Chapter 6
Crew Scheduling Problem

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6.1 Introduction

The crew scheduling problem (CSP) involves assigning crew to trains, while satisfying a variety of Federal Railway Administration (FRA) regulations and trade-union work rules. Train crew work together to move a train from its origin to its destination. As the train travels over its route, it goes through numerous crew districts. In each crew district, the train is manned by an engineer and a conductor who are qualified to operate the train within that district. The objectives of crew scheduling are therefore to assign crew to the trains, while minimizing the cost of operating trains, improving crew quality of life, and satisfying all FRA regulations and work rules.

The crew scheduling problem is a difficult problem to solve because the deployment of crew on trains is governed by many regulations. Crews cannot be assigned outside their crew districts and they need to have minimum rest between assignments. Each crew has a home location and an away location, and there are rules that govern how often a crew must return to its home location. If a crew is detained at an away location for more than certain duration, the railroad needs to pay detention costs. Further, crew need to be assigned to trains in a First-In-First-Out (FIFO) manner. Also, the number of incoming trains and outgoing trains may be imbalanced, which may necessitate crew deadheading on trains or repositioning via taxi so that they may be available to work at a different location. All these constraints and decisions make the problem hard to solve.

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B.W. Patty (ed.), *Handbook of Operations Research Applications at Railroads*,
International Series in Operations Research & Management Science 222,
DOI 10.1007/978-1-4899-7571-3_6

Several researchers have worked on airline and passenger rail crew scheduling (for example, Barnhart et al. 1994, 2003; Caprara et al. 1997; Chu and Chan 1998; Freling et al. 2004). Most of the railroad crew scheduling literature is related to European and Asian railroads; these settings do not have the FIFO requirements and are therefore very different from that in North America. The two articles that have been written specific to the North American railroad crew scheduling are due to Gorman and Sarrafzadeh (2000) and Vaidyanathan et al. (2007). Gorman and Sarrafzadeh (2000) used dynamic programming to solve CSPs where the districts are single-ended (all crew have the same home location); single-ended districts are the simplest crew district configuration. Vaidyanathan et al. (2007) developed a crew scheduling model that works for double-ended and other complicated crew district configurations; their work reports the most comprehensive crew scheduling model to date. Hence, the mathematical model and the solution approach described in this chapter are based on Vaidyanathan et al. (2007), though the rest of the paper deals with crew scheduling in general.

6.2 Background on Crew Scheduling

This section gives an overview of the CSP and defines some of the terminology needed to understand the problem. It also gives an overview of some of the typical regulations which govern crew management.

6.2.1 Terminology

Crew District: The railroad's network is divided into numerous *crew districts*; a crew district constitutes a subset of terminals. Each crew district is a geographic corridor over which trains can travel with one crew. A typical network for a major railroad in the U.S. is divided into as many as 200–300 crew districts. As a train follows its route, it goes from one crew district to another, picking up and dropping off crew at *crew change terminals*.

Crew Pools: Within a crew district, there are several types of crew called *crew pools* or *crew types*, which may be governed by different trade-union rules and regulations. For example, a crew pool may have preference over the trains operated in a pre-specified time window. In some cases, a crew pool consisting of senior crew personnel is assigned only to pre-designated trains so that crews in that pool know their working hours ahead of time.

Home and Away Terminals: The terminals where crews from a crew pool change trains are designated as either *home terminals* or *away terminals*. The railroad does not incur any lodging cost when a crew is at its home terminal. However, the railroad has to make arrangements for crew accommodation at their away terminals. A crew district with one home terminal and one away terminal is called a *single-ended crew district*. The other type of crew district is a *double-ended crew district*, in which more than one terminal is a home terminal for different crew pools. Some of the other crew district configurations are crew districts with one home terminal and several away terminals, and crew districts with several home terminals and corresponding sets of away terminals.

Crew Detention: Once a crew reaches its away terminal and rests for the prescribed hours, the crew is ready to head back to its home terminal. However, if there is no train, then the crew may have to wait in a hotel. According to the trade-union rules, once a crew is at the away terminal for more than a pre-specified number of hours (generally 16 h), the crew earns wages (called *detention costs*) without being on duty.

Crew Deadheading: This refers to the repositioning of crew between terminals. A crew normally operates a train from its home terminal to an away terminal, rests for a designated time, and then operates another train back to its home terminal. Sometimes, at the away terminal, there is no return train projected for some time, or there is a shortage of crews at another terminal. Thus, instead of waiting for train assignment at its current terminal, the crew can take a taxicab or a train (as a passenger) and deadhead to the home terminal. Similarly, the crew may also deadhead from a home terminal to an away terminal in order to rebalance and better match the train demand patterns and avoid train delays.

On-duty and Tie-up Time: When a crew is assigned to a train, it performs some tasks to prepare the train for departure, and hence crews are called on-duty before train departure time. The time at which the crew has to report for duty is called the *on-duty time.* Similarly, a crew performs some tasks after the arrival of the train at its destination, and hence crews are released from duty after the train arrival. The time at which the crew is released from duty is called *tie-up time*. The duty duration before train departure is referred to as *duty-before-departure* and the duty duration after train arrival as *duty-after-arrival.* Hence, the total duty time (or *duty period*) of a crew assigned to a train is the sum of the *duty-before-departure*, the *duty-after-arrival*, and the travel time of the train.

Duty Period: In most cases, duty period of a crew assigned to a train is the total duration between the *on-duty time* and the *tie-up time*. In some cases when a crew rests for a very short time at an away location before getting assigned to a train, the rest time and the duration of the second train may also be included in the duty period of the crew.

Dead Crews: By federal law, a train crew can only be on duty for a maximum of 12 consecutive hours, at which time the crew must cease all work and it becomes *dead or dog-lawed.*

Train Delays: When a train reaches a crew change location and there is no available crew qualified to operate this train, the train must be delayed. Train delays due to crew unavailability are quite common among railroads. These delays are very expensive and can be reduced significantly through better crew and train scheduling.

6.2.2 Regulatory and Contractual Requirements

Assignment of crews to trains is governed by a variety of Federal Railway Administration (FRA) regulations and trade-union rules. The regulations vary from district to district and from crew pool to crew pool. Some examples are listed below:

- Duty period of a crew cannot exceed 12 h.
- When a crew is released from duty at the home terminal or has been deadheaded to the home terminal, they can resume duty only after 12 h of rest (10 h rest followed by 2 h call period) if duty period is greater than 10 h, and after 10 h of rest (8 h rest followed by 2 h call period) if duty period is less than or equal to 10 h.
- When a crew is released from duty at the away terminal, they can typically resume duty only after 8 h rest.
- Crews belonging to certain pools must be assigned to trains in a FIFO order.
- A train can only be operated by crews belonging to pre-specified pools.
- Every train must be operated by a single crew.
- Crews are guaranteed a certain minimum pay per month regardless of how much they work.

Figure 6.1 gives an example of the decision process that needs to be followed by railroad crew planners.

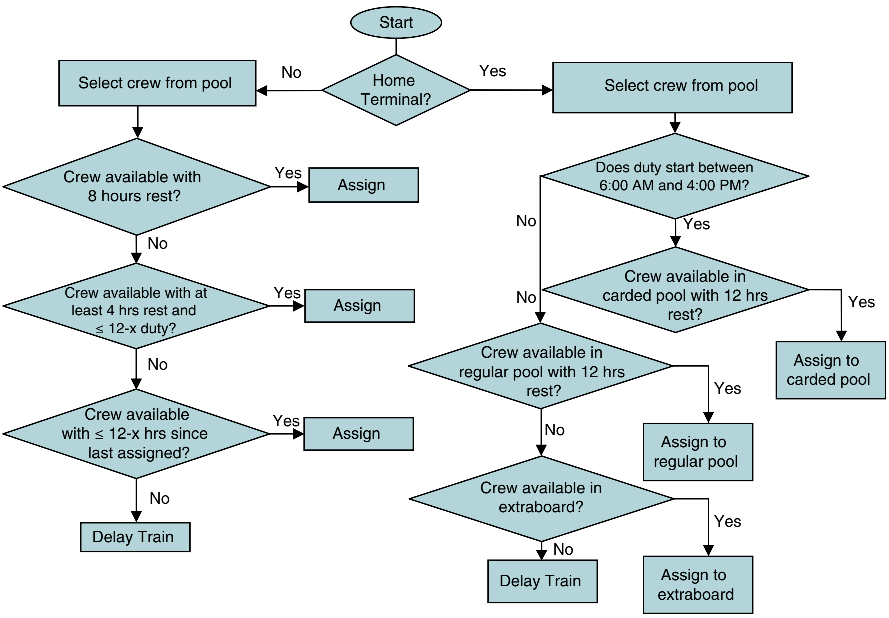

Fig. 6.1 An example of crew scheduling decision tree

6.3 Mathematical Models for Crew Scheduling

We now describe the mathematical formulation of the crew scheduling problem. Since crews do not work outside their crew districts, this means that the problem can be solved as an independent problem for each crew district. We first describe the inputs that are required to define the problem. Then, we describe the network that is used to model the problem. Finally, we describe the mathematical formulation and solution approaches.

6.3.1 Model Inputs

The inputs that go into the mathematical formulation of the crew scheduling problem are:

- Train Schedule: The train schedule provides information about the departure time, arrival time, on-duty time, tie-up time, departure location, and arrival location for every train in each crew district it passes through.
- Crew Pool Attributes: This includes the home location, the away locations, minimum rest time, and train preferences for each crew pool.
- Crew Initial Position: This provides the position of each crew at the beginning of the planning horizon, and includes the terminal at which a crew is released from duty, the time of release, the number of hours of duty done in the previous assignment, and the crew pool of the crew.
- Train-Pool Preferences: The train-pool preferences specify the set of trains that can be operated by a crew pool.
- Away Terminal Attributes: This includes the rest rules and detention rules for each crew pool at each away terminal.
- Deadhead Attributes: This specifies the travel time by taxi between two terminals in a crew district.
- Cost parameters: Cost parameters are used to set up the objective function. They consist of crew wage per hour, deadhead cost per hour, detention cost per hour, and train delay cost per hour.

6.3.2 Space–Time Network Construction

The CSP is solved as a separate problem for each crew district. The schedule of crew is modeled as the flow of commodities on a space–time network (refer to Ahuja et al. (1993) for more about networks). Each node in the network corresponds to a crew event and has two defining attributes: location and time. The events that are modeled while constructing the network are departure of trains, arrival of trains, departure of deadheads, arrival of deadheads, initial positions and availability of

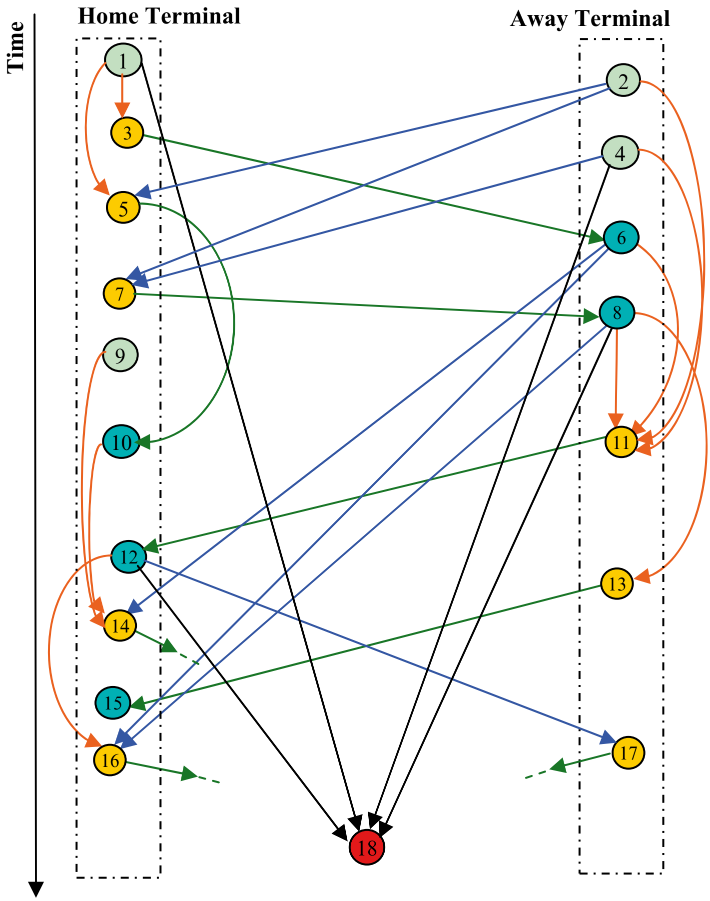

Fig. 6.2 Space–time network for a single-ended district with a single crew type. *Node legend*: *green* (supply), *blue* (arrival), *yellow* (departure), *red* (demand). *Arc legend*: *green* (train), *orange* (rest), *blue* (deadhead), *black* (demand)

crew, and end of the planning horizon. Figure 6.2 presents an example of the space–time network in a crew district (for the sake of clarity, this network only represents a subset of all the arcs).

For each crew, a supply node whose time corresponds to the time at which this crew is available for assignment, and whose location corresponds to the terminal from which the crew is released for duty is created. Each supply node is assigned a supply of one unit and corresponds to a crew. The network also has a common sink node for all crews at the end of the planning horizon. This sink has no location attribute and has the time attribute equal to the end of the planning horizon. The sink node has a demand equal to the total number of crew in the district.

For each train l passing through a crew district, a *departure node*, l', is created at the first crew change terminal and an arrival node, l'', is created at the last crew change terminal in the crew district. Each arrival or departure node has two attributes: place and time. For example, *place* (l') = *departure-station* (l) and *time* (l') = *on-duty-time* (l); and similarly, *place* (l'') = *arrival-station* (l) and *time* (l'') = *tie-up-time* (l).

Train arc (l', l'') is created for each train l connecting the departure node and arrival node of train l. *Deadhead arcs* are constructed to model the travel of crew by taxi. A deadhead arc is constructed between a train arrival or crew supply node at a location and a train departure node at another location. All the deadhead arcs which satisfy the contractual rules and regulations are created. *Rest arcs* are constructed to model resting of a crew at a location. A rest arc is constructed between a train arrival node or a crew supply node at a location and a train departure node at the same location. Rest arcs are created in conformance to the contractual rules and regulations. All rest arcs which satisfy the contractual rules and regulations are constructed. Since the contractual regulations are often crew pool specific, deadhead arcs and rest arcs are created specific to a crew pool. Finally, *demand arcs* are created from all train arrival nodes and crew supply nodes to the sink node. Each arc in the network has an associated cost equivalent to the crew wages, deadhead costs, or detention costs, as the case might be. All contractual requirements other than the FIFO constraint are easily handled in the network construction.

So far, the network does not model the scenario when qualified crews are not available for assignment to a train, which causes train delays. Train delays are modeled by the construction of additional arcs. To do this rest arcs and deadhead arcs which do not honor the rest regulations are also constructed and flows on these arcs are penalized to ensure that flows on these arcs occur only when qualified crews are not available for assignment. If the solution contains nonzero flows on these arcs, it implies that the associated train will be delayed until crew becomes qualified for train operation. Since the delay of a train could have propagating effect in the availability of crews in subsequent assignments, it is assumed that the crew assigned to a delayed train has sufficient slack in the rest time at the train arrival node to make it qualified for subsequent assignments.

6.3.3 Mathematical Formulation

The CSP is formulated as an integer multi-commodity flow problem on the space–time network described in the previous section. Each crew pool represents a commodity. Crews enter the system at crew supply nodes, travels on a sequence of connected train, rest, and deadhead arcs before finally reaching the sink node (Table 6.1).

Decision Variables

x_c^l: Flow of crew pool $c \in C$: On each train arc $l \in L$.
x_d: Flow on deadhead arc $d \in D$.
x_r: Flow on rest arc $r \in R$.

Table 6.1 Notation

N	Set of nodes in the space–time network	i_c^+	Set of outgoing arcs specific to crew pool c at node i
L	Set of train arcs in the network, indexed by l	i_c^-	Set of incoming arcs specific to crew pool c at node i
D	Set of deadhead arcs in the network, indexed by d	Ar	Set of arcs on which flow will violate FIFO constraint if there is flow on rest arc r
R	Set of rest arcs in the network, indexed by r	f	Total number of available crew
A	Set of arcs in the space–time network, indexed by a	M	A very large number
G (N, A)	Space–time network	c_c^l	Cost of crew wages for crew pool $c \in C$ on train arc $l \in L$
N_s	Set of crew supply nodes	c_d	Cost of deadhead arc $d \in D$
N_d	Sink node	c_r	Cost of rest arc $r \in R$
C	Set of crew pools in the system, indexed by c	tail(l)	The node from which arc l originates
i^+	Set of outgoing arcs at node i	head(l)	The node at which arc l terminates
i^-	Set of incoming arcs at node i		

Objective Function

$$\text{Min} \sum_{l \in L} \sum_{c \in C} c_l^c x_l^c + \sum_{d \in D} c_d x_d + \sum_{r \in R} c_r x_r$$

Constraints

$$\sum_{c \in C} x_l^c = 1, \quad \text{for all } l \in L \tag{6.1}$$

$$\sum_{a \in i^+} x_a = 1, \quad \text{for all } i \in N_s \tag{6.2}$$

$$\sum_{a \in N_d^-} x_a = f \tag{6.3}$$

$$x_l^c = \sum_{a \in \text{tail}(l)_c^-} x_a, \quad \text{for all } l \in L, c \in C \tag{6.4}$$

$$x_l^c = \sum_{a \in \text{head}(l)_c^+} x_a, \quad \text{for all } l \in L, c \in C \tag{6.5}$$

$$\sum_{r' \in A_r} x_{r'} - M(1 - x_r) \le 0, \quad \text{for all } r \in R \tag{6.6}$$

$$x_l^c \in \{0,1\} \text{ and integer}, \quad \text{for all } l \in L, c \in C \tag{6.7}$$

$$x_d \in \{0,1\} \text{ and integer}, \quad \text{for all } d \in D \tag{6.8}$$

$$x_r \in \{0,1\} \text{ and integer}, \quad \text{for all } r \in R \tag{6.9}$$

Constraint (6.1) is the train cover constraint, which ensures that every train is assigned a qualified crew to operate it. Constraint (6.2) ensures flow balance at a crew supply node. Constraint (6.3) ensures the flow balance at the sink node. Constraints (6.4) and (6.5), respectively, ensure flow balance at train departure and arrival nodes. Constraint (6.6) ensures that the crew assignment honors the FIFO constraint. Constraints (6.7)–(6.9) specify that all the decision variables in the model are binary. The objective function is constructed to minimize the total cost of crew wages, deadheading, detentions, and train delays. Note that the detention and delay costs are taken into account while calculating the cost of rest arcs.

Most crew districts have two terminals, and a typical train schedule has around 500 trains running in 2 weeks in a crew district. Each crew district could have two to four crew types and around 50 crews. Therefore, the space–time network could have around $50+2\times500=1{,}050$ nodes. The number of deadhead arcs is typically around 25,000, and the number of rest arcs is around 100,000.

Since the number of rest arcs for a typical problem is of the order of 100,000, and as each rest arc has one FIFO constraint, the number of FIFO constraints in the model is around 100,000, which is very large. Also, these constraints spoil the structure of the problem and a direct approach using commercial solvers to solve the CSP suffers from intractability and does not converge to a feasible solution even after several hours of computation. However, the integer programming problem with FIFO constraints relaxed (*Relaxed Problem*) can be solved to optimality within minutes. In the next section, we describe efficient methods to solve the CSP.

6.3.4 Solution Methods

6.3.4.1 Successive Constraint Generation (SCG)

The SCG algorithm is very simple. The algorithm works by iteratively pruning crew assignments which violate the FIFO constraints from the current solution of a more relaxed problem. First, the relaxed CSP without any FIFO constraints is solved. Then, the algorithm checks for violations of the FIFO constraint. If there are no violations, then the optimal solution to the CSP has been determined, and the algorithm terminates. If there are FIFO violations, the algorithm adds the violated constraints and resolves the problem. This procedure is repeated until an optimal solution that does not violate the FIFO constraints is found.

6.3.4.2 Quadratic Cost-Perturbation (QCP) Algorithm

While the SCG is an exact algorithm, the running time of this algorithm could be quite high. The cost perturbation-based algorithm described in this section is a heuristic but works extremely well in practice. This algorithm penalizes FIFO violations, so that the FIFO constraints do not need to be explicitly considered while

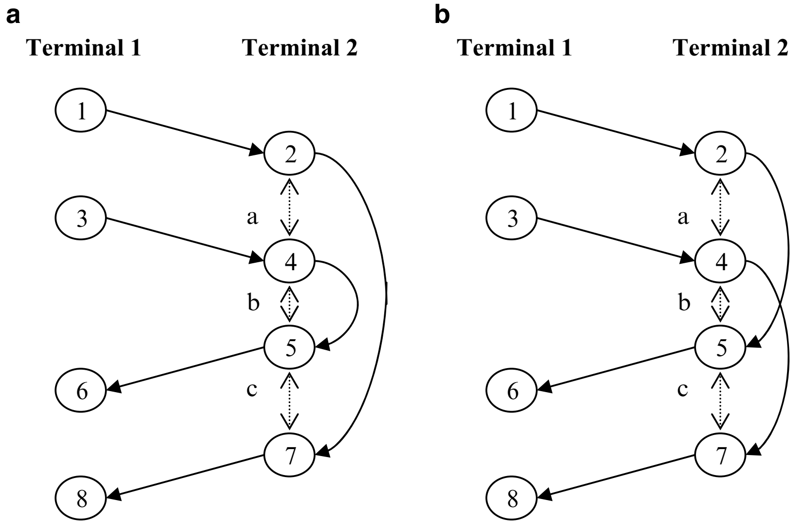

Fig. 6.3 Illustrating the FIFO assignments. (**a**) Invalid assignment. (**b**) Valid assignment

solving the problem. In other words, the costs of arcs are perturbed by a small amount so that the solution to the relaxed CSP is automatically FIFO compliant.

The cost perturbation strategy is presented through the illustration shown in Fig. 6.3 for the case when there is only one crew pool type. In case (a), crew assignments are made in a non-FIFO manner, and in case (b), the assignments are made in a FIFO manner. Consider the case when crews are detained at the Terminal 2. Then, due to the nature of detention costs, the cost of the assignment (b) would definitely be less than or equal to the cost of assignment (a), and hence the solution to the relaxed CSP would honor FIFO constraints. On the other hand, suppose all the rest arcs had a cost of zero; then both the assignments would have the same cost, and the relaxed CSP would have no cost incentive to choose assignment (b) over assignment (a). Thus, a solution to the relaxed problem may violate the FIFO constraints. In order to provide an incentive to the relaxed CSP to choose case (b) over case (a), the cost assignments on rest arcs are perturbed.

The cost perturbation scheme that is used is a function of the duration of rest arcs. Suppose that the time duration between events corresponding to nodes 2 and 4, 4 and 5, and 5 and 7 are a, b, and c, respectively. Consider a cost assignment which is proportional to the square of the duration of rest arcs. The constant of proportionality is represented by k (k is set to a very small value).

Then, cost of assignment

$$\begin{aligned}(\mathrm{a}) &= k\left(\text{duration arc } (2,7)\right)^2 + k\left(\text{duration arc } (4,5)\right)^2 = k\left(a+b+c\right)^2 + kb^2 \\ &= k\left(a^2 + 2b^2 + c^2 + 2ab + 2bc + 2ca\right),\end{aligned}$$

and cost of assignment

$$(\mathrm{b}) = k\left(\text{duration arc } (2,5)\right)^2 + k\left(\text{duration arc } (4,7)\right)^2 = k\left(a+b\right)^2 + k\left(b+c\right)^2$$
$$= k\left(a^2 + 2b^2 + c^2 + 2ab + 2bc\right).$$

The cost of assignments in case (b) is less than that in case (a). Hence, when the rest arcs have zero costs, the quadratic cost perturbation scheme gives FIFO compliant assignments, without having to explicitly add FIFO constraints to the model.

The solution time of QCP is comparable to that of the relaxed CSP. As reported in Vaidyanathan et al. (2007), the QCP method produced solutions with objective function values almost the same as those for the relaxed CSP. This implies that FIFO constraints can be satisfied with little or no impact on the solution cost. Thus, QCP can be used to obtain excellent quality solutions very fast. Due to its attractive running times and solution quality, this method has the potential to be used in both the planning and the real-time environment.

6.4 Applications of the Model

The crew scheduling model has many applications in the tactical, planning, and strategic environments, and some examples are provided in this section.

6.4.1 Tactical Benefits

The model has several benefits in the tactical scheduling environment such as:

- Assignment of crew to trains: The output of the model gives the assignment of crew to trains.
- Recommend which crews to place in hotels and which crews to deadhead home: When a crew arrives at an away terminal, the crew callers have to decide whether the crew should deadhead back home or go to a hotel for rest. The model can be used to mathematically look ahead and evaluate the trade-off between different costs such as crew wages, deadhead cost, detention costs, and rest violation costs.
- Minimize train delays due to shortage of crew: Train delays are potentially very costly because the delay of a train may lead to the unavailability of crew to operate another train in the future and may have a negative domino effect on network-wide operations. By creating several deadhead arcs while constructing the space–time network, the possibility of train delays is reduced.
- Disruption management: The model can be used as a tool to bring back disrupted operations to normalcy. Suppose at some point in time the operations are disrupted. The current state or snapshot of the system gives us the location of each crew and the hours of duty already done. Using this information and the information about the future train schedule, the model can be used to optimally re-assign crew to trains.

6.4.2 Planning Benefits

The essence of the crew planning problem is to determine how many crews should be in each crew pool. Railroads typically solve the pool sizing problem based on historical precedent and rules-of-thumb, through negotiation with the union, and by trial and error. The network flow model can satisfy the need for a structured approach that captures all of the considerations, quantifies the various costs, and recommends the best way to define and staff crew pools. Some of the applications of the model in the planning environment are:

- Develop and evaluate crew schedules: The crew scheduling model can be used to compare the current crew schedule used with the model-generated schedule on the basis of several criteria such as average rest time at the home location, average rest time at the away location, average deadhead time, etc. By suitably changing the model cost parameters, schedules with different characteristics can be obtained.
- Size of crew pools: The crew scheduling model can be used to study the impact of varying the crew pool size on the solution quality. For example, suppose the objective is to minimize the number of crew used. While formulating the problem, large cost incentives can be given to flow on the demand arcs from crew supply nodes to the sink node.

6.4.3 Strategic Benefits

Strategic management involves development of policies and plans and allocating resources to implement these plans. The timeframe of strategic management extends over several months or even years. Strategic crew problems include forecasting future head-count needs and evaluating major policy changes such as negotiating changes to trade-union rules or changing the number and location of crew change points on a network. The model can be used to quickly calibrate efficient frontiers for each crew district and show what number of crews minimizes the sum of train delay costs and crew costs.

Some applications of the CSP in the strategic environment are:

- Determining the number of crew districts and territory of crew districts: The model can be used to re-optimize and test different crew district configurations. For example, suppose crew district 1 operates trains between location A and location B, and crew district 2 operates trains between location B and location C. The model could be used to evaluate the benefit of merging all three stations into a single crew district.
- Effect of changing crew trade-union rules: The crew scheduling problem is a complex optimization problem due to strict trade-union rules related to crew operation. The change of any of these rules will face a lot of resistance from the labor union. At the same time, change of any of these rules has the potential to impact crew costs substantially. Using the crew scheduling model, the impact of changing the trade-union rules on the crew cost can be evaluated.

- Forecasting crew requirement: The model can be used to forecast crew requirement by running it with a very large number of available crew. Since the crew supply is more than what is required, many crews will directly flow from the crew supply to the sink node. The total crew supply minus the number of unused crews will give an idea of the number of crews required based on the forecasted train schedule.

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