranking ranges, 16

Ratio form DEA, 124
Reference DMU, 300
Relational network model, 92
Research and development (R&D) Organizations
- capital stock, 181
- Capital Stock, 181–183
- CSIST, 170
- data selection and description, 174–175
- DEA
  - and ANP, 171
  - applications, 171–172
- human resources, 181
- OECD, 168
- performance, 169
- SMEs, 170
- in Taiwan, 170
- technological development, 168
- three-stage value-creation process, 172

Returns-to-scale (RTS)
- administrative regions, 375, 379
- using algorithm I, 320
- using algorithm II, 320
- BCC model, 194
- CCR model, 194, 318, 319
- CIs, 209–211
- CRS, 190
- DEA envelopment models, 189, 190
- DEA-CCR and BCC frontiers, 190, 191
- definition, 316–318
- DMUs, 189, 362
- efficiency score, 192
- efficient frontier, 365
- equiproportionate, 366
- FDH, 190–192
- input and output oriented models, 193
- minimization form of model, 320
- models, 375, 376, 378
- MPSS, 362, 363
- multi-component, 366, 367
- multiplier models, 190, 191
- non-linear model, 363
- numerical example, 319–321
- optimal solution, 365
- orientation, 193, 194
- RTS determination algorithms, 327
- scale efficiency, 366
- scale indicator values, 194
- scaling, 192–193
- single-output technology, 362
- stochastic parametric frontier approach, 189
- virtual input and output, 190
- VRS, 190

Robustness testing, 133, 139
RTS statuses
- DMUs, Algorithm I, 320
- DMUs, Algorithm II, 321
- efficient schools, 322, 323
- inefficient schools, 322–325

S

SBB technology, 246
Scale efficiency, 127, 128
Schools evaluation, RAM model, 322
Score matrix, 70
SEC financial statement data sets, 386
Series production system, 231–232
SFA parameters, 137
Shephard's input distance function, 356
Simple additive weighting (SAW), 6
Single-Output BCC-O, 128–129
Single-output translog function, 132–135
Sistema Integrado de Información Universitaria (SIIU), 147
Slack-based superefficiency models, 488
Slacks-based measure (SBM), 169
Small and medium enterprises (SME), 168
Social sustainability, 449
Sound mathematical model, 54
Spanish public universities, 154
Standard deviation, 139
Stochastic frontier analysis (SFA), 51, 55, 125–127, 171
Strategic pattern identification, 73–82
Strong complementary slackness conditions (SCSCs), 442
Strong disposability, 450
Sub-decision making units (SDMUs), 354–356
Supply chain management, 491
Supply chain operations reference (SCOR) model, 490
SUV Segment, 344

T

Taiwan Institute of Economic Research (TIER, 174
Target aggression model, 224
Tchebycheff radius methodology, 488
Technological advancement patterns, 339–345
Technological forecasting, 346
Technology forecasting using DEA (TFDEA), 332
Technometrics, 332
TFDEA parameters, 335–337
Thick Frontier Approach (TFA), 52, 55

Three-stage value-creation process, 172
Times Higher Education (THESQS), 2
Tobit regression, 118, 488
TOPSIS, 6
Translog function, 126
Triangular fuzzy numbers, 100
TU cooperative games (TUCG)
  core and concepts, 221
  DEA production games, 229
  ED setting, 227
  Genetic Algorithm (GA), 227
  multistage production systems, 231–232
  Nash Decomposition, process efficiency, 231–232
  normalized cross efficiency 385 matrix, 227
  production games, 233–235
  Shapley value, 221–223, 229, 230
  Tchebycheff metric, 227
Two-seaters and compact segments, 340–341

U
Undesirable congestion (UC), 415
Unified efficiency (UE), 421–424, 435–437
Unified efficiency under managerial disposability (UEM), 425
Unified efficiency under natural and managerial disposability (UENM), 426–429
Unified efficiency under natural disposability (UEN), 424
Universidad Internacional de Andalucía (UNIA), 149
Universidad Nacional de Educación a Distancia (UNED), 149

V
Value-creation process, 178
Variable returns to scale (VRS), 22–24, 61, 135, 148, 190, 304, 334
Variables and regulatory constraints, 260–262

W
Ward's method, 79
Weight restrictions, 90, 91, 98
Wharton's Research Data Services, 119
Window analysis, 168

Z
Zadeh's extension principle, 95