# Multilevel Response-to-Intervention Prevention Systems: Mathematics Intervention at Tier 2

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As with reading, poor mathematics learning in school is associated with serious, lifelong difficulties (e.g., National Mathematics Advisory Panel (NMAP) 2008; Rivera-Batiz 1992), and the prevalence of mathematics difficulty is as high, with 5–9% of the population experiencing such problems (e.g., Dirks et al. 2008; Shalev et al. 2000). Moreover, although the prevalence and lifelong debilitating consequences are similar for learning disabilities in mathematics and reading, mathematics has received much less emphasis than reading. This is the case not only in research but also in school practice. That is, schools are much more likely to provide Tier 2 intervention in reading than in mathematics.

At the same time, mathematics difficulty may require more attention than reading difficulty in both research and schools. This is because mathematics difficulty is potentially more complicated and difficult to address, given that the mathematics curriculum is organized into many more strands that are presumed to represent different component skills. In reading, measurement studies (e.g., Mehta et al. 2005) provide the basis for five-component reading skills: phonological awareness, decoding, fluency, vocabulary, and comprehension. In mathematics, measurement studies are yet to be conducted, but the assumption reflected in the curriculum is that many more component skills exist. For example, just considering the elementary school grades, one major focus is whole numbers, which is subdivided into curricular strands: concepts, numeration, basic facts, algorithmic computation, and word problems. Another major focus is fractions, which includes common fractions, decimals, and proportions and has its own set of subdomains: part–whole understanding, measurement interpretation, calculations, and word problems. As reflected in the Common Core State Standards (National Governors Association Center for Best Practices 2010), however, algebra is yet another major curricular strand at the elementary school level. This is a complicated curricular scope, and it is unclear whether strengthening performance in one domain can be expected to transfer to other components. Failure to produce strong performance across curricular components, as has been sometimes assessed and demonstrated, creates additional challenges for Tier 2 mathematics intervention (beyond those that are relevant for Tier 2 reading intervention) to circumvent the need for ongoing Tier 2 support in mathematics.

In this chapter, we focus is on the mathematics side of Tier 2 intervention. First, we provide an overview of the design principles involved in effective Tier 2 intervention and illustrate their application in a validated tutoring program for addressing students' difficulty with word problems. Then, we discuss more recent innovations in Tier 2 intervention by focusing on early arithmetic skill at first grade and on conceptual understanding and procedural skill

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with fractions at fourth grade. The chapter concludes with a discussion of the limitations of Tier 2 intervention research in mathematics and identifies areas for future research.

## Principles of Effective Tier 2 Intervention

### Six Design Principles

As conceptualized elsewhere (Fuchs et al. 2008), six design principles are central to the provision of effective Tier 2 intervention in mathematics. The first is *instructional explicitness*. Many typically developing students profit from the general education mathematics program that typically relies, at least in part, on a constructivist, inductive approach to instruction, in which teachers avoid explicitly explaining the nature of concepts or procedures for solving mathematics problems. Instead, teachers encourage students to discover conceptual understanding and methods for problem solution, with the hope that this will lead to deeper understanding and longer retention. Students who accrue serious mathematics deficits, however, fail to profit from such an approach in ways that result in understanding of the structure, meaning, and operational requirements of mathematics. A meta-analysis of 58 mathematics studies (Kroesbergen and Van Luit 2003) revealed that students with mathematics difficulty benefited more from explicit instruction than from discovery-oriented methods. Therefore, effective intervention for students with mathematics requires explicit, didactic instruction in which the teacher provides detailed explanations.

Explicitness is important, but it is not sufficient. A second and often overlooked principle of effective mathematics intervention is *instructional design that minimizes the learning challenge*. The goal is to anticipate and eliminate misunderstandings with precise explanations and with the use of carefully sequenced instruction so that the achievement gap can be closed as quickly as possible. This is especially important given the ever-changing and multiple demands of the mathematics curriculum.

The third principle of effective mathematics intervention is the requirement that instruction provide a *strong conceptual basis* for procedures. Supplemental intervention has a history of emphasizing *systematic practice,* a critical and fourth principle of effective practice. Yet, supplemental intervention has sometimes neglected the conceptual foundation of mathematics, and such neglect can cause confusion, learning gaps, and a failure to maintain and integrate previously learned content. In terms of systematic practice, note that this practice needs to be rich in *cumulative review,* the fifth principle of effective intervention.

The sixth principle concerns the need to incorporate *motivators to encourage students to work hard and regulate their attention and behavior*. Students with learning difficulty often display attention, motivation, and self-regulation difficulties, which may adversely affect their behavior and learning (e.g., Fuchs et al. 2005, 2006). By the time students enter Tier 2 intervention, they have experienced failure, causing many to avoid the emotional stress associated with learning mathematics. They no longer try for fear of failing. Therefore, intervention must incorporate systematic self-regulation strategies and motivators; for many students, tangible reinforcers are required. See Table 1 for a summary of the six design principles along with examples from each of the three Tier 2 mathematics interventions discussed below.

**Illustrating the Six Design Principles with Pirate Math** To illustrate these six design principles for effective Tier 2 intervention, we describe a validated tutoring program called Pirate Math (e.g., Fuchs et al. 2009). Pirate Math is designed to address arithmetic as well as word-problem difficulty, while building procedural calculation and early algebraic knowledge. We incorporated a pirate theme because in this schema-broadening instructional program, students are taught to represent the underlying structure of word-problem types using algebraic equations. "They find X, just like Pirates find X on treasure maps."

**Table 1** Implications for practice

| Six principles of effective intervention | Examples from Pirate Math (grades 2–3) | Examples from Galaxy Math (grade 1) | Ex3amples from fraction challenge (grade 4) |
|---|---|---|---|
| Instruction is systematic and explicit | Scaffolded instruction on underlying structure of three word-problem types and solution strategies; all lessons include modeling, guided practice, independent practice, and corrective feedback; simple language is used for explanations, with frequent checks on understanding | Scaffolded instruction on arithmetic principles with manipulatives, number lines, and paper and pencil activities; all lessons include modeling, guided practice, independent practice, and corrective feedback; simple language is used for explanations, with frequent checks on understanding | Scaffolded instruction on fraction magnitude with manipulatives, number lines, and paper and pencil activities; all lessons include modeling, guided practice, independent practice, and corrective feedback; simple language is used for explanations, with frequent checks on understanding |
| Instructional design minimizes the learning challenge | Task analysis provides the most efficient procedures for teaching efficient problem-solving strategies | Task analysis provides the most efficient counting methods, taking into account first graders' fine motor skills | Task analysis provides the basis for instructional strategies for chunking and segmenting complex fraction comparison tasks |
| Instruction has a strong conceptual basis, while providing students with efficient procedural strategies | Instruction on each word-problem type begins with real-life examples and pictorial aids to help students understand the underlying structure of the problem type, while teaching students the most efficient approach word-problem solution strategies | Instruction dominantly focuses on the number knowledge foundational to arithmetic, while teaching students efficient counting strategies to solve addition and subtraction problems | Instruction focuses dominantly on the measurement interpretation of fractions, while teaching students procedural strategies (e.g., "bigger denominator, bigger fraction," and "fewer parts, bigger fraction") |
| Instruction includes systematic practice | In each lesson, "warm-ups" provide flash card practice for applying counting strategies to solve arithmetic problems; daily practice in solving word problems | In each lesson, speeded practice is provided on basic addition and subtraction facts | In each lesson, the Speed Game provides practice on fraction concepts (e.g., circling fractions equivalent to ½) |
| Instruction includes cumulative review. | All independent practice activities include both new material and review | All independent practice activities include both new material and review | All independent practice activities include both new material and review |
| Instruction includes motivators to reinforce positive behavior | Timer is used to monitor and award "gold coins" for on-task behavior on an interval basis; bonus problems (announced at the end of lessons) earn students coins; students spend coins on prizes | Stickers are provided for on-task behavior and hard work, which are affixed to "Galaxy Math Sticker Chart"; when chart is completed, students select a prize | Timer is used to monitor and award half or quarter "dollars" for on-task behavior; bonus problems (announced at end of lessons) earn students half or quarter dollars; students spend fraction money at "Fraction Store." |

Pirate Math comprises four units: an introductory unit, which addresses mathematics skills foundational to solving word problems, and three word-problem units, each focused on a different type of word problem. Every tutoring lesson is scripted. Scripts are studied; they are not read or memorized. Pirate Math runs for 16 weeks, with 48 sessions (three per week).

Each session lasts 20–30 min. The instruction, as outlined below, is *systematic and explicit*; it is designed with care to *minimize the learning challenge*; it is rich in *concepts*; it incorporates *systematic practice* as well as *cumulative review*; and it relies on *systematic reinforcement* to encourage good attention, hard work, and accurate performance.

The introductory unit addresses mathematics skills foundational to word problems. Tutors teach strategic counting for deriving answers to arithmetic problems, review algorithms for double-digit addition and subtraction procedural calculations, teach methods to solve for “X” in any position in simple algebraic equations (i.e., $a+b=c$; $d-e=f$), and teach strategies for checking work in word problems.

A single strategic counting lesson is designed to address arithmetic deficits. Students are taught that if they “just know” the answer to an arithmetic problem, to “pull it out of your head.” If, however, they do not know an answer immediately, they “count up.” Strategic counting for addition and subtraction is introduced with the number line. *The “min strategy” is a mathematics counting strategy in which students start with the larger number and count up and the answer to the addition problem is the last number spoken. This is a more efficient counting strategy than the “max strategy” in which students start with the first number in an addition problem and count up (regardless of whether the first addend is the bigger or smaller number). The “min strategy” is referred to across the mathematics literature and is defined in the chapter.*

Practice in strategic counting is then incorporated into subsequent lessons. The tutor begins each session by asking the student, “What are the two ways to find an answer to a simple math problem?” The student responds, “Know it or count up.” Then, the student explains how to count up an addition problem and how to count up a subtraction problem. Next, the tutor requires the student to count up two addition and two subtraction problems. Then the tutor conducts a flash card warm-up activity, in which students have 1 min to answer arithmetic problems. If they respond incorrectly, the tutor requires them to count up until they derive the correct answer. At the end of 1 min, the tutor counts the cards, and the student then has another minute to beat the first score. Also, throughout the lesson, whenever the student makes an arithmetic error, the tutor requires the student to count up. Finally, when checking the paper–pencil review, the tutor corrects arithmetic errors by demonstrating the counting strategy.

Each of the three word-problem units focuses on one word-problem type and, after the first problem-type unit, subsequent units provide systematic, mixed cumulative review that includes previously taught problem types. The word-problem types are Total (two or more amounts are combined; e.g., Doris has two flowers. Her sister has five. How many flowers do they have?), Difference (two amounts are compared; e.g., Doris is 2 years old. Her sister is 5. How much older is her sister?), and Change (initial amount increases or decreases; e.g., Doris had two pennies. Then she got two more. How much money does she have now?). Each word-problem session comprises six activities. The first is the counting strategies review and flash card warm-up already described, which lasts 5 min. Word-problem warm-up is the next activity, which lasts about 2 min and begins during the first word-problem unit. The tutor shows the student the word problem that the student had solved during the previous day’s paper-and-pencil review. The student explains to the tutor how he or she solved the problem.

Conceptual and strategic instruction is the next activity. It is the heart of the lesson, lasting 15–20 min. Tutors provide scaffolded instruction in the underlying structure of and in solving the three types of word problems (i.e., developing a schema for each problem type), along with instruction on identifying and integrating transfer features (to broaden students’ schema for each problem type). The tutor relies on role-playing, manipulatives, instructional posters, modeling, and guided practice. In each lesson, students solve three word problems, with decreasing amounts of support from the tutor.

Total is the first problem type addressed. In the total unit, tutors teach students to *run* through a problem: a three-step strategy prompting students to read the problem, underline the question, and name the problem type. Students use the *run* strategy across all three problem types. Next, for each problem type (i.e., schema), students are taught an algebraic equation to represent the underlying structure of that problem type. Students fill in slots of the equation as they identify and circle relevant information in the problem narrative. For example, for Total problems, students circle the item (e.g., flowers) being combined and the numerical values representing that item (e.g., 2 and 5), and then label the circled numerical values as "$P1$" (for part one; e.g., 2), "$P2$" (for part two; e.g., 5), and "T" (for the total; e.g., indicated with $x$—the missing value). Students then construct an equation representing the underlying mathematical structure of the problem type. For Total problems, the equation takes the form of "$P1+P2=T$," and "$x$" can appear in any of the three variable positions. Students are taught to solve for $x$, to provide a word label for the answer, and to check the reasonableness and accuracy of their work.

The strategy for Difference problems and Change problems follows similar steps but uses variables and equations specific to those problem types. For Difference problems, students are taught to look for the bigger amount (labeled "B"), the smaller amount (labeled "s"), and the difference between amounts (labeled "D"), and to use the algebraic equation "B − s = D" to represent the problem type. For Change problems, students are taught to locate the starting amount (labeled "St"), the changed amount (labeled "C"), and the ending amount (labeled "E"); the algebraic equation for Change problems is "St ± C = E" (± depends on whether the change is an increase or decrease in amount).

For each problem type, explicit instruction to broaden schemas occurs in six ways. First, students are taught that because not all numerical values in word problems are relevant for finding solutions, they should identify and cross out irrelevant information as they identify the problem type. Second, students learn to recognize and solve word problems with the missing information not only in the traditional, third slot of the equation, but also in the first or second position of the algebraic equation representing the underlying structure of the problem type. Third, students learn to apply the problem-solving strategies to word problems that involve addition and subtraction with double-digit numbers with and without regrouping. Fourth, students learn to solve problems involving money. Fifth, students are taught to find relevant information for solving word problems in pictographs, bar charts, and pictures. Finally, students learn to solve two-step problems that involve two problems of the same problem type or that combine problem types. Across the three problem-type units, previously taught problem types are included for review and practice.

Sorting word problems is the third activity and takes 5 min. Tutors read aloud flash cards, each displaying a word problem. The student identifies the word-problem type, placing the card on a mat with four boxes labeled "Total," "Difference," "Change," or "?." Students do not solve word problems; they sort them by problem type. To discourage students from associating a cover story with a problem type, the cards use similar cover stories with varied numbers, actions, and placement of missing information. After 2 min, the tutor notes the number of correctly sorted cards and provides corrective feedback for up to three errors.

In paper-and-pencil review, the final activity, students have 2 min to complete nine number sentences asking the student to find x. Then, they have 2 min to complete one word problem. Tutors provide corrective feedback and note the number of correct problems on the paper. Tutors require students to count up arithmetic errors, and keep the paper-and-pencil review sheet for the next day's word-problem warm-up activity.

Pirate Math includes a systematic reinforcement program. Throughout each Pirate Math session, tutors use a timer, which is set to ring three times at unpredictable intervals. If all students in the group are "on task" when the timer rings, each student earns a gold coin. (If one or more students are not on task, no one earns a coin.) Students can also earn gold coins for completing bonus problems correctly. Students do not know which problems are bonus problems until the end of the lesson. This encourages hard work throughout the session. At the end of the lesson, each earned gold coin is placed on the student's individual "treasure map." Sixteen coins lead to a picture of a treasure box and, when reached, the student chooses a small prize from a treasure box. The student keeps the old treasure map and receives a new map in the next lesson.

**Effectiveness of Pirate Math** The efficacy of Pirate Math has been demonstrated and replicated (e.g., Fuchs et al. 2009, 2010a). For example, Fuchs et al. (2009) identified third-grade students with substantial difficulty in computation and word problems in the Nashville-Metropolitan Public Schools and in the Houston Independent School District. Then, these children were randomly assigned to receive 13 weeks (3 times per week) of Pirate Math tutoring or Math Flash (Fuchs et al. 2003) tutoring (a validated program focused entirely on computation) or control (the school program without any research-based mathematics tutoring). Each session was audiotaped. A representative sample of lessons was coded for fidelity against the tutoring scripts, and fidelity was strong for both tutoring interventions. Students were pre- and posttested on computation and word-problem measures.

Pirate Math and Math Flash students improved comparably on computation and significantly more than students in the control group. The effect size (ES) comparing Math Flash to the control group was large (0.85 standard deviations). The ES comparing Pirate Math to the control group was similar, but somewhat smaller (0.72). But given that Pirate Math allocated only 5 min of every session to computation (whereas Math Flash spent 20–30 min per session on computation), Pirate Math's effects on computation are noteworthy. At the same time, however, effects on word problems clearly favored Pirate Math. The ES comparing Pirate Math to the control group was large (0.89), and there was no significant difference between Math Flash and the control group. Most impressively, the ES comparing Pirate Math to Math Flash was 0.72. This indicates that when Pirate Math, a tutoring program designed to incorporate the six principles of effective Tier 2 intervention, is implemented as designed, it benefits at-risk (AR) students' word-problem learning in dramatic ways on computation and word problems.

## Innovations in Tier 2 Intervention

This section provides examples of innovations in Tier 2 intervention, relying on some of our more recent research (while providing an overview of prior related intervention work). The first focus is on early arithmetic skill (i.e., adding and subtracting single-digit numbers) at first grade. Early arithmetic skill (at the start of first grade) predicts mathematics learning through the end of fifth grade (Geary 2011) and is an indicator of risk for long-term learning disabilities (Geary et al. 2012b). The NMAP (2008) concluded that early mastery of simple arithmetic is a critical step toward eventual mastery of high-school algebra, a gateway for later entry into mathematics-intensive fields. These factors point to the importance of early mathematics intervention and the need for a strong focus on arithmetic in that intervention. Yet, despite its foundational importance, few randomized control trials have been conducted with early arithmetic Tier 2 intervention as its focus and, prior to our most recent arithmetic study, none had sought to isolate the added value of an intervention component. After describing our innovation in Tier 2 intervention for early arithmetic difficulty, we move to fraction intervention at fourth grade. Half of middle and high school students in the USA are still not proficient with the ideas and procedures taught about fractions in the elementary grades (e.g., Behr et al. 1984; Hiebert and Wearne 1985; National Council of Teachers of Mathematics (NCTM) 2007; NMAP 2008; Ni 2001). Yet, competence with fractions is considered foundational for learning algebra, for success with more advanced mathematics, and for competing successfully in the American workforce (NMAP 2008; Geary et al. 2012b). For these reasons, NMAP (2008) recommended that high priority be assigned to improving performance on fractions, a theme reflected in the Common Core State Standards (National Governors Association Center for Best Practices 2010) (http://www.corestandards.org). Therefore, improving common fraction performance for fourth graders at risk for poor outcomes is a critical focus. Yet, prior to this study, no prior work on Tier 2 intervention at the elementary school level was identified.

## Early Arithmetic Skill at First Grade

**Development of Arithmetic Competence and Need for Early Intervention** By the time children enter first grade, most have a rudimentary understanding of addition and subtraction and can count to solve these problems (Geary 1994). For addition, they typically count both addends. For subtraction, they represent the beginning quantity (the minuend) with objects and sequentially separate the number of objects to be subtracted (the subtrahend); then they count the remaining set (e.g., Groen and Resnick 1977). As understanding of cardinality and the counting sequence develops, children discover the number-after rule for adding with 1. They also understand the sum of $5+2$ cannot be 6 but instead is two numbers beyond 5. In this way, children discover the efficiency of counting from the first addend and rely on more efficient counting procedures. For addition, the most efficient counting procedure involves the min strategy: starting with cardinal value of the larger addend and counting up the number of times equal to the smaller addend (e.g., $4+3=$ "four: five, six, seven"). For subtraction, the most efficient strategy relies on the missing addend: starting with the subtrahend and counting to the minuend (e.g., $5-2=$ "two: three, four, five"; the number of counts is the answer). Frequent use of efficient counting procedures reliably results in the correct association between problem and answer, which produces long-term memories (Fuson and Kwon 1992; Siegler and Robinson 1982; Siegler and Shrager 1984). This enables direct retrieval of answers, and the commutativity of addition facilitates retrieval of related addition problems (Rickard et al. 1994). Subtraction, which is not commutative, is more difficult, but can be facilitated by retrieval of related addition facts (e.g., $8-5=3$, based on $5+3=8$; LeFevre and Morris 1999), once children understand the inverse relation between addition and subtraction (Geary et al. 2008).

Students with mathematics learning disabilities show consistent delays in the adoption of efficient counting procedures, make more counting errors during their execution, and fail to make the shift toward memory-based retrieval (e.g., Geary et al. 2012a; Goldman et al. 1988). Most of these children eventually catch up to peers in skilled use of counting procedures, but difficulty with retrieval tends to persist (Geary et al. 2012b; Jordan et al. 2003). They retrieve fewer answers from memory and when they do retrieve answers, they commit more errors (e.g., Geary et al. 1991; Geary et al. 2007). So, simple arithmetic fluency may be a signature deficit of mathematics learning disability (e.g., Geary et al. 2012b; Goldman et al. 1988; Jordan et al. 2003), and remediating arithmetic deficits in older students can be difficult (Fuchs et al. 2010a, c). For these reasons, there is a pressing need for early Tier 2 intervention.

**Prior Work and Purpose of Our Innovation** Four randomized control trials assessing Tier 2 intervention efficacy for first-grade students at risk for poor mathematics outcomes were located. Fuchs et al. (2006) conducted a randomized control trial to assess the efficacy of practice alone. An addition or subtraction problem with its answer briefly flashed on a computer screen; then students generated the problem and answer from short-term memory. This is based on the assumption that practice strengthens retrieval when problems and answers are simultaneously active in working memory (Geary 1993). Compared to an analogous computer-assisted spelling practice condition, arithmetic practice (10 min, twice weekly for 18 weeks) produced significantly better performance for addition but not subtraction; ESs were 0.95 and −0.01. Other mathematics outcomes were not assessed.

Two randomized control trials combined number knowledge tutoring with practice and assessed a broader range of outcomes. In Fuchs et al. (2005), tutoring occurred three times per week for 16 weeks. Each session included 30 min of tutor-led instruction designed to build number knowledge plus 10 min of computerized arithmetic practice, as just described. Results favored tutoring over a no-tutoring control group on measures of concepts and applications (ES = 0.67), procedural calculations (ES = 0.40-0.57), and word problems (ES = 0.48), but effects were not reliable on simple arithmetic (ESs = 0.15–0.40). Bryant et al. (2011) also integrated tutoring on number knowledge with practice (four times per week for 19 weeks). In each session, 20 min were devoted to number knowledge; 4 to practice, which focused on arithmetic problems, as well as reading numerals, counting on/back, writing dictated numerals, and writing 3-number sequences. Effects were significantly stronger for tutoring compared to a no-tutoring control group on simple arithmetic (ES = 0.55), place value (ES = 0.39), and number sequences (ES = 0.47). But tutoring did not enhance word-problem outcomes (ES = −0.05 and 0.07).

In the only randomized control trial to focus exclusively on number knowledge, Smith et al. (2013) evaluated Math Recovery (MR, Wright et al. 2002; Wright 2003), in which tutors adapt lessons to meet student needs as reflected on MR assessments. Tutors introduce tasks and have students explain their reasoning, but practice is not provided. Tutoring was to occur 4–5 times per week, 30 min per session across 12 weeks, but the median number of sessions was 32. At end of first grade, effects favored MR over the control group on fluency with simple arithmetic (ES = 0.15), concepts and applications (ES = 0.28), quantitative concepts (ES = 0.24), and mathematical reasoning (ES = 0.30). Effects were stronger for students who began tutoring below the 25th percentile (0.31–0.40), but are generally smaller than in studies that combined number knowledge tutoring with practice. Comparisons are, however, difficult because this study allowed fidelity to vary, whereas the other studies tried to ensure fidelity.

These four studies suggest potential for Tier 2 intervention, compared to no tutoring, for enhancing some forms of mathematics learning among AR first graders. Prior work does not, however, provide the basis for understanding whether the effects of Tier 2 intervention are simply a matter of more instruction (i.e., prior studies only include no-tutoring control groups rather than incorporating a contrasting tutoring condition). In this way, prior work also fails to inform practitioners about what components of Tier 2 intervention contribute to positive effects.

For these reasons, our innovation focused on the effects of number knowledge tutoring with contrasting forms of practice on AR first graders' emerging competence with simple arithmetic (Fuchs et al. 2013a). Two-digit calculation, number knowledge, and word-problem outcomes were also assessed. The major emphasis in tutoring was developing interconnected knowledge of number, but a small portion of each session was devoted to practice. In one condition, practice was designed to reinforce the relations and principles that serve as the basis of reasoning strategies that support fact retrieval. The other form of practice was more rote: It was designed to promote quick responding and use of efficient counting procedures to generate many correct responses and thereby form long-term representations to support retrieval. Both practice conditions occurred on the same content, encouraged strategic behavior, and provided immediate corrective feedback. So the two major distinctions between the two forms of practice were as follows. First, one condition encouraged a variety of number-principle strategies (e.g., relying on number lists, arithmetic principles such as cardinality, commutative principle, subtraction as the inverse of addition, and efficient counting procedures), whereas the other condition only encouraged efficient counting strategies. Second, in the condition that encouraged a variety of strategic behavior, practice did not involve speeded execution of the chosen strategy; the focus instead was on executing strategies thoughtfully to emphasize number knowledge. By contrast, in the condition that relied exclusively on counting strategies, practice was speeded. In this chapter, the terms *nonspeeded practice* and *speeded practice* are used to refer to these conditions.

To understand the efficacy of number knowledge tutoring when combined with speeded versus nonspeeded practice, we compared each tutoring condition against an AR no-tutoring control group that received the same classroom instruction as the tutored groups. To understand how the type of practice affects learning and whether the effects of tutoring are attributable to more than simply providing extra instructional time, we contrasted the two tutoring conditions against each other. To provide insight into whether different forms of tutoring help narrow the achievement gap, we included a group of low-risk classmates, who received the same classroom instruction as the AR tutored and control groups. We recruited forty schools and two hundred and thirty-three classes, in which we screened students to identify students with and without risk. Attrition was minimal, with 190 number knowledge tutoring + nonspeeded tutoring, 195 number knowledge tutoring + speeded practice, 206 AR no-tutoring control, and 300 low-risk control completing the study.

**Nature of Tutoring** Tutoring occurred for 16 weeks, three times per week, 30 min per session. In both tutoring conditions, 25 of each 30-min session were the same, designed to foster number knowledge. The last 5 min, which involved practice, differed. To foster engagement, the program uses a space theme (e.g., children are encouraged to “blast off into the math galaxy” by improving their mathematics knowledge; some manipulatives are shaped as space rockets). Tutors/students refer to the program as Galaxy Math (Fuchs et al. 2010b), which is related to Number Rockets (Fuchs et al. 2005).

Segment 1 focuses on number knowledge, with five units. Unit 1 (lessons 1–18) addresses basic number knowledge. Unit 2 (lessons 19–20) focuses on arithmetic doubles ($0+0$ through $6+6$; $2-2$ through $12-6$). Unit 3 (lessons 21–52) addresses arithmetic problem sets 5 through 12 (e.g., the 5 set includes all problems with sums or minuends of 5; the term *sets* is used, as in the *5 set*). Unit 4 (lessons 53–66) focuses on tens concepts. Unit 3 comprises approximately half the program (not counting the review unit). It focuses on partitioning number into constituent sets and number families (e.g., for the 5 set, $0+5$, $1+4$, $2+3$, $5-0$, $5-1$, $5-2$, etc.). Toward this end, five activities are conducted in each lesson. First, the tutor and student use unifix cubes to explore how the target number (e.g., 5 in the 5 set) can be partitioned in different ways to derive the adding and subtracting problems comprising that set. The second activity also focuses on part–whole knowledge, but with number families (problems using the same three numbers, e.g., $2+3=5$, $3+2=5$, $5-2=3$, and $5-3=2$, in that set). The tutor relies on visual displays that group families in the set and uses blocks to help the student rely on part–whole knowledge to understand how/why four problems make a family. Third, the student generates all addition and subtraction problems (with answers) in the target set, while using rockets to show the problems. Fourth, the tutor and student work together to solve a word problem on that set; they produce the answer and explain why the word problem is specific to the set. Fifth, the student reviews previous sets, orally stating answers to problems with corrective feedback. Between one and four lessons are allocated to each set, with a mastery test determining if and when students can advance before all four lessons on that are conducted.

Segment 2 involves practice. The content addressed in the final 5-min segment was the same in both conditions, addressing content covered in that day’s number knowledge tutoring lesson. In the nonspeeded practice condition, students played games with space-themed manipulatives. Games were designed to provide contextualized review of the content addressed in the day's lesson. For example, to reinforce $n \pm 1$ lessons, the student used a spinner on a dial segmented from 1 to 19 to identify the number of "rockets called out to explore the math galaxy" and counted this number of rockets onto the board. Then, the tutor informed the student that either one more rocket was needed or that one rocket was called back to the space station, so the student added one rocket to the board or took one away. The student then stated the answer and number sentence. For an 8-set lesson, the student was informed how many rockets constituted the fleet and wrote that numeral as the total. The student then rolled a die to find the first group of rockets released from the space station, counted that number of rockets onto the game board, and wrote the numeral as an addend. The student then determined how many more rockets were needed to complete the fleet, wrote that numeral as an addend, and read the number sentence. Next, the student rolled the die to find how many rockets were called back to the space station. The student wrote numerals to generate and read a number sentence. Games differed for each day on the same topic. Throughout nonspeeded practice and lessons, tutors encouraged students to know the answer or to rely on a variety of number-principle strategies including (but not limited to) using number lists, relying on arithmetic principles (e.g., cardinality, commutative principle, inverse relation between addition and subtraction), and efficient counting strategies. "Knowing the answer right off the bat" was the preferred strategy when students were sure of answers.

In the speeded practice condition, students completed the "Meet or Beat Your Score" activity, with which students had 90 s to answer a stack of flash cards. For example, for $n \pm 1$ lessons, flashcards were all $n+1$ and $1+n$ problems ($n = 0$–18); for 8-set lessons, flashcards were all addition problems with the sum 8 and all subtraction problems with the minuend 8. Tutors corrected student errors immediately and required the student to use counting to produce the correct response. Therefore, students answered each problem presented correctly. The 90 s continued to elapse as the student used the counting procedure (as many times as needed). In this way, careful but quick responding increased the number of correct responses, which were counted and charted on a Rocket Chart at the end of 90 s. Then, the student had two chances to meet or beat that score. Throughout speeded practice and lessons, tutors required the student to know the answer (i.e., retrieve it from memory, if confident) or use the efficient counting strategies they had been taught. Tutors explained that "knowing the answer right off the bat" was preferred, if the child was sure of the answer. The counting strategies were simplified versions of the counting strategies already explained for Math Flash.

**Findings** In terms of fluency with simple arithmetic, number knowledge tutoring with nonspeeded practice produced significantly better learning compared to AR control students who did not receive tutoring. The ES was 0.38. Nonspeeded practice was designed to reinforce the relations and principles that were emphasized in the number knowledge portion of tutoring and that serve as the basis of reasoning strategies to support arithmetic skill. Findings lend theoretical support to studies indicating the important role number knowledge plays in developing competence with simple arithmetic (e.g., Baroody 1988, 1999; Butterworth and Reigosa 2007; De Smedt et al. 2009; Duncan et al. 2008; Koontz and Berch 1996; Rousselle and Noel 2007).

At the same time, nonspeeded practice did not help AR students narrow the achievement gap (ES = 0.07 favoring low-risk classmates). By contrast, incorporating speeded practice in number knowledge tutoring produced superior improvement in simple arithmetic compared to low-risk classmates, with an ES of 0.39, thereby narrowing the achievement gap. Speeded practice was also substantially more effective than number knowledge tutoring with nonspeeded practice (ES = 0.51) and produced an ES of 0.87 over AR control. In this way, this study extended earlier randomized control trials by isolating the effects of speeded practice, delivered in the context of tutoring to build number knowledge. Results indicate a substantial role for speeded practice in promoting simple arithmetic learning. It is noted that, in contrast to how practice is sometimes configured in schools (i.e., without sufficient scaffolding in number knowledge, in massed doses, and without support for correct responding), speeded practice was delivered in the context of number knowledge instruction and formulated practice to help children generate many correct responses, support development of fluency with efficient counting strategies, require students to immediately correct errors with an efficient counting procedure, and encourage strategic meta-cognitive behavior. So, findings should be generalized only to speeded practice that incorporates similarly sound, theoretically motivated instructional design.

Although findings suggest the value of speeded practice in supporting AR children's development of arithmetic competence, they do not address the possibility that speeded practice, with its focus on rote responding, is detrimental to other forms of mathematics learning. This possibility was investigated by assessing two-digit calculations, number knowledge, and word problems and found no evidence to support such a hypothesis. In fact, on complex calculations, speeded practice combined with number knowledge tutoring produced stronger learning compared to number knowledge tutoring with nonspeeded practice (ES = 0.21) or AR control (ES = 0.69). This was the case even though Unit 4's focus on multi-digit number knowledge was identical in the speeded and nonspeeded practice conditions. Moreover, improvement on complex calculations was comparable for speeded practice students and their low-risk classmates (ES = 0.01), even as improvement on more complex calculations for low-risk students exceeded that of nonspeeded practice students (ES = 0.19) and AR control students (ES = 0.57).

It is important to note, however, that effects on number knowledge or word-problem learning did not favor the speeded practice condition—although there was no indication that speeded practice was detrimental to these forms of mathematics learning. On number knowledge, the tutored groups developed comparably (ES = 0.11, favoring speeded practice) and better than AR control (ESs = 0.29 and 0.19 for the speeded and nonspeeded practice conditions, respectively). On word problems, the tutored groups again developed comparably (ES = 0.04, this time favoring nonspeeded practice) and better than AR control (ESs = 0.22 and 0.27 for speeded and nonspeeded practice, respectively).

In sum, findings suggest that number knowledge tutoring, with nonspeeded or speeded practice, is effective for enhancing arithmetic, complex calculations, number knowledge, and word-problem learning over no tutoring. At the same time, well-designed speeded practice, delivered in the context of tutoring to build number knowledge, is more effective than nonspeeded practice in promoting complex calculations as well as simple arithmetic, a core mathematical competence. Effects favoring speeded over nonspeeded practice on simple arithmetic (and complex calculations) were substantial, and results showed that the advantage for speeded over nonspeeded practice may occur by helping students compensate for the demands on reasoning ability, which an instructional focus on number knowledge creates. At the same time, no evidence that speeded practice inhibits development of number knowledge or word-problem skill was found, despite that rote responding was involved in speeded practice. In fact, both number knowledge tutoring conditions produced comparable number knowledge and word-problem learning, which was superior to AR control students (Fuchs et al. 2013a).

## Conceptual Understanding and Procedural Skill with Fractions at Fourth Grade

**Importance of Conceptual Understanding of Fractions** Conceptual understanding is important for learning and maintaining accurate procedures with fractions (e.g., Byrnes and Wasik

1991; Hecht et al. 2003; Mazzocco and Devlin 2008; Ni and Zhou 2005; Rittle-Johnson et al. 2001). In this next study, we assessed the efficacy of a small-group tutoring program designed to foster understanding of fractions (Fuchs et al. 2013b). The intervention focused on two types of conceptual knowledge (Kieren 1993). The first is part–whole understanding, with which a fraction is understood as a part of one entire object or a subset of a group of objects. This type of understanding is typically represented using an area model, in which a region of a shape is shaded or a subset of objects is distinguished from the remaining objects. The second type of conceptual knowledge, the measurement interpretation of fractions, reflects cardinal size (Hecht 1998; Hecht et al. 2003) and is often represented with number lines (e.g., Siegler et al. 2011). In American schools, fractions are taught primarily via area models that underpin part–whole understanding. Measurement understanding is assigned a subordinate role (addressed later and with less emphasis).

Our study was innovative because our major emphasis was the measurement interpretation of fractions, although a smaller amount of time on part–whole conceptualizations to build on students' incoming understanding of fractions was also incorporated, as addressed in their classrooms. In emphasizing the measurement interpretation, we sought to avoid the understanding of fractions exclusively as part–whole relationships, which may create difficulty for conceptualizing improper and negative fractions (NMAP 2008). A focus on the measurement model is also in keeping with NCTM standards (2006) that instruction be designed to foster understanding of fraction magnitudes in terms of the number line. It is also an explicit emphasis of the fourth-grade Common Core State Standards on understanding of fraction equivalence and ordering (http://www.corestandards.org).

**Prior Work and Purpose of Our Innovation** Six studies assessing Tier 2 intervention efficacy on fractions were located. Bottge and colleagues conducted two studies in which they contrasted video-based real-world problem-solving instruction against conventional word-problem instruction. Both conditions required conceptual and procedural knowledge of fractions to solve problems. However, instruction on fractions, which occurred only in the service of problem-solving, was inductive and incidental. Bottge (1999) randomly assigned 17 eighth-grade students in a remedial mathematics class to the two ten-session conditions. Bottge et al. (2002) randomly assigned 42 seventh graders in two mathematics classes to 12 sessions and separately reported data for the eight students with prior mathematics difficulty. In both studies, there was little evidence of improvement on the fraction outcome, which required calculations, and no significant difference between conditions.

Other studies incorporated a more explicit instructional approach, reflecting the six design principles outlined earlier in this chapter. Relying on a multiple-baseline design, Joseph and Hunter (2001) demonstrated experimental control for a cue-card strategy across three eighth-grade AR students. A teacher initially taught students to use the cue card, which supported a three-pronged strategy for adding or multiplying fractions with and without common denominators and for reducing answers. After students showed competence in using the strategy, they were instructed to use the cue card while solving problems on daily fraction probes. In a maintenance phase, the cue card was removed. All three students showed substantial improvement with introduction of the cue card strategy, and maintenance (i.e., performance without the cue card) was strong. The study focus was, however, entirely procedural in terms of instruction and outcome.

Kelly et al. (1990) also took an explicit approach to fraction instruction, but focused simultaneously on procedures and concepts. They randomly assigned 28 high-school AR students from one remedial and one general mathematics class to ten sessions of teacher-mediated videodisc-supported instruction or teacher-mediated conventional textbook instruction. Although direct instruction was employed in both conditions, videodisc instruction differed by providing mixed problem-type instruction, separating highly confusable concepts and terminology during early instructional stages, and providing a broader range of examples to avoid misconceptions (e.g., introducing proper and improper fractions in the first lesson). Both groups improved substantially from pretest (40 % on a 12-item test) to posttest (96 % vs. 82 %), with the videodisc group improving significantly more. Yet, despite the instructional focus on concepts and procedures, the fraction measure was largely procedural.

By contrast, intervention in the next two studies focused primarily on understanding of fractions and assessed outcomes on concepts as well as procedures. Butler et al. (2003) contrasted two explicit instruction conditions with 50 sixth- through eighth-grade AR learners. Both conditions carefully transitioned students from a conceptual emphasis, largely based on part–whole understanding, to algorithmic rules for handling fractions, and from visual to symbolic representations. Only one condition, however, included concrete manipulatives. Both groups significantly improved across ten sessions. On one measure, in which students circled fractional parts of sets, those who received 3 days of manipulatives improved significantly more; on the other four measures, the difference between conditions was not significant, providing mixed evidence regarding the importance of concrete representations within an explicit instructional approach. Without random assignment or a control group, however, conclusions are tentative.

Hecht (2011) expanded on Butler et al. (2003) by employing random assignment, including a control group, and doubling the duration of intervention. Seventh-grade AR students ($n = 43$) were randomly assigned to control or 1:1 tutoring using 23 Rational Number Project lessons (Initial Fraction Ideas; Cramer et al. 2009). These lessons rely on area models to teach part–whole relations, the concept of the unit, order and equivalence, and addition/subtraction, and apply these concepts in the context of computation, word problems, and estimation of sums/differences. Intervention students improved significantly more on a range of procedural and conceptual measures, including a number line task rooted in measurement understanding of fractions, despite that none of the lessons addressed the measurement model or strategies for using the number line.

These studies provide the basis for only tentatively concluding that Tier 2 intervention, based on part–whole understanding of fractions, enhances fraction learning among middle- and high-school AR students. Each of these studies was, however, small and relied exclusively on experimental measures, closely aligned with instruction, to assess outcomes (an exception is Butler et al. 2003, who included a commercial criterion-referenced measure in addition to experimental tasks). More importantly, none of these studies addressed the earlier grades, when the foundation for understanding fractions is developed, and none operationalized risk with the clarity needed to understand the level of students' mathematics performance.

Our major innovation, therefore, was to examine the effects of early intervention, with the clarity required to understand what "risk" entailed (Fuchs et al. 2013b). We focused on fourth grade, when the curriculum emphasizes understanding of fractions. Conceptual understanding and calculation skill were assessed using experimental tasks as well as an external, widely accepted measure of competence with fractions: the pool of easy, medium, and hard fourth-grade and easy eighth-grade released fraction items from the National Assessment of Educational Progress (NAEP; 18 items were selected from the pool of items released between 1990 and 2009).

In Fuchs et al. (2013b), we operationalized risk as whole-number calculation skill below the 35th percentile. Whole-number skill was used to define risk because the range of performance on fraction measures is limited at the start of fourth grade and because prior work (Hecht and Vagi 2010; Seethaler et al. 2011) identifies whole-number calculation skill as a predictor of fraction learning. A cutoff at the 35th percentile is, however, high. So, we designed the study to examine whether response to intervention differs for students with more versus less severe incoming deficits with whole numbers (< 17th vs. 18th–34th percentile). This question, whether intervention is differentially effective depending on the severity of mathematics difficulty, is important for designing instruction that addresses the full range of students with mathematics difficulty. Yet, the authors were unable to locate experimental studies on this topic. Finally, to contextualize results, we compared year-end performance for the more and less severe groups of tutored students against low-risk (> 34th percentile) classmates. This is important given that few fraction measures provide a normative framework or thorough behavior sampling of fourth-grade fraction skill.

**Nature of Tutoring** The fraction intervention program, Fraction Challenge (Fuchs and Schumacher 2010), described in Fuchs et al. (2013b), includes 36 lessons taught over a 12-week period (three 30-min lessons per week). As with Galaxy Math, scripts provide a model of the lessons and key explanatory language. Tutors review scripts prior to delivering lessons; however, to promote teaching authenticity and responsiveness to student difficulty, tutors do not memorize or read scripts.

As mentioned, tutoring relies primarily on the measurement conceptualization of fractions, with content focused primarily on representing, comparing, ordering, and placing fractions on a 0–1 number line. This focus is supplemented by attention to part–whole understanding (e.g., showing objects with shaded regions) and fair shares representations to build on classroom instruction. In this way, number lines, fraction tiles, and fraction circles are used throughout the lessons, with stronger emphasis on part–whole representations in beginning lessons. Lesson 22 (of 36) introduces fraction computation. Throughout the program, we focus on proper fractions and fractions equal to one. Improper fractions greater than 1 are introduced with addition and subtraction of fractions. To reduce computational demands, denominators do not exceed 12 and exclude 7, 9, and 11. This tutoring content mirrored classroom instruction with the following exceptions. The focus was narrower, with greater emphasis on measurement understanding, whereas classroom instruction emphasized part–whole understanding more than intervention; calculations substantially less than classroom instruction were focused; and a more limited pool of denominators was used.

More specifically, during the first 2 weeks of Fraction Challenge, the focus is on understanding of fraction magnitude. We begin by addressing “what is a fraction” and teach relevant vocabulary (e.g., *numerator*, *denominator, unit*). We rely on a combination of part/whole relations, measurement, and equal sharing to explain the fraction magnitudes. Instruction emphasizes the role of the numerator and denominator and how they work together to constitute the fraction, which is one number, even though it comprises two whole numerals.

In the 3rd week, tutors review material presented in the first six lessons. Students practice naming fractions, reading fractions, and comparing two fractions when the denominators are the same or when the numerators are the same. In this review, two types of flashcards are used to build fluency with the meaning of fractions. The first type shows flashcards with one fraction; students read and state the meaning of the fraction. For example, for $\frac{1}{4}$ students say, “one-fourth, one of four equal parts.” Students take turns over a 2-min period, responding to as many fractions as they can. The tutor keeps track of the group total for each lesson; the students’ goal is to meet or beat the previous day’s score. The second type shows two fractions. Students determine if the fractions pairs fit one of three categories: same numerators (different denominators), same denominators (different numerators), or different numerator and different denominator. Students categorize flashcards in this way for 1 min. Then, the tutor gives each student two fraction cards; for each, students place the greater than or less than sign between fractions and explain their rationale to the group.

In weeks 4 and 5, students learn about fractions equivalent to $\frac{1}{2}$ (e.g., $\frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \frac{6}{12}$). They also learn to compare two fractions in which the numerators and denominators both differ, using $\frac{1}{2}$ as a benchmark for comparison and writing the greater than, less than, or equal sign between the fractions. Then, two activities are introduced: placing two fractions on the 0–1 number line, marked with $\frac{1}{2}$, and ordering three fractions from smallest to largest.

In week 6, tutors introduce fractions representing a collection of items and fractions equivalent to 1, while continuing to work on comparing two fractions, ordering three fractions, and placing fractions on the 0–1 number line, now without the $\frac{1}{2}$ marker. Students are encouraged, however, to think about where $\frac{1}{2}$ goes on the number line in relation to placing other fractions. Week 7 (lessons 19–21) was cumulative review on all concepts and skills.

Weeks 8 and 9 focus on simple calculations. Addition with like denominators is introduced first; then subtraction with like denominators; then mixed addition and subtraction; and then addition with unlike denominators and then subtraction with unlike denominators. Concepts and procedures are addressed. When introducing unlike denominators, tutors limit the pool of problems. In all cases, one fraction is equivalent to $\frac{1}{2}$ or 1 so students can write equivalent fractions they already learned. The last 2 weeks are cumulative review.

Each lesson comprises four activities: introduction of concepts or skills, group work, the speed game, and individual work. The first activity, introduction of new concepts and skills, lasts 8–12 min. Concrete manipulatives (e.g., fraction tiles, fraction circles), visual representations, and problem-solving strategies are presented. In group work, which lasts 8–12 min (the introduction plus group work lasted 20 min for each lesson), students rehearse and apply concepts and practiced strategies addressed in the introduction. Students take turns leading the group through problems, while all students show their work for each problem. The third activity, the speed game, is designed to build fluency on a previously taught concept or skill. For instance, to build fluency on fractions equivalent to $\frac{1}{2}$, tutors give each student a paper showing 25 fractions, and students have 1 min to circle fractions equivalent to $\frac{1}{2}$. Sometimes, the Speed Game requires computation, and students are given specific instructions on which items to solve. For example, students might be told to solve only addition problems or solve only problems with like denominators. In this way, students discriminate between problem types. The fourth activity is individual work for which students independently complete a two-sided practice sheet. One side presents problems taught in that day's lesson; the other side is cumulative review. This activity lasts approximately 8 min, for a total of 30 min per session.

As with Galaxy Math, the program also encourages students to regulate their attention/behavior and to work hard. Tutors teach students that on-task behavior means listening carefully, working hard, and following directions and that on-task behavior is important for learning. Tutors set a timer to beep at three unpredictable times during each lesson. If all students are on task when the timer beeps, all students receive a checkmark. To increase the likelihood of consistent on-task behavior, students cannot anticipate time intervals. Also, on each practice sheet, 2 of 16 problems are bonus problems. As the tutors score the practice sheet, they reveal which problems are bonus items. Students receive a checkmark for each correctly answered bonus problem. At the end of the lesson, tutors tally checkmarks for each student and award them with a "half dollar" per checkmark. At the end of each week, students shop at the "fractions store" to spend money earned during tutoring. All items in the store are listed in whole dollar amounts at three price points so students must exchange half dollars for whole dollars and determine what they can afford. In this way, to use the fraction store, students must rely on their fraction knowledge, while exercising judgment about buying a less expensive item versus saving for a more expensive one. In lesson 19, half dollars are replaced with quarter dollars.

**Findings** In our study, we stratified by risk severity and classroom when randomly assigning students to tutoring versus control conditions. Participants were fourth graders from 53 classrooms in 13 schools. Of these students, 259 were at risk: 129 tutored students (60 more severe and 69 less severe) and 130 control students (66 more severe and 64 less severe). The other 292 students were low-risk classmates. With this sample, we included two measures that isolated the type of understanding on which we primarily focused: measurement understanding. On comparing fractions (in which students place a greater than, less than, or equal sign between two fractions), the ES favoring AR intervention students over control children was 1.82 SDs, and the achievement gap between AR tutored students and their low-risk classmates narrowed, while the gap for AR control students increased. On fraction number line (in which students place a fraction on a 0–1 number line), the ES was 1.14. Although fraction number line data on low-risk classmates (only group-administered data on low-risk students were collected) were not collected, the posttest performance of AR tutored students was at the 75th percentile for a normative sample of sixth graders, as per Siegler et al. (2012).

Because the alignment for comparing fractions and fraction number line was greater for intervention than for classroom instruction, we also considered effects on NAEP. NAEP was not aligned with intervention, and it focused with comparable emphasis on measurement and part–whole understanding. Here, effects were also significant and strong. The ES favoring AR intervention students over control was 0.94 SDs, and the achievement gap favoring low-risk classmates over AR control students remained large, while the achievement gap for AR intervention students decreased substantially or was eliminated.

Moreover, although classroom instruction focused on calculations more than intervention, effects again favored intervention students over control. Here, the ES favoring AR intervention students was 2.51; the achievement gap between AR tutored students and their low-risk classmates narrowed, while the gap for AR control students increased; and AR tutored students' posttest performance actually exceeded that of low-risk classmates. Given that classroom instruction allocated substantially more time to calculations, this suggests that understanding fractions, perhaps specifically the measurement understanding of fractions, transfers to procedural skill, at least with respect to adding and subtracting fractions (Hecht et al. 2003; Mazzocco and Devlin 2008; Ni and Zhou 2005; Rittle-Johnson et al. 2001; Siegler et al. 2011).

In these ways, this study innovatively extends Tier 2 intervention on fractions by focusing primarily on understanding magnitude (rather than part–whole understanding as in earlier work) and targeting younger students (rather than middle or high school students). Another interesting extension to the literature concerns the focus on risk severity. We found that response to intervention was comparable for students with more versus less severe risk, when risk was defined in terms of whole-number deficits. That is, there were no significant interactions between risk severity and intervention condition, and ESs were similar for more versus less severe student groups. By any standard, the effects of intervention designed to foster understanding of fraction magnitude for AR fourth graders were strong, with the achievement gap for AR learners substantially narrowed or eliminated. Of course, classroom instruction's failure not only to address the needs of a substantial majority of AR learners in a more successful manner but also to promote stronger learning among low-risk classmates raises questions about the quality and nature of classroom fraction instruction. This in part explains widespread difficulty with fractions (e.g., Behr et al. 1984; NCTM 2007; Ni 2001) and highlights the pressing need to improve the quality of fraction instruction and learning in this country (NMAP 2008).

## Need for Future Research

Studies described in this chapter, from our own research programs as well as prior work, provide the basis for thoughts about the power as well as the limitations of Tier 2 intervention. First, we focus on the power. As the research described in this chapter illustrates, the literature indicates it is possible to design tutoring programs to enhance the outcomes of students who are at risk of poor mathematics development. Interventions that incorporate explicit instruction, provide students with a strong conceptual foundation and efficient procedural strategies, and embed regular, strategic, and cumulative practice are generally efficacious. Results clearly demonstrate that students AR for poor mathematics development suffer reliably and substantially less positive mathematics outcomes if left in the general education program without such tutoring (as represented in the control conditions in these studies). When AR students do not receive these preventative tutoring services, the gap between their level of mathematics performance and those of low-risk classmates grows, making it increasingly difficult for these children to profit from classroom instruction. By contrast, AR students as a group, who receive high-quality tutoring, make progress toward catching up to classmates and, for some of these children, the scaffolding provided through such Tier-2 validated intervention creates a strong foundation for them to experience long-term success in mathematics. Clearly, reliable screening of risk to identify students for 12–20 weeks of accurately implemented, validated tutoring, as represented in Tier 2 of multilevel RTI prevention systems, is a valuable and important service.

At the same time, it is important for policymakers and schools to recognize the limitations of Tier 2 intervention for dramatically reducing the need for ongoing and intensive services for some segment of the school population. We focus on two such limitations: lack of universal response and questions about transfer across the components of the mathematics curriculum. In terms of lack of universal response, in the three studies from our own research program described in this chapter, not all students respond. This phenomenon has been documented before, not only for mathematics but also for reading As O’Connor and Fuchs (2013) described, for example, the modal rate of unresponsiveness on the components of the curriculum targeted for intervention approximated in our prior studies approximates 4 % of the general population. This is similar to the prevalence of learning disabilities in the USA when intelligence quotient (IQ)–achievement discrepancy is used as the method of identification (although research indicates that the groups of students identified via RTI methods of identification versus the IQ–achievement discrepancy differ; Fuchs et al. 2005a; Fuchs et al. 2008). This rate of unresponsiveness suggests the limitations of Tier 2 intervention for dramatically reducing the need for ongoing, intensive services for students traditionally identified as having a learning disability. This is the case when 12–20 weeks of small-group tutoring are provided. If it were possible to provide a longer duration of tutoring or deliver that tutoring individually, it may be possible to reduce the rate of unresponsiveness further. But with longer runs of one-to-one tutoring, services begin to resemble the level of intensity expected in special education, and this prompts concerns about due process and how schools might fund such a level of intensity without special education resources.

At the same time, it is important to consider that the rate of unresponsiveness in efficacy studies, which control the quality of implementation, probably underestimates the actual percentage when Tier 2 intervention is practiced in schools. In actual practice, it is likely that fidelity of implementation will be lower, with reduced effects. In addition, as students continue in school, the effects of tutoring can be expected to diminish, and without additional support, some responders will reemerge with difficulty.

The second issue that represents a challenge to preventing long-term mathematics difficulty with Tier 2 intervention concerns questions about transfer across components of the mathematics curriculum. As our studies described in this chapter illustrate, although transfer may occur across some domains it is decidedly limited across others. For example, in Fuchs et al. (2013a), we found clear indications of transfer from simple arithmetic tutoring to more complex calculations. However, transfer to word problems was more limited, with word-problem achievement gaps growing over the course of intervention for both tutoring conditions. In a similar way, in Fuchs et al. (2013b), although fraction intervention had dramatic effects in decreasing the achievement gap for more and less severe risk fourth graders, fraction tutoring had no effect on students' whole-number calculation skill. As already discussed, mathematics, more than reading, is potentially complicated by the fact that the elementary school curriculum comprises multiple components within and across the grades. This problem becomes more complicated over the course of high school, where the components of the mathematics curriculum (e.g., geometry, trigonometry, calculus, as well as algebra) diverge more dramatically than in the earlier grades.

Clearly, additional research on other components of the mathematics curriculum, at early and later stages of mathematics development, is required to elucidate where prior Tier 2 intervention creates protection against further risk and where one can expect new forms of risk to emerge. New risk may emerge due to lack of transfer from earlier intervention. Alternatively, new topics in the mathematics curriculum may create risk for students whose prior mathematics performance has been adequate. All this creates the need not only for additional intervention work, with a focus on long-term outcomes, but also for additional research on screening for risk on topics at the intermediate grades and at the middle- and high-school levels.

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