Chapter 8
DEA Performance Assessment of Mutual Funds

Antonella Basso and Stefania Funari

Abstract The objectives of this paper are manyfold. First we present a comprehensive review of the literature of DEA models for the performance assessment of mutual funds. Then we discuss the problem of the presence of negative returns in DEA modeling for mutual funds and we identify a DEA model that is financially justified and tackles the issue of negative returns in a natural way. Moreover, we present an empirical application on real market data, considering different risk measures. We consider also different holding periods, which include both a period of financial crisis and one of financial recovery. Moreover, we compare the results of the DEA performance measure with those obtained with traditional financial indicators.

Keywords DEA • Mutual fund performance evaluation • Negative data • Sharpe index • Sortino index

8.1 Introduction

The applications of data envelopment analysis (DEA) to the assessment of the performance of mutual funds have become more and more numerous in the last years. If we consider the applications to conventional mutual funds, socially responsible investment (SRI) mutual funds, Islamic funds, pension funds, exchange-traded funds (ETFs), hedge funds, commodity trading advisors (CTAs) and managed future funds, the number of papers published on international journals and books totals about 100 (data referred to the end of 2014).

However, if we look at the performance indicators used in the financial practice to compare mutual funds on the basis of their historical results, we find that the

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indicators most often used do not include any of the performance scores obtained with a DEA model.

We may wonder what is the reason of this lack of “financial visibility” with both practitioners and big data providers working in the financial market. In our opinion, we may find two main reasons.

The first explanation is connected to the difficulty of providing a clear financial interpretation of the DEA indicators which can be plainly grasped by financial professionals.

A second possible explanation lies in the relative sophistication of the DEA models, compared to some of the traditional financial indicators, such as the Sharpe, Treynor and Sortino ratios. This is especially true for many of the advanced DEA models proposed in the more recent literature.

We have to raise an issue that often affects the financial data on mutual funds and can cause a drawback in DEA modeling: the presence of negative mean returns, which is often observed, especially in periods of financial crisis, and requires the usage of special devices in the formulation of the DEA models.

The main objectives of this paper are manyfold and may be summarized as follows.

First, we present a comprehensive review of the literature of DEA models for the evaluation of the performance of mutual funds (Sect. 8.2).

Secondly, we discuss the problem of the presence of negative returns in the DEA modeling for mutual funds (Sect. 8.3).

Thirdly, we identify a DEA model that is financially well justified and tackles the issue of negative returns in a natural way, being inspired by financial considerations rather than derived from mathematical technicalities. At the same time, it relies on one of the basic DEA models, so that it is relatively simple to implement (Sect. 8.4).

In Sect. 8.5 we outline some of the traditional indicators that are most widely used in finance to evaluate the mutual fund performance; they also suffer from serious drawbacks in presence of negative mean returns.

A fourth objective of this contribution is to present an empirical application of the DEA model chosen on real market data, considering different risk measures. The empirical investigation considers also different holding periods and is carried out both on a period of financial crisis and on a period of financial recovery. Moreover, we compare the results of the DEA performance measure with those obtained with the traditional financial indicators (Sect. 8.6).

8.2 DEA Literature on Mutual Funds

The issue of the assessment of the performance of mutual funds was not among the first applications of the DEA methodology. Indeed, the first contributions which proposed a DEA model to study the performance of mutual funds were published only a little more than 15 years ago. Among the pioneering papers we find Murthi et al. (1997), McMullen and Strong (1998), Basso and Funari (2001), Choi and Murthi (2001), Tarim and Karan (2001) and Galagadera and Silvapulle (2002).

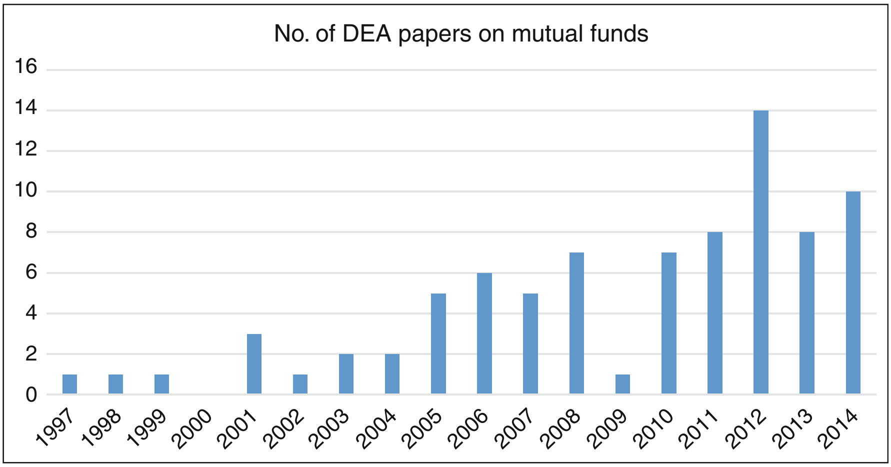

Fig. 8.1 Number of published papers on DEA models for the evaluation of the performance of mutual funds (including SRI funds and Islamic funds), pension funds and ETFs by year of publication

The number of papers published yearly on this subject begins to be considerable starting from 2003, and becomes copious in the more recent years. The situation is summarized in Fig. 8.1, which illustrates the number of papers published in scientific journals or books that adopt a DEA model for the evaluation of the performance of mutual funds by year of publication. The number of publications considered in Fig. 8.1 includes the papers which focus on traditional mutual funds, socially responsible investment (SRI) mutual funds and Islamic funds, but also pension funds and exchange-traded funds (ETFs).

Along the same line, Fig. 8.2 displays the number of published papers that use a DEA approach for the assessment of the performance of hedge funds, commodity trading advisors (CTAs) and managed futures funds.

As can be seen from Figs. 8.1 and 8.2, the overall number of papers on DEA assessment of mutual funds published up to now is quite relevant. Table 8.1 presents the various contributions in detail, reporting for each paper the kind of DEA models proposed/used, a brief summary of the main features of the study and the main characteristics of the empirical analysis carried out (the geographical area and time period of the data and the number of funds considered).

Analogously, Table 8.2 presents the papers on the DEA performance evaluation of socially responsible investment (SRI) mutual funds, which exploits the ability of DEA to take into account not only the financial features but also a measure of the degree of social responsibility of mutual funds.

Moreover, in the more recent years the DEA approach has been applied also to a special kind of SRI funds: the funds that follow the rules of Islamic finance (Shariah compliant); for a review of the contributions on Islamic mutual funds see Table 8.3.

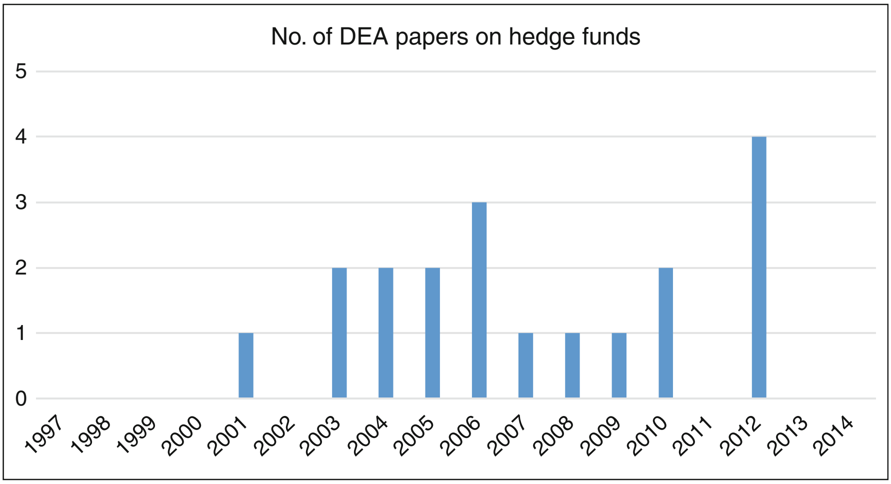

Fig. 8.2 Number of published papers on DEA models for the evaluation of the performance of hedge funds, CTAs and managed futures funds by year of publication

Furthermore, the review of the literature on DEA models for pension funds is presented in Table 8.4, while Table 8.5 considers the papers which try to apply DEA to exchange-traded funds (ETFs). With regard to the applications of DEA to ETFs, however, we must bear in mind that the aim of ETFs is to track a stock or a bond index, rather than to beat the market.

Another field of application of the DEA methodology in finance is the assessment of the performance of hedge funds. Table 8.6 displays the contributions of the DEA literature on hedge funds.

In addition, Table 8.7 reports the contributions that apply DEA to measure the performance of commodity trading advisors (CTAs) and managed futures funds.

On the other hand, mutual funds are actually stock and/or bond portfolios managed by financial professionals. Therefore, some DEA models that have been proposed for portfolio selection may also be useful for the performance evaluation of mutual funds. Table 8.8 presents the papers which use the DEA methodology for portfolio selection. As can be seen, the number of such papers has become considerable mainly in the last years; see also Fig. 8.3, that displays the number of published papers by year.

8.3 The Problem of Negative Returns in DEA Modeling

In classical DEA models it is common to assume that all the input and output values are non negative. This is indeed a crucial assumption in the measurement of performance with the DEA technique. However, it is far from being always satisfied

Table 8.1 Stylized review of the literature on DEA models for the assessment of the performance of mutual funds

Paper	DEA models	Features	Empirical analysis
Adeli O. (2013)	CCR	Uses various risk measures. Inputs: standard deviation, beta, half-variance, costs of issuance and cancellation. Outputs: downside, best time	Iran, 29 funds
Alexakis P., Tsolas I. (2011)	BCC	Empirical paper. Inputs: standard deviation, beta, assets, front-end load, back-end load. Output: return	Greece, 2001–2004, 55 funds
Ali Asghar M. J.-K., Afza T., Bodla M.A. (2013)		Empirical paper. Inputs: management fee, business services, equity. Outputs: investments, returns	Pakistan, 2005–2010, 100 funds
Anderson, R.I., Brockman, C.M., Giannikos, C., McLeod, R.W. (2004)	CCR	Empirical analysis on real estate mutual funds. Inputs: standard deviation, front load, deferred load, 12b-1 fees, other expenses. Output: return	1997–2001, 110 real estate funds
Babalos V., Caporale G.M., Philippas N. (2012)	DEA-based Malmquist index	Evaluates the funds' total productivity change using the DEA-based Malmquist index. Employs a panel logit model to analyse the relationship between the probability of being efficient and funds' size. Inputs: standard deviation, total expense ratio, capital invested. Output: terminal value of the investment	Greece, 2003–2009, 31 funds
Babalos V., Doumpos M., Philippas N., Zopounidis C. (2015)	CCR, BCC	Combines DEA with MCDA. First uses DEA, then adopts a multicriteria decision aid (MCDA) approach to develop an overall performance measure on the basis of the DEA efficiency results. Inputs: gross expense ratio, turnover rate, standard deviation. Outputs: 1 +DMR (deviation from the median return), capital flow	US, 2003–2010, more than 500 funds

(continued)

Table 8.1 (continued)

Paper	DEA models	Features	Empirical analysis
Baghdadabada M.R.T., Tanha F.H., Halid N. (2013)		Evaluates funds managers' efficiency in terms of management style. Analyzes the benefits of the DEA approach in the DARA, CARA and IARA framework. Inputs: variance, turnover, expense ratio, loads. Outputs: return, stochastic dominance indicators	US, 2005–2010, 17,686 funds
Basso A., Funari S. (2001)	CCR	Proposes a single-output DEA performance measure that generalizes Sharpe, Treynor and reward-to-half-variance indices and a multiple output performance measure that takes into account a stochastic dominance indicator that reflects the investors' preference structure and the time occurrence of the returns. Inputs: some risk measures, subscription and redemption costs. Outputs: a return measure (expected return or expected excess return), stochastic dominance indicator	Italy, 1997–1999, 47 funds
Basso A., Funari S. (2005a)	CCR, cross-efficiency	Proposes a model that synthesizes the results of the traditional risk adjusted indices in a global performance measure which takes into account also the subscription and redemption costs and suggests an average cross-efficiency measure. Inputs: subscription and redemption costs, unit invested amount. Outputs: different traditional risk adjusted performance indices (Sharpe, reward to half-variance, Treynor, Jensen), stochastic dominance indicator	Italy, 1997–2001, 50 funds

Basso A., Funari S. (2014c)	BCC-O	Discusses the role of fund size in DEA performance evaluation and wonders whether it is appropriate to include size information among the input/output variables of DEA models. Moreover, analyzes the nature of returns to scale in mutual fund performance. Inputs: capital invested net of the initial fees, beta. Output: final value of the investment net of the exit fee	Europe, 2006–2009, 279 funds
Batra D., Batra G. (2012)		Compares systematic investment plan with lump sum investment using DEA. Inputs: standard deviation, beta, minimum investment. Outputs: 3 year return, 5 year return, Sharpe ratio	
Brandouy O., Kerstens K., Van de Woestyne I. (2015)	Shortage function	Explores the potential benefits of a series of non-parametric frontier-based fund rating models. Adopts a comparative approach based on a backtesting methodology	Europe, 2005–2011, 814 funds
Chang K.-P. (2004)	MCIRS	Uses a minimum convex input requirement set (MCIRS) approach to evaluate the performance of mutual funds. Inputs: beta, standard deviation, assets. Output: return	US, 1992–1996, 248 funds
Chang J.F. (2010)	CCR (DPEI model of Murthi, Choi, Desai, 1997)	Provides a method to create funds of funds, as well as strategies to choose sub-funds (with the DPEI index and a genetic algorithm) and a way to allocate weighted capital. Input and output variables are those of DPEI model of Murthi et al. (1997)	Taiwan, 2004–2006, 49 funds

(continued)

Table 8.1 (continued)

Paper	DEA models	Features	Empirical analysis
Chen Y.-C., Chiu Y.-H., Li M.-C. (2011)	System BCC model	Compares the results obtained with the BCC model and a system BCC model (with two subgroups of funds, stock and balanced) using as outputs the classical indicators of mutual fund performance. Inputs: standard deviation, purchasing turnover rate, direct transaction cost rate, selling expense rate. Outputs: Treynor index, Sharpe index, Jensen index, return rate	Taiwan, 2007, 278 funds
Chen Z., Lin R. (2006)	BCC-I	Considers several risk measures. Treats the same fund during different time periods as different funds. Inputs: investment costs, some risk measures, including value at risk and conditional value at risk. Outputs: a return measure (expected return or expected excess return), stochastic dominance indicator, traditional performance indices (Treynor index, Sharpe index, Jensen index)	China, 2000–2002, 33 funds; 1999–2000, 14 funds
Choi Y.K., Murthi B.P.S. (2001)	BCC	Compares DEA scores with the values obtained using the classical fund performance indices; analyzes the importance of fund's size and the returns to scale. Inputs: standard deviation, expense ratio, loads, turnover. Output: return	731 funds of different categories
Daraio C., Simar L. (2006)	DEA, FDH for comparison	Proposes a robust nonparametric approach for analysing mutual funds based on an order-m frontier and on a probabilistic approach; compares the results with those obtained with DEA and FDH. Inputs: standard deviation, expense ratio, turnover ratio. Output: return	US

de Guzman M.P.R., Cabanda E.C. (2005)	DEA-based Malmquist index	Empirical paper. Inputs: redemption costs, operating expenses. Outputs: total assets, number of accounts, total sales	Philippines, 1999–2003, 10 funds
Devaney M., Weber W.L. (2005)	Directional output distance function with CRS and VRS	Models risk as an undesirable output. Estimates from a directional output distance function are used to construct a risk/return frontier that defines the best-practice management technology for real estate investment trusts. Inputs: book value of assets, expenses. Desirable output: return. Undesirable output: risk (either variance or beta)	US, 1995–2000, 77 real estate investment trusts
Galagadera D.U.A., Silvapulle P. (2002)	CCR, BCC	Uses logistic regression to examine the dependence of the DEA efficiency scores on fund attributes, management strategy and operating environment. Seven inputs: four standard deviations of the 1, 2, 3, 5 year gross performance, sales charges, operating expenses, minimum initial investment. Four outputs to capture short, medium and long term performance	Australia, 1995–1999, 257 funds
Gökgöz F. (2010)	CCR, BCC	Empirical paper	Turkey, 2006–2007, 36 mutual and 41 pension funds
Gökgöz F., Çandarli D. (2011)	CCR, BCC-I	Compares technical, pure and scale efficiency of pension funds and mutual funds. Inputs: standard deviation, beta, expense ratios, turnovers. Output: excess return	Turkey, 2009, 36 mutual and 36 pension funds
Gregoriou G.N. (2006a)	CCR, super-efficiency, cross-efficiency	Empirical paper. Inputs: standard deviation, downside deviation, maximum drawdown. Outputs: return, monthly percentage profitable	US, 1990–2005, 25 funds
Gregoriou G.N. (2007)		Examines the efficiency of large US stocks, bonds and balanced funds using DEA. Compares the efficiency of the funds of the various DEA models with the Sharpe index	US

(continued)

Table 8.1 (continued)

Paper	DEA models	Features	Empirical analysis
Guo J., Ma C., Zhou Z. (2012)	BCC-I	Considers the higher order moment characteristics of funds return which reflect the preference of investors. Inputs: net assets, unit cost ratio, standard deviation, kurtosis. Outputs: accumulative daily net asset value, growth rate, skewness	China, 2010, 27 funds
Hanafizadeh P., Khedmatgozar H. R., Emrouznejad A., Derakhshan M. (2014)		Uses neural network back-propagation DEA in measurement of mutual funds efficiency	
Haslem J.A., Scheraga C.A. (2003)	BCC-I	Empirical paper. Inputs: cash, expense ratio, stocks, P/E ratio, P/B ratio, total asset. Output: Sharpe index	1999, 80 funds
Haslem J.A., Scheraga C.A. (2006)	BCC-I	Examines the efficiency of mutual fund portfolio management; follows Haslem, Scheraga (2003). Inputs: cash, expense ratio, stocks, P/E ratio, P/B ratio, number of securities held, portfolio turnover. Output: total asset	2001, 58 funds
Hsu C.-S., Lin J.-R. (2007)	CCR	Tests performance persistence. Inputs: standard deviation, ratio of management fee, ratio of load fee, turnover ratio. Output: excess return	Taiwan, 1999–2003, 192 funds
Hu J.-L., Yu H.-E., Wang Y.-T. (2012)	BCC-I	Uses an input oriented BCC model to adjust for a negative output; the truncated regression model is used to estimate effects of environmental variables on input slacks in a second stage. Inputs: standard deviation, expense ratio. Output: return	Taiwan, 2006–2010, 60 funds

Hu J.-L., Chang T.-P. (2008)	CCR	Applies a three-stage DEA approach to decompose mutual fund underperformance and to obtain pure managerial performance. Inputs: standard deviation, expense ratio. Output: return	Taiwan, 2005–2006, 156 funds
Joro T., Na P. (2006)		Proposes a non-linear DEA-like framework where the correlation structure among the units is taken into account and the diversification effects are calculated. Input: variance. Outputs: excess return, skewness	1995–2000, 54 funds
Kerstens K., Mounir A., Van de Woestyne I. (2011)	Shortage function	Argues in favor of the use of the shortage function and discusses three crucial specification issues in using non parametric methods for fund evaluation: (1) returns to scale issue (2) - higher-order moments and cost components issues (3) convexity issue	US, Europe, 2004–2009, 1070 funds
Khedmatgozar H.R., Kazemi A., Hanafizadeh P. (2013)	VEA, BCC-O	Uses value efficiency analysis (VEA) to incorporate the investor's judgements and compares the results with those of general DEA models. Input: variance. Outputs: return, skewness	US, 2005–2007, 22 funds
Lamb J.D., Tee K.-H. (2012a)	Input oriented NIRS, diversification-consistent DEA model	Discusses the assumptions needed for a DEA model of investment funds. Models diversification through a series of linear approximation to an ideal nonlinear model. Shows how the convex programme might be solved through an iterative procedure. Uses coherent risk measures	2000–2004, 30 hedge funds
Lamb J.D., Tee K.-H. (2012b)	Input and output oriented NIRS, diversification-consistent DEA model	Studies uncertainty in DEA efficiency estimates; investigates consistency and bias and uses bootstrap to develop stochastic DEA models for funds	2000–2004, 30 hedge funds

(continued)

Table 8.1 (continued)

Paper	DEA models	Features	Empirical analysis
Lin R. (2009)		Uses data envelopment windows analysis and considers the different period separately. Includes conditional value at risk as risk measure in the DEA model	
Lin R., Chen Z. (2008)		Uses value at risk and conditional value at risk as risk measures and treats the same fund during different time periods as different funds	US
Lozano S., Gutiérrez E. (2008a)	Additive metric	Proposes six DEA-like linear programming models consistent with second-order stochastic dominance, with different choices of input and output variables	Spain, 2002–2005, 108 funds
Lozano S., Gutiérrez E. (2008b)		Proposes three DEA-based models consistent with third-order stochastic dominance. Each model considers an appropriate risk measure as input and an appropriate return measure as output	Spain, 2002–2005, 33 funds
Majid M.S.A., Maulana H. (2010)	DEA-based Malmquist index	Empirical paper. Inputs: front-end loads, redemption fees, expense ratio; Output: return	Indonesia, 2004–2007, 23 mutual funds
Malhotra D.K., Malhotra R. (2012)	CCR	Empirical paper. Inputs: standard deviation, beta, load, 12b-1 charges, expense ratios. Output: return	2010–2011, 189 aggressive growth mutual funds
Margaritis D.M., Otten R., Tourani-Rad A. (2007)	BCC-I	Uses a Tobit regression to analyze the dependence of the efficiency scores on some fund attributes not included in the DEA analysis. Inputs: standard deviation, expenses ratio, load factor. Output: return	New Zealand, 1998–2003, 52 funds
Matallín-Sáez J.C., Soler-Domínguez A., Tortosa-Ausina E. (2014)	FDH, order-m, order-α partial frontiers	Applies DEA FDH, order-m and order-α partial frontiers. Inputs: standard deviation, kurtosis, expense ratio, beta. Outputs: return, skewness	US, 2001–2011, 1450 funds

McMullen P.R., Strong R.A. (1998)	BCC-I, BCC-O	Rescales the input and output variables so that they start from zero. Inputs: standard deviation, sales charge, minimum initial investment, expense ratio. Outputs: 1 year return, 3 year return, 5 year return	135 funds
Morey M.R., Morey R.C. (1999)	Quadratic programming	Presents two approaches, which take inspiration from data envelopment analysis, for identifying those funds that are strictly dominated. Inputs: variances over different horizons. Outputs: returns over different horizons	1985–1995, 26 funds
Murthi B.P.S., Choi Y.K., Desai P. (1997)	CCR	Proposes one of the first DEA models for the evaluation of mutual funds, the DEA portfolio efficiency index (DPEI). Inputs: standard deviation, expense ratio, loads, turnover. Output: return	731 funds of different categories
Pendaraki K. (2012)	BCC-I	Analyzes the effect of inclusion of higher moments as variables in DEA performance risk-return framework by applying DEA twice. i) Input: standard deviation. Outputs: return, asset. ii) Inputs: standard deviation, kurtosis. Outputs: return, asset, skewness	Greek, 2007–2010, 43 funds
Premachandra I.M., Zhu, J., Watson J., Galagadera D.U.A. (2012)	Two-stage DEA	Uses two-stage DEA models that decomposes the overall efficiency into two components: operational efficiency and portfolio efficiency. Stage 1 has two inputs (management fees, 12b-1 fees) and one output (net asset value). Stage 2 has five inputs (fund size, net expense ratio, turnover, standard deviation, adjusted net asset value) and one output (return)	US, 1993–2008, 66 families (1269 mutual funds)

(continued)

Table 8.1 (continued)

Paper	DEA models	Features	Empirical analysis
Simar L., Vanhems A., Wilson P.W. (2012)	FDH and DEA estimators, directional distance function, VRS	Studies statistical properties of directional estimators; establishes that the directional DEA estimators share the properties of the traditional radial DEA estimators. Inputs: standard deviation, expense ratio, turnover ratio. Output: return	2001, 129 aggressive-growth funds
Soongswang A., Sanohdontree Y. (2011a)	CCR	Empirical paper	Thailand, 2002–2007, 138 funds
Soongswang A., Sanohdontree Y. (2011b)		Empirical paper	Thailand, 2002–2007, 138 funds
Tarim S.A., Karan M.B. (2001)	CCR with restrictions on weights	Uses the DPEI approach of Murthi et al. (1997) with the introduction of upper and lower bounds on the weights of the input variables. Inputs: standard deviation, expense ratio, turnover ratio. Output: return	Turkey, 1998, 191 funds
Tavakoli Baghdadabad M.R., Noori Houshyar A. (2014)	CCR, DEA-based Tornqvist index	Assesses the changes of mutual funds' total productivity using the DEA-based Tornqvist productivity index. Moreover, uses a panel logit model to study the relationship between efficiency and productivity and some funds' features	US, 2000–2012, 11,522 funds
Tsolas I.E. (2014)	BCC-I	Uses a two-stage procedure with first a BCC model and then a Tobit model. Inputs: standard deviation, management expense ratio, front load, deferred load. Output: return	62 precious metal funds
Zhao X., Shi J. (2010)		Uses a coned context DEA model with expected shortfall modeled under an asymmetric Laplace distribution. Input: expected shortfall. Outputs: 1 year return, 3 year return, 5 year return	China, 2003–2007, 17 funds

Zhao X.-J., Wang S.-Y. (2007)	CCR, BCC-O	Empirical paper. Inputs: comparative standard deviation, percentage of negative monthly return, operational fees. Outputs: final value, return	China, 2004–2005, 78 funds (24 open-end and 54 closed-end funds)
Zhao X., Wang S., Lai K.K. (2011)		Proposes two quadratic-constrained DEA models to evaluate mutual funds, based on endogenous benchmarks	China, 2005–2006, 25 funds
Zhao X., Yue W. (2008)		Proposes a coned context dependent DEA model	China, funds
Zhao X., Yue W. (2012)	MFDEA	Proposes a multi-subsystem fuzzy data envelopment analysis (MFDEA) model to evaluate mutual funds management companies' core competence. Considers two subsystems with different choices of input and output variables	China, 2004–2008, 32 mutual funds management companies

Table 8.2 Stylized review of the literature on DEA models for the assessment of the performance of socially responsible investment (SRI) mutual funds

Paper	DEA models	Features	Empirical analysis
Abdelsalam O., Duygun-Fethi M., Matallín J.C., Tortosa-Ausina E. (2014a)	Two variants of FDH	Combines two variants of FDH methods (order-m and order-alpha) in the first stage of analysis and quantile regression in the second stage. Inputs: standard deviation, kurtosis, expenses. Outputs: gross return, skewness	With a geographic focus (East/West/Middle/global), 2001–2011, 138 Islamic and 636 SRI funds
Abdelsalam O., Duygun M., Matallín-Sáez J.C., Tortosa-Ausina E. (2014b)	Two variants of FDH	Uses two efficiency models based on FDH (order-m and order-alpha) in the first stage of analysis, a recursive investment approach in the second stage; persistence performance is evaluated in the third stage. Inputs: standard deviation, kurtosis, expenses. Outputs: gross return, skewness	With a geographic focus (East/West/Middle/global), 2001–2011, 138 Islamic and 636 SRI funds
Basso A., Funari S. (2003)	CCR, exogenously fixed output, categorical variable model	Proposes three different DEA models for the performance evaluation of SRI mutual funds: a basic model, an exogenously fixed output model and a categorical model. Inputs: standard deviation, beta, subscription costs, redemption costs. Outputs: expected return, ethical indicator	Simulated data
Basso A., Funari S. (2005b)	Exogenously fixed output	Proposes a DEA model for the performance evaluation of ethical funds with non negative outputs and an exogenously fixed ethical level. Inputs: initial investment, standard deviation, initial fees, exit fees. Outputs: final value of the investment, ethical level	UK, 2002–2005, 32 SRI and 25 - non-SRI funds
Basso A., Funari S. (2008)	CCR, exogenously fixed output	Proposes a measure of the ethical level of SRI funds which takes into account the main socially responsible features and three DEA models that adjust the ones proposed in Basso	Europe, 2002–2005, 159 SRI and 110 - non-SRI funds

		and Funari (2003) to tackle the problem of the presence of negative returns. Inputs: initial investment, standard deviation, initial fees, exit fees. Outputs: final value of the investment, ethical level	
Basso A., Funari S. (2014a)	CCR, BCC-O, exogenously fixed output	Proposes some models which can be computed in all phases of the business cycle. Tests the presence of returns to scale on European SRI equity funds; provides a measure of the degree of social responsibility of SRI funds; compares the performances between SRI and non-SRI funds. Inputs: initial payout invested in the fund, beta. Outputs: final value of the investment, ethical level	Europe, 2006–2009, 189 SRI and 90 - non-SRI funds
Basso A., Funari S. (2014b)	Models with a constant input; CCR, BCC-O, exogenously fixed output	Proposes constant and variable returns to scale DEA models which consider a constant initial capital and standard deviation as inputs and the final value of the investment and ethical level as outputs. The implications of the presence of a constant input in DEA models are studied	Europe, with special focus on Sweden, 2006–2009, 189 SRI and 90 non-SRI funds
Basso A., Funari S. (2014d)	CCR, exogenously fixed output	Evaluates the performance of SRI equity funds in the main European countries with three different DEA models, compares the performance of SRI and non-SRI mutual funds in the various countries with a series of statistical tests and compares the performance obtained by SRI mutual funds among different countries. Input and output variables are those considered in Basso and Funari (2008)	Europe, 2006–2009, 190 SRI and 91 - non-SRI funds

(continued)

Table 8.2 (continued)

Paper	DEA models	Features	Empirical analysis
Einolf K.W. (2007)	CCR	Uses DEA in an application of SRI portfolio development to find the best financially and socially performing companies within each industry sector (with a best-in-class optimization approach in place of screening). Inputs: IW financial environment, social and governance scores. Outputs: alpha, Morningstar star rating	US, 2007, 978 stocks
Jeong S.O., Hoss A., Park C., Kang K.-H., Ryu Y. (2013)	DEA, FDH; input oriented approach	Provides a stock portfolio screening tool for socially responsible investment based upon corporate social responsibility and financial performance. Compares DEA and FDH results. Inputs: inverted ESG (environmental, social, governance) scores. Outputs: return on assets, return on equity, operating profit percentage	Korea, 2006–2009, 253 stocks
Pérez-Gladish B., Méndez Rodríguez P., M'zali B., Lang P. (2013)	Output oriented VRS	Integrates the role of financial criteria and the influence of social responsibility considerations in investment decisions within a DEA framework consistent with second-order stochastic dominance. Compares the performance of conventional and SRI funds. Inputs: conditional value at risk, turnover ratio, expense ratio, deferred loads, front loads. Outputs: return, social and environmental responsibility index, quality of socially responsible investment management	US, 2007, 25 SRI and 21 conventional funds

Table 8.3 Stylized review of the literature on DEA models for the assessment of the performance of Islamic mutual funds

Paper	DEA models	Features	Empirical analysis
Abdelsalam O., Duygun-Fethi M., Matallín J.C., Tortosa-Ausinad E. (2014a)	Two variants of FDH	Combines two variants of FDH methods (order-m and order-alpha) in the first stage of analysis and quantile regression in the second stage. Inputs: standard deviation, kurtosis, expenses. Outputs: gross return, skewness	With a geographic focus (East/West/ Middle/global), 2001–2011, 138 Islamic and 636 SRI funds
Abdelsalam O., Duygun M., Matallín-Sáez J.C., Tortosa-Ausina E. (2014b)	Two variants of FDH	Uses two efficiency models based on FDH (order-m and order-alpha) in the first stage of analysis, a recursive investment approach in the second stage; persistence performance is evaluated in the third stage. Inputs: standard deviation, kurtosis, expenses. Outputs: gross return, skewness	With a geographic focus (East/West/ Middle/global), 2001–2011, 138 Islamic and 636 SRI funds
Majid M.S.A., Maulana H. (2012)		Investigates the influence of the mutual funds companies' characteristics on efficiency measures using DEA and the generalized least square estimation. Compares the performance of Islamic and conventional mutual funds companies	Indonesia, 2004–2007
Rubio J.F., Hassan M.K., Merdad H.J. (2012)	BCC-I, Russell model	Performance comparison of Islamic and non Islamic mutual funds. Inputs: standard deviation, lower partial momentum, maximum drawdown period. Outputs: expected return, upper partial momentum, maximum period of consecutive gain	Indonesia
Saad N.M., Majid M.S.A., Kassim S., Hamid Z., Yusof, R.M. (2010)	DEA-based Malmquist index	Investigates the efficiency of conventional and Islamic unit trust companies. Inputs: management expenses ratio, portfolio turnover ratio. Output: return	Malaysia, 2002–2005, 5 Islamic and 22 unit trust companies

Table 8.4 Stylized review of the literature on DEA models for the assessment of the performance of pension funds

Paper	DEA models	Features	Empirical analysis
Andreu L., Sarto J.L., Vicente L. (2013)	SBM	Applies four variants of the slacks-based measure (SBM) to evaluate the efficiency of strategic asset allocation in pension fund management	
Barrientos A., Boussofiane A. (2005)	CCR, BCC	Studies the performance of pension fund management market over time using DEA. Inputs: marketing and sales costs, office personnel and executive pay, administration and computing costs. Outputs: total revenue, number of contributors	Chile, 1982–1999, pension funds
Gökgöz F. (2010)	CCR, BCC	Empirical paper	Turkey, 2006–2007, 36 mutual and 41 pension funds
Gökgöz F., Çandarli D. (2011)	CCR, BCC-I	Compares technical, pure and scale efficiency of pension funds and mutual funds. Inputs: standard deviation, beta, expense ratios, turnovers. Output: excess return	Turkey, 2009, 36 mutual and 36 pension funds
Guillén G.B. (2008)	CCR	Uses DEA to compare pension fund institutions of nine Latin American countries. Inputs: administrative cost, sale cost. Outputs: total revenue, number of contributors	Latin America, 2005–2007, 876 pension fund institutions of 9 countries
Medeiros Garcia M.T. (2010)	DEA-based Malmquist index	Estimates the change in total productivity, the technically efficient change and the technological change by means of DEA-Malmquist index. Inputs: value of pensions paid, number of workers, net assets, contributions received. Outputs: number of funds managed, value of the funds managed, profits paid to stockholders, number of participants	Portugal, 1994–2007, 12 pension funds management companies
Pestana Barros C., Medeiros Garcia M.T. (2006)	Cross-efficiency, super-efficiency	Uses DEA to evaluate the performance of pension funds management companies with the cross-efficiency and the super-efficiency DEA model. Inputs: number of full time equivalent workers, fixed assets, contributions received from participants or sponsors. Outputs: number of funds managed, value of the funds managed, pensions paid to subscribers	Portugal, 1994–2003, 12 pension funds management companies

Table 8.5 Stylized review of the literature on DEA models for exchange-traded funds (ETFs)

Paper	DEA models	Features	Empirical analysis
Chu J., Chen F., Leung P. (2010)	RDM	Applies DEA to evaluate the performance of IShare World exchange-traded funds with a range directional measure (RDM)	2006–2009, IShare World ETFs
Prasanna P.K. (2012)	Cross-efficiency, super-efficiency	Uses cross-efficiency and super-efficiency DEA models to evaluate the performance of ETFs. Inputs: standard deviation, maximum drawdown, downside deviation. Outputs: percentage of months with positive returns, compounded monthly return	India, 2006–2011, 82 ETFs
Tsolas I.E. (2011)	Generalized proportional distance function	Applies a two-stage procedure. In the first stage, the generalized proportional distance function in the DEA context is used to measure the relative efficiency of ETFs. In the second stage, a Tobit model is employed to identify the drivers of performance. Inputs: portfolio price/cash flow, portfolio price/book, total expense ratio. Output (first DEA run): Sharpe index. Output (second DEA run): Jensen's alpha. Output (third DEA run): Sharpe ratio, Jensen's alpha	2008–2010, 15 natural resources ETFs
Tsolas I.E., Charles V. (2015)	RAM	Appraises the performance of a sample of ETFs using the range-adjusted measure (RAM), the RAM-BCC model and a common set of weights of RAM	

Table 8.6 Stylized review of the literature on DEA models for the assessment of the performance of hedge funds

Paper	DEA models	Features	Empirical analysis
Ammann M., Moerth P. (2008)		Discusses the impact of fund size on the performance of funds of hedge funds. Inputs: standard deviation, drawdown, kurtosis, modified value at risk. Outputs: return, skewness, proportion of positive months, omega, Sortino ratio, kappa, upside potential ratio, Calmar ratio, alpha	2000–2005, 167 funds of hedge funds
Eling M. (2006)	CCR, BCC, super-efficiency	Presents ten DEA applications with different choices of input and output variables. In some applications the choice of inputs and outputs is based on two selection rules (Spearman's rank correlation, principal component analysis)	1996–2005, 30 hedge funds
Gregoriou G.N. (2003)	BCC, cross-efficiency, super-efficiency	Uses three DEA models to evaluate the management quality of funds of hedge funds. Inputs: lower mean semi-skewness, lower mean semi-variance, mean lower return. Outputs: upper mean semi-skewness, upper mean semi-variance, mean upper return	1997–2001, 168 funds of hedge funds
Gregoriou G.N., Rouah F., Sedzro K. (2003)		Chapter 4: Fund of hedge fund performance using data envelopment analysis	
Gregoriou G.N., Sedzro K., Zhu J. (2005)	BCC, cross-efficiency, super-efficiency	Uses three DEA models to obtain insights due to problems encountered when using multifactor models to predict hedge fund returns. Inputs: lower mean semi-skewness, lower mean semi-variance, mean lower return. Output: upper mean semi-skewness, upper mean semi-variance, mean upper return	1997–2001, 614 hedge funds

Gregoriou G.N., Zhu, J. (2005)	CRS, VRS, context-dependent DEA, fixed- and variable-benchmark models	Book on DEA performance of hedge funds and CTAs. Different types of models with different input and output variables	1998–2004, top 20 hedge funds, funds of hedge funds and CTAs from various categories
Gregoriou G.N., Zhu J. (2007)	BCC-I	Applies DEA to funds of hedge funds. Inputs: standard deviation, downside deviation, maximum drawdown. Outputs: percentage of profitable months, return, number of successive months with positive returns	1994–2004, 25 funds of hedge funds
Kumar U.D., Roy A., Saranga H., Singal K. (2010)	SBM, super-SBM	Uses slack-based measure (SBM) and super-SBM models. Analyzes the hedge fund strategies using a variety of classical risk return measures. Based on Spearman's rank correlation the following inputs and outputs are considered: standard deviation of drawdown and value at risk as inputs, higher partial moment of order 0, skewness as outputs. Moreover, a principal component analysis is carried out	1995–2007, 4730 hedge funds
Lamb J.D., Tee K.-H. (2012a)	Input oriented NIRS, diversification-consistent DEA model	Discusses the assumptions needed for a DEA model of investment funds. Models diversification through a series of linear approximation to an ideal nonlinear model. Shows how the convex programme might be solved through an iterative procedure. Uses coherent risk measures	2000–2004, 30 hedge funds
Lamb J.D., Tee K.-H. (2012b)	Input and output oriented NIRS, diversification-consistent DEA model	Studies uncertainty in DEA efficiency estimates; investigates consistency and bias and uses bootstrap to develop stochastic DEA models for funds	2000–2004, 30 hedge funds
Lamb J., Tee K.-H. (2012c)	Input oriented NIRS, diversification-consistent DEA model	Discusses about hedge funds risk. Uses the diversification-consistent DEA model of Lamb and Tee (2012a)	2000–2004, 30 hedge funds
Sedzro K. (2009)	BCC	Compares the rankings obtained with the stochastic dominance method and DEA	1998–2003; 615 hedge funds

Table 8.7 Stylized review of the literature on DEA models for the assessment of the performance of commodity trading advisors (CTAs) and managed futures funds

Paper	DEA models	Features	Empirical analysis
Darling G., Mukherjee K., Wilkens K. (2004)	BCC-I	Investigates the performance of CTAs with a DEA model and Tobit regressions of the scores on various features. Inputs: standard deviation, proportion of negative returns. Outputs: return, minimum return, skewness, kurtosis	1998–2002, 216 CTAs
Diz F., Gregoriou G.N., Rouah F., Satchell S.E. (2004)	BCC-I, cross-efficiency, super-efficiency	Studies the efficiency of CTAs using DEA; computes also cross-efficiencies and super-efficiencies. Inputs: lower mean semi-skewness, lower mean semi-variance, mean lower return. Outputs: upper mean semi-skewness, upper mean semi-variance, mean upper return	1997–2001, 150 CTAs
Glawischnig M., Sommersguter-Reichmann M. (2010)	BCC-I, assurance region model, Russell measure, RAM	Compares various existing DEA models for the evaluation of the performance of alternative investment fund industry. Inputs: lower partial moments 0–3, maximum drawdown periods. Outputs: upper partial moments 1–3	2004–2007, 167 managed futures funds
Gregoriou G.N. (2006b)	BCC-I, cross-efficiency, super-efficiency	Computes DEA efficiency, cross-efficiency and super-efficiency of CTAs. Inputs: beginning assets under management, number of futures contracts traded per million dollars net of all brokerage fees. Outputs: compound return, ending assets under management	1995–2004, 90 CTAs
Gregoriou G.N., Chen Y. (2006)	Input oriented VRS	Investigates the performance of CTAs using fixed and variable benchmarking models. Inputs: standard deviation, semi-deviation, percentage of negative returns, periods to recover from first largest maximum drawdown. Outputs: compounded return, percentage of positive returns	1998–2004, 143 CTAs

Gregoriou G.N., Henry S.C. (2015)	BCC-O with undesirable outputs	Investigates the efficiency of CTAs using undesirable outputs. Inputs: margin to equity ratio, incentive fee, round turn, management fee. Outputs: Sharpe index, cumulative return, maximum drawdown, standard deviation, semi-deviation	2009–2013, 50 CTAs
Gregoriou G.N., Zhu, J. (2005)	CRS, VRS, context-dependent DEA, fixed- and variable-benchmark models	Book on DEA performance of hedge funds and CTAs. Different types of models with different input and output variables	1998–2004, top 20 hedge funds, funds of hedge funds and CTAs from various categories
Tokic D. (2012)	BCC-O	Uses DEA to evaluate the relative performance efficiency of CTAs	1992–2010, 30 CTAs
Wilkens K., Zhu J. (2001)	BCC-I	Applies DEA to commodity trading advisor returns, using the input oriented BCC to deal with negative outputs. Inputs: standard deviation, proportion of negative returns. Outputs: average return, minimum return, skewness	1999, 11 CTAs

Table 8.8 Stylized review of the literature on DEA models for portfolio selection and stock indices

Paper	DEA models	Features	Empirical analysis
Adamauskas S., Krusinskas R. (2012)	CCR	Proposes a model to evaluate the efficiency of private investor's decisions with five stages; the fourth stage, regarding the investment instruments selection, uses DEA. Considers different input and output variables for stocks and funds evaluation	Lithuania, 2004–2010, 18 investment portfolios
Bahrani R., Khedri N. (2013)	CCR, BCC-I, BCC-O	Creates a portfolio of efficient companies using DEA. Inputs: assets, salaries, sale costs. Outputs: income, operating profit	Iran, 2001–2007, companies
Banihashemi S., Sanei M. (2013)		Evaluates the performance of portfolios and asset allocations using DEA and DEA/AHP (Analytic Hierarchy Process) ranking model. Input: variance. Output: return	
Branda M. (2011)	Quadratic programming	Proposes models related to the third-degree stochastic dominance (TSD), based on necessary conditions for TSD and on related mean-risk models; these models draw their inspiration from the DEA methodology and lead to quadratic programming problems	World, 2006–2010, 25 financial indices
Branda M. (2013a)	Mixed-integer linear programming	Proposes efficiency models which draw their inspiration from the DEA methodology and take into account portfolio diversification; they lead to mixed-integer linear programming problems. Uses general deviation measures as inputs and return measures as outputs	World, 2006–2010, 25 financial indices
Branda M. (2013b)		Presents numerically tractable formulations of the diversification-consistent models proposed in others contributions of Branda	US, 2002–2011, 46 industries portfolios
Branda M. (2013c)	Same model used in Branda (2013a)	Presents a proof of equivalence	
Branda M., Kopa M. (2012a)	CRS	Compares DEA, mean-risk and stochastic dominance on a set of financial indices. Inputs: several risk measures. Output: return	World, 2006–2010, 25 financial indices

Branda M., Kopa M. (2012b)	CRS, VRS	Compares several DEA models for portfolio efficiency and a diversification-consistent model. Inputs: several risk measures. Output: return	US, 1982–2011, 48 industries portfolios
Branda M., Kopa M. (2014)	CRS, VRS	Empirically compares CRS and VRS DEA models and diversification-consistent models. Inputs: conditional value at risk at several probability levels. Output: return	US, 1982–2011, 48 industries portfolios
Briec W., Kerstens K., Jokung O. (2007)	Shortage function	Extends the shortage function to the mean-variance-skewness space to account for a preference for positive skewness in addition to a preference for returns and an aversion to risk	1997–1999, 35 assets
Briec W., Kerstens K., Lesourd J.B. (2004)	Shortage function	Studies existing nonparametric efficiency measurement approaches for single period portfolio selection from a theoretical perspective and generalizes currently used efficiency measures into the mean-variance space	26 funds analyzed in Morey and Morey (1999)
Chen H.-H. (2008)		Uses DEA to construct stock portfolios	Taiwan, stocks
Dia M. (2009)		Presents a four-step methodology for portfolio selection based on DEA	Stocks
Ding H. Zhou Z., Xiao H., Ma C., Liu W. (2014)	BCC-I, BCC-O	Develops DEA models to evaluate the performance of portfolios with margin requirements; the BCC frontiers approximate the exact frontier. Input: variance. Output: expected return	Simulations
Edirisinghe N.C.P., Zhang X. (2007)	GDEA	Proposes a generalized DEA approach (GDEA) for stock portfolios in which the selection of inputs and outputs is sought iteratively. Potential inputs and outputs: 18 financial parameters through a range of performance perspectives (profitability, asset utilization, liquidity, leverage, valuation, growth)	US, 1996–2002, 230 stocks
Edirisinghe N.C.P., Zhang X. (2010)	Iterative two-stage optimization	Presents a methodology for selecting input and output variables endogenously to the DEA model in the presence of expert's knowledge	US, 1997–2005, 827 stocks

(continued)

Table 8.8 (continued)

Paper	DEA models	Features	Empirical analysis
Galagedera D.U.A. (2013)	CCR, cross-efficiency	Estimates the cross-efficiency of equity markets in a multi-dimensional risk-adjusted return framework. Inputs: standard deviation, beta, downside deviation. Outputs: two return factors (two positive variables based on observed excess returns)	World, 2003–2011, 40 equity markets
Gnanasekar I.F., Arul R. (2013)		Case study on financial risk tolerance using DEA	
Gnanasekar I.F., Arul R. (2014)		Studies the efficiency of portfolio investors with respect to the financial risk tolerance using DEA	
Hsu C.-M. (2014)	CCR	Proposes an integrated procedure using DEA, ant colony optimization for continuous domains and gene expression programming, in which DEA is used to select stocks in the first stage. Inputs: total assets, total equity, cost of sales, operating expenses. Outputs: net sales, net income	Taiwan, 2007–2011, 48 stocks
Huang C.-Y., Chiou C.-C., Wu T.-H., Yang S.-C. (2015)		Proposes an integrated DEA-MODM method for portfolio optimization. Uses DEA to select the portfolio and develops a multi-objective decision-making (MODM) model to determine the allocation of capital to each stock in the constructed portfolio	Taiwan
Huang T.H., Leu Y.H. (2014)		Presents a method to construct a profitable portfolio of mutual funds. In the first stage, the DEA, Sharpe and Treynor indices and the monthly rates of return are used to select a mutual fund portfolio. In the second stage, the linear regression model, the fruit fly optimization algorithm and the general regression neural network are used to construct a prediction model	
Ismail M.K.A., Salamudin N., Rahman N. M.N.A., Kamaruddin B.H. (2012)		Employs DEA to evaluate the firms' efficiencies, which are then used to form a portfolio	Malaysia, 2004–2005

Kadoya S., Kuroko T., Namatame T. (2008)	DEA, inverted DEA	Proposes an index for an investment strategy to capture the return-reversal effect using both DEA and inverted DEA. DEA inputs (inverted DEA outputs): three surprise indices calculated using sales, operating profit, ordinary profit. DEA outputs (inverted DEA inputs): 1 year return, 3 year return, 5 year return	Japan, 2000–2004, 1146 stocks
Kumar Singh A., Sahu R., Bharadwaj S. (2010)	CCR	Compares ordered weighted averaging (OWA)-heuristic algorithm and basic DEA for asset selection. Input: variance. Output: return	India, 2005–2007, 45 stocks
Lim S., Oh K.W., Zhu, J. (2014)		Uses DEA cross-efficiency evaluation in portfolio selection. Sixteen financial metrics within various performance perspectives are used as input and output variables. Input perspectives: asset utilization, liquidity, leverage. Output perspectives: profitability, growth	Korea, 2002–2011, stocks
Lopes A., Lanzer E., Lima M., da Costa N. Jr. (2008)	CCR	Defines a multi-period investment strategy based on DEA to select efficient stocks. Inputs: price to earnings ratio, beta, volatility. Outputs: 1 year return, 3 year return, 5 year return	Brazil, 2001–2006, stocks
Pai V. (2012)		Explores the application of an extended ant colony optimization algorithm for the solution of a risk budgeted portfolio optimization problem. Compares the results with those obtained by two other metaheuristic optimization methods. Uses DEA to compare the efficiencies of the optimal risk budgeted portfolios obtained by the three approaches	Bombay and Tokyo Stock Exchange, 2001–2006
Pätäri E., Leivo T., Honkapuro S. (2010)		Uses DEA as a basis of value portfolio selection criterion. The performance of portfolios is evaluated on the basis of average return and several risk-adjusted performance metrics	Finland, non-financial stocks

(continued)

Table 8.8 (continued)

Paper	DEA models	Features	Empirical analysis
Pätäri E., Leivo T., Honkapuro S. (2012)	CCR, super-efficiency, cross-efficiency	Uses DEA as a basis of selection criteria for equity portfolios, integrating the benefits of both value investing and momentum investing. Considers three variants of DEA models and four combinations of input and output variables	Finland, 1994–2010, 126 - non-financial stocks
Powers J., McMullen P.R. (2000)		Uses DEA to determine efficient stocks. Inputs: standard deviation, beta, price to earnings ratio. Outputs: 1 year return, 3 year return, 5 year return, 10 year return, earnings per share	185 large cap stocks
Premachandra I., Powell J.G., Shi J. (1998)	SDEA	Uses a spreadsheet-based numerical stochastic data envelopment analysis (SDEA) model estimated with a simulation package to study the selection of portfolios by fund managers. Inputs: total value initially invested in risky holdings, value of each portfolio's initial risk-free investments. Output: portfolio's total market value at the end of a time period minus the comparison benchmark return	New Zealand, 1975–1992
Sahoo B.K., Meera E. (2008)	SBM, CRS, VRS	Empirical paper on large cap market securities which uses different DEA models. Inputs: standard deviation, beta, price to earnings ratio. Outputs: 1 year return, 3 year return, 6 year return, earning per share	India, 93 stocks
Sengupta J.K. (2003)	BCC	Develops a set of nonparametric tests which includes the convex hull method and the stochastic dominance criteria. Different selections of input and output variables	1988–1998, 60 fund portfolios
Zamani L., Beegam R., Borzoian S. (2014)	BCC-I, super-efficiency	Uses DEA to build portfolios. Inputs: beta, modified 5 year beta, debt to equity ratio. Outputs: return on equity, return on capital employment, net profit margin, earning per share	India, 2013, 43 stocks

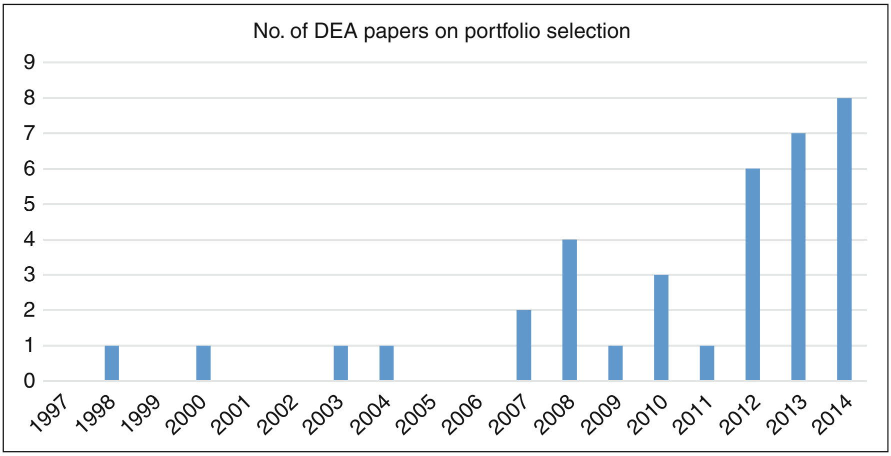

Fig. 8.3 Number of published papers on DEA models for portfolio selection and stock indices by year of publication

in the empirical applications to the performance evaluation of mutual funds when, as is usual, an output variable is chosen as either the mean return or the mean excess return.

On the other hand, it is well known in the literature that the DEA performance measure may give non satisfactory results when some output variables may take negative values; see for example Silva Portela et al. (2004). The reason can easily be grasped from a quick analysis of the following stylized examples, in which we consider a DEA model with constant returns to scale.

Let us consider the problem of evaluating the performance of four decision making units (DMUs) U_1, U_2, U_3, U_4, with one input x and two outputs y_1 and y_2. Let the values of the outputs of the four DMUs, normalized with respect to the input value, be as follows:

$$U_1 = \left(\frac{y_{11}}{x_1}, \frac{y_{21}}{x_1}\right) = (5, 1) \tag{8.1}$$

$$U_2 = \left(\frac{y_{12}}{x_2}, \frac{y_{22}}{x_2}\right) = (3, 2) \tag{8.2}$$

$$U_3 = \left(\frac{y_{13}}{x_3}, \frac{y_{23}}{x_3}\right) = (2, 3) \tag{8.3}$$

$$U_4 = \left(\frac{y_{14}}{x_4}, \frac{y_{24}}{x_4}\right) = (-1, a), \tag{8.4}$$

with $a \in \mathbf{R}^+$.

In the CCR model with constant returns to scale, the DEA performance measure for DMU j_0, with $j_0 \in \{1, 2, 3, 4\}$, is the optimal value of the following linear fractional programming problem

$$\max_{v,\, u_1,\, u_2} \frac{u_1 y_{1j_0} + u_2 y_{2j_0}}{v x_{j_0}} \tag{8.5}$$

s.t.

$$\frac{u_1 y_{1j} + u_2 y_{2j}}{v x_j} \leq 1 \quad j = 1,2,3,4 \tag{8.6}$$

$$v, u_1, u_2 \geq \varepsilon, \tag{8.7}$$

where v, u_1, u_2 are the weights associated to the input and output variables, respectively, and ε is a non-Archimedean constant. The optimal solution can be found by solving the following equivalent output oriented linear program

$$\min_{v,\, u_1,\, u_2} \ v x_{j_0} \tag{8.8}$$

s.t.

$$u_1 y_{1j_0} + u_2 y_{2j_0} = 1 \tag{8.9}$$

$$u_1 y_{1j} + u_2 y_{2j} \leq v x_j \quad j = 1,2,3,4 \tag{8.10}$$

$$v, u_1, u_2 \geq \varepsilon, \tag{8.11}$$

which is equivalent to the following reduced linear problem

$$\min_{v,\, u_1,\, u_2} \ v x_{j_0} \tag{8.12}$$

s.t.

$$u_1 \frac{y_{1j_0}}{x_{j_0}} + u_2 \frac{y_{2j_0}}{x_{j_0}} = \frac{1}{x_{j_0}} \tag{8.13}$$

$$u_1 \frac{y_{1j}}{x_j} + u_2 \frac{y_{2j}}{x_j} \leq v \quad j = 1,2,3,4 \tag{8.14}$$

$$u_1, u_2 \geq \varepsilon. \tag{8.15}$$

If we restrict the analysis to the set of DMUs U_1, U_2, U_3, we have a classical DEA problem in which all the input and output values are positive. The efficiency frontier of such an instance is represented in Fig. 8.4, where the cartesian axes represent the normalized output values $\frac{y_{1j}}{x_j}$ and $\frac{y_{2j}}{x_j}$.

The efficient frontier is the upper-right line which connects the efficient DMUs, i.e. the DMUs with a DEA performance measure equal to 1. Figure 8.4 shows that DMUs U_1 and U_3 are efficient, while U_2 is inefficient and its DEA performance measure is equal to $\frac{dist\ (O, U_2)}{dist\ (O, P_2)} = 0.923$. As is well known, the point P_2 represents

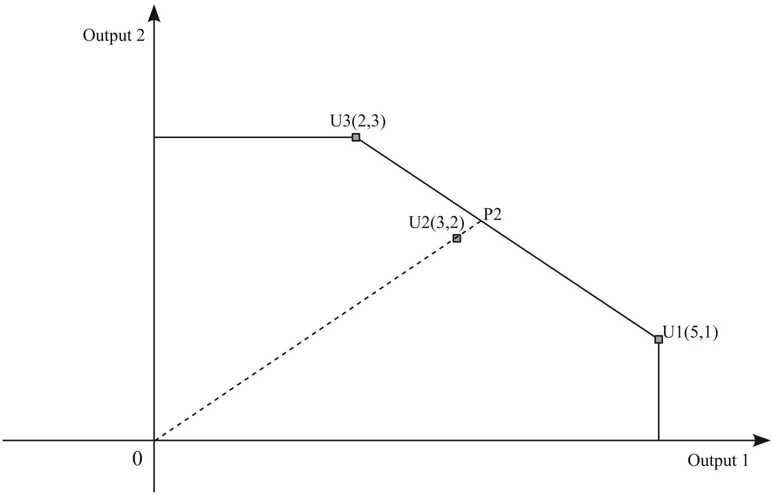

Fig. 8.4 Efficient frontier of the example with DMUs U_1, U_2, U_3 (normalized output values)

Table 8.9 DEA efficiency scores for DMUs U_1, U_2, U_3, U_4 for different values of $a \in \mathbf{R}^+$, the second output of U_4

a	Score of U_1	Score of U_2	Score of U_3	Score of U_4
1	1.000	0.923	1.000	0.333
2	1.000	0.923	1.000	0.667
3	1.000	0.923	1.000	1.000
4	1.000	0.923	1.000	1.000
5	1.000	0.923	1.000	1.000
6	1.000	0.871	0.903	1.000
7	1.000	0.833	0.833	1.000
8	1.000	0.805	0.780	1.000
9	1.000	0.783	0.739	1.000
10	1.000	0.765	0.706	1.000

the virtual unit which has the same input and output orientation as U_2 and lies on the efficient frontier. This virtual unit suggests that unit U_2 might improve its output values while keeping the input value fixed, by moving along the dashed line $\overline{OP_2}$ towards the efficient frontier, till its reaches efficiency.

If we include in the analysis also DMU U_4, which has a negative value of output 1, puzzling results can be obtained, so that the DEA fractional problem (8.5)–(8.7) does not give a reasonable efficiency measure any longer. Table 8.9 displays the values of the efficiency measure obtained for the four DMUs for different values of the second output of U_4, a; Figs. 8.5, 8.6, 8.7, and 8.8 show the efficient frontier obtained in some relevant cases.

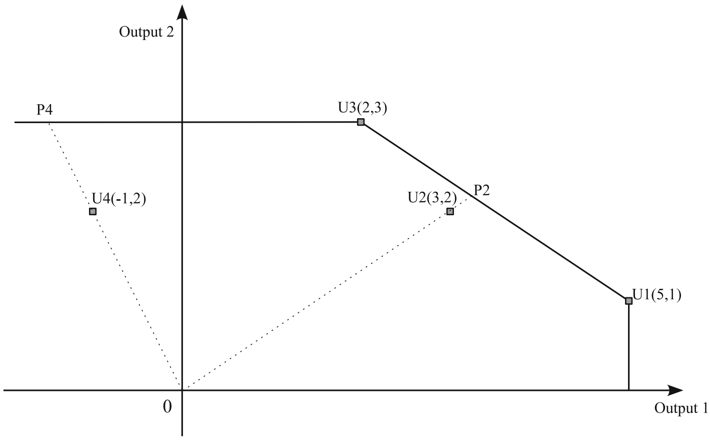

Fig. 8.5 Efficient frontier of the example with DMUs U_1, U_2, U_3, U_4 in the case $U_4 = (-1, 2)$ (normalized output values)

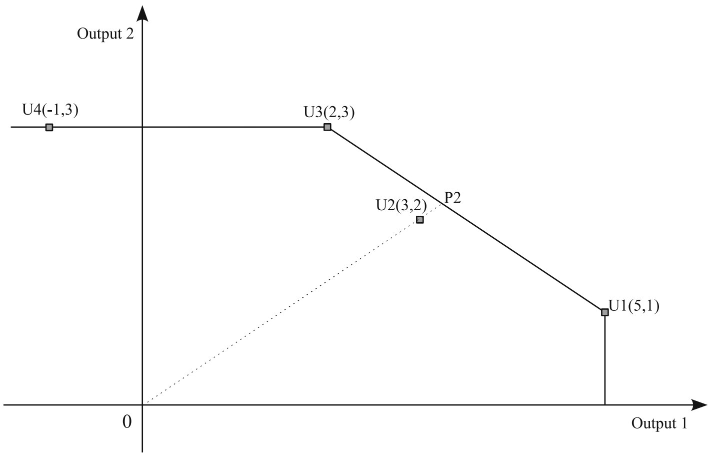

Fig. 8.6 Efficient frontier of the example with DMUs U_1, U_2, U_3, U_4 in the case $U_4 = (-1, 3)$ (normalized output values)

As can be seen, in the cases with $a \leq 3$ the inclusion of U_4 in the analysis does not modify the part of the efficient frontier which envelops U_1, U_2, U_3: this part is exactly the same as in the case without U_4; this is important because it entails that the efficiency scores of U_1, U_2, U_3 does not change either. For $a < 3$ (see Fig. 8.5) U_4 does not lie on the efficient frontier and therefore it is not efficient, while for $a = 3$ U_4 reaches the efficient frontier, as shown in Fig. 8.6, and therefore it becomes efficient.

In the cases with $3 < a \leq 5$, represented in Fig. 8.7, the displacement of U_4 upwards does modify the efficient frontier; however this shift does not alter the

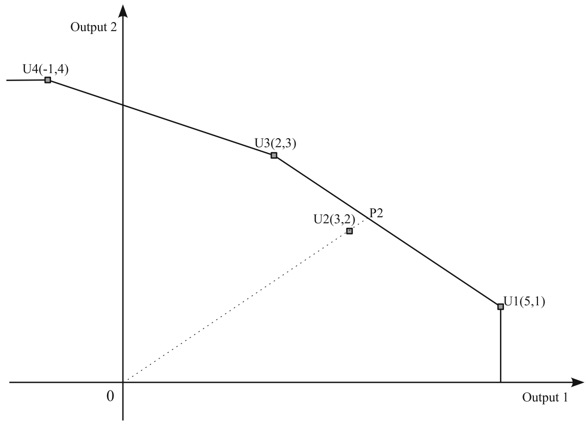

Fig. 8.7 Efficient frontier of the example with DMUs U_1, U_2, U_3, U_4 in the case $U_4 = (-1,4)$ (normalized output values)

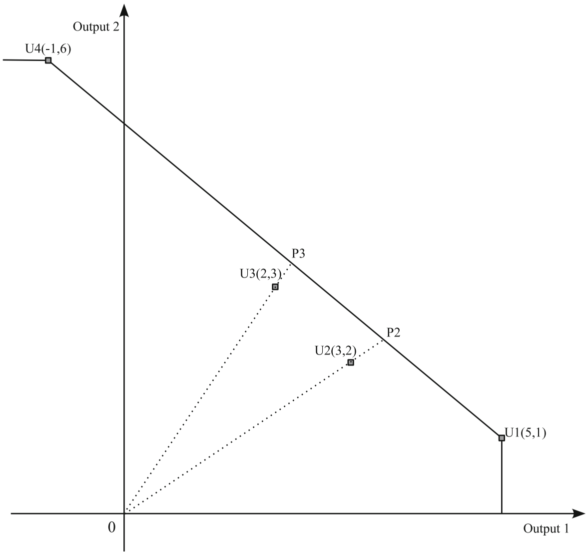

Fig. 8.8 Efficient frontier of the example with DMUs U_1, U_2, U_3, U_4 in the case $U_4 = (-1,6)$ (normalized output values)

section of the efficient frontier that determines the efficiency scores of U_1, U_2, U_3, so that their performance measures do not change. On the other hand, for $a \geq 3$ U_4 lies on the efficient frontiers and hence it is efficient.

Figure 8.8 shows that for $a > 5$ the raising of U_4 moves the efficient frontier away from DMUs U_2 and U_3, causing a worsening of their efficiency scores; this shift makes U_3 become inefficient. Hence, a sufficiently high value of the second output can compensate for the negative value of the first output, in such a way as to make U_4 become efficient when the value of the second output is high enough.

On the other hand, let us keep the values of both the input and the second output constant while decreasing the value of the first (negative) output. In particular, let us analyze the behavior of the efficiency score of U_4 as the value of the negative

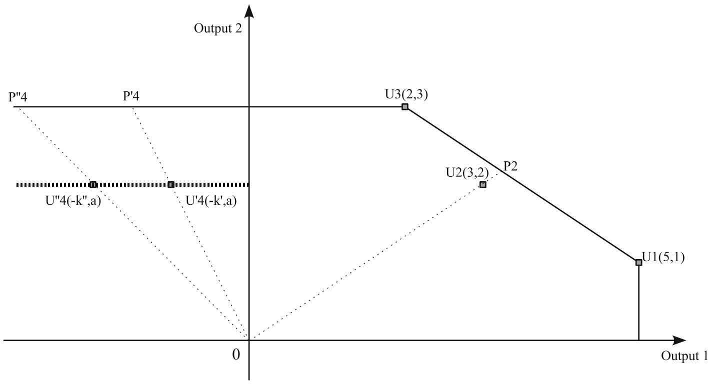

Fig. 8.9 Distance from the efficient frontier of DMU $U_4(-k,a)$ for different values of k and $a=2$ (normalized output values)

output worsens. In such a case, a good performance measure should exhibit a decreasing efficiency score for U_4 as the negative output value worsens. However, this is not what happens.

Actually, let us consider $U_4(-k,a)$ for $0<a\leq 3$ as $k>0$ increases; Fig. 8.9 shows the situation for $k'=1$ and $k''=2$, and $a=2$. It is easy to see that the Cartesian coordinates of the virtual unit P_4 on the efficient frontier are the following

$$P4\left(-\frac{3k}{a},3\right), \tag{8.16}$$

so that the DEA efficiency score of U_4 turns out to be constant and equal to

$$\frac{dist\,(O,U_4)}{dist\,(O,P_4)}=\frac{\sqrt{(-k)^2+a^2}}{\sqrt{\left(-\frac{3k}{a}\right)^2+3^2}}=\frac{a}{3}, \tag{8.17}$$

no matter the value of the first output. This means that the efficiency measure of U_4 is the same for all values of the first (negative) output, and thus the value of the second output (besides that of the input) is the only thing that matters.

In the context of the measurement of the performance of mutual funds, this fact has an unrealistic consequence, that does not satisfy the usual economic assumptions on the investors preferences. Actually, if the first output represents the average rate of return of the mutual fund (or its average excess return), this entails that when the average rate of return is negative, its value is indifferent for investors, whether it is only slightly less than zero or it entails a heavy loss. In this case only the value of the second output would be relevant, which is clearly in contrast with the economic principle that, all other things equal, a higher expected value is always preferred.

On the other hand, if we consider a single output that may take negative values (and any number of inputs), things are not better. Actually, it can be easily seen from the CCR-O model

$$\min_{u, v_1, \ldots, v_p} \sum_{i=1}^{p} v_i x_{ij_0} \tag{8.18}$$

s.t.

$$uy_{j_0} = 1 \tag{8.19}$$

$$\sum_{i=1}^{p} v_i x_{ij} \geq uy_j \qquad j = 1, 2, \ldots, n \tag{8.20}$$

$$u, v_1, v_2, \ldots, v_p \geq \varepsilon, \tag{8.21}$$

that if y_{j_0} is negative a solution does not even exist; in such a case the feasible region is empty, since the first constraint (8.19) has no solution with $u \geq \varepsilon$.

Things only partially improve introducing variable returns to scale with a BCC model. Indeed, a BCC-I model is translation invariant with respect to outputs (see Pastor 1996), which means that the DEA efficiency measure is invariant for translations of the original output values consequent to an addition of a constant to the original data. Therefore, with a BCC-I model the problem of negative returns (or excess returns) can be easily overcome by adding a suitable constant (greater than the absolute value of the lowest mean return) to the negative output; for an overview on this subject see also Pastor and Ruiz (2007).

However, since the performance results depend on the orientation of the model, the orientation should be carefully chosen on the basis of financial considerations. In our opinion, the most appropriate orientation for the assessment of the performance of mutual funds is the output orientation, since investors usually seek to maximize the value of mean returns and (eventual) other output variables without increasing the value of the input variables. But the BCC-O model is not translation invariant with respect to outputs, so that with this model we face drawbacks analogous to those encountered with CCR models; for further remarks see for instance Silva Portela et al. (2004).

On the other hand, we could try to adopt a suitable DEA model which is translation invariant (Ali and Seiford 1990; Lovell and Pastor 1995). In particular, a well known DEA model with such a property is the additive model, and actually this model is often used in order to tackle the problem of negative data in DEA analyses. However, an additive DEA model discriminates between efficient and inefficient DMUs, but it cannot gauge the depth of eventual inefficiencies: indeed, the efficiency measure given by an additive model does not provide a radial efficiency measure such as that given by the basic CCR and BCC models (for the additive model see e.g. Cooper et al. 2011), and its financial interpretation is far from being straightforward.

Another approach, the range directional model proposed in Silva Portela et al. (2004), treats the problem of negative data in DEA models by modifying the efficiency measure used, but neither this approach is directly connected to radial efficiency. Other contributions recently appeared in the literature propose different approaches which modify, in one way or another, the efficiency measure. Among them, we find the semi-oriented radial measure used by Emrouznejad et al. (2010), a generalised proportional distance function used by Kerstens and Van de Woestyne (2011), a shortage function adopted in Kerstens et al. (2012), a probabilistic characterization of directional distances devised by Simar and Vanhems (2012); Cheng et al. (2013) also proposes a way to deal with negative data.

Notwithstanding, these methodologies make the efficiency measures difficult to interpret, especially from a financial point of view. If we aim to assert the validity of a DEA efficiency measure for the evaluation of the performance of mutual funds and wish to spread its adoption in the financial practice, these approaches may well be met with distrust in the financial world. For this reason, we need a model which is at the same time simple, financially meaningful and able to deal with both positive and negative returns.

Actually, only some of the DEA models used in the literature to assess the performance of mutual funds can cope with the presence of negative mean returns. Table 8.10 reports the papers on mutual and hedge funds that explicitly tackle this problem and highlights the different solutions that they adopt to allow for the presence of negative returns in the DEA models.

In addition, as can be seen looking at Tables 8.1, 8.2, 8.3, 8.4, 8.5, 8.6, and 8.7, even without making any remark on the problem of negative data, some of the papers use either an input oriented BCC model or an additive model which are able to handle the problem.

On the other hand, let us point out that the presence of negative mean returns poses a problem not only in DEA modeling. Also most of the traditional indicators used to evaluate the performance of mutual funds run into serious problems that prevent a sensible usage in the presence of negative mean returns, as will be discussed in Sect. 8.5.

8.4 A DEA Model for the Performance Assessment in Periods of Financial Crisis

We adopt a performance measure recently proposed in Basso and Funari (2014a) which is simple to implement, meaningful from a financial point of view and able to deal with the negative returns characterising the periods of financial crisis. This model is focused on the objectives of investors and takes the point of view of a representative investor who has to pick the best mutual fund in the set $\{1, 2, \ldots, n\}$ of mutual funds analyzed.

Table 8.10 Negative data

Paper	Methodology adopted to allow for negative returns
Basso A., Funari S. (2005b)	Uses the mean capitalization factor, which is always positive, instead of mean return
Basso A., Funari S. (2008)	Uses the mean capitalization factor, which is always positive, instead of mean return
Basso A., Funari S. (2014a)	Uses the final value of the investment, which is always positive, instead of mean return
Basso A., Funari S. (2014b)	Uses the final value of the investment, which is always positive, instead of mean return
Basso A., Funari S. (2014c)	Uses the final value of the investment, which is always positive, instead of mean return
Basso A., Funari S. (2014d)	Uses the mean capitalization factor, which is always positive, instead of mean return
Gregoriou G.N., Zhu J. (2005)	Uses a model that is translation invariant with respect to outputs (BCC-I)
Hu J.-L., Yu H.-E., Wang Y.-T. (2012)	Uses a model that is translation invariant with respect to outputs (BCC-I)
Kumar U.D., Roy A., Saranga H., Singal K. (2010)	Uses slack-based models
Lozano S., Gutiérrez E. (2008a)	Uses an additive model
Lozano S., Gutiérrez E. (2008b)	Uses an additive model
Simar L., Vanhems A., Wilson P.W. (2012)	Uses a model with directional distance and an input oriented VRS model
Tavakoli Baghdadabad M.R., Noori Houshyar A. (2014)	Uses the mean capitalization factor, which is always positive, instead of mean return

Let K_j be the initial payout required by fund j to start with an initial capital equal to 1, net of the initial fee c_{Ij}:

$$K_j = \frac{1}{1 - c_{Ij}} \qquad j = 1, 2, \ldots, n. \tag{8.22}$$

This is the first input variable of the model; in addition, we consider also one or more risk measures $\{q_{1j}, q_{2j}, \ldots, q_{hj}\}$ that may shed light on different features of the financial risk of fund j. For example, we may include widely used measures, such as the historical volatility σ_j (the standard deviation of the returns of fund j) and the β-coefficient β_j (the ratio of the covariance between fund j and the market returns to the variance of the market return), but also some of the other measures proposed in the recent literature (see the review presented in Sect. 8.2).

As for the outputs, we consider a single output model in which the output variable is the final value M_j of the investment, net of the exit fee c_{Ej}; more precisely, M_j is defined as follows:

$$M_j = (1 + R_j)^T (1 - c_{Ej}) \qquad j = 1, 2, \ldots, n, \tag{8.23}$$

where R_j denotes the mean rate of return of fund j in the holding period of length T considered, measured on an annual basis using the compound interest regime.

Equivalently, we may compute the final value M_j using the continuous law of interest; as a matter of fact, this is the choice adopted in the empirical analysis that is presented in Sect. 8.6. In such a case, R_j denotes the mean instantaneous rate of return, measured on an annual basis using the continuous compounding, and M_j can be written as:

$$M_j = \mathrm{e}^{R_j T}(1 - c_{Ej}) \qquad j = 1, 2, \ldots, n. \tag{8.24}$$

Note that the final values M_j computed with formulas (8.23) and (8.24) coincide, since the instantaneous rate of return used in formula (8.24) is equal to the natural logarithm of 1 plus the compound rate of return of formula (8.23). Note also that either of the two methods to compute the rate of return, compound or continuous capitalization, may be used in the computation of the risk measures $\{q_{1j}, q_{2j}, \ldots, q_{hj}\}$.

We point out that the output variable is a measure of the overall profitability of the investment and, as such, depends heavily on the choice of the holding period.

On the other hand, we have $M_j \geq 0 \, \forall \, j$, independently of the phase of the business cycle, so that the model can easily be used also in the presence of negative mean returns.

Let us also observe that this choice of the input and output variables enables the model to take into account the initial and exit fees without the need to include them directly as input variables. We may thus avoid the problem of assessing a fund as efficient only because it has low fees.

As we have seen in Sect. 8.2, in the literature on mutual funds we find instances of both models with constant returns to scale (CRS) and others with variable returns to scale (VRS). We formulate both a constant returns to scale model, which will be denoted by DEA-C, and a variable returns to scale model, denoted by DEA-V.

In order to write the DEA-C model, which is a "plain vanilla" CCR model, let us begin with its formulation as a fractional programming problem:

$$\max_{\{u, v_i\}} \quad \frac{u M_o}{v_1 K_o + \sum_{i=2}^{h+1} v_i q_{io}} \tag{8.25}$$

subject to

$$\frac{u M_j}{v_1 K_j + \sum_{i=2}^{h+1} v_i q_{ij}} \leq 1 \quad j = 1, 2, \ldots, n \tag{8.26}$$

$$u \geq \varepsilon, \tag{8.27}$$

$$v_i \geq \varepsilon \quad i = 1, 2, \ldots, h+1 \tag{8.28}$$

where u is the weight assigned to the final value M_j, v_1 is the weight assigned to the initial payout $K_j, v_2, v_3, \ldots, v_{h+1}$ are the weights assigned to the h risk measures and

ε is a non-Archimedean constant. As usual in the data envelopment analysis, the optimal value of the objective function (8.25) gives the efficiency score of mutual fund $o \in \{1, 2, \ldots, n\}$.

As is well known, model (8.25)–(8.28) can be transformed into an equivalent linear programming problem. We may distinguish the case $M_o = 0$ from the case $M_o > 0$. When $M_o = 0$ the efficiency score is clearly equal to 0. When $M_o > 0$ we can adopt the output orientation and write the following equivalent linear program:

$$\min_{\{u, v_i\}} \quad v_1 K_o + \sum_{i=2}^{h+1} v_i q_{io} \tag{8.29}$$

subject to

$$uM_o = 1 \tag{8.30}$$

$$-uM_j + v_1 K_j + \sum_{i=2}^{h+1} v_i q_{ij} \geq 0 \quad j = 1, 2, \ldots, n \tag{8.31}$$

$$u \geq \varepsilon \tag{8.32}$$

$$v_i \geq \varepsilon \quad i = 1, 2, \ldots, h+1 \tag{8.33}$$

In this single output model we may observe that constraint (8.30) makes constraint (8.32) redundant.

We may also consider the dual of problem (8.29)–(8.33):

$$\max \quad z_0 + \varepsilon s^+ + \sum_{i=1}^{h+1} \varepsilon s_i^- \tag{8.34}$$

subject to

$$M_o z_0 - \sum_{j=1}^{n} M_j \lambda_j + s^+ = 0 \tag{8.35}$$

$$\sum_{j=1}^{n} K_j \lambda_j + s_1^- = K_o \tag{8.36}$$

$$\sum_{j=1}^{n} q_{ij} \lambda_j + s_i^- = q_{io} \qquad i = 2, 3, \ldots, h+1 \tag{8.37}$$

$$\lambda_j \geq 0 \quad j = 1, 2, \ldots, n \tag{8.38}$$

$$s^+ \geq 0 \tag{8.39}$$

$$s_i^- \geq 0 \quad i = 1, 2, \ldots, h+1, \tag{8.40}$$

where z_0 is the dual variable associated with the equality constraint (8.30), λ_j are the dual variables associated with the mutual funds constraints (8.31) and s_1^+ and s_i^- are the dual variables connected with the output and input weight constraints (8.32) and (8.33), respectively.

It is known that one of the advantages of the DEA methodology is that it gives further information to the inefficient units, with the indication of the so called "virtual unit", which is a combination of efficient units (the "peers") that is efficient with the inefficient unit's weights. The financial interpretation of the virtual unit is interesting, since it may be seen as an efficient benchmark portfolio with a similar profile, which the inefficient fund can strive to imitate (see Basso and Funari 2001).

This benchmark portfolio is defined as the linear combination of the peers with coefficients given by the optimal values λ_j^* of the dual variables λ_j and has inputs

$$\sum_{j=1}^{n} \lambda_j^* K_j \tag{8.41}$$

and

$$\sum_{j=1}^{n} \lambda_j^* q_{ij} \qquad\qquad i = 2, 3, \ldots, h+1 \tag{8.42}$$

and output

$$\sum_{j=1}^{n} \lambda_j^* M_j. \tag{8.43}$$

Let us now consider a convexity constraint on the "production possibility set" (see Banker et al. 1984); in our analysis of mutual funds, this means that we restrict the set of benchmark portfolios that can be considered to the convex hull of the funds analyzed, so that only convex combinations of the existing funds are allowed. This is obtained by adding the following convexity constraint

$$\sum_{j=1}^{n} \lambda_j = 1 \tag{8.44}$$

to the dual program (8.34)–(8.40). We obtain in this way the corresponding BCC model with variable returns to scale, namely model DEA-V:

$$\max \quad z_0 + \varepsilon s^+ + \sum_{i=1}^{h+1} \varepsilon s_i^- \tag{8.45}$$

subject to

$$M_o z_0 - \sum_{j=1}^{n} M_j \lambda_j + s^+ = 0 \tag{8.46}$$

$$\sum_{j=1}^{n} K_j \lambda_j + s_1^- = K_o \tag{8.47}$$

$$\sum_{j=1}^{n} q_{ij} \lambda_j + s_i^- = q_{io} \quad i = 2, 3, \ldots, h+1 \tag{8.48}$$

$$\sum_{j=1}^{n} \lambda_j = 1 \tag{8.49}$$

$$\lambda_j \geq 0 \quad j = 1, 2, \ldots, n \tag{8.50}$$

$$s^+ \geq 0 \tag{8.51}$$

$$s_i^- \geq 0 \quad i = 1, 2, \ldots, h+1, \tag{8.52}$$

With the addition of constraint (8.44), the primal program associated to the dual program (8.45)–(8.52) requires an additional variable, free in sign, whose sign allows one to perform a local analysis of the returns to scale for the points on the efficient frontier, telling whether they are increasing, constant or decreasing (see Banker et al. 1984).

8.5 More Traditional Indicators of Mutual Fund Performance

Various traditional indicators often used in finance to assess the performance of mutual funds either refer to ratios of the expected value of the excess return over a measure of the risk of the investment or are derived from well known financial models such as the Capital Asset Pricing Model (CAPM). Here the excess return is defined as the difference between the return of the fund and the return of a riskless asset.

As a matter of fact, since the expected values are not known, it is usual to compare the performance of mutual funds over a past period of time by replacing the (ex ante) expected return and the (unknown) value of the risk measure with the mean return R_j and the value of the risk measure computed ex post on the historical data in the period considered.

The most popular indicator defined as a ratio is probably the Sharpe index (see Sharpe 1994). Let $r_{j1}, r_{j2}, \ldots, r_{jT}$ be the rates of return obtained by fund j in the periods $1, 2, \ldots, T$; for example, we could consider the monthly rates of return of the last 3 years. Analogously, let $r_{f1}, r_{f2}, \ldots, r_{fT}$ be the risk-free rates of return in the same periods. The Sharpe index can be computed as follows:

$$I_{j,Sharpe} = \frac{R_j - R_f}{\sqrt{\mathrm{Var}[r_j - r_f]}}, \tag{8.53}$$

where R_j and R_f are the mean return of fund j and the mean risk-free rate in the holding period $[0, T]$, respectively, and $\mathrm{Var}[r_j - r_f]$ is the variance of the differences $r_{jt} - r_{ft}$.

Note that the Sharpe index, as well as the other traditional indicators, does not take into account the initial and exit fees required by the mutual fund.

The standard deviation used as risk measure in the Sharpe index is appropriate when the fund returns are normally distributed and the investor does not possess other risky assets.

When the investor possesses a well diversified portfolio of assets, a more suitable risk measure is the beta coefficient β_j; this is the risk measure used by the Treynor index (see Treynor 1965), which is defined as the ratio:

$$I_{j,Treynor} = \frac{R_j - R_f}{\beta_j}. \tag{8.54}$$

When the fund returns are not normally distributed, risk measures other than the standard deviation may better describe the risk. Among them, it is worth citing the downside risk DR, defined as the lower semi-deviation of the returns from a target value m (also called minimum acceptable return, Sortino and van der Meer 1991):

$$\mathrm{D}R_j = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(\min[r_{jt} - m, 0])^2}. \tag{8.55}$$

This measure of risk translates the statement of fact that investors consider as unfavourable only the returns lower than the target value, not those that exceed it.

The downside risk is used by the Sortino index to define another risk adjusted indicator which is similar to the Sharpe ratio but measures the risk with the downside risk:

$$I_{j,Sortino} = \frac{R_j - m}{\mathrm{D}R_j}. \tag{8.56}$$

When the target is not chosen as a fix predetermined value but as the return of a benchmark, such as for example the return of a riskless asset, the Sortino index may be adjusted by computing the downside risk as follows:

$$\mathrm{D}R_j = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(\min[r_{jt} - r_{ft}, 0])^2} \tag{8.57}$$

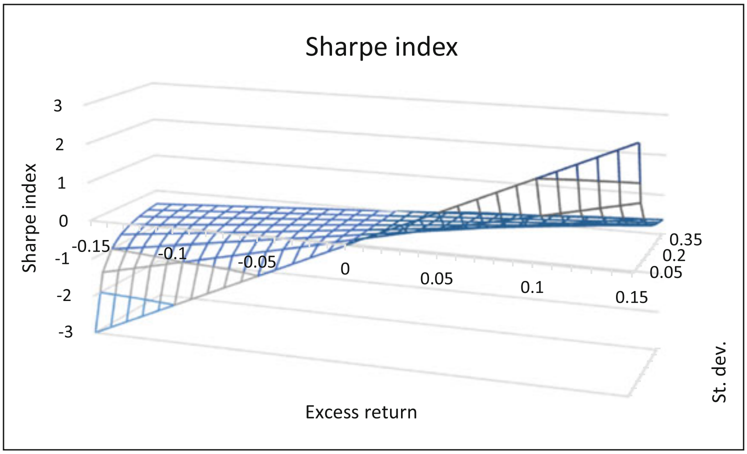

Fig. 8.10 Behavior of the Sharpe ratio as the excess return and the standard deviation vary

and writing:

$$I_{j,Sortino} = \frac{R_j - R_f}{\mathrm{D}R_j}. \tag{8.58}$$

On the other hand, for the funds that exhibit a negative mean excess return, these traditional indicators can give misleading results.

The situation is well depicted in Fig. 8.10 for the Sharpe ratio, but the same drawback is exhibited also by the Treynor and Sortino indices. This figure shows the behavior of the Sharpe ratio as the excess return $R_j - R_f$ and the standard deviation $\sqrt{\mathrm{Var}[r_j - r_f]}$ vary.

It can be noticed that when the excess return is positive, the value of the Sharpe index decreases with the standard deviation, as we expect for a performance indicator. However, when the excess return is negative, the Sharpe index exhibits the opposite behavior as its value increases with the standard deviation. This means that if we compare two mutual funds with a same negative value of the excess return (hence with a rate of return lower than the riskless interest rate), the Sharpe index leads to choose the fund with the highest standard deviation, i.e. that with the highest risk.

Other traditional performance indicators derive from the theory of the well known Capital Asset Pricing Model. The progenitor of these indicators, and still the most widely used among them, is Jensen's alpha index (see Jensen 1968), which measures the portfolio performance through the intercept α_j of the CAPM regression line and can be computed as follows:

$$\alpha_j = (R_j - R_f) - \beta_j(R_m - R_f), \tag{8.59}$$

where R_m is the mean return of the so called "market portfolio" (see Jensen 1968).

Note that the alpha index can be computed and effectively used regardless of the phase of the business cycle. A positive value of α_j indicates that fund j outperformed the market portfolio in the holding period considered, while a negative value denotes that the management of fund j was not able to obtain returns on the line to the market portfolio, once the risk is properly taken into account through the beta coefficient.

8.6 An Empirical Investigation on Different Holding Periods

In order to see how the DEA-V model handles the different phases of the business cycle and to compare the performance scores obtained with this model to the values obtained with the more traditional performance indices, we have carried out an empirical analysis on a large set of European mutual funds from different countries.

To this aim, the investigation is conducted on different holding periods, and the performance measures are computed first on a holding period of 7 years, ranging from 30/11/2006 to 30/11/2013. Then the analysis is performed on two different 3-year subperiods, the first and the last 3 years of the time horizon considered: the first holding period, $[0, 3]$, ranges from 30/11/2006 to 30/11/2009 and is characterized by a negative trend of the economy, the second, $[4, 7]$, ranges from 30/11/2010 to 30/11/2013 and is characterized by a positive trend.

The 312 mutual funds analyzed have been randomly chosen among the mutual funds domiciled in Western Europe; their distribution by country is presented in Table 8.11. Table 8.11 shows also the average values by country of the initial and exit fees and the mean instantaneous return measured on an annual basis.

The rate of returns $r_{j1}, r_{j2}, \ldots, r_{jT}$ have been computed as monthly total returns, accumulating the dividends paid in the holding period considered, and they have been calculated from the fund's net asset value (NAV) extracted from the Bloomberg database, expressed as a per-share amount in euros.

In the analysis we consider three different risk measures, namely the historical volatility σ_j, the β-coefficient β_j computed with respect to the STOXX Europe Total Market Index (TMI, which represents a market portfolio for the Western Europe region), and the downside risk DR_j computed with respect to the 12 month Euribor rate.

Table 8.11 shows all the average values by country and by holding period. Let us remark that the different length of the holding periods affects the final value M_j in different ways.

First, of course the mean return R_j is related to the holding period considered; with regard to this, note that the average value (computed on all the European funds considered) is heavily negative in the holding period $[0, 3]$ (-0.0987, i.e. around $-10\,\%$ per year), while it is strongly positive in the holding period $[4, 7]$ ($9\,\%$ per year); on the whole time interval $[0, 7]$ the overall return is low but positive

Table 8.11 Average values of the fundamental data for the European mutual funds analyzed by country of domicile and by holding period

Country	No. funds	c_I	c_E	β-coeff.	Standard dev.	Downside risk	Mean return	% negative mean ret.
Holding period [0,7]								
Austria	16	0.0400	0.0000	0.9499	0.1872	0.1532	–0.0070	62.5
Belgium	10	0.0300	0.0000	1.0239	0.1913	0.1533	0.0034	40.0
Germany	11	0.0223	0.0023	0.9028	0.1782	0.1405	0.0153	36.4
France	74	0.0307	0.0033	1.0195	0.1858	0.1466	0.0047	37.8
Great Britain	44	0.0232	0.0018	0.8787	0.1871	0.1441	0.0266	13.6
Ireland	11	0.0348	0.0109	0.9464	0.1831	0.1409	0.0312	9.1
Luxembourg	91	0.0231	0.0035	0.9205	0.1828	0.1431	0.0214	19.8
Norway	16	0.0064	0.0013	0.9992	0.1904	0.1487	0.0317	6.3
Sweden	17	0.0065	0.0012	0.9397	0.1826	0.1350	0.0361	11.8
Others	22	0.0111	0.0058	0.7765	0.1593	0.1230	0.0175	27.3
Western Europe	312	0.0238	0.0031	0.9381	0.1832	0.1432	0.0173	25.6
Holding period [0,3]								
Austria	16	0.0400	0.0000	1.0180	0.2356	0.2060	–0.1148	100.0
Belgium	10	0.0300	0.0000	1.0800	0.2383	0.2040	–0.1080	100.0
Germany	11	0.0223	0.0023	0.9388	0.2155	0.1864	–0.1183	100.0
France	74	0.0307	0.0033	1.0159	0.2203	0.1849	–0.0927	100.0
Great Britain	44	0.0232	0.0018	0.9397	0.2327	0.1972	–0.1329	97.7
Ireland	11	0.0348	0.0109	1.0187	0.2278	0.1901	–0.0979	90.9
Luxembourg	91	0.0231	0.0035	0.9703	0.2245	0.1899	–0.0893	96.7
Norway	16	0.0064	0.0013	1.0737	0.2425	0.2031	–0.0790	93.8
Sweden	17	0.0065	0.0012	0.9989	0.2243	0.1808	–0.0916	100.0
Others	22	0.0111	0.0058	0.7888	0.1893	0.1587	–0.0832	100.0
Western Europe	312	0.0238	0.0031	0.9774	0.2239	0.1889	–0.0987	98.1
Holding period [4,7]								
Austria	16	0.0400	0.0000	0.8479	0.1397	0.1014	0.0577	12.5
Belgium	10	0.0300	0.0000	0.9265	0.1421	0.1029	0.0743	10.0
Germany	11	0.0223	0.0023	0.8153	0.1369	0.0934	0.1051	0.0
France	74	0.0307	0.0033	1.0484	0.1537	0.1119	0.0793	1.4
Great Britain	44	0.0232	0.0018	0.7531	0.1345	0.0894	0.1261	0.0
Ireland	11	0.0348	0.0109	0.8168	0.1334	0.0888	0.1083	9.1
Luxembourg	91	0.0231	0.0035	0.8407	0.1381	0.0957	0.0866	8.8
Norway	16	0.0064	0.0013	0.8888	0.1347	0.0913	0.0866	6.3
Sweden	17	0.0065	0.0012	0.8604	0.1387	0.0912	0.0970	0.0
Others	22	0.0111	0.0058	0.7718	0.1276	0.0889	0.0784	9.1
Western Europe	312	0.0238	0.0031	0.8777	0.1404	0.0979	0.0900	5.1

Table 8.12 Correlation coefficients of the performance scores obtained with the DEA-V model using different risk measures (models DEA-V$_{\beta}$, DEA-V$_{\beta,\sigma}$ and DEA-V$_{\beta,DR}$) in the holding period $[0,7]$

	DEA-V$_{\beta}$	DEA-V$_{\beta,\sigma}$	DEA-V$_{\beta,DR}$
DEA-V$_{\beta}$	1		
DEA-V$_{\beta,\sigma}$	0.991	1	
DEA-V$_{\beta,DR}$	0.996	0.997	1

(0.0173). From the last column of Table 8.11 we may observe the presence of a non negligible percentage of funds with a negative mean return even in the "good" periods; for example, in the holding period $[0,7]$ this is observed in as many as one fund in four (25.6 % of the mutual funds considered).

Secondly, the effect of the initial and exit fees on the yearly returns is lessened as the length of the holding period increases. On the other hand, the compounding of interests accentuates the effect of the yearly rate of return on the final value M_j as the length of the holding period increases.

The performance analysis is carried out computing the performance scores with the DEA-V model discussed in Sect. 8.4 and the traditional performance indicators presented in Sect. 8.5. As for the choice of the risk measures to include in the DEA-V model, we have considered primarily the β-coefficient, in order to take into account investors with a well diversified portfolio. In addition, we have also included a dispersion measure, in order to take into consideration also investors without a diversified portfolio.

Table 8.12 shows the values of the correlation coefficients of the performance scores obtained with the DEA-V model by considering only the β-coefficient (model DEA-V$_{\beta}$), β_j and the volatility σ_j (model DEA-V$_{\beta,\sigma}$) and β_j and the downside risk DR_j (model DEA-V$_{\beta,DR}$). All the values are close to 1 (greater than 0.99), so that their results are similar. In the following we analyze more in detail the results obtained with the DEA-V$_{\beta,DR}$ model.

The average results obtained with the different performance measures by country and by holding period are shown in Table 8.13, while Table 8.14 displays the values of their correlation coefficients.

We have seen in Sect. 8.5 that the usage of the traditional Sharpe, Treynor and Sortino indicators should be averted in the presence of negative mean excess returns since they can lead to misleading results. Well, as can be seen from Table 8.13, in the holding period $[0,3]$ the mean excess return is negative for almost all mutual funds (99.4 % of funds): this is evidently a period of financial crisis. As for the holding period $[4,7]$, this is clearly a period of financial recovery; nonetheless, the mean excess return is still negative for a small number of funds (6.4 %). On the whole, in the 7-year holding period $[0,7]$ a good 60 % (more precisely, 60.3 %) of mutual funds exhibit a negative mean excess return, with marked differences among the countries.

From Table 8.14 we may see that the values of the correlation coefficients among the Sharpe, Treynor and Sortino ratios are very high (greater than 0.98 in

Table 8.13 Average values of the performance measures obtained with the DEA-$V_{\beta,DR}$ model and the Sharpe, Treynor, Sortino and alpha indices by country of domicile and by holding period; the mean excess return is also reported

Country	Mean excess ret.	% negative excess ret.	Sharpe index	Treynor index	Sortino index	Alpha index	DEA-$V_{\beta,DR}$ score
Holding period [0,7]							
Austria	−0.0299	93.8	−0.1482	−0.0291	−0.1811	0.0016	0.4689
Belgium	−0.0194	70.0	−0.0933	−0.0181	−0.1102	0.0146	0.5227
Germany	−0.0076	72.7	−0.0095	0.0019	−0.0051	0.0224	0.5995
France	−0.0181	79.7	−0.0903	−0.0163	−0.1098	0.0157	0.5243
Great Britain	0.0038	52.3	0.0229	0.0029	0.0368	0.0329	0.6346
Ireland	0.0083	18.2	0.0484	0.0098	0.0672	0.0397	0.6126
Luxembourg	−0.0014	54.9	−0.0012	0.0006	0.0073	0.0291	0.6158
Norway	0.0088	31.3	0.0521	0.0095	0.0716	0.0420	0.6736
Sweden	0.0132	29.4	0.0613	0.0103	0.0886	0.0444	0.7018
Others	−0.0053	63.6	−0.0227	0.0003	−0.0154	0.0204	0.6388
Western Europe	−0.0055	60.3	−0.0233	−0.0038	−0.0219	0.0256	0.5948
Holding period [0,3]							
Austria	−0.1512	100.0	−0.6553	−0.1515	−0.7534	0.0171	0.6362
Belgium	−0.1444	100.0	−0.6294	−0.1390	−0.7340	0.0341	0.6563
Germany	−0.1547	100.0	−0.6963	−0.1629	−0.8081	0.0005	0.6801
France	−0.1291	100.0	−0.5890	−0.1288	−0.7028	0.0389	0.6973
Great Britain	−0.1693	100.0	−0.7357	−0.1925	−0.8596	−0.0139	0.6475
Ireland	−0.1343	100.0	−0.5882	−0.1307	−0.6997	0.0341	0.6866
Luxembourg	−0.1257	97.8	−0.5853	−0.1354	−0.6902	0.0347	0.7282
Norway	−0.1154	100.0	−0.5138	−0.1139	−0.6139	0.0621	0.7725
Sweden	−0.1280	100.0	−0.6056	−0.1396	−0.7372	0.0372	0.7549
Others	−0.1196	100.0	−0.6559	−0.1677	−0.7904	0.0108	0.7938
Western Europe	−0.1351	99.4	−0.6188	−0.1450	−0.7319	0.0265	0.7077
Holding period [4,7]							
Austria	0.0453	12.5	0.3668	0.0601	0.5335	−0.0060	0.5849
Belgium	0.0618	10.0	0.4615	0.0707	0.6870	0.0058	0.6285
Germany	0.0927	0.0	0.7421	0.1493	1.1730	0.0434	0.7484
France	0.0669	4.1	0.4609	0.0695	0.6662	0.0035	0.6367
Great Britain	0.1137	0.0	0.8698	0.1717	1.3404	0.0682	0.7637
Ireland	0.0959	9.1	0.7404	0.1200	1.1884	0.0465	0.7042
Luxembourg	0.0742	11.0	0.5914	0.1020	0.9477	0.0234	0.6841
Norway	0.0742	6.3	0.6242	0.0936	0.9603	0.0205	0.6955
Sweden	0.0846	0.0	0.6585	0.1090	1.0361	0.0325	0.7178
Others	0.0660	9.1	0.5785	0.1121	0.9293	0.0193	0.6934
Western Europe	0.0776	6.4	0.5990	0.1040	0.9273	0.0245	0.6833

Table 8.14 Correlation coefficients of the performance measures obtained with the DEA-$V_{\beta,DR}$ model and the Sharpe, Treynor, Sortino and alpha indices by holding period

	Sharpe index	Treynor index	Sortino index	Alpha index	DEA-$V_{\beta,DR}$ score
Holding period [0,7]					
Sharpe index	1				
Treynor index	0.980	1			
Sortino index	0.996	0.983	1		
Alpha index	0.948	0.921	0.934	1	
DEA-$V_{\beta,DR}$ score	0.921	0.902	0.924	0.855	1
Holding period [0,3]					
Sharpe index	1				
Treynor index	0.856	1			
Sortino index	0.993	0.848	1		
Alpha index	0.928	0.933	0.926	1	
DEA-$V_{\beta,DR}$ score	0.625	0.481	0.584	0.522	1
Holding period [4,7]					
Sharpe index	1				
Treynor index	0.871	1			
Sortino index	0.977	0.904	1		
Alpha index	0.963	0.898	0.943	1	
DEA-$V_{\beta,DR}$ score	0.877	0.886	0.872	0.892	1

the holding period $[0,7]$) and they are only slightly lower for the correlation between these ratios and the Jensen's alpha index (greater than 0.92 in the holding period $[0,7]$). As for the scores obtained with the DEA-V model, it is interesting to note that the correlations involving the DEA-$V_{\beta,DR}$ score are higher with the Sharpe, Treynor and Sortino ratios (greater than 0.9 in $[0,7]$) and slightly lower with the alpha index (0.855 in $[0,7]$). This may be due to the fact that the DEA scores are expressed as ratios.

Figures 8.11 and 8.12 show in more detail the relationship between the DEA-$V_{\beta,DR}$ score and the value of the Sharpe, Treynor and Sortino indices; more precisely, these figures display the scatter plots of all the mutual funds analyzed with respect to these performance measures for the holding periods $[0,3]$ (Fig. 8.11) and $[4,7]$ (Fig. 8.12). The comparison between the scatter plots in the two holding periods highlights the higher dispersion of the performance results obtained for the mutual funds with the DEA model and a traditional indicator in the holding period of financial crisis.

A similar behavior is observed in Fig. 8.13 for the relationship between the DEA-$V_{\beta,DR}$ score and the value of the Jensen's alpha index in the two holding periods.

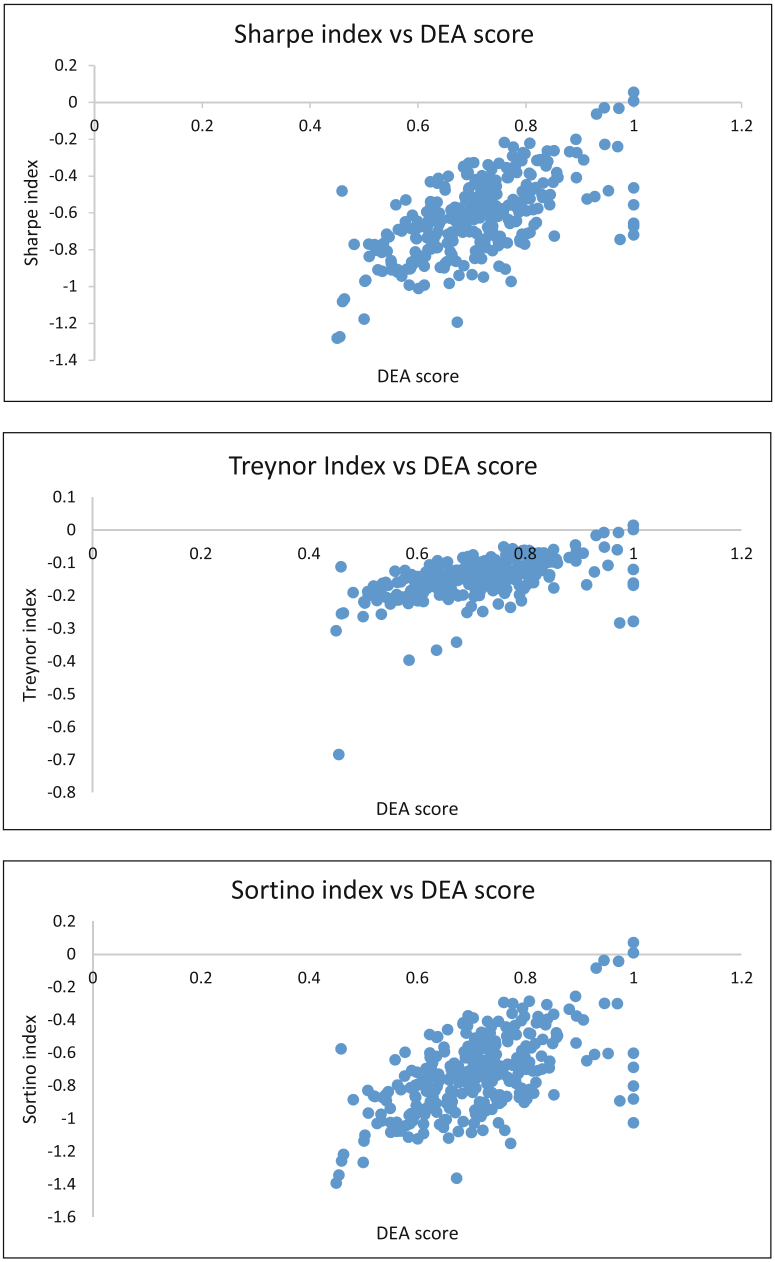

Fig. 8.11 Scatter plots of all mutual funds with respect to the DEA-$V_{\beta,DR}$ score and the Sharpe, Treynor and Sortino indices, respectively, in the holding period $[0, 3]$

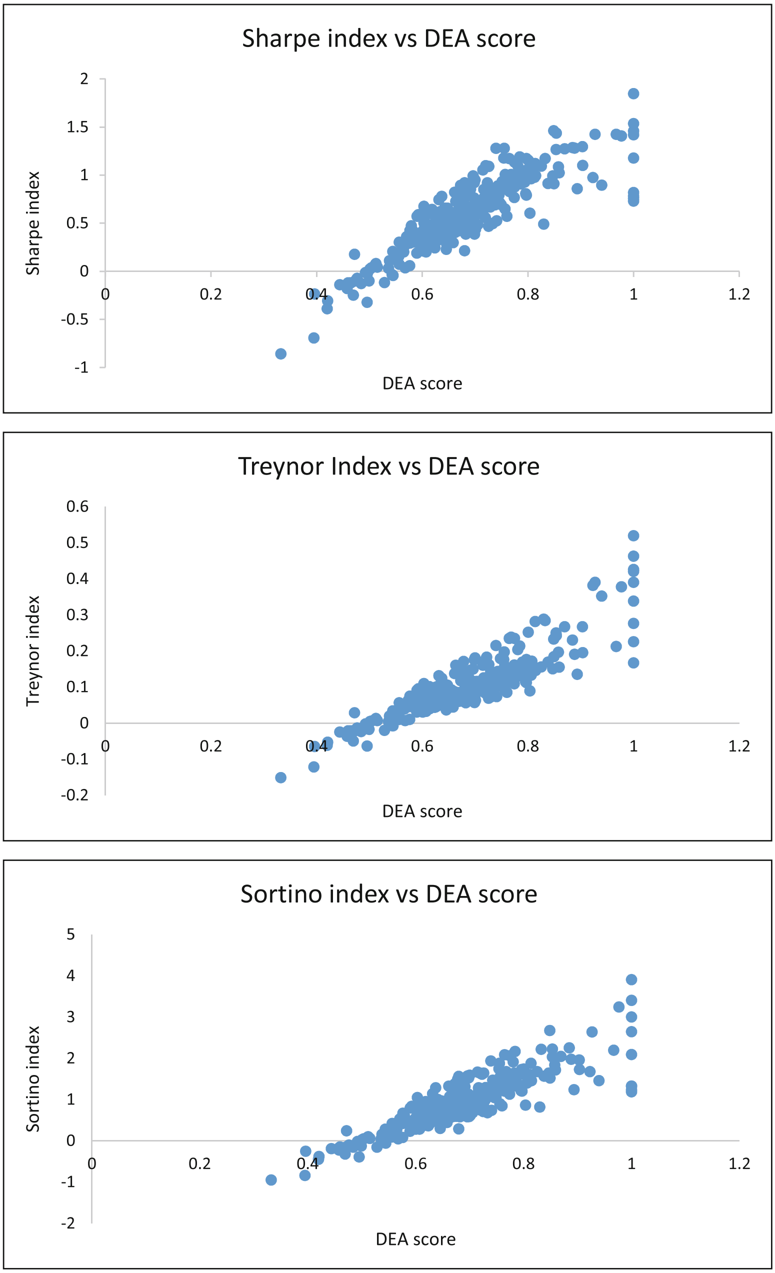

Fig. 8.12 Scatter plots of all mutual funds with respect to the DEA-V$_{\beta,DR}$ score and the Sharpe, Treynor and Sortino indices, respectively, in the holding period [4, 7]

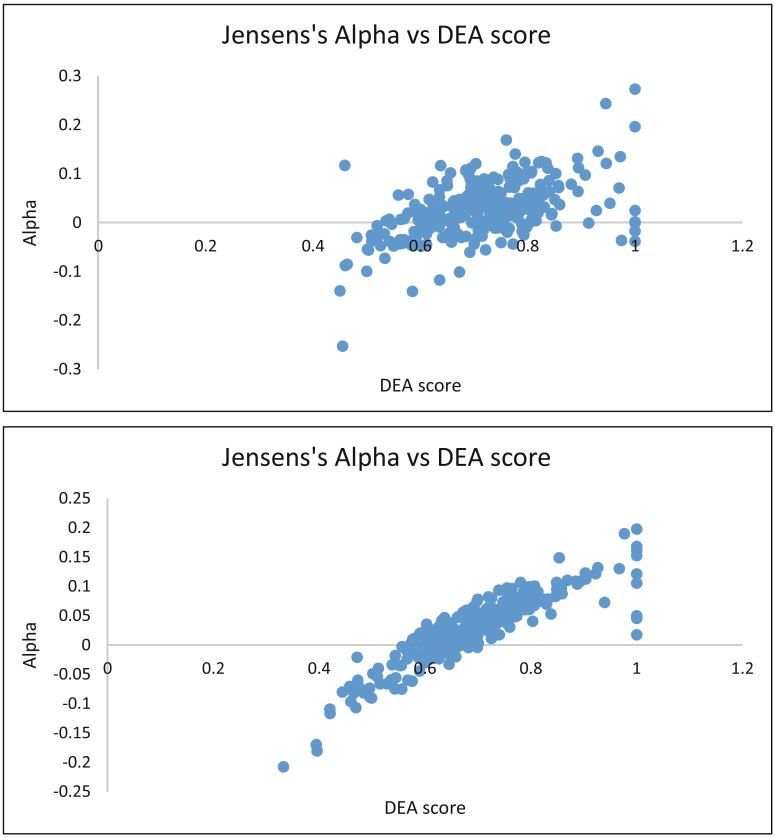

Fig. 8.13 Scatter plots of all mutual funds with respect to the DEA-$V_{\beta,DR}$ score and the Jensen's alpha index in the holding periods $[0, 3]$ and $[4, 7]$, respectively

References

Abdelsalam O, Duygun-Fethi M, Matallín JC, Tortosa-Ausina E (2014a) On the comparative performance of socially responsible and Islamic mutual funds. J Econ Behav Organ 103:S108–S128

Abdelsalam O, Duygun M, Matallín-Sáez JC, Tortosa-Ausina E (2014b) Do ethics imply persistence? The case of Islamic and socially responsible funds. J Bank Financ 40:182–194

Adamauskas S, Krusinskas R (2012) Behavioural finance efficiency under the influence of country's economic cycle. Inzinerine Ekon Eng Econ 23:327–337

Adeli O (2013) Measuring the efficiency of mutual funds. J Oper Res Appl (J Appl Math) 9:27–41

Alexakis P, Tsolas I (2011) Appraisal of mutual equity fund performance using data envelopment analysis. Multinatl Financ J 15:273–296

Ali AI, Seiford LM (1990) Translation invariance in data envelopment analysis. Oper Res Lett 9:403–405

Ali Asghar MJ-K, Afza T, Bodla MA (2013) Efficiency of the mutual funds in Pakistan. Middle-East J Sci Res 18:1055–1064

Ammann M, Moerth P (2008) Performance of funds of hedge funds. J Wealth Manag 2008:46–63

Anderson RI, Brockman CM, Giannikos C, McLeod RW (2004) A non-parametric examination of real estate mutual fund efficiency. Int J Bus Econ 3:225–238

Andreu L, Sarto JL, Vicente L (2014) Efficiency of the strategic style of pension funds: an application of the variants of the slacks-based measure in DEA. J Oper Res Soc 65:1886–1895

Babalos V, Caporale GM, Philippas N (2012) Efficiency evaluation of Greek equity funds. Res Int Bus Financ 26:317–333

Babalos V, Doumpos M, Philippas N, Zopounidis C (2015) Towards a holistic approach for mutual fund performance appraisal. Comput Econ 46:35–53

Baghdadabada MRT, Tanha FH, Halid N (2013) The efficiency evaluation of mutual fund managers based on DARA, CARA, IARA. J Bus Econ Manag 14:677–695

Bahrani R, Khedri N (2013) Evaluation of relative efficiency and performance of companies using data envelopment analysis (DEA) approach. Elixir Financ Manag 56:13299–13304

Banihashemi S, Sanei M (2013) Portfolio optimization and asset allocations using DEA and DEA/AHP. Ind J Sci Res 4:81–86

Banker R, Charnes A, Cooper WW (1984) Some models for estimating technical and scale inefficiencies in data envelopment analysis. Manag Sci 30:1078–1092

Barrientos A, Boussofiane A (2005) How efficient are pension fund managers in Chile? Rev Econ Contemp 9:289–311

Basso A, Funari S (2001) A data envelopment analysis approach to measure the mutual fund performance. Eur J Oper Res 135:477–492

Basso A, Funari S (2003) Measuring the performance of ethical mutual funds: a DEA approach. J Oper Res Soc 54:521–531

Basso A, Funari S (2005a) A generalized performance attribution technique for mutual funds. Cent Eur J Oper Res 13:65–84

Basso A, Funari S (2005b) Performance evaluation of ethical mutual funds in slump periods. Rendiconti per gli Studi Economici Quantitativi 2005:89–105

Basso A, Funari S (2008) DEA models for ethical and non ethical mutual funds. Math Methods Econ Financ 2:21–40

Basso A, Funari S (2014a) Constant and variable returns to scale DEA models for socially responsible investment funds. Eur J Oper Res 235:775–783

Basso A, Funari S (2014b) DEA models with a constant input for SRI mutual funds with an application to European and Swedish funds. Int Trans Oper Res 21:979–1000

Basso A, Funari S (2014c) The role of fund size and returns to scale in the performance of mutual funds. In: Perna C, Sibillo M (eds) Mathematical and statistical methods for actuarial sciences and finance. Springer, Cham, pp 21–25

Basso A, Funari S (2014d) Socially responsible mutual funds: an efficiency comparison among the European countries. In: Mathematical and statistical methods for actuarial sciences and finance. Springer, Cham, pp 69–79

Batra D, Batra G (2012) A DEA comparison of systematic and lump sum investment in mutual funds. Int J Comput Bus Res 3:1–10

Branda M (2011) Third-degree stochastic dominance and DEA efficiency-relations and numerical comparison. In: Dlouhý M, Skočdopolová V (eds) Proceedings of the 29th international conference on mathematical methods in economics. University of Economics in Prague, Jánská Dolina, pp 64–69

Branda M (2013a) Diversification-consistent data envelopment analysis with general deviation measure. Eur J Oper Res 226:626–635

Branda M (2013b) Diversification-consistent DEA-risk tests – solution techniques and an empirical comparison. In: Vojáčková H (ed) Proceedings of the 31th international conference on mathematical methods in economics. College of Polytechnics Jihlava, Jihlava, pp 77–82

Branda M (2013c) Reformulations of input-output oriented DEA tests with diversification. Oper Res Lett 41:516–520

Branda M, Kopa M (2012a) DEA-risk efficiency and stochastic dominance efficiency of stock indices. Czech J Econ Financ 62:106–124

Branda M, Kopa M (2012b) From stochastic dominance to DEA-risk models: portfolio efficiency analysis. In: Sakalauskas L, Tomasgard A, Wallace SW (eds) Proceedings of the international workshop on stochastic programming for implementation and advanced applications. Vilnius Gediminas Technical University, Vilnius, pp 13–18

Branda M, Kopa M (2014) On relations between DEA-risk models and stochastic dominance efficiency tests. Cent Eur J Oper Res 22:13–35

Brandouy O, Kerstens K, Van de Woestyne I (2015) Frontier-based vs.traditional mutual fund ratings: a first backtesting analysis. Eur J Oper Res 242:332–342

Briec W, Kerstens K, Lesourd JB (2004) Single-period Markowitz portfolio selection, performance gauging, and duality: a variation on the Luenberger shortage function. J Optim Theory Appl 120:1–27

Briec W, Kerstens K, Jokung O (2007) Mean-variance-skewness portfolio performance gauging: a general shortage function and dual approach. Manag Sci 53:135–149

Chang JF (2010) Applying DEA investment portfolio efficiency index and GA to the establishment of the fund of funds in Taiwan. Int J Organ Innov 2:225–249

Chang K-P (2004) Evaluating mutual fund performance: an application of minimum convex input requirement set approach. Comput Oper Res 31:929–940

Chen H-H (2008) Stock selection using data envelopment analysis. Ind Manag Data Syst 108:1255–1268

Chen Z, Lin R (2006) Mutual fund performance evaluation using data envelopment analysis with new risk measures. OR Spectr 28:375–398

Chen Y-C, Chiu Y-H, Li M-C (2011) Mutual fund performance evaluation - application of system BCC model. S Afr J Econ 79:1–16

Cheng G, Zervopoulos P, Qian Z (2013) A variant of radial measure capable of dealing with negative inputs and outputs in data envelopment analysis. Eur J Oper Res 225:100–105

Choi YK, Murthi BPS (2001) Relative performance evaluation of mutual funds: a non-parametric approach. J Bus Financ Account 28:853–876

Chu J, Chen F, Leung P (2010) ETF performance measurement-data envelopment analysis. In: International conference on service systems and service management (ICSSSM), 28–30 June 2010, pp 1–6

Cooper WW, Seiford LM, Zhu J (eds) Handbook on data envelopment analysis, 2nd edn. Springer, New York

Daraio C, Simar L (2006) A robust nonparametric approach to evaluate and explain the performance of mutual funds. Eur J Oper Res 175:516–542

Darling G, Mukherjee K, Wilkens K (2004) CTA performance evaluation with data envelopment analysis. In: Gregoriou GN, Karavas VN, Lhabitant F-S, Rouah F (eds) Commodity trading advisors: risk, performance analysis, and selection. Wiley, Hoboken, pp 79–104

de Guzman MPR, Cabanda EC (2005) Performance appraisal of Philippine mutual funds using DEA approach. Labuan Bull Int Bus Financ 3:65–77

Devaney M, Weber WL (2005) Efficiency, scale economies, and the risk/return performance of real estate investment trusts. J Real Estate Financ Econ 31:301–317

Dia M (2009) A portfolio selection methodology based on data envelopment analysis. Inf Syst Oper Res 47:71–79

Ding H, Zhou Z, Xiao H, Ma C, Liu W (2014) Performance evaluation of portfolios with margin requirements. Math Probl Eng 2014:1–8

Diz F, Gregoriou GN, Rouah F, Satchell SE (2004) Simple and cross-efficiency of CTAs using data envelopment analysis. In: Gregoriou GN, Karavas VN, Lhabitant F-S, Rouah F (eds) Commodity trading advisors: risk, performance analysis, and selection. Wiley, Hoboken, pp 129–147

Edirisinghe NCP, Zhang X (2007) Generalized DEA model of fundamental analysis and its application to portfolio optimization. J Bank Financ 31:3311–3335

Edirisinghe NCP, Zhang X (2010) Input/output selection in DEA under expert information, with application to financial markets. Eur J Oper Res 207:1669–1678

Einolf KW (2007) Building a socially responsible equity portfolio using data envelopment analysis. Philosophica 80:71–103

Eling M (2006) Performance measurement of hedge funds using data envelopment analysis. Financ Mark Portfolio Manag 20:442–471

Emrouznejad A, Anouze AL, Thanassoulis E (2010) A semi-oriented radial measure for measuring the efficiency of decision making units with negative data, using DEA. Eur J Oper Res 200:297–304

Galagedera DUA (2013) A new perspective of equity market performance. J Int Financ Mark Inst Money 26:333–357

Galagadera DUA, Silvapulle P (2002) Australian mutual fund performance appraisal using data envelopment analysis. Manag Financ 28:60–73

Glawischnig M, Sommersguter-Reichmann M (2010) Assessing the performance of alternative investments using non-parametric efficiency measurement approaches: is it convincing? J Bank Financ 34:295–303

Gnanasekar IF, Arul R (2013) Financial risk tolerance using data envelopment analysis-a case study. ZENITH Int J Bus Econ Manag Res 3:251–262

Gnanasekar IF, Arul R (2014) Data envelopment analysis: Jeopardy can never be defeat without taking financial risk tolerance. Indian J Appl Res 4:121–124

Gökgöz F (2010) Measuring the financial efficiencies and performances of Turkish funds. Acta Oecon 60:295–320

Gökgöz F, Çandarli D (2011) Data envelopment analysis: a comparative efficiency measurement for Turkish pension and mutual funds. Int J Econ Perspect 5:261–281

Gregoriou GN (2003) Performance appraisal of funds of hedge funds using data envelopment analysis. J Wealth Manag 5:88–95

Gregoriou GN (2006a) Optimisation of the largest US mutual funds using data envelopment analysis. J Asset Manag 6:445–455

Gregoriou GN (2006b) Trading efficiency of commodity trading advisors using data envelopment analysis. Derivatives Use Trading Regul 12:102–114

Gregoriou GN (2007) Efficiency of US mutual funds using data envelopment analysis. In: Gregoriou GN (ed) Performance of mutual funds: an international perspective. Palgrave Macmillan, Basingstoke, pp 152–167

Gregoriou GN, Chen Y (2006) Evaluation of commodity trading advisors using fixed and variable and benchmark models. Ann Oper Res 145:183–200

Gregoriou GN, Henry SC (2015) Undesirable outputs in commodities trading advisers: a data envelopment analysis approach. J Wealth Manag 17:85–92

Gregoriou GN, Zhu J (2005) Evaluating hedge fund and CTA performance: data envelopment analysis approach. Wiley, New York

Gregoriou GN, Zhu J (2007) Data envelopment analysis: a way to assess the efficiency of funds of hedge funds. J Portfolio Manag 33:120–132

Gregoriou GN, Rouah F, Sedzro K (eds) (2003) Performance evaluation of hedge funds. Beard Books, Washington

Gregoriou GN, Sedzro K, Zhu J (2005) Hedge fund performance appraisal using data envelopment analysis. Eur J Oper Res 164:555–571

Guillén GB (2008) Is efficiency the equivalent to a high rate of return for private pension funds? Evidence from Latin American countries. J CENTRUM Cathedra 1:109–117

Guo J, Ma C, Zhou Z (2012) Performance evaluation of investment funds with DEA and higher moments characteristics: financial engineering perspective. Syst Eng Proc 3:209–216

Hanafizadeh P, Khedmatgozar HR, Emrouznejad A, Derakhshan M (2014) Neural network DEA for measuring the efficiency of mutual funds. Int J Appl Decis Sci 7:255–269

Haslem JA, Scheraga CA (2003) Data envelopment analysis of Morningstar's large-cap mutual funds. J Invest 12:41–48

Haslem JA, Scheraga CA (2006) Data envelopment analysis of Morningstar's small-cap mutual funds. J Invest 15:87–92
Hsu C-M (2014) An integrated procedure for resolving portfolio optimization problems using data envelopment analysis, ant colony optimization and gene expression programming. Int J Comput Sci Bus Inform 9:45–65
Hsu C-S, Lin J-R (2007) Mutual fund performance and persistence in Taiwan: a non-parametric approach. Serv Ind J 27:509–523
Hu J-L, Chang T-P (2008) Decomposition of mutual fund underperformance. Appl Financ Econ Lett 4:363–367
Hu J-L, Yu H-E, Wang Y-T (2012) Manager attributes and fund performance: evidence from Taiwan. J Appl Financ Bank 2:85–101
Huang TH, Leu YH (2014) A mutual fund investment method using fruit fly optimization algorithm and neural network. Appl Mech Mater 571–572:318–325
Huang C-Y, Chiou C-C, Wu T-H, Yang S-C (2015) An integrated DEA-MODM methodology for portfolio optimization. Oper Res 15:115–136
Ismail MKA, Salamudin N, Rahman NMNA, Kamaruddin BH (2012) DEA portfolio selection in Malaysian stock market. In: 2012 international conference on innovation management and technology research (ICIMTR), 21–22 May 2012, pp 739–743
Jensen MC (1968) The performance of mutual funds in the period 1945–1964. J Financ 23:389–416
Jeong SO, Hoss A, Park C, Kang K-H, Ryu Y (2013) Portfolio selection for socially responsible investment via nonparametric frontier models. Commun Stat Appl Methods 20:115–127
Joro T, Na P (2006) Portfolio performance evaluation in a mean-variance-skewness framework. Eur J Oper Res 175:446–461
Kadoya S, Kuroko T, Namatame T (2008) Contrarian investment strategy with data envelopment analysis concept. Eur J Oper Res 189:120–131
Kerstens K, Van de Woestyne I (2011) Negative data in DEA: a simple proportional distance function approach. J Oper Res Soc 62:1413–1419
Kerstens K, Mounir A, Van de Woestyne I (2011) Non-parametric frontier estimates of mutual fund performance using C- and L-moments: some specification tests. J Bank Financ 35:1190–1201
Kerstens K, Mounir A, Van de Woestyne I (2012) Benchmarking mean-variance portfolios using a shortage function: the choice od direction vector affects rankings! J Oper Res Soc 63:1199–1212
Khedmatgozar HR, Kazemi A, Hanafizadeh P (2013) Mutual fund performance evaluation: a value efficiency analysis approach. Int J Electron Financ 7:263–280
Kumar UD, Roy A, Saranga H, Singal K (2010) Analysis of hedge fund strategies using slack-based DEA models. J Oper Res Soc 61:1746–1760
Kumar Singh A, Sahu R, Bharadwaj S (2010) Portfolio evaluation using OWA-heuristic algorithm and data envelopment analysis. J Risk Financ 11:75–88
Lamb JD, Tee K-H (2012a) Data envelopment analysis models of mutual funds. Eur J Oper Res 216:687–696
Lamb JD, Tee K-H (2012b) Resampling DEA estimates of investment fund performance. Eur J Oper Res 223:834–841
Lamb JD, Tee K-H (2012c) Data envelopment analysis (DEA) integrated risk assessment technique on hedge funds investment: theory and practical application. In: Risk assessment and management. Academy Publish, Wyoming, pp 73–84
Lim S, Oh KW, Zhu J (2014) Use of DEA cross-efficiency evaluation in portfolio selection: an application to Korean stock market. Eur J Oper Res 236:361–368
Lin R (2009) Mutual fund performance dynamic evaluation using data envelopment windows analysis. In: International conference on management and service science, pp 1–4
Lin R, Chen Z (2008) New DEA performance evaluation indices and their applications in the American fund market. Asia Pac J Oper Res 25:421

Lopes A, Lanzer E, Lima M, da Costa N Jr (2008) DEA investment strategy in the Brazilian stock market. Econ Bull 13:1–10

Lovell CAK, Pastor JT (1995) Units invariant and translation invariant DEA models. Oper Res Lett 18:147–151

Lozano S, Gutiérrez E (2008a) Data envelopment analysis of mutual funds based on second-order stochastic dominance. Eur J Oper Res 189:230–244

Lozano S, Gutiérrez E (2008b) TSD-consistent performance assessment of mutual funds. J Oper Res Soc 59:1352–1362

Majid MSA, Maulana H (2010) Assessing performance of mutual funds in Indonesia. J Econ Coop Dev 31:49–76

Majid MSA, Maulana H (2012) A comparative analysis of the productivity of Islamic and conventional mutual funds in Indonesia: data envelopment analysis (DEA) and general least square (GLS) approaches. Gadjah Mada Int J Bus 14:183

Malhotra DK, Malhotra R (2012) Using data envelopment analysis framework to evaluate mutual fund efficiency. In: 2012 Northeast decision sciences institute conference proceedings, pp 137–143

Margaritis DM, Otten R, Tourani-Rad A (2007) New Zealand equity fund performance appraisal: a non-parametric approach. In: Gregoriou G (ed) Performance of mutual funds: an international perspective. Palgrave-McMillan, New York, pp 17–30

Matallín-Sáez JC, Soler-Domínguez A, Tortosa-Ausina E (2014) On the informativeness of persistence for evaluating mutual fund performance using partial frontiers. Omega Int J Manag Sci 42:47–64

McMullen PR, Strong RA (1998) Selection of mutual funds using data envelopment analysis. J Bus Econ Stud 4:1–12

Medeiros Garcia MT (2010) Efficiency evaluation of the Portuguese pension funds management companies. J Int Financ Mark Inst Money 20:259–266

Morey MR, Morey RC (1999) Mutual fund performance appraisals: a multi-horizon perspective with endogenous benchmarking. Omega Int J Manag Sci 27:241–258

Murthi BPS, Choi YK, Desai P (1997) Efficiency of mutual funds and portfolio performance measurement: a non-parametric approach. Eur J Oper Res 98:408–418

Pai V (2012) Risk budgeted portfolio optimization using an extended ant colony optimization metaheuristic. Int J Appl Metaheuristic Comput 3:25–42

Pastor JT (1996) Translation invariance in data envelopment analysis: a generalization. Ann Oper Res 66:93–102

Pastor JT, Ruiz JL (2007) Variables with negative values in DEA. In: Zhu J, Cook WD (eds) Modeling data irregularities and structural complexities in data envelopment analysis. Springer, New York, pp 63–84

Pätäri E, Leivo T, Honkapuro S (2010) Enhancement of value portfolio performance using data envelopment analysis. Stud Econ Financ 27:223–246

Pätäri E, Leivo T, Honkapuro S (2012) Enhancement of equity portfolio performance using data envelopment analysis. Eur J Oper Res 220:786–797

Pendaraki K (2012) Mutual fund performance evaluation using data envelopment analysis with higher moments. J Appl Financ Bank 2:97–112

Pérez-Gladish B, Rodríguez MP, M'zali B, Lang P (2013) Mutual funds efficiency measurement under financial and social responsibility criteria. J Multi-Criteria Decis Anal 20:109–125

Pestana Barros C, Medeiros Garcia MT (2006) Performance evaluation of pension funds management companies with data envelopment analysis. Risk Manag Insur Rev 9:165–188

Powers J, McMullen PR (2000) Using data envelopment analysis to select efficient large market cap securities. J Bus Manag 7:31–42

Prasanna PK (2012) Performance of exchange-traded funds in India. Int J Bus Manag 7:122–143

Premachandra I, Powell JG, Shi J (1998) Measuring the relative efficiency of fund management strategies in New Zealand using a spreadsheet-based stochastic data envelopment analysis model. Omega Int J Manag Sci 26:319–331

Premachandra I, Zhu J, Watson J, Galagadera DUA (2012) Best-performing US mutual fund family from 1993 to 2008: evidence from a novel two-stage DEA model for efficiency decompostition. J Bank Financ 36:3302–3317

Rubio JF, Hassan MK, Merdad HJ (2012) Nonparametric performance measurement of international and Islamic mutual funds. Account Res J 25:208–226
Saad NM, Majid MSA, Kassim S, Hamid Z, Yusof RM (2010) A comparative analysis of the performance of conventional and Islamic unit trust companies in Malaysia. Int J Manag Financ 6:24–47
Sahoo BK, Meera E (2008) A comparative application of alternative DEA models in selecting efficient large cap market securities in India. Int J Manag Perspect 2:62–74
Sedzro K (2009) New evidence on hedge fund performance measures. Int Bus Econ Res J 8:95–105
Sengupta JK (2003) Efficiency tests for mutual fund portfolios. Appl Financ Econ 13:869–876
Sharpe WF (1994) The sharpe ratio. J Portfolio Manag 21:49–58
Silva Portela MCA, Thanassoulis E, Simpson G (2004) Negative data in DEA: a directional distance approach applied to bank branches. J Oper Res Soc 55:1111–1121
Simar L, Vanhems A (2012) Probabilistic characterization of directional distances and their robust versions. J Econom 166:342–354
Simar L, Vanhems A, Wilson PW (2012) Statistical inference for DEA estimators of directional distances. Eur J Oper Res 220:853–864
Soongswang A, Sanohdontree Y (2011a) Equity mutual fund: performances, persistence and fund rankings. J Knowl Manag Econ Inf Technol 6:1–27
Soongswang A, Sanohdontree Y (2011b) Open-ended equity mutual funds. Int J Bus Soc Sci 2:127–136
Sortino FA, van der Meer R (1991) Downside risk. J Portfolio Manag 17:27–31
Tarim SA, Karan MB (2001) Investment fund performance measurement using weight restricted data envelopment analysis: an application to the Turkish capital market. Russ East Eur Financ Trade 37:64–84
Tavakoli Baghdadabad MR, Noori Houshyar A (2014) Productivity and efficiency evaluation of US mutual funds. Czech J Econ Financ (Finance a úvěr) 64:120–143
Tokic D (2012) Managed futures for long-term investors: a DEA ranking analysis. Aust Econ Rev 45:422–440
Treynor JL (1965) How to rate management of investment funds. Harv Bus Rev 43:63–75
Tsolas IE (2011) Natural resources exchange traded funds: performance appraisal using DEA modeling. J CENTRUM Cathedra 4:250–259
Tsolas IE (2014) Precious metal mutual fund performance appraisal using DEA modeling. Resour Policy 39:54–60
Tsolas IE, Charles V (2015) Green exchange-traded fund performance appraisal using slacks-based DEA models. Oper Res 15:51–77
Wilkens K, Zhu J (2001) Portfolio evaluation and benchmark selection: a mathematical programming approach. J Altern Invest 4:9–19
Zamani L, Beegam R, Borzoian S (2014) Portfolio selection using data envelopment analysis (DEA): a case of select Indian investment companies. Int J Curr Res Acad Rev 2:50–55
Zhao X, Shi J (2010) Evaluation of mutual funds using multi-dimensional information. Front Comput Sci China 4:237–253
Zhao X, Wang S-Y (2007) Empirical study on Chinese mutual funds' performance. Syst Eng Theory Pract 27:1–11
Zhao X, Yue W (2008) Coned context DEA model with application to mutual funds evaluation. In: IEEE international conference on industrial engineering and engineering management, pp 805–809
Zhao X, Yue W (2012) A multi-subsystem fuzzy DEA model with its application in mutual funds management companies' competence evaluation. Proc Comput Sci 1:2469–2478
Zhao X, Wang S, Lai KK (2011) Mutual funds performance evaluation based on endogenous benchmarks. Expert Syst Appl 38:3663–3670