Chapter 15
Mathematics Education in Russia

Alexander Karp

1 Introduction

It is natural to begin a history of mathematics education in Russia in modern times with the reign of Peter I. The changes that took place during this period were recognized even by contemporaries. "What is there to say about arithmetic, geometry, and other mathematical arts, which Russian children today learn with zeal and acquire with joy, displaying what they have acquired in a praiseworthy manner: was it ever so in the past?" exclaimed Feofan Prokopovich, a prominent writer of the era (cited in Polyakova 1997, p. 83). It had, indeed, never been so in the past: the role of government in education and particularly in mathematics education had grown immeasurably. However, while in the countries of Western Europe similar processes were taking place against a background of solidifying and developing capitalist relations, Peter's modernization accompanied a hardening absolutism. Both at that time and in subsequent periods, social and political differences from the countries of Western Europe were responsible for the distinctive characteristics of the development of mathematics education in Russia, which will be the focus of the discussion below.

No sufficiently systematic and complete history of the development of mathematics education in Russia since Peter's time has yet been written, in our view, although prominent Russian historians have paid attention to it including S. M. Soloviev (1993a, b) and P. N. Miliukov (1994). Among the not very numerous works entirely devoted to the history of mathematics and mathematics education in Russia, we would mention old studies by V. V. Bobynin (1899), A. P. Yushkevich (1947–1948), and V. E. Prudnikov (1956), as well as more recent work by T. S. Polyakova (1997, 2002, 2010). The Soviet period (following the Revolution of 1917) has been addressed, for example, in studies by Karp (2009, 2010, 2012) and Abramov (2010). More detailed information concerning certain events and materials examined in this chapter, along with more detailed bibliographies, may be found in the sources listed above.

A. Karp (✉)
Teachers College, Columbia University, New York, USA
e-mail: apk16@columbia.edu

A. Karp and G. Schubring (eds.), *Handbook on the History of Mathematics Education*,
DOI 10.1007/978-1-4614-9155-2_15, © Springer Science+Business Media New York 2014

2 The Formation of the Russian Mathematics Education System: Eighteenth Century

As Miliukov (1994) wrote: "Because it was difficult to master, mathematical knowledge was among the least widespread kinds of knowledge in ancient Rus'; it was acquired only out of necessity, and specialists themselves had only a very imperfect command of it" (p. 223). No organized system for acquiring mathematical knowledge existed – "knowledge was acquired on an individual basis, through reading or learning from master specialists" (p. 223). This knowledge, moreover, was extremely scanty, and not infrequently techniques were employed that were so imprecise that they may be considered simply wrong.

2.1 Peter's Reforms

Striving to strengthen the armed forces and to build a fleet, Peter I invariably needed "professionals" with a much better command of mathematics than anything required in the past. Undoubtedly, one option was simply to invite learned foreigners, and another was to educate Russians abroad. Both of these methods were widely used by Peter. He recognized, however, that this was not enough and spoke about the need to organize a school that would produce "people for all kinds of purposes" (Soloviev 1993a, p. 74). However, there were no teachers, no textbooks, and often even no potential students for such a school.

Peter hired his first teachers abroad. Thus, Aberdeen University professor Henry Fargwarson was invited to teach at the Mathematical and Navigational School, which was founded in 1701. Two other British teachers, who had come with Fargwarson, also taught at this school, as well as the Russian Leonty Magnitsky. It is not clear where Magnitsky himself was educated – perhaps he did receive some kind of schooling in Moscow at the Slavic-Greek-Latin Academy, which had been founded in 1687, although its curriculum devoted only cursory attention to mathematics even in the best of times; most likely, however, he was an autodidact (Polyakova 1997).

The first textbooks published in Russia were written by these teachers themselves. Among them, a special place has traditionally been allotted to Magnitsky's "Arithmetic." This famous Russian textbook was a kind of compilation of Western and old Russian books that contained expositions of elementary arithmetic. Other textbooks that appeared around the same time were also largely repetitions or simply translations of Western textbooks. Such, for example, were the geometry textbooks prepared by Fargwarson and Peter's general Jacob Bruce, who had a British background; these textbooks effectively established the teaching of this subject in Russia.

Finding students was also not easy, although we do possess a letter written in 1703 by the effective head of the education system, Alexey Kurbatov, to his superior Admiral Golovin, in which the former optimistically reports that "many people of all ranks… have discerned the sweetness of that science, send their children to those schools, and some minors and Reiters' children and young clerks from administrative offices even come on their own with no little enthusiasm" (Soloviev 1993a, p. 75). Kurbatov even wondered what should be done if the number of students should exceed 200. And yet in 1722, for example, 127 students ran away from the Navigational School in Moscow – and by that time, senior navigational classes had been transferred to St. Petersburg, where they became a foundation for the Naval Academy, founded in 1715 (Polyakova 1997). Teaching and living conditions at the school were difficult, while the benefit that students themselves derived from the school were not all that clear.

At the Mathematical and Navigational School, students methodically studied reading and writing, arithmetic, and later also geometry, trigonometry, geography, and navigation. Some graduates were subsequently sent to England or the Netherlands for further studies. Sometimes students who had not yet graduated were sent on specific assignments.

Table 15.1 Social background of students

Clergy	931 (45.4 %)
Soldiers' children	402 (19.6 %)
Administrative officials (clerks)	374 (18.2 %)
Townspeople (merchants, craftsmen, etc.)	93 (4.5 %)
Nobility	53 (2.5 %)

Miliukov (1994) notes that a fundamental feature of the system created by Peter was that his schools were organized as professional schools. General education and development was the least of Peter's concerns. The medieval system of liberal arts had penetrated into Russia only to a small degree. And in the system created by Peter, there was nothing "liberal" in any sense of the word. The state needed specific professional experts and it undertook to prepare them. Schooling was not limited to training navigational experts, and soon artillery schools and mining schools, which prepared experts for metallurgical factories, appeared as well. Education in school was seen as a kind of civil service, difficult and to a large extent compulsory.

Life, however, introduced its own correctives: it often turned out to be necessary to employ students who had graduated, say, from the Navigational School not as navigation experts but simply as educated people. This was the case because there were not enough people who possessed even elementary knowledge, and therefore, those who were capable of imparting such knowledge were in demand.

In 1714, a number of so-called arithmetic (*tsifirnye*) schools were founded in provincial cities in Russia's gubernias. The teachers in these schools were ordered to teach arithmetic and elementary geometry, and two Navigational School students were sent to each gubernia for teaching in these schools. By the early 1720s, several dozens of such schools had opened across Russia. By 1727, they had over 2,000 students. In terms of their social backgrounds, these students were distributed as follows (Table 15.1).

This distribution did not last long, however; merchants and craftsmen tended to keep their children at home, in order to help out with the family business, while the clergy (both of their own free will and because they were compelled by their superiors) began to send their children to religious educational establishments. Gradually, arithmetic schools started to close, and by 1744 they were abolished by being merged with so-called garrison schools, which had been attached to regiments since 1732.

There were few children of the gentry among arithmetic school students, nor is this surprising: starting at a certain point, these schools were formally prohibited from accepting noblemen's children – they were intended to serve a different purpose (although teaching noblemen "arithmetic and geometry" was considered so important that a decree of 1714 prohibited noblemen from getting married "before they learn these" – cited in Soloviev 1993a, p. 445). But in "professional" schools in the capitals, noblemen gradually began to predominate – initially, children not from noble families, who had "discerned the sweetness of the sciences," had to be accepted, since there really were no other students available, but even then they often began encountering difficulties as they advanced to higher grades. Subsequently, it became even more difficult for non-nobles to enter professional schools, and most importantly, educational institutions (military schools) began to appear whose names explicitly included the word "noble."

Here, we should also mention the appearance, during Peter's reign, of secondary educational institutions of a completely different character: general educational schools. The first actual gymnasium – Pastor Glück's – appeared in Moscow in 1703, but it met with no success and finally closed in 1715. A different fate awaited the so-called St. Petersburg Academic Gymnasium, established in 1724 in conjunction with the founding of the St. Petersburg Academy of Sciences and its affiliated university. This gymnasium was conceived as a general educational institution to prepare students for the university, which in its turn was supposed to prepare scholars above all "for glory among foreigners" (quoted in Miliukov 1994, p. 258); these goals were obviously different from the other objectives set by Peter in education, and in order to realize them, many outstanding scientists were invited from abroad, including Leonhard Euler. The gymnasium was to provide a thorough grounding in mathematics – arithmetic, geometry, and trigonometry. The school opened in 1726, already after Peter's death.

2.2 The Post-Petrine Period

The system created by Peter functioned poorly or did not function at all. Miliukov considered Peter's attempts to create in Russia elementary schools for low classes fruitless. One may form an impression of how nobles studied mathematics during the period following Peter from the memoirs of M. V. Danilov (1913), who was born in 1722 and ended his career as an artillery major. This man, who had been sent by a decree of 1735 to an artillery school and had obtained, by the standards of the age, quite a full education in mathematics, writes that initially the school had two apprentices only for teaching arithmetic and that subsequently a certain Alabushev – who was then under suspicion for murder – was also sent to teach at the school. Alabushev was "a man who, although he knew something, went over Magnitsky's printed arithmetic and demonstrated some of the geometrical figures to the students, and for this reason presented himself as a learned man, was yet a quarrelsome drunkard" (p. 23). However, later one more teacher appeared at the school, who finally "put things at the school into better order." Danilov, undoubtedly, was lucky – he himself relates how he taught arithmetic to those who had not been taught anything in school.

At the Szlachta Land-Forces Cadet Corps, which opened in 1732, only arithmetic was mandatory; geometry, despite all of Peter's injunctions, was studied in 1733 by only 36 students, while the German language was studied by 237 (Soloviev 1993b, p. 502). Even at the Academic Gymnasium, instruction clearly left much to be desired. Simply finding a teacher who was not a drunkard was not easy. Reports have survived from an even later period in the gymnasium's history, 1767, which contain requests for a new teacher of arithmetic who "is capable of carrying out his duties more assiduously and diligently" than the existing teacher, who has "so fallen into drunkenness [that he] is rarely seen sober" (Tishkin 2001, p. 143).

In addition to the shortage of teachers, there was also a shortage of students. When the Academic Gymnasium first opened, 120 students enrolled in it during its first year, 58 during its second year, 26 during its third year, 74 during its fourth year, and many of these students were foreigners (Miliukov 1994, p. 259). But afterward, the inflow of new students dried up. The school began actively enrolling children of soldiers and workers and offering state stipends, but even these failed to attract a sufficient number of new students.

No functioning system existed, nor could any functioning system probably have emerged all at once – too much had to be done at the same time. And yet, important steps had been taken. Garrison schools and the army in general started producing individuals who, if only in an elementary sense, had command of arithmetic and sometimes even of geometry. Fonvizin's famous play "The Minor," written in 1782, depicts a landowner's son who is taught by three teachers, only one of whom, the teacher of arithmetic, a retired soldier named Tsifirkin ("*tsifra*" being the Russian for "numeral" or "number"), evokes the author's sympathy. Very many children had such Tsifirkins. The great poet Derzhavin (1743–1816) recalled that his mother gave her children

> to be educated, for lack of better teachers of arithmetic and geometry, first to a garrison schoolboy, Lebedev, and then to a bayonet-junker of the artillery, Poletayev; but since they themselves had little knowledge of these sciences… they [taught] arithmetic and geometry without proofs and rules, contented themselves in arithmetic with the first five parts only, and in geometry with the drawing of figures, with no understanding of what purpose anything served. (Derzhavin 2000, p. 11)

The education offered by these Tsifirkins was, admittedly, not brilliant; still, it was better than nothing. As a result, there gradually began appearing in Russia increasing numbers of people who were ready for a more advanced education and people who were capable of bringing elementary education to a more large-scale population. Perhaps even more importantly, through the efforts of the Tsifirkins, although fundamentally as a consequence of the government's will, the idea that the study of mathematics was something useful and even necessary began taking root in the country.

The country also saw the appearance of people who (although not many in number) had a more thorough grounding in mathematics. And this pertained not just to foreigners, whose contribution, of course, was very significant. The educational institutions that had appeared, although they lacked the means to educate even large numbers of students, let alone everyone, nonetheless produced a certain number of quite well-prepared specialists, who were in a position to carry their knowledge further. The man who might be described as the founding father of Russian science, Lomonosov (1711–1765), studied initially at the Slavic-Greek-Latin Academy in Moscow but then became a student at the St. Petersburg Gymnasium and the University. Nikolay Kurganov (1726–1796), the author of very important textbooks in mathematics, himself attended the Navigational School and the Naval Academy. Notable figures in Russian education, including such mathematics educators as Semyon Kotel'nikov (1723–1806) and Stepan Rumovsky (1734–1812), studied in seminaries and then at the Academic University in St. Petersburg (Kotel'nikov attended the gymnasium as well); another famous textbook author, Mikhail Golovin (1756–1790), was schooled at the Academic Gymnasium; and so on. Thus, from a certain point of view, the Academic Gymnasium, which by 1804 had been shut down mainly due to the fact that by this time it had clearly become useless, had justified its existence.

The Academic Gymnasium was also important as a kind of center for the organization and methodology of the teaching of mathematics. It was precisely for the Academic Gymnasium that Euler wrote his so-called *Universal Arithmetic* (it came out initially in German, but in 1740 was published in Russian as well, in a translation by V. Ye. Adodurov, a former student at the gymnasium). Even before this, Euler prepared plans for a reorganization of the gymnasium, in which, among other concerns, he wrote about standards for mathematics textbooks. The development of the aforementioned Russian mathematics educators and the preparation of new textbooks took place under Euler's influence. Polyakova (2010) speaks of the existence of an "Euler school" in methodology.

In 1755, Moscow University was founded, and two gymnasia opened in Moscow shortly thereafter – for nobles and for the *raznochintsy* (a social estate consisting of "people of miscellaneous ranks") – and then also one in Kazan. Various boarding schools and schools for children of nobles began to appear. As a result, the possibility of obtaining a mathematics education of a relatively high level became formally accessible to a larger and larger number of people. Bobynin (1899) noted, however, that even though improvements were made in certain academic curricula, they were not necessarily accompanied by improvements in teaching methodology. Students were often taught, as before, using foreign textbooks and often in German, not in Russian, with instruction that consisted essentially of dictation. Derzhavin (2000), who attended the Kazan gymnasium, wrote that students there were "taught the study of languages: Latin, French, German; arithmetic, geometry, dancing, music, drawing, and fencing…; however, for want of good teachers, with hardly better rules, much as before" (p. 12).

Instruction at the gymnasium, in Derzhavin's opinion, "although it did not make pupils skilled in the sciences, nonetheless gave them humaneness and a certain freedom in conversation" (p. 12). This was, in fact, what most of the noble pupils were looking for, not necessarily having any special need for the sciences – especially after a decree of 1762 freed them from the obligation to enter government service (indeed, education at a school, as opposed to at home with a personal Tsifirkin, was often not necessary at all). On the other hand, the career of a nobleman was usually linked to military service; consequently, general educational institutions, particularly in St. Petersburg, lost out in attracting nobles to privileged military educational institutions.

As for these military educational institutions, they could not do without mathematics. Therefore, both the Naval Cadet Corps, which grew out of the Navigational School and the Naval Academy, and the Szlachta Land-Forces Cadet Corps (subsequently the First Cadet Corps) were and remained important methodological and mathematical centers. The teachers at these schools included the authors of important textbooks, and the courses taught at these schools were therefore models for other educational institutions. One of the most influential figures in mathematics education there was Euler's assistant and relative Nicolas Fuss (1755–1825), permanent secretary to the St. Petersburg Academy of Sciences, who taught at both corps and wrote for their students.

2.3 The Reforms of Catherine II

Catherine II corresponded with French Enlightenment thinkers, and her views on education were far more sophisticated than Peter's. Miliukov identified two periods in her activities in this sphere – initially, in keeping with the spirit of her age, she dreamed of a system that would create a "new man," but eventually she came to the conclusion that what had to be supported was precisely a system of education with a strong emphasis on the word "education." As we have already noted, however, at the beginning of her reign, no such system existed. At a comparatively high level, there were only isolated educational institutions, while at the lowest level, outside of military service and military schools, there were no state institutions at all. The man who founded Moscow University, Shuvalov, had plans to cover the country with a network of schools – elementary schools where students would be taught reading, writing, and arithmetic in small towns and gymnasia in large cities – but these plans came to nothing.

A system for providing people around the entire country with at least some kind of education may be said, with various qualifications, to have been created by Catherine, who relied on Austrian ideas, which in turn drew on Prussian sources. The implementer of the reforms was a Serb, Fyodor Yankovich de Mirijevo, whose plans were adopted in 1786.

These plans established schools of three types: "small" schools with two grades, "middle" schools with three, and "chief" schools with four grades. Mathematics (arithmetic) was to be taught in the second grade, but in addition elementary geometry and certain "mathematical" subjects (something like mathematical geography) were to be taught in the fourth grade in the chief schools.

A Teachers' Seminary was founded for preparing teachers. In general, the role of the teacher in Yankovich de Mirijevo's conception was supposed to consist not so much in giving and checking assignments, as in actually teaching. The idea of "collective instruction" appeared at this time, that is, the idea of teaching the entire class (rather than having students work individually on their own assignments). In his "Instructions for Teachers," Yankovich de Mirijevo noted that, in presenting a rule, teachers must elucidate it using an example and even explain why they proceeded as they did and not differently. Teachers had to be prepared for such work. Thus, albeit in embryonic form, mathematics teacher education appeared in Russia. Textbooks for schools for low classes (*narodnye uchilischa*) appeared as well – translated textbooks and probably some that were written in Russia.

For all these schools, too, students were not easy to find. The same Derzhavin, while serving as governor of Tambov, brought children to schools using the police. The population, particularly in small towns that were supposed to have "small schools," saw no need for them. In general, by 1800 no more than 20,000 students were enrolled in all of these schools around the entire country (Russia's population at that time was of the order of 26 million). It should be noted, too, that although the need for village schools was discussed, practically no village schools appeared at that time, despite the fact that the overwhelming majority of the population were peasants. In general, Catherine's reforms began much more energetically than they were subsequently implemented. Nonetheless, her importance is evident.

3 The Classical Russian System of Mathematics Education: Nineteenth Century

The development of mathematics education in Russia was largely determined by the development of the education system as a whole, and from Catherine's times to the beginning of the twentieth century, the education system as a whole went through major transformations ("four times Russian schools have been subjected to radical restructuring," Miliukov wrote), becoming incomparably more organized and coming to encompass a far greater, although still only a comparatively small, part of the population.

3.1 On the Organization of Education in the Nineteenth Century

In 1802, the Ministry of National Education was founded in Russia. In the course of a series of organizational reforms, the Ministry of National Education founded new universities, each of which was supposed to oversee the less advanced educational institutions that were located in its district. A statute of 1804 established these educational institutions of three sequential types: parish schools, uyezd (district) schools, and gymnasia.

Education in parish schools (including education in elementary arithmetic) was to last 1 year; education in uyezd schools (for which students were prepared by parish schools) was to last 2 years, and students would be taught arithmetic and elementary geometry. Students were to be admitted to the (four-year) gymnasia after uyezd schools or after home schooling, and the curriculum here was to include comparatively elaborate courses in both pure and applied mathematics (Nikoltseva 2000). The reorganization proceeded slowly, however, and old forms coexisted with new ones for some time.

In 1828, a statute introduced a number of changes, some of them in response to a social issue that was perhaps the most important problem in the development of Russian education: in a country that was divided rigidly into social classes, the schools, too, remained in general based on the same principle. The role of social class here, however, was not always the same and not always equally prominent; in the early nineteenth century, gymnasia were at least nominally open to virtually everyone aside from serfs (although along with gymnasia, the so-called boarding schools for nobles, and a few years later "lycees," which gave their graduates considerably greater privileges, enjoyed popularity at this time). The 1828 statute changed this state of affairs: it explicitly referred to an "education appropriate to rank" (Nikoltseva 2000, p. 91). The gymnasium program was now divided into 7 years, and although the new rules allowed for the possibility of entering a gymnasium not just in first grade but even in fourth grade, they did not in any way provide for the possibility of entering gymnasia after graduation from uyezd schools. It is noteworthy that the government was not satisfied by what it had achieved and strove to reduce access to gymnasia for non-nobles, which was small enough to begin with; the Emperor Nikolay I wrote explicitly in 1845: "determine whether there are ways of making it difficult for the *raznochintsy* to enter gymnasia" (Kapterev 2004, p. 249). Concurrently, the government also attacked private educational institutions, making it more difficult to open them and curtailing their rights.

A statute on secondary school passed in 1864, during the period of great reforms under Alexander II (1861 – the abolition of serfdom in Russia), again changed the situation, stating explicitly that "gymnasia and pre-gymnasia accept children from all backgrounds without distinction of the profession or religion of their parents" (Dzhurinsky 2004, p. 257); still, the parents' economic level turned out to play an important role. This statute provided for the establishment of gymnasia of different types: gymnasia with two classical languages, with one classical language, and without any classical languages at all – "real schools."

The opposition between "classical" education and "real" education – which was offered in the so-called real schools (or real gymnasia), which were largely modeled on the German *Realschulen* and which focused on technical and natural scientific disciplines instead of classical languages – endured in one way or another for many years. Champions of classical languages included both Sergey Uvarov, minister under Nikolay I during the years 1833–1849, and Dmitry Tolstoy, who became head of the Ministry in 1866. At certain times, the opposition between classical and real education possessed an openly political character: the "new people" of the 1860s were "natural scientists," such as Bazarov, the hero of Turgenev's famous novel *Fathers and Sons*, and among the revolutionaries of 1860–1890, there were many natural scientists and technical experts as well (although not infrequently, they still had diplomas from classical gymnasia, without which one could not enter the university). On the other hand, classical languages were seen as a defense against liberal influences – classical languages were no longer learned in order to attend lectures at the university in Latin, but for general educational purposes.

A statute passed under Dmitry Tolstoy in 1871 and programs approved in 1872 marked a victory for the opponents of real education. The privileges of graduates from real schools were reduced by comparison with those of gymnasium graduates. A memo that came out in 1887 and subsequently gained notoriety ("On Kitchen Maids' Children") again introduced measures that limited access to education in gymnasia for representatives of the lower classes, as a result of which during the next decade the fraction of gymnasium students whose parents were not nobles or civil servants dropped from 53 % to 44 % (Dzhurinsky 2004, p. 262) – although this figure was still incomparably higher than what it had been at the beginning of the century.

On the other hand, for example, in 1897 in the country as a whole, 58,092 students attended classical gymnasia and 24,279 students attended real schools. This was approximately 15 times greater than the number of students attending gymnasia in 1809 (Miliukov 1994), and yet for a country that at this point had a population of 125 million, these figures were negligibly small. The overwhelming majority of the population – peasants – remained as before without any secondary education and very often without any education at all.

In examining changes in the Russian education system, it is important to note that mathematics played an important role in virtually every type of educational institution, and their variety prior to the Revolution of 1917 was considerable. In classical gymnasia, it was viewed as a formal discipline that facilitated mental development and was not subject to short-term political influences, while in real schools it was a necessity for future technical experts and natural scientists. In elementary public schools, mathematics was a required subject – no competent worker, let alone merchant, could do without it. And finally and most importantly, mathematics was practically the cornerstone of military education, which for many years was the most attractive form of education for those Russian noblemen who wished to pursue a career.

3.2 How Mathematics Was Taught: Contemporaries' Accounts

Many people have left memoirs that describe how mathematics was taught to young noblemen (Karp 2007c). For example, Alexey Galakhov (1999), man of letters and author of school textbooks in literature, recalled how "at first our father himself taught us arithmetic, but then he found somewhere a retired navy officer, who added grammar to arithmetic" (p. 29). Subsequently, Galakhov attended an uyezd school, where virtually no other students came from noble families (since noble families that were better off than his own did not send their children to study at the uyezd school) and where he once again studied arithmetic, but did not study elementary geometry – although he was supposed to study it – since it was not feasible to cover all of the 15 subjects included in the curriculum. After that, he attended a gymnasium, from which he graduated in 1822 at the age of 15 and a half. Here, the mathematics was somewhat more serious: Galakhov recalled how the "algebra teacher, after explaining to us, albeit very hazily, the theory of first-degree equations, posed several problems" (p. 65). The memoirist was unable to figure out these problems and therefore sent for his former arithmetic teacher, who arrived from his village "fortunately sober" and explained everything at once, so that Galakhov "solved all of the problems himself in front of [his teacher]." Although this intellectual triumph did bring him joy, as he himself explained "a gymnasium student, while succeeding in some subjects, could have unsatisfactory results in others" (p. 64) without any administrative repercussions.

Such freedom was even more evident in "institutions for nobles" and in the education of children from noble families. Nikolay Markevich, a future historian and man of letters who attended the St. Petersburg Boarding School for Nobles at around this time, recalled not without pride how he rebuffed his mathematics teacher, Dmitry Chizhov, a future distinguished professor at St. Petersburg University, who demanded that he pay attention in class; and when Chizhov threw Markevich out of the classroom in anger, Uvarov, the supervisor of the St. Petersburg school region and future minister, took Markevich's side and admonished Chizhov (Karp 2007c).

Such practices, however, were impossible in military and military-engineering schools. Here, on the contrary, mathematics found itself in a privileged position. As one graduate from such a school recalled, "For laziness in mathematics, students were punished with the rod; for laziness in other subjects, they were given bread and soup for supper" (Miturich 1888, p. 526). Graduates from military schools did not go on to serve only in the army; for example, among ten ministers of national education from 1802 until the freeing of the serfs in 1861, three had graduated from the Naval Corps, which provided perhaps the finest mathematics education in the country at that time (AS. Shishkov, P.A. Shirinsky-Shikhmatov, Ye.V. Putyatin), and another (Ye.P. Kovalevsky) had graduated from a specialized mining corps. As a result, recognition of the value of mathematics education was sufficiently widespread, penetrating even into women's education and home schooling (Karp 2007c).

Memoirists often recall meaningless rote memorization and a general absence of connection and structure. The historian Pogodin (1868), for example, recalled that "in algebra, we solved various problems well, and in geometry and trigonometry, we proved all theorems clearly, but separately from one another, so that their connections, applications, and significance were not explained" (p. 620). Educators did struggle against rote memorization, however. As early as 1810, Minister Alexey Razumovsky advised the Main School Directorate that "teachers must be required to know not any mechanical methodology, but one that can facilitate a genuine enrichment of the mind with useful and necessary truths" (Nikoltseva 2000, p. 37). And indeed, there are accounts of how, as early as the first third of the nineteenth century, teachers tried, for example, to use letters differently, in order to encourage students to think and not merely to memorize by heart (Karp 2007c).

The stiffening of the rules that took place under Nikolay I to some degree forced even those representatives of the ruling class who had no interest in mathematics to study the discipline. But the overall level of organization should not be overestimated. Sergey Vitte (1960), the future prime minister of Russia, who attended a gymnasium in the 1860s, recalled how nothing was demanded of him initially, since he was the son of an important civil servant. Subsequently, however, he decided to hire a teacher of mathematics and to study with him independently day and night in order to pass his gymnasium exams. Interestingly, because the hired teacher had confirmed that Vitte had considerable mathematical abilities, the director of the gymnasium agreed that if Vitte got perfect scores on his exams in physical and mathematical subjects (arithmetic, geometry, algebra, physics, including geography), he would be given credit for all the other subjects as well.

The goal and structure of the mathematics lesson gradually became more complex. For example, V. Omel'chenko-Pavlenko, who worked as a teacher of mathematics in the 1870s, believed that it was important at the beginning of a lesson through questioning to establish a connection with what has already been covered and subsequently to make sure that the teaching of new material rested on the conscious assimilation of old material. It was customary for teachers to pose leading questions and questions that tested students' understanding, to discuss the steps that had to be taken to solve a problem with the class, to analyze examples and models, and to have the students practice solving problems based on these examples (Ganelin 1954). Such practices can be said to have been fairly widespread in gymnasia.

At the same time, already at the beginning of the twentieth century, the outstanding mathematics educator Andrey Kiselev, defending the importance of exams, complained that at times teachers had no chance to hear a student's voice even once over an entire school year (Karp 2002). And the questions that were posed to test students' understanding – even those offered as examples by the same Omel'chenko-Pavlenko – seem at times somewhat pointless and scholastic. In a humorous story written, again, at the beginning of the twentieth century, a teacher of mathematics, requesting that students define a fraction, insists that the definition begin by indicating that the fraction in question is specifically an "arithmetical fraction," since the Russian word for "fraction" also has other meanings that have no relation to mathematics (Averchenko 1990). Such excessive refinement, although exaggerated in the short story, was sufficiently widespread, which fueled society's negative view of gymnasia as schools for "drills and rote memorization."

3.3 Mathematics Educators

Discussing the position of the teacher during the eighteenth century, Miliukov (1994) wrote that he "could neither move up the social ladder nor leave his job other than by becoming a soldier – for drunkenness and 'unethical conduct'" (p. 273). A grievance letter written in 1806 by Semyon Gur'ev, an academician from the St. Petersburg Academy of Sciences, indicates just how simple ethical norms in the academic world were at the time. Invited to observe an exam at the St. Petersburg Pedagogical Institute, Gur'ev noted that the proofs given by the students were poor. In response to this, as Gur'ev writes, Novoseltsev, the president of the Academy of Sciences and supervisor of the St. Petersburg educational region, who was also present, "remarked that he knows mathematics and, understanding all that is being asked, finds it to be optimal… [and further] berated me as a fool and poorly brought-up person" (p. 1).

Teachers were indeed far from the top of the social ladder, but the differences between its rungs in general were extremely significant. And although between a teacher, particularly one from a non-noble background, and a "person from the first four ranks," that is, a general, there lay an abyss, no less an abyss lay between, say, an officer or any civil servant possessing a rank (the system introduced by Peter I provided for 14 grades of ranks, with the fourteenth at the bottom and the first at the top) and a person who may have even been free but possessed no rank.

The career of a teacher was not an attractive one for a nobleman, but a teacher of mathematics simply by virtue of his connection with the world of the military found himself in a somewhat better position than other teachers. Polyakova (2010) notes that a mathematics teacher at a gymnasium at the beginning of the nineteenth century usually possessed a rank of Grade 9, whereas by the end of the century, Andrey Kiselev, say, had been elevated for his pedagogical work to a rank of Grade 5. In addition, in certain cases former students could become their teachers' patrons, and the students of the same Gur'ev, for example, included Arakcheyev, at one time a virtually all-powerful figure in the Russian government (Prudnikov 1956).

Gur'ev (1766–1813) came from a poor, noble family, received his mathematical education at the Artillery and Engineering Cadet Corps, and subsequently was associated as a teacher with various engineering and naval educational institutions, although not only with them – in particular, he taught mathematics at the St. Petersburg Religious Academy (Prudnikov 1956). Another outstanding mathematics educator and author of numerous textbooks, Fyodor Busse (1794–1859), was not a nobleman by birth – his father was a Lutheran minister. Busse studied at first in a gymnasium and then at the St. Petersburg Pedagogical Institute (which in 1819 was transformed into the University) and then was sent abroad (the aforementioned Dmitry Chizhov (1785–1853) had a similar biography, with the exception that the latter spent some time studying at a seminary prior to enrolling at the Pedagogical Institute). Subsequently, he taught at the Pedagogical Institute and various gymnasia. Vasily Evtushevsky (1836–1888), a mathematics educator active at a later period, who wrote textbooks in arithmetic that were used in many elementary schools, came from a poor noble family; attended a gymnasium, the Pedagogical Institute, and St. Petersburg University; studied abroad; and taught in gymnasia and pedagogical programs (Prudnikov 1956). Andrey Kiselev (1852–1940), who eventually became a kind of icon, came from a merchant background, attended a gymnasium and then St. Petersburg University, and taught at a real school and in the Cadet Corps (Karp 2002).

The family backgrounds and education of these mathematics educators were quite typical. Mathematics teachers initially came from among graduates of military and military-engineering schools and later from among graduates of pedagogical institutes and universities. The first independent pedagogical institute opened in 1804 on the foundation of the aforementioned Teachers' Seminary; in 1819, it was transformed into the University; almost 10 years later, it was reopened once more as a special educational institution; and in 1859 it was again shut down. From that time on, pedagogical programs affiliated with universities began to appear. Teachers for public schools were also prepared in special gymnasium classes. However, the teachers who came out of teacher preparation programs were by no means completely specialized: teachers for public schools were prepared without being divided according to subject, but even a university graduate had to be prepared to teach a rather broad range of disciplines at a gymnasium.

The mathematics education methodology that arose in the eighteenth century developed substantially during the nineteenth. Polyakova (2002) stresses the important role played by Gur'ev as the author of one of the first texts on teaching methodology, "Essays on Improving Elementary Geometry." Fyodor Busse was the author of a number of methodological manuals, the most important of which was probably his "Manual on Teaching Arithmetic for Teachers." Subsequently, numerous similar manuals and even more general works in methodology appeared. The first Russian mathematical-methodological journal – "The Educational Mathematics Journal" (*Uchebnyi matematicheskiy zhurnal*) – began to be published in 1833 in Derpt (now Tartu) by Karl Kupfer. It did not last long, and new mathematical-methodological journals appeared only after several decades, but pedagogical as well as general literary journals regularly devoted a certain amount of attention to issues in mathematics education (Depman 1951).

Foreign publications and sources had a significant influence on the development of methodological thought. At the same time, Russian scholarly literature gradually began to voice a desire to isolate itself from foreign influences and to counter them with Russian practices. A vivid example of this was the discussion about the teaching of arithmetic based on the methods of the German pedagogue Grube (Karp 2006).

Of course, the development of mathematics education in Russia did indeed have its own distinctive features (as was true of other countries). One of them was a very great degree of involvement in education by professional research mathematicians. Mathematicians in other countries were also quite actively involved in school education (e.g., recall French Poisson or Italian Betti), but the Russian system of government control over education assigned a dominant role to universities and hence to research mathematicians, who not infrequently became involved in school administrations and school inspections, which inevitably led also to their participation in solving methodological and substantive problems. Nor must we forget about the initially professional nature of education in Russia, which fostered the involvement of professional mathematicians, and about the tradition that gradually grew out of the foundation laid down already by Euler.

About Perevoshchikov (1788–1880), a professor and at one-time rector of Moscow University, the already-cited Galakhov wrote that, together with his colleagues, he set "the teaching of mathematics at Moscow University, and through it also in the gymnasia of the Moscow educational district, on a rational path. [Previously, teachers] would say what had to be done in proving one or another theorem, but did not explain why this in particular had to be done, as opposed to something else.…

Perevoshchikov dispelled this fog" (p. 78). In the Kazan Gubernia, a significant role was played by Nikolay Lobachevsky (1792–1856). Mikhail Ostrogradsky (1801–1861) wrote and reviewed teaching materials. Pafnuty Chebyshev (1821–1894) supervised the adoption of textbooks for the whole country. Another Moscow University professor, Avgust Davidov (1823–1885), was the author of one of the most popular gymnasium textbooks and so on.

3.4 The Contents of Mathematics Education: The Classical System

One can form an impression of what was studied in a comparatively full course in mathematics in a gymnasium or military school during the first quarter of the nineteenth century by looking at the textbooks of Nicolas Fuss (1755–1825). His "Elementary Foundations of Pure Mathematics" consists of three parts. The first part, "containing the elementary foundations of algebra, extracted from the foundations of this science by the famous Euler" (quoting from the title page of the 1820 edition), consisted of four sections: the first dealt with fractions, roots, exponents, and logarithms; the second addressed operations involving letters along with such topics as the "representation of unextractable roots as infinite series." The third section was devoted to algebraic equations, and the fourth section to relations, proportions, and progressions. The second part of the "Foundations" was geometric and included a course in plane and solid geometry that followed Euclid pretty closely (in the opinion of some, however, this course was not sufficiently rigorous – Polyakova 2002). Finally, the third part contained: "(1) Applications of Algebra to Geometry, (2) Plane Trigonometry, (3) Conic Sections, and (4) Basic Differential and Integral Calculus" (quoting from the 1823 edition). The third part was intended to be used for rather highly specialized preparation.

The material in the textbook was broken up into paragraphs, which contained expositions of theoretical issues. There were also problems – "questions" or "examples" – the solutions to which were immediately analyzed. For example, one question from the second part read as follows: "Find the side of a [regular] tetrahedron inscribed in a sphere with radius R." The end of the book contained drawings to accompany the theoretical material and the problems.

It is worth noting that no less popularity – indeed, as far as we can judge, greater popularity – was enjoyed at the time by foreign textbooks, including those translated into Russian. For example, many memoirists recall how they studied using the textbook of the Frenchman L. B. Francoeur (1809). Also popular were textbooks by A.-M. Legendre, S. Lacroix, and E. Bézout.

Gradually, Russian teaching manuals improved, above all from the methodological point of view. Polyakova (2002), for example, especially notes the complex-based character of Fyodor Busse's approach to the school textbook – his textbook in arithmetic for gymnasia was published along with a special manual for teachers and a collection of problems in arithmetic. Naturally, the gymnasium geometry textbooks of Avgust Davidov, first published in the 1860s, or Andrey Kiselev, first published in the early 1890s, already contained diagrams in the text, included problems for students to solve on their own in each section, and in general organized material incomparably more clearly and conveniently for both student and teacher. There were also differences in the content of the course, of course, but they were not so substantial and conspicuous.

The reviewers of Kiselev's textbook noted that his elementary geometry "relies on the views on the exposition of this subject expressed by the authors of the latest French and German manuals, the former in particular.... It contains nothing that might reveal the author's desire to show off his originality. Nonetheless, it contains much that is new, intended to satisfy existing demands, both theoretical and practical" (Nasha uchebnaya 1893, pp. 26–27). Similar observations may be made about other textbooks of the end of the nineteenth century. Russia became a part of what is called (e.g., by Polyakova 2010) the international classical education system. The basic school mathematics subjects became defined, corresponding to mathematics from the sixteenth to seventeenth century or earlier, but presented in a more modern fashion; the contents of the mathematics curriculum became distributed over different years of schooling in a standard way; and methodological principles were formulated. The teaching of mathematical subjects was no longer conducted in a few isolated institutions: the network of schools, although still far from encompassing all potential students, was nonetheless broad by the end of the century; there were many highly qualified teachers; and a system for the preparation and further training of teachers was in place. The influence of what was happening abroad (above all, in Western Europe) and the resemblance to it was considerable, but Russia no longer simply copied and borrowed foreign findings; rather it created and accumulated its own distinctive manuals, techniques, and textbooks. Arguably, however, the country's most distinctive feature at this time lay not in mathematical-methodological details, but in the position that the mathematical-methodological aspect of education occupied in Russian life and how it developed and penetrated into that life.

4 The Period of Reforms

The need for reforms in mathematics education had been actively discussed in Russian society since the end of the nineteenth century. It was evident that the curriculum was overloaded and that many of its sections were excessively artificial, and it was also clear that greater attention needed to be devoted to issues that had not been included in the course. Reformist ideas in mathematics education were in many ways consonant with what was being developed in the West; indeed, to a certain extent, they were stimulated by these developments.

Probably the greatest evidence of the support for methodological reforms at this time is provided by the proceedings of the All-Russian Congresses of Mathematics Teachers that took place in 1911–1912 and 1913–1914 (Trudy 1913; Doklady 1915). At these congresses, special emphasis was placed on the role of the international movement and ICMI in what was happening in Russia. The range of problems discussed at the congresses was in line with the issues being addressed in international discussions: the need to introduce the study of functions and basic calculus into the school curriculum, the role of visual geometry and laboratory work in mathematics, the importance of a propaedeutic course in geometry, and so on.

Discussions about transforming the mathematics curriculum were part of far broader discussions, which involved a far greater number of participants, about the need to reform education in general and in particular to reorganize or, more precisely, to create a large-scale system of public schools and also to transform the nature of secondary and above all gymnasium education. After the two revolutions of 1917, which successively abolished the monarchy in Russia and handed the government to the Communist Party, education was indeed radically restructured.

In place of all existing types of educational institutions, a statute of 1918 established the so-called unified labor schools. These schools were divided into two stages, and the network of first-stage schools, which were far more numerous to begin with, continued to be intensively developed. (According to the official figures, the number of students in elementary and secondary schools rose from 7,800,000 in 1914 to 20 million in 1931 – Abakumov et al. 1974, p. 156.) The goal was to eliminate from schools anything reminiscent of former discipline and drills, including exams, textbooks, and even separate subjects (including mathematics). The ideas of American progressive educators were taken up and developed in Russia (Soviet Union); schools made use of projects, laboratory work, group work, and, above all, "complexes" (Karp 2012).

"Complexes" had to link through one overarching theme topics that had previously been studied in different subject classes. For example, teachers could use a theme such as "The Post Office" to get their students to do some writing, to perform some computations, to talk about geography, and even to discuss the difficult position of the working class in other countries (Karp 2010, 2012). The themes were varied.

It is now difficult to judge to what degree “complex-based education” and other innovations were actually applied in practice and to what extent they remained mere wishful thinking, while teachers continued to teach as they always had. By all appearances, in first-stage schools, mathematics education was indeed complex-based: as evidence of this, we have the testimony of students who attended these schools as well as a large number of textbooks oriented toward “complex-based” teaching (Karp 2012). In higher grades, however, such an approach became completely unworkable, and educators were consequently willing to regard the mere establishment of links between subjects as a form of “complex-based education,” while very frequently, and perhaps usually, using old textbooks, sometimes somewhat updated, to teach their classes. At the same time, it is clear that many reformist ideas, such as increased attention to the study of functions or the visual element in geometry, were indeed widely applied in teaching.

The ideas promoted in schools by the state – for example, the notion that knowledge should always be derived from experience – were consonant with the views held by many thinking teachers before the Revolution. But now, all teachers in all schools were requested to work in this new manner. In addition, it was constantly explained that “mathematics in itself has no educational value in schools; mathematics is important only insofar as it helps to solve practical problems, since students become aware that command of mathematical methods facilitates their participation in the struggle of life and construction” (GUS 1925, p. 134). The directness and haste with which the reforms were implemented, and their deliberate rejection of existing traditions, clearly did not help their popularity among mathematics educators.

5 Soviet Mathematics Education: After 1931

The period of reforms ended as decisively as it began. Between 1931 and 1936, the Central Committee of the All-Soviet Communist Party (Bolsheviks) issued a series of resolutions that fundamentally transformed the school system.

5.1 Schools Under Stalin

A resolution passed in 1931 stated that the principal shortcoming of school education consisted in the fact that “teaching in the schools does not provide students with a sufficient breadth of general knowledge and does not satisfactorily solve the problem of preparing for vocational schools and colleges sufficiently competent individuals with a sound grasp of the fundamentals of science” (Abakumov et al. 1974, p. 157). Consequently, former innovative techniques were declared to have been left-wing distortions, and they were replaced by a gradual revival of the style and substance of pre-Revolutionary education, often in their more conservative versions; for example, analyzing the curricula for the 1937–1938 school year, Sakharov (1938) wrote: “With a single stroke of the pen, the propaedeutic course in geometry for the fifth grade has been eliminated – a course for which more than one generation of mathematicians had fought” (p. 78).

Much else disappeared as well. This was motivated by the argument that students were overburdened with work and thus failed to assimilate the basic topics in the course (the Central Committee’s resolution of 1932 explicitly singled out the propaedeutic study of three-dimensional geometry in seventh grade as an example of the fact that “a number of subjects are covered hastily, and children fail to acquire a sound grasp of the relevant knowledge and skills” (Abakumov et al. 1974, p. 161).

The changes did not take place overnight, but every year more and more aspects of education became more and more rigid. Standard mandatory textbooks were introduced across the country; after a few trials, the textbooks that became established in this position were textbooks from the pre-Revolutionary period by Andrey Kiselev. Exams returned; gradually, exams started being composed not in individual schools and not even in regional centers, but in Moscow – identical exams for the whole country – something unheard-of prior to the Revolution (Karp 2007a).

Curricula – for example, the ninth grade curriculum for 1935 – included such topics as progressions, the generalization of the notion of an exponent, exponential functions, inscribed and circumscribed polygons, the concept of the limit, the length of a circumference and the area of a square, the relative positions of lines and planes in space, and basic trigonometry. The curricula during the period of reforms were somewhat different, which is partially explained by the fact that ninth grade was the highest grade during this period, while by 1935 a tenth grade had been created and certain topics were moved to the tenth grade curriculum. But more important than such changes was the fact that deviations from curricula, which had previously been viewed as inevitable, since teachers were instructed to take local conditions into account, were now practically prohibited. A teacher's very conduct in class and the structure of the lesson were strictly regulated. The instructions for 1933 stated:

> Homework must be checked during every class for 10-15 minutes.... The teacher must call the student up to the blackboard, take his notebook, and look through it quickly, pointing out mistakes to the student if they are minor. If the teacher sees that the student needs additional instruction, he should arrange a "working with failing students" session. (Berezanskaya 1933, pp. 11–12)

Normative pedagogy and methodology presupposed comprehensive monitoring and control over the work of the schools by government agencies and over the work of teachers by both school administrators and general and specialized subject supervisors. School principals and vice principals had to visit hundreds of classes per year and keep track of possible shortcomings.

The government's goal of preparing vast numbers of students for colleges and technical schools led to the broad recruitment and involvement of research mathematicians in the reorganization of mathematics education. In general, while in the years immediately following the Revolution the role of pedagogues and psychologists was very great, and both mathematicians and mathematics itself were considered to be not all that important, now the situation became completely reversed. Pedagogical psychology was demolished, but attention to teachers' mathematical preparedness conspicuously increased. Teachers were taught and retaught mathematics, and the authorities insisted that the teachers' own classes be substantive, active, and demanding. Inspectors criticized classes for insufficient mathematical content and the failure to use time productively for teaching students how to think:

> the exercises and problems that are given to students are very simple: there is nothing to think about. Such work cannot attain the main objective of mathematics education: the ability to think and reason correctly. Furthermore, it does not teach students to apply theoretical knowledge to solving exercises and problems, and reduces their interest in mathematics. (LenGorONO 1936, p. 31)

A lesson had to employ a variety of methodological techniques, challenging strong students but also teaching the weaker ones; it had to teach them to think and reason but also to provide them with a firm grasp of the basic knowledge and skills prescribed by the curriculum; it had to involve students in active and independent work, but this work had to be done under clear and even rigid supervision from the teacher. For the interested and gifted, the lesson could be extended through so-called extracurricular work, while for failing students as well as for teachers, supplementary lessons were practically mandatory.

We have already noted that, in many respects, school curricula and school practices returned to prerevolutionary models. But what had previously been offered merely to a comparatively small segment of the population now became accessible to an incomparably greater number of students. A Leningrad city school board report (LenGorONO 1938) for the 1937–1938 school year gives the following statistics about the number of students in the city's schools, by year and grade (Table 15.2).

The preliminary selection of students for a serious and challenging course in mathematics was practically eliminated. Meanwhile, requirements in mathematics (at least, official requirements)

Table 15.2 The number of students in Leningrad's schools, by year and grade

School year	Grade 7	Grade 8	Grade 9	Grade 10
1935–1936	22,997	10,993	6,211	2,500
1936–1937	26,984	14,328	7,342	4,533
1937–1938	34,074	19,411	11,360	6,461

constantly increased (Karp 2010). A crucial problem for schools was the struggle against the failing rate, which not infrequently was around 20 %. With no less pathos, however, educators at the same time struggled against "rotten liberalism" in handing out grades.

In reality, of course, things did not always run smoothly. The struggle against failure often turned into harassment of failing students. On the other hand, there are accounts not only of grade inflation, in order to avoid criticism for low pass rates, but also of straightforward violations of rules and instructions, for example, disclosing classified exam materials. Ensuring that students had full understanding of the material which they were learning was seen as a paramount objective, while meaningless drilling was considered unacceptable, but in reality examiners' reports are full of complaints about students' inability to respond to questions that sounded even slightly unfamiliar or to offer examples to illustrate general concepts that they could talk about quite fluently.

And yet, the study of mathematics, which was regarded as the cornerstone of technical (and hence also military-technical) education, was in an incomparably better state than the study of other arears, if only because it was much less subject to ideological pressures. Moreover, it was deliberately emphasized that in mathematics classes, students had to study specifically mathematics, and therefore, even the biggest ideological campaigns affected the teaching of mathematics only to a very limited degree (Karp 2007b).

5.2 More Reforms and Counterreforms

The liberalization of the regime that began in the 1950s after Stalin's death affected schools as well: certain practices that had been introduced by him during the last decade of his life, such as separate schooling for boys and girls, were abolished. It soon became evident that something also had to be done with mathematics, the main school subject. The prerevolutionary textbooks, even those that had been somewhat revised, were simply too poorly suited for the new generations and the new objectives, such as the implementation of universal eight-year schooling. The scientific-technological revolution and scientific-technological rivalry with the West spurred educators to seek new approaches to the preparation of future scientists. To this was added an awareness of the fact that the school curriculum in its existing form differed radically from the spirit, style, and language of contemporary mathematics – a similar line of thought led to New Math in the United States and related movements in other countries. Lastly, it became more and more apparent that a kind of corruption was creeping into the education system: it was simply impossible to administer punishments with the same viciousness as in former years – there were not enough resources – and under such circumstances, "liberalism" in grade-giving, which had been persecuted previously, turned out to be the most natural response to demands for improved student success rates.

The first attempts to reform the content of mathematics education took place in the late 1950s, and in early 1965 a Central Commission for Developing the Content of School Education was established under the aegis of the Academy of Sciences of the USSR and the Academy of Pedagogical Sciences of the USSR, under the chairmanship of A. I. Markushevich, a prominent mathematician and vice president of the Academy of Pedagogical Sciences. The subject commission in mathematics was headed by A. N. Kolmogorov, and although other outstanding mathematicians also took part in the reforms enacted during the 1960s and 1970s, these reforms are rightly often referred to as the Kolmogorov reforms, since the leading role of A. N. Kolmogorov in them was beyond dispute (Abramov 2010).

In substance, the reformers' proposals resembled what was happening in other countries: they built their course in mathematics on a set theoretical foundation, increased attention to functional-theoretical questions (basic calculus had already become a mandatory part of the curriculum in the upper grades somewhat earlier), in geometry offered many proofs that used geometric transformations, and introduced the study of combinatorics. On the other hand, far less attention was devoted to achieving a high level of technical skills, and the most complicated and intricate assignments aimed at honing such skills disappeared from the course.

New textbooks and teaching manuals were systematically prepared for all grade levels and for all mathematics subjects taught in schools. The time allotted for implementing the reforms, however, was very brief. In addition, the new textbooks were still conceived, as before, as being standard and universal for the whole country; meanwhile, in 1973 the government set a goal of making secondary education universal, which did not go well with the heightened scientific character of the mathematics curriculum. The consequence of all this was that already by the late 1970s, voluble objections to the reforms began to be voiced by prominent mathematicians – I. M. Vinogradov, L. S. Pontryagin, and others – who found support among many teachers and parents. In the end, a decision was made, nominally by the Ministry of Education but in reality by the Central Committee of the CPSU, to roll back the reforms. Support was given to new textbooks, which effectively revived the traditional approach, although with rather notable changes.

In the process, some of the textbooks written by Kolmogorov's group, on orders from the authorities, were removed from the schools. Others remained in schools in somewhat revised form, but no longer in their former capacity as the only sanctioned textbooks. This revolutionary change – the permission to use several different textbooks in the country – was the outcome of a countrywide mathematics textbooks competition conducted in 1987–1988, which to some degree took stock of and summed up the preceding two decades (Abramov 2010).

Far more successful was another reform, which was also largely associated with A.N. Kolmogorov: the establishment of schools with an advanced course of study in mathematics. The first such school appeared in Moscow at the end of the 1950s under the banner of the idea of "polytechnization" and the preparation of workers in schools, which was being promoted by the government at that time. The objective of this school, which was created by S.I. Shvartsburd, and of several others, which appeared shortly after it, was to prepare computer programmers. In 1963, physics-mathematics boarding schools appeared at four leading universities. Subsequently, the network of schools with an advanced course of study in mathematics first expanded somewhat, then – under pressure from the government, which saw these schools as hotbeds of dissent – contracted, and finally, during the years of Gorbachev's *perestroika*, expanded once more. Most Russian scientists who today work in the fields of mathematics and physics graduated from these schools. These schools evolved a special curriculum, which included sections not ordinarily studied either in schools or colleges, but even more importantly, which approached the study of relatively traditional material in a new way as well (Karp 2011).

6 Discussion and Conclusion

The changes that began occurring in the country in 1985 led to the collapse of the Soviet Union and dramatic transformations in its largest part, Russia. These changes could not but affect education in general and mathematics education in particular. We will not discuss them here, except to note that the textbooks that took part in and won the 1987–1988 competition are still used, although even they have to some degree been revised and new textbooks have been added to them. The traditional Soviet

(Russian) system is still regarded as a norm with which new developments are compared and in terms of which they are evaluated.

Russian mathematics education evolved as an offshoot of Western European education, but in the course of its development, it acquired its own distinctive traits, including traits that subsequently influenced many other countries. Thanks to Stalin's counterreforms, Soviet (Russian) education preserved many features that over the years disappeared from education in the West. Russian schoolchildren of the 1980s–1990s, and even the schoolchildren of today, spend much more time than their Western counterparts on algebraic transformations and proofs. One may wonder how many of them go on to use this knowledge and to what extent, but it cannot be denied that the Russian (Soviet) system at a certain stage offered the opportunity to obtain a high level of mathematics education if not to all children, then to very many. Gymnasium and real school education before the Revolution was elitist; it was made available to a vastly greater student population; and on the whole, at least for a certain time, it has been successful. By comparison, the designs of the reformers of the early twentieth century, which had also been nurtured in elitist institutions (Karp 2012), clearly proved incapable of making the transition to large-scale schools, despite decisive support from the government during the 1920s. The Kolmogorov reforms were also unquestionably connected with ideas formed in the process of elitist, intensive education, but in this instance, too, the transition from the elitist to the universal failed.

One can discuss the reasons for these failures and these successes – what is clear is that time was required in order to accumulate resources, above all, human resources. Peter I introduced new practices with no less decisiveness than the Soviet leaders, but almost a century passed until enough Tsifirkins had been prepared, who in their turn gave a preliminary education to those who were able to study (often with foreign teachers) and then to teach in gymnasia. From among the graduates of these gymnasia and with their assistance, it eventually became possible to prepare people who went on to teach using Kiselev's textbooks in schools "for the masses." The interrelations between education for the elite and education for the masses were not simple, and the one did not exist without the other.

The "anti-humanistic," professional character of education, which predominated for much of the time after Peter, and the fact that mathematics was almost always the queen among the subjects in the school curriculum were, of course, important for the development and success of mathematics education. The study of mathematics was always seen as necessary and important, and the voice of professional mathematicians was always very significant in its teaching. On the other hand, it may be argued that because the humanities were pushed to the periphery of Soviet consciousness, mathematics assumed many functions of the humanities (this development, too, was to some extent anticipated by the teaching of mathematics before the Revolution). The development of speech, thinking, the ability to construct arguments, and so on came to be seen in Soviet public school practice as objectives specifically for mathematics education. "Mathematics must be studied if only because it puts the mind in order": posters with these words of M.V. Lomonosov's hung in virtually all mathematics classrooms in the country.

On the other hand, it may be said that the deliberate conservatism of the system, typical by no means of the Soviet period alone, not only aided teaching, allowing teachers to accumulate experience with working with the same materials, but also hindered them, preserving many sections whose study could no longer be justified in any way and preventing new ideas from penetrating into the schools. The crisis that mathematics education went through during the post-*perestroika* period was, in part, the price for this conservatism. Preserving the best traditions while adopting and, above all, creating new ideas – that is, the task that Russian mathematics education faces.

Acknowledgment The author wishes to express his gratitude to Dr. Dmitri Gouz'evitch for a useful discussion of this chapter.

References

Abakumov, Alexander A., Nikolay P. Kuzin, Fedor I.Puzyrev, Leonid F. Litvinov. (Eds.) 1974. *Narodnoe obrazovanie v SSSR. Sbornik dokumentov. 1917–1973*. [People's education in the USSR. Collection of documents]. Moscow: Pedagogika.

Abramov, Alexander M. 2010. Toward a history of mathematics education reform in Soviet schools (1960s–1980s). In *Russian mathematics education: History and world significance*, ed. Alexander Karp and Bruce Vogeli, 87–140. London-New Jersey-Singapore: World Scientific.

Averchenko, Arkady T. 1990. Britva v kisele. Izbrannye proizvedeniya [A Razor Blade in the Pudding. Selected Works]. Moscow: Pravda.

Berezanskaya, Elizaveta S. 1933. *Matematika v sredney shkole. Instruktivno-metodicheskie materially NKP RSFSR* [Mathematics in Secondary Schools. Instructional-Methodological Materials. RSFSR Ministry of Education]. Moscow: Uchpedgiz.

Bobynin, Viktor V. 1899. L'Enseignement Mathématique en Russie aperçu historique. *L'Enseignement Mathématique* 1: 77–100.

Danilov, Mikhail V. 1913. *Zapiski* [Memoirs]. Kazan': Molodye sily.

Depman, Ivan Ya. 1951. Russkie matematicheskie zhurnaly [Russian Mathematics Journals]. *Matematika v shkole* 6: 9–23.

Derzhavin, Gavriil R. 2000. *Zapiski [Memoirs]*. Moscow: Mysl'.

Doklady, chitannye na 2-m Vserossiiskom s'ezde prepodavateley matematiki v Moskve [Reports Delivered at the Second All-Russian Congress of Mathematics Teachers in Moscow]. 1915. Moscow.

Dzhurinsky, Alexander N. 2004. *Istoriya obrazovaniya i pedagogicheskoy mysli [The History of Education and Pedagogical Thought]*. Moscow: Vlados.

Francoer, L.-B. 1809. *Cours complet de mathématiques pures*. Paris: Mme. Ve. Courcier.

Galakhov, Alexey D. 1999. *Zapiski cheloveka [The Memoirs of a Man]*. Moscow: Novoe literaturnoe obozrenie.

Ganelin, Sholom I. 1954. *Ocherki po istorii sredney shkoly v Rossii vtoroy poloviny XIX veka [Sketches in the History of Russian Secondary Schools during the Second Half of the Nineteenth Century]*. Moscow: GosUchPedGiz.

Gur'ev, Semyon. 1806. *Dokladnaya zapiska Ministru Narodnogo Prosvescheniya grafu P.V. Zavadovskomu* [Memo to the Minister of National Education, Count P.V. Zavadovsky]. Russian National Library, Department of Manuscripts, f.1000, op. 1, # 691.

GUS. 1925. *Programmy I kontsentra shkoly II stupeni. Programma po matematike* [Curricula for the First Stage of Stage-Two Schools. Curriculum in Mathematics].

Kapterev, Petr F. 2004. *Istoriya russkoy pedagogiki [History of Russian Pedagogy]*. St. Petersburg: Aleteya.

Karp, Alexander. 2002. *Klassik real'nogo obrazovaniya [Classic of Genuine Education]*. St. Petersburg: SMIO-Press.

Karp, Alexander. 2006. "Universal responsiveness" or "splendid isolation"? Episodes from the history of mathematics education in Russia. *Paedagogica Historica* 42(4–5): 615–628.

Karp, Alexander. 2007a. Exams in algebra in Russia: Toward a history of high stakes testing. *International Journal for the History of Mathematics Education* 2(1): 39–57.

Karp, Alexander. 2007b. The Cold War in the Soviet school: A case study of mathematics. *European Education* 38(4): 23–43.

Karp, Alexander. 2007c. We all meandered through our schooling…: Notes on Russian mathematics education in the early nineteenth century. *BSHM Bulletin* 22: 104–119.

Karp, Alexander. 2009. Back to the future: The conservative reform of mathematics education in the Soviet Union during the 1930s–1940s. *International Journal for the History of Mathematics Education* 4(1): 65–80.

Karp, Alexander. 2010. Reforms and counter-reforms: Schools between 1917 and the 1950s. In *Russian mathematics education. History and world significance*, ed. Alexander Karp and Bruce Vogeli, 43–85. London-New Jersey-Singapore: World Scientific.

Karp, Alexander. 2011. Schools with an advanced course in mathematics and schools with an advanced course in the humanities. In *Russian mathematics education. Programs and practices*, ed. Alexander Karp and Bruce Vogeli, 265–318. London-New Jersey-Singapore: World Scientific.

Karp, Alexander. 2012. Soviet mathematics education between 1918 and 1931: A time of radical reforms. *ZDM/The International Journal on Mathematics Education* 44(4): 551–561.

LenGorONO. 1936. *Dokladnye zapiski i otchety o sostoyanii prepodavaniya v shkolakh* [Reports and briefs on the state of teaching in schools]. Central State Archive SPb, f. 5039, op. 3, d. 154.

LenGorONO. 1938. *Dokladnaya zapiska LenGorono ob itogakh uchebno-vospitatel'noy raboty za 1937/38 uchebnyi god v shkolakh Leningrada* [Leningrad city school board report on the outcomes of teaching and instruction for the 1937–38 school year in Leningrad schools]. Central State Archive SPb, f, 5039, op. 3, d, 201.

Miliukov, Pavel N. 1994. *Ocherki po istorii russkoy kul'tury [Outlines of Russian culture]*, vol. 2. Moscow: Progress-Kul'tura.

Miturich, P.V. 1888. Morskoy kadetskii korpus v 1823–1828 godakh [The navy cadet corps in 1823–1828]. *Istoricheskii vestnik,* v 33, # 9: 507–543.

Nasha uchebnaya literatura [Our educational literature]. 1893. *Zhurnal Ministerstva narodnogo prosvescheniya*, august: 17–28.

Nikoltseva, N.F. 2000. *Nachal'noe i srednee obrazovanie v Sankt-Peterburge XIX-nachalo XX veka.* [Elementary and Secondary Education in St. Petersburg during the Nineteenth and Early Twentieth Centuries] St. Petersburg: Liki Rossii.

Polyakova, Tatiana S. 1997. *Istoriya otechestvennovo shkol'novo matematicheskovo obrazovaniya* [The history of Russian school mathematical education]. Two centuries: Book 1 Rostov/D: RGPU.

Polyakova, Tatiana S. 2002. *Istoriya matematicheskovo obrazovaniya v Rossii [The history of mathematical education in Russia]*. Moscow: Moscow University.

Polyakova, Tatiana S. 2010. Mathematics education in Russia before the 1917 revolution. In *Russian mathematics education. History and world significance*, ed. Alexander Karp and Bruce Vogeli, 1–42. London-New Jersey-Singapore: World Scientific.

Pogodin, Mikhail. 1868. Shkol'nye vospominaniya.1814–1820 [School memoirs. 1814–1820]'. *Vestnik Evropy* 4(8): 605–630.

Prudnikov, Vasily E. 1956. *Russkie pedagogi matematiki XVIII-XIX vekov [Russian Mathematics Educators of the XVIII-XIX centuries]*. Moscow: Gosudarstvennoe uchebno-pedagogicheskoe izdatel'stvo.

Sakharov. 1938. Novye programmy po matematike na 1937/1938 uchebnyi god [New Programs in Mathematics for the 1937–38 School Year]. *Matematika v shkole* 1: 77–82.

Soloviev, Sergey M. 1993a. *Sochineniya v 18 tt. Kniga 8 [Works in 18 Volumes. Book 8]*. Moscow: Mysl'.

Soloviev, Sergey M. 1993b. *Sochineniya v 18 tt. Kniga 10 [Works in 18 Volumes. Book 10]*. Moscow: Mysl'.

Tishkin, Grigory A. 2001. *Materialy po istorii Sankt-Peterburgskogo Universiteta XVIII v. Obzor arkhivnykh dokumentov [Materials on the History of St. Petersburg University during the Eighteenth Century. Overview of Archival Documents]. St*. Petersburg: Izdatel'stvo St.Petersburgskogo Universiteta.

Trudy 1-go Vserossiiskogo s'ezda prepodavateley matematiki [Proceedings of the First All-Russian Congress of Mathematics Teachers]. *Volumes I-III*. 1913. St. Petersburg.

Vitte, Sergey Iu. 1960. *Vospominaniya [Memoirs]*. Moscow: Sotsekgiz.

Yushkevich, Adolph P. 1947–1948. Matematika i ee prepodavanie v Rossii v XVII-XIX vekakh [Mathematics and its instruction in Russia in the 17th–19th centuries]. *Matematika v shkole*.